LTP Seminar

Radiative Corrections for MUSE

Marco Rocco

for the McMUSE Team

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Paul Scherrer Institut

26TH JUNE 2023

- higher-order predictions and comparison with precision experiments
- focus on low-energy QED scattering processes
- theoretical background for lepton experiments (Mu3e, MUSE, MUonE. . .)
- all this in

McMule

Monte Carlo for MUons and other LEptons

`https://mule-tools.gitlab.io/`

◇ fully-differential Monte Carlo integrator, not an event generator (yet)

[mules by A. Signer]

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what this talk does *not* contain 

- experimental details
- technical details on higher-order QED calculations
- studies on two-photon-exchange (TPE) corrections

what this talk does contain 

- phenomenology tailored to MUSE
- studies on QED radiative corrections to $\ell^{\pm} p^{*} \to \ell^{\pm} p^{*}$

lepton-proton scattering

$$\ell\, p \to \ell\, p$$

lepton-proton scattering (*known subset*)

$$\ell\,\ell' \to \ell\,\ell'$$

lepton-proton scattering (*one step more*)

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ "single-dipole"

$$\ell\, \mu \to \ell\, \mu$$ "point-like"

lepton-proton scattering @MUSE

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ "single-dipole"

$$\ell\, \boldsymbol{\mu} \to \ell\, \boldsymbol{\mu}$$ "point-like"

- $\ell = \{e^\pm,\, \mu^\pm\}$

lepton-proton scattering @MUSE

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ “single-dipole”

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- $\ell = \{e^\pm, \mu^\pm\}$
- $E_{\text{beam}} = 210$ MeV

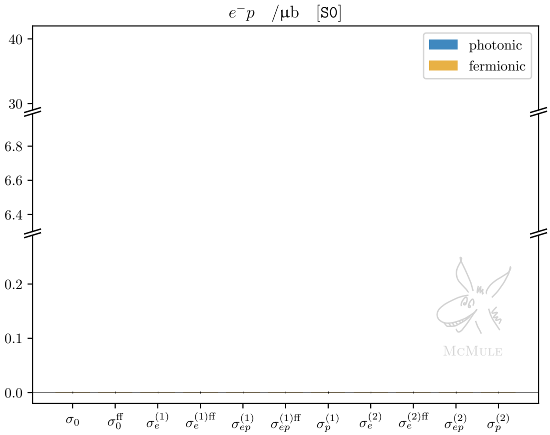
lepton-proton scattering @MUSE

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ “single-dipole”

$$\ell\, \mu \to \ell\, \mu$$ “point-like”

- $\ell = \{e^\pm,\, \mu^\pm\}$
- $E_{\text{beam}} = 210$ MeV
- $20\,\text{deg} < \theta_\ell < 100\,\text{deg}$

Leading-Order QED (α^2)



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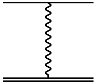

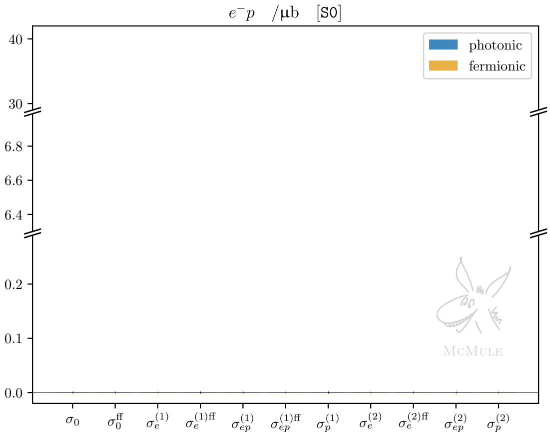

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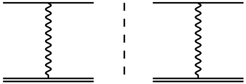
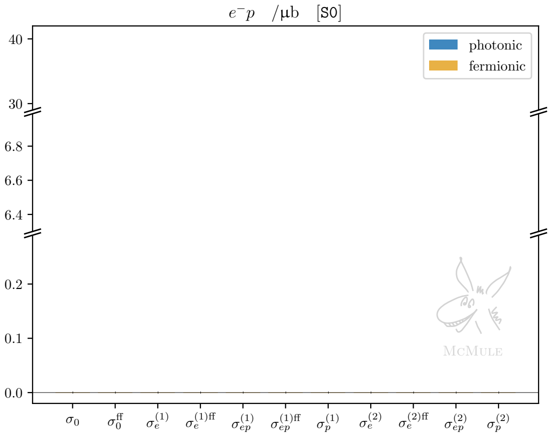
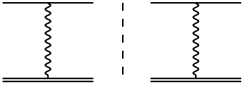
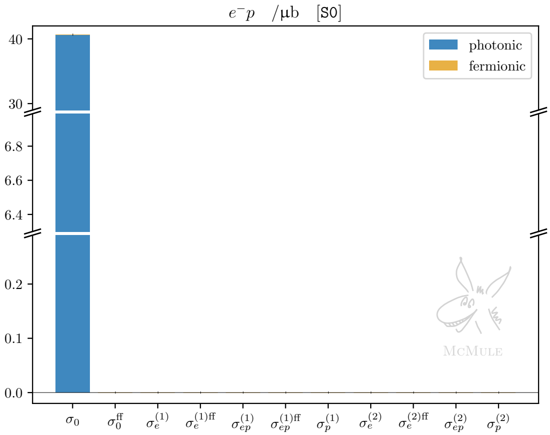




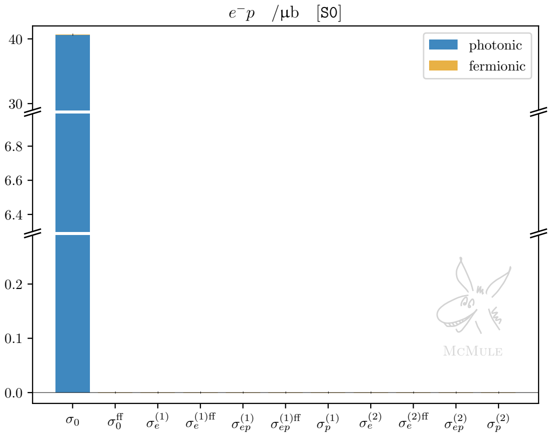
Leading-Order QED (α^2)

$\int_{[d\Phi_2]}$

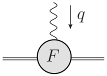




LO QED + single dipole

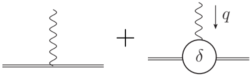


p means proton

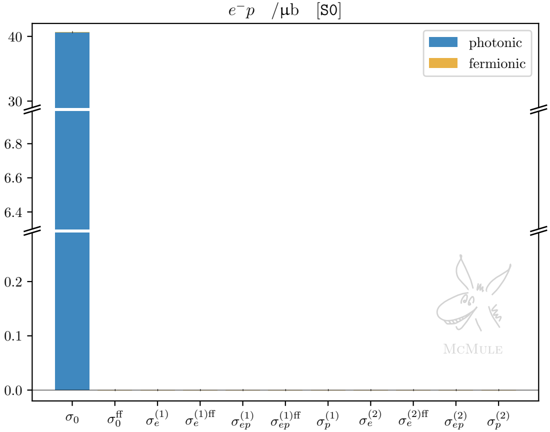


$$e\gamma^\mu F_1(Q^2,\Lambda) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_p} F_2(Q^2,\Lambda)$$

$=$



$$\Lambda^2 = \{0.60, 0.66, 0.71, 0.86\}\,\mathrm{GeV}^2$$



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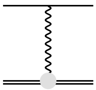

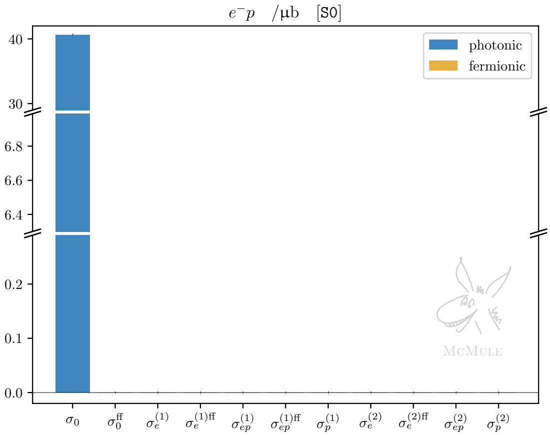

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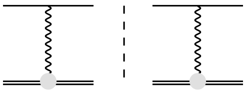

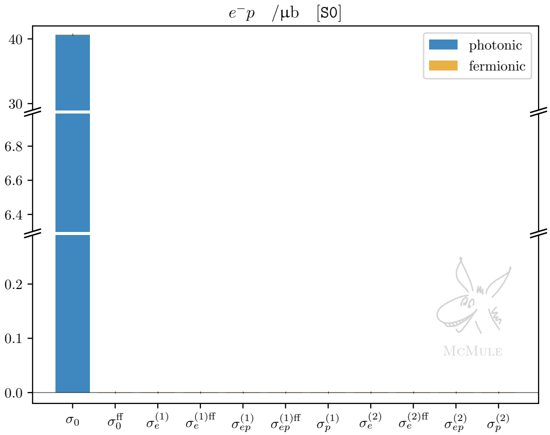

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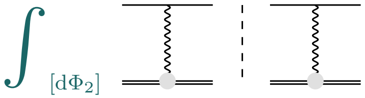

