Accelerator Physics & Modelling

Zuoz Summer School Lecture 1: History & Basics

Andreas Adelmann

Paul Scherrer Institut, Villigen
E-mail: andreaad@ethz.ch

August 4-10, 2024

Program ...

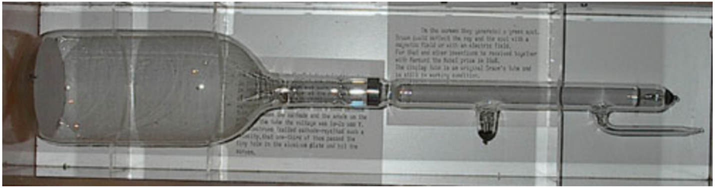
- Lecture 1: History & Basics
- Lecture 2: Linear Maps
- Lecture 3: Non-Linear Maps
- Lecture 4: Collective Effects (& Collisions)

References

M. Berz, *Modern Map Methods in Particle Beam Physics*
A.J. Dragt, *Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics," (2009 ...)*
H. Goldstein, "Classical Mechanics," Addison-Wesley (3nd edition, xxxx).
Equations of Motion and Canonical Transformations are especially relevant.
E. Forest, "Beam Dynamics: A New Attitude and Framework," Taylor and Francis (1998).
A treasure-trove, though somewhat daunting. Highly relevant.
A.W. Chao and M. Tigner (editors), "Handbook of Accelerator Physics and Engineering," World Scientific (1999).
Section 2.3 (various authors) covers nonlinear dynamics.
OPAL - Object Oriented Parallel Accelerator Library
https://gitlab.psi.ch/OPAL/src/-/wikis/home

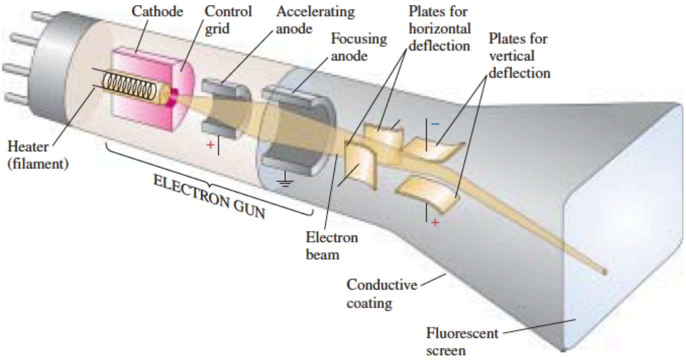
Karl Ferdinand Braun

- first cathode-ray tube in 1879
- 20 kV acceleration voltage
- magnetic & electrostatic focussing and guiding



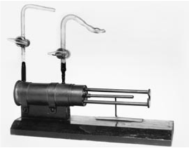
Karl Ferdinand Braun

- first cathode-ray tube in 1879
- 20 kV acceleration voltage
- magnetic & electrostatic focussing and guiding



Ernest Rutherford I

- Ernest Rutherford's historical experiment in 1919: nitrogen nuclei are disintegrated by α-particles coming from radioactive decay of Ra and Th $\Rightarrow$ start of a new era for science
- only few light atoms can be modified using particles from radioactive decays
- the dream of the ancient alchemists!




Ernest Rutherford II

Rutherford in a famous speech at the Royal Society asks for

- accelerators capable to disintegrate heavy nuclei
- theory predicts the threshold for penetration of the nucleus at ≈ 500 keV
- $\Rightarrow$ from 1929 onwards, various labs start developing “particle accelerators” for > 500 keV

Early Times (1928-1930) I

Rolf Rolf Widerøe: a Norwegian student of electrical engineering at Karlsruhe and Aachen. The X-ray transformer that he had chosen for his PhD Thesis at Aachen University did not work, and he was forced to choose quickly another subject. Inspired by a 1924 paper by Ising, a Swedish professor (acceleration of particles using "voltage pulses"), in 1928 he put together for his thesis a device to demonstrate the acceleration of particles by RF fields.

- in 1928, Rolf Widerøe developed the first linear accelerator, which was a pivotal breakthrough in accelerator technology [13]
- Widerøe's work laid the foundation for future advancements in particle acceleration

