

# User-defined Superconductivity Formulations in COMSOL - Tipps for Efficient and Reusable Implementations

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# COMSOL Fully Integrated Software Suite

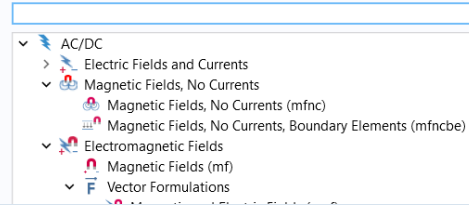
- Multiphysics simulation, from building geometry to results analysis, via a streamlined physics-based modeling workflow
- Application Builder
- Model Manager

The screenshot displays the COMSOL Multiphysics software interface for a simulation titled "sync\_drive\_mon25dfull3d.mph". The interface is divided into several main sections:

- Top Bar:** Contains the menu (File, Home, Definitions, Geometry, Materials, Physics, Mesh, Study, Results, Developer) and a toolbar with icons for various modeling and simulation tasks.
- Model Builder:** A tree view on the left showing the model's structure. The "Nonlinear Magnetic Core 2" component is selected. The tree includes:
  - sync\_drive\_mon25dfull3d.mph (root)
  - Global Definitions
  - Synchronous Electric Drive 2D (comp2d)
  - Synchronous Electric Drive 3D (comp3d)
    - Definitions
    - Geometry 2
    - Materials
    - Moving Mesh
    - Rotating Machinery, Magnetic (Full3D) (rmm3d)
      - Electric Field Transformation 1
      - Ampère's Law 1
      - Mixed Formulation Boundary 1
      - Magnetic Insulation 1
      - Initial Values 1
      - Continuity 1
      - Sector Symmetry 1
      - Periodic Conditions
        - Periodic Condition 1
        - Periodic Condition 2
        - Periodic Condition 3
        - Periodic Condition 4
        - Periodic Condition 5
      - Magnetic Flux Conservation 1
      - Zero Magnetic Scalar Potential 1
      - Gauge Fixing for A-field 1
      - Gauge Fixing for A, Psi Constraint 1
      - Nonlinear Magnetic Core 1
      - Nonlinear Magnetic Core 2
      - Thin Low Permeability Gap 1
      - Magnet Array 1
      - Magnet Array 2
      - Arkko Torque Calculation 1
      - Phase Apos
      - Phase Cneg
      - Phase Bpos
    - Hairpin Circuit (Full3D) (cir3d)
    - Meshes
    - Study 1: Full Transient Analysis (Mon2.5D)
    - Study 2: Full Transient Analysis (Full3D)
    - Results
      - Datasets
      - Views
      - Derived Values
      - Tables
    - Probe Plot: Torque And Experiments

# Typical COMSOL Usage in the HTS Community

## Select Physics



## Built-in Physics Interfaces

- H-formulation (*mfh*)
- A-formulation (*mf*)
- $\phi$  –formulation (*mfnc*)
- Several more...

$$\begin{aligned} \mathbf{J}_c &= \sigma \mathbf{E} \\ \mathbf{E} &= \rho(\nabla \times \mathbf{H} - \mathbf{J}_e) \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} \mathbf{H} &= -\nabla V_m \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}$$

## Manual Coupling of Built-in PI

- Use two or more of built-in interfaces and couple in boundaries to obtain H- $\phi$ , A- $\phi$ , T-A, etc.
- “*straightforward*”

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \Gamma = f$$

$$0 = \int_{\Omega} \text{weak } \delta S$$

## Equation-based Interfaces

- Coefficient-form
- General form
- Weak form
- “*innovative but nerdy*”

# My Topic Today: Further Power Tools Uncovered

1

## Modify Built-in Eqns.

Less need to reinvent the wheel, concentrate on your innovation.

2

## Record Methods

Design reusable *model methods* to avoid repetitive settings work *in one model*.

