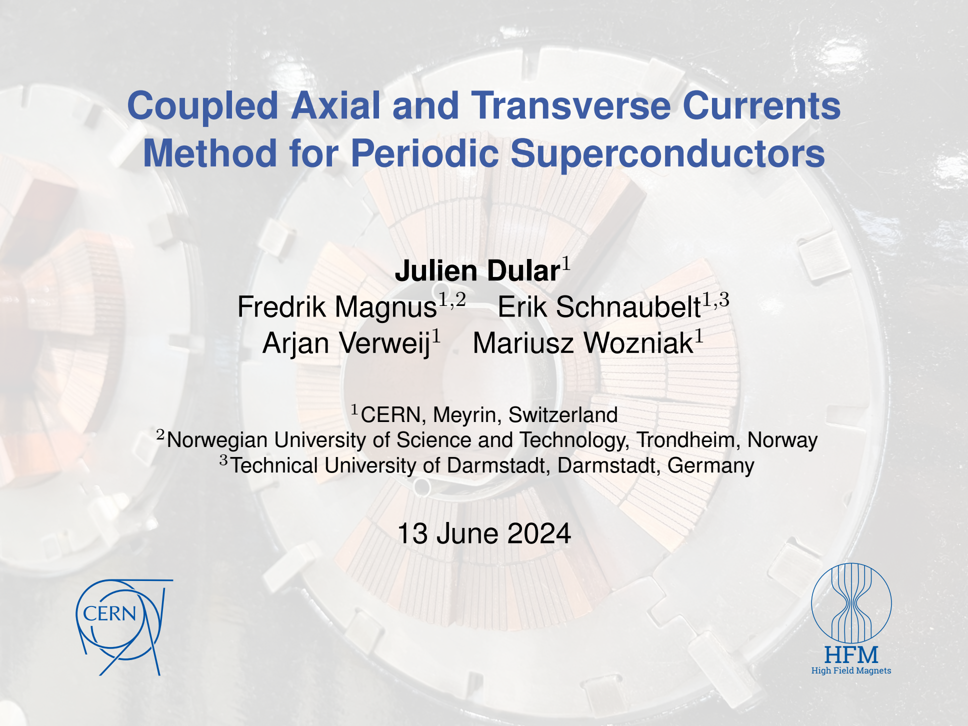

Coupled Axial and Transverse Currents Method for Periodic Superconductors

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Fredrik Magnus[1,2] Erik Schnaubelt[1,3]
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13 June 2024

Outline

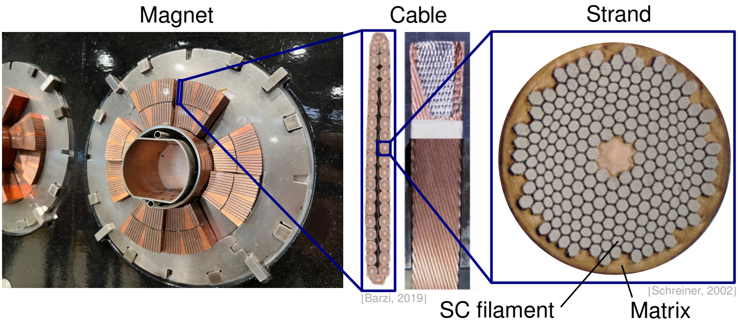
Periodic composite superconductors

Coupled **A**xial and **T**ransverse Currents (**I**) method

Implementation, results and applications

Outlook and conclusions

Superconducting magnet electromagnetic modelling



[Barzi, 2019]

[Schreiner, 2002]

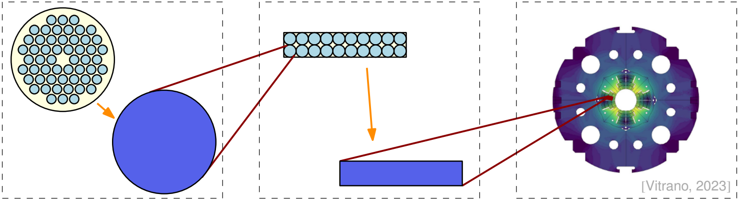
- Quench protection design requires good AC loss models.
- Example: CLIQ (coupling-loss induced quench) devices.
- Magnet geometry is multi-scale. Small-scale, 3D, transient, effects contribute significantly to AC losses.
  $\Rightarrow$ Need for accurate modelling down to the strand level.

