Pepper-pot scan on surface muon beams

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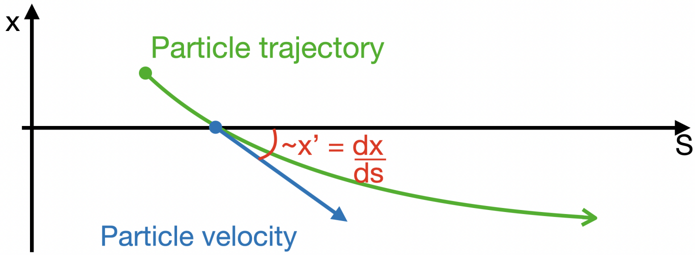
Intro

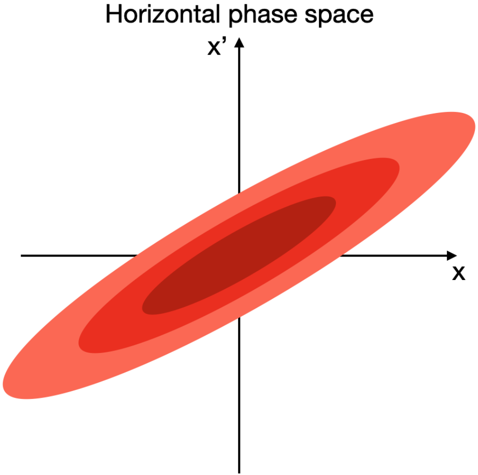
Beam dynamics recap

- A common approach to describe beam dynamics is to use the longitudinal variable s as the independent variable, switching it with time in the Hamiltonian
- with such a transformation the canonical momenta of the transverse variables are the quantities $x'(s_0) = \left(\frac{\mathrm{d}x}{\mathrm{d}s}\right)_{s=s_0}$ and $y'(s_0) = \left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)_{s=s_0}$ (divergence), usually expressed in radians in the **paraxial approximation**



Beam dynamics recap

→The 2D transverse phase space is defined as the transverse coordinate of interest (x or y) vs the corresponding divergence.



Beam dynamics recap

- A common approach to describe beam dynamics is to use the longitudinal variable s as the independent variable, switching it with time in the Hamiltonian
- with such a transformation the canonical momenta of the transverse variables are the quantities $x'(s_0) = \left(\frac{\mathrm{d}x}{\mathrm{d}s}\right)_{s=s_0}$ and $y'(s_0) = \left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)_{s=s_0}$ (divergence), usually expressed in radians in the **paraxial approximation**
- in the linear approximation, the solution of the equations of motion can be expressed using the Transport Matrix formalism which is used to propagate particles through a lattice
- as the Liouville theorem is valid, the phase space volume is conserved: this quantity is usually referred at as **emittance**

It is usually helpful to be able to model the dynamics of a whole beam rather than of a single particle to either tune a beamline or as an input to experiments (MEG II and Mu3e).

Gaussian approximation

A way of simplifying the modeling of a particle beam is assuming its phase space to be gaussian. This is in general a good approximation and allows to model the collective beam dynamics by propagating the widths of the gaussian distribution.

To represent the full phase space, one can use the covariance matrix of the beam:

$$\Sigma_x = \begin{bmatrix} \sigma_x^2 & \sigma_{x,x'} \\ \sigma_{x,x'} & \sigma_{x'}^2 \end{bmatrix} \tag{1}$$

For such a beam, the level surfaces are ellipses, and the 1-σ phase space is defined as:

$$x(s_0)^T \Sigma_x^{-1}(s_0) x(s_0) = 1 \tag{2}$$

Gaussian approximation

$\Sigma_x^{-1}(s_0)$ transforms as a scalar product. If x transforms as $Ux_0 = x_1$, then:

$$\Rightarrow x_0^T \Sigma_{x,0}^{-1} x_0 = x_0^T U^T (U^T)^{-1} \Sigma_{x,0}^{-1} U^{-1} U x_0 \tag{3}$$

$$= x_1^T (U \Sigma_{x,0} U^T)^{-1} x_1 = 1 \tag{4}$$

$$\Rightarrow \Sigma_{x,1} = U \Sigma_{x,0} U^T \tag{5}$$

Gaussian approximation

In the case of a drift L, a single particle would be propagated as follows:

$$M_{drift}(s_0|s_0+L)\,x(s_0) = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix} = \begin{bmatrix} x_0 + L\,x'_0 \\ x'_0 \end{bmatrix} \tag{6}$$

In case of a gaussian beam, the same transport matrix can be used to propagate the covariance matrix of the beam phase space:

$$\Sigma_x(s) = M_{drift}(s_0|s_0+L)\Sigma_x(s_0)M_{drift}(s_0|s_0+L)^T, \quad \text{with } \Sigma_x = \begin{bmatrix} \sigma_x^2 & \sigma_{x,x'} \\ \sigma_{x,x'} & \sigma_{x'}^2 \end{bmatrix} \tag{7}$$

In case of elements which don't introduce coupling between horizontal and vertical motion, the dynamics in the two transverse directions can be expressed independently, as in the case of the drift above.

How can one characterize the transverse phase space?

