NON-PERTURBATIVITY IN THE CONTEXT OF THE NEUTRON ELECTRIC DIPOLE MOMENT

LTP(🍕)hD SEMINAR

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OUTLINE

1) EFFECTIVE FIELD THEORIES

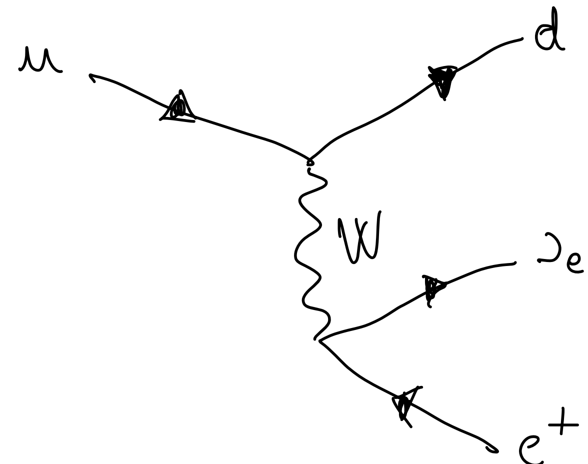
2) RENORMALIZATION/NON-PERTURBATIVITY

3) GRADIENT FLOW FORMALISM

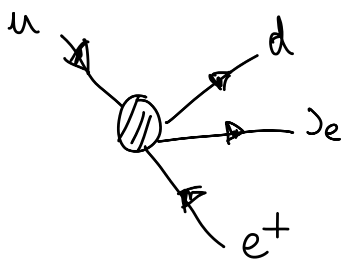
EFFECTIVE FIELD THEORIES

- 1933 Fermi introduced a 4-fermion interaction for β-decay.

  Now we know that it happens through



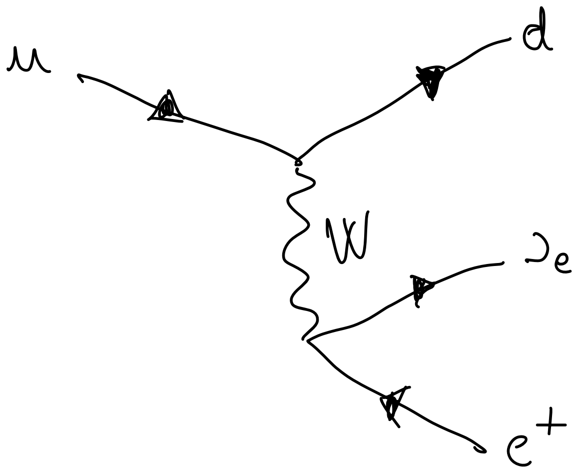
- But when there was no W boson (no SM) what do you observe?



$$\mathcal{L} = (\bar{q} q)(\bar{\ell} \ell) \qquad D = 4 \cdot \frac{3}{2} = 6$$

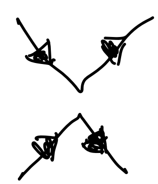
$\Rightarrow$ Not renormalizable in the traditional sense

- Not renormalizable ⇒ Valid only at low energies. But you use it to postulate the W!



$$\sim \frac{1}{p^2-M_W^2} = \frac{-1}{M_W^2}\cdot\left(1+\frac{p^2}{M_W^2}+\mathcal{O}\left(\frac{p^4}{M_W^4}\right)\right)$$

