

NON-PERTURBABILITY
IN THE CONTEXT OF
THE NEUTRON ELECTRIC
DIPOLE MOMENT

LTP() AND SEMINAR

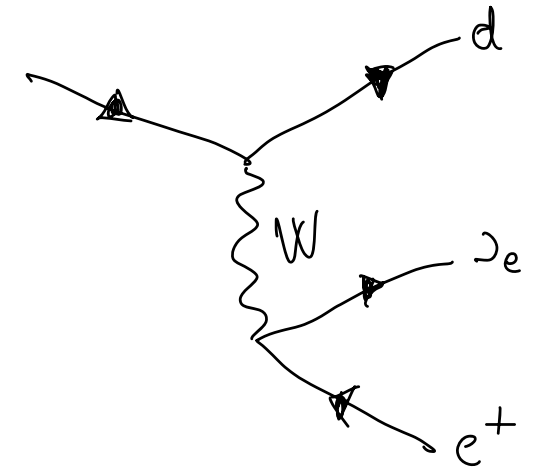
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UZH AND PSI

OUTLINE

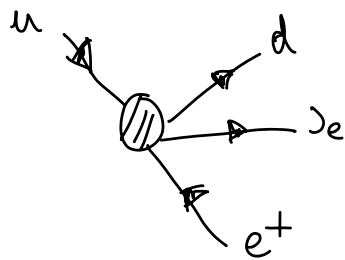
- 1) EFFECTIVE FIELD THEORIES
- 2) RENORMALIZATION/NON-PERTURBABILITY
- 3) GRADIENT FLOW FORMALISM

EFFECTIVE FIELD THEORIES

- 1933 Fermi introduced a 4-fermion interaction for β -decay. Now we know that it happens through u



- But when there was no W boson (no SM) what do you observe?



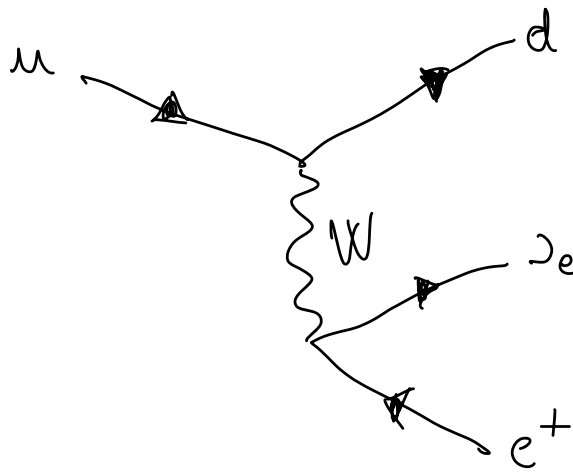
$$\mathcal{L} = (\bar{q}q)(\bar{l}l)$$

$$D = 4 \cdot \frac{3}{2} = 6$$

\Rightarrow Not renormalizable in the traditional sense

EFFECTIVE FIELD THEORIES

- Not renormalizable \Rightarrow Valid only at low energies. But you use it to postulate the W!



$$\sim \frac{1}{p^2 - M_W^2} = \frac{-1}{M_W^2} \cdot \left(1 + \frac{p^2}{M_W^2} + \mathcal{O}\left(\frac{p^4}{M_W^4}\right) \right)$$