From strands to magnet

Fully discretized magnet models are very heavy to solve.

⇒ Lightweight, intermediate models are necessary.

Homogenization of small-scale properties:



[Vitrano, 2023]

Homogenized parameters:

- magnetization, lumped resistance and inductance.

From strands to magnet

Fully discretized magnet models are very heavy to solve.

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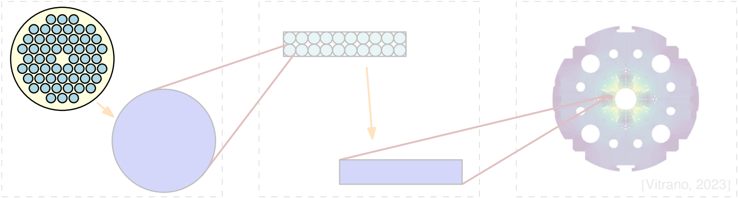
Homogenization of small-scale properties:


[Vitrano, 2023]

Homogenized parameters:

- magnetization, lumped resistance and inductance.

Back to the small-scales: AC losses in strands (and cables).

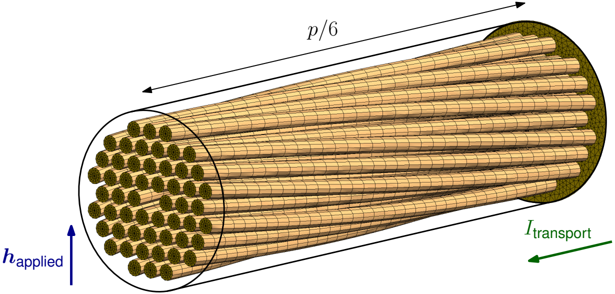
⇒ CATI method

Problem statement - Multifilamentary strand

Multifilamentary strand subject to:

- transport current,
- transverse magnetic field,

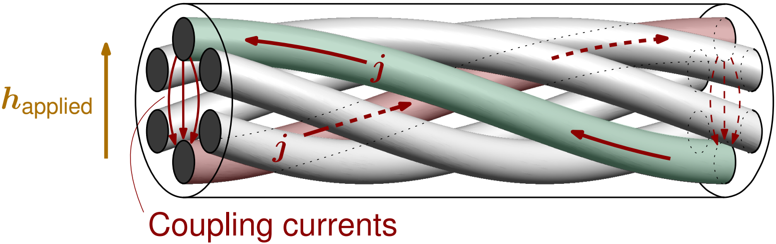
with a range of frequencies and amplitudes.



SC filaments
Cond. matrix
Twist pitch p

Output: transient AC losses and magnetization.

Composite strand dynamics



Loss contributions:

- Coupling currents in the matrix.

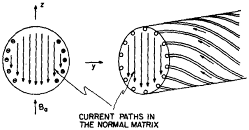
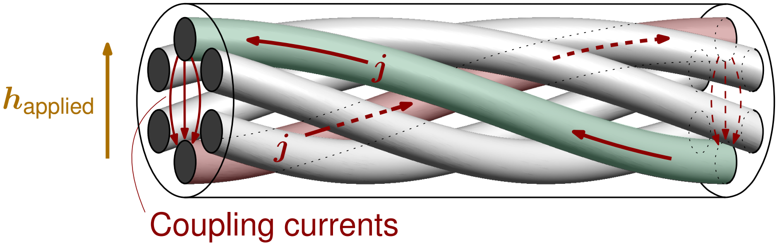


Fig. 2. Current paths in some of the superconducting filaments at the surface and the normal metal matrix of a twisted, multifilament wire which is exposed to a uniform changing field. The interior filaments are not shown since they carry no current.

