Energy Integrating Ratio Analysis of Anomalous Precession Frequency in Fermilab Muon g-2 Experiment

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LTP Seminar

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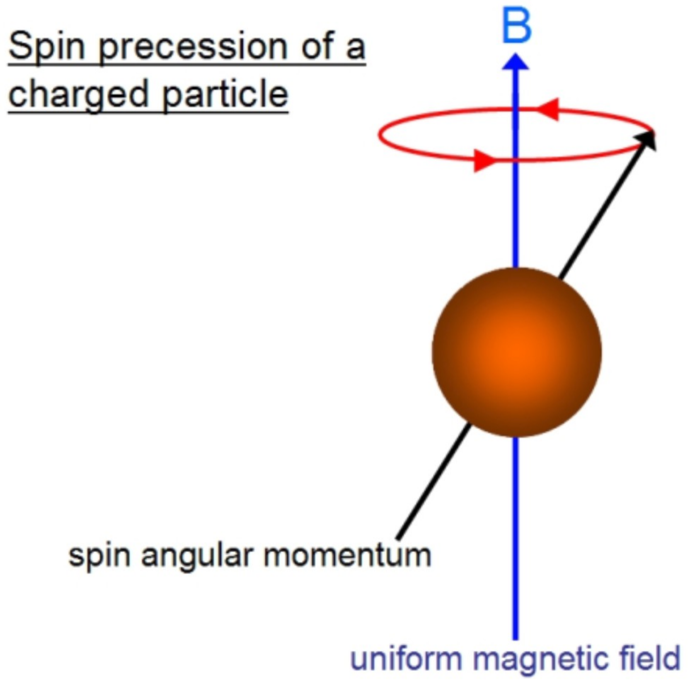
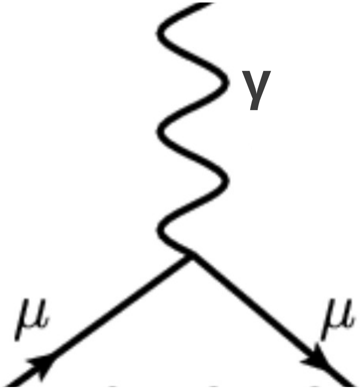
Motivation



$$\vec{\mu} = g\frac{q}{2m}\vec{S}$$



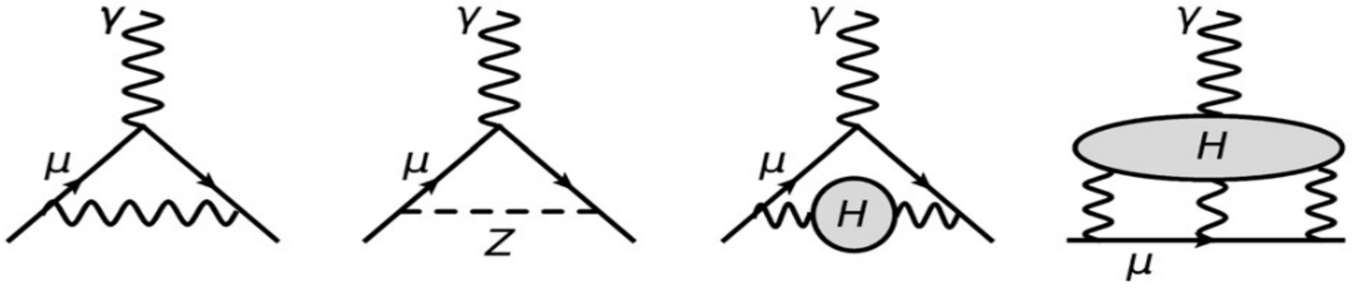
- g denotes the strength of spin coupling with external magnetic field
- Dirac theory → g = 2

Motivation

- But g ≠ 2 !

- Corrections from Quantum Electrodynamics (QED), Electro-weak (EW), Hadronic....

$$a_\mu = \frac{g-2}{2} > 0$$



QED EW Hadronic

- Experimental measurement of g-2 in BNL in 2001 with precision of 540 ppb

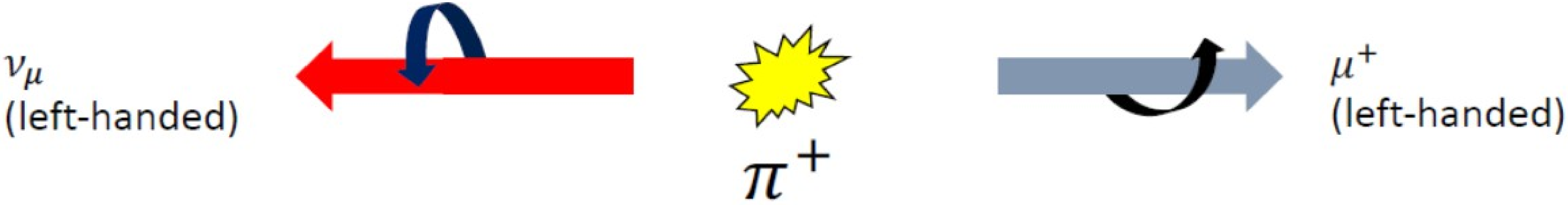
$$a_\mu = \frac{g-2}{2} = 11659208.0(6.3) \times 10^{-10}$$

- Greater than 3 σ discrepancy → Possible new physics? → More precise experimental measurement needed → Fermilab g-2 experiment with target precision of 140 ppb

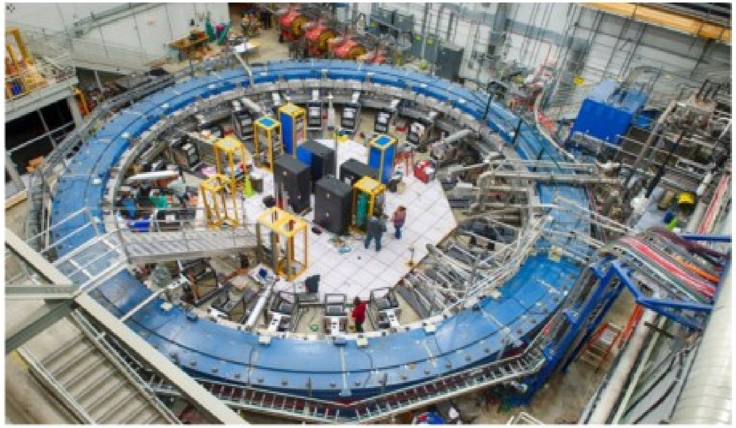
Fermilab Muon g-2 Experiment

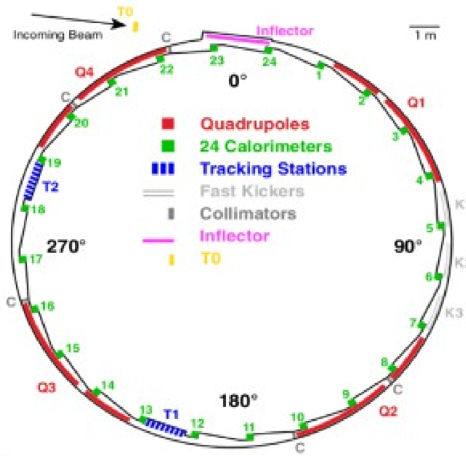
- An 8 GeV proton beam is collided on a fixed target to produce pion
- Pions produce longitudinally polarized muons via parity violating weak decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

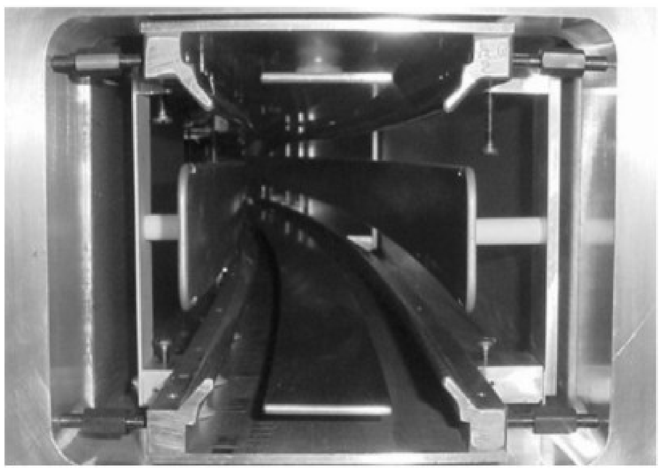


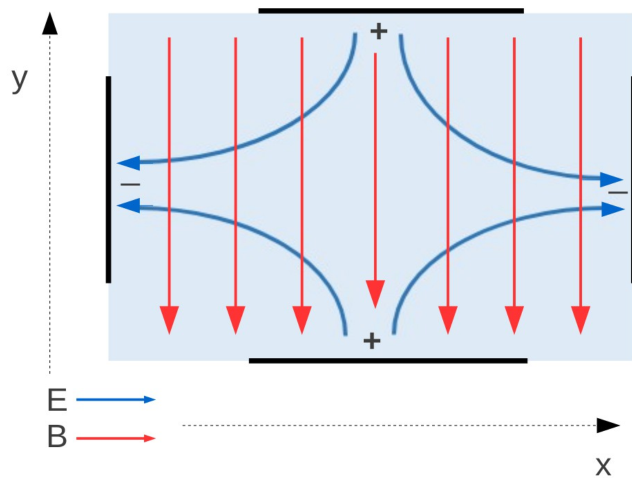
Experimental Set-up





Top view



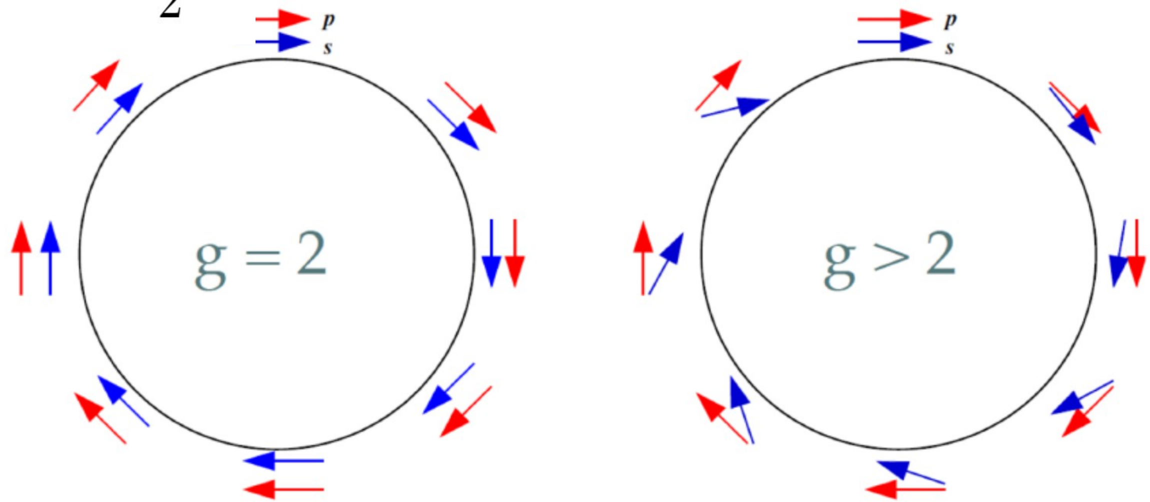


Cross-sectional view

- Muons are stored in a vertical B-field with a ESQ (Electro-static Quadrupole) system

Measurement Principle

- If g = 2, cyclotron and spin precession frequency are equal to each other
- The $a_\mu = \frac{g-2}{2}$ is responsible for the anomalous spin precession $\overrightarrow{\omega_a}$



$$\vec{\omega_a} = \vec{\omega_s} - \vec{\omega_c} = -\frac{q}{m_\mu}[a_\mu \vec{B} - a_\mu(\frac{\gamma}{\gamma+1})(\vec{\beta}.\vec{B})\vec{\beta} - (a_\mu - \frac{1}{\gamma^2-1})\frac{\vec{\beta}\times\vec{E}}{c}]$$

[1] [2]

- [1] ~ 0 since $\overrightarrow{\beta}\,.\,\overrightarrow{B} = 0$
- We can make [2] → 0, by selecting $\gamma = \sqrt{1 + \frac{1}{a_\mu}} \approx 29.3$
- So, only need to measure $\overrightarrow{\omega_a}$ and $\overrightarrow{B}$ in the experiment

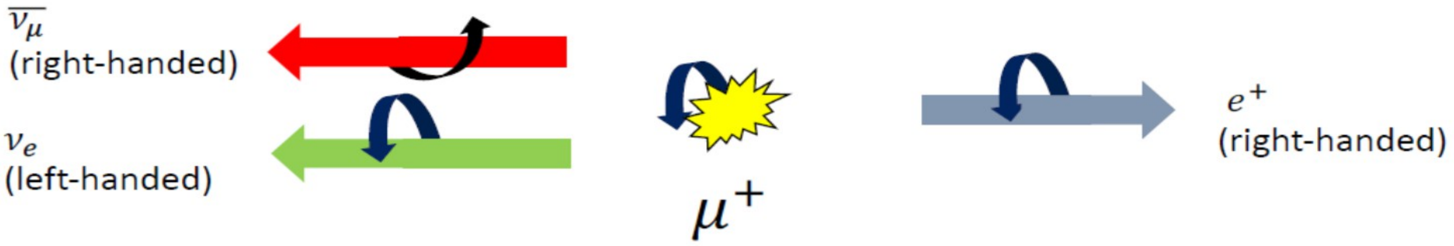
Measurement Principle

$\overrightarrow{\omega_a}$ measurement

- In the storage ring the muons decay into positrons

$$\mu^+ \rightarrow \bar{\nu_\mu} + e^+ + \nu_e$$

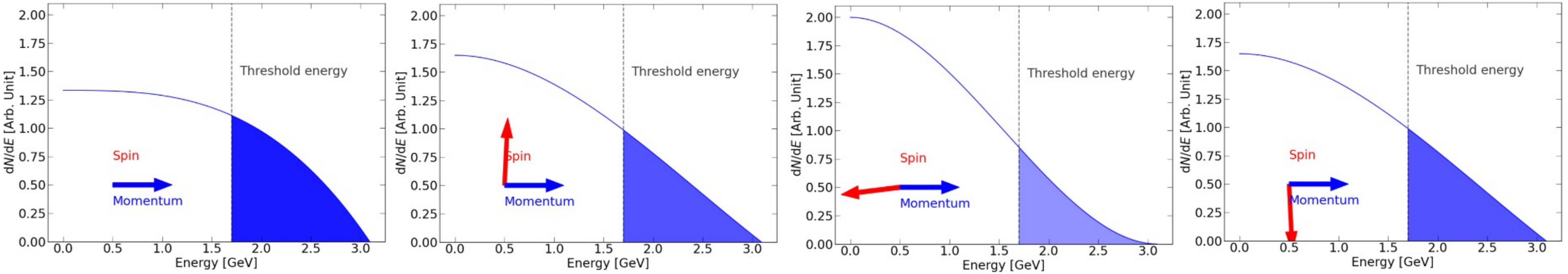
- Positrons carry information about muon spin direction due to parity violating weak decay



(In muon rest frame)