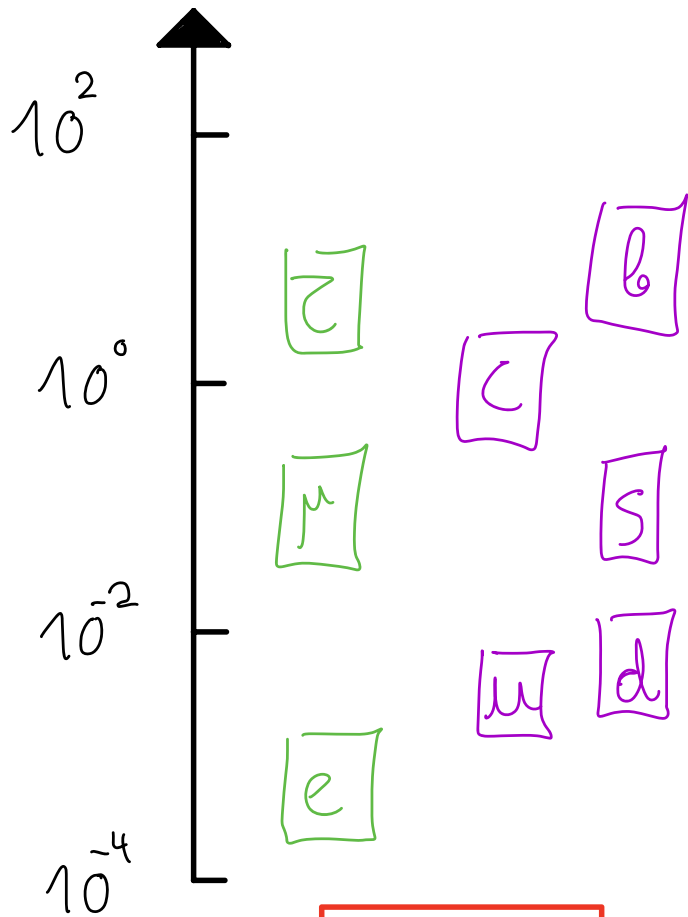
$$= \frac{-1}{M_W^2} \cdot \left[\begin{array}{c} \text{---} \diagdown \text{---} \\ \text{---} \diagup \text{---} \end{array} \right] + \text{dimension 8} + \dots$$

operator

EFFECTIVE FIELD THEORIES

"1934"

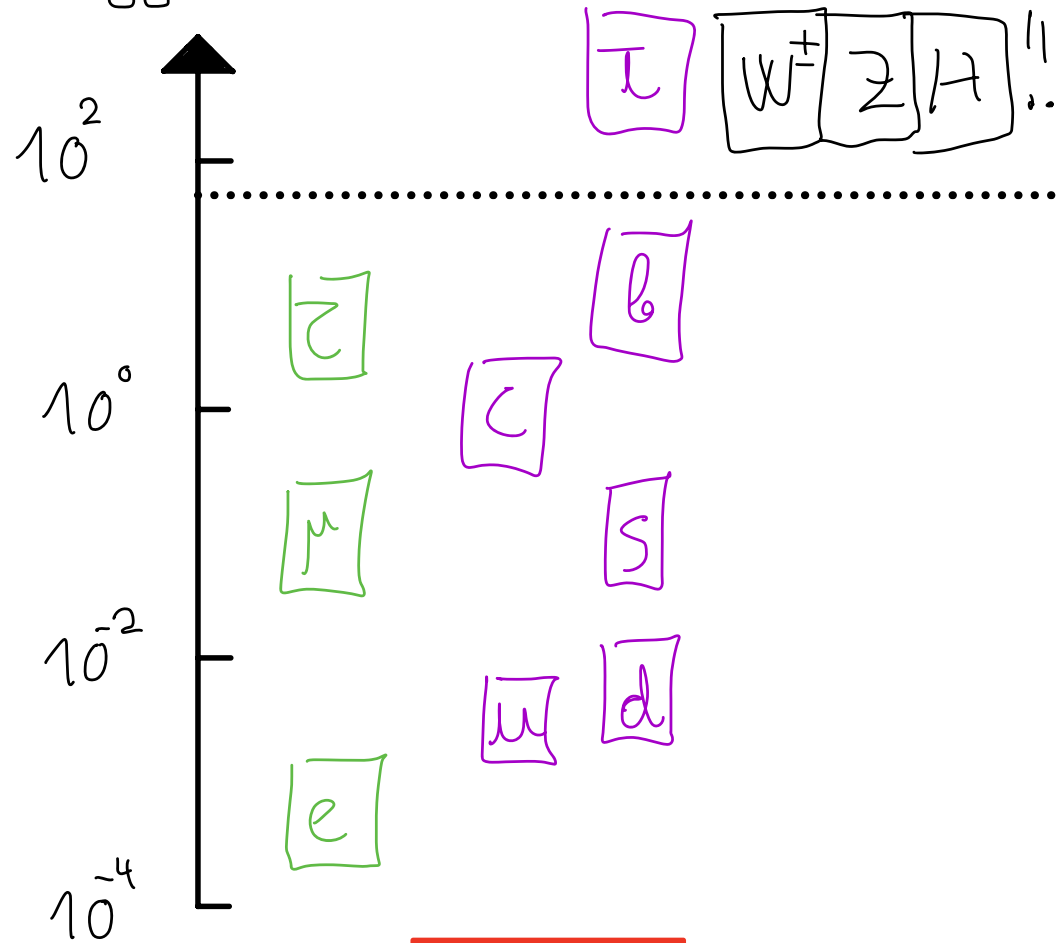
Energy (GeV)