[Morgan, 1970]

Composite strand dynamics



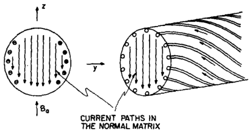


FIG. 2. Current paths in some of the superconducting filaments at the surface and the normal metal matrix of a twisted, multifilament wire which is exposed to a uniform changing field. The interior filaments are not shown since they carry no current.

[Morgan, 1970]

Loss contributions:

- Coupling currents in the matrix.
- Eddy current in the matrix.
- Hysteresis in SC filaments.

Magnetization contributions:

- Matrix $+$ filaments.

These different contributions interact with each other.

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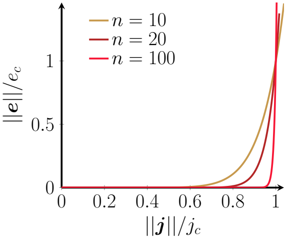
Equations and FE formulation

Magneto-quasistatic equations and constitutive laws (T is fixed):

$$\boxed{\begin{cases} \operatorname{div} \boldsymbol{b} = 0, & \text{(Gauss)} \\ \mathbf{curl}\, \boldsymbol{h} = \boldsymbol{j}, & \text{(Ampère)} \\ \mathbf{curl}\, \boldsymbol{e} = -\partial_t \boldsymbol{b}, & \text{(Faraday)} \end{cases} \text{ with } \begin{cases} \boldsymbol{b} = \mu_0 \boldsymbol{h}, \\ \boldsymbol{e} = \rho(\boldsymbol{j}, \boldsymbol{b})\, \boldsymbol{j}, \end{cases}}$$

with the (nonlinear) power law for the resistivity in SC filaments:

$$\rho(\boldsymbol{j}, \boldsymbol{b}) = \frac{e_c}{j_c(\boldsymbol{b})} \left(\frac{\|\boldsymbol{j}\|}{j_c(\boldsymbol{b})} \right)^{n-1}.$$



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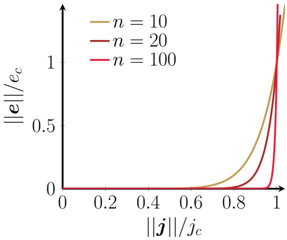


Efficient choice for SC: $\boxed{h\text{-}\phi\text{-formulation}}$

- Weak form of Faraday's law.
- Find $\boldsymbol{h} \in \mathcal{H}(\Omega)$ such that, $\forall \boldsymbol{h}' \in \mathcal{H}_0(\Omega)$:

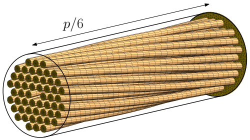
$$\left(\partial_t(\mu_0 \boldsymbol{h}), \boldsymbol{h}'\right)_\Omega + \left(\rho\, \mathbf{curl}\, \boldsymbol{h}, \mathbf{curl}\, \boldsymbol{h}'\right)_{\Omega_c} = 0.$$

- It ensures $\mathbf{curl}\, \boldsymbol{h} = \mathbf{0}$ in Ω_c^C ("cuts").

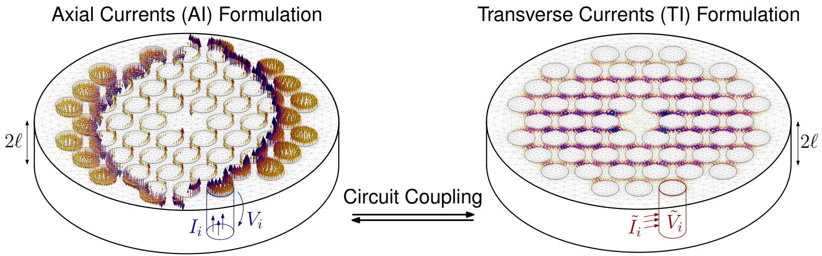
CATI method

- 3D model: computationally expensive.
- We propose a pair of 2D models.

