$E2$ and $E4$ Deformations in ^{233,234,235,238}U

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Precise intrinsic quadrupole and hexadecapole moments of ^{233,234,235,238}U have been determined from muonic K, L, M, and N x rays. For ^{233,235}U seven $E2$ matrix elements were independently determined. These $E2$ matrix elements are in good agreement with the adiabatic rotational model; this agreement is further improved if a correction for $\Delta K = 1$ band mixing is included. The measured hexadecapole moments are in good agreement with shell-correction calculations and Hartree-Fock calculations.

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As a first approximation, the excited states of deformed nuclei can be understood by regarding the intrinsic and rotational degrees of freedom as separate and uncoupled.[1] In this adiabatic approximation the transition matrix elements that connect rotational states of a single intrinsic configuration are related by purely geometrical factors. Tests of the validity of this notion have, for the lack of sufficient and precise experimental data, been rather limited, especially in the case of odd-A nuclei.

Measurements of the muonic $2p$ and $3d$ hyperfine-splitting energies in high-Z nuclei constitute a sensitive means for determining certain diagonal and off-diagonal $E2$ matrix elements.[2] We have exploited this sensitivity to determine *independently* as many $E2$ matrix elements for the uranium isotopes as possible. In contrast, most previous analyses of muonic actinide spectra[3,4] have provided only a single (intrinsic) electric quadrupole parameter and essentially no information regarding possible departures from adiabatic rotational behavior. Sufficient information was available from our measurements of the muonic hyperfine spectra of the odd-A isotopes ^{233}U and ^{235}U to determine independently the electric quadrupole matrix elements connecting the four lowest states. Moreover, we also determined the intrinsic nuclear hexadecapole ($E4$) moments of both even- and odd-A isotopes with a precision better than 10%, which considerably exceeds the precision achieved from Coulomb excitation measurements.[5]

The muonic x-ray spectra of ^{233,234,235,238}U were

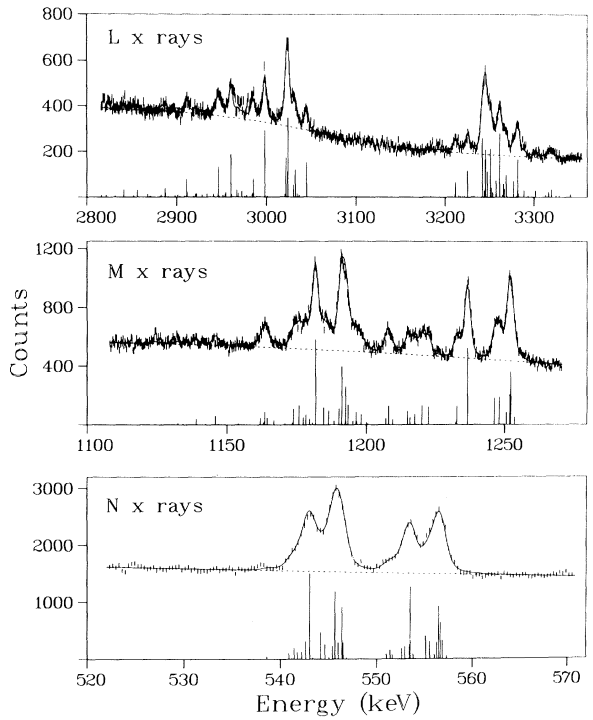

FIG. 1. The ^{235}U muonic L, M, and N x rays and the calculated spectrum. The vertical lines at the bottom of each spectrum indicate the energies and relative intensities of the individual x-ray transitions.

measured at the biomedical channel of the Clinton P. Anderson Meson Physics Facility (LAMPF). The ^{233}U, ^{234}U, ^{235}U, and ^{238}U targets were enriched to 99.47%, 99.83%, 97.64%, and > 99.9%, respectively. The target arrangement, Ge(Li) spectrometer, and data-acquisition system have been described in previous papers.[6,7] Figure 1 shows the muonic L, M, and N x-ray spectra for ^{235}U along with the computed hyperfine spectrum that results from our analysis.