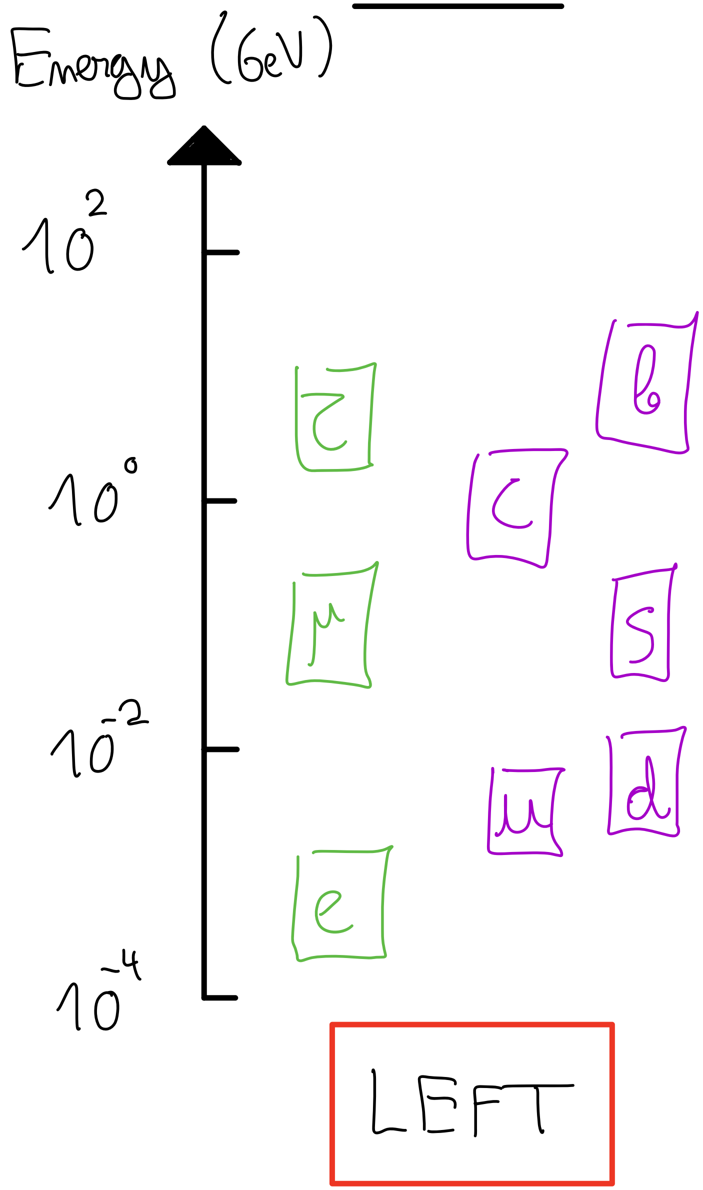
$$= \frac{-1}{M_W^2}\cdot$$



