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Precision Higgs Physics, QCD & Parton Showers [2/3]

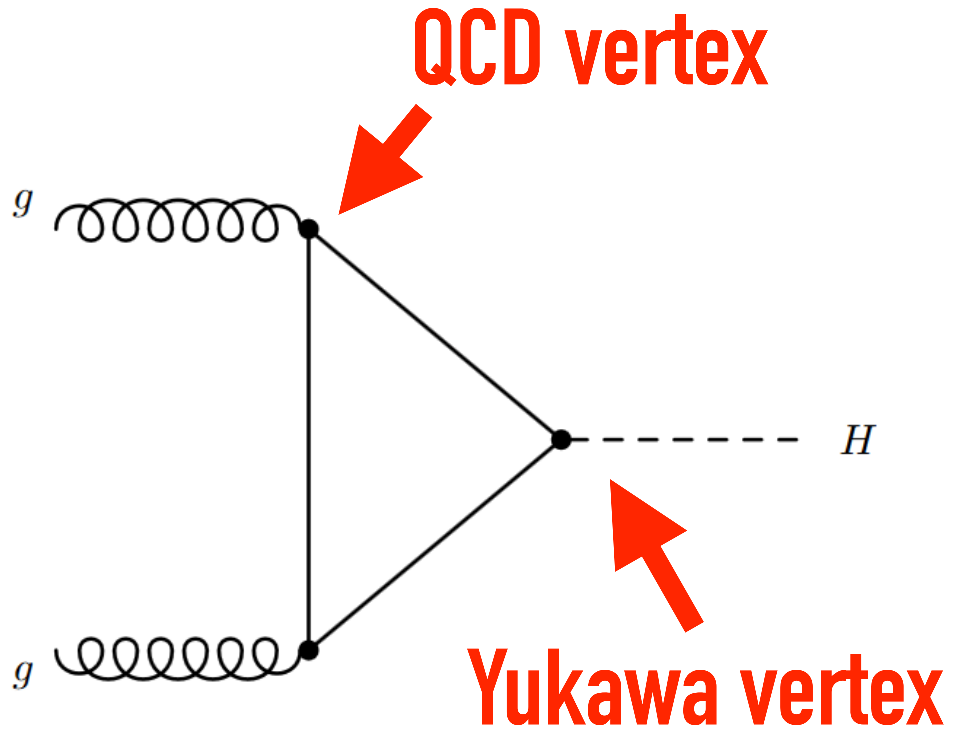
PSI summer school - Zuoz, Aug 2024

Radiative corrections & Collider Observables

The Higgs total cross section (cont’d)

Recap: The leading order (LO) cross section

- After averaging over colour & spin states, the partonic XS reads $\hat{\sigma}_0 = A_{gg}\,\delta(1-z)$



$$A_{gg} = \frac{\alpha_s(\mu_R)^2}{\pi}\frac{1}{256v^2}\left|\sum_{q\in\text{loop}} \tau_q(1+(1-\tau_q)f(\tau_q))\right|^2, \quad z = \frac{m_h^2}{\hat{s}}$$

$$f(\tau_q) = -\frac{1}{4}\left(\ln\frac{1+\sqrt{1-\tau_q}}{1-\sqrt{1-\tau_q}} - i\pi\right)^2 \quad \text{if } \tau_q < 1$$

$$\tau_q = 4m_q^2/m_h^2$$

$$f(\tau_q) = \arcsin^2\left(\sqrt{1/\tau_q}\right) \quad \text{if } \tau_q \geq 1$$

- Total cross section is simply given by

$$\sigma_0 = \int_0^1 dx_1 dx_2 f_g(x_2,\mu_F) f_g(x_1,\mu_F)\, m_h^2\, A_{gg}\,\delta(\hat{s}-m_h^2) = m_h^2\, A_{gg}\,\mathscr{L}_{gg}\left(\frac{m_h^2}{s}\right)$$

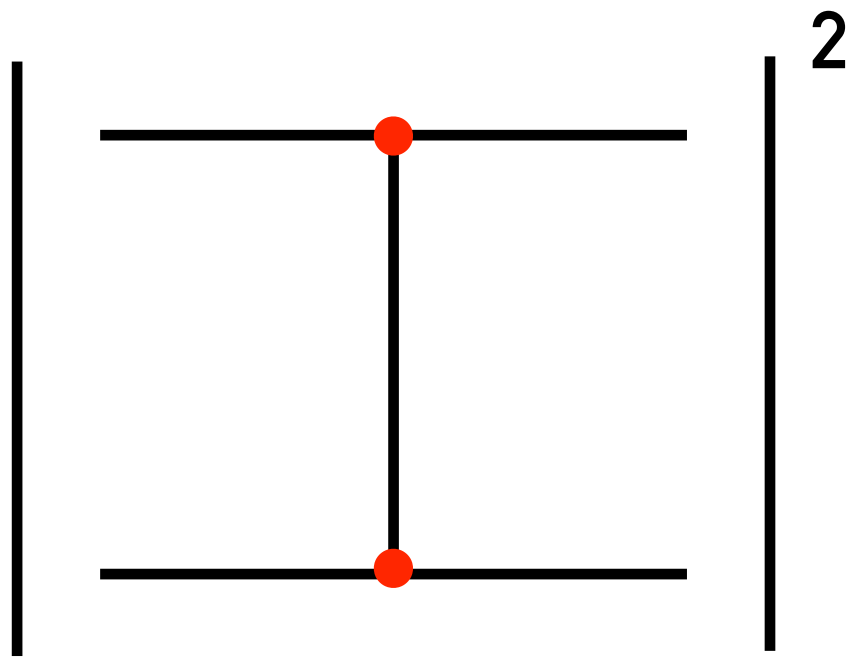
$$\mathscr{L}_{ij}(\tau) = \int_\tau^1 \frac{dx}{x} f_i(x,\mu_F) f_j\left(\frac{\tau}{x},\mu_F\right)$$

$\hat{s} = x_1 x_2 s$

Parton (gluon) luminosity

Recap: NLO calculations (e.g. for $2 \to 2$ partonic process)

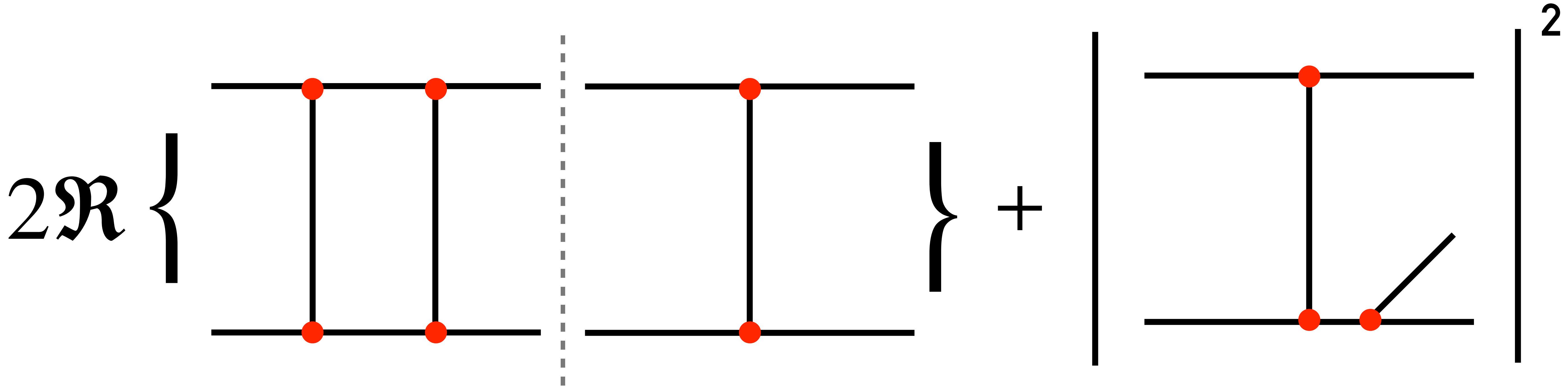
LO‡
(only tree-level diagrams)



$\sim \mathcal{O}(g_s^4) \sim \mathcal{O}(\alpha_s^2)$

NLO‡
1) Add a virtual loop to the LO process and expand the squared norm

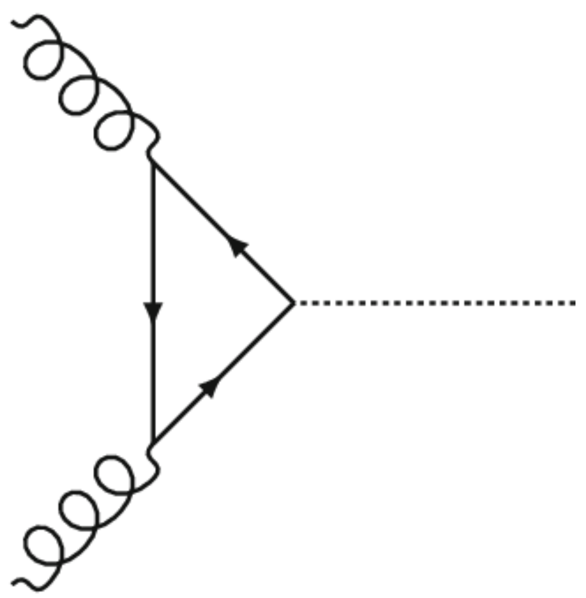
2) Add a real emission to the LO process



$\sim \mathcal{O}(\alpha_s^3)$

$$|\mathscr{A}^{(0)} + \alpha_s \mathscr{A}^{(1)}|^2 = |\mathscr{A}^{(0)}|^2 + \alpha_s\, 2\mathfrak{R}\mathscr{A}^{(0)}(\mathscr{A}^{(1)})^\dagger + \ldots$$

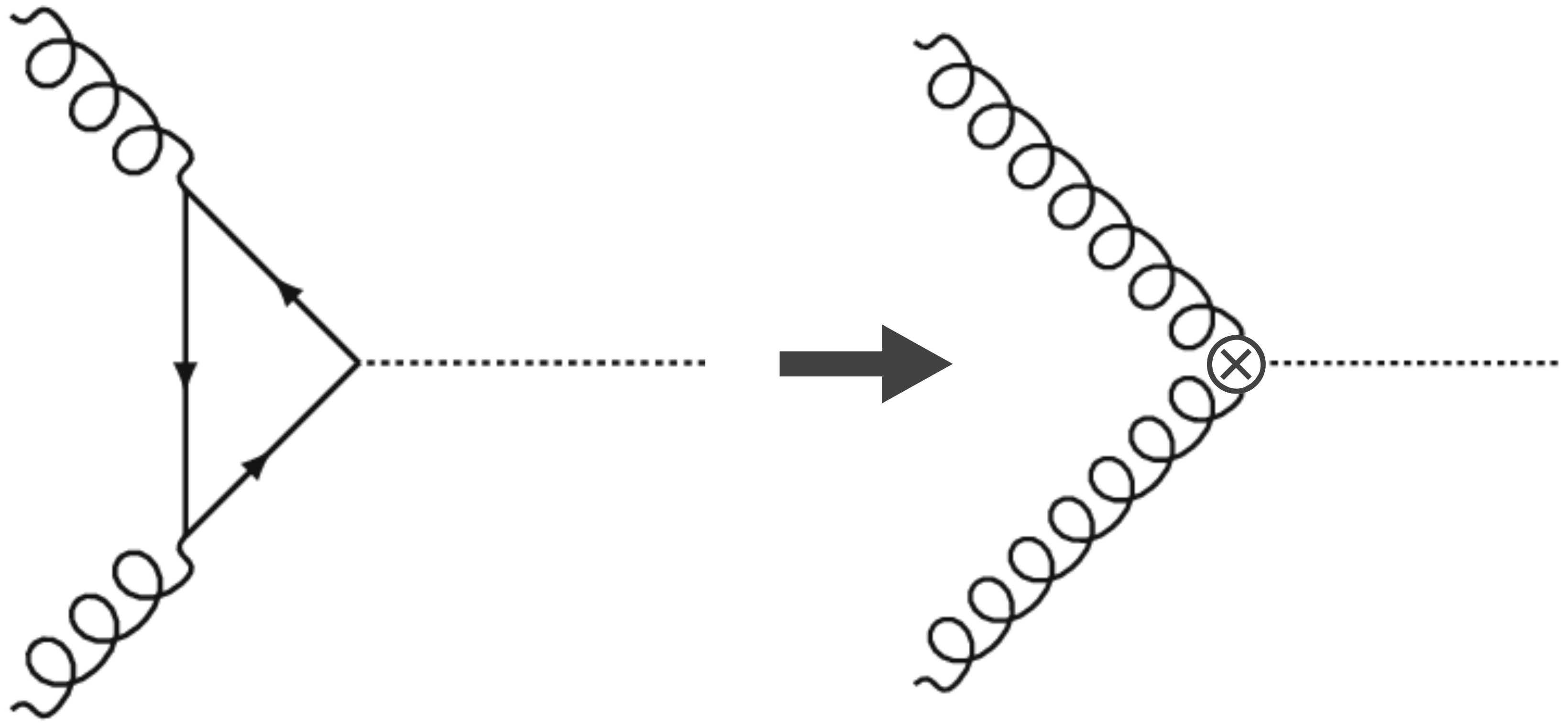
- In our case the Born process is given by $gg \to h$, i.e. the one-loop diagram