Early Times (1928-1930) II

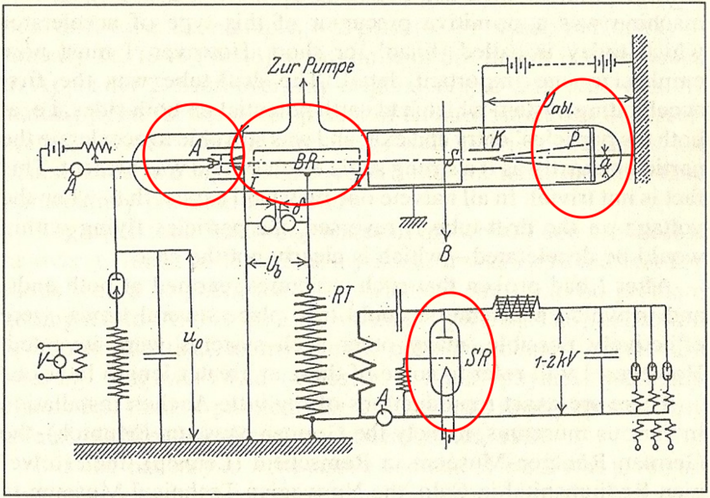
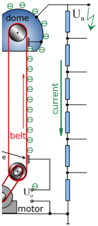
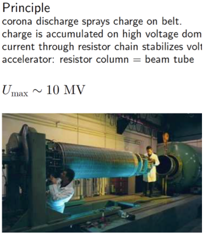


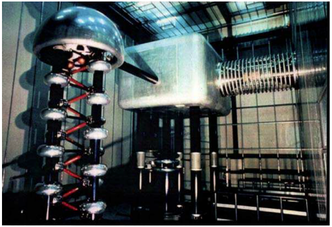
Fig. 3.6: Acceleration tube and switching circuits [Wi28].

Early Times (1928-1930) III

- the Van de Graaff generator, invented by Robert J. Van de Graaff in 1929, uses a moving belt to transfer electric charge to a high-voltage terminal [12]
- potentials in the range of millions of volts
- developed by John Cockcroft and Ernest Walton in the early 1930s, the Cockcroft-Walton accelerator uses a voltage multiplier circuit to generate high voltages [3]

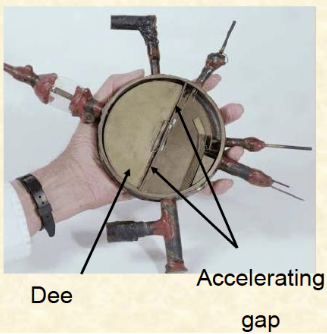




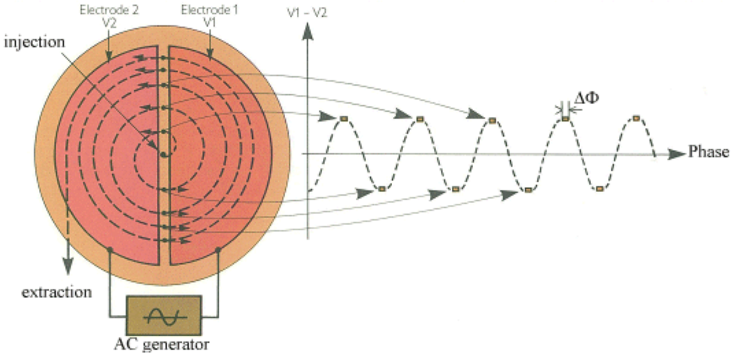


Early Times (1928-1930) IV

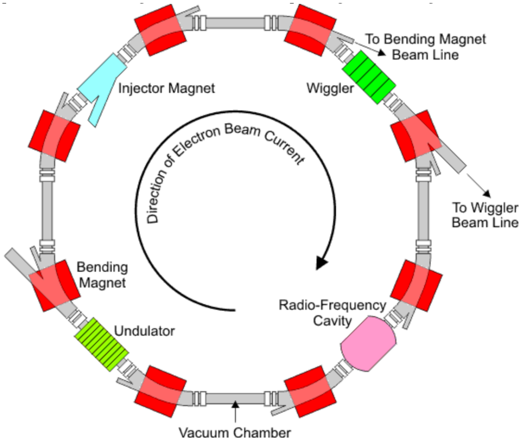
- in the 1930s, Ernest O. Lawrence invented the cyclotron, a type of circular accelerator. The cyclotron accelerated particles along a spiral path using a constant magnetic field and a rapidly varying electric field [6]
- this invention enabled particles to reach much higher energies compared to previous linear accelerators



Early Times (1928-1930) V



More Modern Times I



- the next major advancement came in the 1940s with the development of the synchrotron