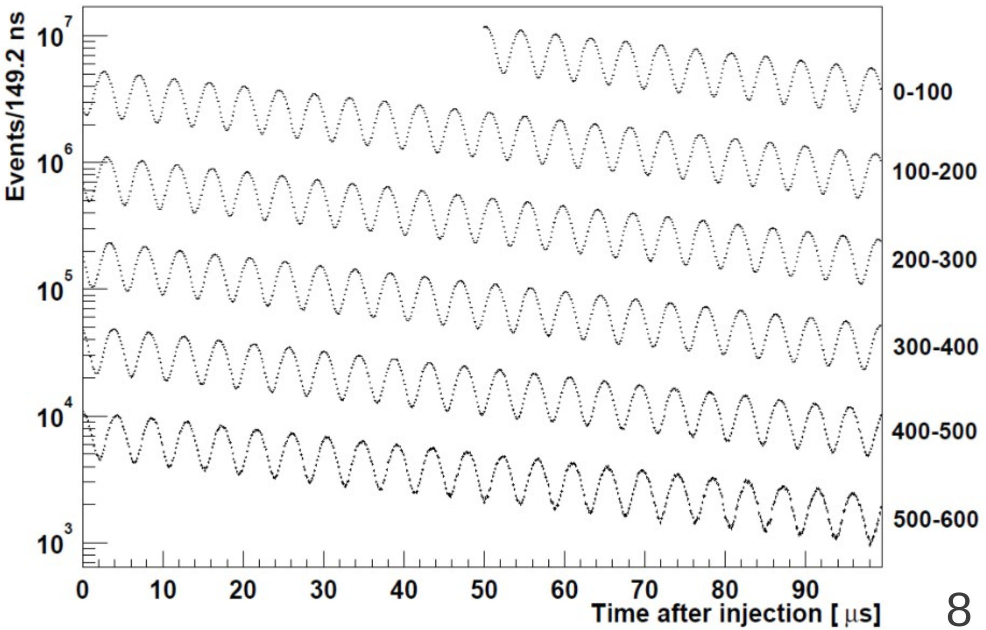
Measurement Principle

- In lab frame, more high-energy positrons emitted when spin parallel to momentum

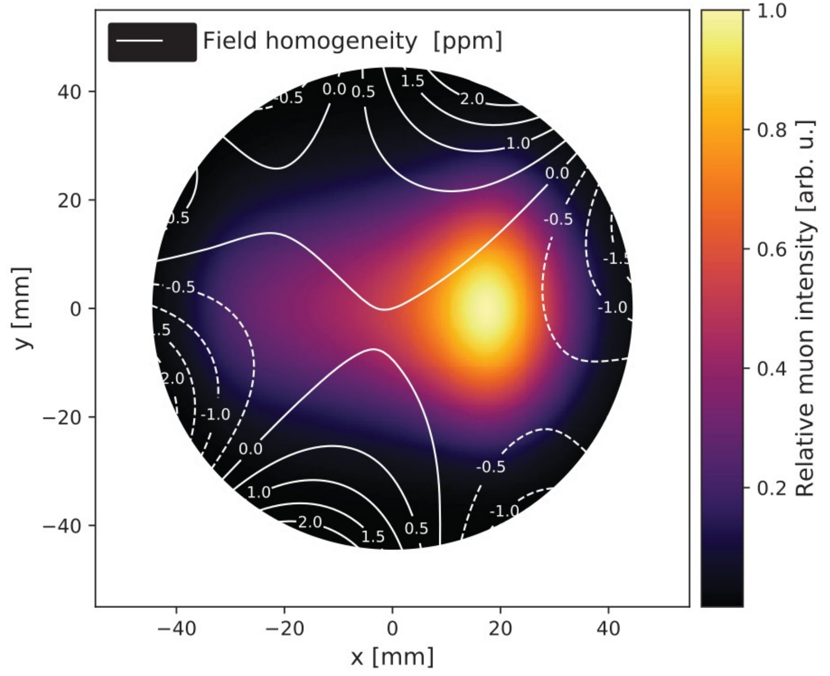


- Count of high energy positrons above a threshold is modulated by $\vec{\omega_a}$

$$N(t) = N_0 e^{-t/\gamma\tau_\mu}(1 + A(E_{\rm th})\cos(\omega_a t + \phi_0))$$



Measurement Principle



<u>B-field measurement</u>

- Pulsed proton NMR
- Precession frequency of proton related to B-field

$$\hbar\omega'_p = 2\mu_p B$$

- After some rearrangement

$$a_\mu = \frac{\omega_a}{\tilde{\omega}_p}\frac{m_\mu}{m_e}\frac{\mu_p}{\mu_e}\frac{g_e}{2}$$

22 ppb 3 ppb 0.2 ppt

- All other quantities are measured experimentally with high precision

ω_a Analysis

- Count of high energy positrons modulated by ω_a
- BNL experiment only used the positron counting method

- Positrons deposit energy on EM calorimeters
- 9X6 PbF_2 crystal array
- produce Cherenkov shower

→

- SiPMs detect the photons
- Produce voltage signals → digitized at rate of 800 MSPS
- Recorded over a fill (700 μs)

→ 40 μs islands neigh-boring pulses with energy above 50 MeV for all crystals, are saved by the DAQ: **Positron Counting (threshold) Method** or **T-Method**

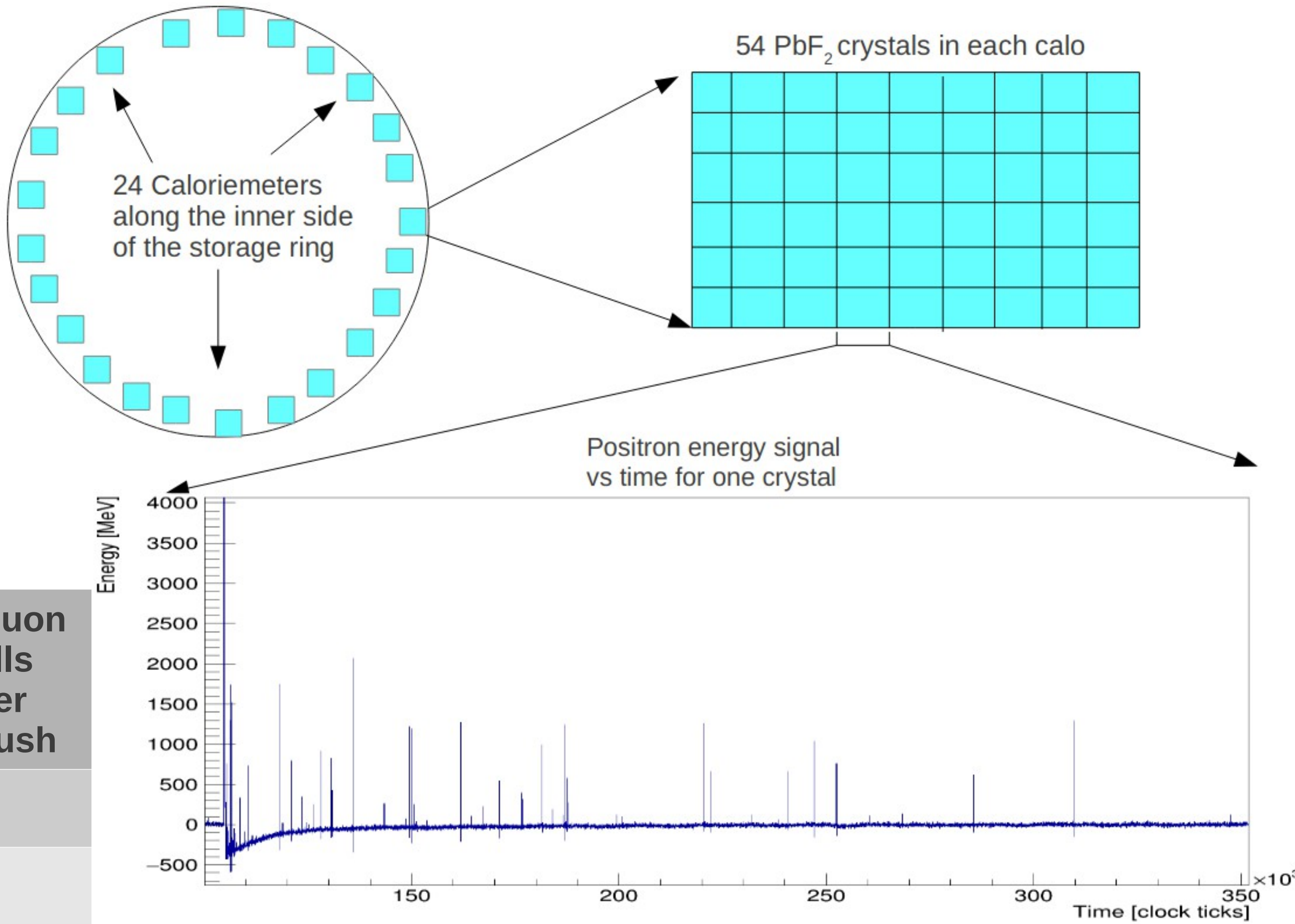
→ Continuous streams of digitized signal are saved over a muon fill in time bins of fixed width: **Energy Integrating Method** or **Q-Method**

- Energy of the positrons also modulated by ω_a
- Integrated energy time spectrum can also be used to determine ω_a

Energy Integrating Analysis of ω_a

- Energy Integrating method or **Q-method** uses continuous digitized waveforms
- To manage stored data memory usage, the end time of Q-method fill less than T-method
- Clock-tick time bins are decimated by some factor
- Several fills are summed together into a flush

Runs	Start time (μs)	End time (μs)	Time deci-matio -n	Muon fills per flush
1	-6	231	60	1
2-3	-6	309	15	4
4-6	-6	556	30	4



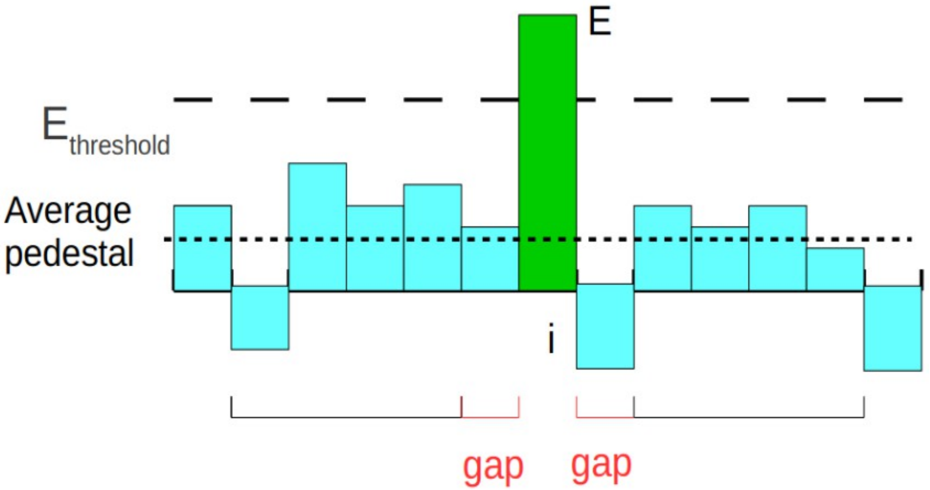
1 clock-tick ~ 1.25 ns

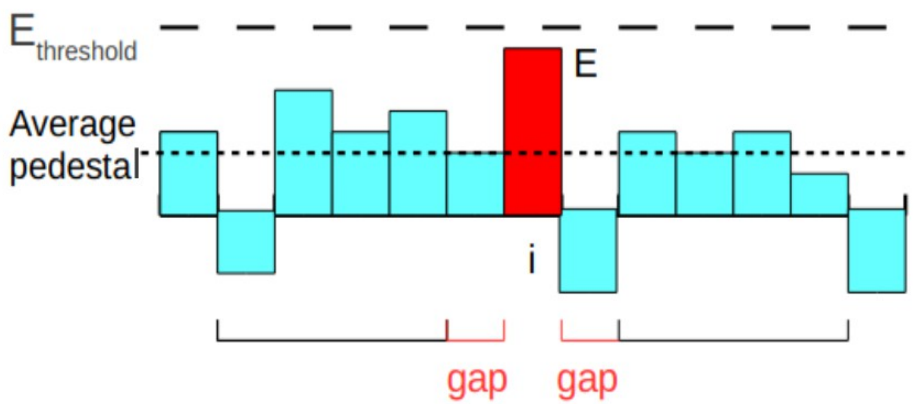
Q-Method Pedestal Subtraction

- Background drift effects and noise should be mitigated for unbiased ω_a determination.
- Average pedestal calculation, window and gap

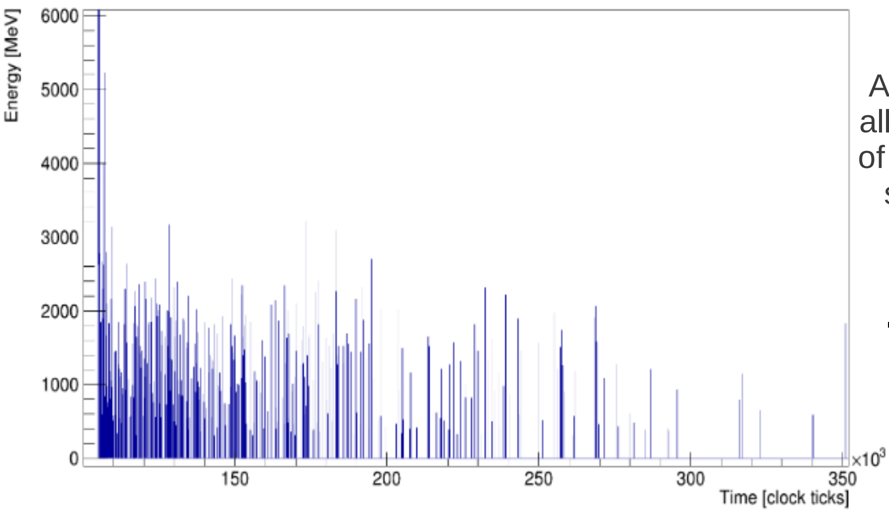
  Run 1 → 4 bins with 1 bin gap both sides
  Run 2-3 → 8 bins with 1 bin gap both sides

- Compare against small threshold ~ 300 MeV
- Reject above threshold pulse from average pedestal calculation