Inspired from [Satiramatekul, Bouillault, 2005].



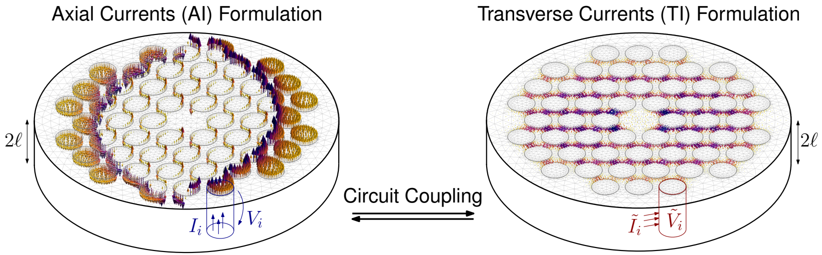
Coupled **A**xial and **T**ransverse currents (**I**), or $\boxed{\text{CATI method}}$


Axial Currents (AI) Formulation

Transverse Currents (TI) Formulation

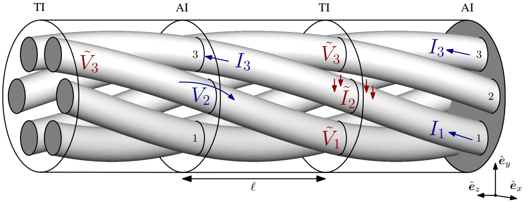
- 2D models written on $2\ell = 2p/6$.
- Global quantities: I_i, V_i, $\tilde{I}_i$, $\tilde{V}_i$ (currents and voltages).
- Coupling via circuit equations.

Circuit coupling equations



Axial Currents (AI) Formulation

Transverse Currents (TI) Formulation



Exploiting periodicity:

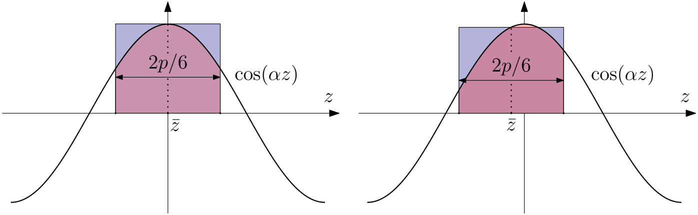
$$\tilde{I}_2 = I_3 - I_1,$$

$$V_2 = \tilde{V}_3 - \tilde{V}_1.$$

Length correction factor

Without correction: overestimation of flux and current.

- Currents and voltages vary continuously along the wire.
- We are assuming constant values over 2ℓ:



- We reduce ℓ to ℓ^* to account for this:

$$\ell^* = \frac{\sin(2\pi\ell/p)}{2\pi\ell/p}\ \ell.$$

- For $2\ell = 2p/6$ (common hexagonal pattern), $\ell^*/\ell = 0.8270$.

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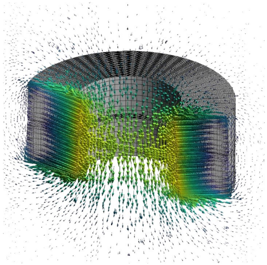
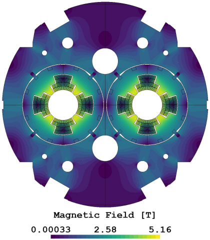
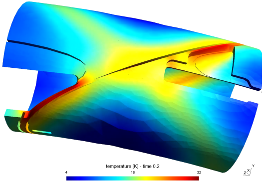
Implementation - GetDP and FiQuS

- Free and open-source **Fi**nite element **Qu**ench **S**imulator.
- Coded in Python, uses the FE framework Gmsh & GetDP.
- HTS Pancake coils, multipole magnets, CCT magnets…