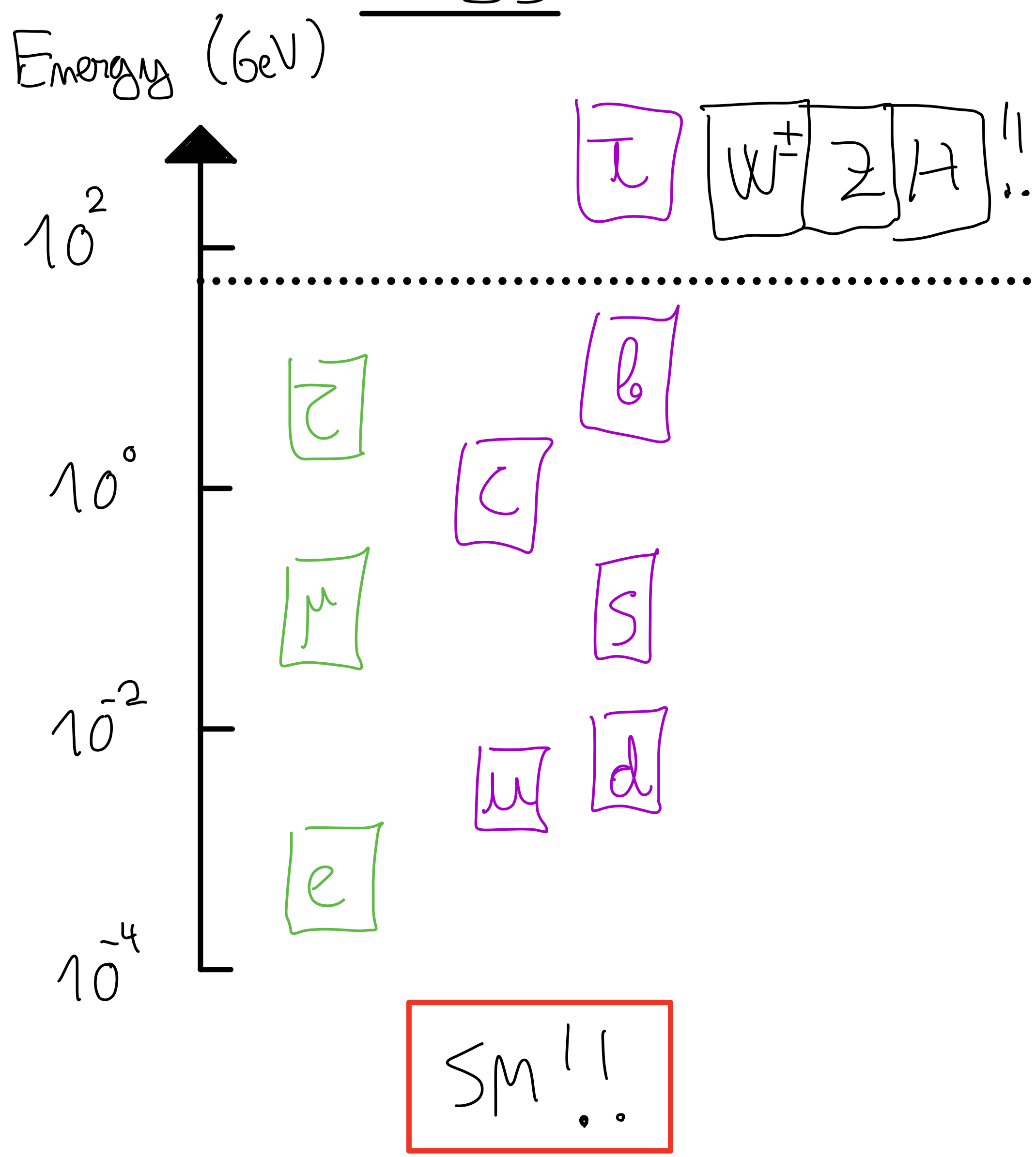
+ dimension 8 operator + ...

EFFECTIVE FIELD THEORIES

"1934"