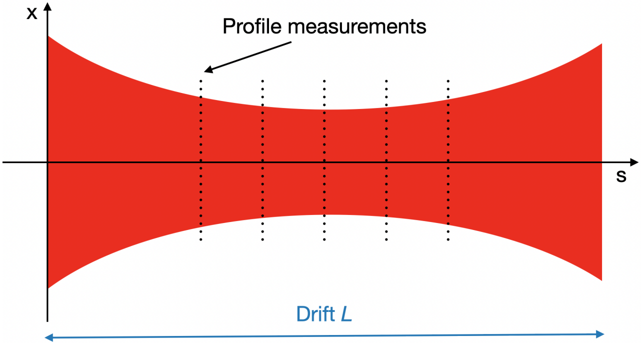
Phase space measurement

The tricky part of measuring a phase space is clearly determining its divergence: an easy way of doing it would be to measure the beam profile in the two transverse direction at many positions along a drift. Three measurements fully constrain the problem.

This approach comes with some issues:

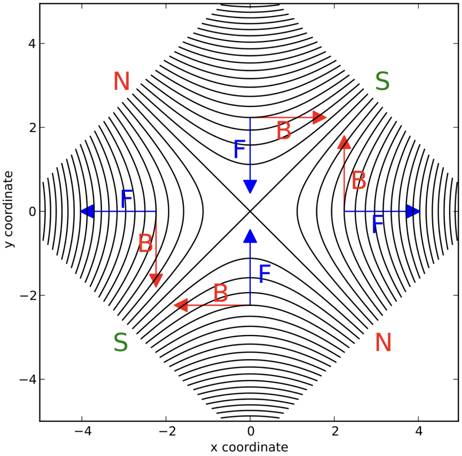
- the whole drift must be in vacuum
- a long drift might be needed (a few meters)

This is not ideal in cases were detectors are not installed along the beamline and not enough space is available to let the beam drift.



Quadrupole scan

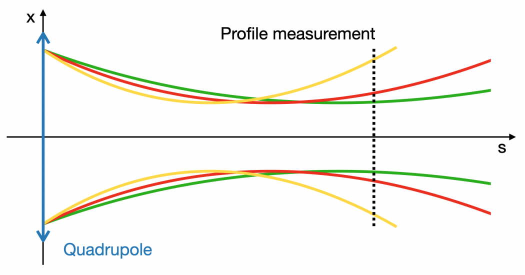
Everything becomes easier if the beam profile is measured at a single location after a quadrupole magnet. In general a quadrupole magnet behaves as a linear lens which is focusing in one transverse direction and defocusing in the other.



Quadrupole scan

Everything becomes easier if the beam profile is measured at a single location after a quadrupole magnet. In general a quadrupole magnet behaves as a linear lens which is focusing on one transverse direction and defocusing on the other.

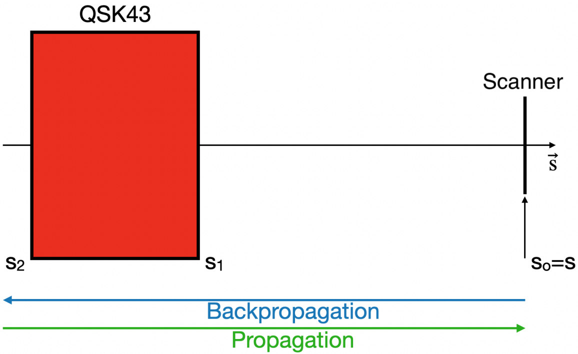
- the beam size depends on the strength of the quadrupole
- the beam size depends on the phase space upstream to the quadrupole
- with three measurements the phase space can be fully characterized
- only one detector is needed for the measurement



Quadrupole scan

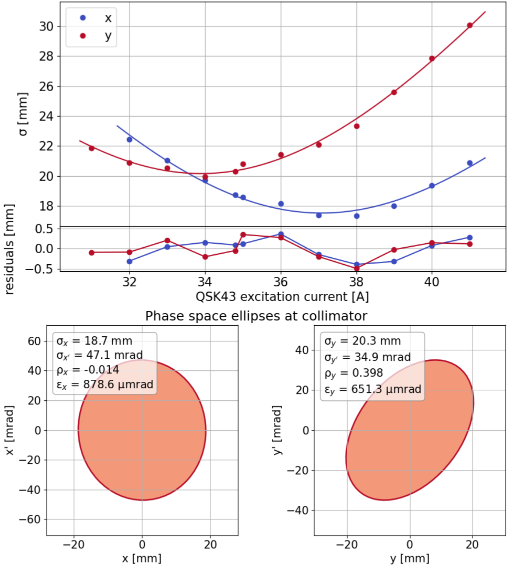
In practice, one can express the beam width with the matrix formalism. In πE5 we use the QSK43 quadrupole for such measurement:

$$M(s_0|s) = M_{\text{drift}}(s_1|s)M_{\text{QSK43}}(s_2|s_1)M^{-1}_{\text{QSK43,opt}}(s_1|s_2)M^{-1}_{\text{drift}}(s_1|s_0) \tag{8}$$