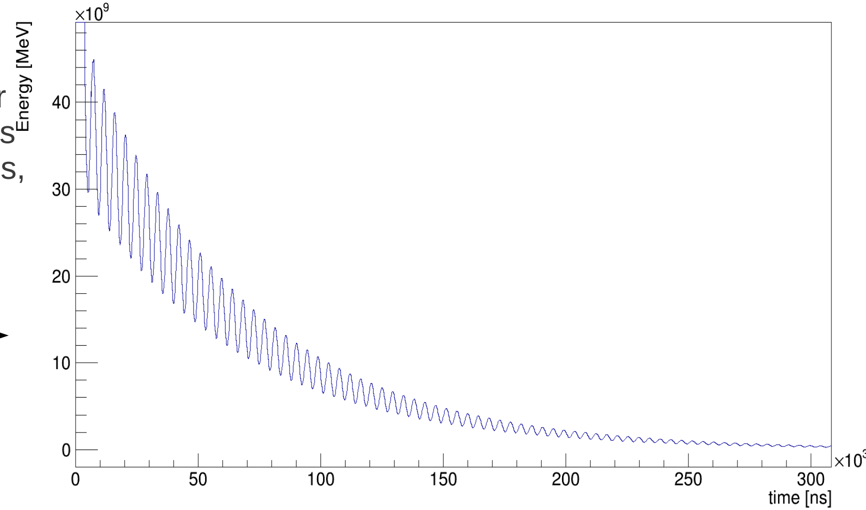


Q-Method Time Spectrum



Add together all the crystals of all the calos, stack many muon fills together

1 clock-tick ~ 1.25 ns



Bin content of each Q-method time bin

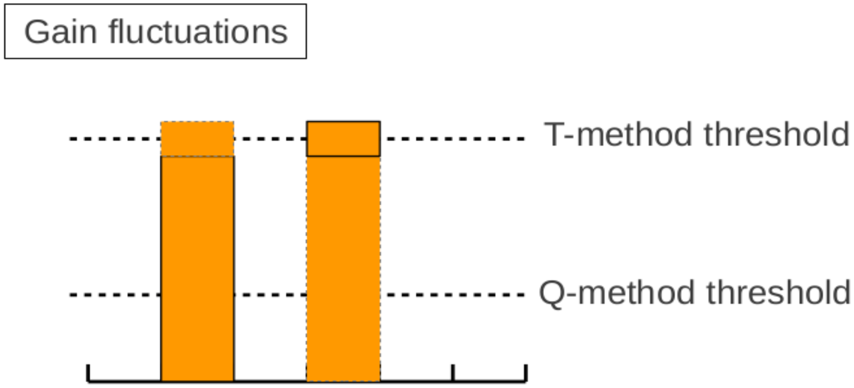
$$E_{total} = n_1 E_1 + n_2 E_2 + n_3 E_3 + ...$$

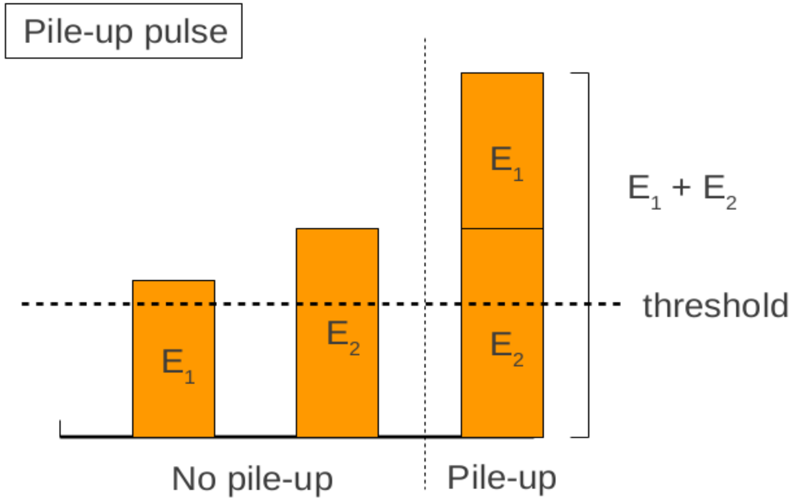
Uncertainty assigned

$$\Delta E_{total} = \sqrt{(E_1)^2 + (E_2)^2 + (E_3)^2} + ...$$

Q-Method Analysis: Advantages

- Different sensitivity to systematic effects
- Gain: Fluctuations in signal amplitude due to detector effects. Low threshold of Q-method → less sensitivity
- Pile-up: Two or more positrons events miscounted as one. T-method signal distortion, Q-method records total energy → no distortion
- Despite being statistically less powerful, different sensitivity to gain/pile-up → important cross-check of ω_a



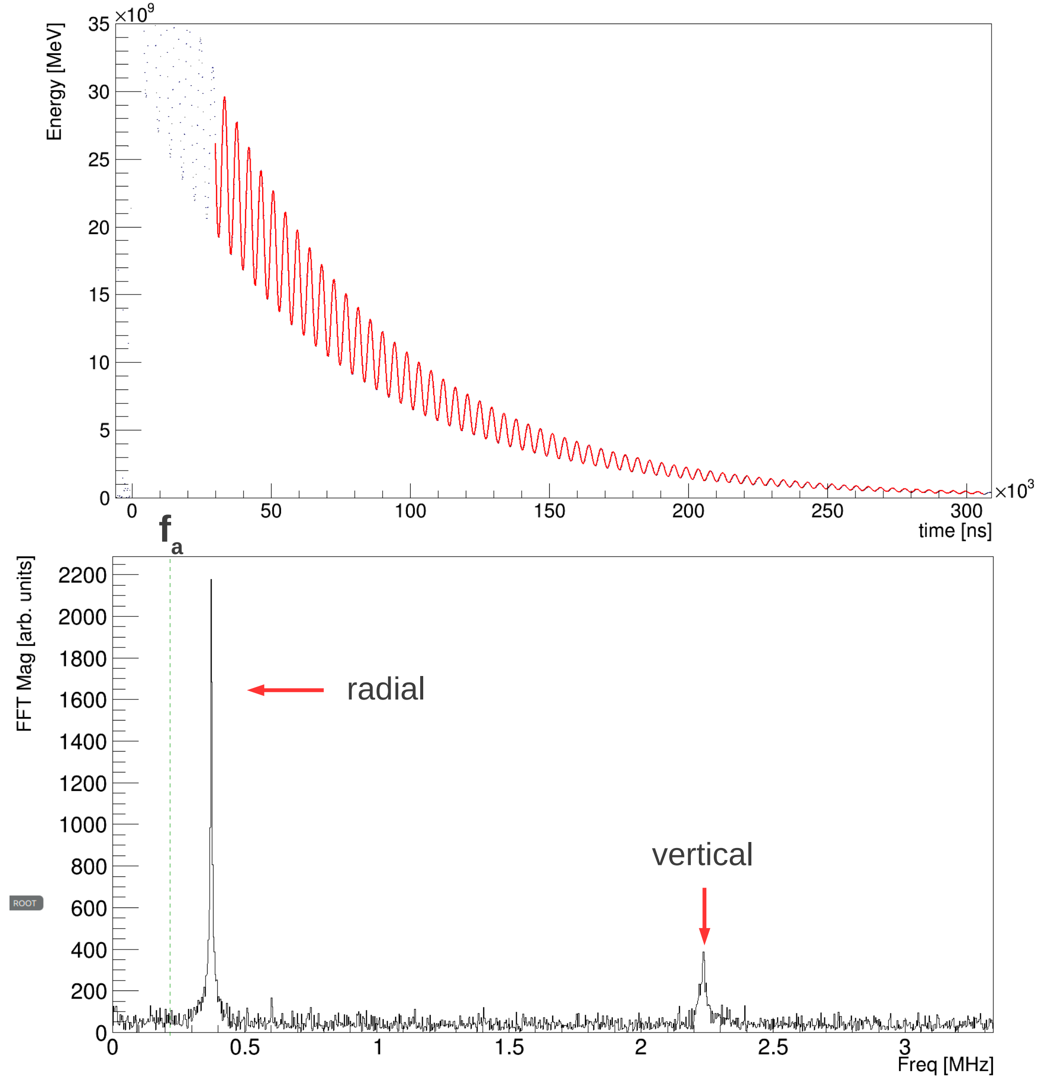


Q-Method Analysis: Fitting

- Fit function

$$f(t) = N_0 e^{\frac{-t}{\gamma\tau}}(1 + Acos(\omega_a t - \phi))$$

- Fit range → 30 μs to 305 μs
- FFT of data-fit shows peaks at ~ 0.37 MHz and ~ 2.22 MHz
- Beam oscillation in radial and vertical direction
- Modulation by cyclotron frequency



Q-method Full Fit-function

- Updated function to fit Q-method histogram:

$$f(t) = N_0 N_{cbo}(t) N_{vw}(t) N_y(t) \Lambda(t) e^{\frac{-t}{\tau}} (1 + A_0 A(t) cos(\omega_a t - \phi_0 - \phi(t)))$$

where

Radial beam oscillation (normalization)

$$N_{cbo}(t) = 1 + A_{cbo_N} e^{\frac{-t}{\tau_{cbo}}} cos(\omega_{cbo} t - \phi_{cbo_N}) + A_{2cbo_N} e^{\frac{-2t}{\tau_{cbo}}} cos(2\omega_{cbo} t - \phi_{2cbo_N})$$

Vertical beam Oscillation (normalization)

$$N_{vw}(t) = 1 + A_{vw} e^{\frac{-t}{\tau_{vw}}} cos(\omega_{vw} t - \phi_{vw})$$

$$N_y(t) = 1 + A_y e^{\frac{-t}{\tau_y}} cos(\omega_y t - \phi_y)$$

Radial beam oscillation (asymmetry and phase)

$$A(t) = 1 + A_{cbo_A} e^{\frac{-t}{\tau_{cbo}}} cos(\omega_{cbo} t - \phi_{cbo_A})$$

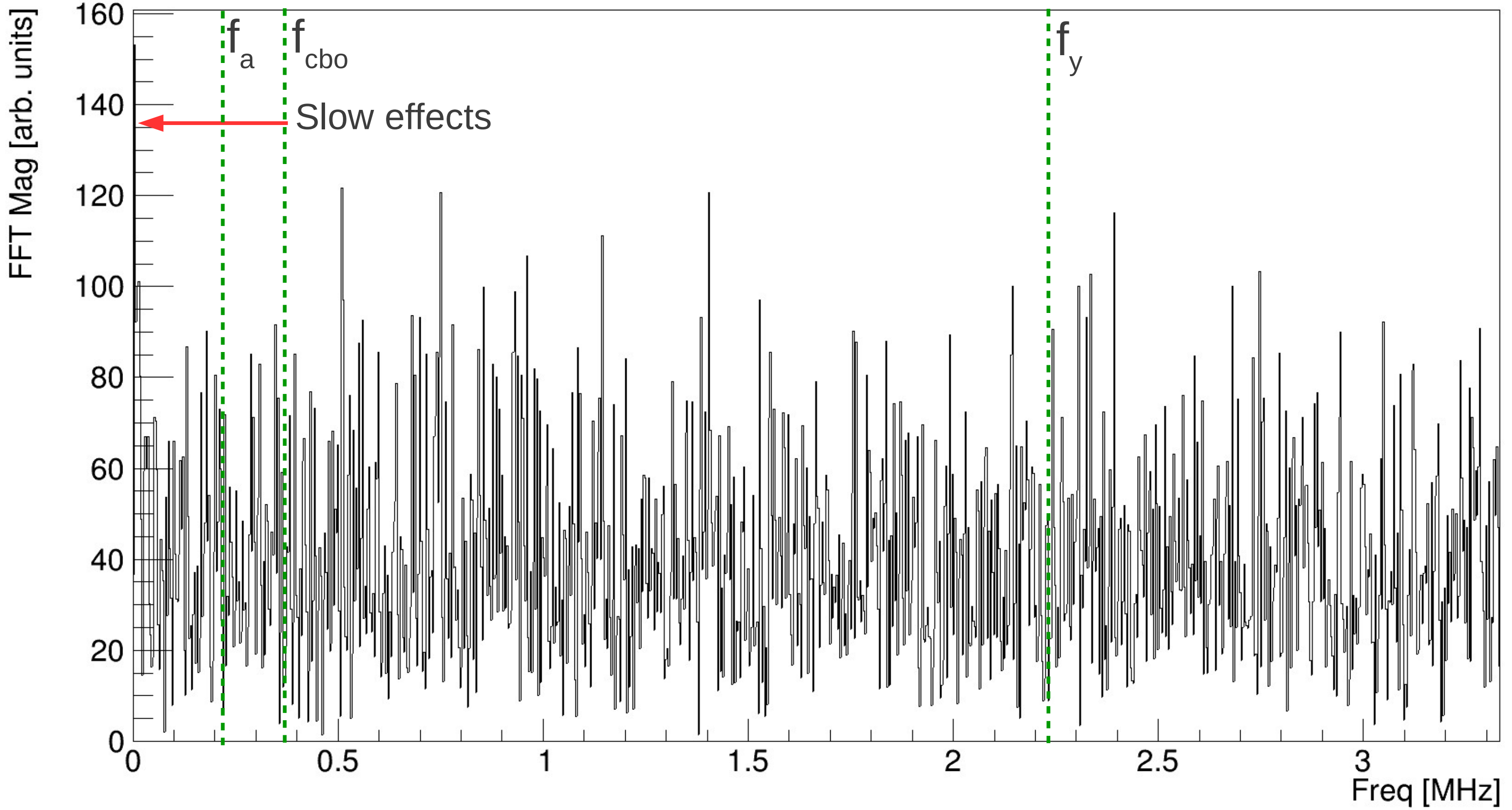
$$\phi(t) = A_{cbo_\phi} e^{\frac{-t}{\tau_{cbo}}} cos(\omega_{cbo} t - \phi_{cbo_\phi})$$

Muon loss

$$\Lambda(t) = 1 - \kappa_{loss} \int_0^t L(t') e^{\frac{t'}{\tau}} dt'$$

Fourier Transform

Updated fit-function fit FFT:



- In Run 1 slow effects accounted for 300 ppb systematic error in Q-method result

Ratio Method Analysis

- Create 4 histograms from reconstructed data

$$v_1 = v_2 = N(t)$$ $$u_- = N(t - \frac{T_a}{2}), u_+ = N(t + \frac{T_a}{2})$$

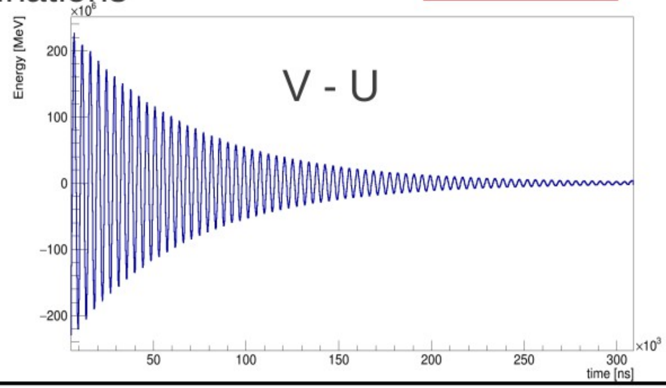
- Move 2 of these earlier and later in time by ½ of anomalous frequency period, T_a

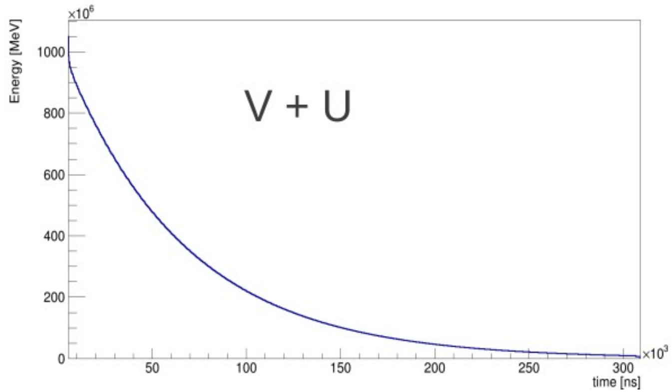
where $N(t) = N_0 e^{\frac{-t}{\gamma\tau}}(1 + A cos(\omega_a t - \phi))$