‡ We use representative diagrams, the actual number of Feynman diagrams explodes with the perturbative order

Heavy-top effective field theory

- A way to simplify the NLO calculation is to consider the limit $m_t \to \infty$
  - Effective field theory (EFT) Lagrangian reads



Wilson coefficient $\mathcal{O}(\alpha_s)$

$$\mathscr{L}_{ggH} = \frac{1}{2v} C\left(\frac{\mu^2}{m_t^2}\right) h\, G^a_{\mu\nu} G^{\mu\nu}_a$$

We reduce the complexity of the problem by a loop order!

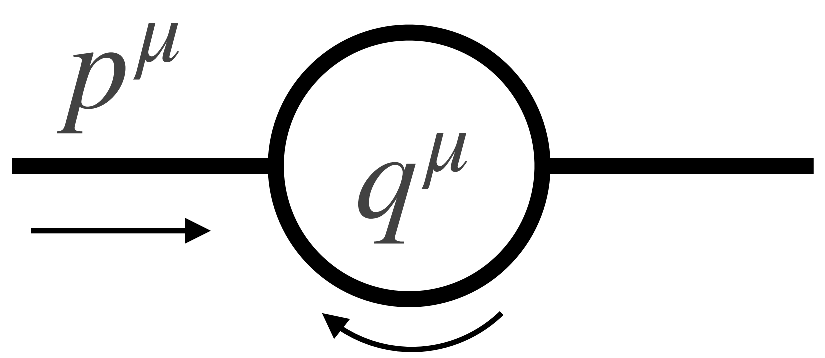
  - Excellent approximation for the <u>total XS</u> up to an overall rescaling

$$\hat{\sigma}_0 \to \hat{\sigma}_0^{\text{EFT}} = \frac{\alpha_s^2}{\pi} \frac{1}{576 v^2} \delta(1-z) \quad \Rightarrow \quad \frac{\hat{\sigma}_0}{\hat{\sigma}_0^{\text{EFT}}} \simeq 1.066$$

Since $m_h < 2\, m_t$ we can calculate radiative corrections in the EFT and then rescale by the ratio of LO cross sections.

Path to NLO: UV divergences and running of α_s

- QCD is a renormalisable gauge theory



$$\sim \mu^{2\epsilon} \int \frac{d^D q}{q^2 (q+p)^2} \sim \left(\frac{\mu^2}{-p^2}\right)^\epsilon \left(\frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

  - Loop integrals diverge at most logarithmically in the UV ($q^2 \to \infty$)
  - All divergences can be systematically absorbed into the Lagrangian bare parameters (m_q, α_s)

- To regularise the divergences (preserving Lorentz invariance), we continue to $D = 4 - 2\epsilon$ space-time dimensions: divergences show up as poles at $\epsilon = 0$

$\epsilon > 0 \; (D < 4)$ to regularise theory in the UV

UV divergence (pole)

$$\int \frac{d^4 q}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^D q}{(2\pi)^D} \quad \Rightarrow \quad \alpha_s^{(0)} \to \alpha_s(\mu) Z_g \,, \quad Z_g = f(\epsilon)\left(1 - \frac{\beta_0}{\epsilon}\alpha_s(\mu) + \mathcal{O}(\alpha_s^2)\right)$$

Unphysical renormalisation scale

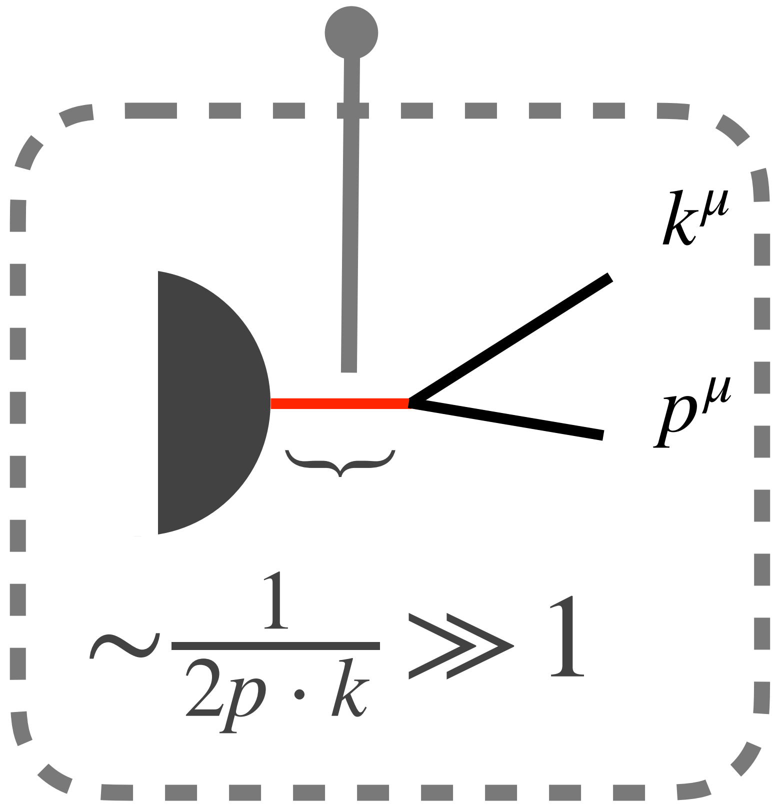
$$\frac{d\alpha_s(\mu)}{d \ln \mu^2} = \beta(\alpha_s(\mu)) = -\beta_0 \, \alpha_s^2(\mu) + \mathcal{O}(\alpha_s^3)$$

Renormalisation is what drives the running of α_s

Path to NLO: Infrared & collinear divergences

- Phase-space (& loop) integrals diverge logarithmically in the soft ($E \to 0$) and/or collinear ($\theta \to 0$) limits. At the squared amplitude level this leads to the following factorisation theorems (here @ lowest order)

Propagator nearly on-shell $\longrightarrow$ **factorisation**



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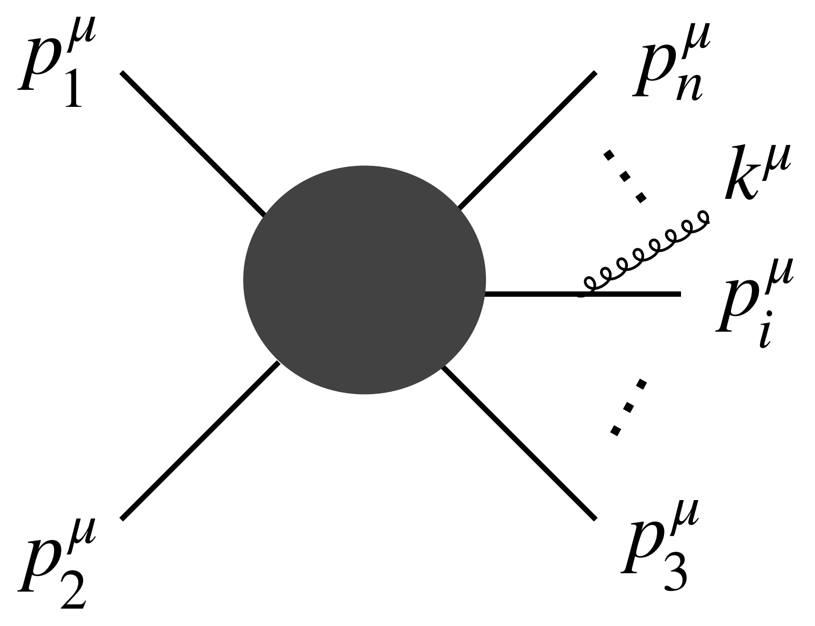
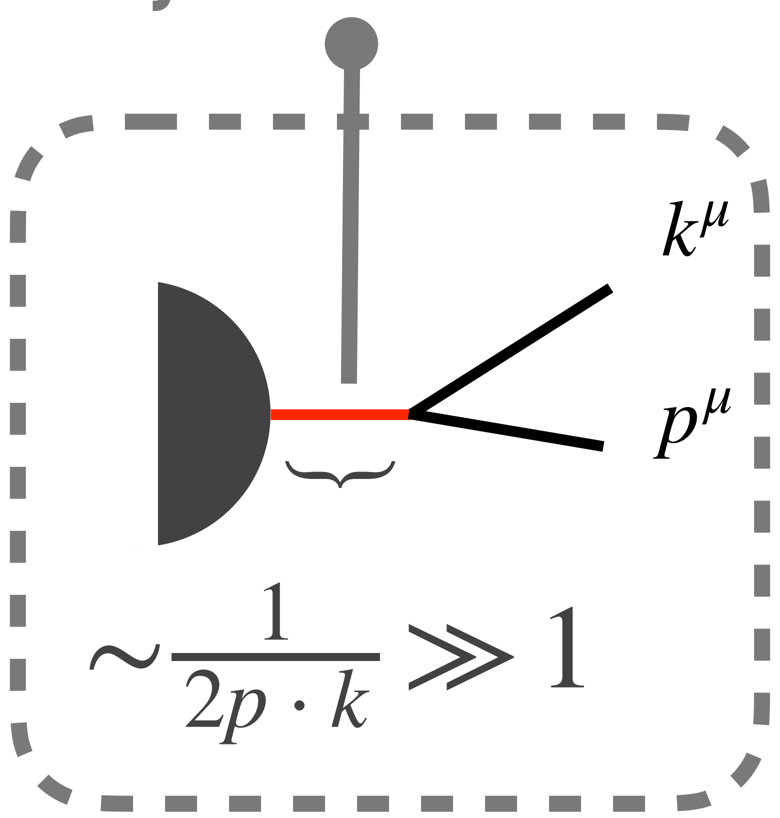
Soft factorisation



Propagator nearly on-shell $\longrightarrow$ factorisation



$$\mathscr{A}_{n+1}\mathscr{A}^\dagger_{n+1} \underset{k^\mu \to 0}{\cong} -4\pi\mu^{2\epsilon}\alpha_s \sum_{i,j=1}^{n} \frac{p_i\cdot p_j}{p_i\cdot k\, p_j\cdot k}\mathscr{A}_n(\mathbf{T}_i\cdot\mathbf{T}_j)\mathscr{A}^\dagger_n$$