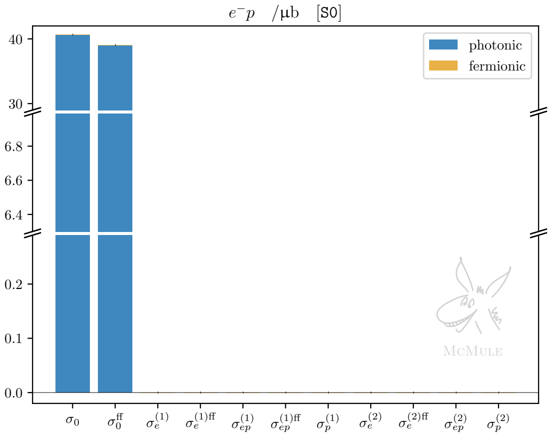

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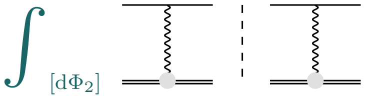
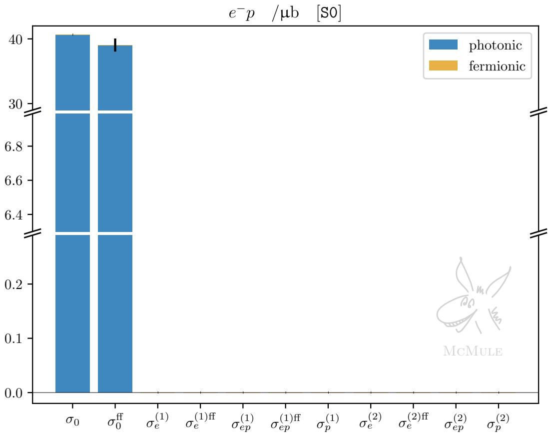
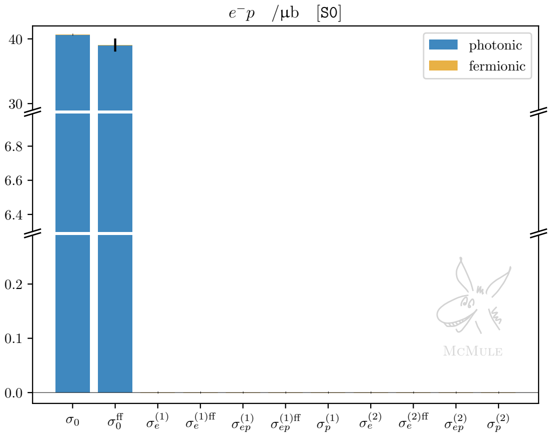




NLO QED (α^3)



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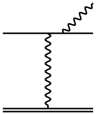

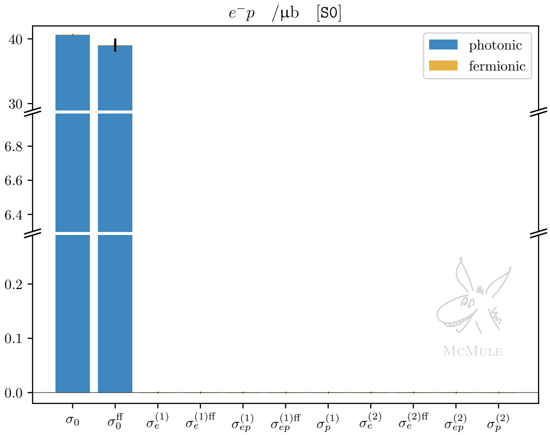

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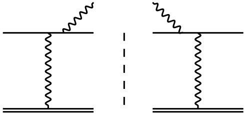

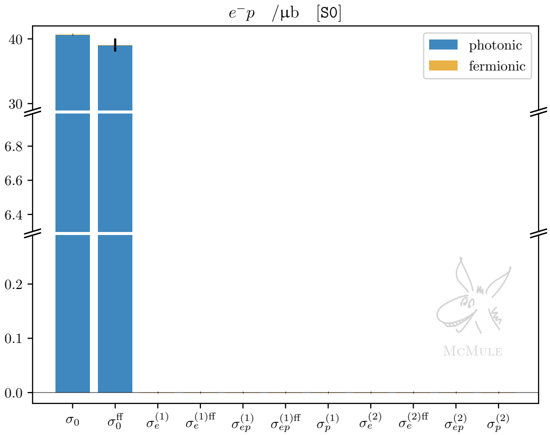

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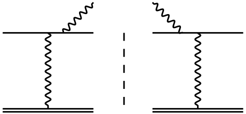
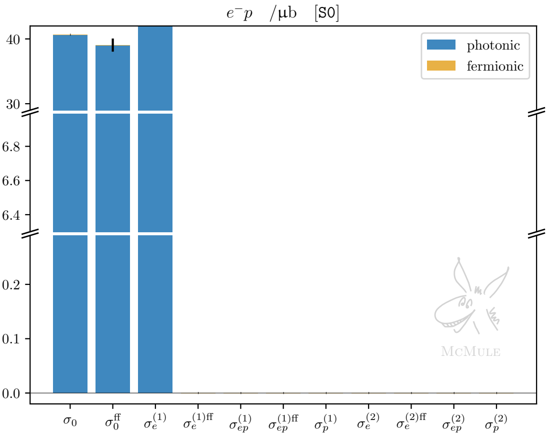
$\int_{[\mathrm{d}\Phi_3]}$





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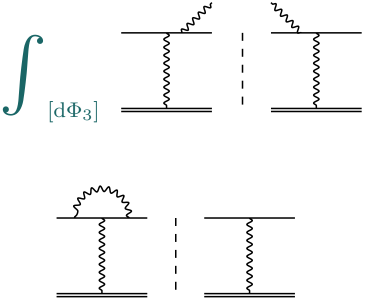

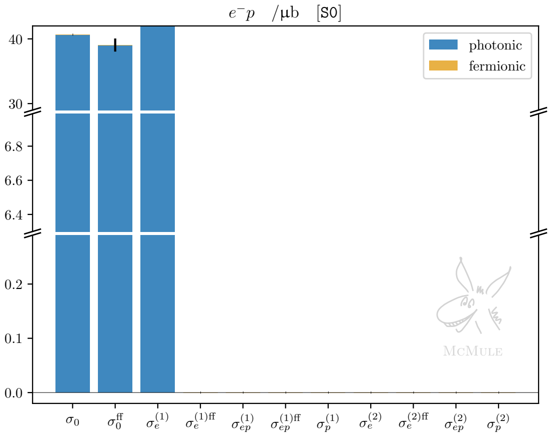

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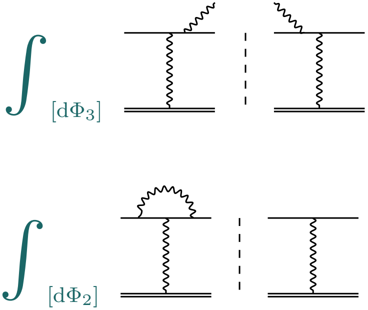
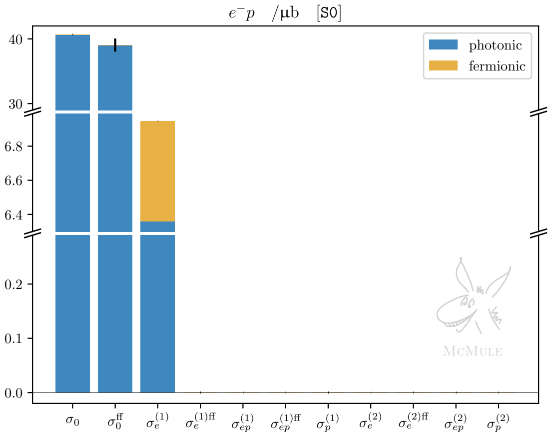
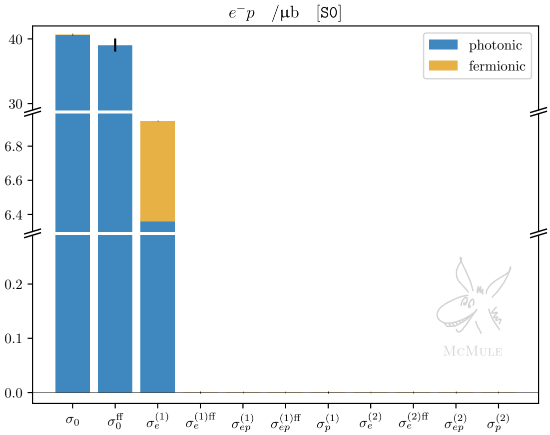




NLO QED + single dipole



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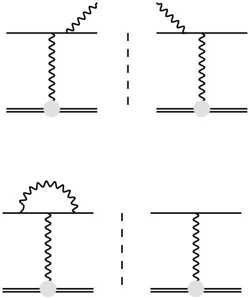