- Make combinations

$V = v_1 + v_2$ and $U = u_+ + u_-$

$$R = \frac{V - U}{V + U} =$$





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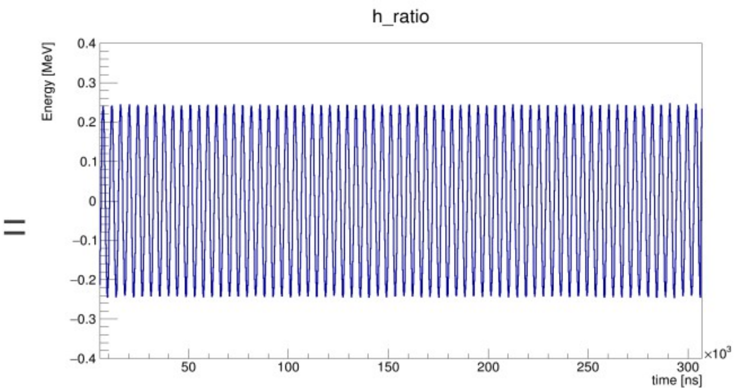
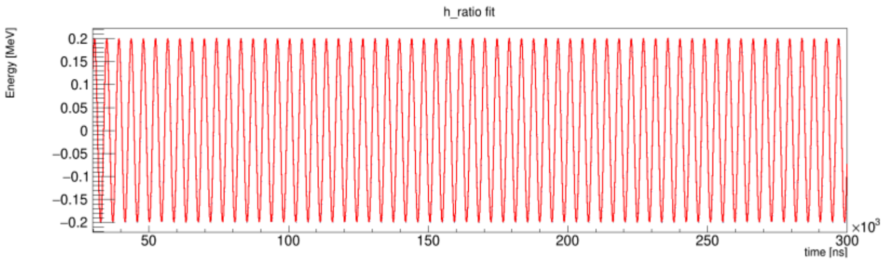


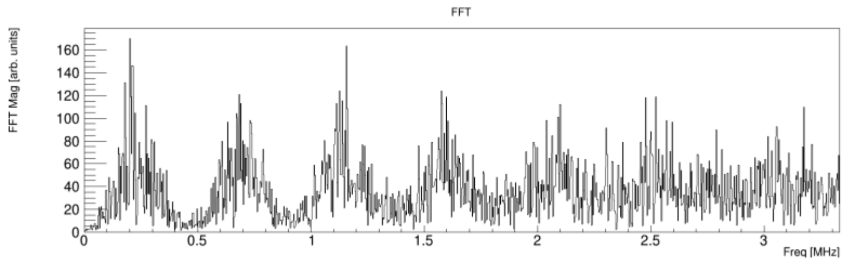
This eliminates the need for having a term for slow effects in fit function

Ratio Method Analysis

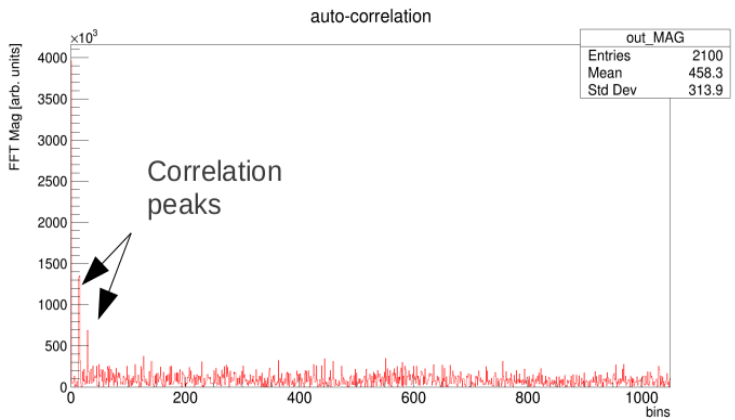
Constructing Ratio histograms

Making 4 copies of same histogram

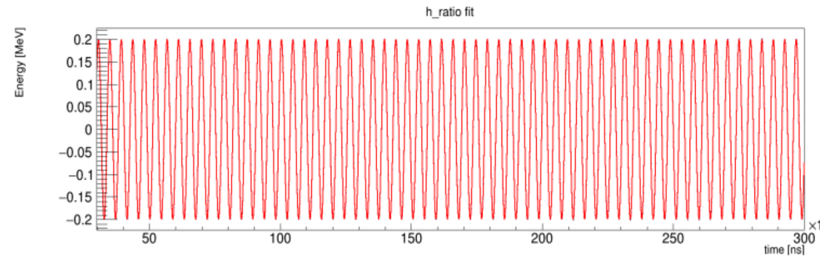


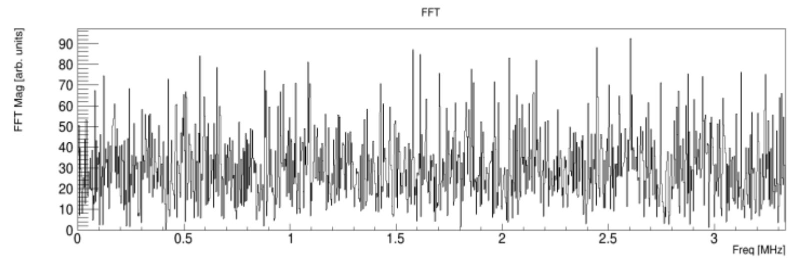


- Introduces bin-to-bin correlation which complicates fitting

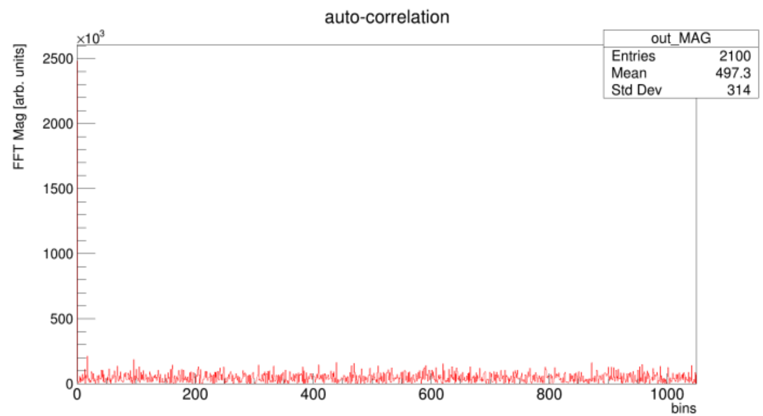


Randomly splitting data into 4 subsets





- Introduces noise which requires running over many random seeds



Covariance Matrix Calculation

Copy Ratio Method: Covariance matrix calculation:

- Each bin in ratio histogram has contribution from neighboring bins:

$$y_{R_i} = \frac{2y_i - y_{i+\delta} - y_{i-\delta}}{2y_i + y_{i+\delta} + y_{i-\delta}}$$

- Correlated bins are i±δ and i±2δ, δ is $T_a/2$ in Q-method time bins

- Need to calculate expectation values, not trivial for ratio fucntion

$$cov(y_i, y_j) = E(y_i y_j) - E(y_i)E(y_j)$$

- Used Taylor expansion of the ratio bin-content about the true mean of it's constituent bin content for calculation of $E(y_i)$, $E(y_j)$ and $E(y_i y_j)$

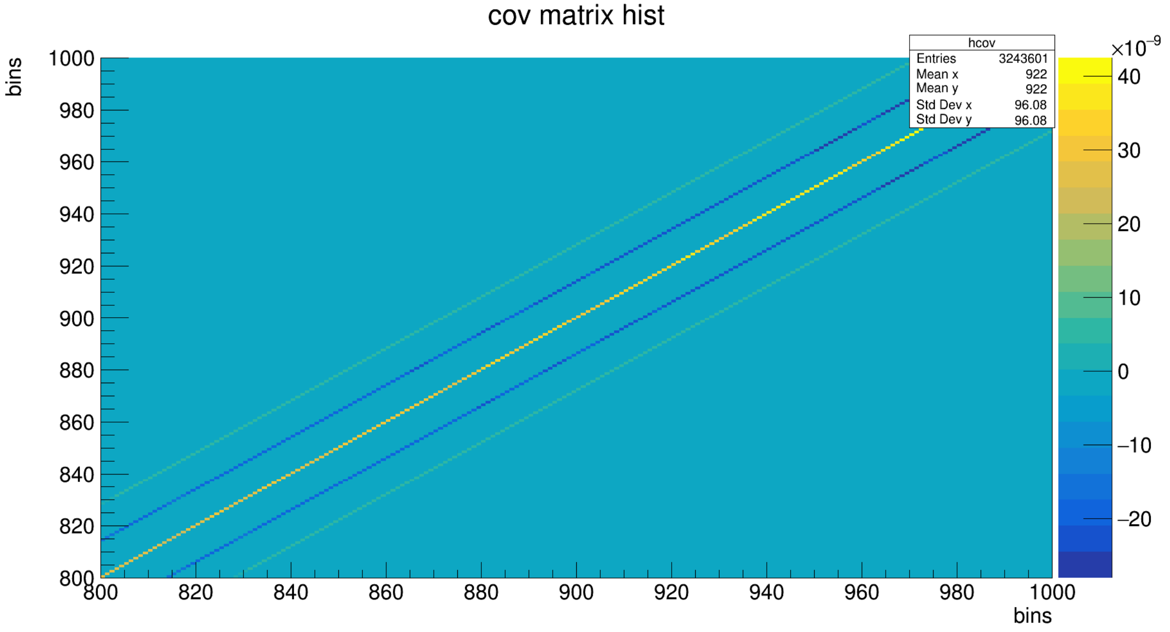
$$\begin{aligned}
E(y_{R_i}) = & E(y_{R_i}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}} + \frac{\partial y_{R_i}}{\partial y_i}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_i - \mu_{y_i}) + \frac{\partial y_{R_i}}{\partial y_{i+\delta}}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_{i+\delta} - \mu_{y_{i+\delta}}) \\
& + \frac{\partial y_{R_i}}{\partial y_{i-\delta}}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_{i-\delta} - \mu_{y_{i-\delta}}) + \frac{1}{2!}[\frac{\partial^2 y_{R_i}}{\partial y_i^2}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_i - \mu_{y_i})^2 \\
& + \frac{\partial^2 y_{R_i}}{\partial y_{i+\delta}^2}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_{i+\delta} - \mu_{y_{i+\delta}})^2 + \frac{\partial^2 y_{R_i}}{\partial y_{i-\delta}^2}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_{i-\delta} - \mu_{y_{i-\delta}})^2 \\
& + 2\frac{\partial^2 y_{R_i}}{\partial y_i \partial y_{i+\delta}}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_i - \mu_{y_i})(y_{i+\delta} - \mu_{y_{i+\delta}}) \\
& + 2\frac{\partial^2 y_{R_i}}{\partial y_{i+\delta} \partial y_{i-\delta}}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_{i+\delta} - \mu_{y_{i+\delta}})(y_{i-\delta} - \mu_{y_{i-\delta}}) \\
& + 2\frac{\partial^2 y_{R_j}}{\partial y_{i-\delta} \partial y_i}|_{\mu_{y_i},\mu_{y_{i+\delta}},\mu_{y_{i-\delta}}}(y_{i-\delta} - \mu_{y_{i-\delta}})(y_i - \mu_{y_i})])
\end{aligned}$$

Fitting with Covariance Matrix

- Fitting histogram with bin-to-bin correlation
- Chi-squared function for minimization:

$$\chi^2 = \sum_{i,j} [yR_i - f(x_i)][cov]_{ij}^{-1}[yR_j - f(x_j)]$$

- Non-zero covariances are at diagonal ±δ and ±2δ



Ratio Q-Method Fit-function

Function for fitting ratio histogram:

$$F(t) = \frac{2f(t) - f(t+\frac{T_a}{2}) - f(t-\frac{T_a}{2})}{2f(t) + f(t+\frac{T_a}{2}) + f(t-\frac{T_a}{2})}$$

where $$f(t) = N_{cbo}N_{vw}N_y(1 + A(t)cos(\omega_a t - \phi_0 - \phi(t)))$$

with

Radial beam Oscillation (normalization)

$$N_{cbo} = 1 + A_{cbo_N} e^{-\frac{t}{\tau_{cbo}}} \cos(\omega_{cbo} t - \phi_{cbo_N}) + A_{2cbo_N} e^{-\frac{2t}{\tau_{cbo}}} \cos(2\omega_{cbo} t - \phi_{2cbo_N})$$

Vertical beam Oscillation (normalization)

$$N_{vw} = 1 + A_{vw} e^{-\frac{t}{\tau_{vw}}} \cos(\omega_{vw} t - \phi_{vw})$$

$$N_y = 1 + A_y e^{-\frac{t}{\tau_y}} \cos(\omega_y t - \phi_y)$$

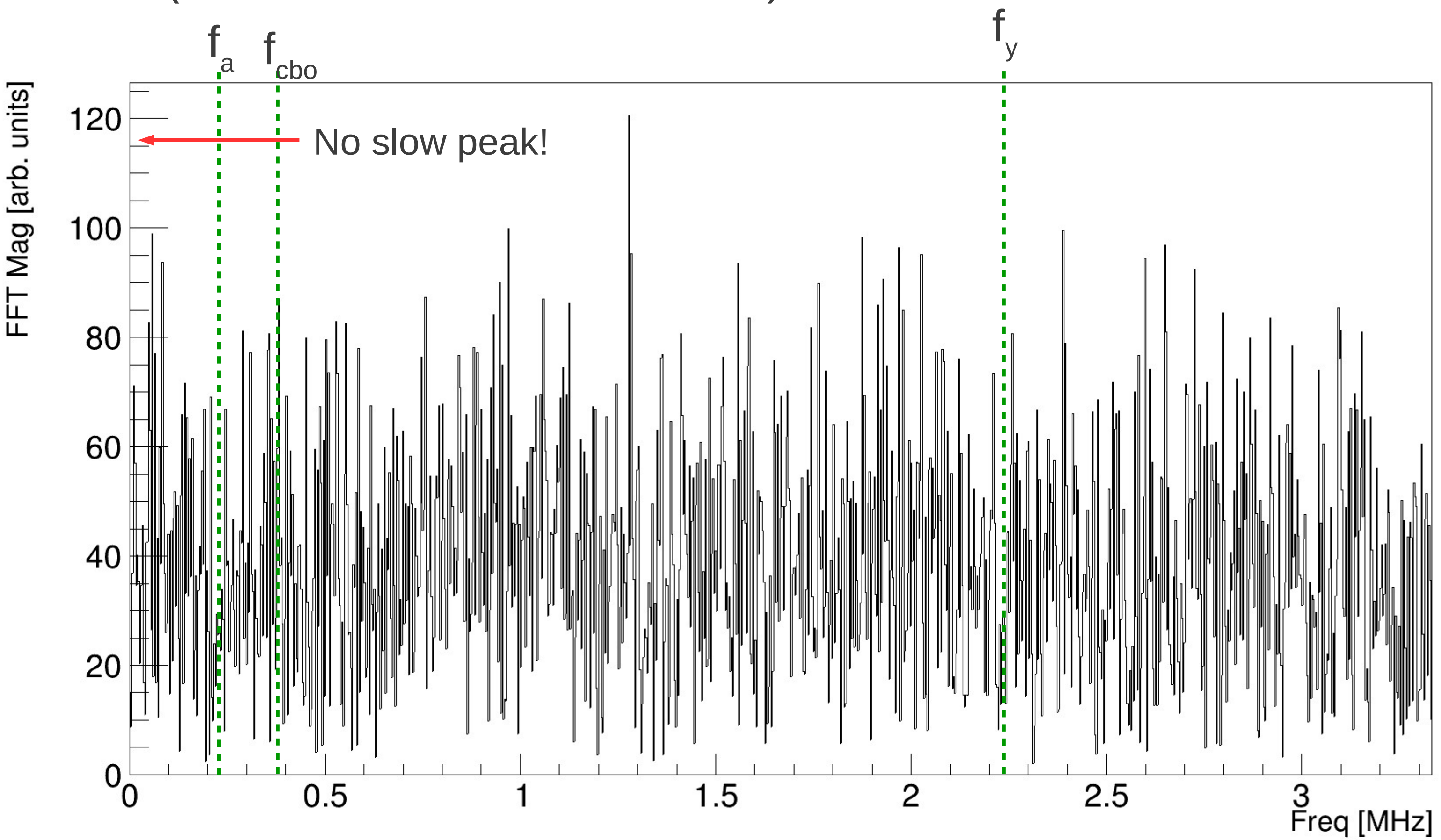
Radial beam Oscillation (asymmetry And phase)

