The gradient flow formalism for CP-violating operators

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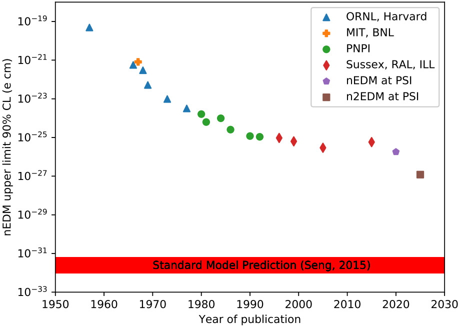
CP Violation Beyond the Standard Model

- CP violation present in the Standard Model (CKM phase and a possible theta term) is not enough to explain matter antimatter asymmetry.
- Electric Dipole Moments violate T $\implies$ CP violation (CPT theorem).

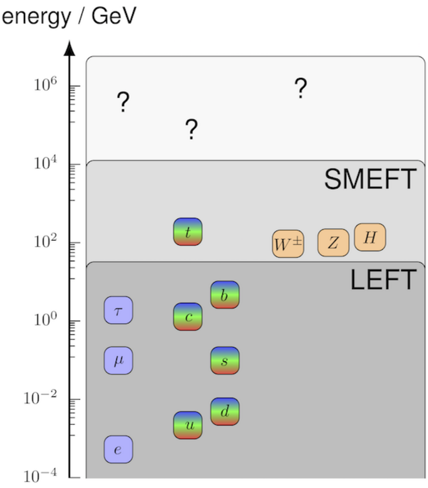
Neutron Electric Dipole Moment



Two possibilities:

- Detecting a signal in the unexplored region $\implies$ CP violating New Physics.
- Not detecting a signal, lowering the bound. Still very interesting! Constrains the shape of New Physics Models.

Effective Field Theories



- Effective Field Theories contain only the relevant degrees of freedom at a certain energy scale.
- Model independent way to encode the effects of all heavy particles (both BSM and SM).

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QCD+QED}} + \sum_{d\geq 5}\sum_{i=1}^{n_d} L_i^{(d)} \mathcal{O}_i^{(d)}$$

$$d_n \sim \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle \tag{1}$$

$$\begin{aligned} d_n \;=\; & -(1.5\pm0.7)\cdot 10^{-3}\,\bar\theta\, e\,\mathrm{fm} - (0.20\pm0.01)d_u \\ & + (0.78\pm0.03)d_d + (0.0027\pm0.0016)d_s - (0.55\pm0.28)e\tilde d_u \\ & - (1.1\pm0.55)e\tilde d_d + (50\pm40)\,\mathrm{MeV} e\,\tilde d_G\ . \end{aligned} \tag{2}$$

where d_q denotes the EDM of a quark q, $\tilde d_q$ denotes its chromo EDM, and $\tilde d_G$ denotes the gluon-chromo EDM. Lessons learned:

- We need to measure multiple EDMs.
- Uncertainties in matrix elements are too big, we should aim for at most 25% uncertainty.
- Lattice is key!

Lattice Field Theory and scheme translations

$$d_N \sim \sum_i L_i^{\overline{\mathrm{MS}}} \langle N|\mathcal{O}_i^{\overline{\mathrm{MS}}}|N\gamma\rangle \implies D = 4 - 2\epsilon \tag{3}$$

However, lattice is tied to integer dimensions! We require a scheme translation:

$$\langle N|\mathcal{O}_i^{\overline{\mathrm{MS}}}|N\gamma\rangle = \sum_j C_{ij} \langle N|\mathcal{O}_j^{\mathrm{GF}}|N\gamma\rangle \tag{4}$$

Why the gradient flow?

- No need for renormalization, the gradient flow renders all Green's Functions finite.
- It is widely used for scale setting.
- It smears statistical noise in the lattice.

The gradient flow

We extend D-dimensional Euclidean QCD by introducing an extra artificial dimension called flow-time t. The flowed field satisfies the flow equation

$$\partial_t B_\mu(x,t) = D_\nu B_{\nu\mu}(x,t) \tag{5}$$

together with the boundary condition that implies agreement with QCD at $t = 0$

$$B_\mu\,(x, t = 0) = G_\mu(x) \tag{6}$$

The flow equation is turned into an integral equation and solved perturbatively, which we express in terms of Feynman diagrams.