LEFT

2023

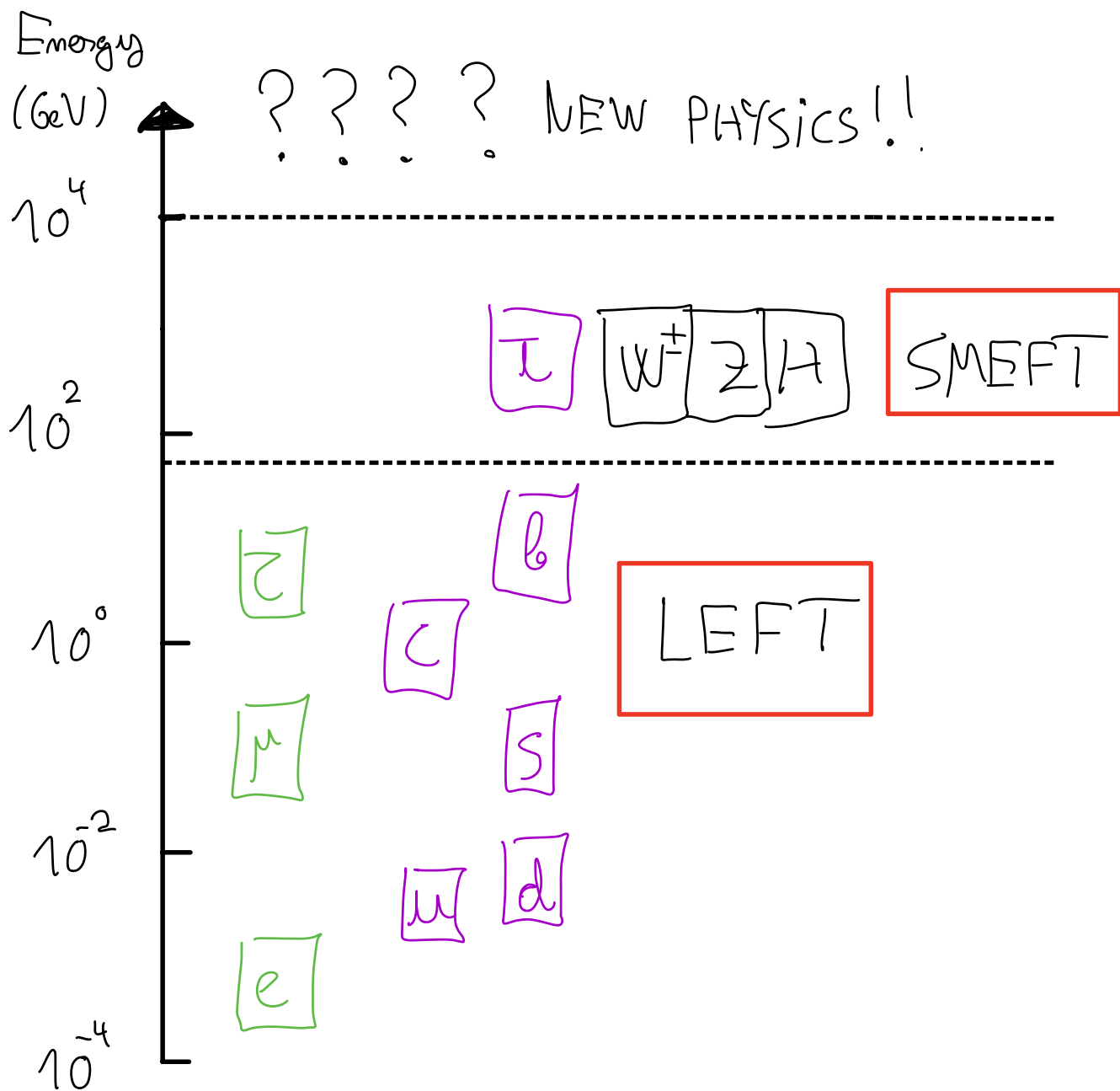
Energy (GeV)