$$A(t) = A_0(1 + A_{cbo_A} e^{-\frac{t}{\tau_{cbo}}} \cos(\omega_{cbo} t - \phi_{cbo_A}))$$

$$\phi(t) = A_{cbo_\phi} e^{-\frac{t}{\tau_{cbo}}} \cos(\omega_{cbo} t - \phi_{cbo_\phi})$$

Ratio Q-Method Fourier Transform

Ratio fit FFT (randomized ratio construction):

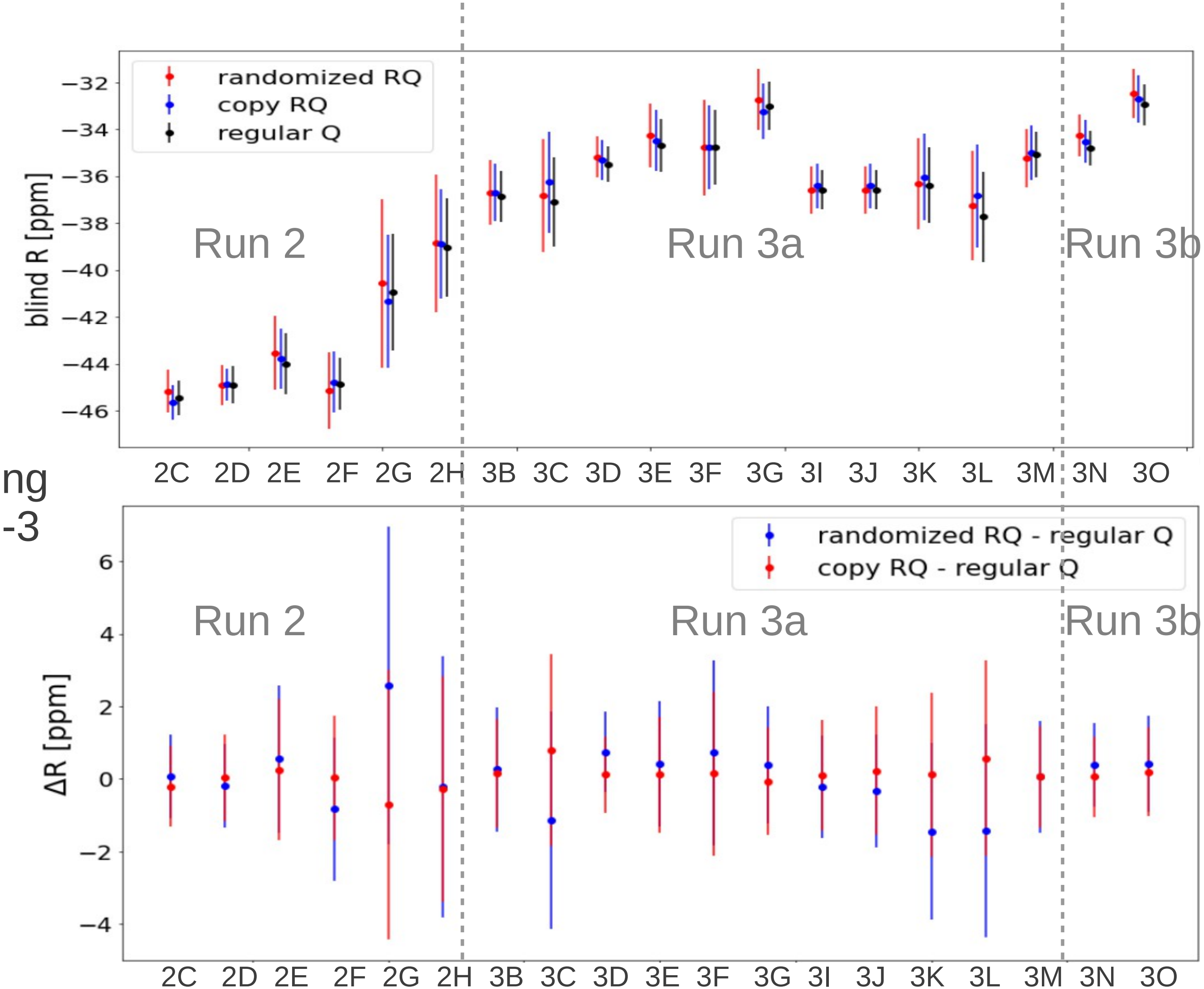


Ratio Method Analysis : Central Fit Results

Blinded R versus Run 2 and 3 datasets for Regular Q-method, and Ratio Q-method (copy and randomized)

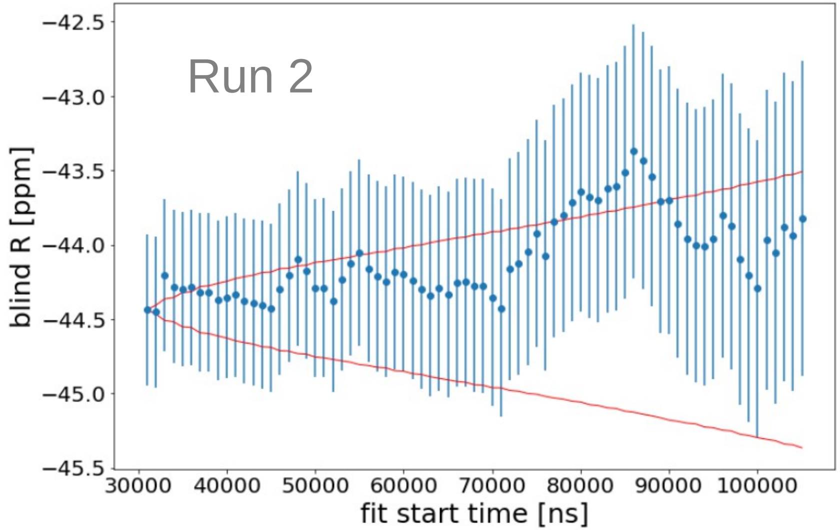
Different hardware blinding between Run-2 and Run-3

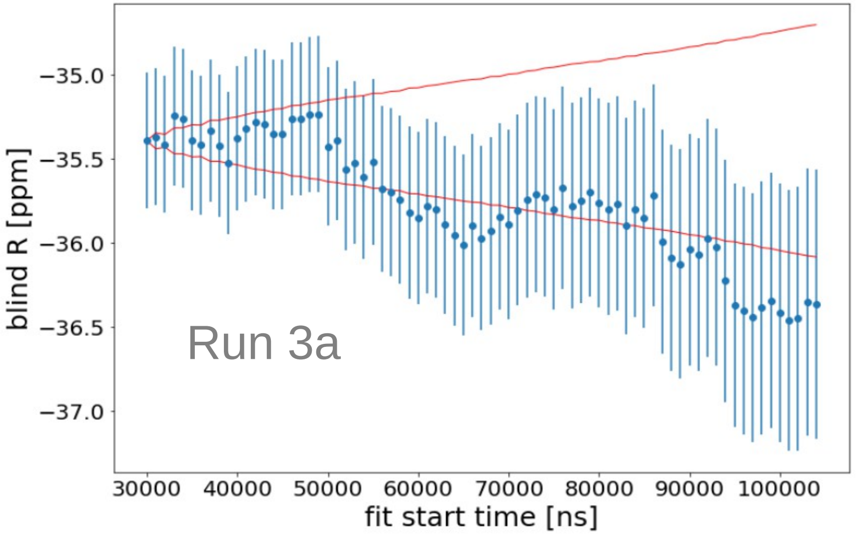
Difference between Regular Q-method and Ratio Q-method blinded R values

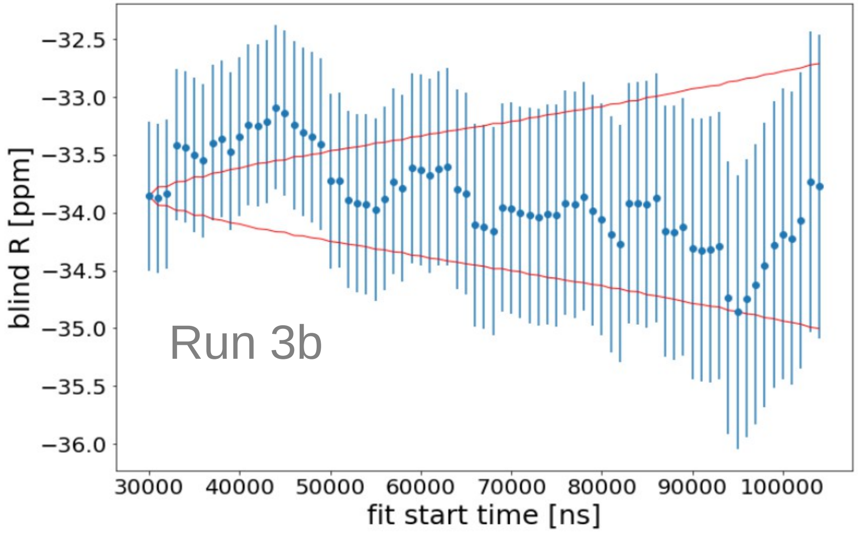


Ratio Q-Method Analysis : Start time Scan

- R versus Fit start times
- Start time → 30 to 105 μs
- Red band → 1 σ

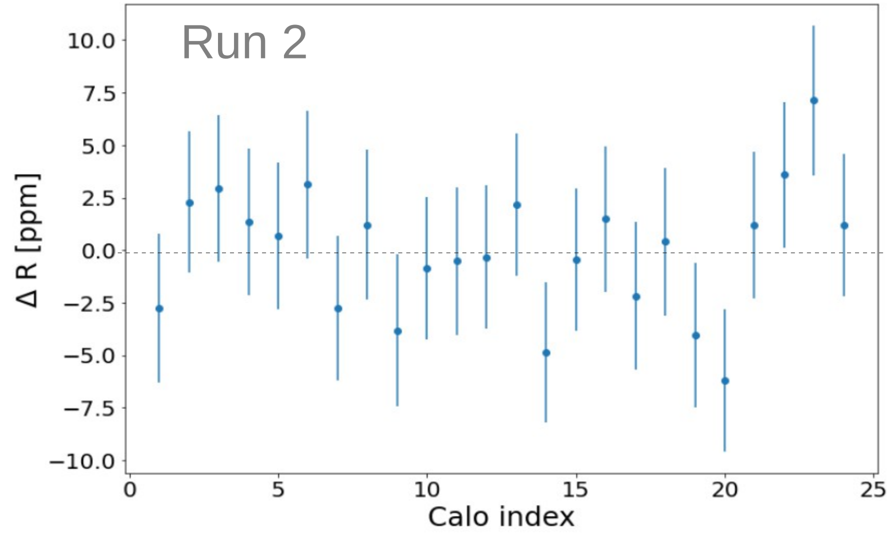


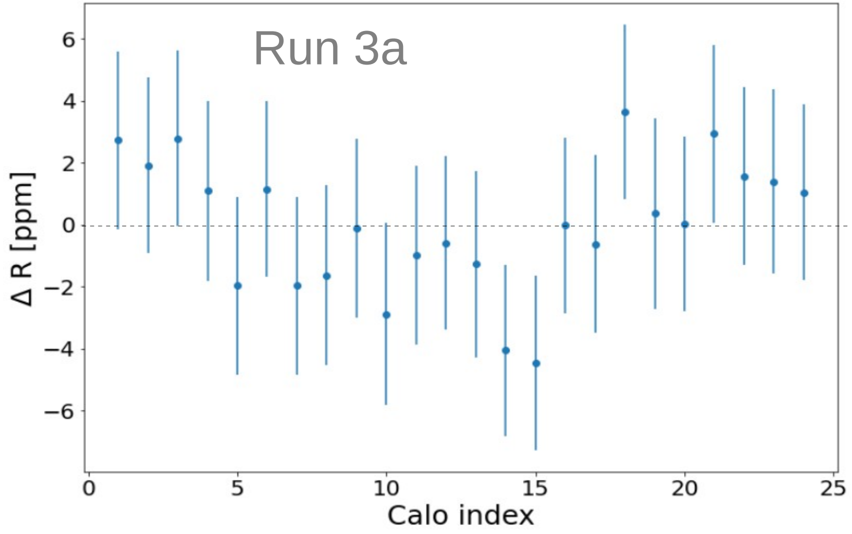


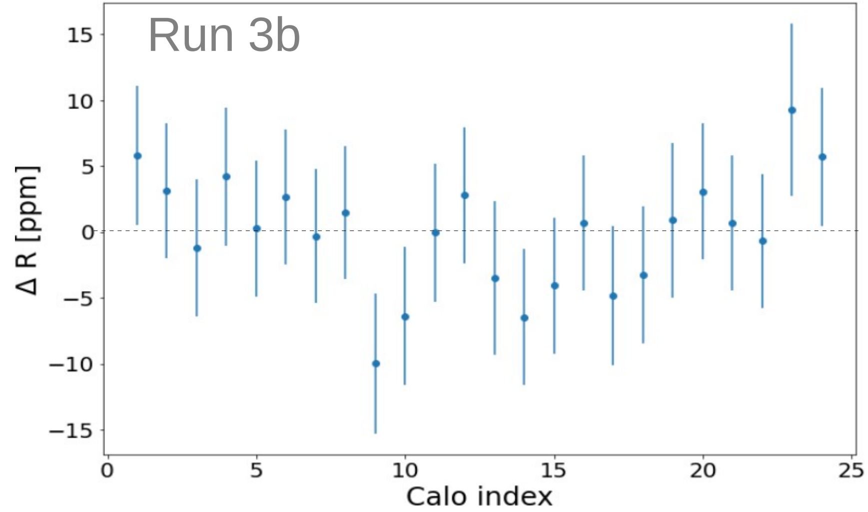


Ratio Q-Method Analysis: Calorimeter Scan

- Blinded ω_a versus calorimeters
- Difference between the average R versus calorimeter indices