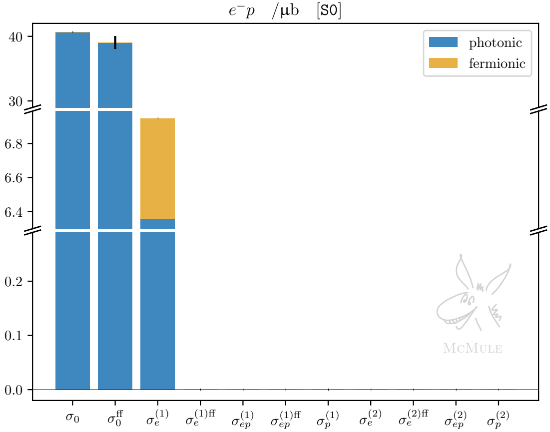

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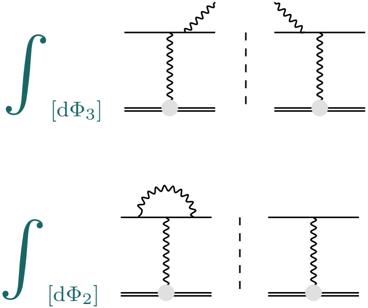

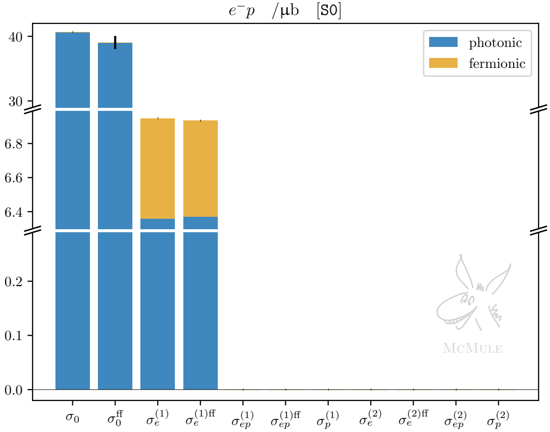

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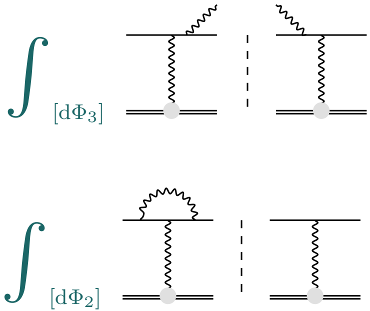
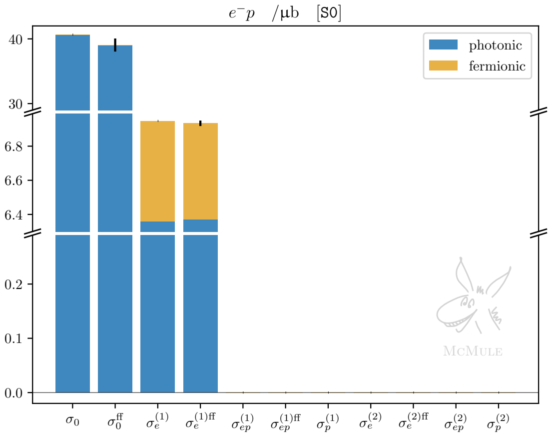
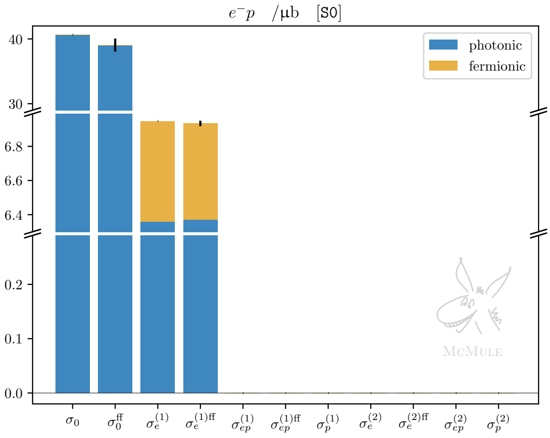




NLO QED pt. 2


e^-p /μb [S0]
photonic
fermionic
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σ_0
σ_0^{ff}
$\sigma_e^{(1)}$
$\sigma_e^{(1)\text{ff}}$
$\sigma_{ep}^{(1)}$
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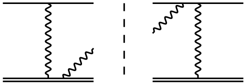
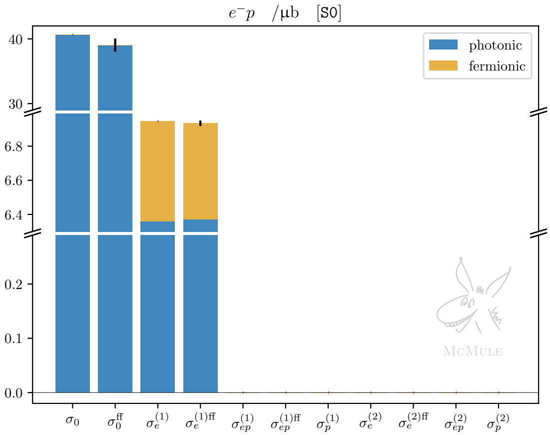
McMule

NLO QED pt. 2

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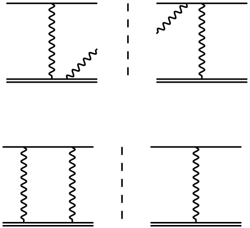
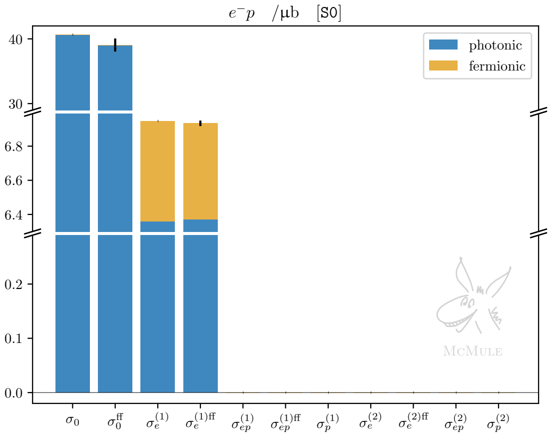
McMule

NLO QED pt. 2

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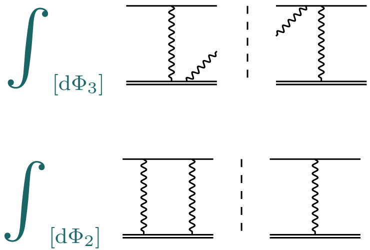
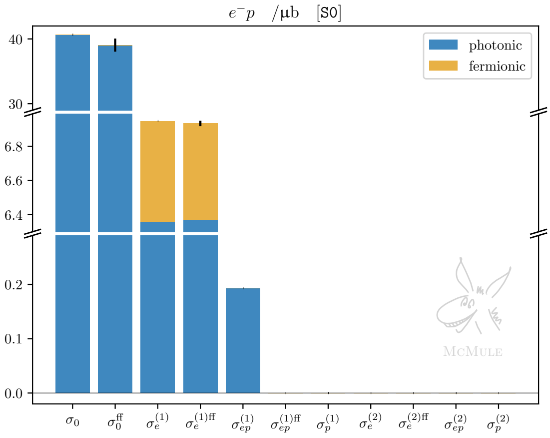
McMule

NLO QED pt. 2


$\int_{[d\Phi_3]}$
$\int_{[d\Phi_2]}$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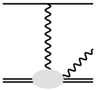
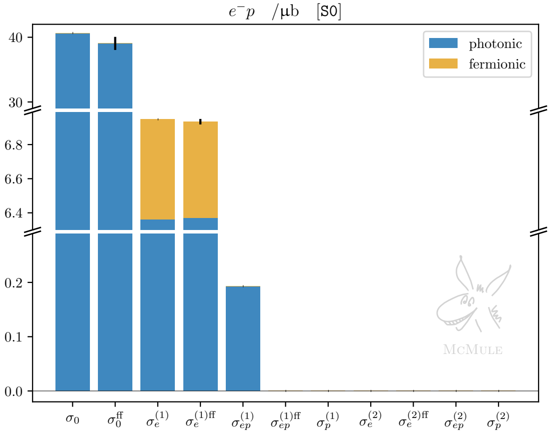
McMule

NLO QED + mess

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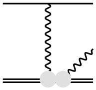
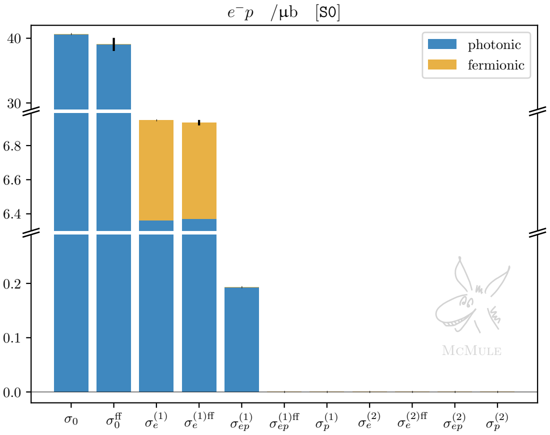
McMule