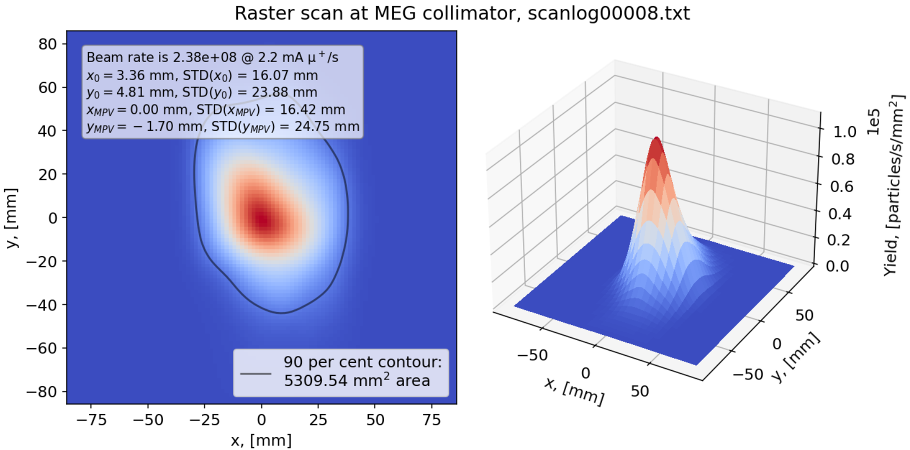
Quadrupole scan

This is a measurement we performed in June 2022 in πE5.
We must keep in mind that: **we are assuming a gaussian beam** and that this measurement depends on our knowledge of the quadrupole magnet.



πE5 beam

The gaussian approximation in πE5 is not perfect. Here is a measurement performed in πE5 in May 2023.

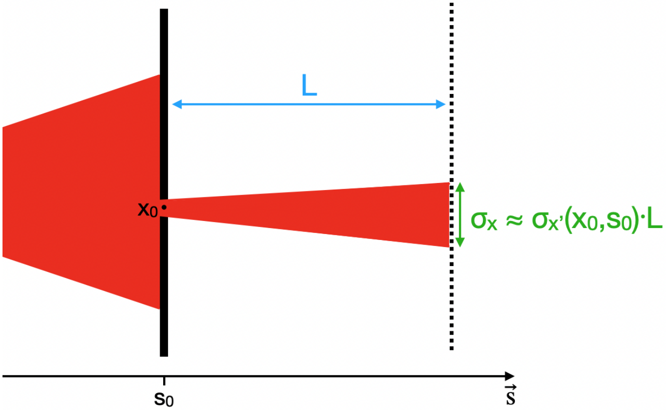


Pepper-pot measurement

Single-slit measurement

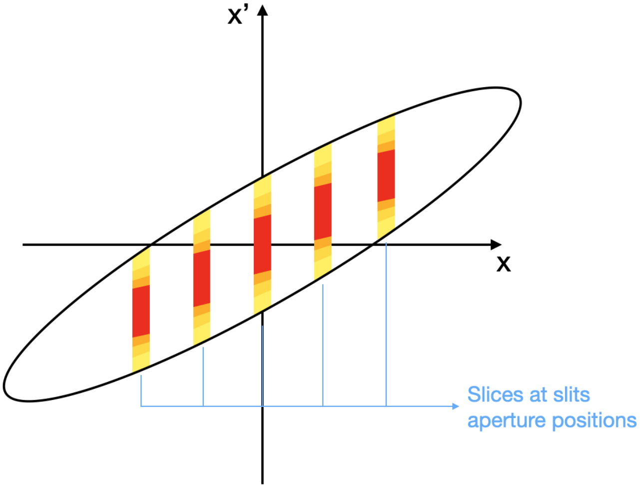
A common approach to measure the emittance of low energy proton beams is to measure the beam spot in presence of an adjustable silts system with a small aperture.

- the slits are thick enough to dump the beam
- the aperture is small enough to consider the resulting beamlet as coming from a point-like source
- after a drift the beamlet is measured and its profile is proportional to the divergence at the aperture location



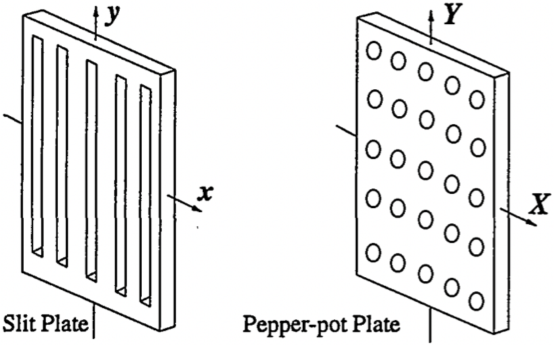
Single-slit measurement

By changing the slits settings one can move the aperture to measure the divergence for different positions and then combine them in a phase space. **This is a direct measurement of the phase space at the slits position.**



Pepper-pot scan

Slits systems are in general bulky and allow only for one scan at a time, so a common approach is to drill the apertures directly on a plate. This allows also for a faster measurement as many beamlets are measured at once. If one uses circular holes instead of "slit-like" apertures the full 4D transverse phase space can be measured at once: this is the **pepper-pot** technique.

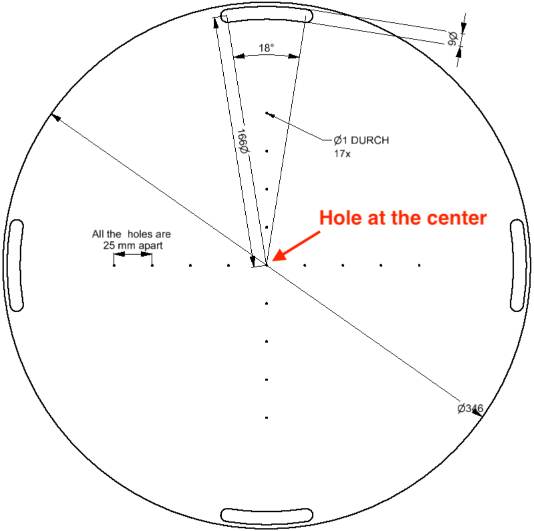


Pepper-pot plates

The ideal device for such a measurement would be a luminophore screen. As this was the first time we did it with a small scintillator: only the axes could be scanned.