3

## Add-ins

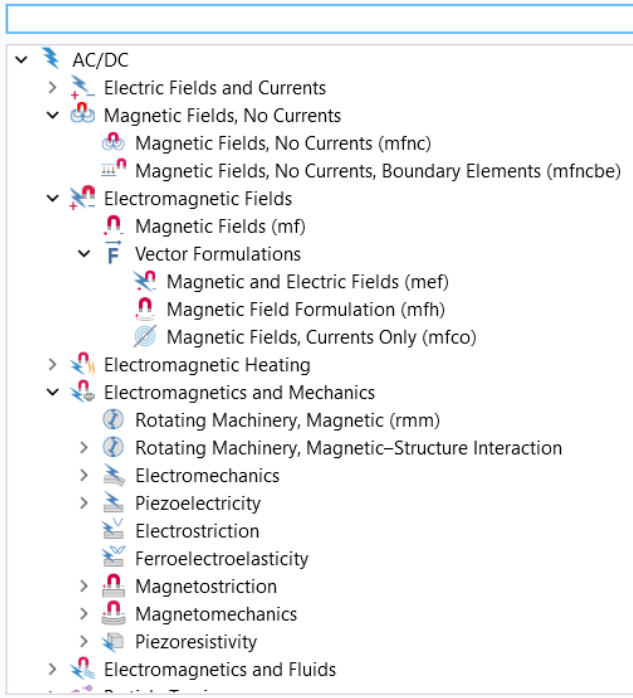
Publish code snips to be re-used in *several models*.

4

## Physics Builder

Design your own tailor-made physics interfaces that everybody can use and plug them into COMSOL.

## Select Physics



## Built-in Interfaces

### Advantages

- Well-established and maintained technology
- Constantly extended by new features
- Easy coupling with any other existing technology

### Recent News

- (*mfco*) Biot-Savart formulation
- (*mfncbe*) Boundary Elements formulation
- Time-periodic FEM solver
- Etc...

# Manual Coupling

- Activate 2 or more physics
- Define domains of validity
- Couple at interfaces
- Enjoy many built-in features, e.g. discontinuities, constitutive relations, solver defaults

superconducting\_wire\_H\_vs\_HPPhi\_vs\_TA\_RESET.mph - COMSOL

File Home Definitions Geometry Sketch Materials Physics Mesh Study Results Developer

Model Builder

Type filter text

- superconducting\_wire\_H\_vs\_HPPhi\_vs\_TA\_RESET.mph (ro)
  - Global Definitions
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
    - Magnetic Field Formulation (H-Form) (mfh)
      - Magnetic Field Formulation 2 (H-PHI) (mfh2)
        - Faraday's Law 1
        - Magnetic Gauss' Law 1
        - Magnetic Insulation 1
        - Initial Values 1
        - Faraday's Law 2
        - Magnetic Field 1
      - Magnetic Fields, No Currents (H-PHI) (mfnc)
        - Magnetic Flux Conservation 1
        - Magnetic Insulation 1
        - Initial Values 1
        - Magnetic Scalar Potential Discontinuity 1
        - Magnetic Flux Density 1
        - Magnetic Scalar Potential 1
      - Magnetic Field Formulation 3 (H-A, Domain-co
      - Magnetic Fields (H-A, Domain-coupled) (mf)
      - Magnetic Field Formulation 4 (mfh4)
      - Magnetic Fields 2 (mf2)
      - Multiphysics
      - Mesh 1
    - Study 1 (mfh, H-Form)
    - Study 2 (mfh + mfnc, H-PHI)
    - Study 3 (mfh + mf, H-A Domain-coupled)
    - Study 4 (mfh + mf, H-A bnd coupled)
    - Results

H

 $\phi$ 

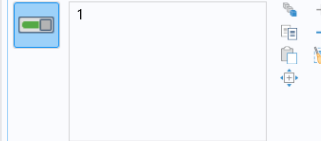
## Settings

### Magnetic Scalar Potential Discontinuity

Label: Magnetic Scalar Potential Discontinuity

#### Boundary Selection

Selection: Manual



#### Override and Contribution

#### Equation

Show equation assuming:

Study 1 (mfh, H-Form), Time Dependent

$$V_{m,d} - V_{m,d} = V_{m,d}$$

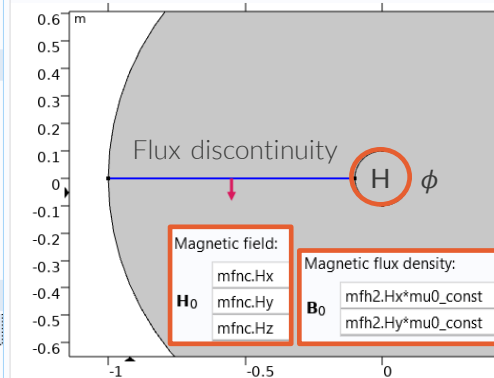
#### Magnetic Scalar Potential Discontinuity

