Laboratoire de Supraconductivité et Magnétisme

POLYTECHNIQUE MONTRÉAL TECHNOLOGICAL UNIVERSITY

A physics-guided recurrent machine learning model for long-time prediction of quench dynamics

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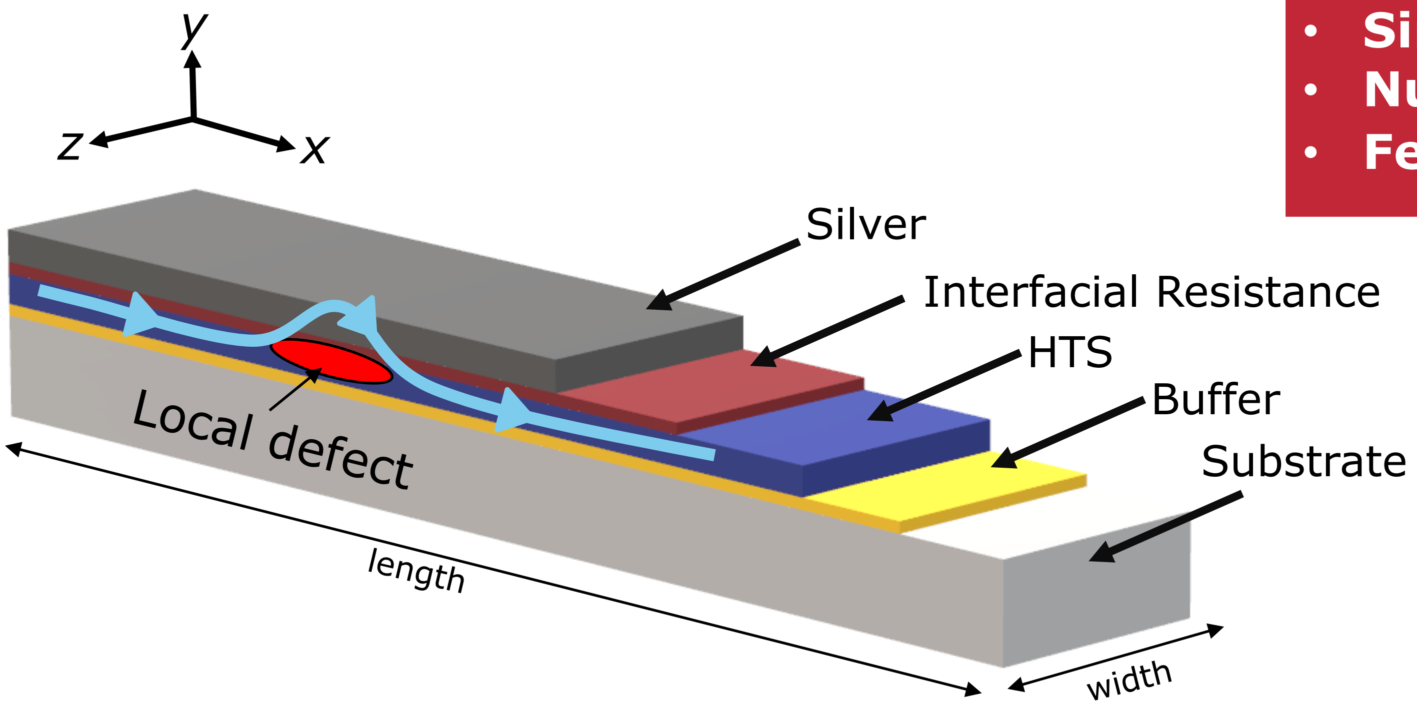
PROBLEMATIC: HIGH COMPUTATIONAL COST FOR 3D SIMULATIONS



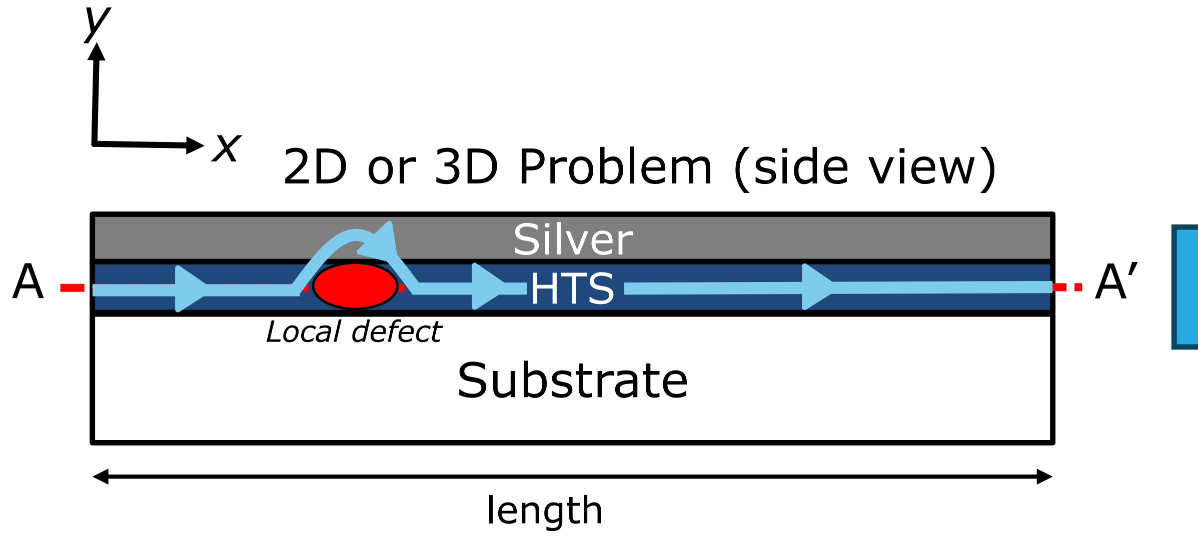
Exemple of HTS architecture
(not to scale)

Main problems with FEM:
- **Simulation time: *Several hours***
- **Numerical instabilities**
- **Few meters tape simulation: *Impratical***

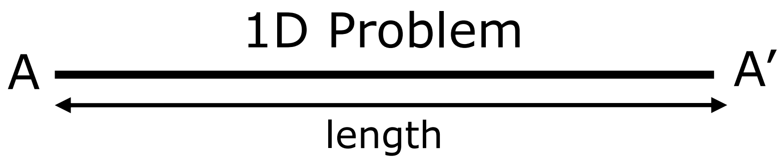
Modelling approach:
- **Electrothermal (FEM)**
- **Nonlinear E-J relation**
- **Local defect in HTS (quench)**
- **Solve for T(x,y,z,t) and V(x,y,z,t)**

A continuation of J.-H. Fournier-Lupien's work. Heat problem only.

HOW TO SPEED UP SIMULATION: THE MAIN IDEA



Dimension Reduction



Low Computational cost

In practice: length ~ m, width ~ mm, and heigth ~ µm

How to maintain accuracy?