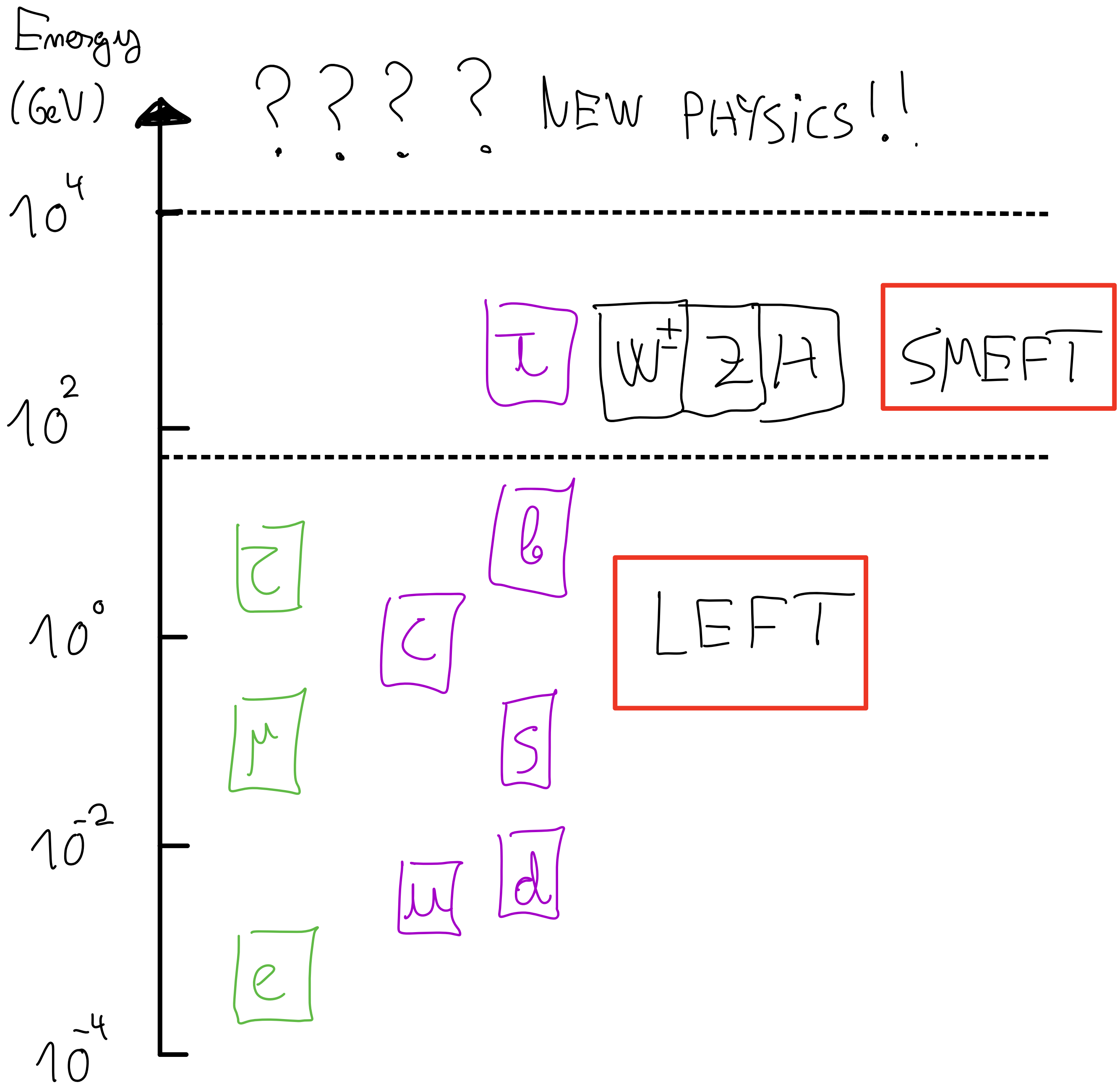


2023