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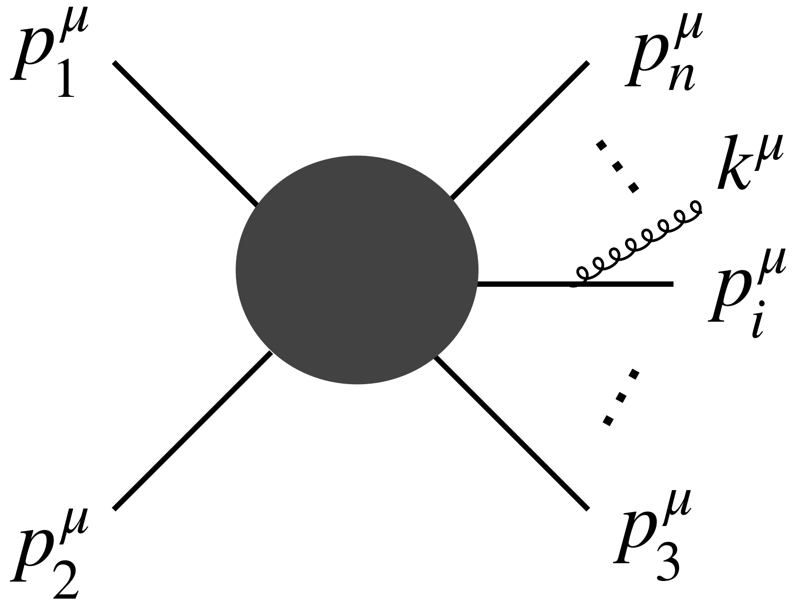

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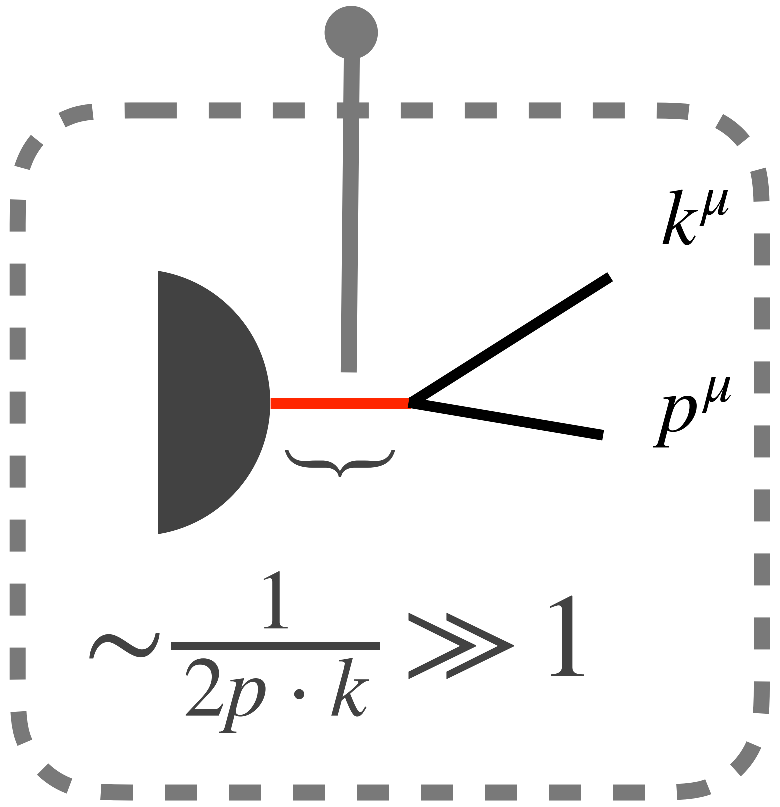
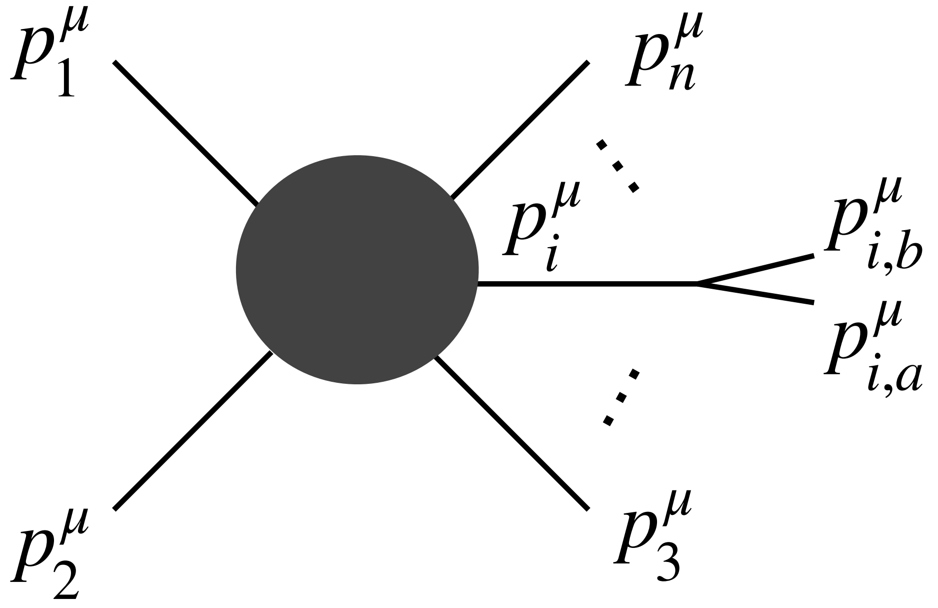
Propagator nearly on-shell ⟶ factorisation



Collinear factorisation‡



$$\mathcal{A}_{n+1}\mathcal{A}^\dagger_{n+1} \underset{\theta_{ab}\to 0}{\cong} \frac{8\pi}{s_{ab}}\mu^{2\epsilon}\alpha_s P_{ai}(z,\epsilon)\,\mathcal{A}_n\mathcal{A}_n^\dagger$$

‡ Here we neglect spin correlations between the hard squared amplitude and the splitting kernel

Path to NLO: Infrared & collinear divergences

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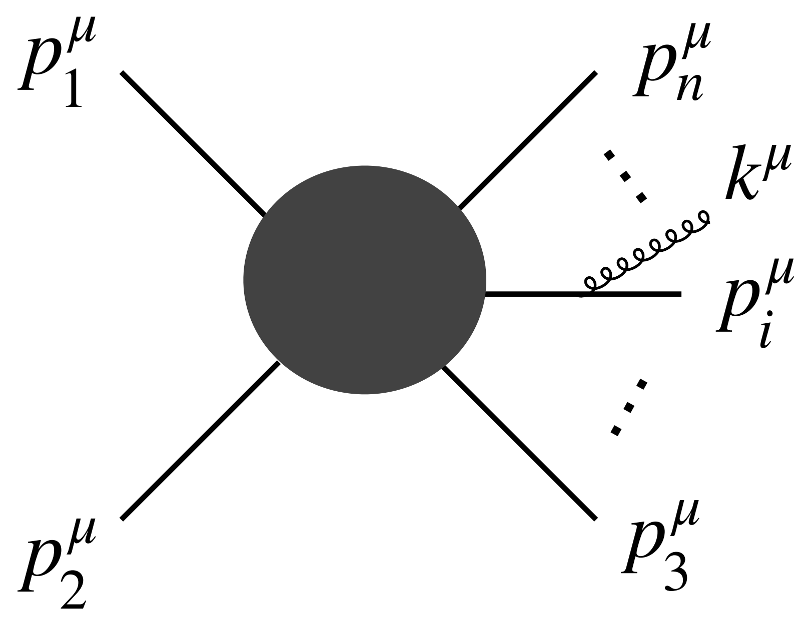
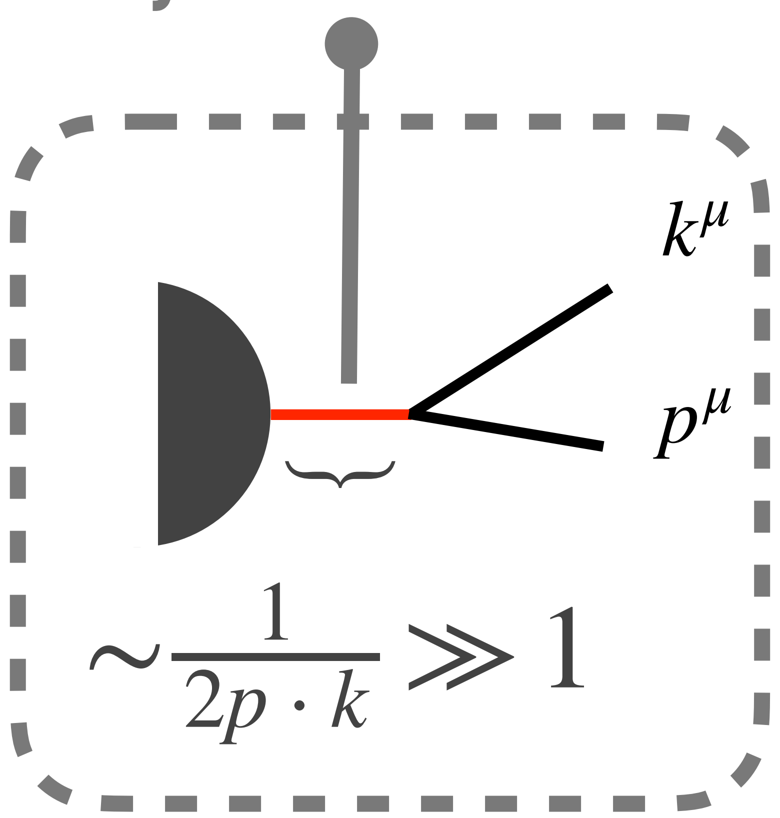
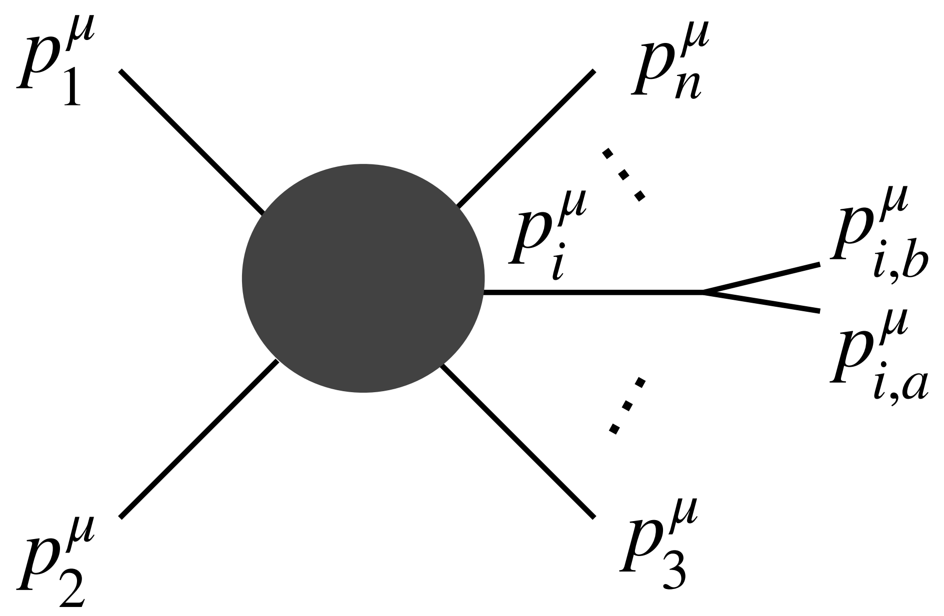


Propagator nearly on-shell → factorisation



Collinear factorisation‡



$$\mathscr{A}_{n+1}\mathscr{A}^\dagger_{n+1} \underset{k^\mu \to 0}{\cong} -4\pi\mu^{2\epsilon}\alpha_s \sum_{i,j=1}^{n} \frac{p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k} \mathscr{A}_n (\mathbf{T}_i \cdot \mathbf{T}_j) \mathscr{A}^\dagger_n$$

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- The Kinoshita-Lee-Nauenberg (KLN) theorem guarantees the cancellation of IRC divergences when summing over all possible physical states (real & virtual corrections)

- Dimensional regularisation can be used to regularise IRC divergences too, this time with $\epsilon < 0\,(D > 4)$

‡ Here we neglect spin correlations between the hard squared amplitude and the splitting kernel

Virtual corrections

We use the conventions adopted in Nucl.Phys.B 359 (1991) 283-300.