- **Solution inside the HTS (thin layers)**
- **T(x,t) and V(x,t)**

Machine learning: Learn physics & Dimension reduction impacts

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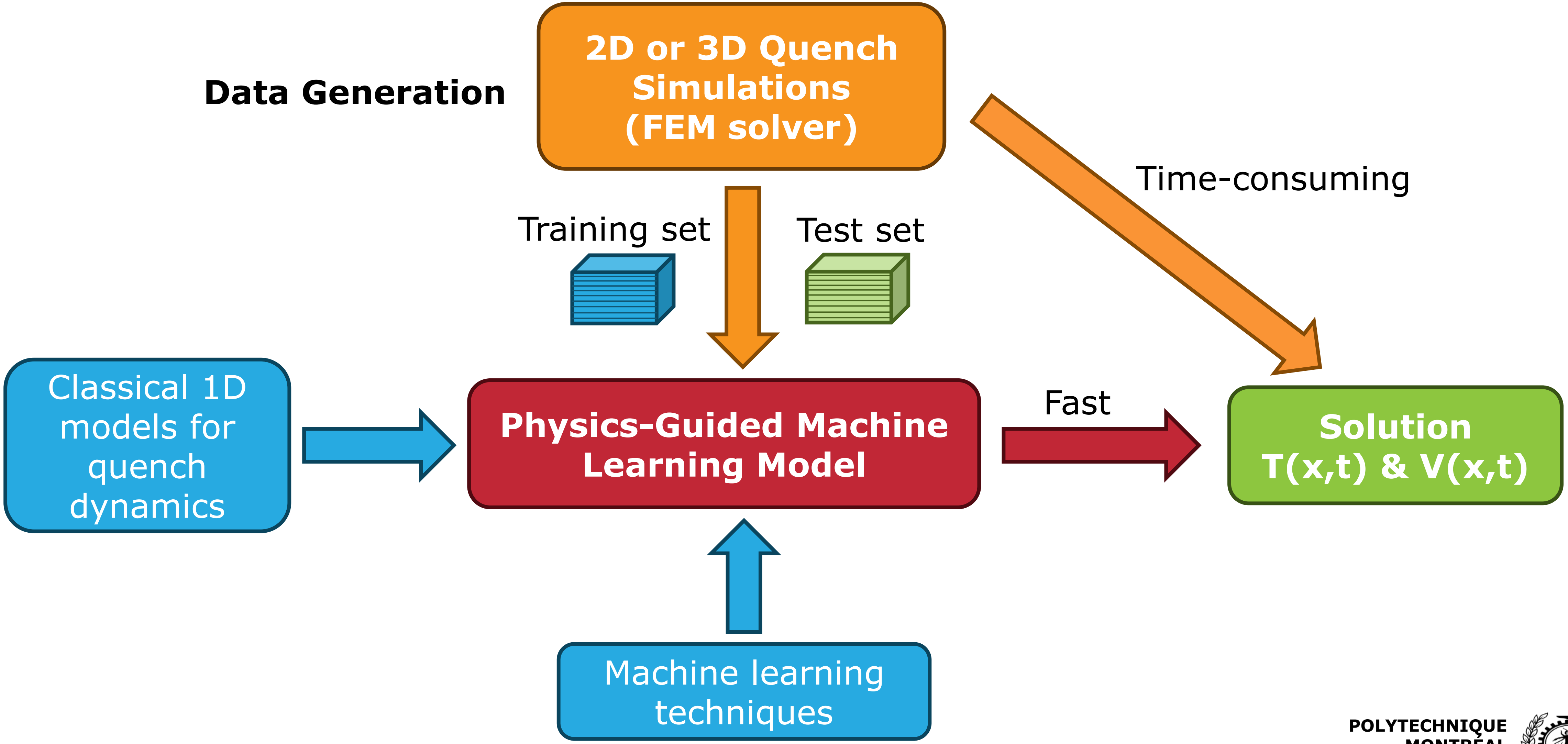
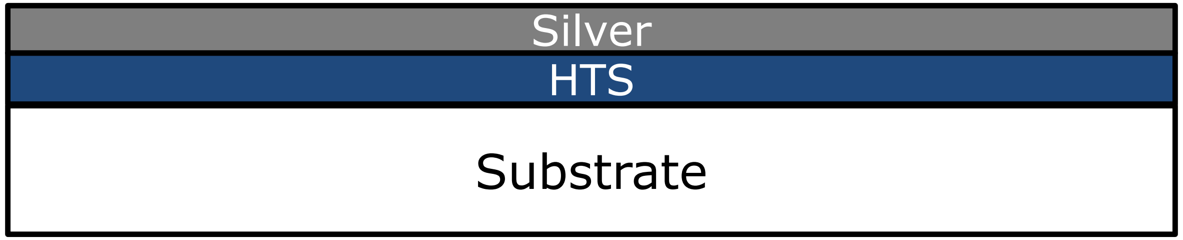
OVERVIEW: WORKFLOW AND SPEED UP



PROBLEM DEFINITION FOR DATA GENERATION



2-D *Problem to solve with FEM:*

Electrical: Current continuity equation

$$\nabla \cdot (-\sigma(T, V)\nabla V) = 0$$

Thermal: Heat Equation

$$\rho_m C_p(T)\frac{\partial T}{\partial t} + \nabla \cdot (-k(T)\nabla T) = E \cdot J + Q_{conv}$$

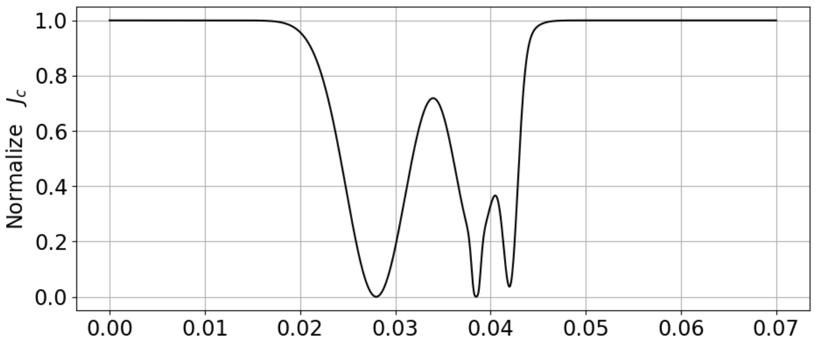
$$J = \sigma(T, V)E$$

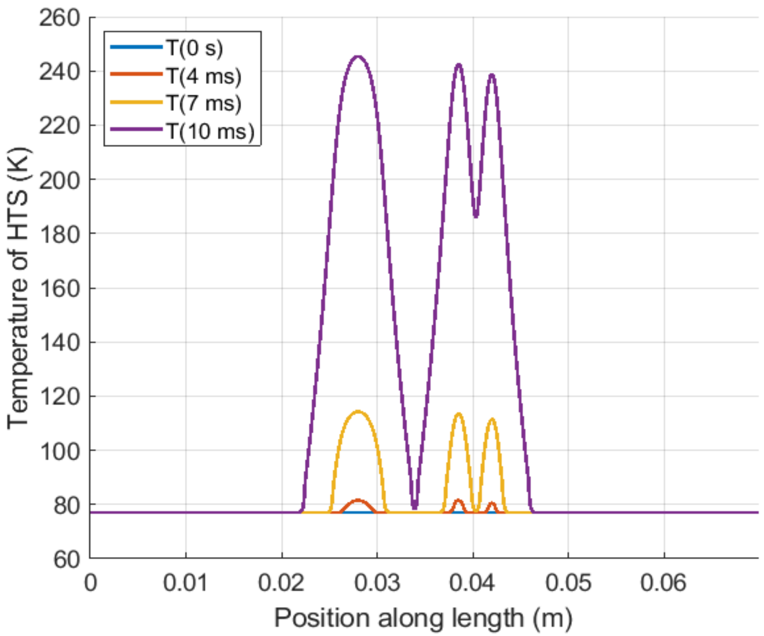
$$E = -\nabla V$$

$$\sigma = \frac{J_c(x, T)}{E}\left(\frac{E}{E_0}\right)^{\frac{1}{n}}$$

Coupled and nonlinear system

DATA GENERATION: RANDOMNESS & PARAMETERS





Example of 2-D FEM solution

Random inhomogeneities in critical current:

- **Position**
- **Numbers**
- **Amplitude**
- **Width**

Tape Architecture:

- **Width : 0.04m**
- **Height : 2 µm (HTS, ag) & 50 µm hastelloy**
- **R_{inter} : 1 µΩcm^2**

FEM Model information:
Δx = 58 µm, N_x = 1201
Δt = 8 µs, N_t = 1201
Simulation time ~ 1h30min

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CLASSICAL FDM 1D MODEL

2-D problem (Silver, YBCO & Hastelloy layers) represented as 1-D problem:

G.A Levin., K.A. Novak and P.N. Barnes. “The effects of superconductor-stabilizer interfacial resistance on the quench of a current-carrying coated conductor”, *Supercond. Sci. Technol.*, 23(1), p. 014021, 2010.