Magnetic scalar potential discontinuity:

$$V_{m,d} \quad |0 \times (1 - \exp(-t/\tau)) \quad A$$

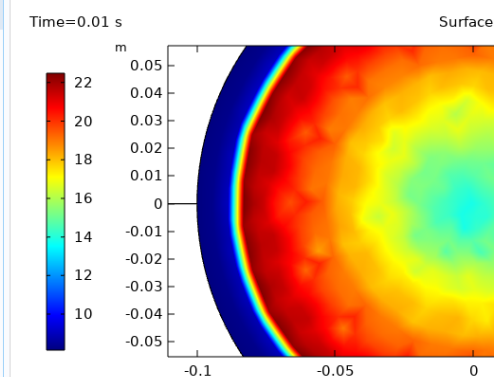
## Graphics

Convergence Plot 1



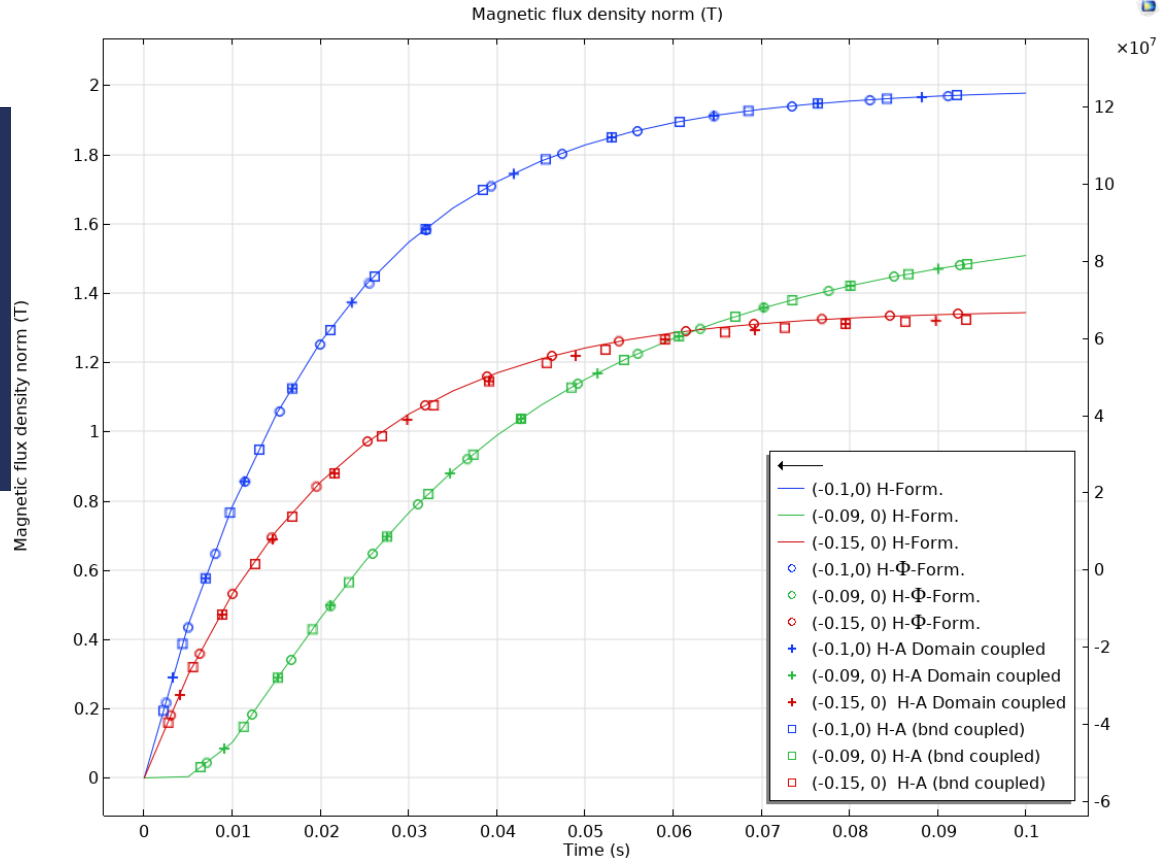
## Plot 3

Time=0.01 s



# Manual Coupling Results














- Proven success in many publications
- Research topics are:
  - Order of shape function
  - Accuracy vs. speed
  - Domain- or boundary coupling