More Modern Times II

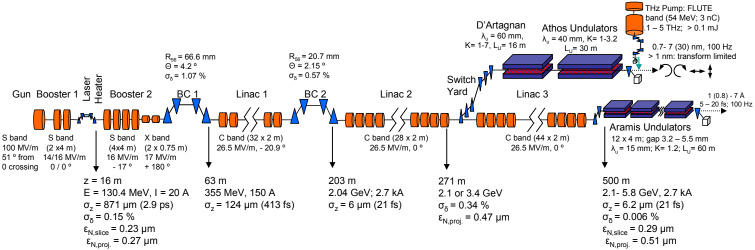
- - the synchrotron kept the particles moving along a fixed circular path, with the magnetic field strength increasing synchronously with the particles' energy [8]
  - this allowed particles to achieve even higher energies, facilitating more advanced research in particle physics
  - a specific type of synchrotron, the electron synchrotron, was developed to accelerate electrons to high energies
  - he first successful electron synchrotron was built in 1949 at the University of California, Berkeley, reaching energies of 300 MeV [2]
- following Widerøe's design, linear accelerators (linacs) evolved
  - the Stanford Linear Accelerator Center (SLAC), established in the 1960s, is one of the most notable examples, with a two-mile-long accelerator capable of reaching energies up to 50 GeV [9]
- in the latter half of the 20th century
  - Large Electron-Positron Collider (LEP) from 1989 to 2000

More Modern Times III

- Tevatron (1983 until its shutdown in 2011)
- Large Hadron Collider (LHC) first beam September 2008, start experimental program March 2010
- the LHC, located at CERN, is currently the world's largest and most powerful particle accelerator, capable of accelerating protons to energies of 7 TeV [5]

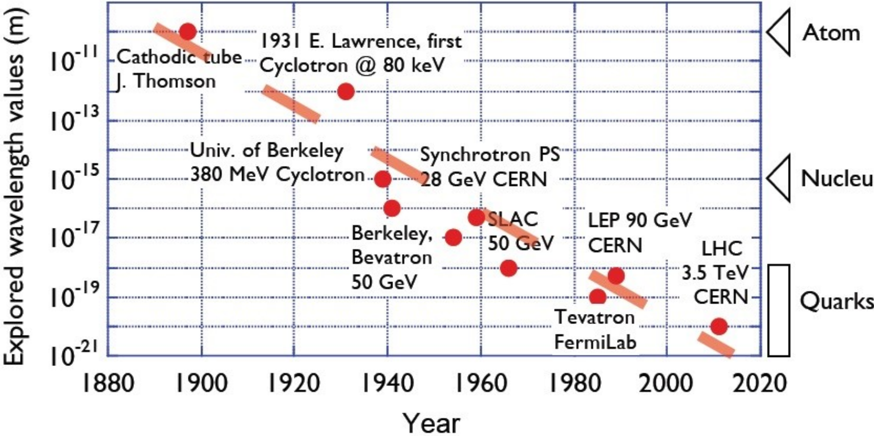
- Free-electron lasers (FELs) are a type of accelerator that produce high-intensity coherent light across a wide range of wavelengths

More Modern Times IV



- unlike conventional lasers, FELs use a beam of relativistic electrons moving through a magnetic structure (undulator) to generate light
- the unique properties of FELs make them invaluable in fields such as material science, chemistry, and biology for probing the structure of matter at atomic scales [7]

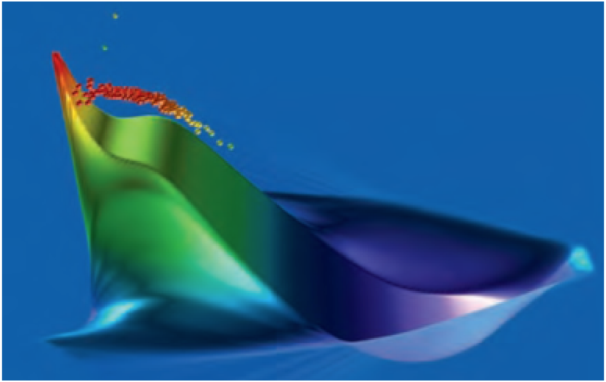
More Modern Times V



Advanced Concepts - Laser Wakefield Accelerator I

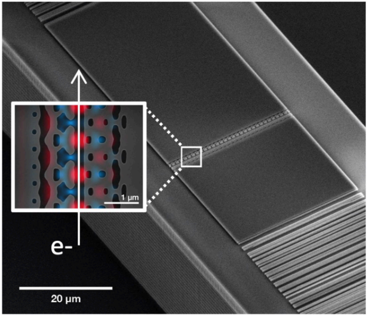
T. Tajima, J.W.Dawson Phys. Rev. Let. **43** (1979) 267

- conventional structures can accelerate with $20 \ldots \mathcal{O}(100)$ MV/m
- In Plasma Wakefield Accelerators (PWA) an ionised gas is formed by a laser or electron beam



Advanced Accelerator Concepts

- dielectric laser accelerators (DLAs) leverage the high electric fields generated by lasers interacting with dielectric structures to achieve high acceleration gradients over short distances [10].
- DLAs (Snowmass) are promising avenue for miniaturizing
- achieved acceleration gradients are as high as 1 GV/m