SM !!

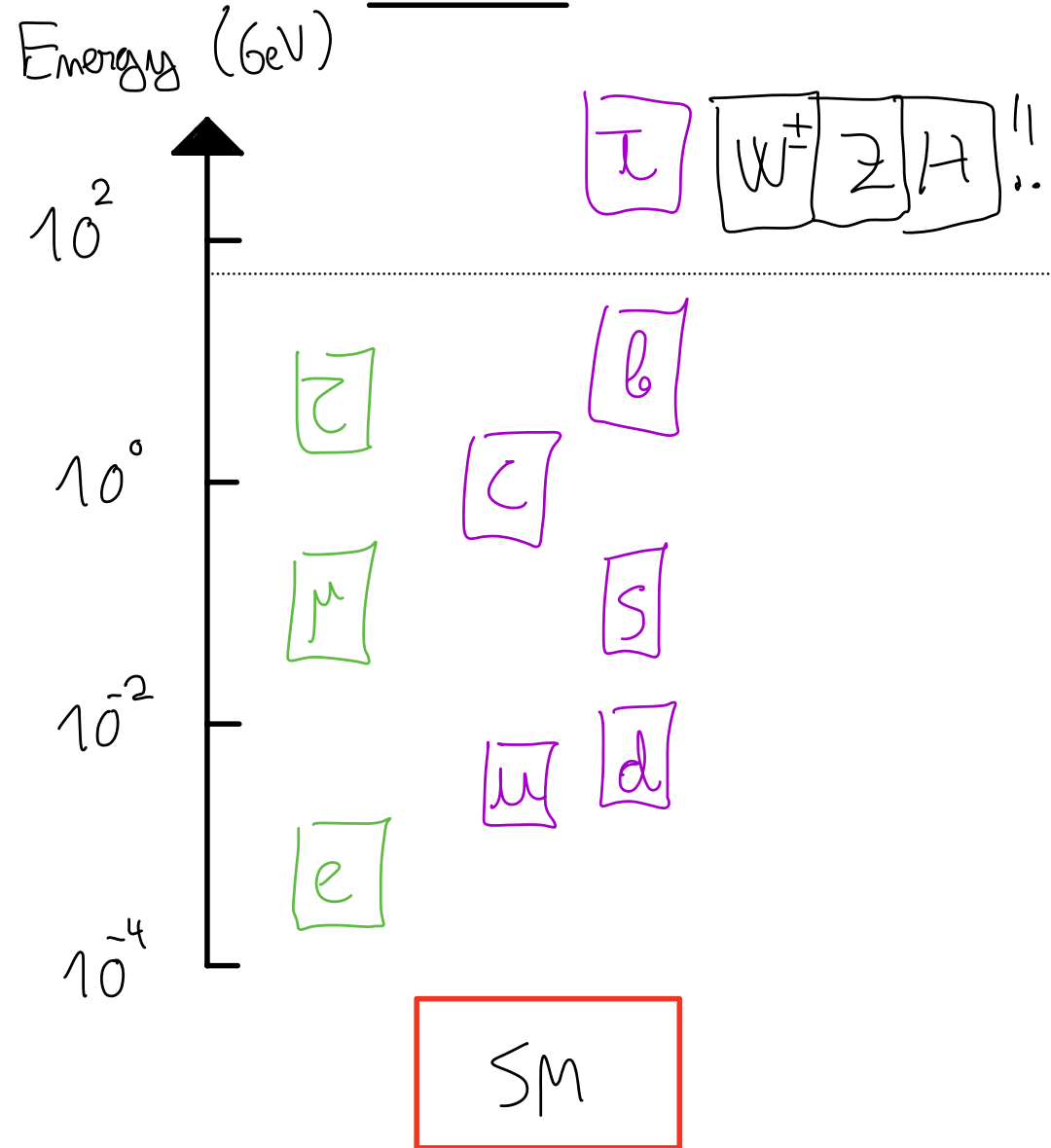
EFFECTIVE FIELD THEORIES

Is this all?