Why would one use anything else? 🤔



## Select Physics

- >  Fluid Flow
- >  Heat Transfer
- >  Optics
- >  Plasma
- >  Radio Frequency
- >  Semiconductor
- >  Structural Mechanics
- ▼  $\Delta u$  Mathematics
  - ▼  $\Delta u$  PDE Interfaces
    - $\Delta u$  Coefficient Form PDE (c)
    - $\Delta u$  General Form PDE (g)
    - $\Delta u$  Wave Form PDE (wahw)
    - $f_{\text{w}}$  Weak Form PDE (w)
    - $\Delta u$  PDE, Boundary Elements (pdebe)
  - >  $\Delta u$  Lower Dimensions
  - >  $\frac{d}{dt}$  ODE and DAE Interfaces
  - >  Optimization and Sensitivity
  - >  $\nabla^2$  Classical PDEs
  - >  Moving Interface
  - >  Deformed Mesh
  -  Wall Distance (wd)
  -  Mathematical Particle Tracing (pt)
  -  Curvilinear Coordinates (cc)

## Equation-Based Interfaces

### Pros

- Full control for testing
- Textbook notation

### Cons

- No default units
- Simple solver defaults
- No constitutive relations
- Basic boundary conditions
- Reuse is “nerdy”

# Equation-Based Modeling

- Used in many publications
- Access to weak form
- 3D, 2D, 1D entities

Settings  
General Form PDE

Label:

Domain Selection

Override and Contribution

Equation

Show equation assuming:

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \Gamma = f$$

$$\mathbf{u} = [u_1, u_2]^T$$

$$\nabla = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right]$$

Conservative Flux

-u1x	x	1/m
-u1y	y	
-u2x	x	1/m
-u2y	y	

Source Term

<input type="text" value="1"/>	1/m <sup>2</sup>
<input type="text" value="1"/>	1/m <sup>2</sup>

Damping or Mass Coefficient

<input type="text" value="1"/>	s/m <sup>2</sup>	<input type="text" value="0"/>	s/m <sup>2</sup>
<input type="text" value="0"/>	s/m <sup>2</sup>	<input type="text" value="1"/>	s/m <sup>2</sup>

Mass Coefficient

<input type="text" value="0"/>	s <sup>2</sup> /m <sup>2</sup>	<input type="text" value="0"/>	s <sup>2</sup> /m <sup>2</sup>
<input type="text" value="0"/>	s <sup>2</sup> /m <sup>2</sup>	<input type="text" value="0"/>	s <sup>2</sup> /m <sup>2</sup>

Settings  
Weak Form PDE

Label:

Domain Selection

Selection:

Override and Contribution

Equation

Show equation assuming:

$$0 = \int_{\Omega} \text{weak} \, \delta S$$

Weak Expressions

weak

# Example: Magnetodynamic H- $\phi$ Formulation [1]

Second, we implemented the H- $\phi$  formulation with Faraday's law in nonconducting domains, such that the equations are given by [5]

$$\begin{aligned} \text{In } \Omega_c^C : \quad & \frac{\partial \mu \nabla \phi}{\partial t} = 0 \\ \text{In } \Omega_c : \quad & \nabla \times (\rho \nabla \times \mathbf{H}) = -\frac{\partial \mu \mathbf{H}}{\partial t}. \end{aligned} \quad (5)$$

We refer to this case as the magnetodynamic H- $\phi$  formulation (H- $\phi$ /D).

The weak equations for this formulation are given by

$$\frac{\partial}{\partial t} \mu \int_{\Omega_c^C} \nabla \phi \cdot \nabla \tilde{\phi} \, d\Omega - \int_{\Gamma} \hat{\mathbf{n}} \times \mathbf{E} \cdot \nabla \tilde{\phi} \, d\Gamma = 0 \quad (6)$$

$$\begin{aligned} \int_{\Omega_c} \rho \nabla \times \mathbf{H} \cdot \nabla \times \tilde{\mathbf{H}} \, d\Omega + \frac{\partial}{\partial t} \mu \int_{\Omega_c} \mathbf{H} \cdot \tilde{\mathbf{H}} \, d\Omega \\ - \int_{\Gamma} \underbrace{\hat{\mathbf{n}} \times \mathbf{E}}_{\mathbf{E}_t} \cdot \tilde{\mathbf{H}} \, d\Gamma = 0. \end{aligned} \quad (7)$$

$$\underbrace{\int_{\Omega_c^C} -\mu \left( \frac{\partial \phi_r}{\partial t} \tilde{\phi}_r + \frac{\partial \phi_z}{\partial t} \tilde{\phi}_z \right) d\Omega}_{\text{weak term}} \quad \underbrace{\frac{\partial}{\partial t} (\nabla \cdot \mathbf{B}) = 0}_{\text{strong form}}$$