`cern.ch/fiqus`



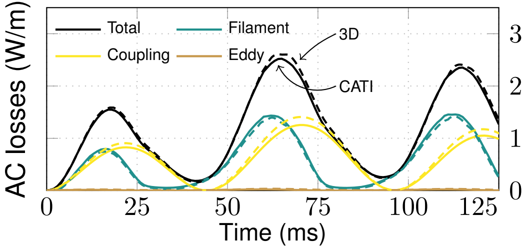


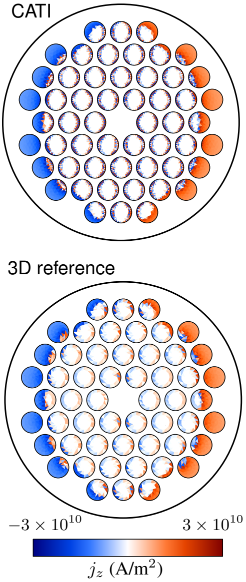


Verification with 3D reference

CATI method vs. 3D reference model:

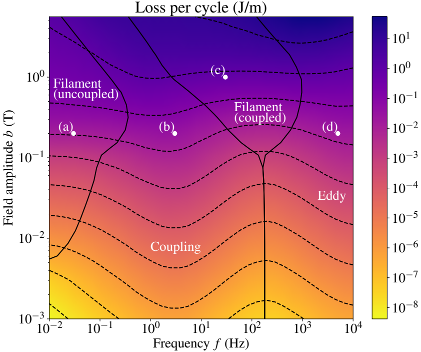
- 54 Nb-Ti filaments, $p = 19$ mm.
- $j_{\text{c,Nb-Ti}}(\boldsymbol{b})$, $\sigma_{\text{Cu}}(\boldsymbol{b})$, $T = 1.9$ K.
- Field $b_{\text{max}} = 0.5$ T, $f = 10$ Hz.
- 3D: 970k DOFs, 150h.
  CATI: 30k DOFs, 1h.
- Relative difference on AC losses $< 5\%$.

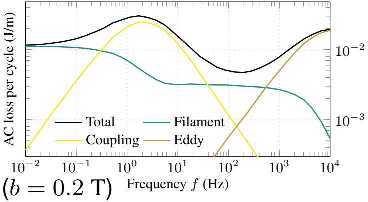




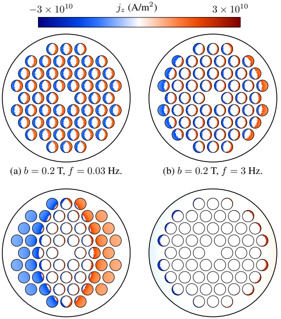
Loss map with no transport current

The CATI method is fast and accurate ⇒ Parameter sweeps.





($b = 0.2$ T)



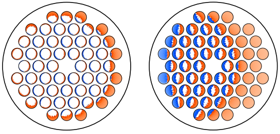
(a) $b = 0.2$ T, $f = 0.03$ Hz.

(b) $b = 0.2$ T, $f = 3$ Hz.

(c) $b = 1$ T, $f = 30$ Hz.

(d) $b = 0.2$ T, $f = 5$ kHz.

With (in-phase) transport current $I_t = 1$ kA:



(e) $b = 0.2$ T, $f = 3$ Hz.

(f) $b = 0.8$ T, $f = 3$ Hz.

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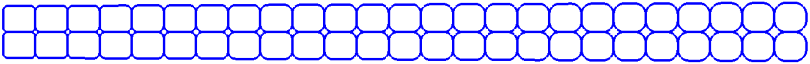
Outlooks and extensions

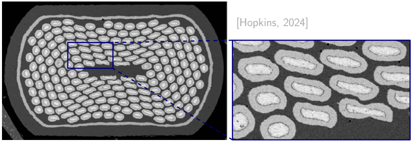
Extension to Rutherford cable geometries



- Homogenized strands from detailed CATI results?
- Tilted current density vectors?