Current sharing equation

$$\frac{\partial}{\partial x}\left(R_{inter} h_{ag} \frac{\partial J_{ag}}{\partial x}\right) = \rho_{ag}(T) J_{ag} - \rho_s(x, T, J_s) J_s$$

→ *Newton-Raphson method*

Heat equation

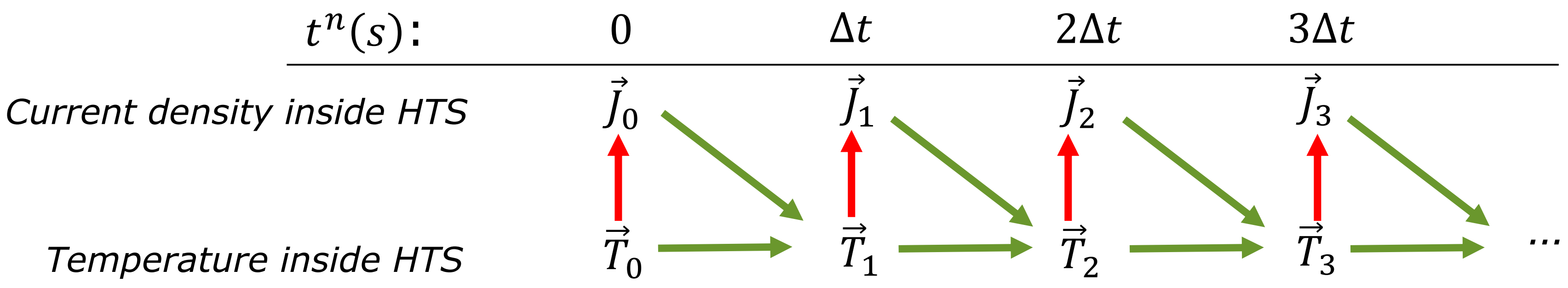
$$C(T)\frac{\partial T}{\partial t} + \frac{\partial}{\partial x}\left(-K(T)\frac{\partial T}{\partial x}\right) = Q\left(J_{ag}, \frac{\partial J_{ag}}{\partial x}, T\right)$$

→ *Dufort-Frankel discretization scheme*

Homogenization

▪▪▪▪▪▪▪: Our Implementation

Sequential decoupled resolution:



PHYSICS-GUIDED MACHINE LEARNING MODEL

Dufort-Frankel discretization:

$$Q_i^n = Q_{ag,i}^n + Q_{s,i}^n + Q_{conv,i}^n + Q_{inter,i}^n$$

Heat equation

$$T_i^{n+1} = \frac{1}{1+A_i}\left\{(1-A_i)T_i^{n-1} + A_i(T_{i+1}^n + T_{i-1}^n) + B_i(T_{i+1}^n - T_{i-1}^n)^2 + \gamma_i Q_i^n\right\}$$

$$A_i = \frac{2K(T_i^n)\Delta t}{C(T_i^n)\Delta x^2} \qquad B_i = \frac{\Delta t}{2C(T_i^n)\Delta x^2}\frac{\partial K(T_i^n)}{\partial T} \qquad \gamma_i = \frac{2\Delta t}{C(T_i^n)}$$

Function of T

Incorporation of Machine learning:

Machine learning: Only for heat equation

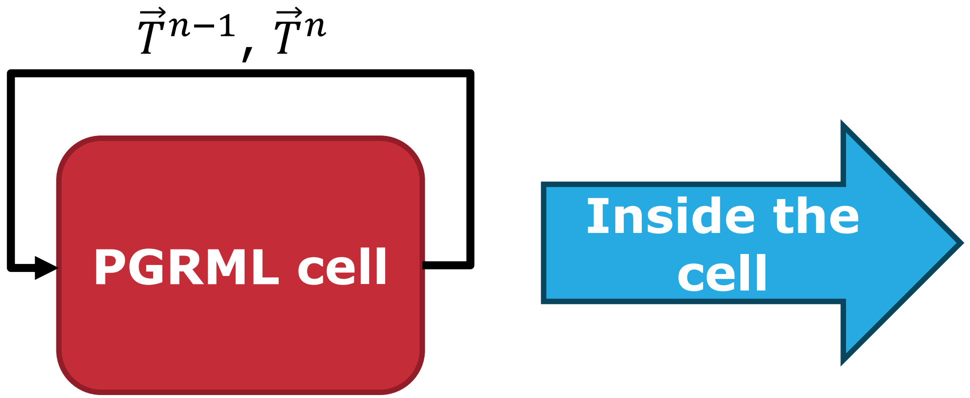
Let's re-define:

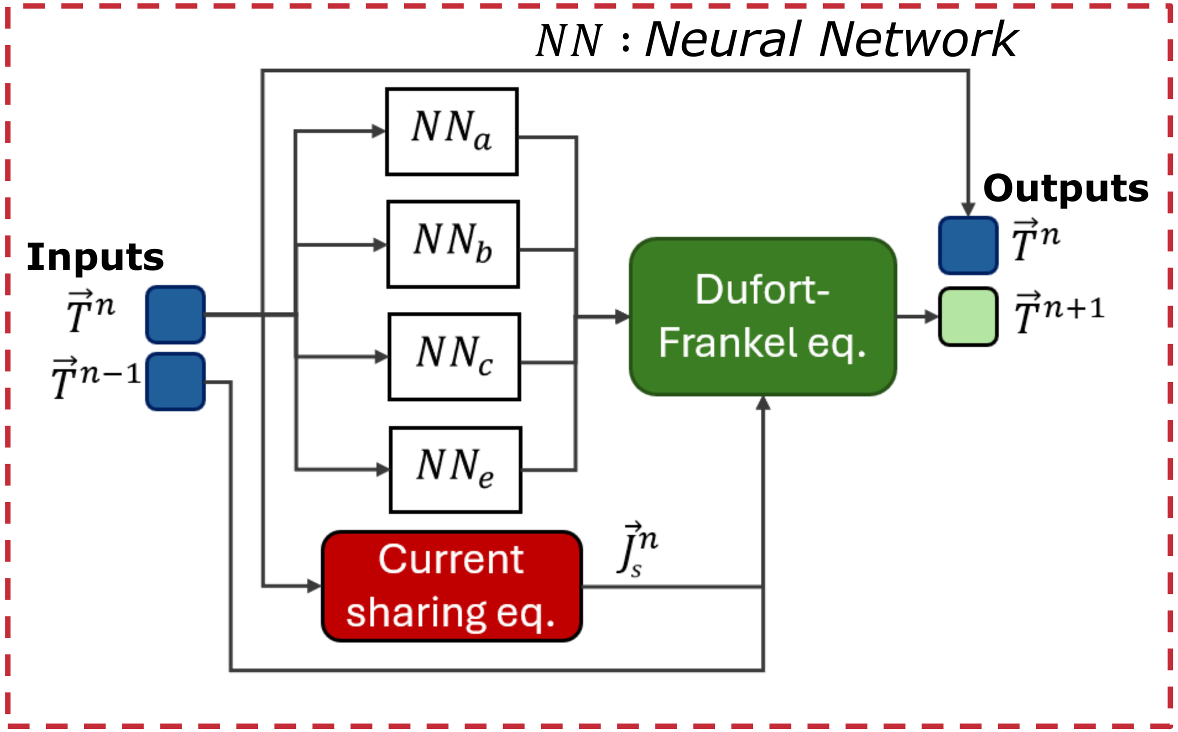
$$A_i \leftarrow A_i\hat{a}(T) \qquad B_i \leftarrow B_i\hat{b}(T) \qquad \gamma_i Q_i^n \leftarrow \gamma_i\hat{c}(T)(Q_{ag,i}^n + Q_{s,i}^n + Q_{conv,i}^n) + \gamma_i\hat{e}(T)Q_{inter,i}^n$$

Prefactors : Functions learned during training!