NLO QED + double dipole





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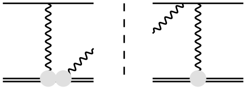

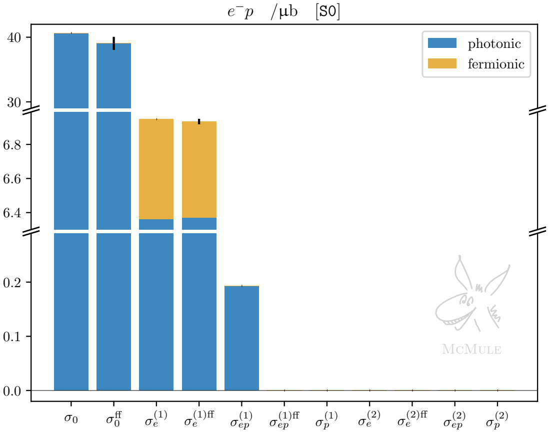

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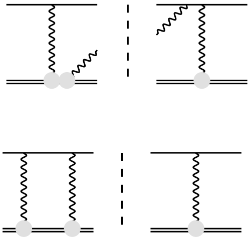

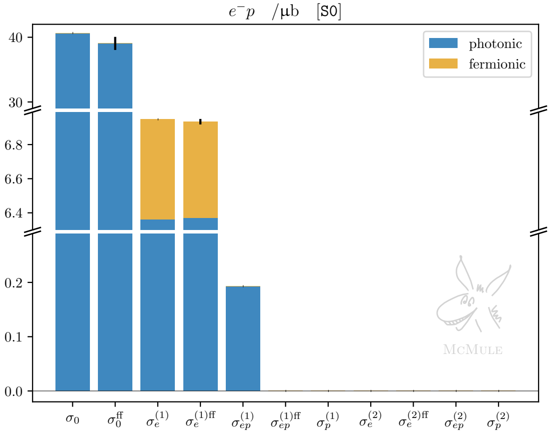

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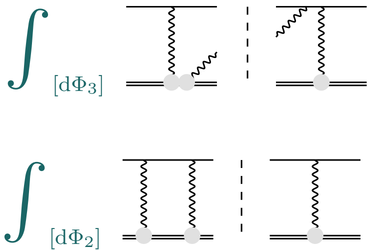

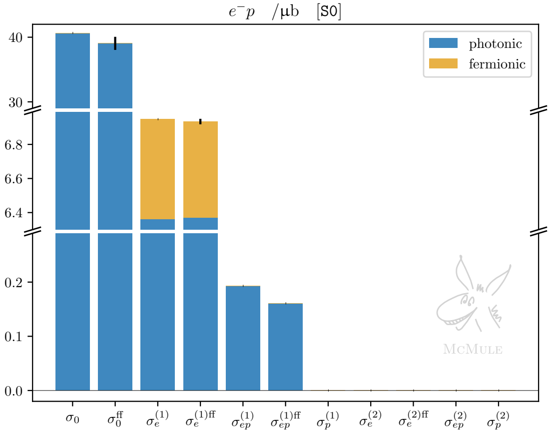

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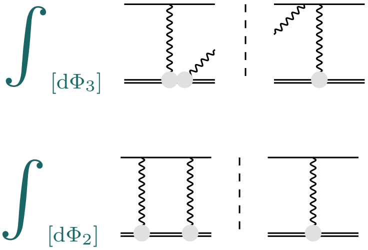
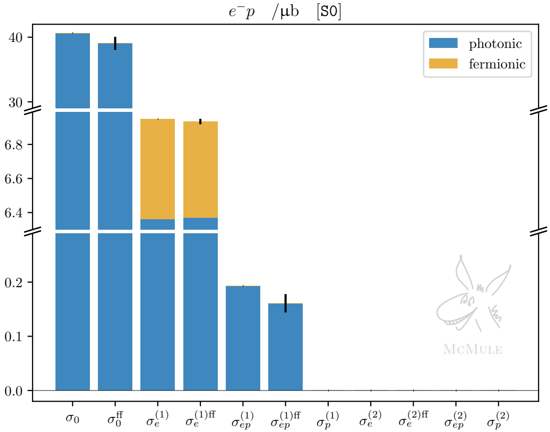
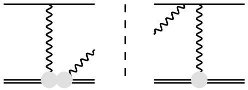


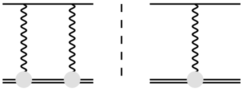


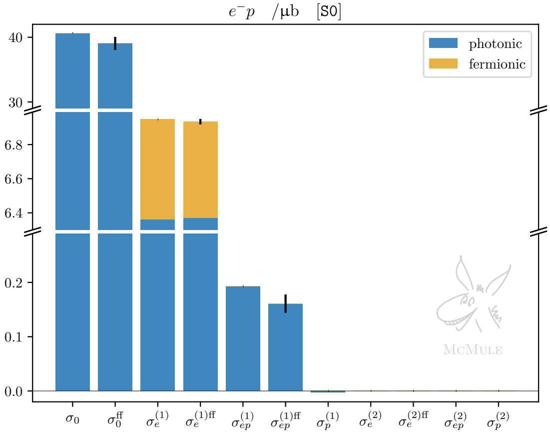
NLO QED + double dipole

$$\int_{[d\Phi_3]}$$

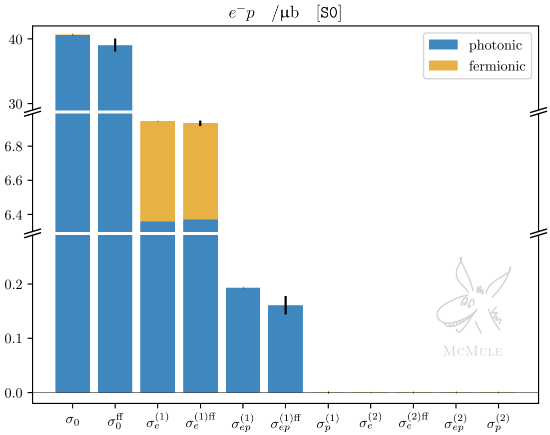


$$\int_{[d\Phi_2]}$$

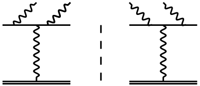


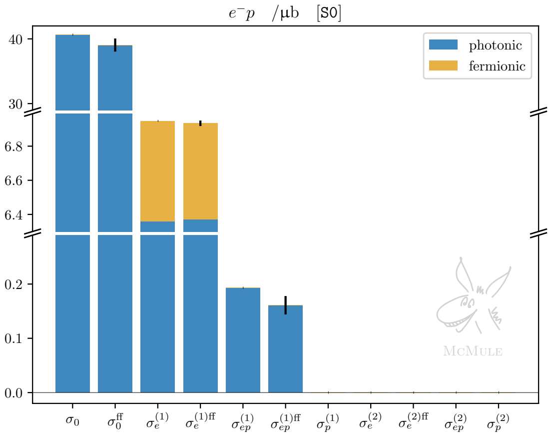


NNLO QED (α^4)

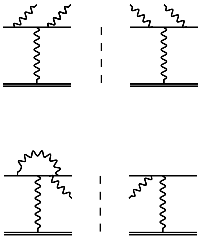


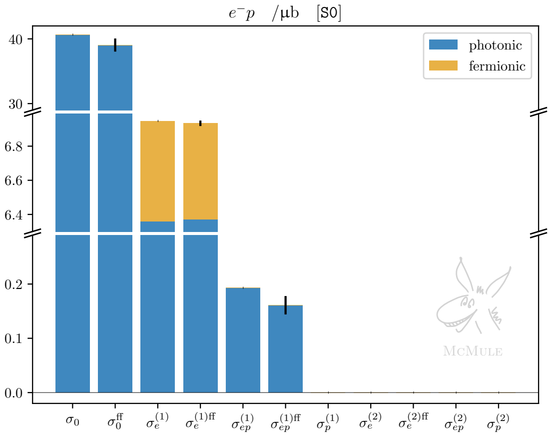
NNLO QED (α^4)



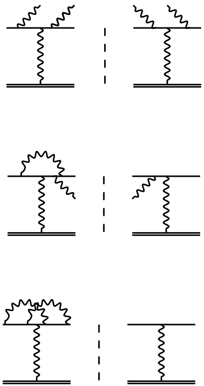


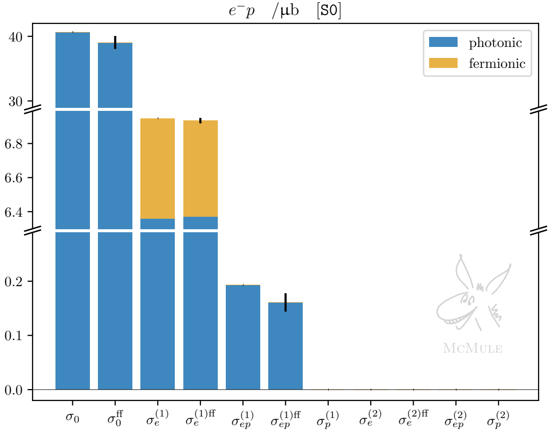
NNLO QED (α^4)