We have three pepper-pot plates (3 mm thick aluminum):

- 1 mm **diameter holes on a cross spaced by** 25 mm**. Central hole on axis**
- as above with holes staggered by 12.5 mm. No hole on axis
- blind plate with no holes for background measurements



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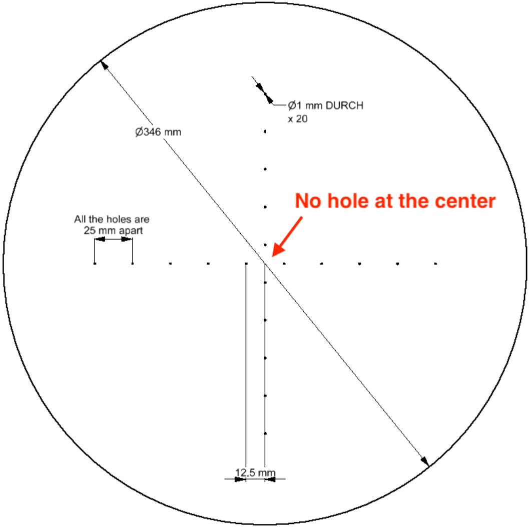
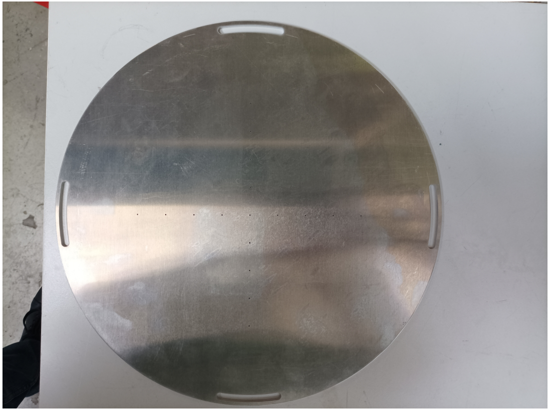
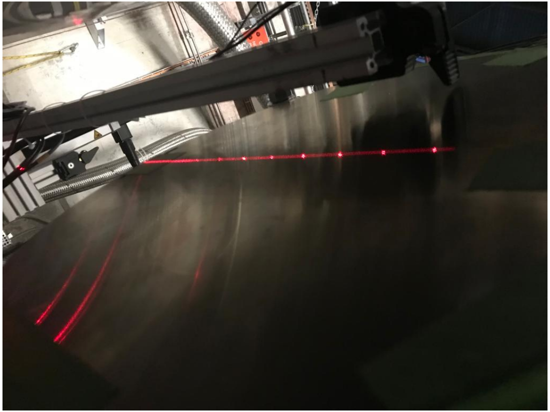


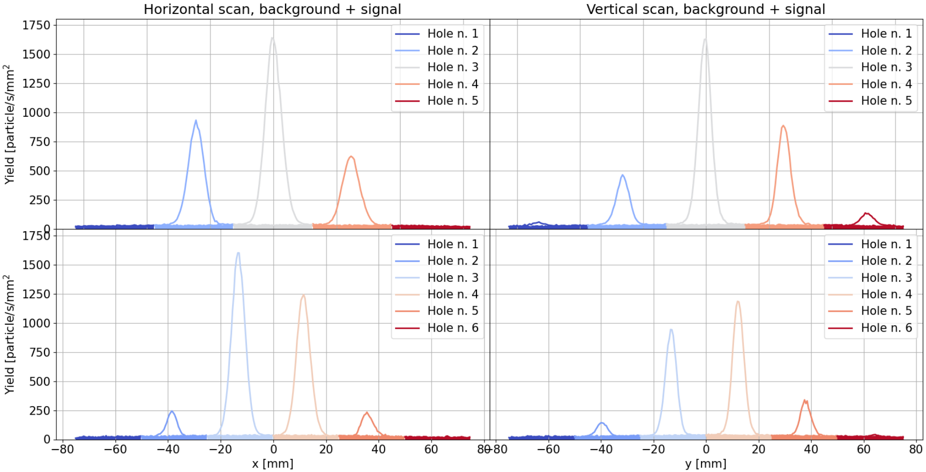
Pepper-pot measurement





Pepper-pot measurement

The scan was performed on the horizontal and vertical axes with an $0.5\,\text{mm}$ pitch. The area filled in the plots below are the measured background.