## Weak Contribution

Weak expression:

```
mu0_const*d(-phir,t)*test(-phir)+mu0_const*d(-phiz,t)*test(-phiz)
```

1

# Modify Built-in Equations

- Use  $\phi$  –formulation (*mfnc*) instead of weak form
- Change weak term only
- Enjoy all the goodies of built-in formulation

The screenshot displays the COMSOL Multiphysics interface for a model named *mfh\_mfnc\_mod\_tuned.mph*. The **Model Builder** pane on the left shows the hierarchy: *mfh\_mfnc\_mod\_tuned.mph (root)* > *Component 1 (comp1)* > *Magnetic Fields, No Currents (mfnc)* > *Magnetic Flux Conservation 1* > *Equation View*. The **Settings** pane on the right is set to **Equation View**. Under **Weak Expressions**, the **Weak expression** field contains the modified equation:  $mfnc.d*(-mfnc.Bx*test(Vmx)-mfnc.By*test(Vmy))$ . A blue arrow points from the **Weak Expressions** section in the **Settings** pane to the **Weak Expressions** section in the **Model Builder** pane. Another blue arrow points from the **Weak expression** field in the **Settings** pane to the **Weak expression** field in the **Model Builder** pane. The **Graphics** pane on the right shows a 2D plot of the magnetic field distribution, with a yellow shaded region and several vertical bars representing field components.

# Modify Built-in Equations

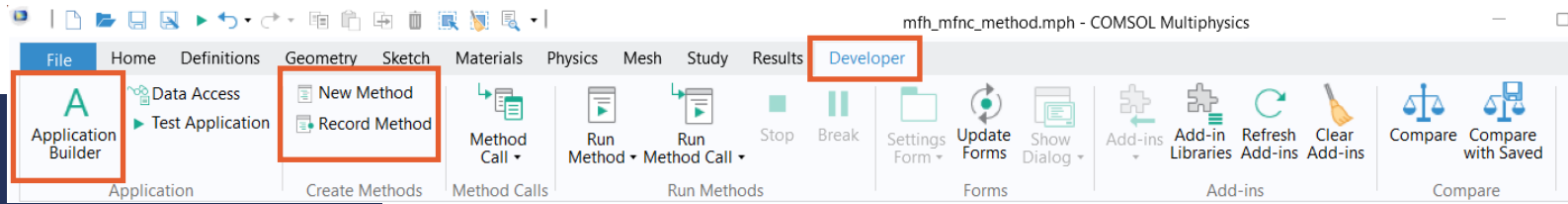
- Compare compute time
  - PDE Version: 36 s
  - Modified mfnc: 56 s (+55%)
- The PDE version uses less robust solver defaults; need to change those, too.

Solver Level	Setting	PDE	mfnc	runtime [s]
<b>Direct solver</b>	Check error estimate	No	Auto	56
<b>Advanced</b>	Reuse sparsity pattern	off	on	61
<b>Fully Coupled</b>	Jacobian Update	minimal	once per time-step	53
<b>Fully Coupled</b>	Max. No of iterations	4	15	38
<b>Fully Coupled</b>	Tolerance factor	1	0.2	38
<b>Time stepping</b>	Steps taken by solver	free	strict	36
<b>Time stepping</b>	Error estimation	Include algebraic	Exclude algebraic	36
<b>Direct solver</b>	Change from MUMPS to	PARDISO	PARDISO	20

**Do I really need to change  
10 settings manually, again? 🤔**

# Record Method

- COMSOL allows to record GUI operations as code from its Java<sup>®</sup> API



```

1 // Modify weak form to Magnetodynamic Phi
2
3 model.component("comp1").physics("mfnc").feature("mfc1").featureInfo("info")
4   .set("root.comp1.mfnc.mfc1.weak$1", new String[]{"mfnc.d*(-d(mfnc.Bx,t)*test(Vmx)-d(mfnc.By,t)*test(Vmy))", "2"});
5 model.component("comp1").physics("mfnc").feature("mfc2").featureInfo("info")
6   .set("root.comp1.mfnc.mfc2.weak$1", new String[]{"mfnc.d*(-d(mfnc.Bx,t)*test(Vmx)-d(mfnc.By,t)*test(Vmy))", "2"});
7
8 // Set Solver to PDE defaults
9 model.sol("sol3").feature("t1").feature("dDef").set("errorchk", false);
10 model.sol("sol3").feature("t1").feature("aDef").set("cachepattern", false);
11 model.sol("sol3").feature("t1").feature("fc1").set("jtech", "minimal");
12 model.sol("sol3").feature("t1").feature("fc1").set("maxiter", 4);
13 model.sol("sol3").feature("t1").feature("fc1").set("ntolfact", 1);
14 model.sol("sol3").feature("t1").set("tstepsbdf", "free");
15 model.sol("sol3").feature("t1").set("estrat", "include");
16 model.sol("sol3").feature("t1").feature("dDef").set("linsolver", "pardiso");
17
18 // Rename Physics and Study
19 model.component("comp1").physics("mfnc").label("Magnetic Fields, No Currents, modified");
20 model.study("std2").label("Study 2 mfh + mfnc_mod");