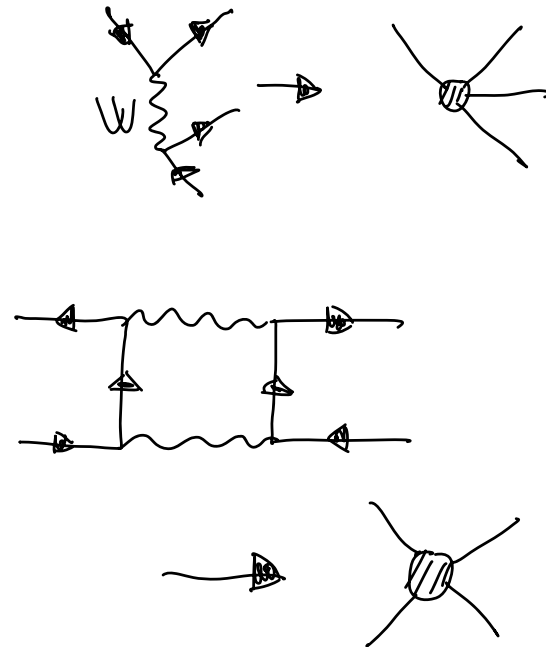
EFFECTIVE FIELD THEORIES

2023

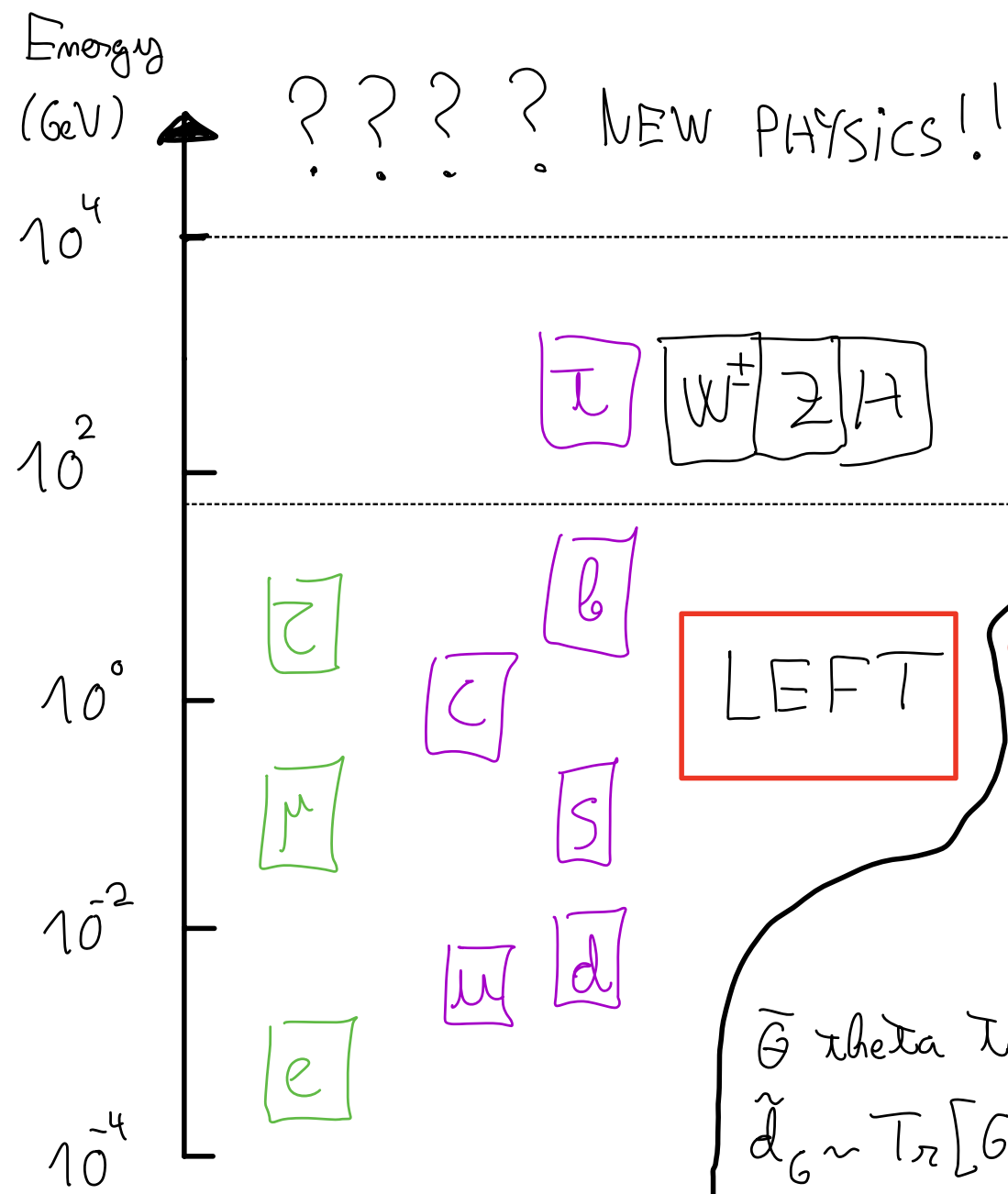


- Also useful for calculations.

If I am below the EW Scale, why would I include W, Z in my calculations?



EFTs APPLIED TO THE NEDM



- $\mathcal{L}_{LEFT} = \mathcal{L}_{QCD+QED} + \sum_{d \geq 5} \sum_{i=1}^{m_d} L_i^{(d)} \mathcal{O}_i^{(d)}$

- $d_m \sim \sum_i L_i \langle N | \mathcal{O}_i | N \rangle$

- $d_m = -1.5 \times 10^{-3} \bar{\theta} e g_m - 0.2 d_u + 0.78 d_d + 0.0027 d_s - 0.55 e \tilde{d}_u - 1.1 e \tilde{d}_d + 50 \text{ MeV} \cdot e \tilde{d}_G$

$\bar{\theta}$ theta term, d_q quark EDM, \tilde{d}_q chromo EDM
 $\tilde{d}_G \sim \text{Tr}[G_{\mu\nu} G_{\nu\lambda} \tilde{G}_{\lambda\mu}]$

EFTs APPLIED TO THE NEDM

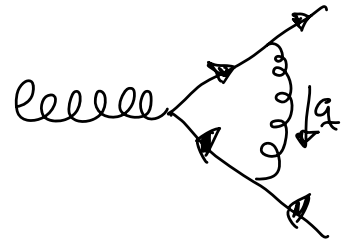
- $d_n = -(1.5 \pm 0.7) \cdot 10^{-3} \bar{\theta} e \cdot g_m - (0.20 \pm 0.01) d_n$
 $+ (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e \tilde{d}_n$
 $- (1.1 \pm 0.55) e \tilde{d}_d + (50 \pm 40) \text{MeV} e \cdot \tilde{d}_6$

Lessons learned:

- 1) We need to measure multiple EDMs
- 2) Uncertainties are too big!
- 3) Lattice is key!

RENORMALIZATION

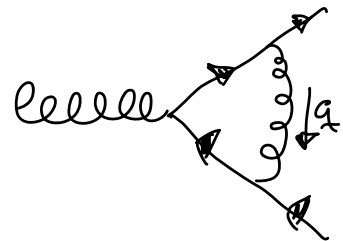
- In theory calculations, we encounter unphysical infinities ☹



A Feynman diagram showing a triangle loop of fermions. The left side is a fermion line with a wavy photon line attached. The top and right sides are fermion lines forming a loop. The bottom side is a fermion line with a wavy photon line attached to it. The photon line has a self-energy correction loop. The diagram is labeled with g and g^2/g .

$$\text{Diagram} = \int_g \text{something} = \infty$$

- The most convenient way to deal with this is to promote the number of space-time dimensions to be D (instead of 4), and then expand $D = 4 - 2\epsilon$.