For NLO calculation in the full theory see Nucl.Phys.B 453 (1995) 17-82

Mathematica code available at this URL

- The one-loop corrections to the ggh vertex lead to the following contribution to the partonic XS
  - ➡ Include UV counter-term stemming from strong coupling renormalisation‡

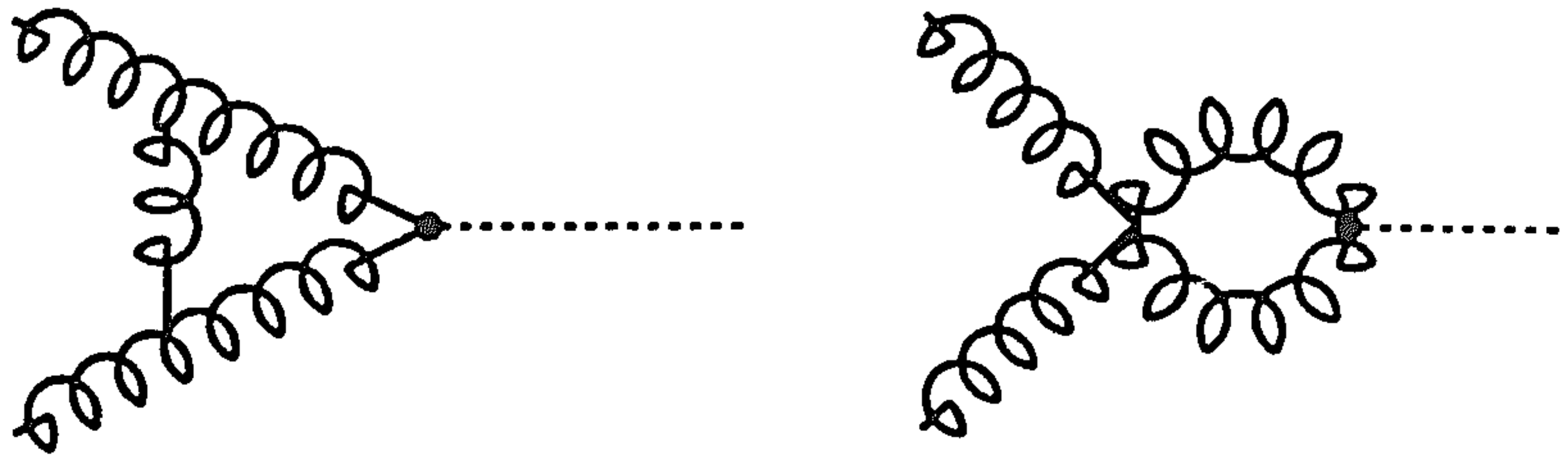
$$\hat{\sigma}_1^{\text{virt}} = \frac{\alpha_s^3}{576\,\pi^2 v^2}\left(\frac{\mu^2}{\hat{s}}\right)^{\epsilon}\left(-\frac{3}{\epsilon^2} - \frac{3}{\epsilon} + \text{regular bits}\right)\delta(1-z) + \hat{\sigma}_1^{\text{UV c.t.}}\delta(1-z)$$

After UV renormalisation:

Double pole: soft AND collinear divergence
Single pole: collinear (OR soft) divergence

Term due to renormalisation of α_s in the LO cross section

$$\hat{\sigma}_1^{\text{UV c.t.}} = -2\hat{\sigma}_0\,\alpha_s\frac{\beta_0}{\epsilon}\,, \quad \beta_0 = \frac{23}{12\pi}$$



‡ Since the top quark decouples in the EFT, the only effect of the m_t renormalisation is encoded in the Wilson coefficient

Real corrections

We use the conventions adopted in Nucl.Phys.B 359 (1991) 283-300.

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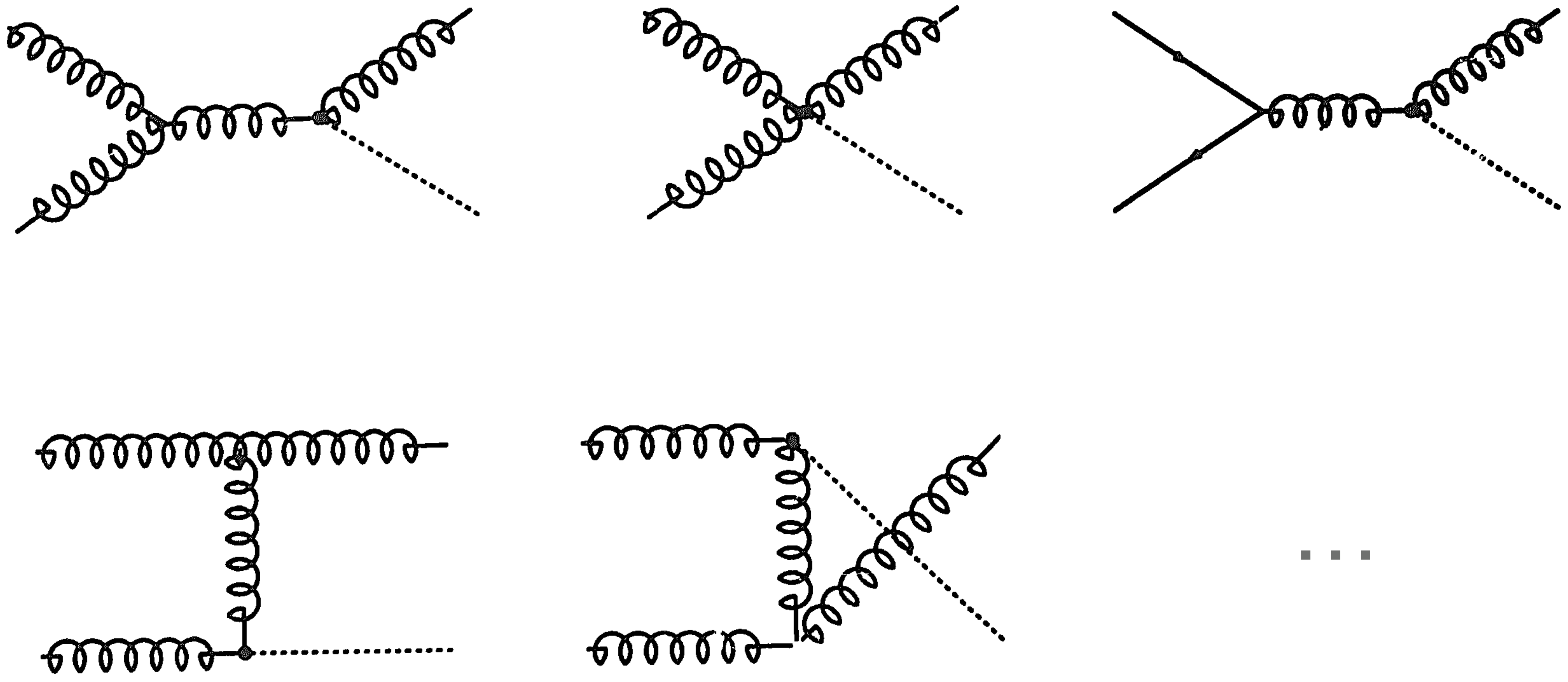
Mathematica code available at this URL

- Real corrections receive contributions from $gg \to gh$ as well as new channels ($gq \to qh, q\bar{q} \to gh$)
  - $q\bar{q} \to gh$ is finite (no poles)
  - $gq \to qh$ is divergent



$$\hat{\sigma}_1^{gq\to qh} = \frac{\alpha_s^3}{1152\,\pi^2 v^2}\left(\frac{\mu^2}{\hat{s}}\right)^{\epsilon}\left(-z\hat{P}_{gq}(z)\frac{1}{\epsilon} + \text{regular bits}\right)$$

$$\hat{P}_{gq}(z) = C_F \frac{1+(1-z)^2}{z}$$

  - $gg \to gh$ is divergent

$$\hat{\sigma}_1^{gg\to gh} = \frac{\alpha_s^3}{576\,\pi^2 v^2}\left(\frac{\mu^2}{\hat{s}}\right)^{\epsilon}\left(\left(\frac{3}{\epsilon^2}+\frac{3}{\epsilon}+\text{regular bits}\right)\delta(1-z) - z\hat{P}_{gg}(z)\frac{1}{\epsilon} + \text{regular bits}\right)$$

$$\hat{P}_{gg}(z) = 2\,C_A\left(\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right) + 2\pi\,\beta_0\delta(1-z)$$

Singularities with a $\delta(1-z)$ cancel against the virtual corrections.

What happens to the remaining single poles??

Collinear divergences & PDFs evolution

- Left-over singularity is related to our inability to sum over physical states (as required by the KLN theorem); a partonic initial state is unphysical.

  - However, we can absorb the divergence into the bare PDFs (similarly to renormalisation). This procedure can be extended systematically to higher orders, leading to collinear factorisation‡

  - Collinear divergences make PDFs evolve with μ much like UV divergences make α_s evolve with μ.

$$f_i(z) = \sum_j \Gamma_{ij}(z) \otimes f_j(z,\mu)\,, \quad \Gamma_{ij}(z) = \delta(1-z)\delta_{ij} + \frac{\alpha_s(\mu)}{2\pi}\hat{P}_{ij}(z)\frac{1}{\epsilon} + \mathcal{O}(\alpha_s^2)$$

Establishes our ability to predict the structure of ISR divergences order by order in perturbation theory

$$\frac{df_i(z,\mu)}{d\ln\mu^2} = \frac{\alpha_s(\mu)}{2\pi}\hat{P}_{ij}(z) \otimes f_j(z,\mu)$$

Structure of divergences leads to flavour mixing in DGLAP evolution, and predicts the evolution of the content of the proton!

‡ No rigorous proof of its validity is known in the most general case!