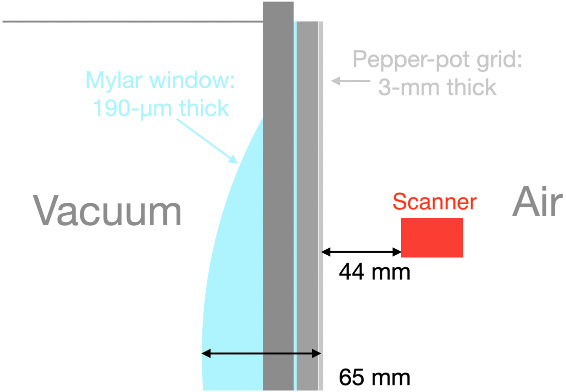


Phase space parametrization

Measurement set-up

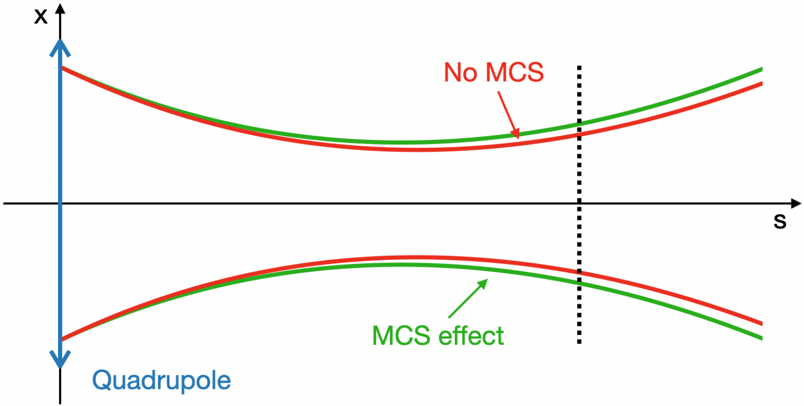
The measurement was performed by employing our standard set-up: the vacuum in the beamline was interrupted with a Mylar window and the scanner was measuring in air.

This are the conditions we had in 2022 during the measurements: even though they are the same as for the quadrupole scan, **in this case Multiple Coulomb Scattering (MCS) is not negligible**.



MCS effect

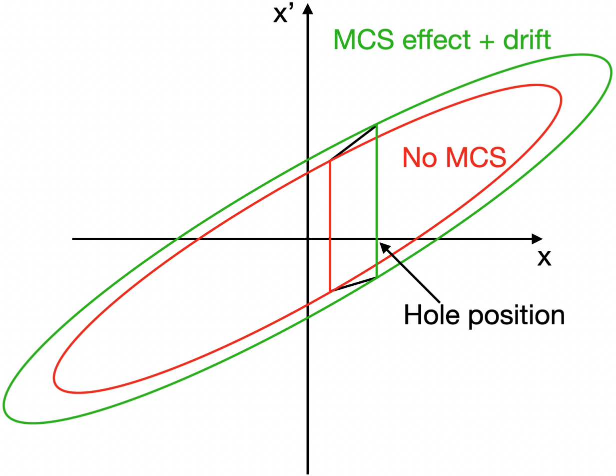
The MCS affects the beam by increasing its divergence. This effect is negligible in our quadrupole scan as the quantity of interest is the beam spot size. An increase in divergence causes an increase in the beam size depending on the length of the drift. In our case this effect is $\sim 3\,\%$.



MCS effect

In case of a pepper-pot scan, the beamlets profiles are proportional to the divergence distribution at the plate location:

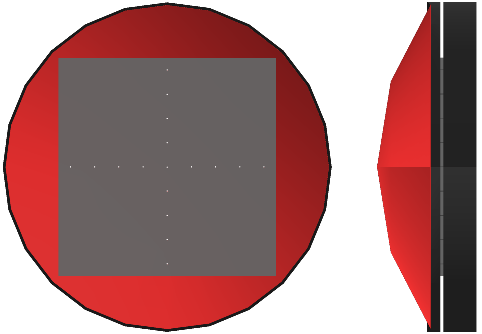
1. each beamlet is expected to be broadened by MCS the same way as the divergence is increased up to the plate
2. after the plate each beamlet will increase in size as in the quadrupole case
3. each beamlet is expected to be displaced depending on the hole position



MCS contribution evaluation

The MCS contribution was obtained by propagating the phase space as measured with the quadrupole scan through the pepper-pot plate in a G4beamline simulation including the full material budget.

1. the beam is propagated without material budget and the beamlets are fitted
2. the beam is propagated with material budget and only the MCS contribution is fitted
3. the fitted MCS contribution is fixed in the fit to the measured data to unfold them



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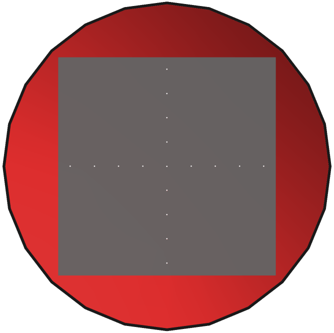
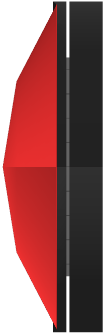
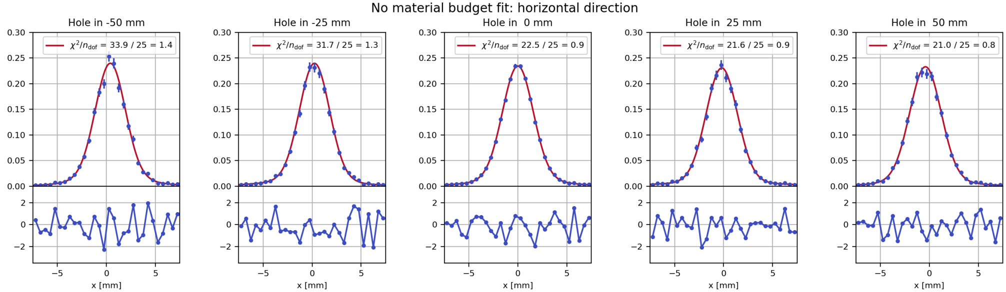




MCS contribution evaluation

At first the simulation was run in absence of the Mylar window and in absence of air and each beamlet was independently fitted by the sum of two gaussians. Here the horizontal profiles are shown.