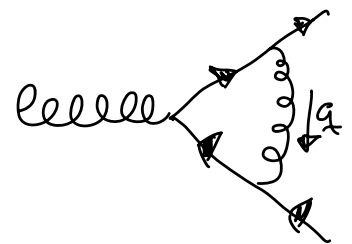


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$$\text{Diagram} = \int_g \text{something} \sim \frac{1}{\epsilon} + (\text{finite as } \epsilon \rightarrow 0)$$

RENORMALIZATION

- The most convenient way to deal with this is to promote the number of space-time dimensions to be D (instead of 4), and then expand $D = 4 - 2\epsilon$.



A Feynman diagram showing a fermion loop (represented by a triangle with arrows) and a ghost loop (represented by a triangle with arrows pointing in the opposite direction). The ghost loop is connected to the fermion loop by two ghost lines, each labeled with g/ϵ . The diagram is equated to an integral over g of something that diverges as $1/\epsilon$ plus a finite term as $\epsilon \rightarrow 0$.

$$\text{Diagram} = \int_g \text{Something} \sim \frac{1}{\epsilon} + \text{finite as } \epsilon \rightarrow 0$$

- Still, if $\epsilon \rightarrow 0$, we get ∞ !!

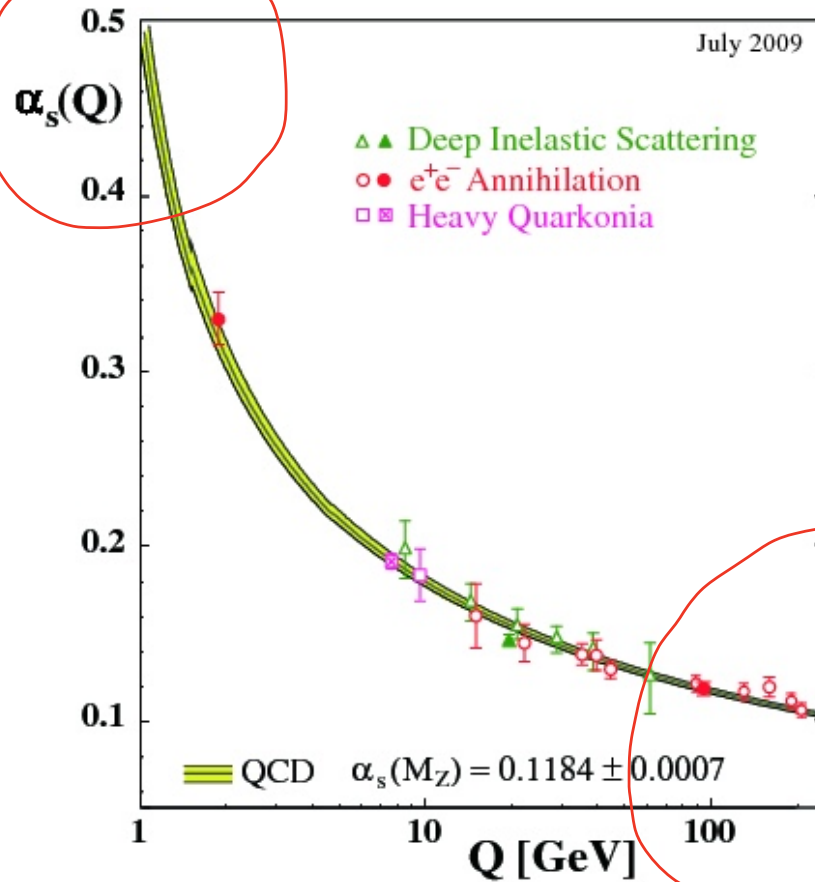
$\mathcal{L} = g \bar{\psi} \gamma_\mu \psi A^\mu$, g is not physical, it is a Lagrangian parameter.