NNLO QED (α^4)



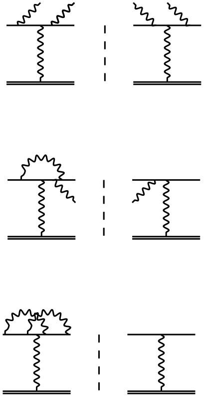


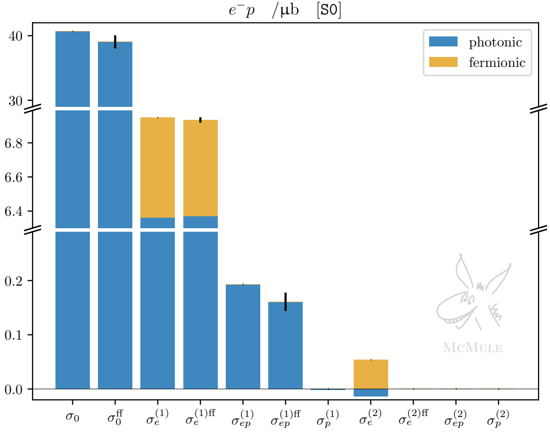
NNLO QED (α^4)

$\int_{[d\Phi_4]}$

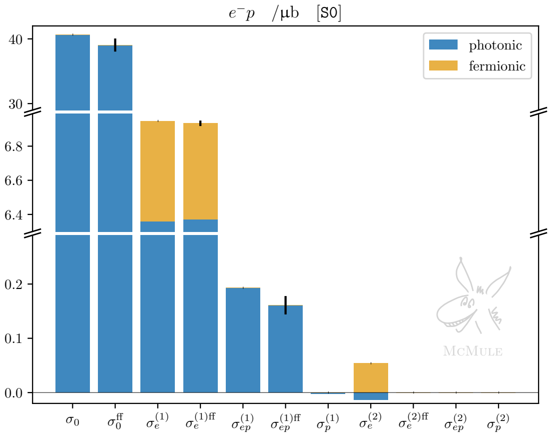
$\int_{[d\Phi_3]}$

$\int_{[d\Phi_2]}$

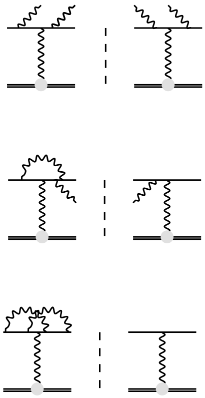


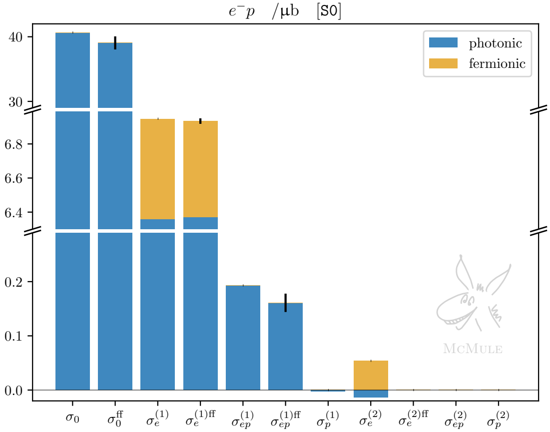


NNLO QED + single dipole

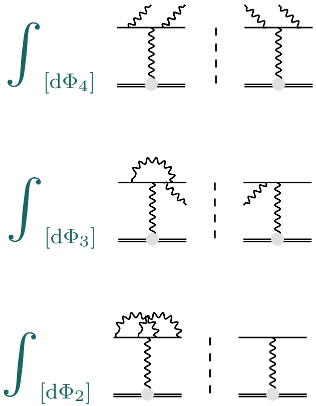


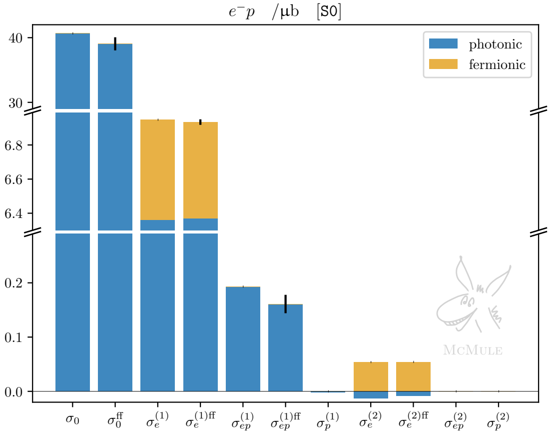
NNLO QED + single dipole



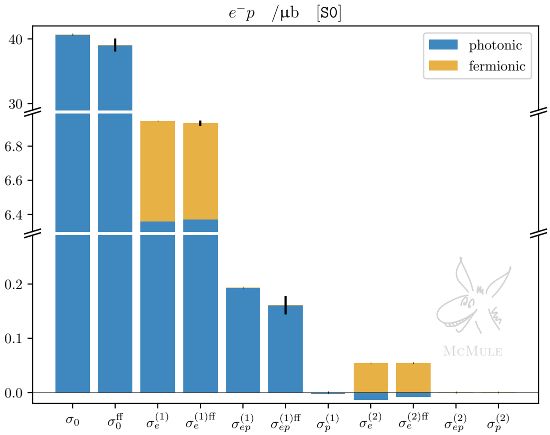


NNLO QED + single dipole

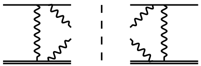


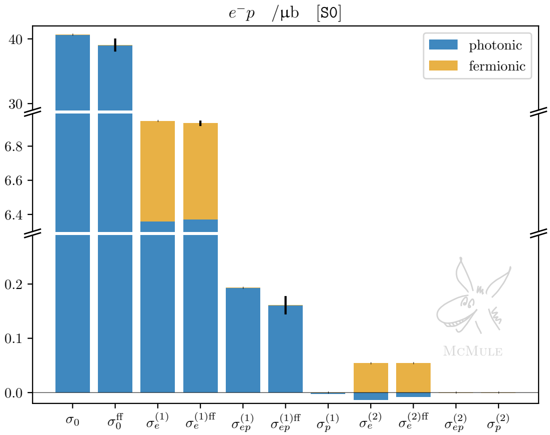


NNLO QED pt. 2


e^-p /µb [S0]
40
30
6.8
6.6
6.4
0.2
0.1
0.0
photonic
fermionic
McMule
σ_0
σ_0^{ff}
$\sigma_e^{(1)}$
$\sigma_e^{(1)\mathrm{ff}}$
$\sigma_{ep}^{(1)}$
$\sigma_{ep}^{(1)\mathrm{ff}}$
$\sigma_p^{(1)}$
$\sigma_e^{(2)}$
$\sigma_e^{(2)\mathrm{ff}}$
$\sigma_{ep}^{(2)}$
$\sigma_p^{(2)}$

NNLO QED pt. 2




e^-p /μb [S0]
photonic
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σ_0
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$\sigma_p^{(1)}$
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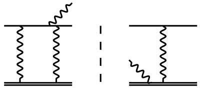
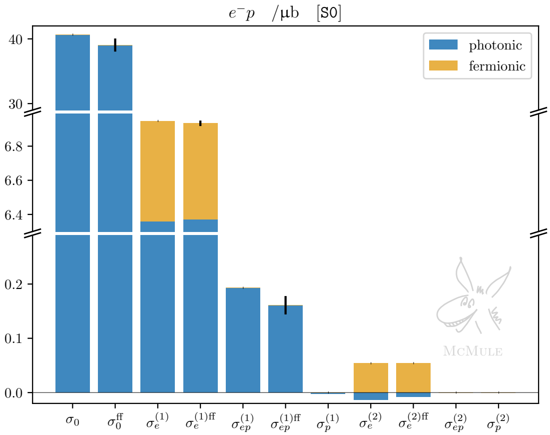
$\sigma_p^{(2)}$
McMule

NNLO QED pt. 2

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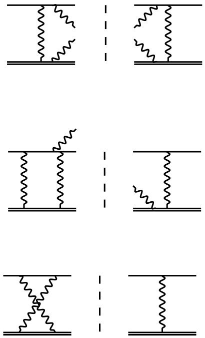
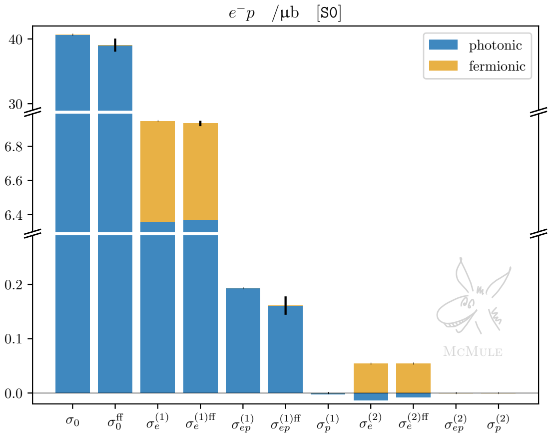
NNLO QED pt. 2