SCHEMATIC VIEW OF THE MODEL

Physics-Guided Recurrent Machine Learning (PGRML) model



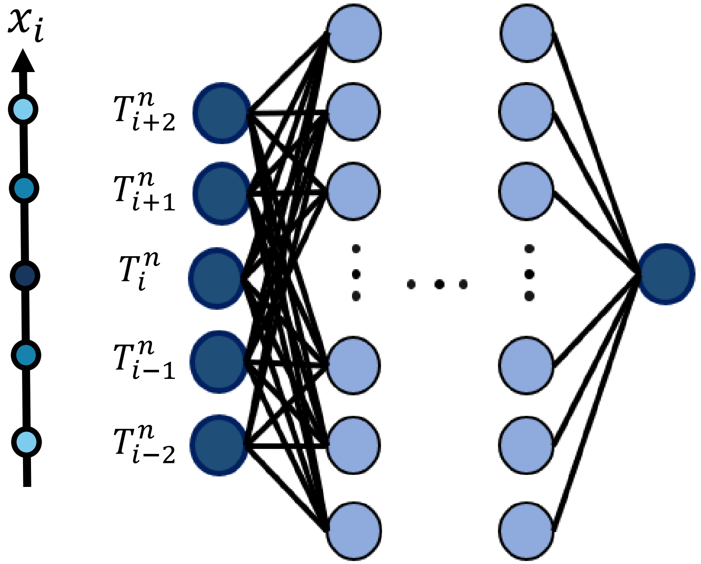


Key attributes:
- **General for <u>any initial states</u>**
- **<u>Independant</u> of tape length (for a given architecture)**
- **Only dependant on temperature seen in training**

MODEL SIZE & RELATED INFORMATION

For each NN:

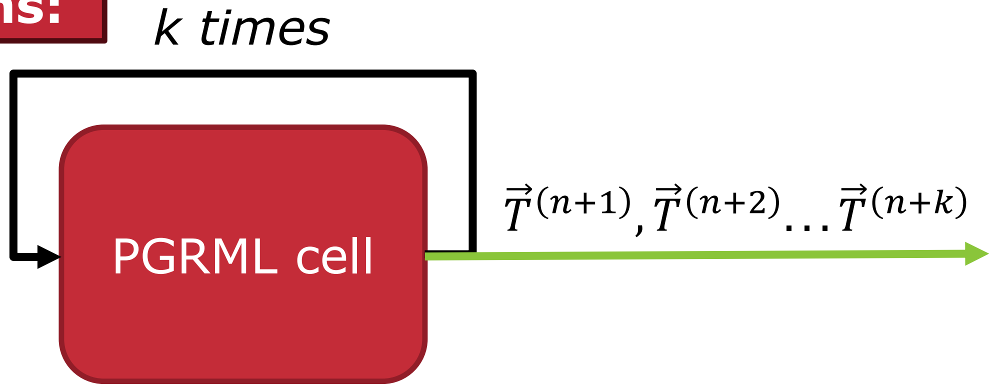
[5,60,60,60,1]



- **Training time: ~12h**
- **Activation function: ReLU**
- **Dropout for hidden layers**
- **MinMax Normalization**
- **Not a fine tuned model**

31 simulations:
- **23 training**
- **1 validation**
- **7 test**

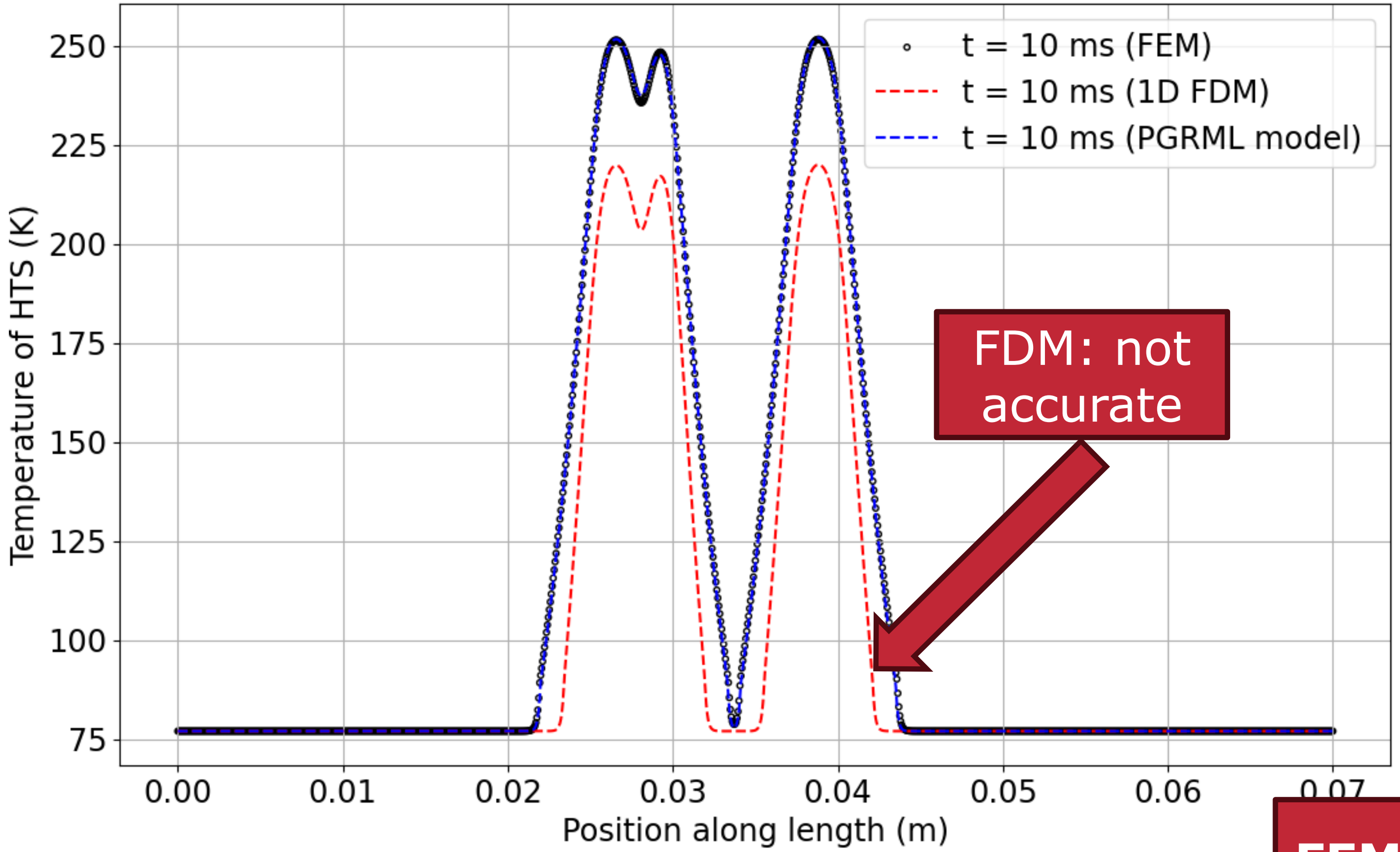
Loss function & recurrent predictions:

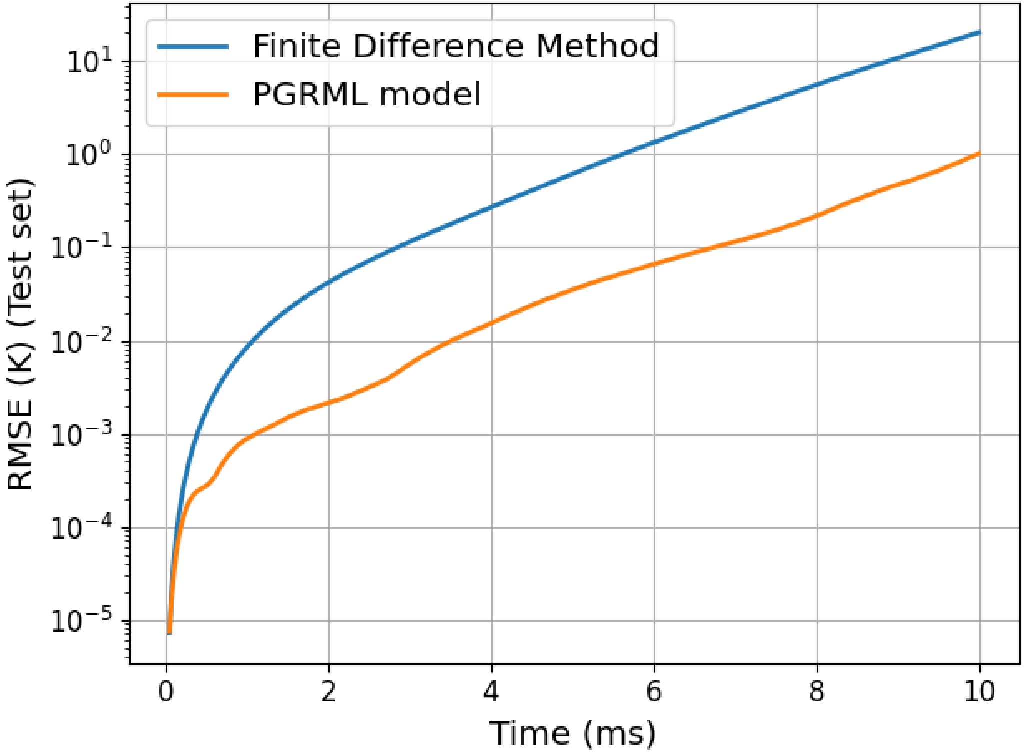


$$L = \frac{1}{N}\sum_{k=1}^{N} \|\vec{T}^{(n+k)} - \vec{T}_{true}^{(n+k)}\|$$

Optimization with respect to T !

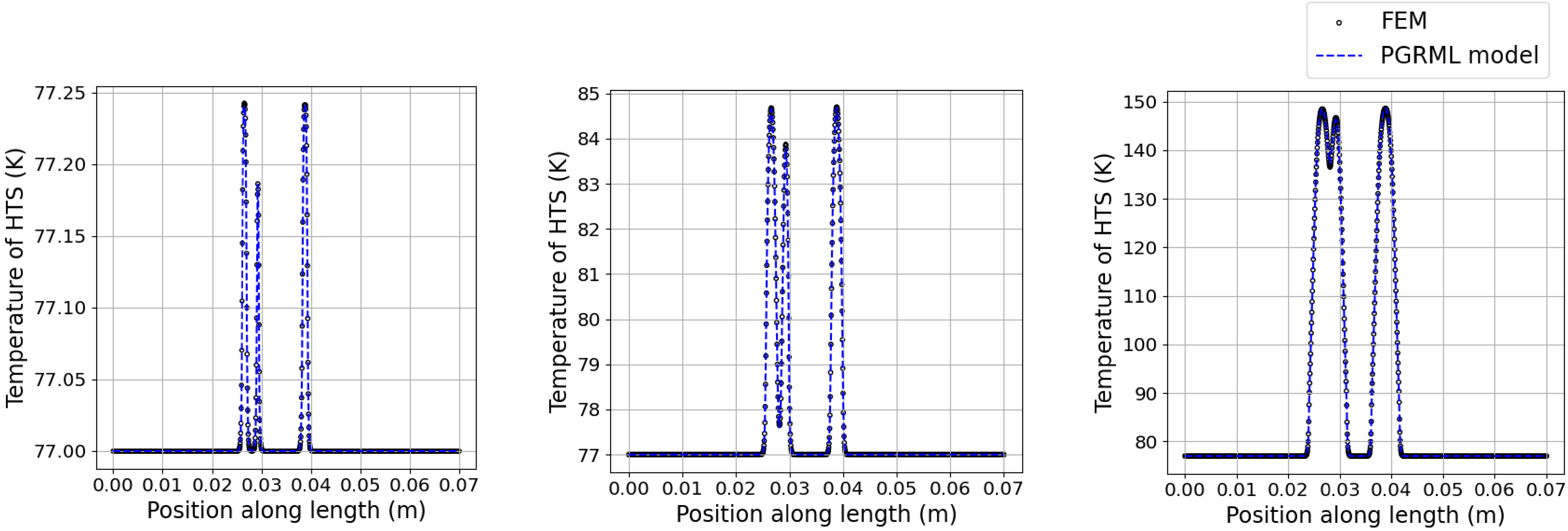
SOLUTION ON TEST SET: PGRML VS FDM





FEM ~ 1h30min : (Δx , Δt) = (58 µm, 8 µs)
PGRML ~ 45s : (Δx , Δt) = (58 µm, 25 µs)