N. Sapra, et al., Science **367**, 6473 (2020)

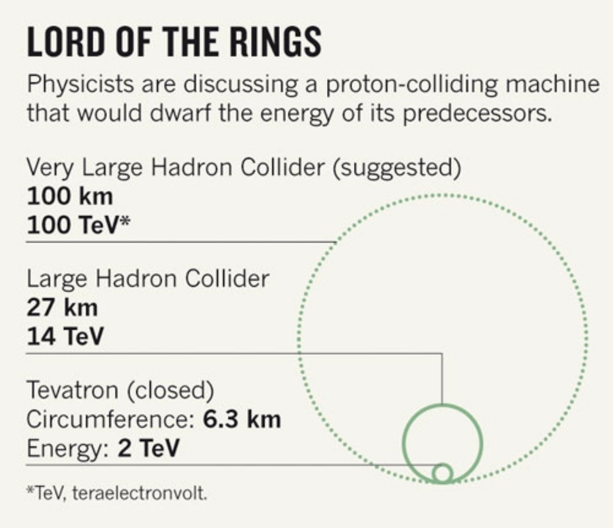
Future Trends in Accelerator Technology

The future of particle accelerators is likely to be shaped by several key trends and innovations aimed at enhancing their capabilities and expanding their applications.

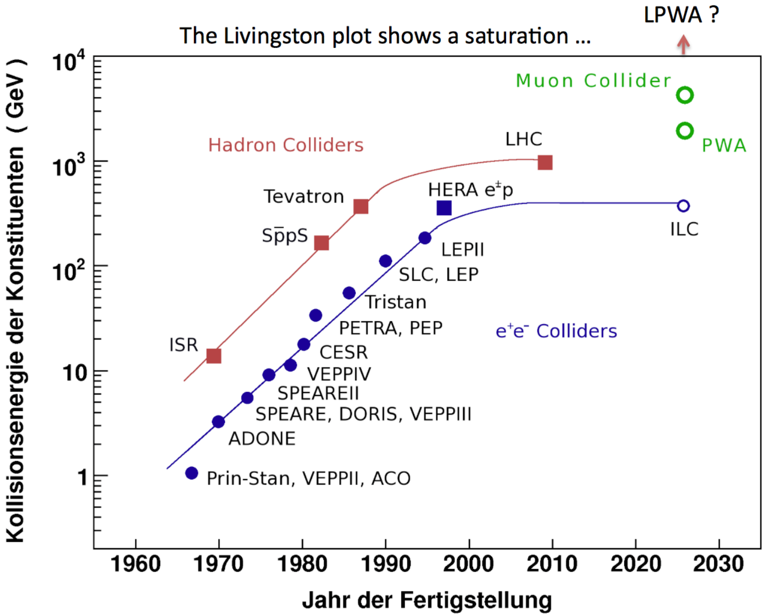
- more applications of particle accelerators: [11]
- compact accelerators: there is significant interest in developing more compact accelerators that can achieve high energies in smaller footprints and could lead to more accessible and cost-effective accelerators for research, medical, and industrial applications [4]
  - dielectric laser accelerators
  - plasma wakefield accelerators
- higher energy colliders: the quest for higher energy collisions continues, with proposals for next-generation colliders such as the Future Circular Collider (FCC) at CERN and the International Linear Collider (ILC) [1]

Future Trends in Accelerator Technology

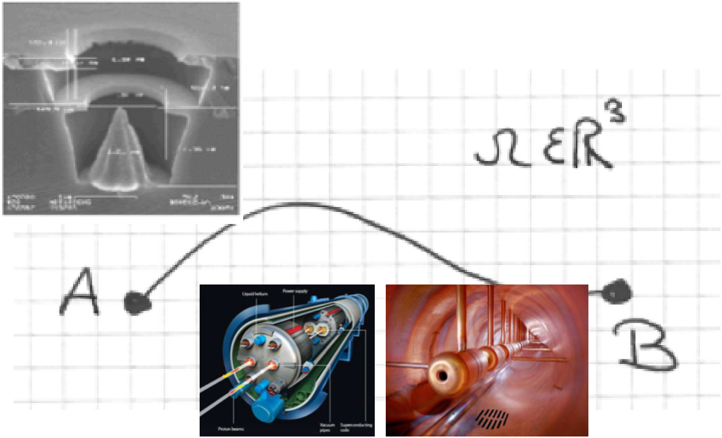
VLHC http://www.nature.com/polopoly_fs/1.14149!/menu/main/topColumns/topLeftColumn/pdf/503177a.pdf



Livingston Plot

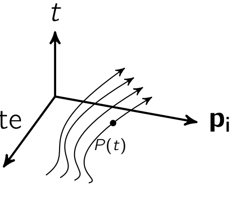


Basics



$$F(\mathbf{x}, t) = \frac{d\mathbf{p}}{dt} = q\left[\mathbf{E}(\mathbf{x}, t) + \mathbf{v} \times \mathbf{B}(\mathbf{x}, t)\right]$$