```

# Method Call

- Embed method calls into the Model Builder tree.
- Methods can be refined to detect existing feature nodes.

The screenshot displays the COMSOL Model Builder interface. The top bar shows navigation icons and a search field labeled "Type filter text". The main tree view is expanded to show the "Global Definitions" folder, which contains several "Parameters" (Parameters 1 through 5), "Default Model Inputs", "Materials", and two "Set" nodes: "Set Magnetostatic Phi" and "Set Magnetodynamic Phi". The "Set Magnetodynamic Phi" node is currently selected. Below the tree, the "Component 1 (comp1)" folder is expanded, showing various physics and meshing nodes such as "Definitions", "Geometry 1", "Materials", "Magnetic Field Formulation (mfh)", "Weak Form PDE (w)", "Magnetic Fields, No Currents (mfnc)", "Multiphysics", and "Mesh 1". At the bottom, there are "Study 1 mfh + weak", "Study 2 mfh + mfnc", and "Results" nodes.

A "Confirm" dialog box is overlaid on the right side of the screen. It contains a question mark icon and the text: "Modify (mfnc) to Magnetodynamic Phi Formulation and activate low-level solver defaults?". There are two buttons at the bottom: "Yes" and "No".



**Can I reuse methods in other files? 🤔**

# Add Ins

- Make methods and forms reusable in other models.
- Load Add-in from library.
- Make sure that the Add-in is robust!

The screenshot shows the COMSOL Multiphysics interface for a model named 'mfh\_mfnc\_method.mph'. The 'Developer' tab is active, and the 'Add-ins' menu is highlighted with a red box. The 'Add-ins' menu contains 'Add-ins', 'Add-in Libraries', and 'Refresh Add-ins'.

The 'Model Builder' window shows the following structure:

- mfh\_mfnc\_method.mph (root)
  - Global Definitions
    - Parameters 1
    - Parameters 2
    - Parameters 3
    - Parameters 4
    - Parameters 5
    - Default Model Inputs
    - Materials
    - Magnetodynamic Add-In
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
    - Magnetic Field Formulation (mfh)
    - Weak Form PDE (w)
    - Magnetic Fields, No Currents (mfnc)
    - Multiphysics
    - Mesh 1
    - Study 1 mfh + weak
    - Study 2 mfh + mfnc
    - Results

The 'Settings' window shows the 'Magnetodynamic H-Phi Add-In' with two options:

- Set Magnetodynamic:  $\frac{\partial}{\partial t}(\nabla \cdot \mathbf{B}) = 0$
- Set Magnetostatic:  $\nabla \cdot \mathbf{B} = 0$

The 'Graphics' window shows a plot of the magnetic field distribution. The y-axis ranges from -0.04 to 0.05, and the x-axis ranges from -0.08 to -0.04. The plot shows a complex field distribution with several peaks and valleys.

**But what if I want to do a 3D model?  
Rewrite the Add-In? 🤔**

4

# Physics Builder

- Build your own physics interfaces, and use them just like standard COMSOL physics.

The screenshot shows the COMSOL software interface. The top menu bar includes File, Home, Definitions, Geometry, Materials, Physics, Mesh, Study, Results, and Developer. The 'New' dialog box is open, showing options for Model (Model Wizard, Blank Model) and Physics (Physics Builder). The 'Physics Builder' button is highlighted, and a blue arrow points to it from the text 'I have never seen that button!' with a surprised face emoji. The 'Preferences' dialog box is also open, showing the 'Physics Builder' preference checked under the 'Physics Builder' category. The 'Physics Builder' preference is highlighted, and the text 'Enable it!' with a sunglasses emoji is displayed next to it.