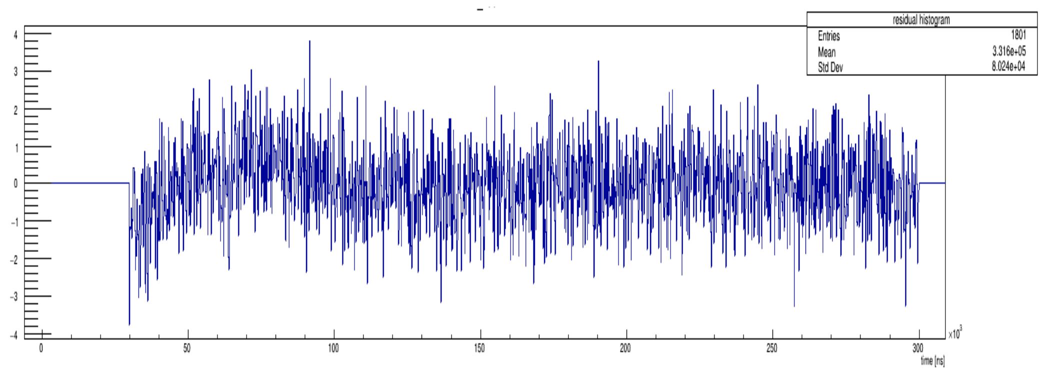




Systematic Uncertainties: Slow drift

- Early to late slow effect, present in fit residual
- Run-1 contribution from broken ESQ resistors
- Q-method more susceptible
- Empirical functional form:

$$A\, e^{-t/Ta} + B\, e^{-t/Tb}$$



Run 1c: Data – fit function

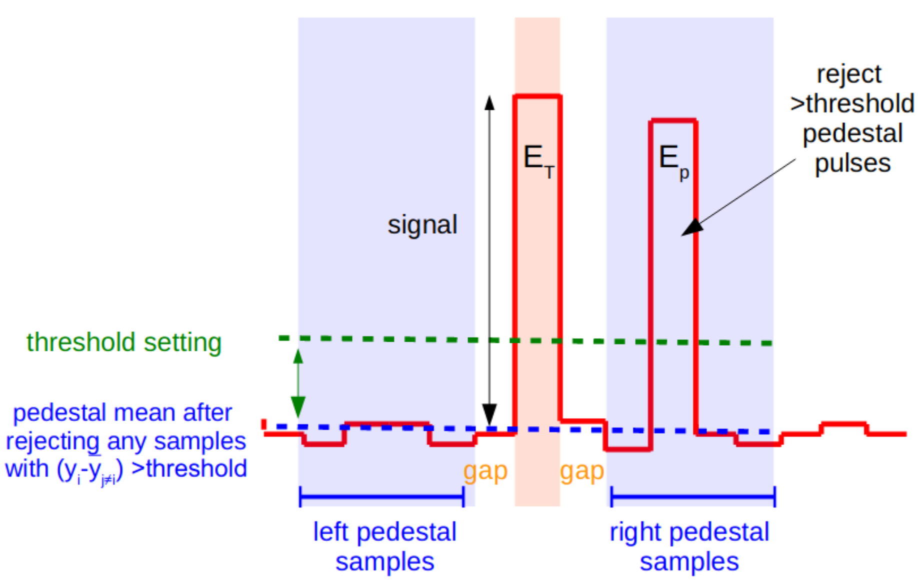
Run 1 Uncertainty

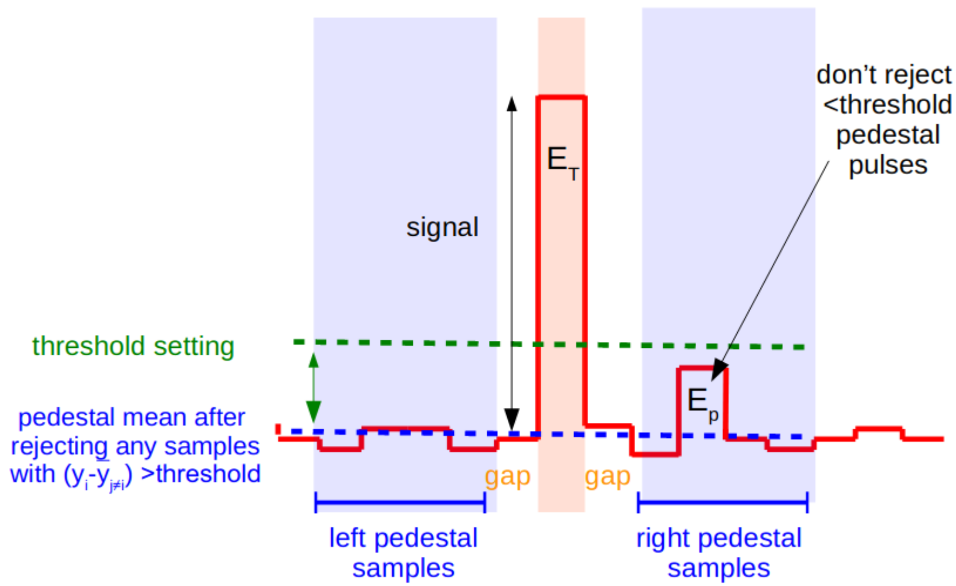
Run 1a	Run 1b	Run 1c	Run 1d
198 ppb	200 ppb	342 ppb	208 ppb

Run 2+3 Uncertainty

Run 2	Run 3a	Run 3b
1 ppb	13 ppb	38 ppb

Systematic Uncertainties: Pile-up





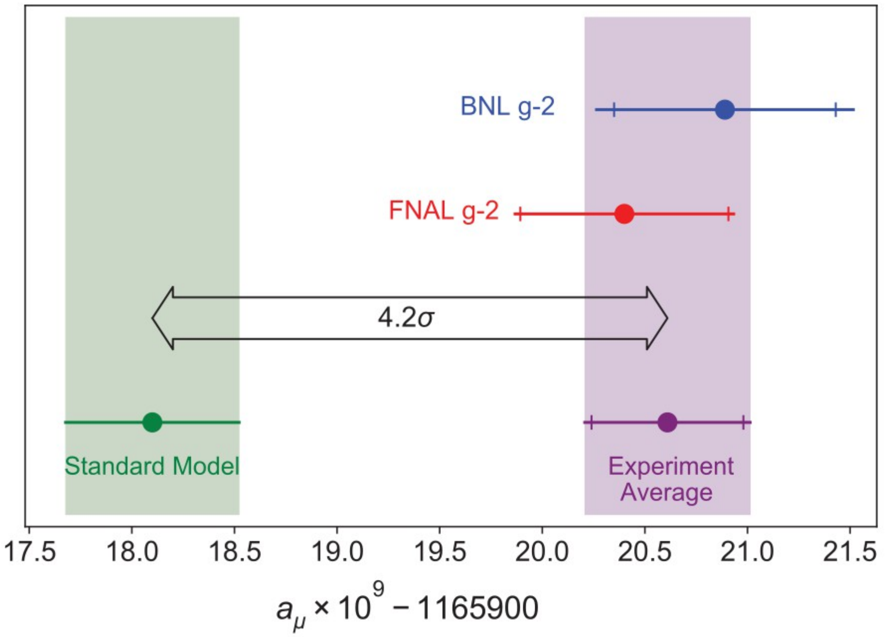
- Overestimation of pulse energy when below threshold pile-up pulse on trigger sample
- Underestimation of pulse energy when below threshold pulse on pedestal window
- These two effects largely cancel out → residual inaccuracies
- Determined by simulations, uncertainty ~ 1ppb

Conclusion and Outlook

- Run 1 results with precision of 460 ppb were released in April 2021.

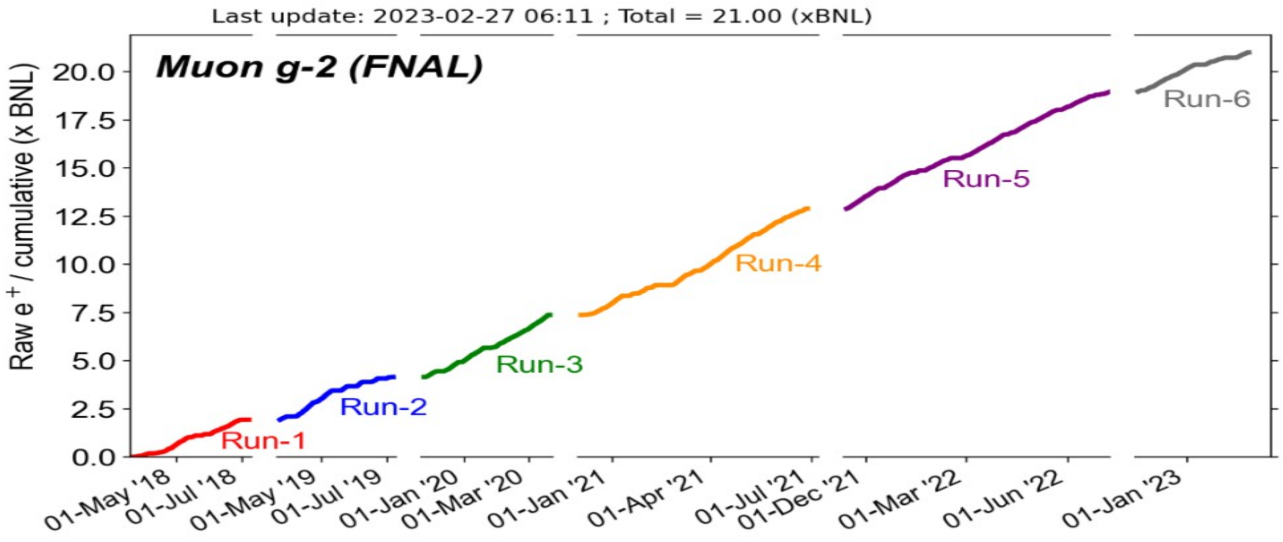
$$a_\mu = \frac{g-2}{2} = 11659204.0(5.4) \times 10^{-10}$$

- It confirmed BNL results
- It also strengthened the discrepancy to 4.2 σ
- Q-method is an alternative determination of ω_a, different sensitivity to some systematics



Conclusion and Outlook

- Largest Q-method systematic uncertainty from slow effects
- Using Ratio method we can mitigate this uncertainty
- Run 2 and 3 ω_a analysis done, good fits, sanity checks look fine
- Systematic uncertainties have been estimated. Slow effects reduced by factor ~ 10.
- Run 2+3 release expected sometime this year. Projected statistical uncertainty to be ~200 ppb, factor of 2 improvement over Run 1.
- Collecting Run 6 data right now. Already reached TDR goal of 21xBNL!



Back up

Full expression for R

$$\mathcal{R}'_\mu \equiv \frac{\omega_a}{\tilde{\omega}'_p(T_r)}.$$

$$\mathcal{R}'_\mu \approx \frac{f_{\mathrm{clock}}\,\omega_a^m (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\mathrm{calib}}\langle \omega_p(x,y,\phi) \times M(x,y,\phi)\rangle (1 + B_k + B_q)}$$

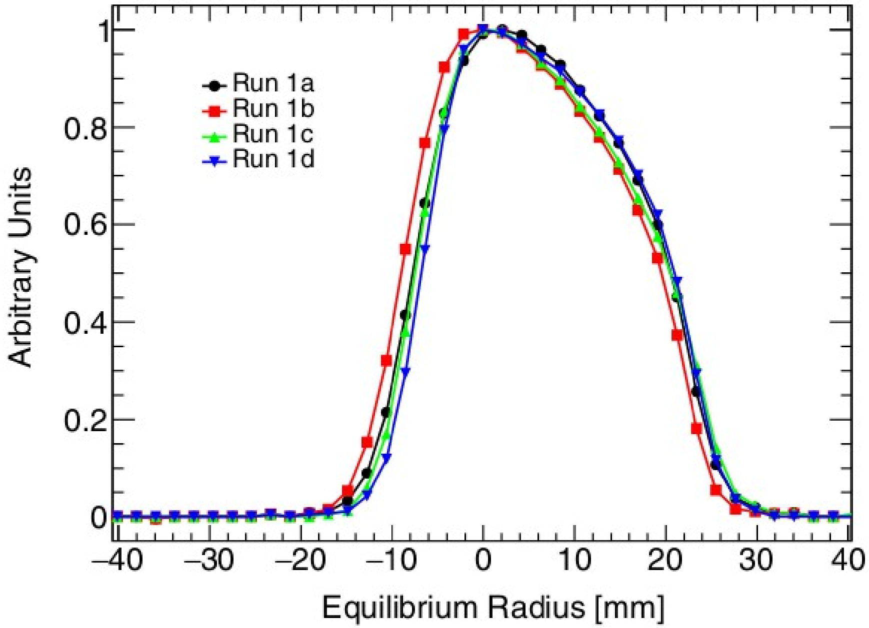
Run	$\omega_a/2\pi$ [Hz]	$\tilde{\omega}'_p/2\pi$ [Hz]	$\mathcal{R}'_\mu \times 1000$
1a	229 081.06(28)	61 791 871.2(7.1)	3.707 300 9(45)
1b	229 081.40(24)	61 791 937.8(7.9)	3.707 302 4(38)
1c	229 081.26(19)	61 791 845.4(7.7)	3.707 305 7(31)
1d	229 081.23(16)	61 792 003.4(6.6)	3.707 295 7(26)
Run-1			3.707 300 3(17)

Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	$\cdots$	434
ω_a^m (systematic)	$\cdots$	56
C_e	489	53
C_p	180	13
C_{ml}	–11	5
C_{pa}	–158	75
$f_{\mathrm{calib}}\langle \omega_p(x,y,\phi) \times M(x,y,\phi)\rangle$	$\cdots$	56
B_k	–27	37
B_q	–17	92
$\mu'_p(34.7°)/\mu_e$	$\cdots$	10
m_μ/m_e	$\cdots$	22
$g_e/2$	$\cdots$	0
Total systematic	$\cdots$	157
Total fundamental factors	$\cdots$	25
Totals	544	462