The NLO cross section & scale uncertainty

Predictions here obtained with the ggHiggs public code

- Assembling all pieces we obtain the NLO cross section
  - Very large correction due to new channels opening at NLO

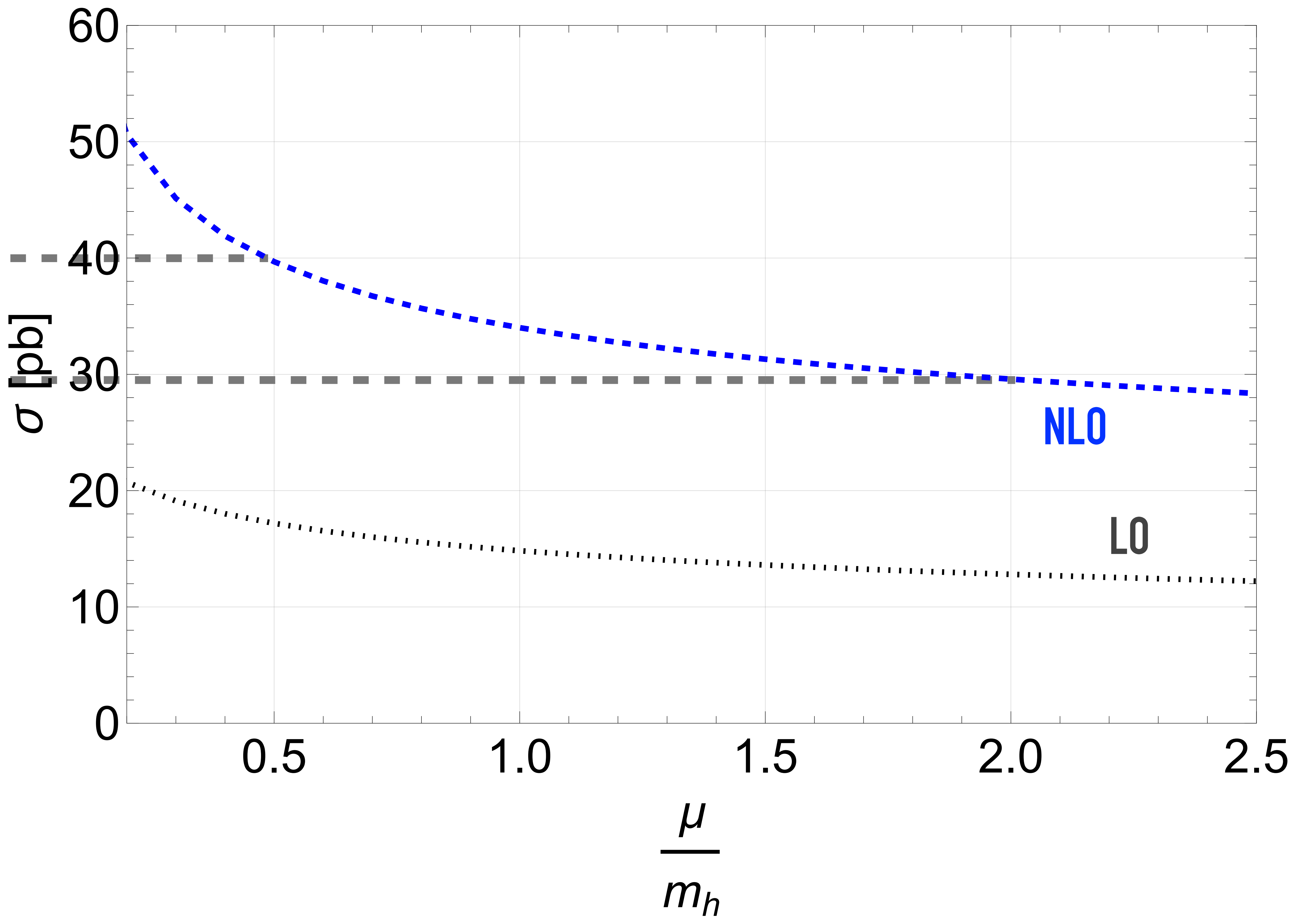
$$K = \frac{\sigma_{\rm NLO}}{\sigma_0} \sim 2$$

  - Unphysical scales provide a handle on theory uncertainty (gets smaller with higher orders)

$$\sigma_{\rm NLO} = 34^{+17\%}_{-13\%}\,{\rm pb}$$

$$\mu_F = \mu_R = \mu\,, \quad 1/2 \le \mu/m_h \le 2$$

  - Commonly μ_R (scale of the coupling) and μ_F (scale of the PDFs) are varied independently for a more conservative error estimate



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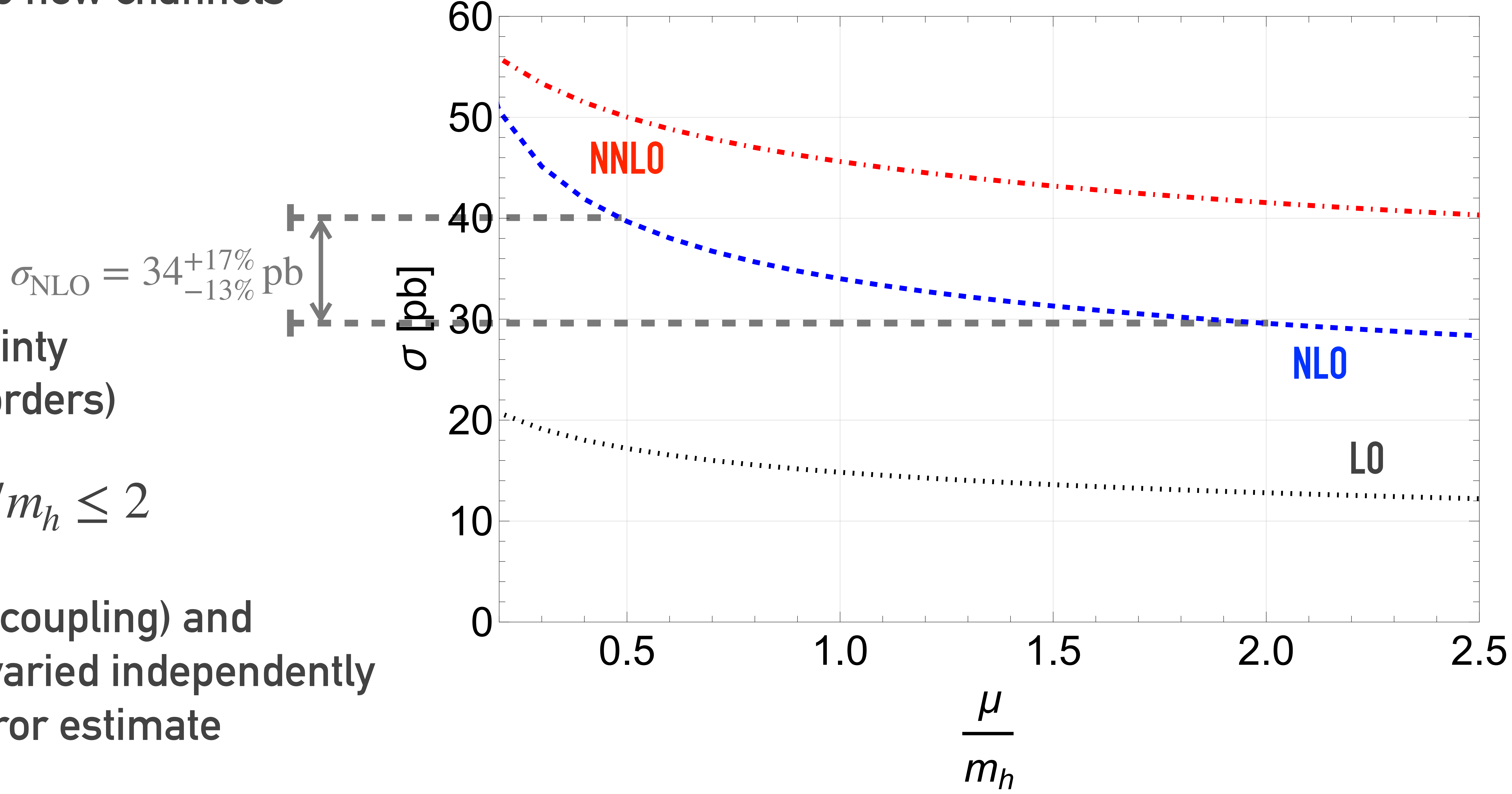

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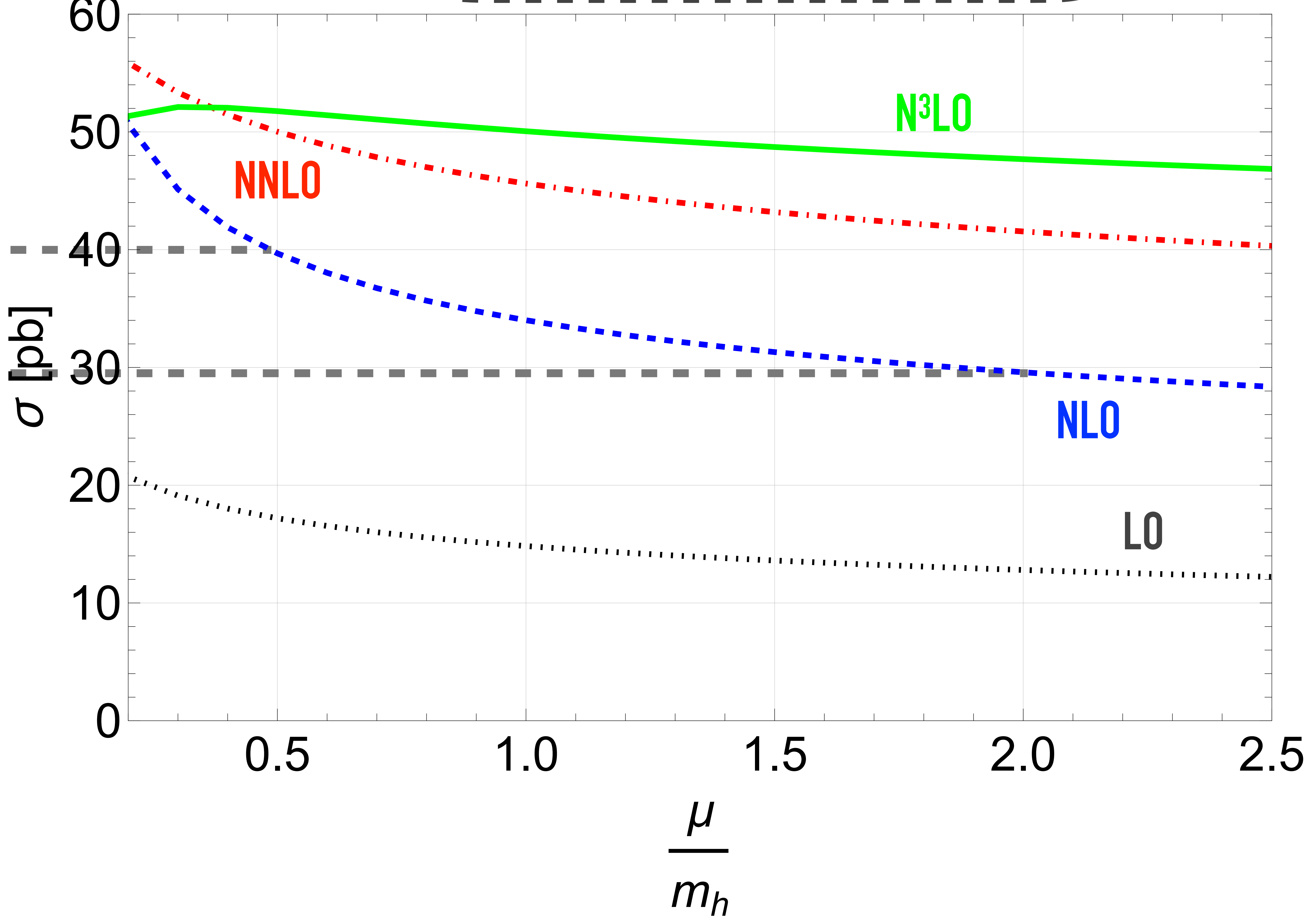
NB: N³LO necessary for few-% control over perturbative error!