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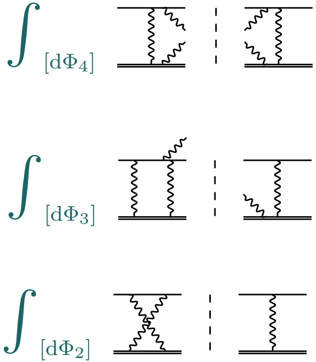
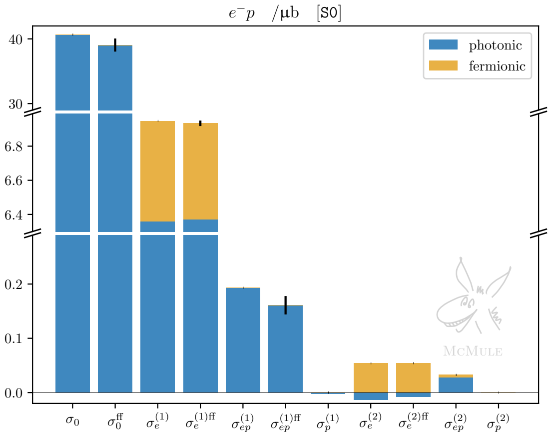
NNLO QED pt. 2


$\int_{[d\Phi_4]}$
$\int_{[d\Phi_3]}$
$\int_{[d\Phi_2]}$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0.0
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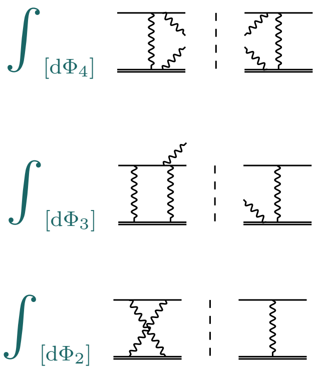
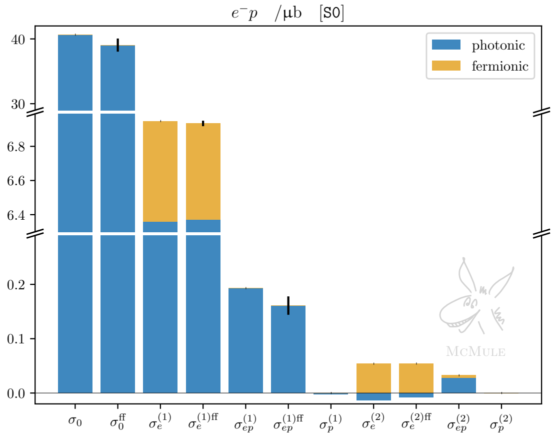
$\sigma_{ep}^{(1)}$
$\sigma_{ep}^{(1){\rm ff}}$
$\sigma_p^{(1)}$
$\sigma_e^{(2)}$
$\sigma_e^{(2){\rm ff}}$
$\sigma_{ep}^{(2)}$
$\sigma_p^{(2)}$
McMule

NNLO QED pt. 2


$\int_{[d\Phi_4]}$
$\int_{[d\Phi_3]}$
$\int_{[d\Phi_2]}$


e^-p /μb [S0]
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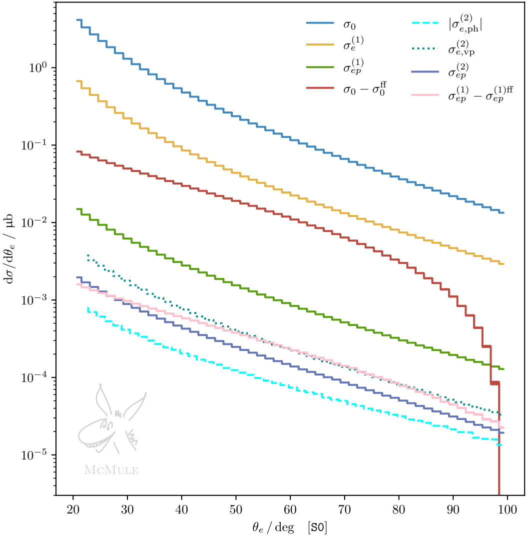
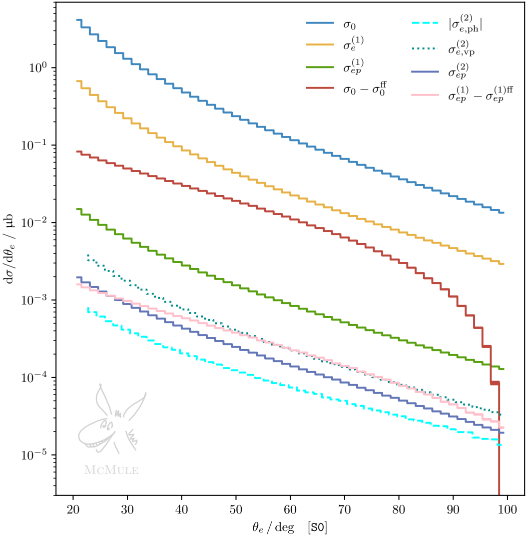
$\sigma_p^{(1)}$
$\sigma_e^{(2)}$
$\sigma_e^{(2)\text{ff}}$
$\sigma_{ep}^{(2)}$
$\sigma_p^{(2)}$
McMule

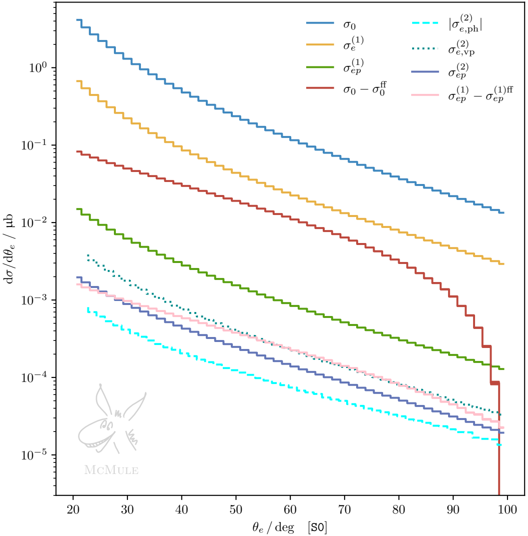
differential recap



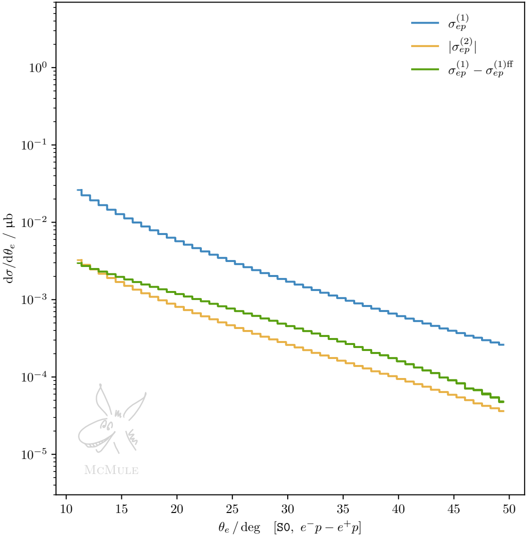


◇ NLO QED $\gtrsim$ LO hadronic

differential recap

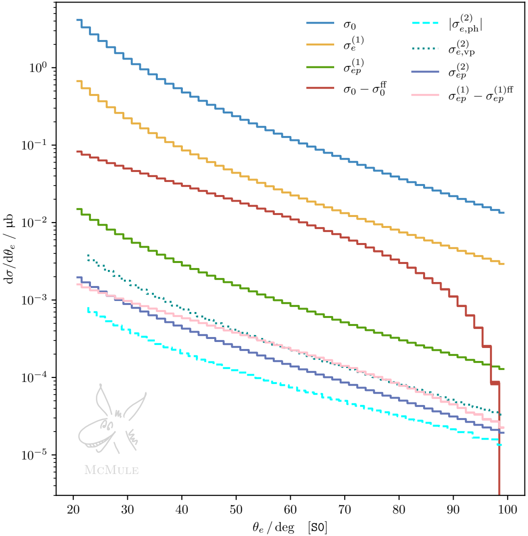


- ◇ NLO QED $\gtrsim$ LO hadronic
- ◇ NNLO QED $\sim$ TPE hadronic



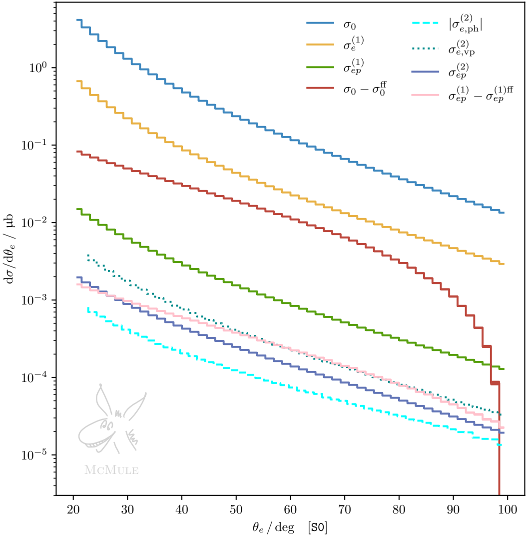
- ◇ NLO QED $\gtrsim$ LO hadronic
- ◇ NNLO QED $\sim$ TPE hadronic