Momentum distribution

Incoming beam momentum spread 1.6%

Momentum distribution, magic momentum is taken to be at 0 mm



SM contributions

Theory Initiative white paper values

$$a_\mu^{SM} = 116592089(63) \times 10^{-11}$$

$$a_\mu^{QED} = 116584718.931(104) \times 10^{-11}$$

$$a_\mu^{EW} = 153.6(1.0) \times 10^{-11}$$

$$a_\mu^{HVP,LO} = 6931(40) \times 10^{-11}$$

$$a_\mu^{HLbL,LO} = 92(19) \times 10^{-11}$$

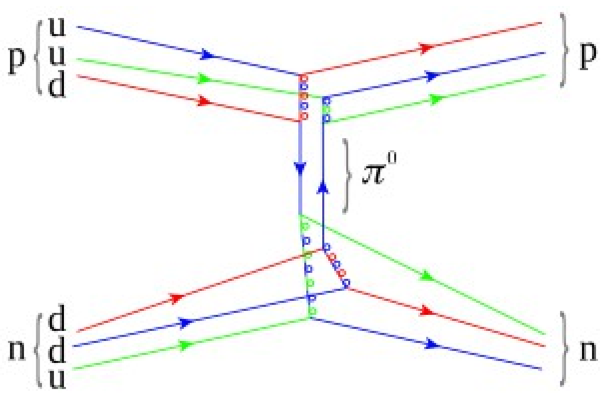
Pion production

Proton beam collides with a stationary target

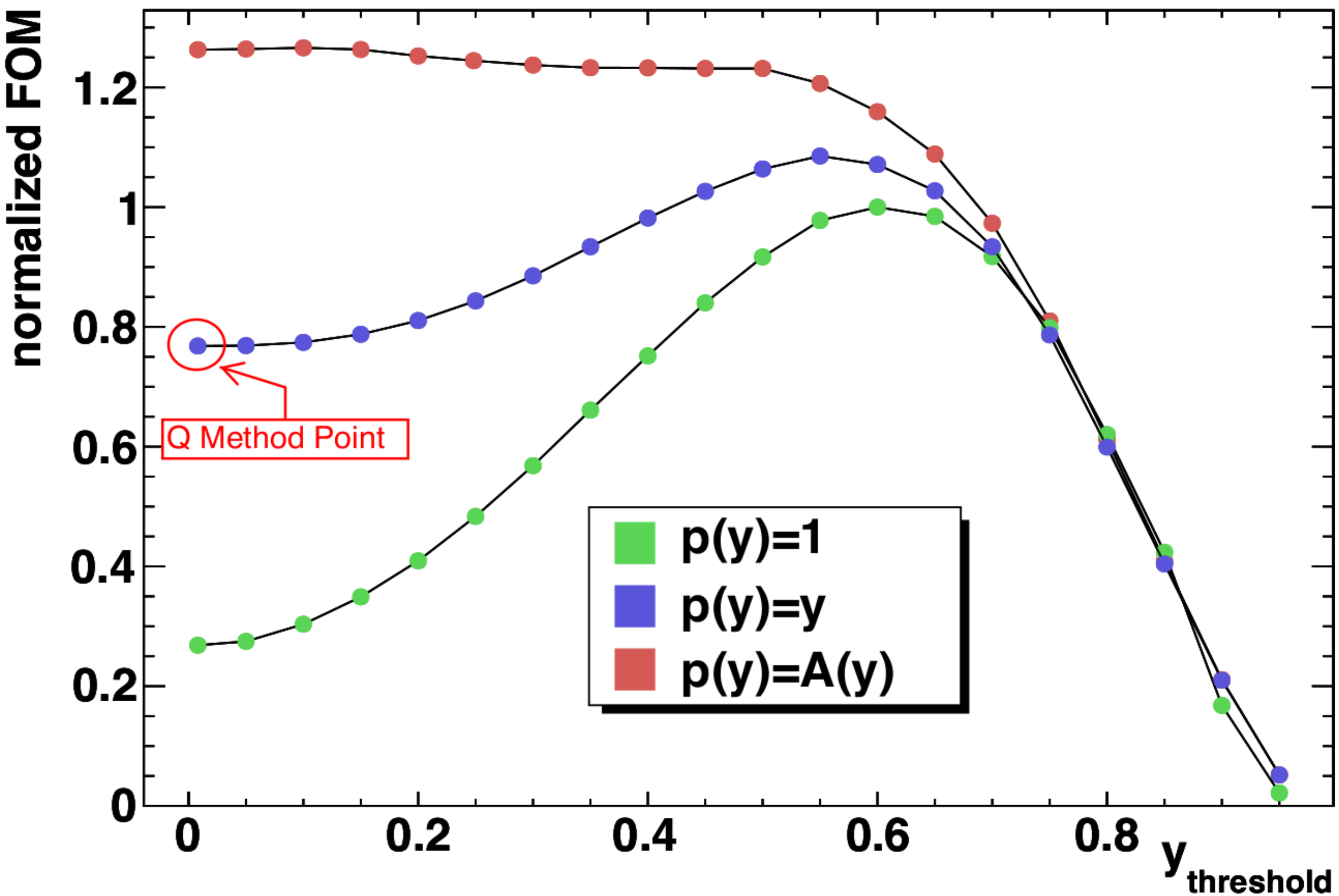
Interaction with protons on the nucleus

$$p + p \to p + p + \pi^0,$$

$$p + p \to p + n + \pi^+.$$



Q-Method Statistical Sensitivity



Q-Method Error Assignment

Ignoring the contribution from the fluctuation of energy per pulse, ΔE_i, the uncertainty for the corresponding bin would be

$$\Delta E_{total} = \sqrt{(E_1 \Delta n_1)^2 + (E_2 \Delta n_2)^2 + (E_3 \Delta n_3)^2} + ... \tag{3.3}$$

Assuming Poisson statistics and $\Delta n_i = \sqrt{n_i}$,

$$\Delta E_{total} = \sqrt{(E_1 \sqrt{n_1})^2 + (E_2 \sqrt{n_2})^2 + (E_3 \sqrt{n_3})^2} + ... \tag{3.4}$$

This is approximated as

$$\Delta E_{total} = \sqrt{(E_1)^2 + (E_2)^2 + (E_3)^2} + ... \tag{3.5}$$

where the effects from *pulse splitting*, that is sharing of a pulse energy between adjacent time bins, are ignored.

Ratio-Method weighting factors

$$u_+(t) : u_-(t) : v_1(t) : v_2(t) = e^{T/2\tau} : e^{-T/2\tau} : 1 : 1$$

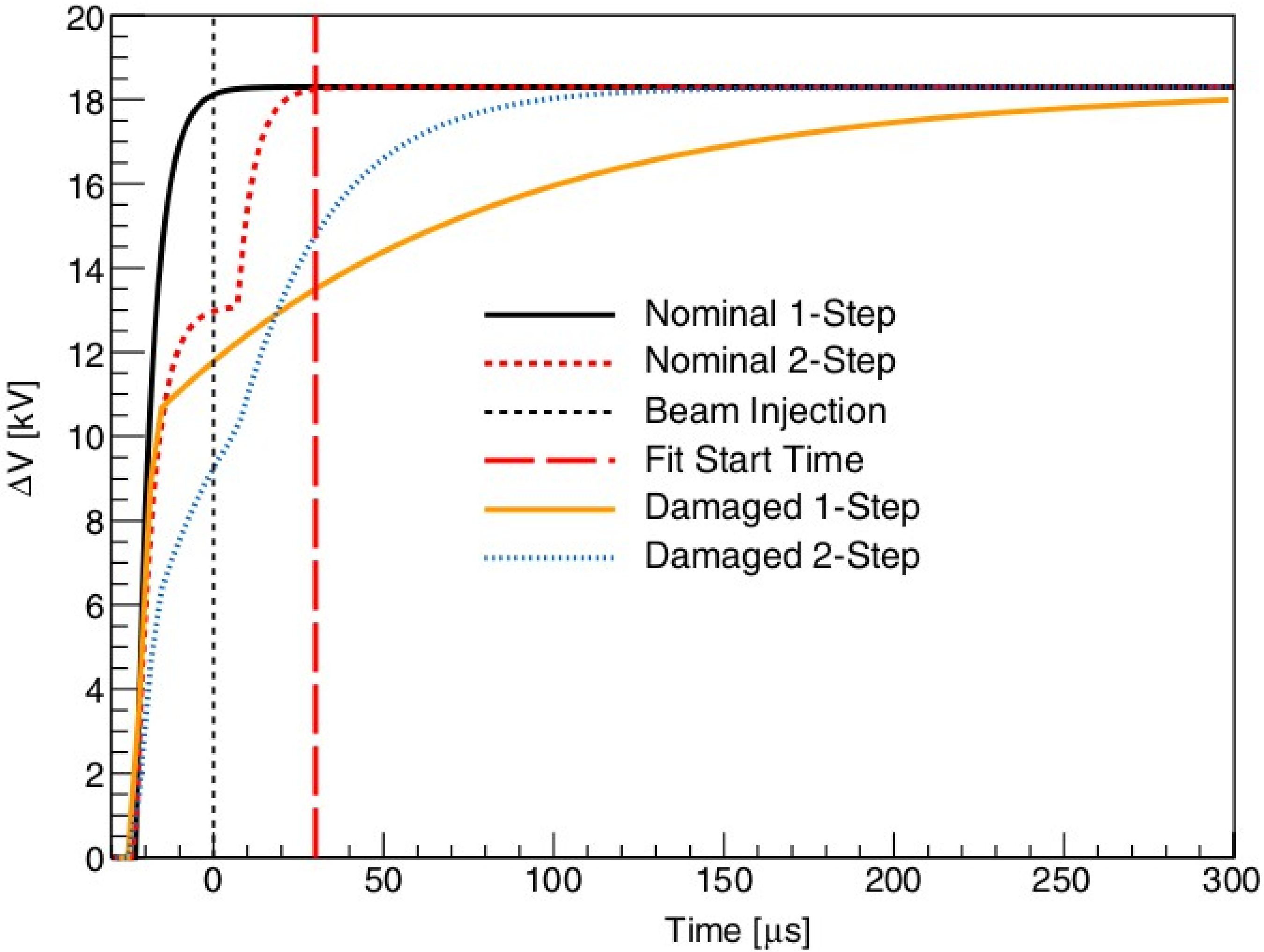
$$u_+(t) = \frac{e^{T/2\tau}}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t + T/2),$$
$$u_-(t) = \frac{e^{-T/2\tau}}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t - T/2),$$
$$v_1(t) = \frac{1}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t),$$
$$v_2(t) = \frac{1}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t).$$

Software Blinding

$$\omega_a = \omega_{ref}[1 - (R - \Delta R) \times 10^{-6}]$$

where $\omega_{ref} = 2\pi \times 0.2291$ MHz.