- Nb_3Sn strands (diffusion barriers, unreacted materials).
- MgB_2 strands (ferromagnetic matrix and loss).
- Bi-2212 or Bi-2223 wires.

- AC losses change due to strand cabling deformation.

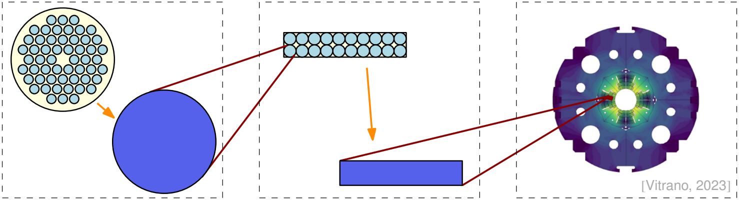


[Hopkins, 2024]

Outlook - Homogenization of full-scale magnets

Strand macroscopic response: magnetization and losses.

⇒ Homogenization of small-scale properties:


[Vitrano, 2023]

with homogenized parameters: magnetization, lumped R and L.

- Either identified *a priori* (simple homogenization),
- or resolved on-the-fly (multi-scale resolution).

Conclusions

Efficient 2D+2D model for AC losses (h-ϕ-formulation)

- Magnetodynamics (eddy currents).
- Nonlinear material properties.
- Fast and accurate.

Implemented in FiQuS, with GetDP/Gmsh

- Open-source, modular, and highly flexible.

Outlooks

1. Extend the CATI method to cables.
2. Investigate different LTS/HTS wire and cable geometries.
3. Exploit the results in homogenized models.

Joint work with the STEAM team at CERN!

References

Analytical AC losses

- Morgan, G. H. (**1970**). *Theoretical behavior of twisted multicore superconducting wire in a time-varying uniform magnetic field*. Journal of Applied Physics, 41(9), 3673-3679.
- Campbell, A. M. (**1982**). *A general treatment of losses in multifilamentary superconductors*. Cryogenics, 22(1), 3-16.

Helicoidal transformation and coupled model first papers

- Nicolet, A., Zolla, F., and Guenneau, S. (**2004**). *Modelling of twisted optical waveguides with edge elements*. The European Physical Journal Applied Physics, 28(2), 153-157.
- Satiramatekul, T., and Bouillault, F. (**2005**). *Magnetization of coupled and noncoupled superconducting filaments with dependence of current density on applied field*. IEEE Transactions on Magnetics, 41(10), 3751-3753.
- Satiramatekul, T., Bouillault, F., Devred, A., and Leroy, D. (**2007**). *Magnetization modeling of twisted superconducting filaments*. IEEE TAS, 17(2), 3737-3740.
- Dular, J., Henrotte, F., Nicolet, A., Wozniak, M., Vanderheyden, B., and Geuzaine, C. (**2023**). *Helicoidal transformation method for finite element models of twisted superconductors*. Submitted to IEEE TAS.

GetDP and FiQuS (Finite Element Quench Simulator)

- Dular, P., Geuzaine, C., Henrotte, F., and Legros, W. (**1998**). A general environment for the treatment of discrete problems and its application to the finite element method. IEEE Transactions on Magnetics, 34(5), 3395-3398.
- Vitrano, A., Wozniak, M., Schnaubelt, E., Mulder, T., Ravaioli, E., and Verweij, A. (**2023**). An open-source finite element quench simulation tool for superconducting magnets. IEEE TAS, 33(5), 1-6.

CATI method - Main contribution

- Dular, J., Magnus, F., Schnaubelt, E., Verweij, A., and Wozniak, M. (**2024**). *Coupled axial and transverse currents method for periodic superconductors finite element modelling*. Submitted to SUST.

`https://arxiv.org/abs/2404.09775`

HFM

High Field Magnets