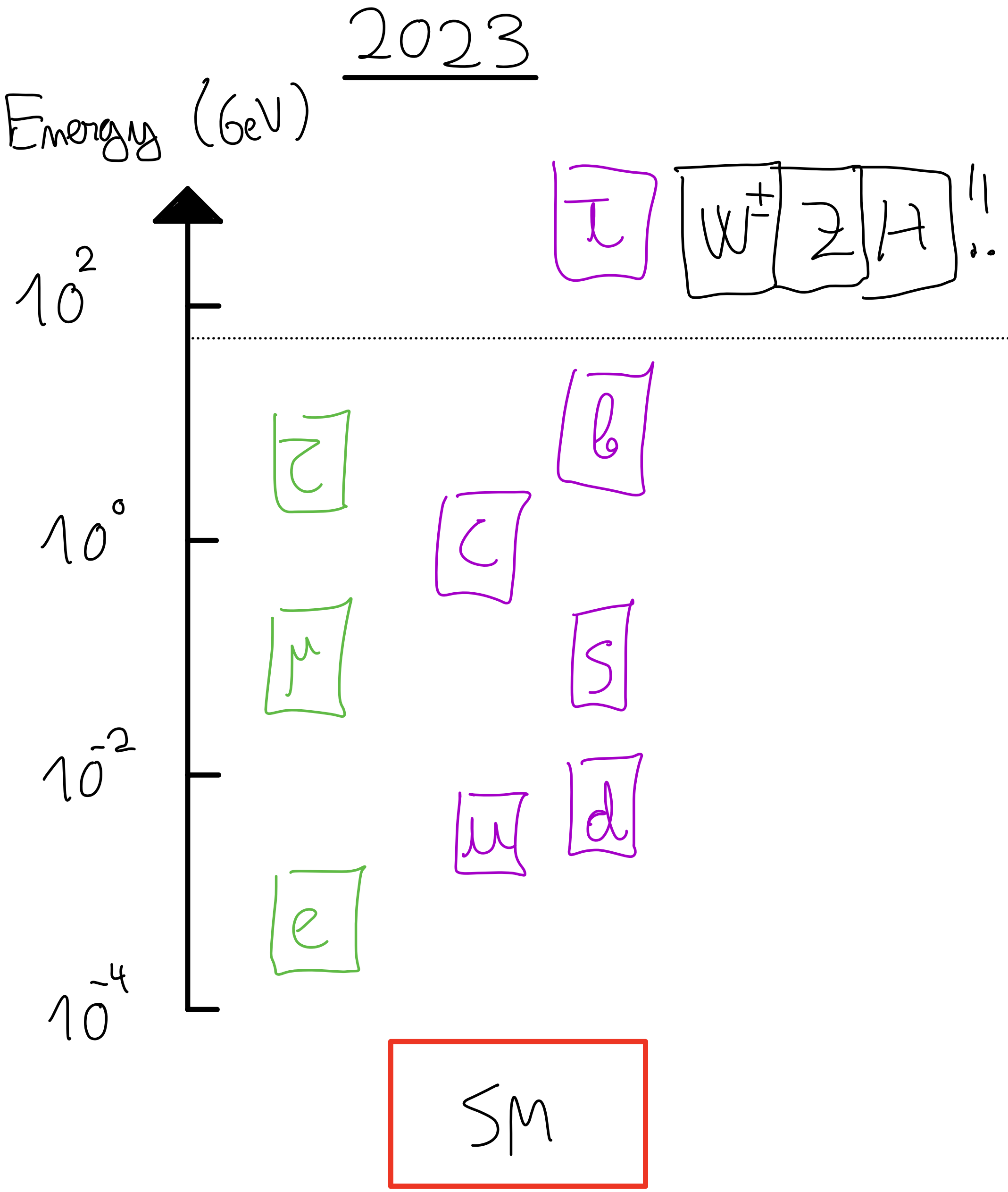


EFFECTIVE FIELD THEORIES

Is this all?