I have never seen that button! 🤖

Preferences

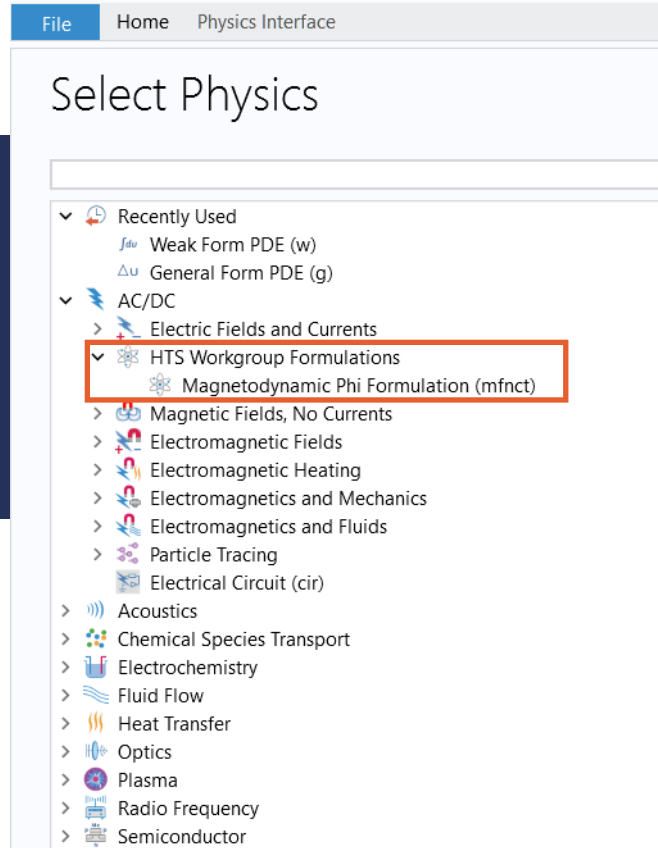
Physics Builder

Enable Physics Builder

**Enable it!** 😎

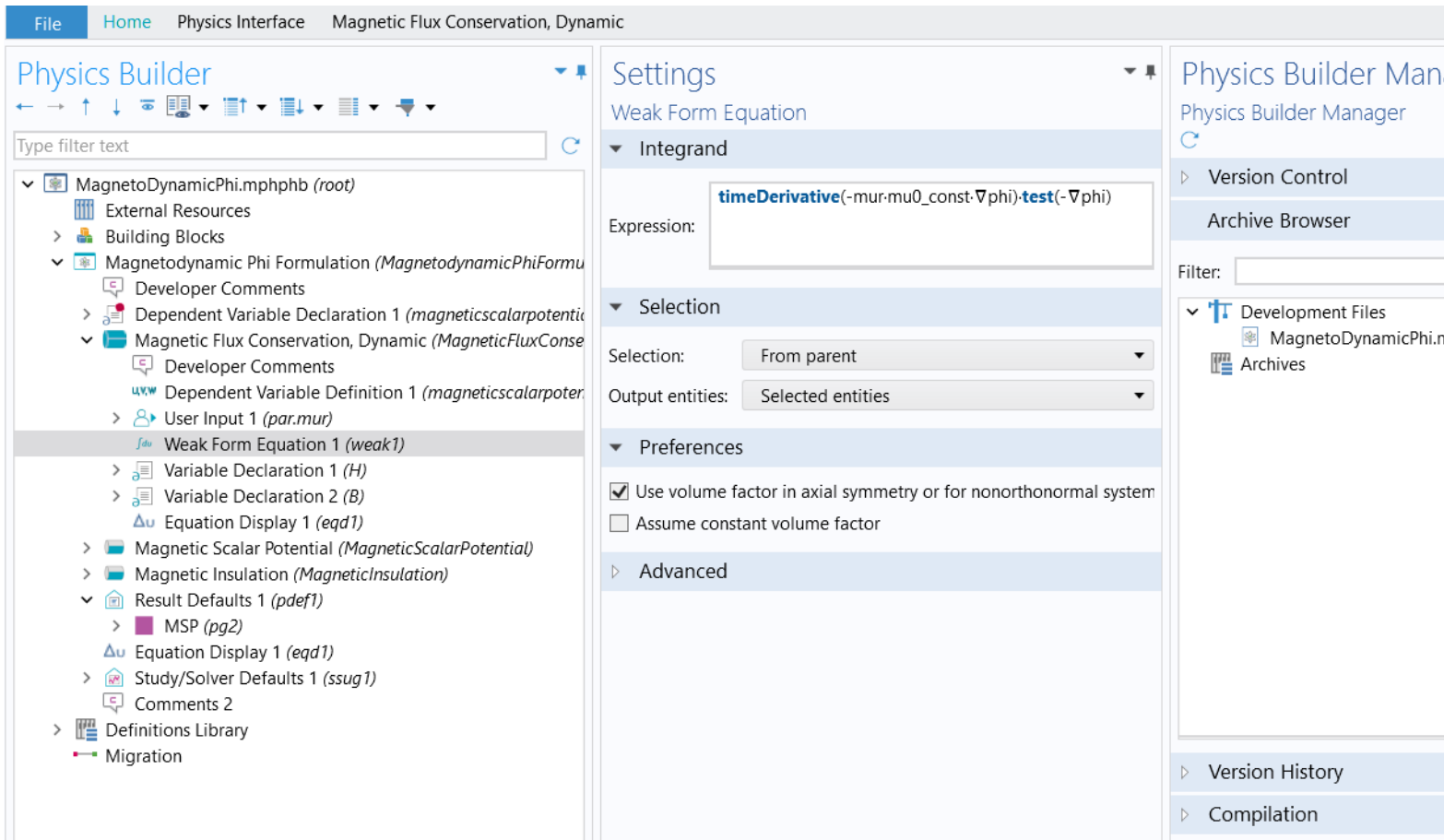
# Physics Builder

- Physics appear in the regular *Select Physics* dialog



- Definition of weak form in textbook notation.
- Works in all permitted space dimensions, including extra dimensions.
- Default domain and boundary conditions.
- Default solver settings.
- Default Plot groups.
- Version history.
- Compilation.

Try it! 



The screenshot displays the Physics Builder interface for a "Magnetic Flux Conservation, Dynamic" study. The left pane shows a tree view of the model structure, with "Weak Form Equation 1 (weak1)" selected. The right pane shows the settings for this equation, including the integrand expression, selection options, and preferences.

**Physics Builder**

File Home Physics Interface Magnetic Flux Conservation, Dynamic

Type filter text

- ▼ MagnetoDynamicPhi.mphpb (root)
  - External Resources
  - > Building Blocks
  - ▼ Magnetodynamic Phi Formulation (MagnetodynamicPhiFormu)
    - Developer Comments
    - > Dependent Variable Declaration 1 (magneticsscalarpotentic)
      - ▼ Magnetic Flux Conservation, Dynamic (MagneticFluxConse)
        - Developer Comments
        - uww Dependent Variable Definition 1 (magneticsscalarpoter)
          - > User Input 1 (par.mur)
            - Weak Form Equation 1 (weak1)**
              - > Variable Declaration 1 (H)
              - > Variable Declaration 2 (B)
              - Δu Equation Display 1 (eqd1)
              - > Magnetic Scalar Potential (MagneticScalarPotential)