Classical Phase Space



Consider a system of N identical particles and denote by $\xi_i(t) = (\mathbf{q}_i(t), \mathbf{p}_i(t) \in \mathbb{R}^d \times \mathbb{R}^d$ the phase space of the i$^{\text{th}}$ particle. In Hamiltonian mechanics generalized coordinates and momenta are used instead

$$\mathbf{q} = (q_1, \ldots, q_{Nd}) \quad \text{and } \mathbf{p} = (p_1, \ldots, p_{Nd}).$$

The phase space is a $2Nd$ dimensional manifold whose each point $P(t)$ corresponds to a possible **microscopic state** of the system. Then the microscopic state of the system is equal to <u>one</u> point in the $2 \cdot d \cdot N$-dimensional phase space with coordinates

$$\xi = (\mathbf{q}, \mathbf{p}), \tag{1}$$

known as the Γ-space (with $d \cdot N$-degrees of freedom). This definition can be easy extended to include spin (or other properties).

Classical Phase Space I

The time evolution of points is associated with the N-particle Hamiltonian $H(\xi, t)$ through Hamilton's equations

$$\frac{d\mathbf{q}_i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i}, \qquad \frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{q}_i}. \tag{2}$$

Consider a system with conservative forces we get for H

$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j}^{N} \phi_{i,j}(\mathbf{q}_i, \mathbf{q}_j) + \sum_{i=1}^{N} U(\mathbf{q}_i). \tag{3}$$

The Coulomb potential energy is given by $\phi_{i,j} = e^2/|\mathbf{q}_i - \mathbf{q}_j|$ and $U(\mathbf{q})$ is the potential energy associated with the external field.

Lie operators I

General Concept

Suppose we have a function f of the phase space variables, coordinates $\mathbf{q}$ and conjugate momenta $\mathbf{p}$:

$$f = f(\mathbf{q}, \mathbf{p}). \tag{4}$$

Suppose we evaluate f at the location in phase space for a particle whose dynamics are governed by a Hamiltonian H. The time evolution of f using Hamilton's equations becomes:

$$\frac{df}{dt} = \frac{\partial H}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{q}} - \frac{\partial H}{\partial \mathbf{q}} \frac{\partial f}{\partial \mathbf{p}}. \tag{5}$$

We define the Lie operator $:g:$ for any function $g(\mathbf{q}, \mathbf{p})$:

$$:g: = \frac{\partial g}{\partial \mathbf{q}} \frac{\partial}{\partial \mathbf{p}} - \frac{\partial g}{\partial \mathbf{p}} \frac{\partial}{\partial \mathbf{q}}. \tag{6}$$

Lie operators II

General Concept

Constructing a Lie operator from the Hamiltonian, we can write:

$$\frac{df}{dt} = -:H:f = \frac{\partial H}{\partial \mathbf{p}}\frac{\partial f}{\partial \mathbf{q}} - \frac{\partial H}{\partial \mathbf{q}}\frac{\partial f}{\partial \mathbf{p}}. \tag{7}$$

Writing the time evolution of f in the form (7) suggests that we can write the value of f at any time t as:

$$f(t) = e^{-:H:t} f(0) = \sum_{n=0}^{\infty} \frac{1}{n!}(-t:H:)^n \cdot f(0),$$

where the exponential of the Lie operator is defined in terms of a series expansion:

$$e^{-:H:t} = 1 - t:H: + \frac{t^2}{2}:H:^2 - \frac{t^3}{3!}:H:^3 + \cdots . \tag{8}$$

Ensembles of Particles I

Remarks

- *Individual trajectories can be represented by a path*
- *beam (or bunch of particles) is usually represented by a distribution of particles, hence statistical quantities are appropriate characteristics.*

Given $f \subset \mathbb{R}^d \times \mathbb{R}^d$ and $\rho(\mathbf{X}) \in f$ a bunch of N particles, we identify the following statistical moments:

- 0^{th} order moment $\int d\mathbf{X}\rho(\mathbf{X}) = N$
- 1^{th} order moment, the centroid $\mathbf{C} = \frac{1}{N}\int d\mathbf{X}\rho(\mathbf{X})\mathbf{X} = \langle \mathbf{X} \rangle$
- 2^{th} order moment, the beam matrix $\boldsymbol{\Sigma} \equiv \langle (\mathbf{X} - \mathbf{C}) \cdot (\mathbf{X} - \mathbf{C})^T \rangle = \langle \mathbf{X} \cdot \mathbf{X}^T \rangle - \mathbf{C} \cdot \mathbf{C}^T$