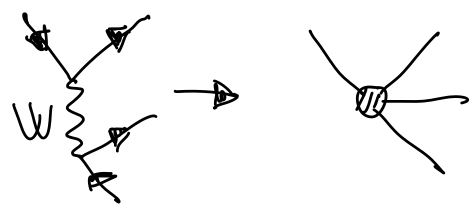


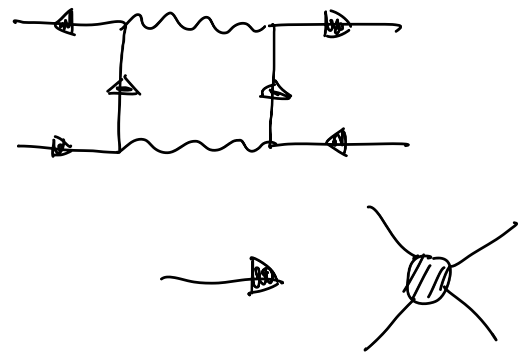
EFFECTIVE FIELD THEORIES



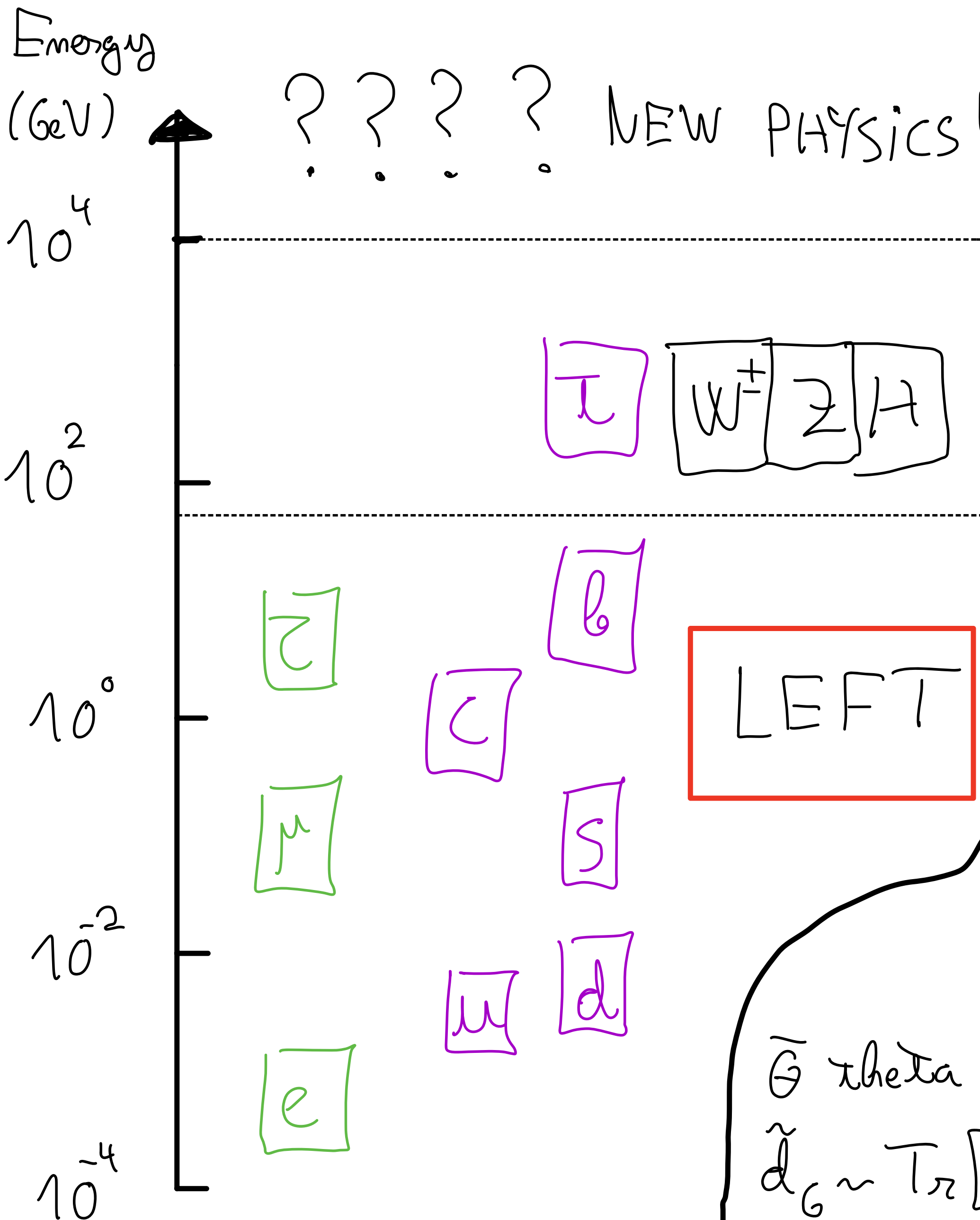
- Also useful for calculations.

If I am below the EW scale, why would I include W, Z in my calculations?





EFTs APPLIED TO THE NEDM



- $\mathcal{L}_{LEFT} = \mathcal{L}_{QCD+QED} + \sum_{d\geq 5}\sum_{i=1}^{n_d} L_i^{(d)}\,\mathcal{O}_i^{(d)}$
- $d_n \sim \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$
- $d_n = -1.5\times10^{-3}\,\bar\theta\, e\cdot fm - 0.2\, d_u + 0.78\, d_d + 0.0027\, d_s - 0.55\, e\cdot\tilde d_u - 1.1\, e\cdot\tilde d_d + 50\,\text{MeV}\cdot e\, d_{\tilde G}$

$\bar\theta$ theta term, d_q quark EDM, $\tilde d_q$ chromo EDM

$d_{\tilde G} \sim \mathrm{Tr}[G_{\mu\nu}G_{\nu\lambda}\tilde G_{\lambda\mu}]$

EFTs APPLIED TO THE NEDM

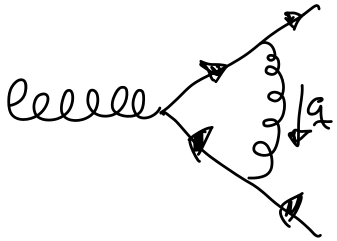
- $$d_n = -(1.5 \pm 0.7) \cdot 10^{-3}\, \bar{\theta}\, e \cdot \mathrm{fm} - (0.20 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e \tilde{d}_u - (1.1 \pm 0.55) e \cdot \tilde{d}_d + (50 \pm 40)\, \mathrm{MeV}\, e \cdot \tilde{d}_G$$

Lessons learned:

1) We need to measure multiple EDMs
2) Uncertainties are too big!
3) Lattice is key!