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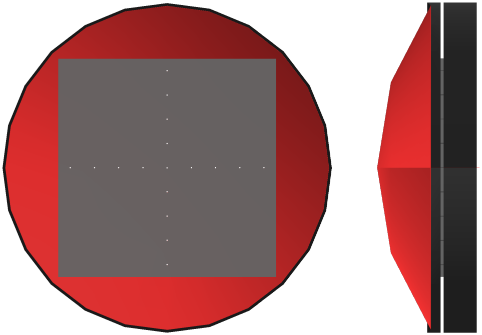
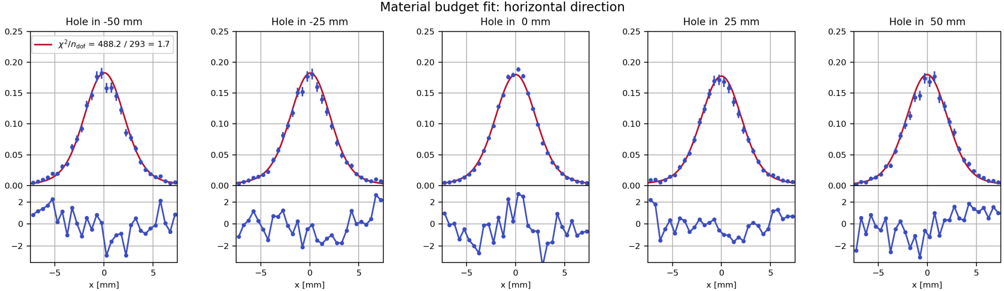


MCS contribution evaluation

Then, the simulation was run with the full material budget. The MCS was included by means of the convolution with the sum of two gaussians and by including a linear displacement to the center of the beamlets. The MCS parameters were constrained to be the same for each beamlet in both transverse planes.

$\sigma_{m,1}$ [mm]	$\sigma_{m,2}$ [mm]	f_m	k
1.288(12)	7.7(9)	0.939(3)	0.372(14)

This parameters are then fixed in the fit to the measured scan, where each beamlet is fitted again independently.



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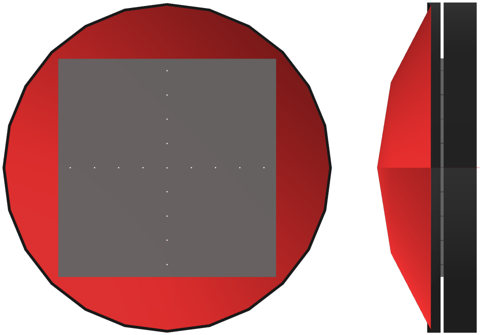
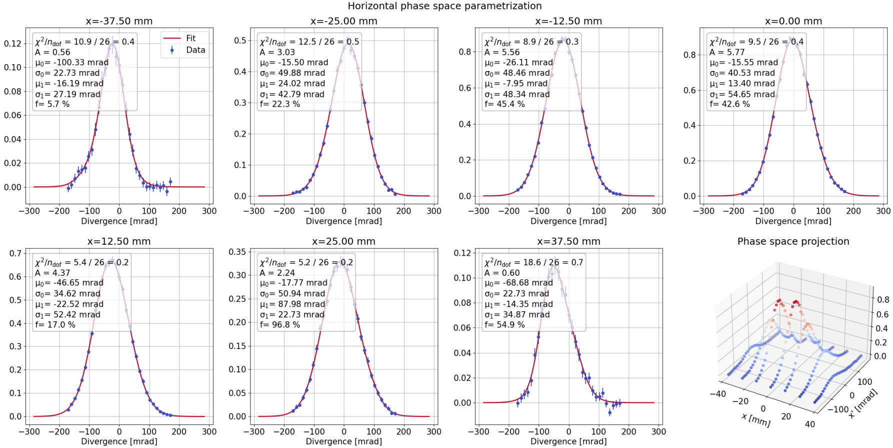


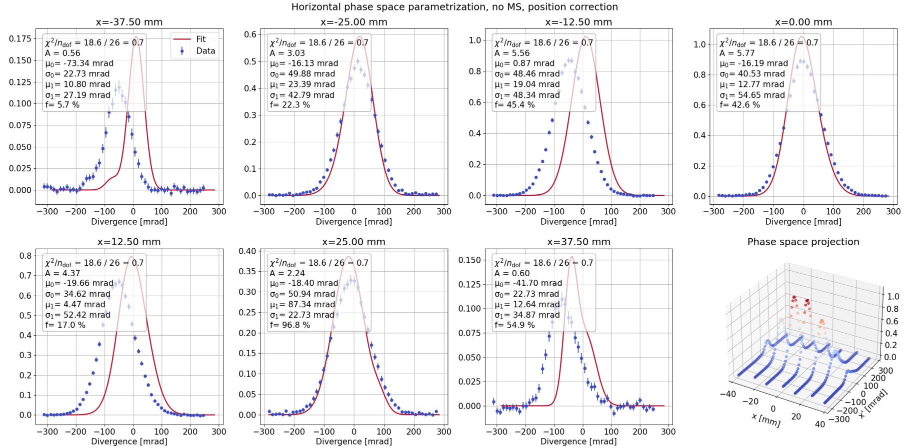
Phase space parametrization - MCS included: horizontal

Here the fit to the measured data in the horizontal direction: the MCS is included.