SOLUTION ON TEST SET: PGRML OVER TIME

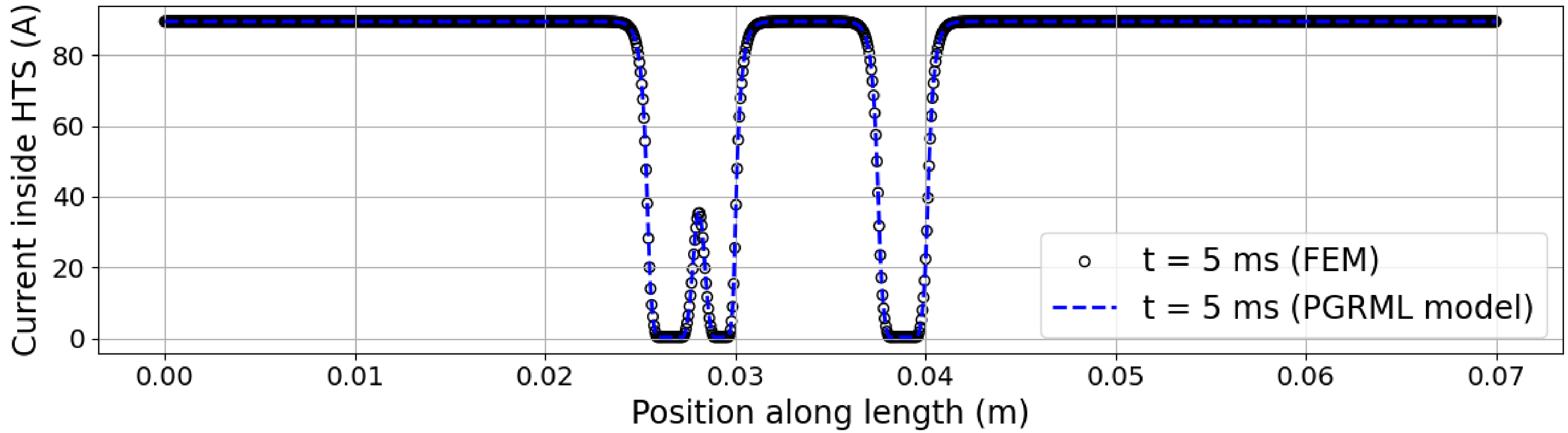


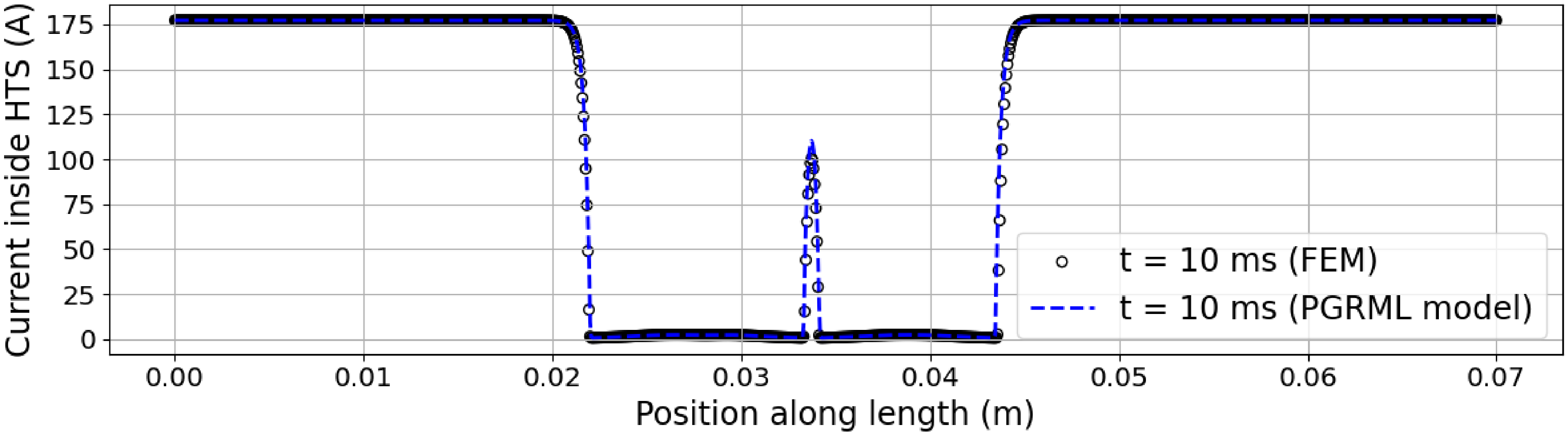
t = 1.2 ms

t = 4.0 ms

t = 8.0 ms

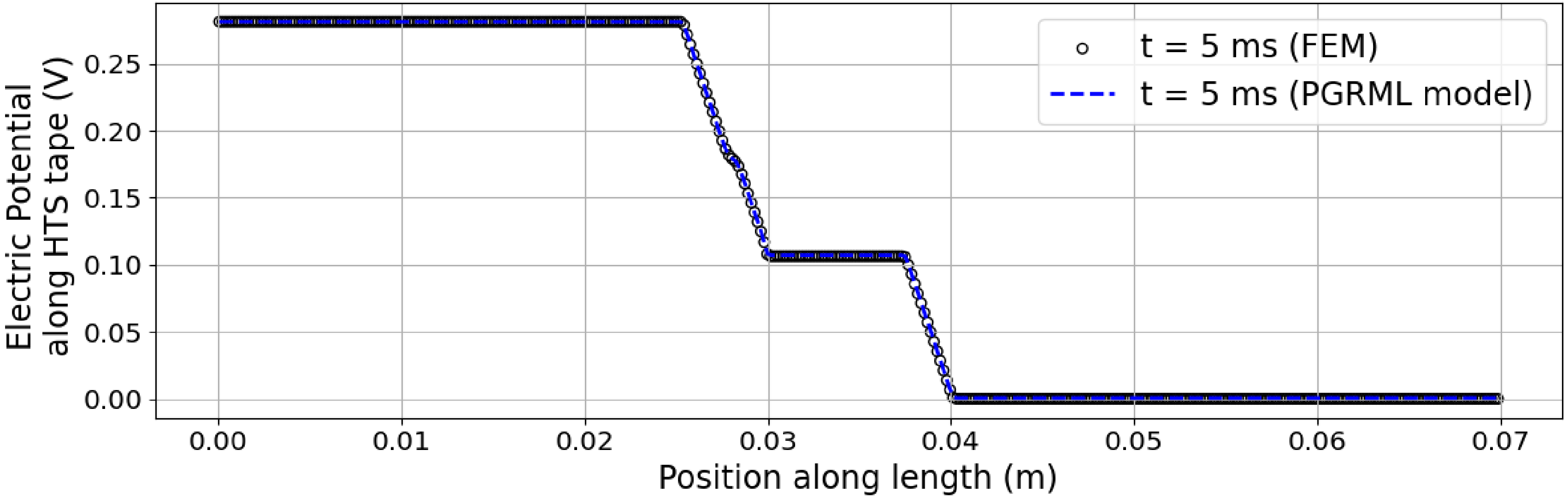
SOLUTION ON TEST SET: CURRENT SHARING

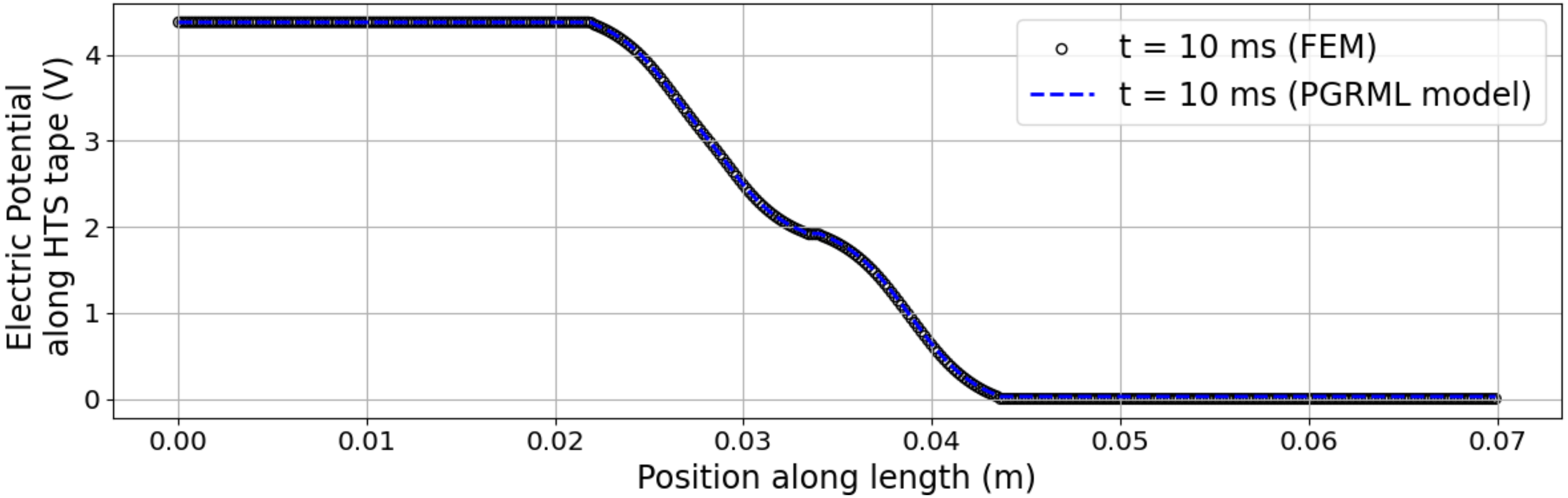




PGRML model <u>is not</u> trained on the current profile!