In our analysis the nuclear charge distribution was represented by a deformed Fermi distribution:

$$\rho(\vec{r}) = \rho_0\{1+\exp[(r-R)/a]\}^{-1}, \quad (1)$$

where

$$R = c[1+\beta_2 Y_{20}(\theta,\phi)+\beta_4 Y_{40}(\theta,\phi)].$$

This form for the charge distribution is a reasonably realistic approximation for the intrinsic shape of highly deformed nuclei and provides a convenient *Ansatz*[8,9] for generating approximate forms for the $E2$ and $E4$ transition charge densities that are needed to calculate hyperfine-splitting energies. Table I lists the values of the monopole charge-distribution parameters c and a and the rms radii determined by fitting the present data.

To gauge the magnitude of possible model errors

TABLE I. Uranium charge parameters. The parameters c, a, and $\langle r^2\rangle^{1/2}$ are given in units of femtometers, Q_0 in units of $e\cdot$b, and H_0 in units of $e\cdot$b^2.

	Nucleus	^{233}U	^{234}U	^{235}U	^{238}U
This work	c[a]	6.9518(16)	6.9703(13)	6.9859(17)	7.0110(12)
	a[a]	0.5125(12)	0.5089(10)	0.5029(13)	0.5046(9)
	Q_0[b]	10.294(59)	10.610(57)	10.630(59)	11.188(58)
	H_0[b]	2.55(30)	2.49(14)	2.64(10)	2.28(11)
	β_2[a]	0.2431(40)	0.2507(18)	0.2485(13)	0.2653(14)
	β_4[a]	0.091(15)	0.0843(71)	0.0913(45)	0.0672(49)
	$\langle r^2\rangle^{1/2}$[a]	5.8158(66)	5.8289(31)	5.8343(28)	5.8604(23)
Bemis *et al.* (Ref. 5)	Q_0	· · ·	10.47(5)	· · ·	11.12(7)
(α,α')	H_0	· · ·	$3.31(^{+44}_{-51})$	· · ·	$1.96(^{+47}_{-63})$
Close *et al.* (Ref. 3)	Q_0	· · ·	· · ·	10.51(6)	11.15(5)
μ^- x rays	H_0	· · ·	· · ·	0.34(2)	0.95(9)
de Wit *et al.* (Ref. 4)					
μ^- x rays	Q_0	10.3(3)	· · ·	10.6(2)	11.25(15)

[a]Model-dependent analysis (statistical uncertainties only).
[b]Q_0 and H_0 include 0.5% and 2.0% model uncertainty, respectively.

TABLE II. Spectroscopic quadrupole moments and $B(E2)$ values for ^{233,235}U. Spectroscopic and intrinsic moments (q and Q, respectively) are given in units of $e \cdot b$, and the $B(E2)$ values are given in $(e \cdot b)^2$. Errors do not include model uncertainties.

	^{233}U ($I=\frac{5}{2}$)			^{235}U ($I=\frac{7}{2}$)		
		Theory			Theory	
	Present experiment	Adiabatic rotation	Rotation plus $\Delta K=1$ mixing	Present experiment	Adiabatic rotation	Rotation plus $\Delta K=1$ mixing
$q(I)$	3.663(8)	3.677	3.666	4.936(6)	4.955	4.939
$B(E2;I \rightarrow I+1)$	5.041(16)	5.020	5.034	4.834(16)	4.757	4.816
$q(I+1)$	0.642(30)	0.686	0.649	1.870(30)	1.931	1.851
$B(E2;I \rightarrow I+2)$	1.756(26)	1.757	1.781	1.189(41)	1.223	1.267
$B(E2;I+1 \rightarrow I+2)$	3.969(37)	3.993	3.973	4.653(73)	4.611	4.613
$B(E2;I+1 \rightarrow I+3)$	2.729(41)	2.683	2.720	2.120(51)	2.117	2.193
$B(E2;I+2 \rightarrow I+3)$	2.974(60)	2.948	2.905	3.778(96)	3.842	3.789
Q_{20}		10.295(12)	10.303(13)		10.619(10)	10.651(12)
ζ		0.0	$-1.4(6)\times10^{-3}$		0.0	$-2.5(5)\times10^{-3}$
χ^2/DF		1.5	0.5		6.4	1.6