Ensembles of Particles II

$$\Sigma = \frac{1}{N}\int d\mathbf{X}\rho(\mathbf{X})(\mathbf{X}-\mathbf{C})\cdot(\mathbf{X}-\mathbf{C})^T$$

Example: The Gaussian density distribution, in even d dimensions ($d = 2, 4, 6$)

$$\rho_G(\mathbf{X}) = \frac{N}{\sqrt{(2\pi)^d \det \Sigma}} \exp\left(-\frac{1}{2}(\mathbf{X}-\mathbf{C})\cdot\Sigma^{-1}\cdot(\mathbf{X}-\mathbf{C})^T\right)$$

If we assume $\mathbf{C} = 0$ and $\Sigma = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_{p_x}^2 \end{pmatrix}$ in $d = 2$ we obtain the usual definition of a Gauss distribution

$$\rho_G(x, p_x) = \frac{N}{2\pi\sigma_x\sigma_{p_x}} \exp\left(-\frac{x^2}{\sigma_x^2} - \frac{p_x^2}{\sigma_{p_x}^2}\right) \tag{9}$$

It is the beam distribution is popular in beam tracking simulation codes to probe the beam transport properties

Emittance I

The emittance ε represents the **phase-space volume** occupied by the particles of the beam. It is one of the most important measures of particle accelerator physics because:

1. the emittance is an invariant of motion
2. the emittance is a beam quality concept reflecting the process of bunch preparation (the injector chain), extending all the way back to the source for hadrons
3. a low emittance particle beam is a beam where the particles are confined to a small distance and have nearly the same momentum
4. in a colliding beam accelerator, keeping the emittance small means that the likelihood of particle interactions will be greater resulting in higher luminosity

Emittance II

Remark

- *the lower the emittance, the easier it is to manipulate the beam through beam pipe apertures, collimators, focal points and radio frequency (RF) devises, to name the most important*
- *in case the emittance is to large, the beam must be cooled wither with cooling rings (synchrotron radiation) electron cooling (intra-beam scattering) with an external low emittance electron beam or ionisation cooling, where the beam is passing gazes or solid matter*

Assuming a Gaussian distribution, the phase space volume is then given by

$$\text{Area} = \pi\sqrt{\det \Sigma} = \pi\varepsilon \tag{10}$$

with ε the so called Emittance.

Emittance III

Depending on the transport topology we can decouple the phase spaces and get for (x, p_x)

$$\Sigma_x = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \rightarrow (x, x')^T \Sigma_x^{-1} (x, x') = 1. \tag{11}$$

In beam dynamics it is custom to write the ellipse equation in the so called Courant-Snyder variables: α, β and γ

$$\Sigma_x = \varepsilon_x \mathbf{T} \text{ with } \mathbf{T} \equiv \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} \tag{12}$$

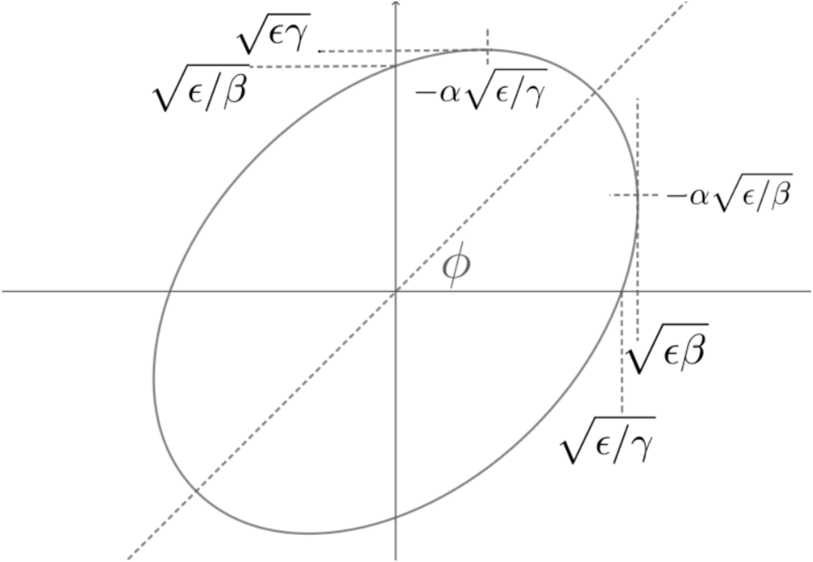
and $\mathbf{det}\,\mathbf{T} = 1$. The equation of the ellipse (Courant-Snyder invariant) is then generated by

$$\varepsilon_x^2 = (x, x')^T \mathbf{T}^{-1} (x, x') = \gamma x^2 + 2\alpha x x' + \beta x'^2 \tag{13}$$

Emittance IV

with x and x' the second moments of the distribution. This describes a special case in which the sub-phase space Γ_x is decoupled from the other degrees of freedom.