Perturbative solution of the flow equation

$$\partial_t B_\mu(t) = D_\nu B_{\nu\mu} \implies \partial_t B^a_\mu(t) = \partial_\nu \partial_\nu B^a_\mu + \underbrace{R^a_\mu}_{\text{non-linear}} \tag{7}$$

Solution to the linear part:

$$B^a_\mu(t,x) = \int_y K_{\mu\nu}(t, x-y) G^a_\nu(y)\,, \quad \widetilde{K}_{\mu\nu}(t,p) = e^{-tp^2} \tag{8}$$

Full solution:

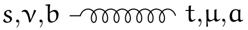
$$B^a_\mu(t,x) = \int_y K_{\mu\nu}(t, x-y) G^a_\nu(y) + \int_y \int_0^t ds\, K_{\mu\nu}(t-s, x-y) R^a_\nu(s,y) \tag{9}$$

Perturbative solution of the flow equation

Full solution:

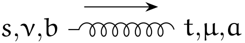
$$B_\mu^a(t,x) = \int_y K_{\mu\nu}(t,x-y)G_\nu^a(y) + \int_y \int_0^t ds\, K_{\mu\nu}(t-s,x-y)R_\nu^a(s,y) \tag{10}$$

Gluon two point function at LO: You just get an extra exponential due to the heat kernel:



$$= g_0^2 \delta^{ab} \frac{1}{p^2} \delta_{\mu\nu} e^{-(s+t)p^2}$$

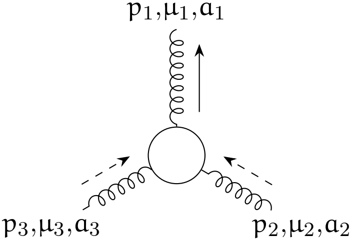
At higher orders, you can take the R_ν part, what will give additional vertices. We view the heat kernel that brings you to the vertex as a generalized propagator (flow line):



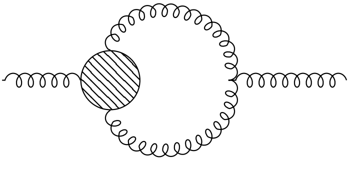
$$= g_0^2 \delta^{ab} \theta(t-s) \delta_{\mu\nu} e^{-(t-s)p^2}$$

which will always connect to a flow vertex.

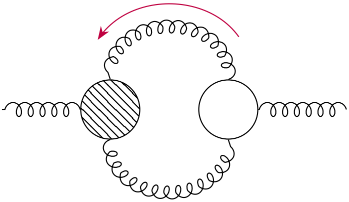
Flow vertices and Feynman diagrams



$$= -if^{a_1a_2a_3}\int_0^{+\infty} dt\Big(\delta_{\mu_2\mu_3}(p_2 - p_3)_{\mu_1} + 2\delta_{\mu_1\mu_3}p_{3,\mu_1} - 2\delta_{\mu_1\mu_2}p_{2,\mu_3}\Big)$$



(a) Linear part: "QCD"



(b) Non-linear part: R_ν

Short flow-time expansion

- Goal: express renormalized flowed operators in terms of renormalized MS operators through a Short Flow-Time Expansion (SFTE):
$$\mathcal{O}_i^R(x,t) = \sum_j C_{ij}(t,\mu)\mathcal{O}_j^{MS}(x,\mu) + \mathcal{O}(t) \tag{11}$$
with the hard scale being $\Lambda = t^{-1/2}$.
- To extract the matching coefficients C_{ij} we consider insertions of the flowed operators $O_i^R(t)$ in suitable Green's functions.

CP-odd Three-gluon operator

Short Flow-Time Expansion of the CP-odd three-gluon operator: (PLB 847 (2023) 138301)

$$\mathcal{O}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\mu\nu}(x,t) G_{\nu\lambda}(x,t) \widetilde{G}_{\lambda\mu}(x,t)] \tag{12}$$

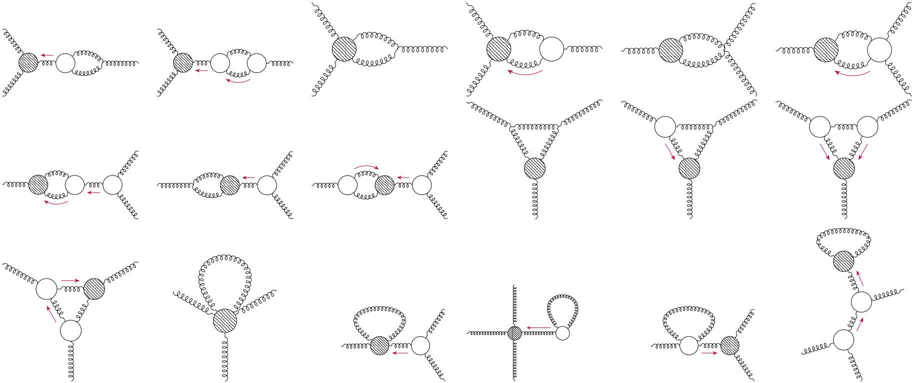
which reads

$$\mathcal{O}^{R}_{\widetilde{G}}(x,t) = \sum_i C_i(t,\mu) \mathcal{O}_i^{\overline{\text{MS}}}(x,\mu) + \sum_i C_{\mathcal{N}_i}(t,\mu) \mathcal{N}_i^{\overline{\text{MS}}}(x,\mu) + \sum_i C_{\mathcal{E}_i}(t,\mu) \mathcal{E}_i^{\overline{\text{MS}}}(x,\mu) \,. \tag{13}$$