Beam Recovery, Fit start time



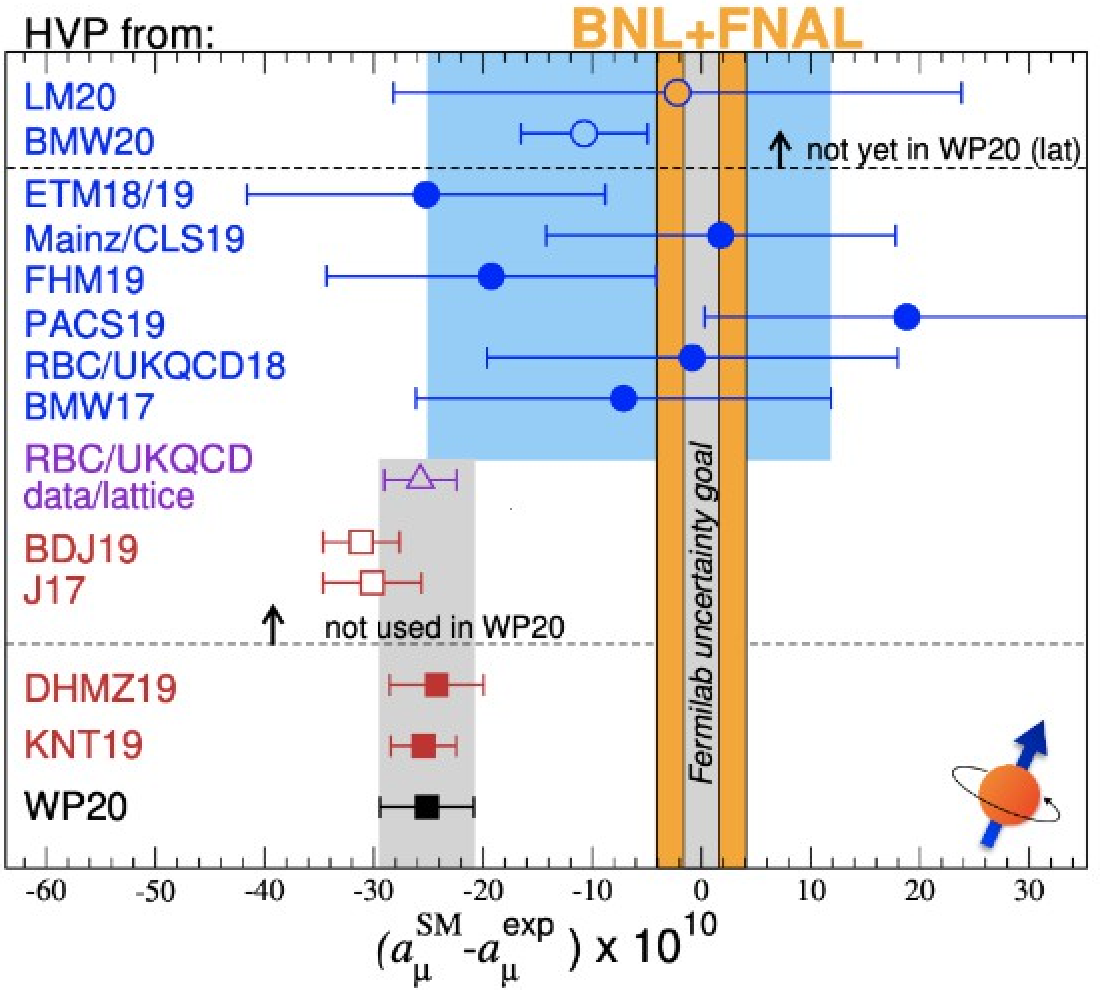
Lattice Calculation for HVP

Lattice

Data-based Dispersive

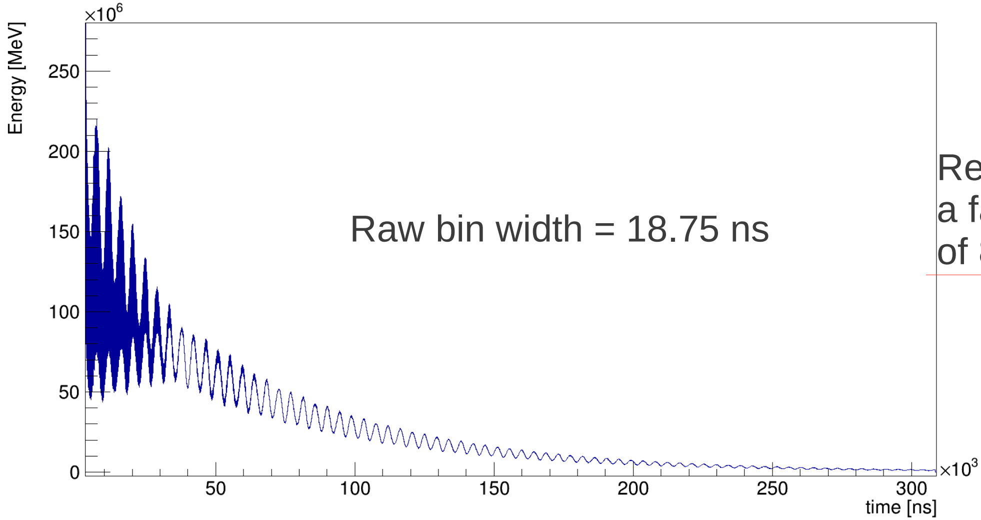
Lattice and Data

Official WP20

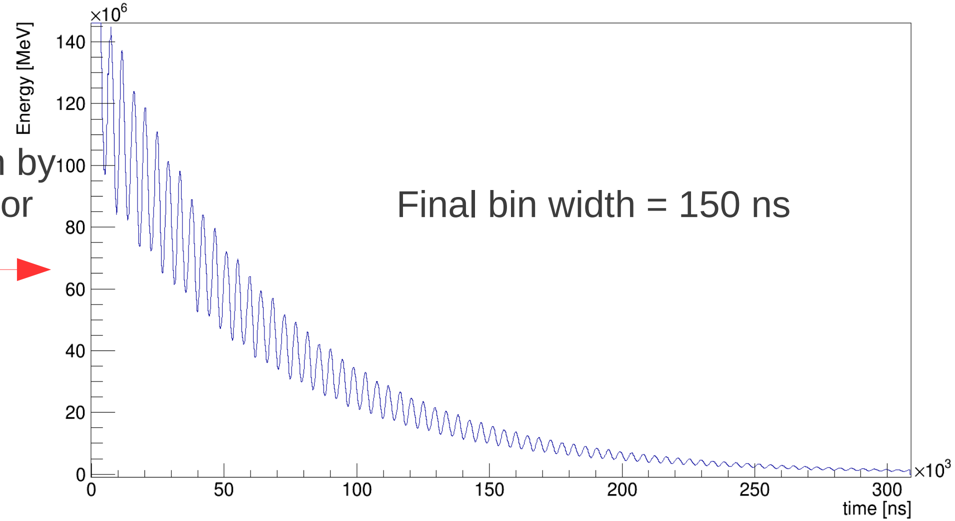


Correcting Cyclotron Frequency Modulation

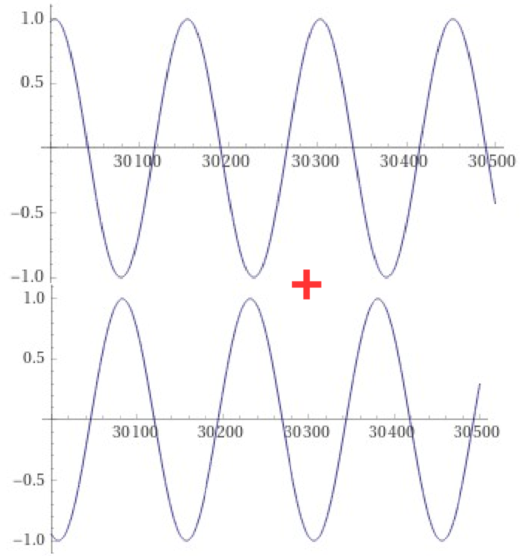
Modulation by cyclotron frequency at early times (T_c= 149.2 ns):

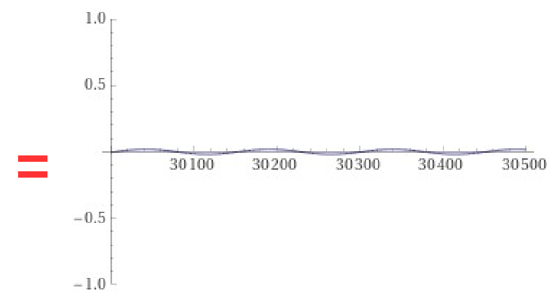


Rebin by a factor of 8



- Rebin raw bins by factor 4
- Make a copy histogram; shift by 1 bin
- Superimpose shifter and un-shifted; Cyclotron phase cancels *
- Rebin by 2 → 150 ns bins





* bins are now correlated

Beam Dynamics

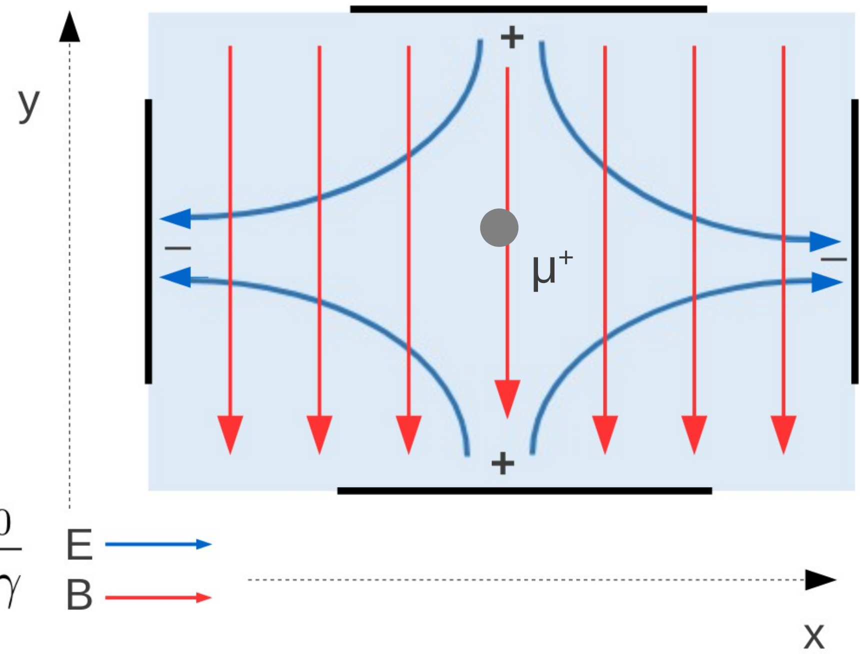
Beam oscillations due to ESQ:

- Equations of motion for muon in storage ring

$$\ddot{x} = -\omega_c^2 (n-1) x$$
$$\ddot{y} = -\omega_c^2 n y$$

where $n \equiv \dfrac{kR_0}{v_0 B_0}$ and $k = \dfrac{\partial E_x}{\partial x} = \dfrac{\partial E_y}{\partial y}$

with cyclotron frequency of muon $\omega_c = \dfrac{eB_0}{m_\mu \gamma}$



- Oscillations in x and y directions:

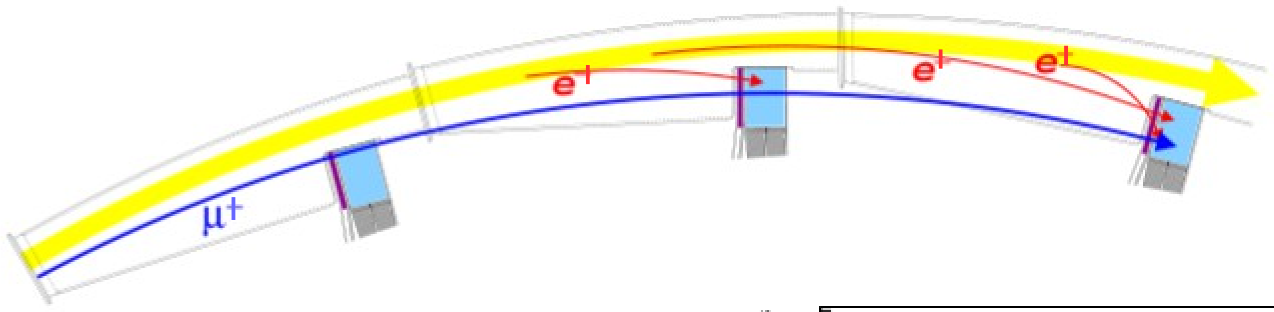
$$\omega_x = \omega_c \sqrt{1-n} \quad , \quad \omega_y = \omega_c \sqrt{n}$$

- Since the signal is discreetly sampled by 24 calos, we observe aliasing

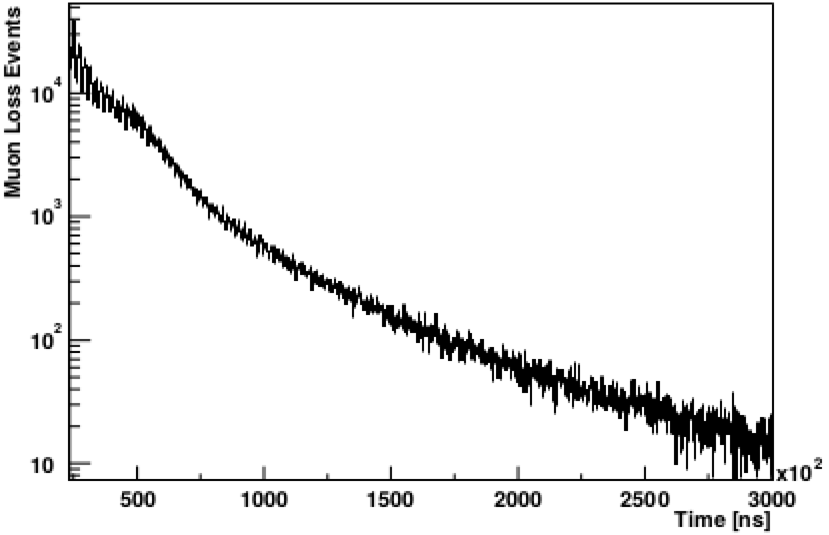
$$\omega_{CBO} = 2\pi f_{cbo} = 2\pi (f_c - f_x)$$
$$\omega_{VW} = 2\pi f_{VW} = 2\pi (f_c - 2 f_y)$$

Muon Loss Correction

- Muons lost from ideal orbit also get detected by calos

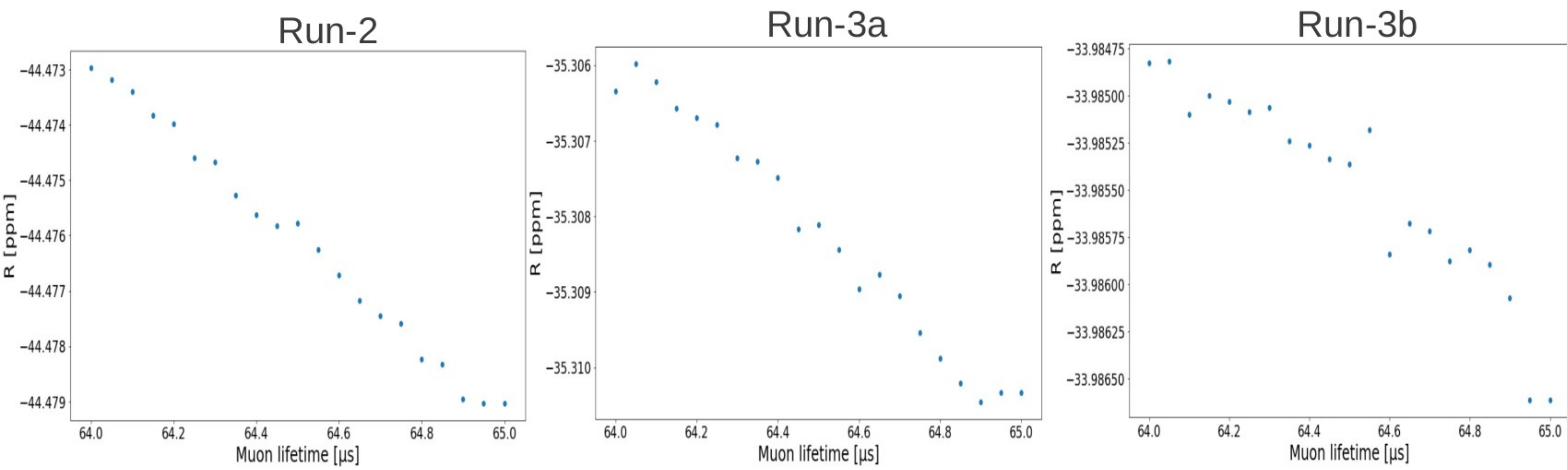


- Distort positron count/energy spectrum
- Correction includes identification and construction of loss spectrum
- Muon loss spectrum is used in fit-function to account for the lost muons



Systematic Uncertainties : Choice of Muon Lifetime

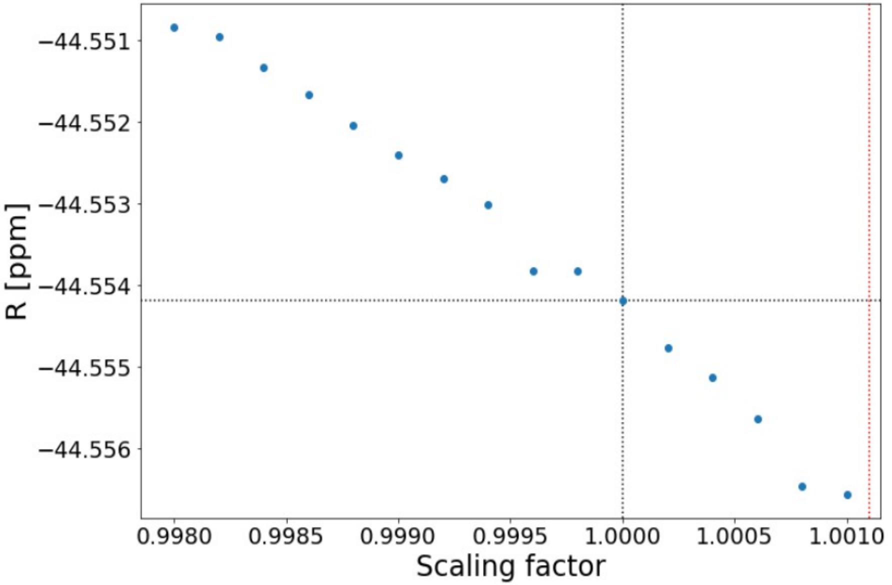
- Muon lifetime fixed at 64.44 μs
- Scan range 64 to 65 μs



Run-2	Run-3a	Run-3b
6 ppb	4 ppb	2 ppb

Systematic Uncertainties: Covariance Matrix

- Off-diagonal elements of covariance matrix calculated using Taylor expansion
- Scaling factor for off-diagonal elements, range 0.998-1.001
- Red dotted line → matrix no longer positive definite
- Change in R over this whole range → 5 ppb



Run-2

Systematic Uncertainty Table

Table 1: Various systematic uncertainties on R in Run-2 and Run-3 analysis using ratio Q-method

	Run-2 [ppb]	Run-3a [ppb]	Run-3b [ppb]
In-Fill Gain Amplitude	3	3	2
In-Fill Gain Time Constant	0	1	1
Q-method Pile-up Error Correction	0.1	0.1	0.1
Q-method Pile-up Simulation	1	1	1
Muon loss: Different Models	3	10	16
CBO Frequency Change	76	26	75
CBO Decoherent Envelople	5	6	12
CBO Time Constant	18	5	7
Muon precession period	5	5	5
Muon Life-time	6	4	2
Slow term	1	13	38