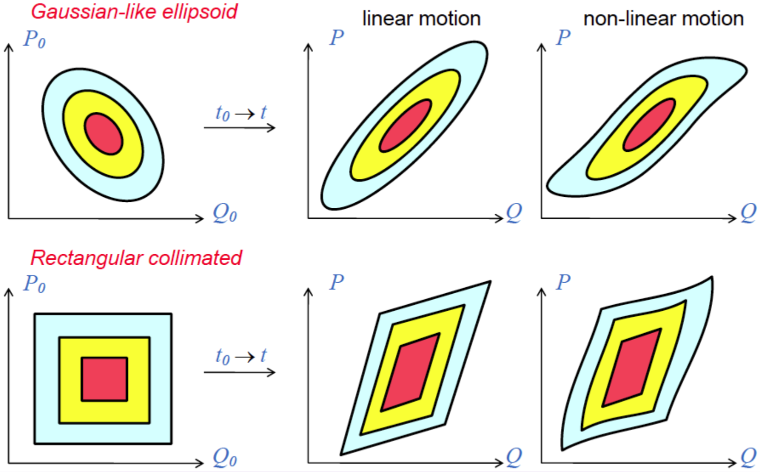
Emittance V



$\sqrt{\epsilon\gamma}$
$\sqrt{\epsilon/\beta}$
$-\alpha\sqrt{\epsilon/\gamma}$
$-\alpha\sqrt{\epsilon/\beta}$
ϕ
$\sqrt{\epsilon\beta}$
$\sqrt{\epsilon/\gamma}$

Liouville Theorem

The Phase Space Volume ε occupied by a beam population is invariant of motion



Study Complicated Phase Space! I

P. Berger MSc thesis

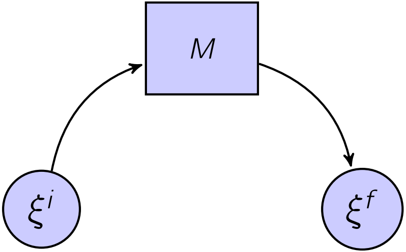
http://amas.web.psi.ch/people/aadelmann/ETH-Accel-Lecture-1/projectscompleted/phys/thesisBerger.pdf

Modelling Challenges

- Structure Design (we will not discuss)
- Single particle dynamics
  - linear optics
  - non-linear optics
- Collective effects
  - Maxwell + Particles in large and complicated structures
  - Multi-scale, multi-physics modelling
  - Large N-body problems (precision), requires HPC

Single particle dynamics I

Consider the state vector $\xi \in \mathbb{R}^6$ of a particle and the superscript i and f having the meaning initial and finale state respectively



and M a linear map.

This picture describes a very general concept of modelling a dynamic system which can be equivalently viewed as solving an ordinary differential equation (ODE). The vector ξ defines positions, momenta or a quantity such as the particle's spin or charge.

Vlasov-Maxwell I

When neglecting collisions, the Vlasov-Maxwell equation describes the (time) evolution of the phase space $f(\mathbf{x}, \mathbf{p}, t) > 0$.
Now define $\mathbf{v} := \mathbf{v}(\mathbf{p}) = c\mathbf{p}/\sqrt{m^2c^2 + \mathbf{p}^2}$ and introduce the incompressible-transport equation

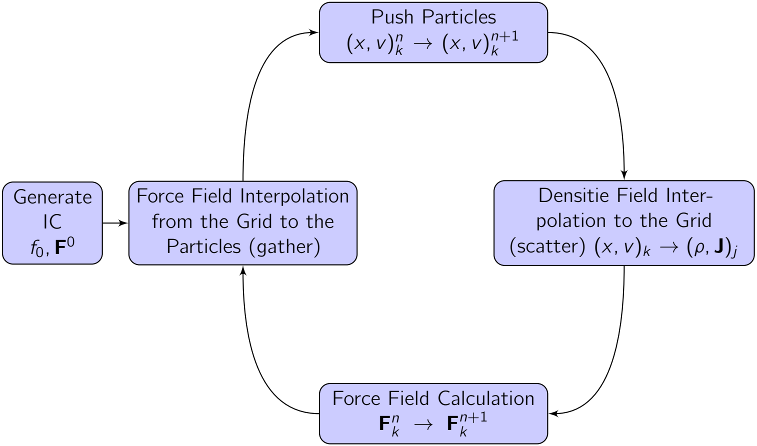
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + \frac{q}{m}(\mathbf{E}(\mathbf{x}, t) + \mathbf{v} \times \mathbf{B}(\mathbf{x}, t)) \cdot \nabla_v f = 0$$

with **E** and **B** the self-consitent fields.