in our analysis, we have compared[9] the quadrupole transition densities computed from Eq. (1) with realistic densities obtained from a Fourier-Bessel analysis of inelastic-electron-scattering cross sections[10] for ^{238}U. We find that the model error is quite small: The quadrupole matrix elements for ^{238}U derived from the two transition densities differ by less than 1%. A similar test is not possible in the hexadecapole case since the electron-scattering data do not allow extraction of a model-independent hexadecapole transition density; however, a density derived from a Tassie-model–based (e,e') analysis yielded hexadecapole matrix elements that differed from those based on Eq. (1) by less than 2%.

Our analysis of the spectra employed methods that have become standard in the Los Alamos muonic-atom group and have been discussed previously.[8,11] However, in the present case both diagonal and nondiagonal nuclear matrix elements for three multipolarities ($M1$, $E2$, and $E4$) were included in the calculations, whereas, in analyses of lighter nuclei, inclusion of $M1$ and $E4$ static moments for only the ground state was, in general, an entirely adequate approximation. The most poorly understood aspect of the muonic analysis probably involves nuclear polarization corrections; we find[9] that any reasonable change in the parameters of nuclear polarization provides results that are within the experimental errors of the values listed in Table I. This is also true for the results given in Table II which we discuss below.

Our results for the individual $E2$ matrix elements of ^{233}U and ^{235}U are given in Table II. In the rigid-rotor limit (adiabatic rotation) the intraband $E\lambda$ matrix elements are related to the intrinsic electric moment of order λ, $Q_{\lambda 0}$, by[12]

$$\langle I_2||M(E\lambda)||I_1\rangle = (2I_1+1)^{1/2}[(2\lambda+1)/16\pi]^{1/2}\langle I_1 K \lambda 0|I_2 K\rangle Q_{\lambda 0}. \quad (2)$$

Figure 2 displays for ^{233}U and ^{235}U the percentage deviation of each of the seven separately determined $E2$ matrix elements from the predictions of Eq. (2). The adiabatic rotational model agrees with experiment to within $\pm 3\%$ in all cases!

Even in a highly deformed nucleus such as uranium, the ground-state rotational band involves admixtures of higher excited states. The effect of Coriolis coupling, the primary admixture mechanism, on the matrix elements of the ground-state band has been computed in Ref. 1 (Eq. 4-183). If a correction for the Coriolis effect ($\Delta K=1$ mixing) is included in the rotational-model calculation, the agreement with experiment is further improved (see Table II). The value of the mixing parameter ζ obtained from the present matrix element data is consistent with that deduced[1] from Coulomb excitation probabilities of excited bands.

For ^{234,238}U we could independently determine only three $E2$ matrix elements [$B(E2;0^+ \rightarrow 2^+)$, $q(2^+)$, and $B(E2;2^+ \rightarrow 4^+)$] from the muonic data. Although these matrix elements agree[9] well with Eq. (2), their limited number makes them a