              - > Magnetic Insulation (MagneticInsulation)
              - ▼ Result Defaults 1 (pdef1)
                - > MSP (pg2)
                - Δu Equation Display 1 (eqd1)
                - > Study/Solver Defaults 1 (ssug1)
                  - Comments 2
                - > Definitions Library
                - Migration

**Settings**

Weak Form Equation

▼ Integrand

Expression: `timeDerivative(-mur-mu0_const*∇phi)·test(-∇phi)`

▼ Selection

Selection: From parent

Output entities: Selected entities

▼ Preferences

Use volume factor in axial symmetry or for nonorthonormal system

Assume constant volume factor

▶ Advanced

**Physics Builder Manager**

Physics Builder Manager

▶ Version Control

Archive Browser

Filter:

▼ Development Files

- MagnetoDynamicPhi.n
- Archives

▶ Version History

▶ Compilation

# Take Home Messages

1

## Modify Built-in Eqns

Less need to reinvent the wheel, concentrate on your innovation.

2

## Record Methods

Design reusable *model methods* to avoid repetitive settings work *in one model*.

3

## Add-ins

Publish methods and user interfaces to be re-used in *several models*.

4

## Physics Builder

Design your own tailor-made physics interfaces that everybody can use and plug them into COMSOL.

Enjoy the next level of Multiphysics Modeling using *user-defined* formulations.

# Resources

- [11 h course: Modeling with Partial Differential Equations in COMSOL Multiphysics®](#)
- [Model Methods](#)
- [Add Ins](#)
- [Physics Builder](#)



**Who has the first question or comment?**



## Contact Us

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