Other sources of theoretical uncertainty (gluon fusion)

- Higgs XS Working Group recommendation:‡

cf. chapter I.4 of 1610.07922

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\,(\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\,(\text{PDF+}\alpha_s)\,.$$

PDF uncertainty & parametric uncertainty in α_s

- Besides the scale uncertainty, the theory error also involves other sources

δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
±1.16%	±1%	±0.83%	±1%

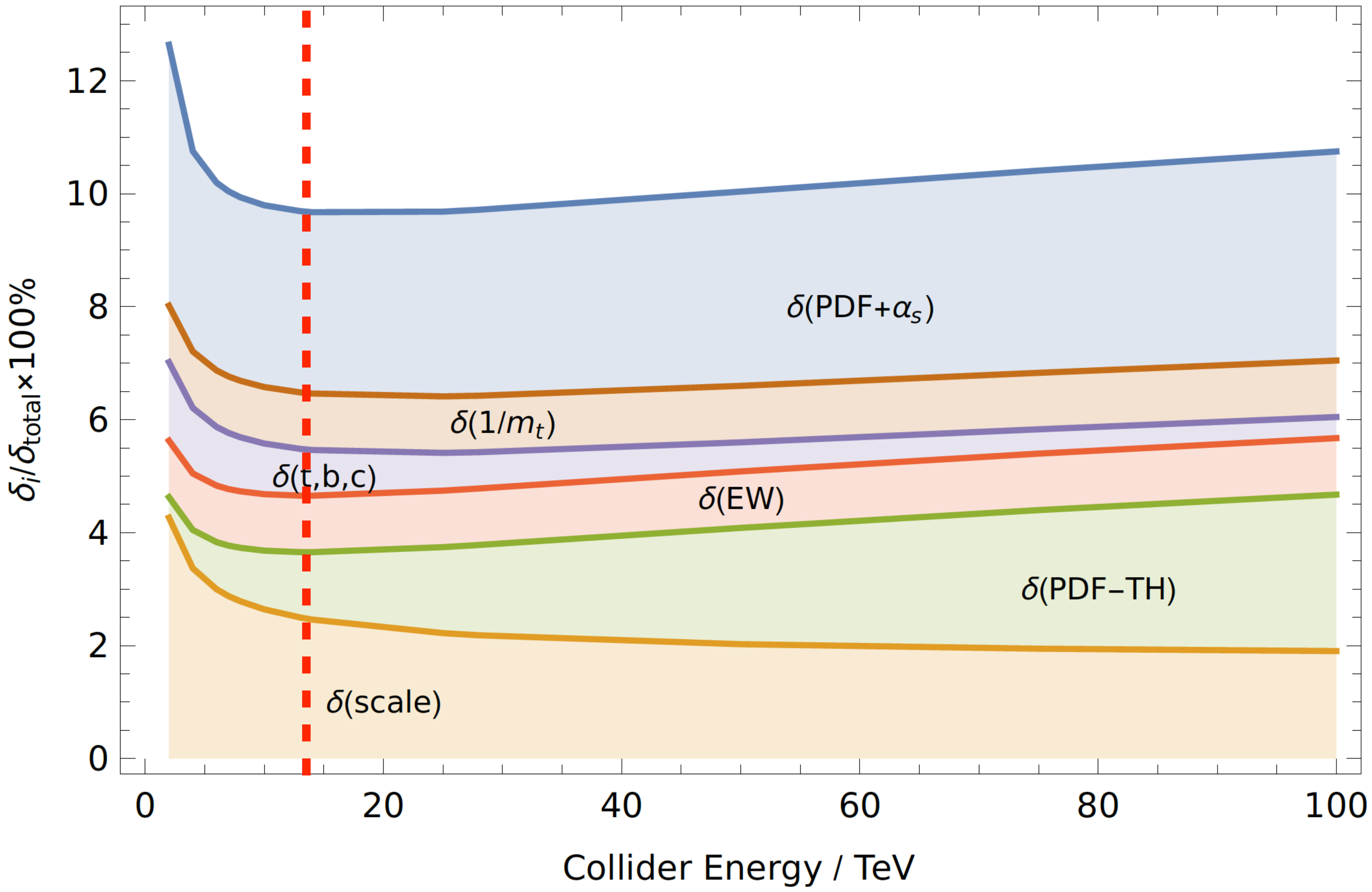
Estimate of effect of missing N³LO PDFs (NNLO PDFs used here) [δ(PDF-TH)]

Estimate of impact of NLO mixed QCD-EW correction. Now known [δ(EW)]

Estimate of uncertainty in the effect of finite quark masses beyond NLO. Now known up to NNLO [$\delta(t,b,c)$, $\delta(1/m_t)$]

TH uncertainty breakdown at N3LO (gluon fusion) 1901.00134

Predictions from Anastasiou et al. 1602.00695



‡ Central scales $\mu_R = \mu_F = m_h/2$

Back to our comparison to LHC data

- Inclusion of radiative corrections and other production channels leads to good agreement with data

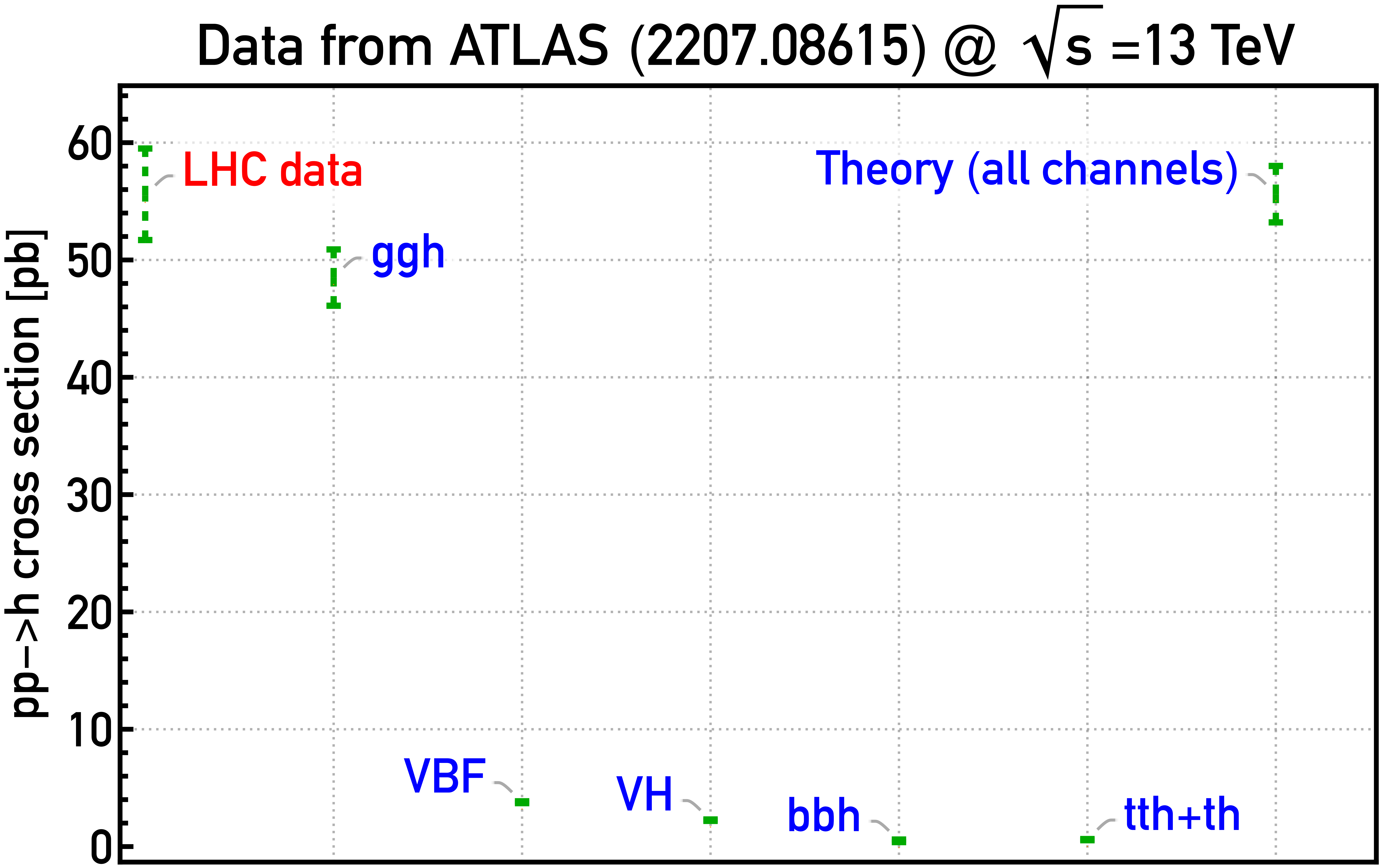
Production XS's from 1610.07922

$$\sigma_{\rm VBF} = 3.78 \pm 0.08\,{\rm pb}$$

$$\sigma_{\rm VH} = 2.25 \pm 0.06\,{\rm pb}$$

$$\sigma_{\rm b\bar{b}h} = 0.49 \pm 0.11\,{\rm pb}$$

$$\sigma_{\rm t\bar{t}h+th} = 0.59 \pm 0.05\,{\rm pb}$$



Differential collider Observables

Going more differential: IRC safety

- To ensure calculability in perturbation theory, we must guarantee the cancellation of IRC divergences between real and virtual corrections. This imposes a criterion on observables known as IRC safety

  - Two conditions on the observable $\mathcal{O}(k_1, k_2, \ldots, k_n)$

$$\mathcal{O}(k_1, \ldots, k_{i-1}, k_i, k_{i+1}, \ldots, k_n) \underbrace{\rightarrow}_{k_i || k_{i+1}} \mathcal{O}(k_1, \ldots, k_{i-1}, k_i + k_{i+1}, \ldots, k_n)$$

$$\mathcal{O}(k_1, \ldots, k_{i-1}, k_i, k_{i+1}, \ldots, k_n) \underbrace{\rightarrow}_{k_i \to 0} \mathcal{O}(k_1, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n)$$

Observable is insensitive to very soft emissions or very collinear splittings

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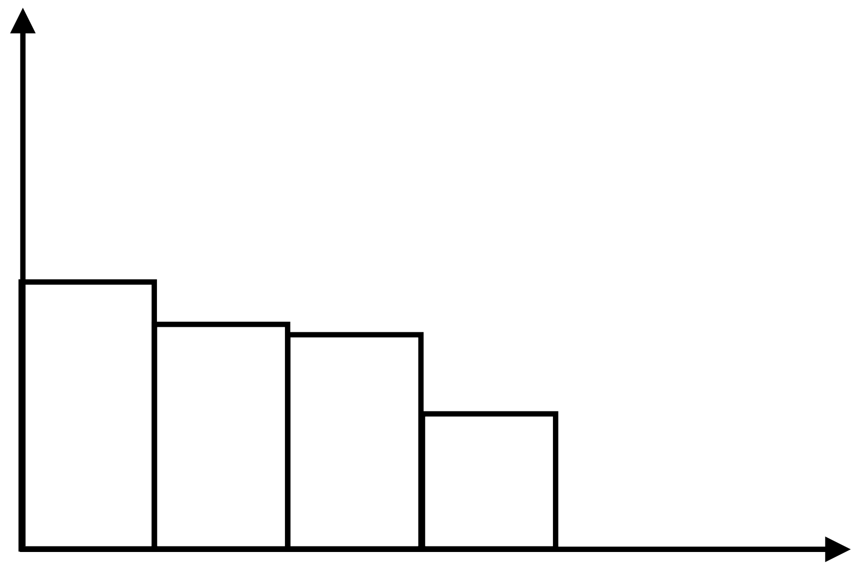
  - Consider a histogram for a given observable:



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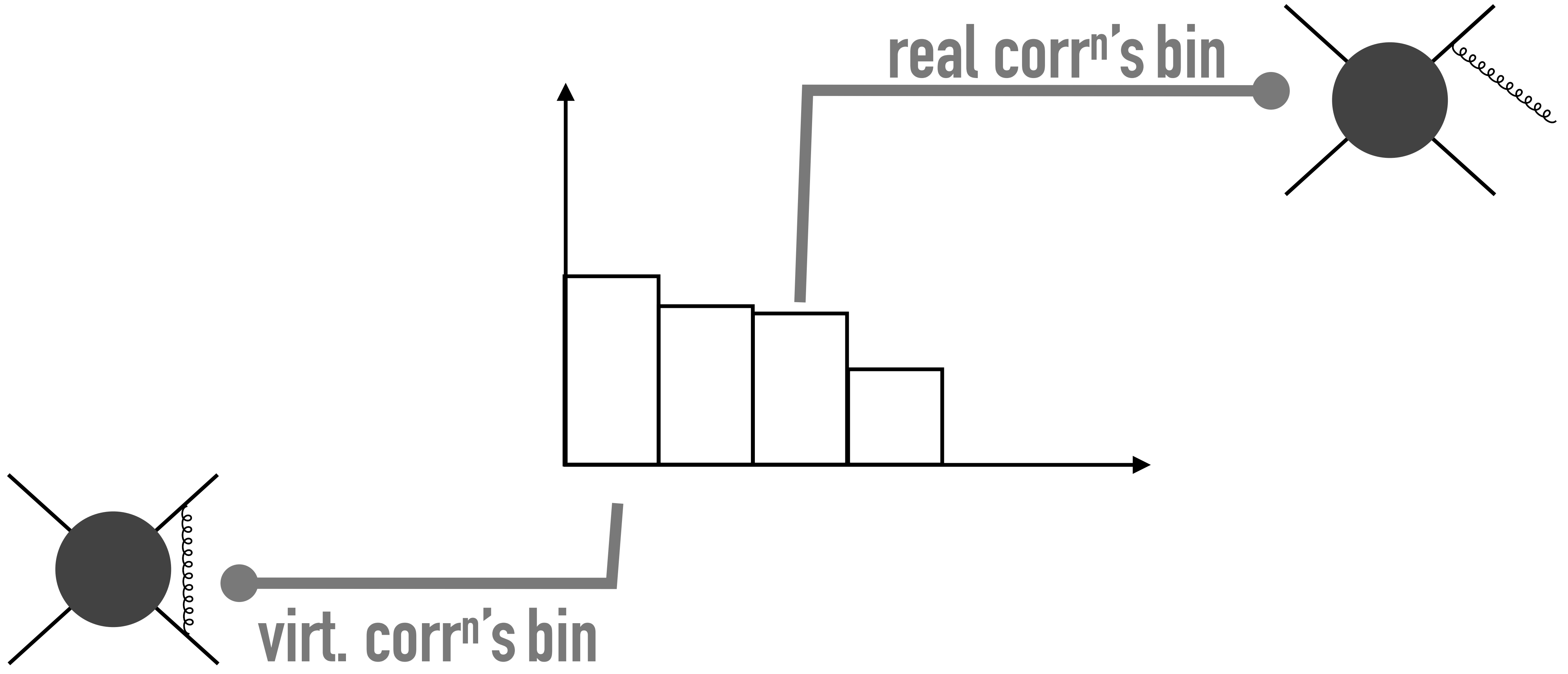


Going more differential: IRC safety

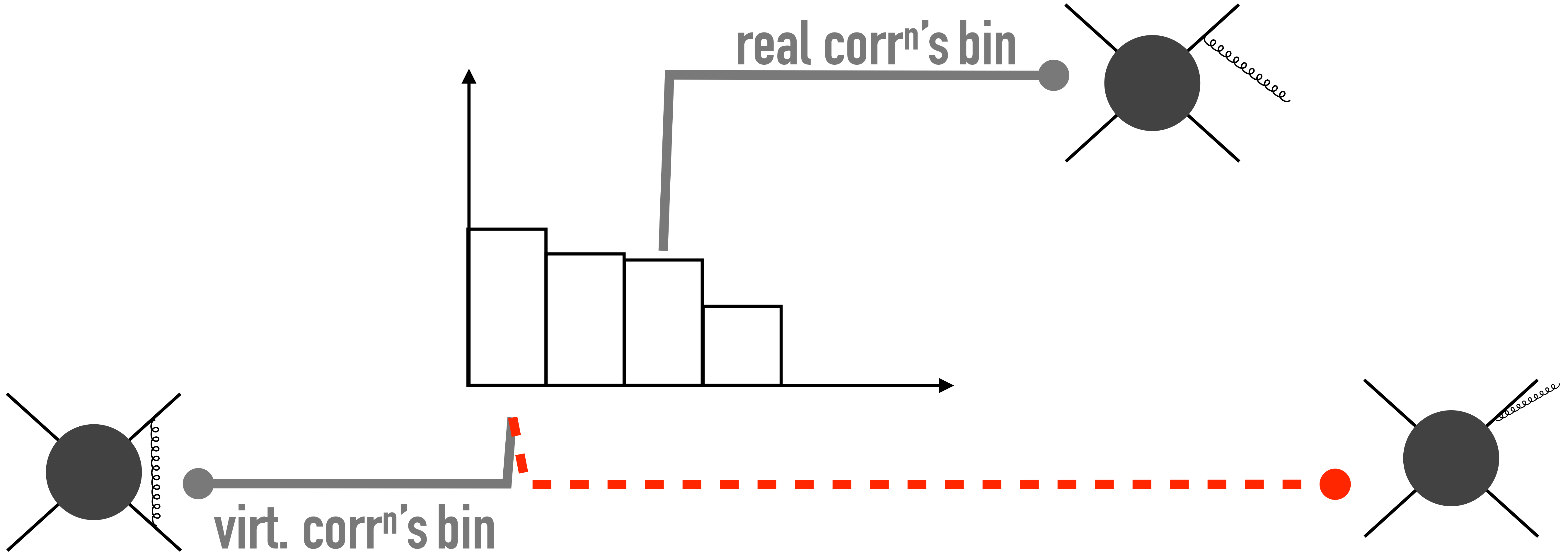
- To ensure calculability in perturbation theory, we must guarantee the cancellation of IRC divergences between real and virtual corrections. This imposes a criterion on observables known as IRC safety
  - Two conditions on the observable $\mathcal{O}(k_1, k_2, \ldots, k_n)$

$$\mathcal{O}(k_1, \ldots, k_{i-1}, k_i, k_{i+1}, \ldots, k_n) \underbrace{\rightarrow}_{k_i || k_{i+1}} \mathcal{O}(k_1, \ldots, k_{i-1}, k_i + k_{i+1}, \ldots, k_n)$$

$$\mathcal{O}(k_1, \ldots, k_{i-1}, k_i, k_{i+1}, \ldots, k_n) \underbrace{\rightarrow}_{k_i \rightarrow 0} \mathcal{O}(k_1, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n)$$

Observable is insensitive to very soft emissions or very collinear splittings

  - Consider a histogram for a given observable:



The reals must end up in the same bin as the virtuals when radiation is soft and/or collinear

Other IRC safe observables

- Make it always customary to check whether an observable is IRC safe. Some examples:
  - Is the energy of a quark/gluon/hadron IRC safe?
  - Is the Higgs rapidity IRC safe?
  - Is the transverse momentum of the Higgs boson IRC safe?
  - Are jet observables IRC safe (e.g. leading-jet transverse momentum)?
  - […]

Other IRC safe observables

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  - [...]

Other IRC safe observables

- Make it always customary to check whether an observable is IRC safe. Some examples:
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Other IRC safe observables

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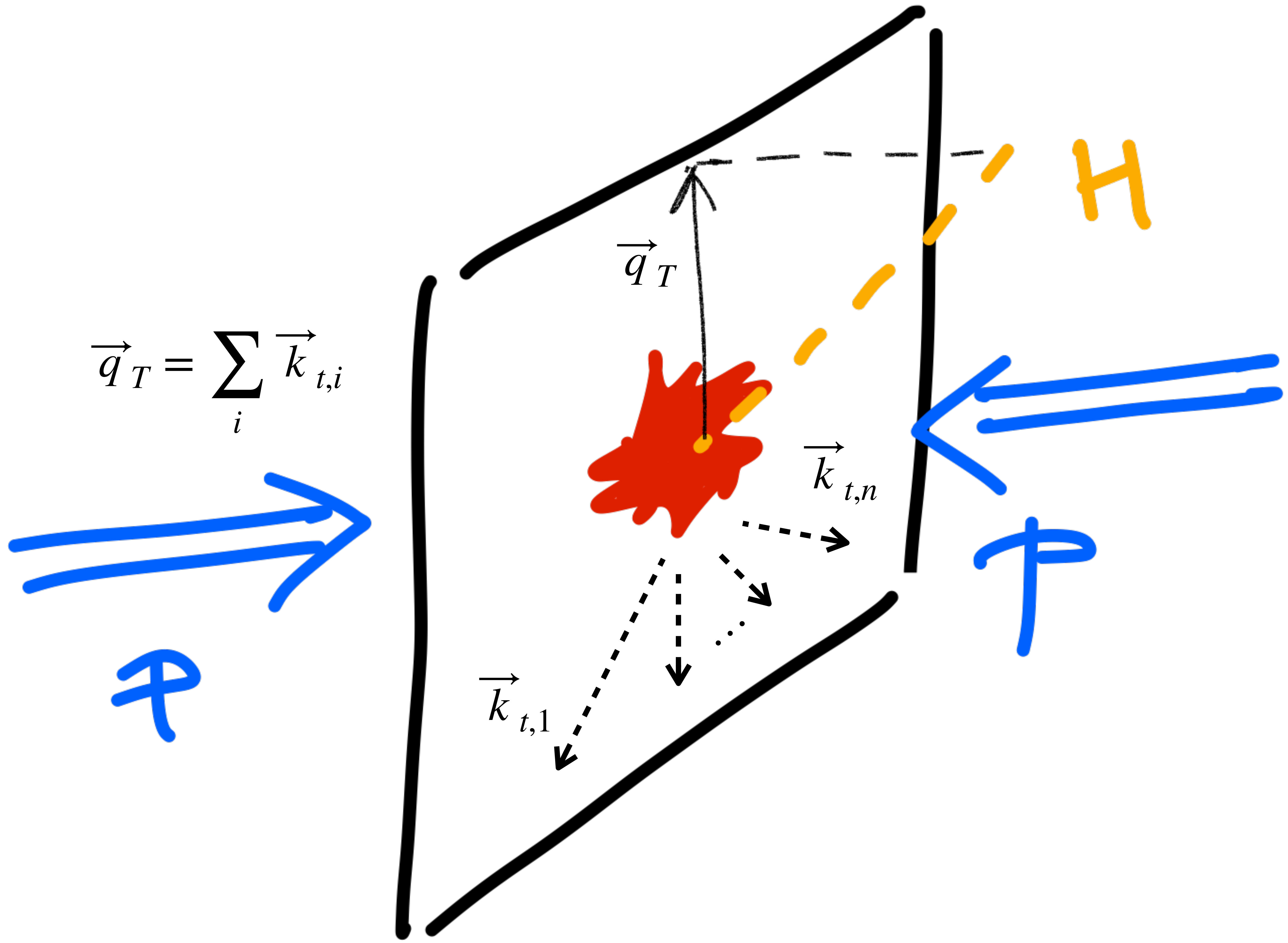
  - Are jet observables IRC safe (e.g. leading-jet transverse momentum)? [?]
  - [...]

An example: The Higgs transverse momentum spectrum

- Theoretically/experimentally precise: recoil of Higgs against radiation

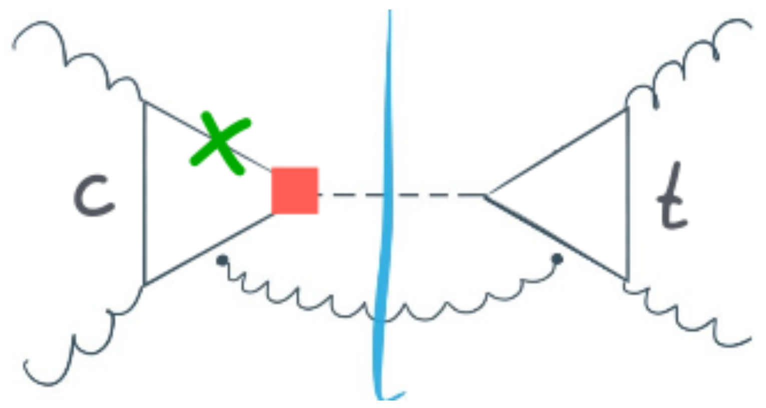


Why is it interesting?