$$\partial_t \mathbf{E}(\mathbf{x}, t) - c^2 \mathbf{curl}\,\mathbf{B}(\mathbf{x}, t) = -\frac{\mathbf{J}}{\varepsilon_0}, \qquad \nabla \cdot \mathbf{E}(\mathbf{x}, t) = \frac{\rho}{\varepsilon_0}$$
$$\partial_t \mathbf{B}(\mathbf{x}, t) + \mathbf{curl}\,\mathbf{E}(\mathbf{x}, t) = 0, \qquad \nabla \cdot \mathbf{B}(\mathbf{x}, t) = 0$$

- this is an example of a class of partial differential equations PDE's known as kinetic equations
- much simpler version, the Vlasov-Poisson equation, is obtained, in the limit of vanishing magnetic fields.

Coupling Particle Dynamics with Electromagnetic Fields



In case of precise particle accelerator modelling, resolving losses at very low level, large N-body problems have to be solved.

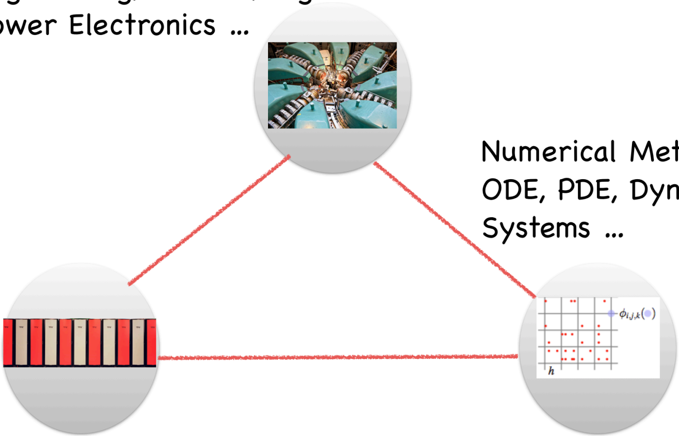
Modeling Challenges

- Multiscale / Multiresolution
  - Maxwell's equations or reduced set combined with particles
  - N-body problem $n \sim 10^9$ per bunch in case of PSI
  - Spatial scales: $10^{-4} \dots 10^4$ (m) $\rightarrow \mathcal{O}(1e5)$ integration steps
  - $v \ll c \dots v \sim c$
  - Large (complicated structures)
- Multiphysics
  - Particle mater interaction: monte carlo
  - Secondary particles i.e. multi specis
- Large Parameter Space to Optimize
  - Multi Objective Optimization in a Pareto optimal sense

The Particle Accelerator Modelling Universe

An Interdisciplinary Field of Science

Physics, Electrical Engineering, Vacuum, High-Power Electronics ...



Numerical Methods, ODE, PDE, Dynamical Systems ...

High Performance Computing

References I

[1] Michael Benedikt et al.
Future circular collider study.
CERN, 100, 2015.

[2] WM Brobeck.
The 300-MeV electron synchrotron at the university of california.
Proceedings of the Institution of Radio Engineers, 38(11):1321–1325, 1950.

References II

[3] John D. Cockcroft and Ernest T.S. Walton.
Experiments with high velocity positive ions. (i) further developments in the method of obtaining high velocity positive ions.
Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 136(830):619–630, 1932.

[4] RJ England, RL Noble, KM Bane, et al.
Dielectric laser accelerators.
Reviews of Modern Physics, 86(4):1337–1389, 2014.

[5] Lyndon Evans and Philip Bryant.
LHC machine.
Journal of Instrumentation, 3(08):S08001, 2008.

References III

[6] Ernest O Lawrence.
Method and apparatus for the acceleration of ions.
US Patent, 1948384, 1931.

[7] John M. J. Madey.
Stimulated emission of bremsstrahlung in a periodic magnetic field.
Journal of Applied Physics, 42(5):1906–1913, 1971.

[8] EM McMillan.
The synchrotro – a proposed high energy particle accelerator.
Physical Review, 68(5-6):143, 1945.

[9] Wolfgang KH Panofsky.
The stanford two-mile accelerator.
Scientific American, 219(1):38–53, 1968.

References IV

[10] E. Peralta, K. Soong, R.J. England, et al.
Demonstration of electron acceleration in a laser-driven dielectric microstructure.
Nature, 503(7474):91–94, 2013.

[11] Suzie Sheehy.
Applications of particle accelerators, 2024.
2407.10216, https://arxiv.org/abs/2407.10216.

[12] Robert J. Van de Graaff.
A 1,000,000-volt electrostatic generator.
Journal of the American Institute of Electrical Engineers, 49(2):150–154, 1931.

[13] Rolf Widerøe.
Über ein neues Prinzip zur Herstellung hohen Spannungen.
Archiv für Elektrotechnik, 21(4):387–406, 1928.