So we do $g \rightarrow \underbrace{g}_{\text{now physical}} - \frac{1}{\epsilon}$ to cancel the infinity!! 😊

RENORMALIZATION / NON-PERTURBATIVE

- Coupling constants acquire scale dependence

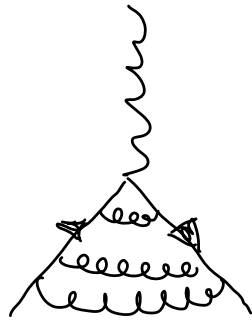
↙
Perturbation
theory
breaks down
☹



↖
Perturbation theory
works well!!
😊

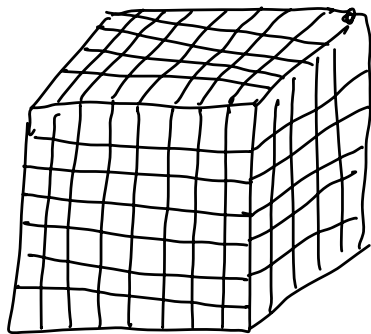
NON-PERTURBABILITY

- We are not able to use perturbation theory for QCD at low energies

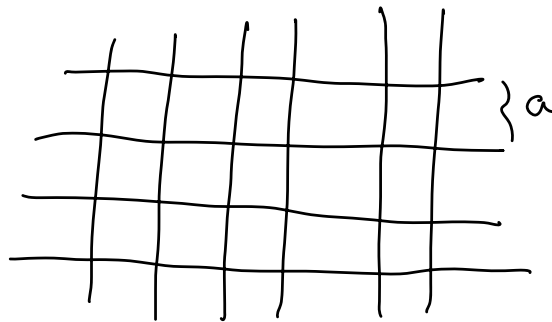


doesn't converge

- But we can calculate/simulate everything on a computer, discretizing spacetime! \Rightarrow Lattice Field Theory



Volume V



Spacing a

$$D = 4 !!$$

THE GRADIENT FLOW FORMALISM

- $d_m \sim \sum_i \bar{L}_i^{\overline{MS}} \langle N | G_i^{\overline{MS}} | N \rangle \Rightarrow D = 4 - 2\epsilon$ (Scheme)

but this is not what we get from lattice people...

- $\langle N | G_i^{\overline{MS}} | N \rangle = \sum_j C_{ij} \langle N | G_j^{GF} | N \rangle$ Translation between

lattice scheme (Gradient Flow) and perturbative scheme (Minimal Subtraction)

- Why the gradient flow?

1) No need for renormalization, the gradient flow renders all

Green's functions finite

2) Used for scale setting

3) It smears statistical noise on the lattice

THE GRADIENT FLOW FORMALISM

- We extend D -dimensional QCD by introducing an extra artificial dimension called flow-time τ . The flowed gauge field satisfies

$$\partial_\tau B_\mu(x, \tau) = D_\nu B_{\nu\mu}(x, \tau)$$

$$B_\mu(x, \tau=0) = G_\mu(x)$$

- Then, this flow equation is turned into an integral equation and solved perturbatively

THE GRADIENT FLOW FORMALISM

- $\partial_\tau B_\mu = D_\nu B_{\nu\mu} \Rightarrow \partial_\tau B_\mu^a = \partial_\nu \partial_\nu B_\mu^a + \underbrace{R_\mu^a}_{\text{non-linear}}$

Solution to the linear part:

$$B_\mu^a(\tau, x) = \int_{\mathcal{Y}} k_{\mu\nu}(\tau, x-y) G_\nu^a(y), \quad \tilde{k}_{\mu\nu}(\tau, p) = e^{-\tau p^2}$$

Full solution:

$$B_\mu^a(\tau, x) = \int_{\mathcal{Y}} k_{\mu\nu}(\tau, x-y) G_\nu^a(y) + \int_{\mathcal{Y}} \int_0^\tau ds k_{\mu\nu}(\tau-s, x-y) R_\nu^a(s, y)$$

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