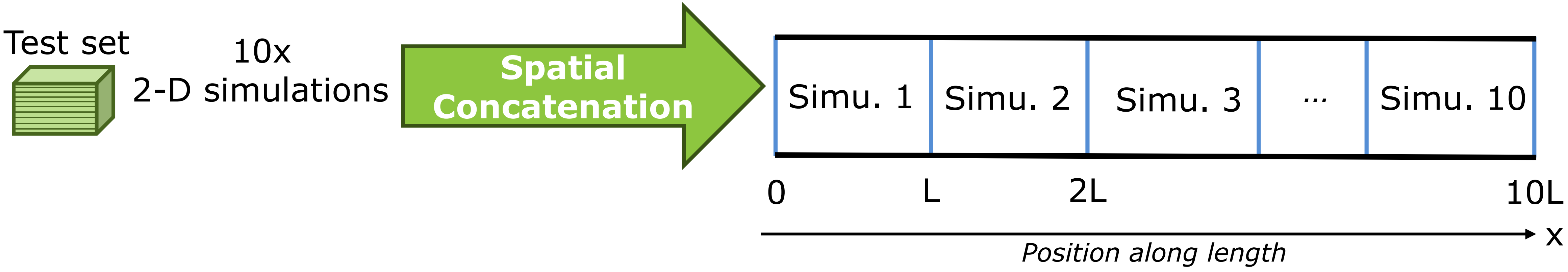
SOLUTION ON TEST SET: ELECTRIC POTENTIAL PROFILE



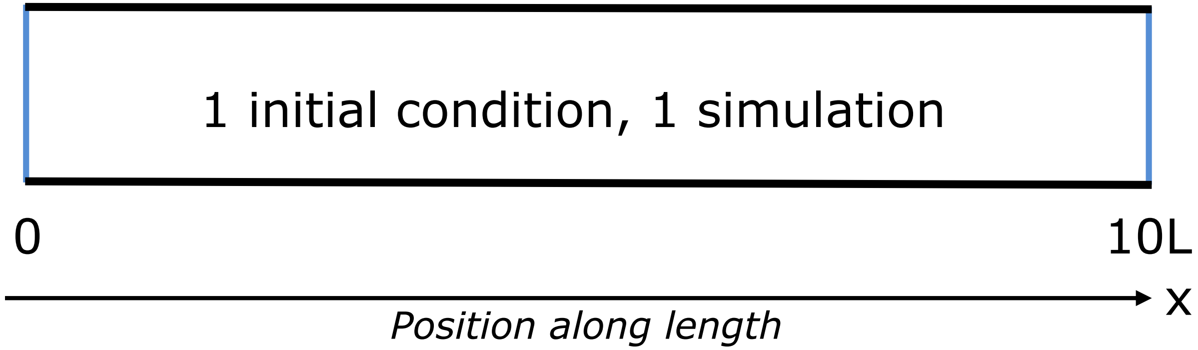


PGRML model <u>is not</u> trained on the electrical potential !

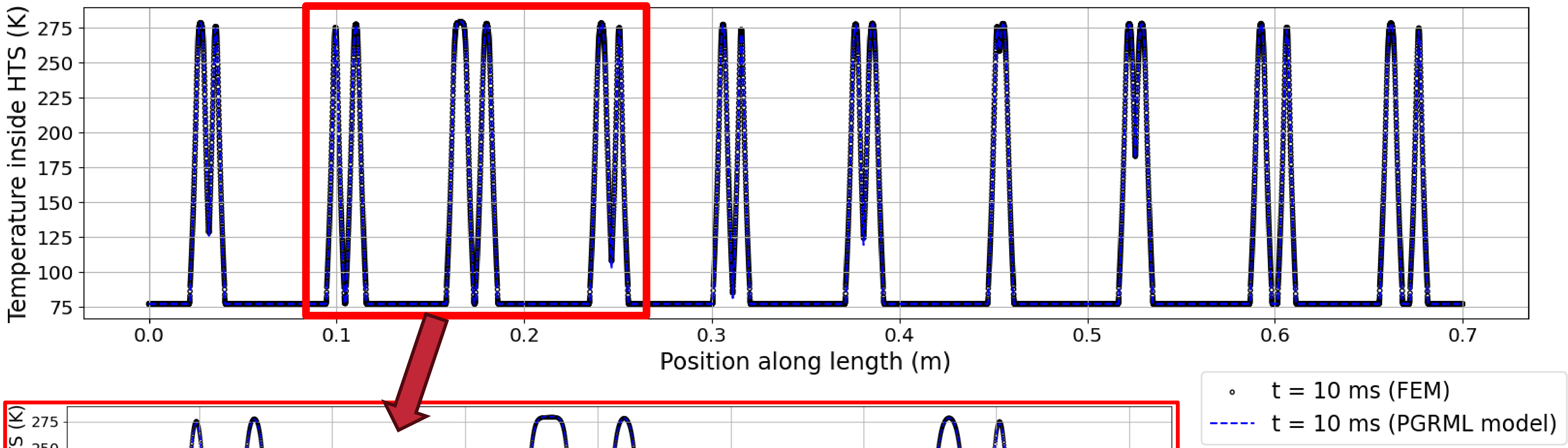
LONG TAPE PREDICTION WITH SOLUTION: METHODOLOGY

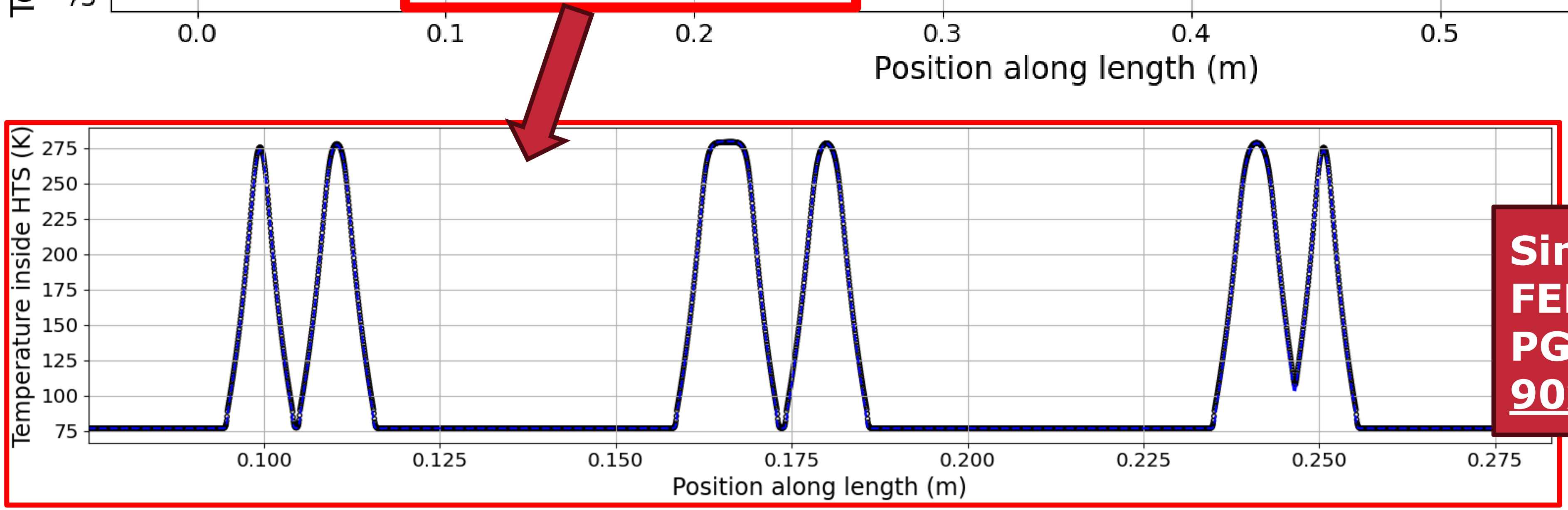


PGRML prediction as a whole:



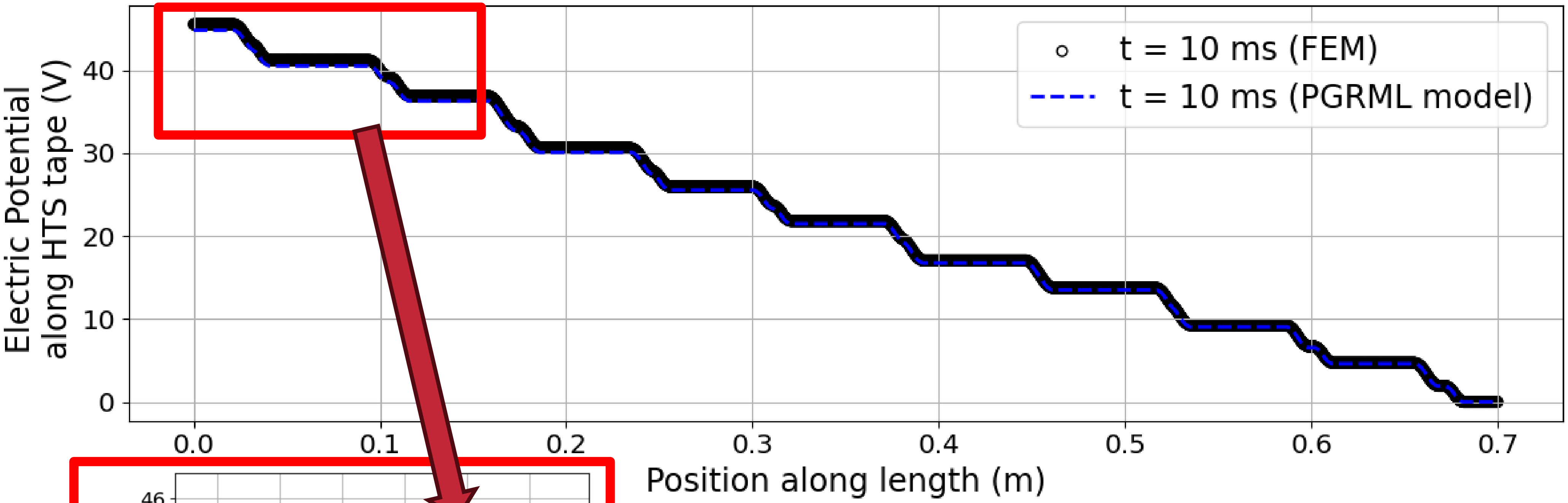
LONG TAPE PREDICTION WITH SOLUTION: TEMPERATURE

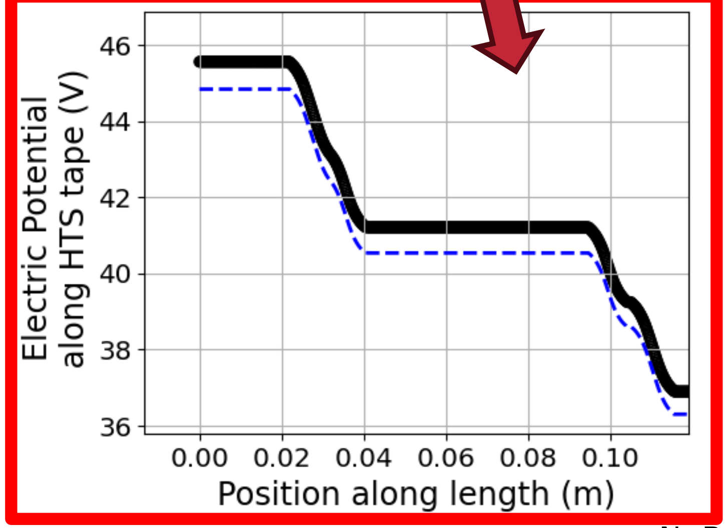




Simulation time:
FEM : 10x 1h30 = 15h
PGRML: 10min
<u>90x Faster !</u>

LONG TAPE PREDICTION WITH SOLUTION: ELECTRIC POTENTIAL





Error accumulation:

$$V(x) = \int_x^L \mathrm{E}(x')dx'$$

$$max\left(\frac{|V_{FEM} - V_{PGRML}|}{V_{FEM}}\right) \approx 1.6\%$$

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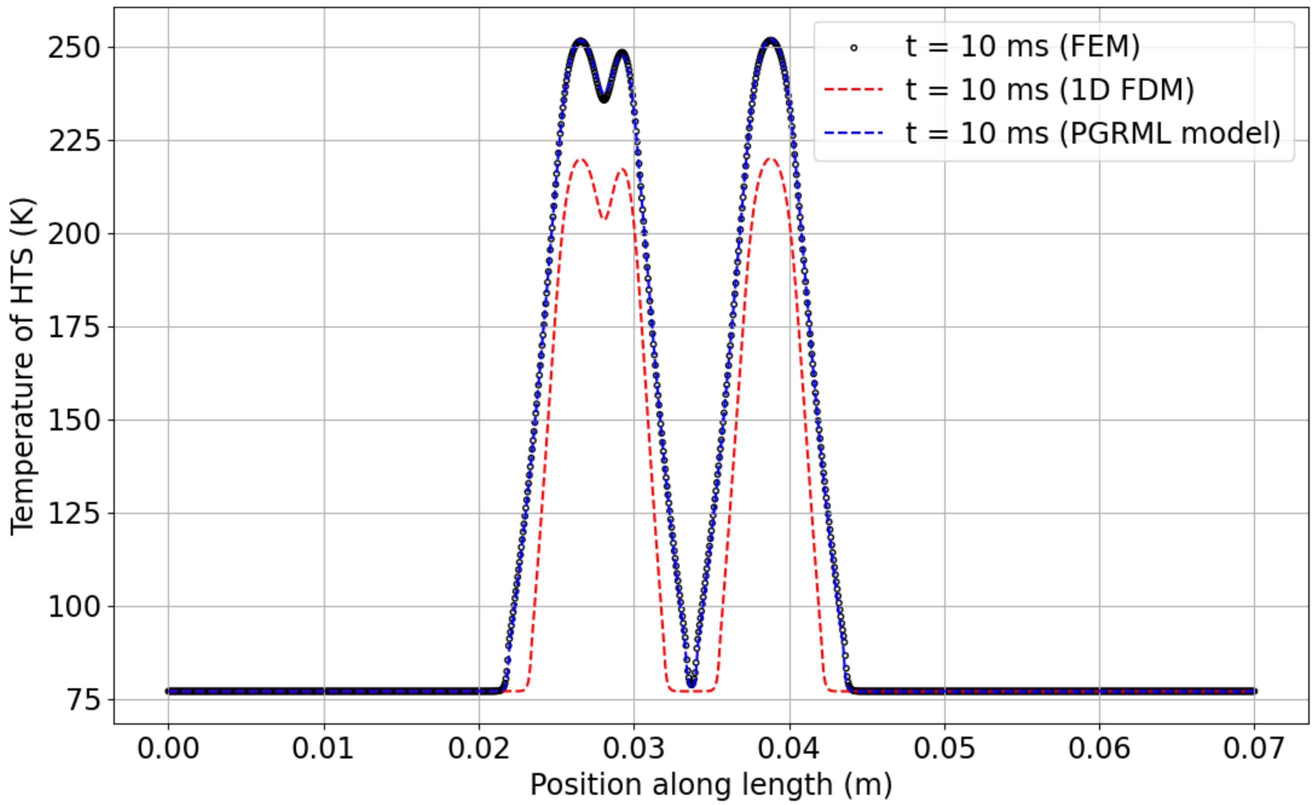
CONCLUSION

Highlights:

- Model based on a reduction in physical dimensionality & Machine learning
- General for any initial state
- No re-training required for length changes given an architecture
- **90x times** faster than FEM

What’s next:

- Complete cooling after hot spots appear
- Variable interfacial resistance
- Real critical current density distribution
- 3-D simulations and CFD architecture



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