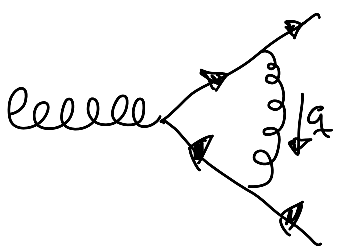
RENORMALIZATION

- In theory calculations, we encounter unphysical infinities :(



$$= \int_q \text{something} = \infty$$

- The most convenient way to deal with this is to promote the number of space-time dimensions to be D (instead of 4), and then expand $D = 4 - 2\epsilon$.



$$= \int_q \text{something} \sim \frac{1}{\epsilon} + \left(\text{finite as } \epsilon \to 0\right)$$

RENORMALIZATION

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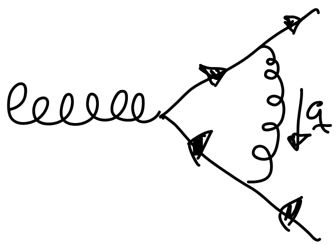


$$= \int_q \text{something} \sim \frac{1}{\epsilon} + \text{finite as } \epsilon \to 0$$

- Still, if $\epsilon \to 0$, we get ∞!!

  $\mathcal{L} = g \cdot \bar{q}\gamma_\mu q A^\mu$, g is not physical, it is a Lagrangian parameter.

  So we do $g \to g - \frac{1}{\epsilon}$ to cancel the infinity!! :)