The physical operators are

$$\mathcal{O}_\theta \sim Tr\left[G_{\mu\nu}\widetilde{G}_{\mu\nu}\right], \quad \mathcal{O}_{\widetilde{G}}, \quad \mathcal{O}_{CE} = m\left(\bar{q}\tilde{\sigma}_{\mu\nu}T^a q\right) G^a_{\mu\nu},$$
$$\mathcal{O}_{\partial G} \sim \partial_\nu Tr\left[\left(D_\mu G_{\mu\lambda}\right)\widetilde{G}_{\nu\lambda}\right], \quad \mathcal{O}_{\Box\theta} \sim \Box Tr\left[G_{\mu\nu}\widetilde{G}_{\mu\nu}\right] \tag{14}$$

Sample diagrams to be computed



Generalized Loop Integrals

The integrals that we have to compute are

$$\begin{aligned}
&\int_k e^{-\beta t k^2} \left(k^2\right)^{\alpha} \\
&\int_0^t dt_1 \int_k e^{-\beta t k^2 - \gamma t_1 k^2} \left(k^2\right)^{\alpha} \\
&\int_0^t dt_2 \int_0^t dt_1 \int_k e^{-\beta t k^2 - \gamma t_1 k^2 - \tau t_2 k^2} \left(k^2\right)^{\alpha} \\
&\int_0^t dt_2 \int_0^{t_2} dt_1 \int_k e^{-\beta t k^2 - \gamma t_1 k^2 - \tau t_2 k^2} \left(k^2\right)^{\alpha}
\end{aligned} \tag{15}$$

We can make use of normal IBPs and flowed-IBPs:

$$\int_k \frac{\partial}{\partial k_\mu} f_\mu = 0\,, \quad \int_0^t dt_1 \partial_{t_1} f\left(t_1, \ldots\right) = f\left(t, \ldots\right) - f\left(0, \ldots\right) \tag{16}$$

Results

$$
\begin{aligned}
C_\theta &= -\frac{9C_A\alpha_s}{16\pi t}\\
C_{\widetilde{G}} &= \frac{3C_A\alpha_s \log\left(8\pi\mu^2 t\right)}{2\pi} + (1-\delta)\,\frac{C_A\alpha_s}{12\pi}\\
C_{CE} &= \frac{3iC_A\alpha_s \log\left(8\pi\mu^2 t\right)}{32\pi} + \frac{31iC_A\alpha_s}{192\pi} + \delta\frac{iC_A\alpha_s}{96\pi}\\
C_{\partial G} &= -\frac{179C_A\alpha_s}{96\pi} - \delta\frac{C_A\alpha_s}{24\pi}\\
C_{\Box\theta} &= 0
\end{aligned}
\tag{17}
$$

With a perturbative uncertainty of $\sim$40%!

Evanescent operators

$$\mathcal{O}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\overline{\mu}\nu} G_{\nu\overline{\lambda}} \widetilde{G}_{\overline{\lambda}\overline{\mu}}] = C\overline{\mathcal{O}}^{MS} + C_{\mathcal{E}}\mathcal{E}^{MS}$$

vs.

$$\overline{\mathcal{O}}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\overline{\mu}\,\overline{\nu}} G_{\overline{\nu}\,\overline{\lambda}} \widetilde{G}_{\overline{\lambda}\overline{\mu}}] = C'\overline{\mathcal{O}}^{MS} + C'_{\mathcal{E}}\mathcal{E}^{MS}$$

In the D-dimensional scheme, we get a tree level contribution to

$$\mathcal{E}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\overline{\mu}\hat{\nu}} G_{\hat{\nu}\overline{\lambda}} \widetilde{G}_{\overline{\lambda}\overline{\mu}}] \tag{18}$$

Renormalized by imposing

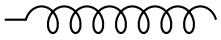
$$\langle \mathcal{E}_{\widetilde{G}} \rangle_{phys} = 0 \implies \text{counterterm from } \mathcal{O}^{MS}!! \implies C' = C \tag{19}$$

Summary

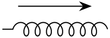
- Electric Dipole Moments are excellent places to look for CP violating New Physics.
- Effective Field Theories parametrize New Physics in a model independent way.
- We require matrix elements from Lattice Field Theory, and a corresponding translation to Minimal Subtraction.
- At one-loop, the SFTE of all operators contributing to the nEDM is now known: (2111.11449), (2304.00985), (2308.16221)
- Precision is key if we want to disentangle the different sources of CP violation $\Longrightarrow$ we need to compute higher orders.

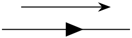
Back up slides

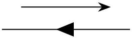
Flow lines

s,ν,b  t,μ,a $= g_0^2 \delta^{ab} \frac{1}{p^2} \delta_{\mu\nu} e^{-(s+t)p^2}$

s,β  t,α $= \delta^{\alpha\beta} \frac{-i\not{p} + m}{p^2 + m^2} e^{-(s+t)p^2}$

s,ν,b  t,μ,a $= g_0^2 \delta^{ab} \theta(t-s) \delta_{\mu\nu} e^{-(t-s)p^2}$

s,β  t,α $= \delta^{\alpha\beta} \theta(t-s) e^{-(t-s)p^2}$

s,β  t,α $= \delta^{\alpha\beta} \theta(t-s) e^{-(t-s)p^2}$