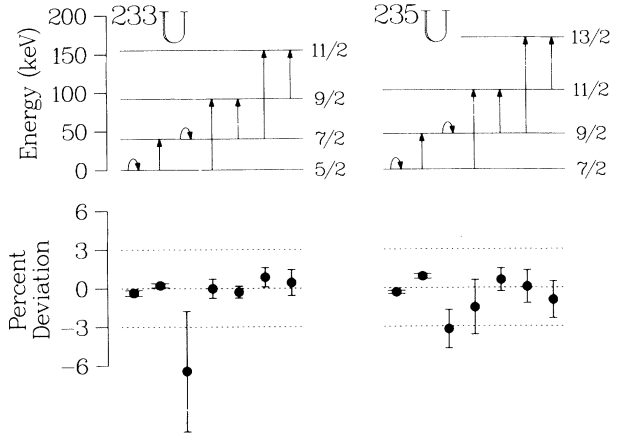

FIG. 2. The $E2$ matrix elements of ^{233}U and ^{235}U independently determined in the present experiment. In the upper section of the figure a circular line closing on a given level represents a spectroscopic quadrupole moment and a line connecting two levels represents a transitional $E2$ matrix element. In the lower part of the figure the deviation of each matrix element from the value calculated with use of Eq. (2) with the value of Q_{20} given in Table II is represented.

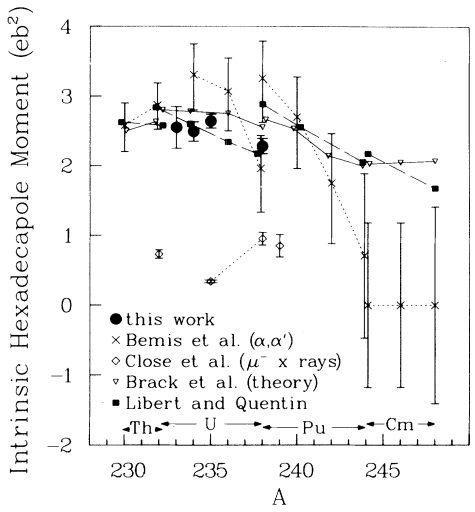

FIG. 3. The intrinsic hexadecapole moment vs A for thorium, uranium, plutonium, and curium isotopes. The legend indicates the sources of the data plotted. Values for isotopes of the same element are connected by dashed or solid lines.

less interesting test of the rotational model than the odd-A cases. Table I gives, for the even isotopes, only the values of Q_{20}.

The magnitude of the muonic hyperfine splitting caused by the $E4$ interaction is only about 1 keV in the uranium $3d$ states (i.e., $\sim \frac{1}{30}$th of the $E2$ splitting). It was therefore not practical to fit individual $E4$ matrix elements as was done in the quadrupole case. Instead, we reverted to fitting a single intrinsic hexadecapole parameter from which all $E4$ matrix elements were computed via Eq. (2). Both static and transitional $E4$ interactions were included in this analysis.

The values of the intrinsic hexadecapole moments that we determined for the four uranium isotopes are given in Table I, along with results from other experiments. Figure 3 graphically displays these data as well as the theoretical predictions of Brack *et al.*[13] and those of Libert and Quentin.[14] Our results are of sufficient precision to provide the first verification that current Hartree-Fock calculations[14] can accurately predict both the magnitude of the nuclear hexadecapole moments and their variation with neutron number. The present data are also in reasonable agreement with the less precise Coulomb excitation results. However, the muonic-atom data reported by Close, Malanify, and Davidson[3] are in marked disagreement. We believe that the neglect of off-diagonal $E4$ interactions in the latter analysis may be responsible for the very small $E4$ moments that they reported.

The Coulomb excitation data indicate that the hexadecapole deformations decrease sharply in the heavier actinides (Pu,Cm). This behavior is consistent with the trend of hexadecapole deformation in lower shells[15] (e.g., the deformed rare-earth elements), for which there seems to be a basic shell-model explanation.[16,17] However, the $E4$ deformations predicted by the shell-correction calculations of Brack *et al.* and the Hartree-Fock calculations of Libert and Quentin do not appear to decrease significantly near midshell, and increasingly disagree with Coulomb excitation results toward the higher mass isotopes.

In view of the precision that the muonic-atom technique brings to the determination of $E4$ moments, as evidenced by the present results for uranium, such measurements on the plutonium and curium isotopes should be valuable in exploring the discrepancy discussed above and, additionally, in establishing reliable nuclear deformation parameters for heavy nuclei against which microscopic calculations can be compared.

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