hard cut on hard photons



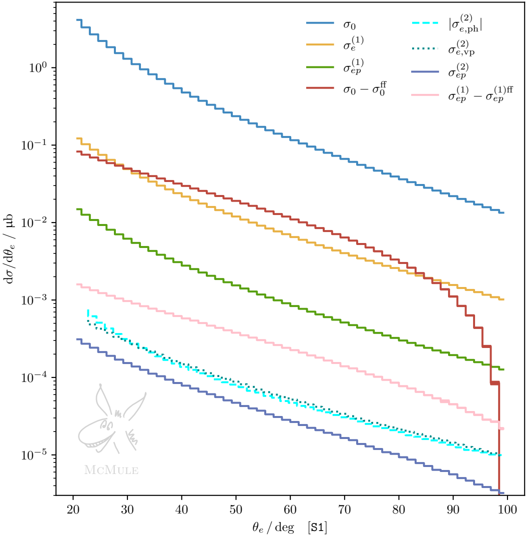
a closer look at MUSE

hard cut on hard photons



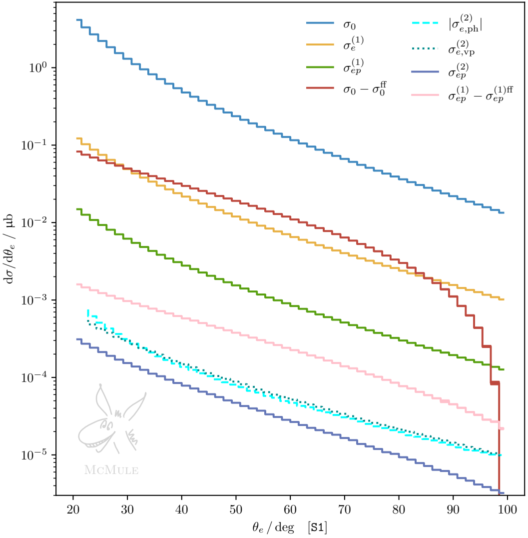
– forward calorimeter ($\sphericalangle \sim 100$ mrad)

hard cut on hard photons



- forward calorimeter ($\sphericalangle \sim 100$ mrad)
- remove events inside w/ $E_\gamma^{\rm tot} > 0.4p$

  (@MUSE $p = 210$ MeV)

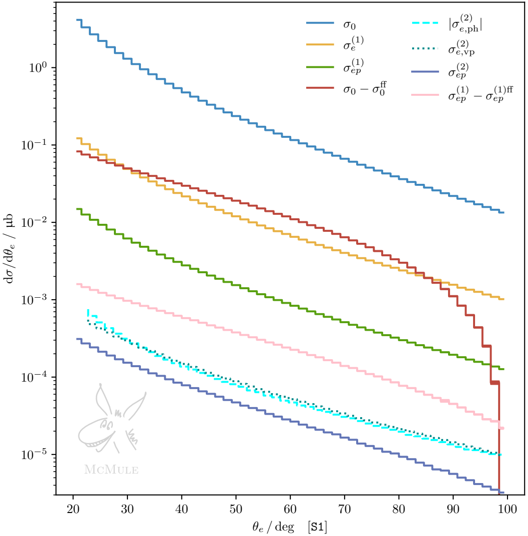


– forward calorimeter ($\sphericalangle \sim 100$ mrad)

– remove events inside w/ $E_\gamma^{\rm tot} > 0.4p$ (@MUSE $p = 210$ MeV)

◇ NLO QED $\sim$ LO hadronic

hard cut on hard photons



– forward calorimeter ($\sphericalangle \sim 100$ mrad)

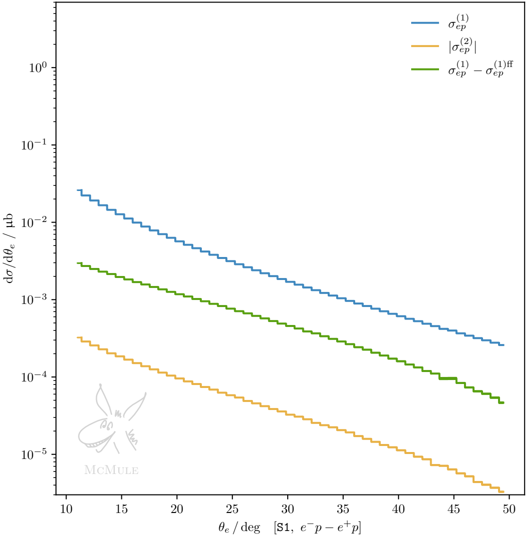
– remove events inside w/ $E_\gamma^{\text{tot}} > 0.4p$

(@MUSE $p = 210$ MeV)

◇ NLO QED $\sim$ LO hadronic

◇ NNLO QED $\lesssim$ TPE hadronic

hard cut on hard photons



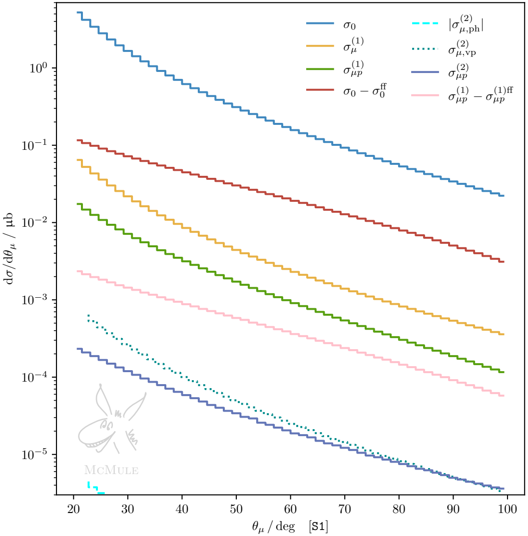
– forward calorimeter ($\sphericalangle \sim 100$ mrad)

– remove events inside w/ $E_\gamma^{\mathrm{tot}} > 0.4p$ (@MUSE $p = 210$ MeV)

◇ NLO QED $\sim$ LO hadronic

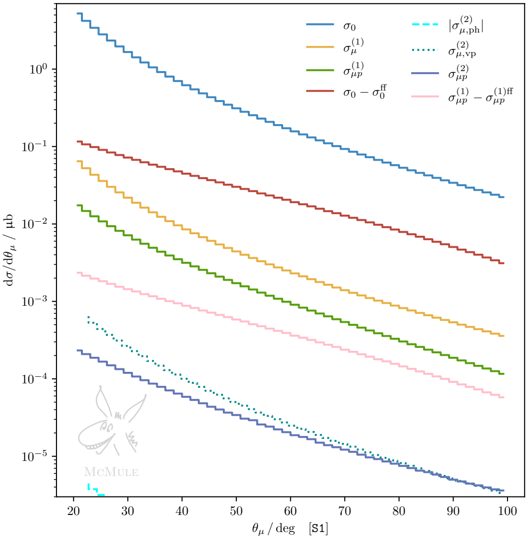
◇ NNLO QED $\lesssim$ TPE hadronic

muons?

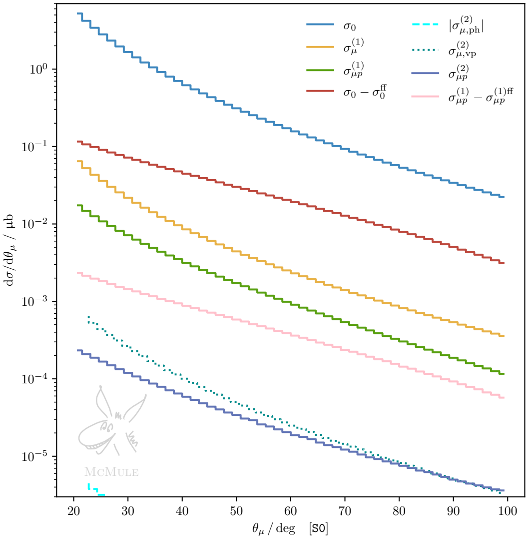


muons are available at MUSE

muons?

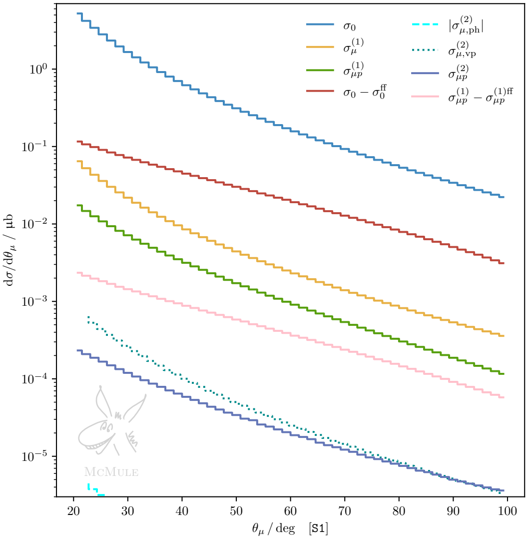


– calorimeter is still there



- calorimeter is still there
- no changes w/out it

muons?