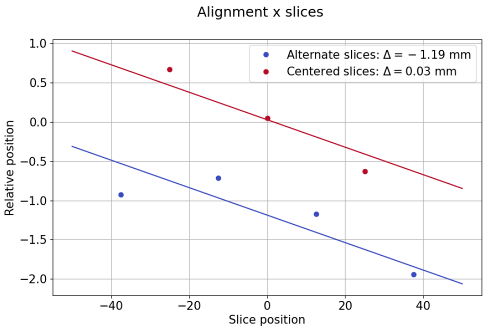
Phase space parametrization - MCS unfolded: horizontal

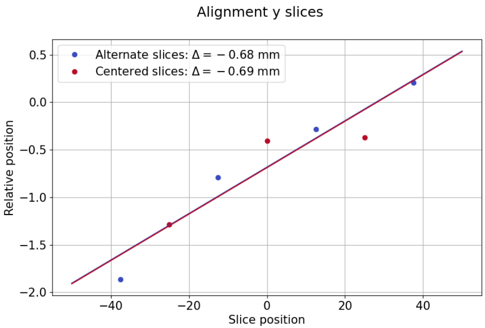
Here the unfolded beamlets in the horizontal direction as compared to the data.



Phase space parametrization: plate alignment

A last thing to be taken into account is the possible displacement between the normal and staggered pepper-pot plate. This was compensated by fitting two straight lines to the beamlets positions.

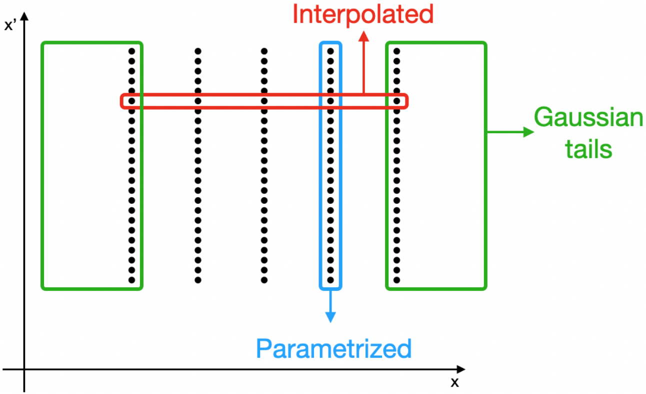




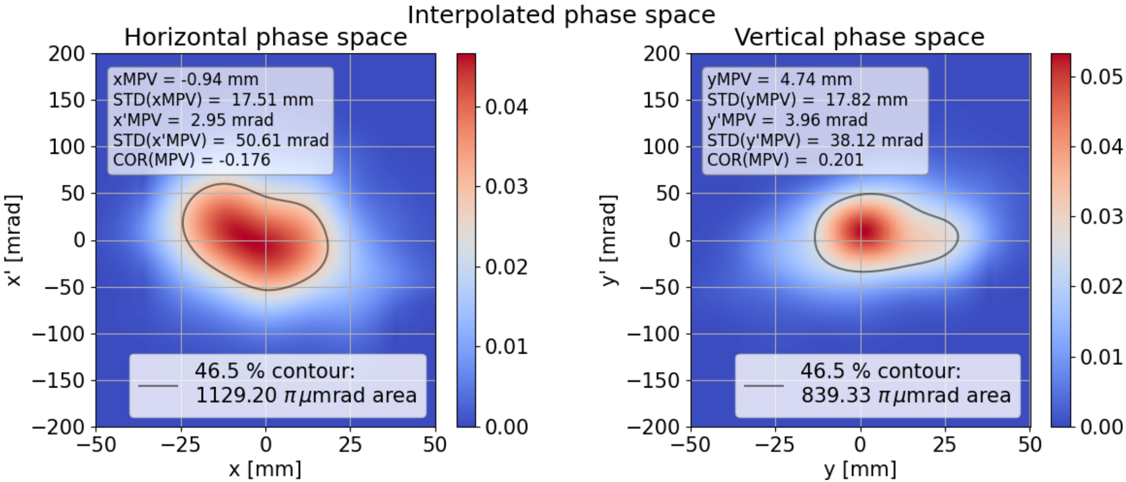
Reconstructed phase space

At this stage, only the divergence at the aperture positions are available. To fill in the remaining phase space an interpolation is applied:

- the parametrized slices are evaluated in the full wanted range
- a cubic spline interpolates the missing area
- a gaussian tail is fitted to each side of the interpolated curve



Pepper-pot measurement results



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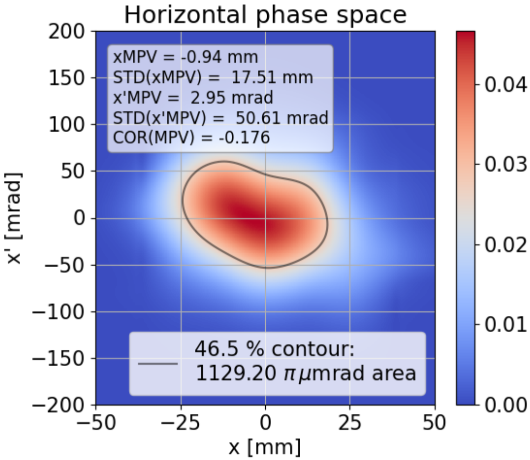