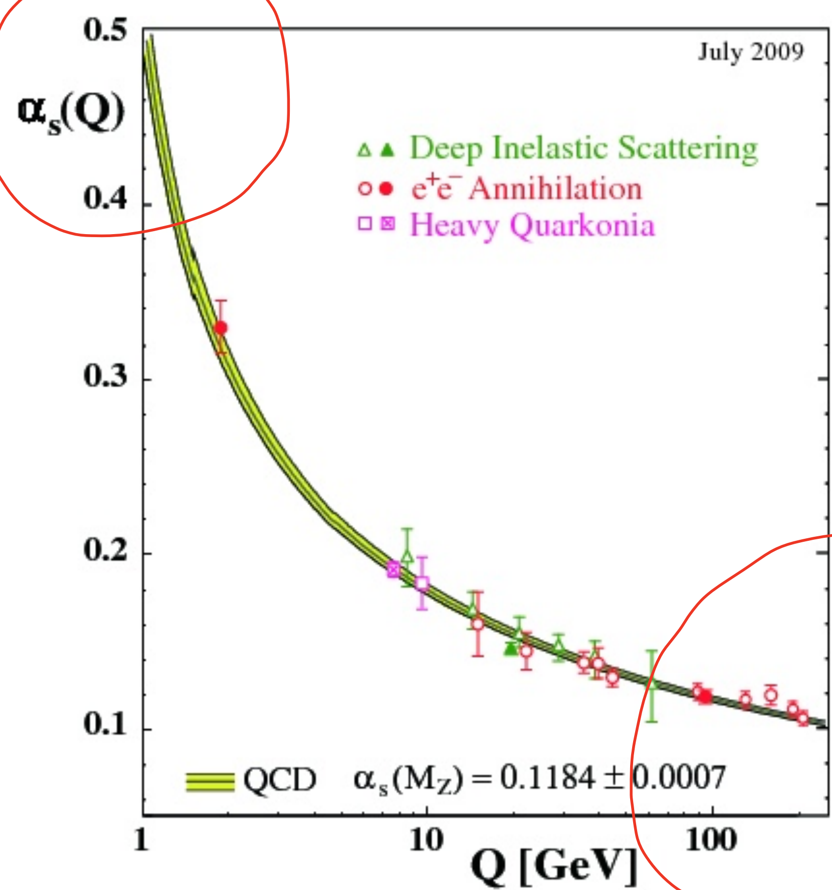
  now physical

RENORMALIZATION / NON-PERTURBATIVITY

- Coupling constants acquire scale dependence

Perturbation theory breaks down

:(

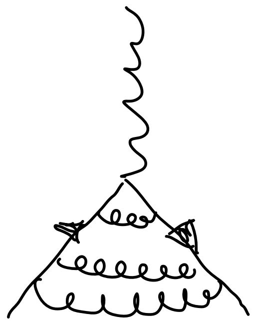


Perturbation theory works well!!

:)

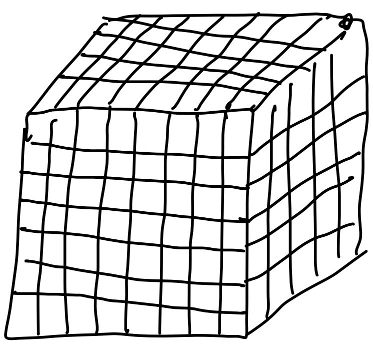
NON-PERTURBATIVITY

- We are not able to use perturbation theory for QCD at low energies

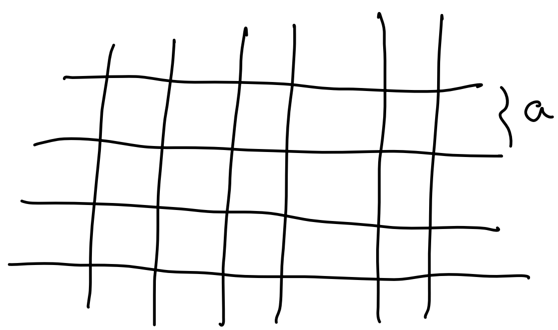


doesn't converge

- But we can calculate/simulate everything on a computer, discretizing spacetime! $\Rightarrow$ Lattice Field Theory



Volume V



Spacing a

$\boxed{D = 4}$!!

THE GRADIENT FLOW FORMALISM

- $d_n \sim \sum_i L_i^{\overline{MS}} \langle N | \mathcal{O}_i^{\overline{MS}} | N\gamma \rangle \Rightarrow D = 4 - 2\epsilon$ (Scheme)

  but this is not what we get from lattice people...

- $\langle N | \mathcal{O}_i^{\overline{MS}} | N\gamma \rangle = \sum_i C_{ij} \langle N | \mathcal{O}_j^{GF} | N\gamma \rangle$ Translation between lattice scheme (Gradient Flow) and perturbative scheme (Minimal Subtraction)

- Why the gradient flow?

  1) No need for renormalization, the gradient flow renders all Green's functions finite
  2) Used for scale setting
  3) It smears statistical noise on the lattice

THE GRADIENT FLOW FORMALISM

- We extend D-dimensional QCD by introducing an extra artificial dimension called flow-time τ. The flowed gauge field satisfies

$$\partial_\tau B_\mu(x,\tau) = D_\nu B_{\nu\mu}(x,\tau)$$

$$B_\mu(x,\tau=0) = G_\mu(x)$$

- Then, this flow equation is turned into an integral equation and solved perturbatively

THE GRADIENT FLOW FORMALISM

- $$\partial_\tau B_\mu = D_\nu B_{\nu\mu} \Longrightarrow \partial_\tau B^a_\mu = \partial_\nu \partial_\nu B^a_\mu + \underbrace{R^a_\mu}_{\text{non-linear}}$$

Solution to the linear part:

$$B^a_\mu(\tau, x) = \int_y K_{\mu\nu}(\tau, x-y)\, G^a_\nu(y) \,, \quad \tilde{K}_{\mu\nu}(t, p) = e^{-\tau p^2}$$

Full solution:

$$B^a_\mu(\tau, x) = \int_y K_{\mu\nu}(\tau, x-y)\, G^a_\nu(y) + \int_y \int_0^\tau ds\, K_{\mu\nu}(\tau - s, x-y)\, R^a_\nu(s, y)$$