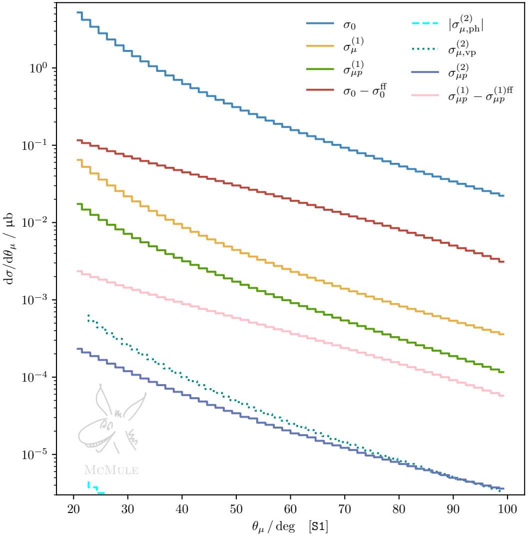


– calorimeter is still there

– no changes w/out it

◇ NLO QED $\lesssim$ LO hadronic

muons?



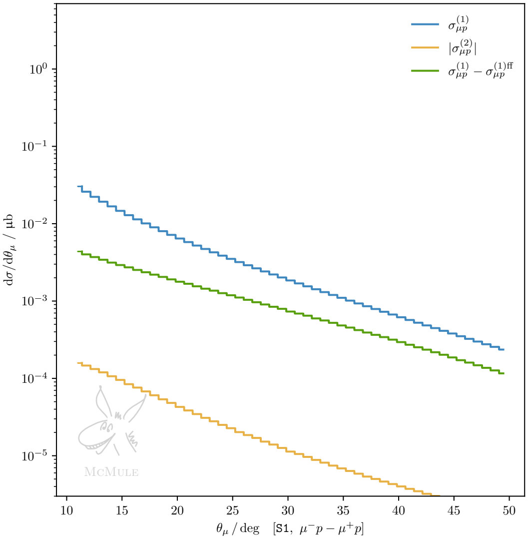
— calorimeter is still there

— no changes w/out it

◇ NLO QED $\lesssim$ LO hadronic

◇ NNLO QED $<$ TPE hadronic

muons?

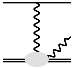


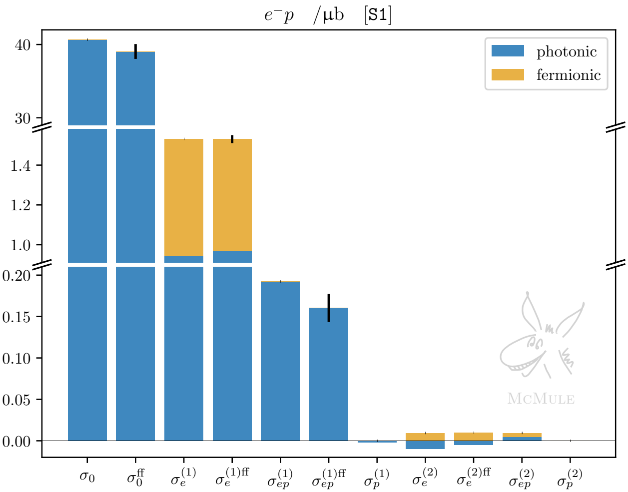
— calorimeter is still there

— no changes w/out it

◇ NLO QED $\lesssim$ LO hadronic

◇ NNLO QED $<$ TPE hadronic

- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic) for MUSE
  ↪ make sure you are taking care of photons → McMule allows you to!
- ballpark estimate for TPE for a simple dipole, with error band
- the mule is currently attacking  (or even fancier things) for more insights



e^-p /μb [S1]
photonic
fermionic
40
30
1.4
1.2
1.0
0.20
0.15
0.10
0.05
0.00
σ_0
σ_0^{ff}
$\sigma_e^{(1)}$
$\sigma_e^{(1)\mathrm{ff}}$
$\sigma_{ep}^{(1)}$
$\sigma_{ep}^{(1)\mathrm{ff}}$
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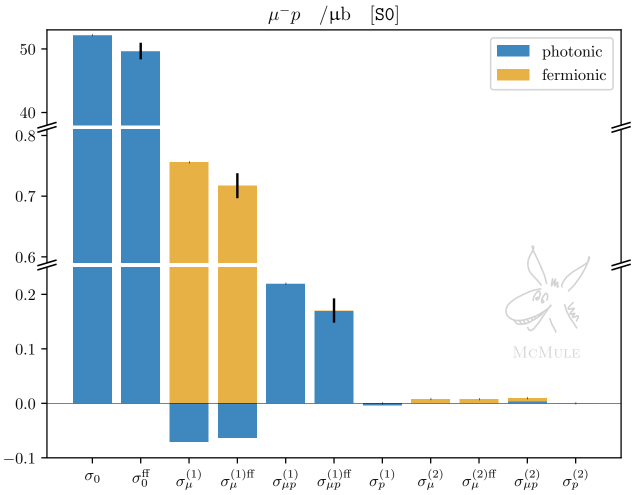
$\sigma_e^{(2)\mathrm{ff}}$
$\sigma_{ep}^{(2)}$
$\sigma_p^{(2)}$
McMule



$\mu^- p$ /µb [S0]
photonic
fermionic
50
40
0.8
0.7
0.6
0.2
0.1
0.0
−0.1
σ_0
$\sigma_0^{\rm ff}$
$\sigma_\mu^{(1)}$
$\sigma_\mu^{(1)\rm ff}$
$\sigma_{\mu p}^{(1)}$
$\sigma_{\mu p}^{(1)\rm ff}$
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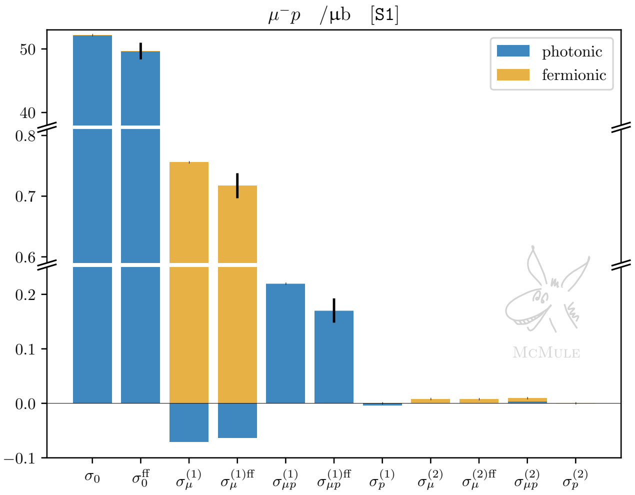
$\sigma_{\mu p}^{(2)}$
$\sigma_p^{(2)}$
McMule


$\mu^- p$ /µb [S1]
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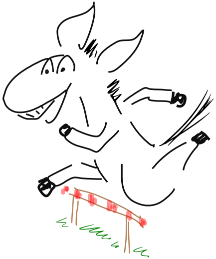
$\sigma_p^{(1)}$
$\sigma_\mu^{(2)}$
$\sigma_\mu^{(2)\mathrm{ff}}$
$\sigma_{\mu p}^{(2)}$
$\sigma_p^{(2)}$
McMule

full muone 2-loop amplitude with $M \neq 0,\, m = 0 \rightarrow$ [Bonciani et al. 21]

full muone 2-loop amplitude with $M \neq 0,\, m \neq 0 \rightarrow$ [??]



$\rightarrow$ exploit scale hierarchy $m^2 \ll M^2, Q^2$

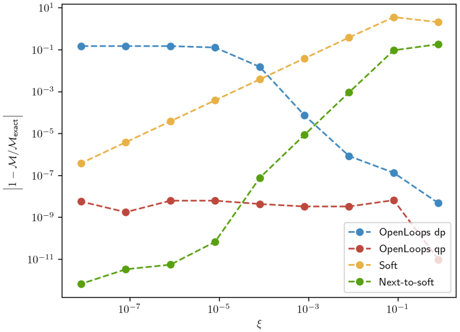
- ◇ massification: $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

  [Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18]

OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

LBK theorem [LBK 58-61, Engel, Signer, Ulrich 21, Engel 23]

$$\mathcal{M}_{n+\gamma} \overset{E_\gamma\to 0}{=\!=} \mathcal{E}\,\mathcal{M}_n + \big(D_{\text{LBK}} + \mathcal{S}\big)\mathcal{M}_n + \mathcal{O}(E_\gamma^0)$$



◇ introduce NTS stabilisation [McMule 21, 22]