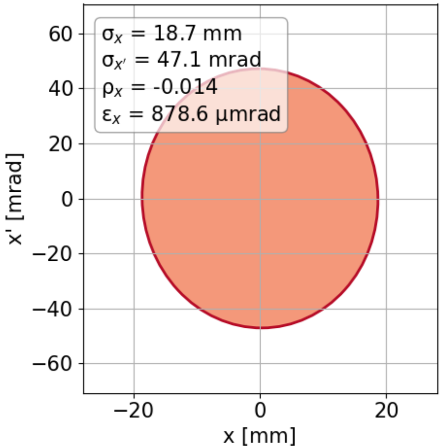

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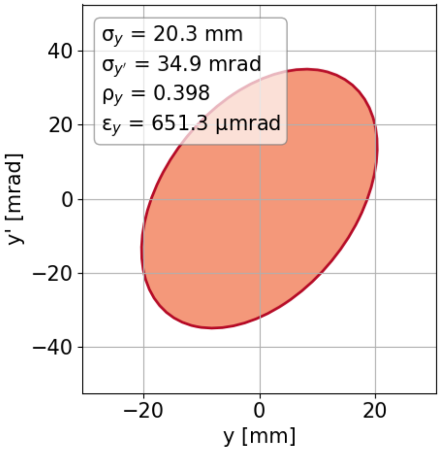

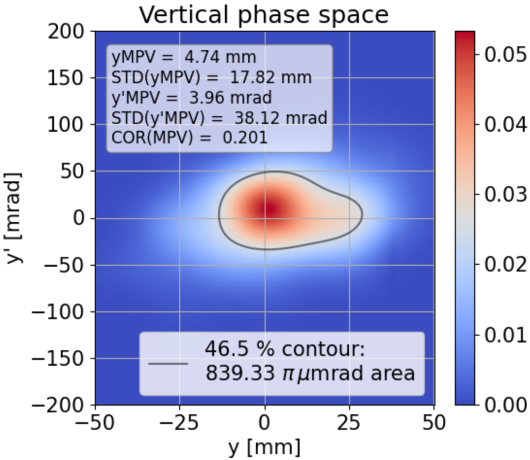

Conclusions and Outlook

- A direct measurement of the transverse phase space in πE5 has been made during the 2022 MEG II beam time.
- The standard deviations obtained with such measurement are consistent with the quadrupole scan results at the $10\,\%$ level
- The reconstructed phase space can be now used to simulate beam propagation in πE5 for both MEG II and Mu3e.

Back-up

Quadrupole scan comparison - Horizontal scans

The comparison b/w the quadrupole scan and the pepper-pot scan is not trivial, because of the assumptions made for the quadrupole scan. Here a comparison between the profiles measured at collimator during the quadrupole scans and of those obtained by transmitting the pepper-pot phase space.

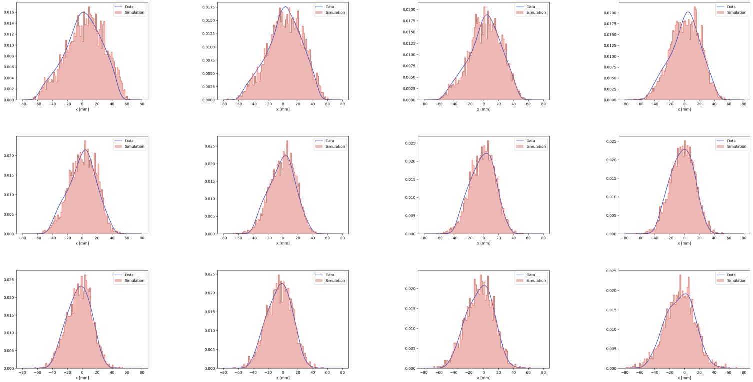

Figure: Current increased in 1 A steps b/w 30 A and 41 A. Horizontal profiles.

Quadrupole scan comparison - Vertical scans

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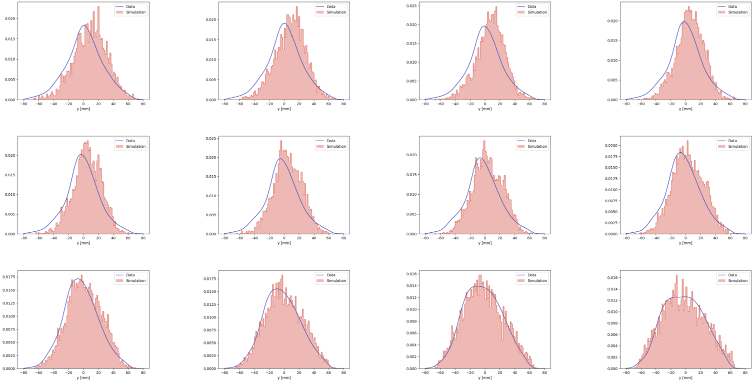

Figure: Current increased in 1 A steps b/w 30 A and 41 A. Vertical profiles.