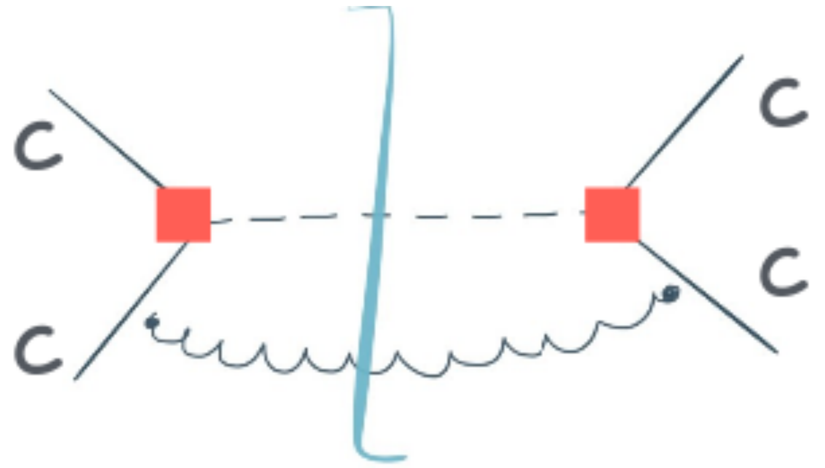
- Spans wide range of momentum scales: sensitivity to a variety of effects in different regimes

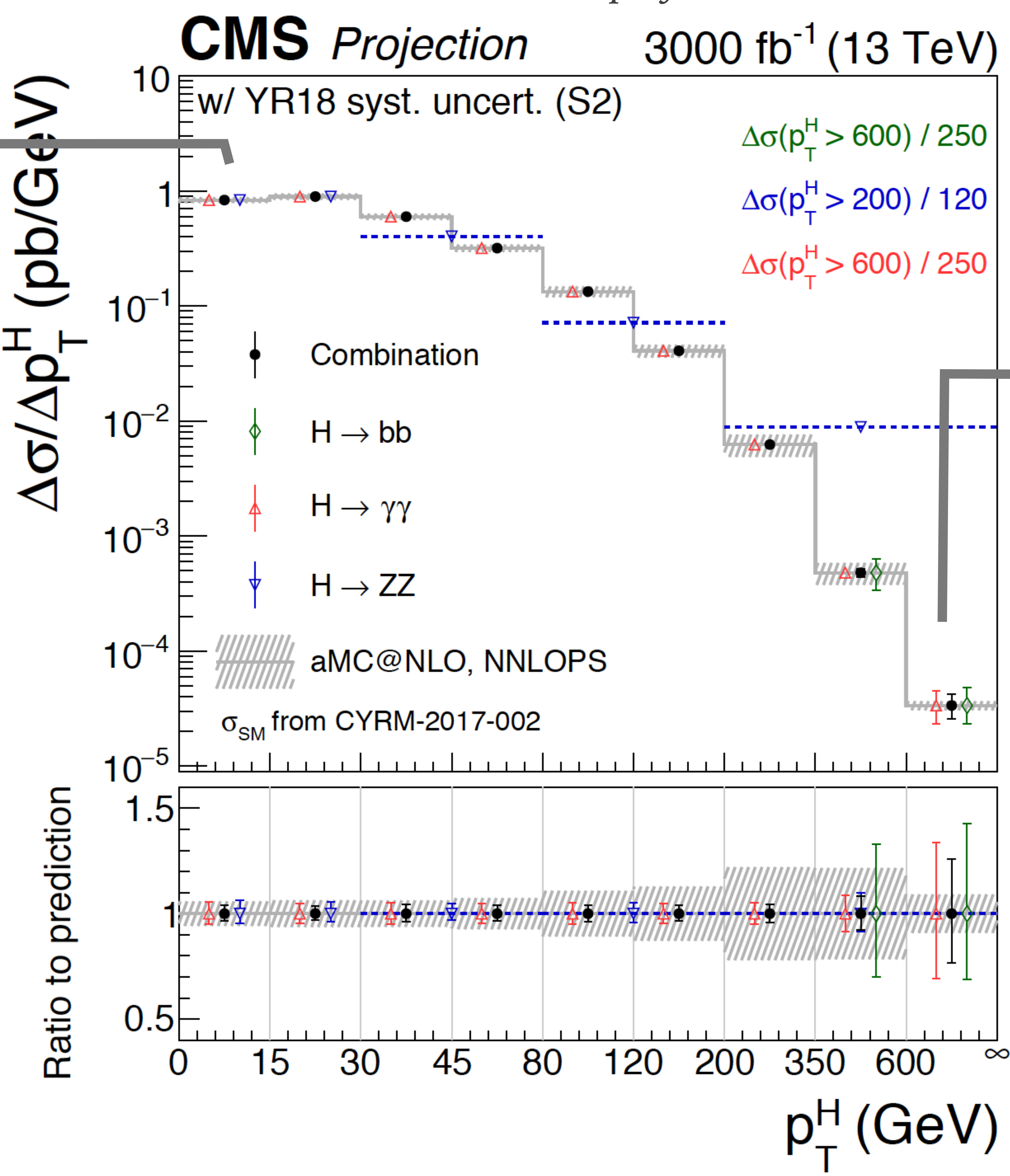
HL-LHC projection 1902.00134

Low-q_T regime can be exploited to infer indirect bounds on light-quark Yukawa couplings (e.g. charm quark)

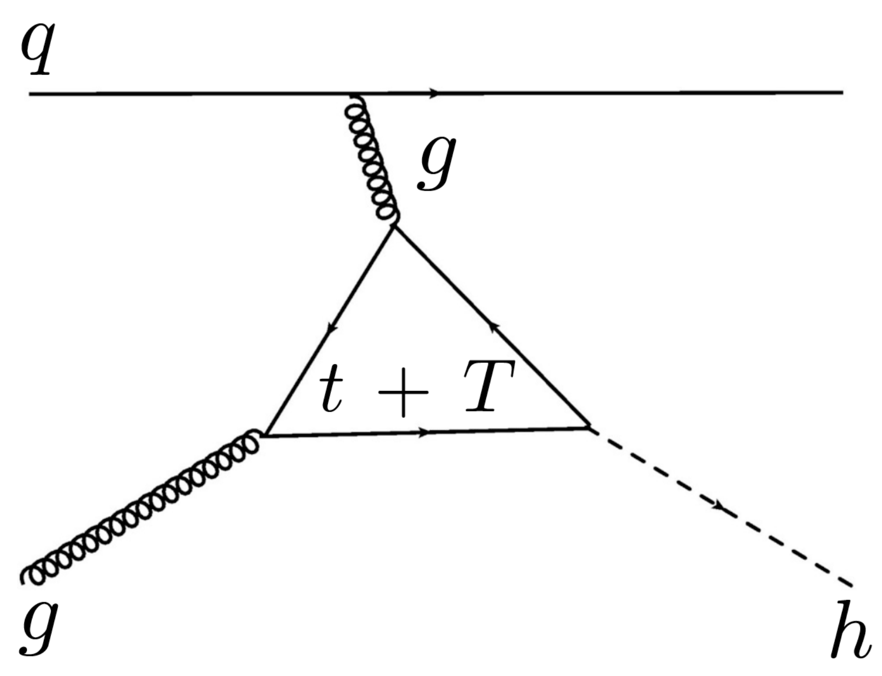


vs.





High-q_T (boosted) regime can be exploited to set bounds on couplings to top quark and gluon (e.g. heavy top partners)



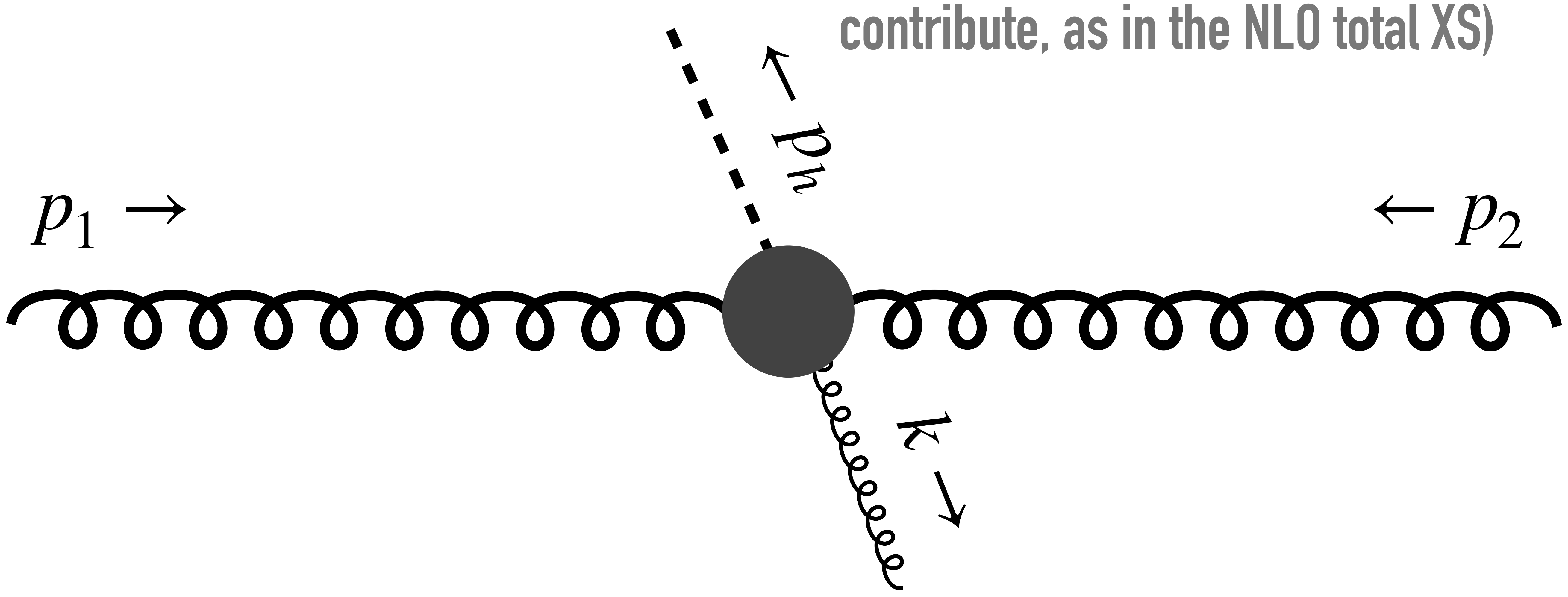
Calculating the Higgs q_T distribution at leading order

- As for the total XS, we can compute the spectrum in the large-m_t EFT and rescale by the ratio of the LO cross section in the full theory to the EFT one. We choose the following parametrisation for the kinematics

$$p_1^\mu = \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \qquad k^\mu = k_t\,(\cosh\eta, 1, 0, \sinh\eta)$$

$$p_2^\mu = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1) \qquad p_h^\mu = \left(\sqrt{m_h^2 + q_T^2\cosh^2\eta}, -q_T, 0, -q_T\sinh\eta\right)$$

e.g. gg → gh (also other flavour channels contribute, as in the NLO total XS)



The relative azimuthal angle between the Higgs and the radiation can be integrated out exploiting Lorentz invariance

Calculating the Higgs q_T distribution at leading order

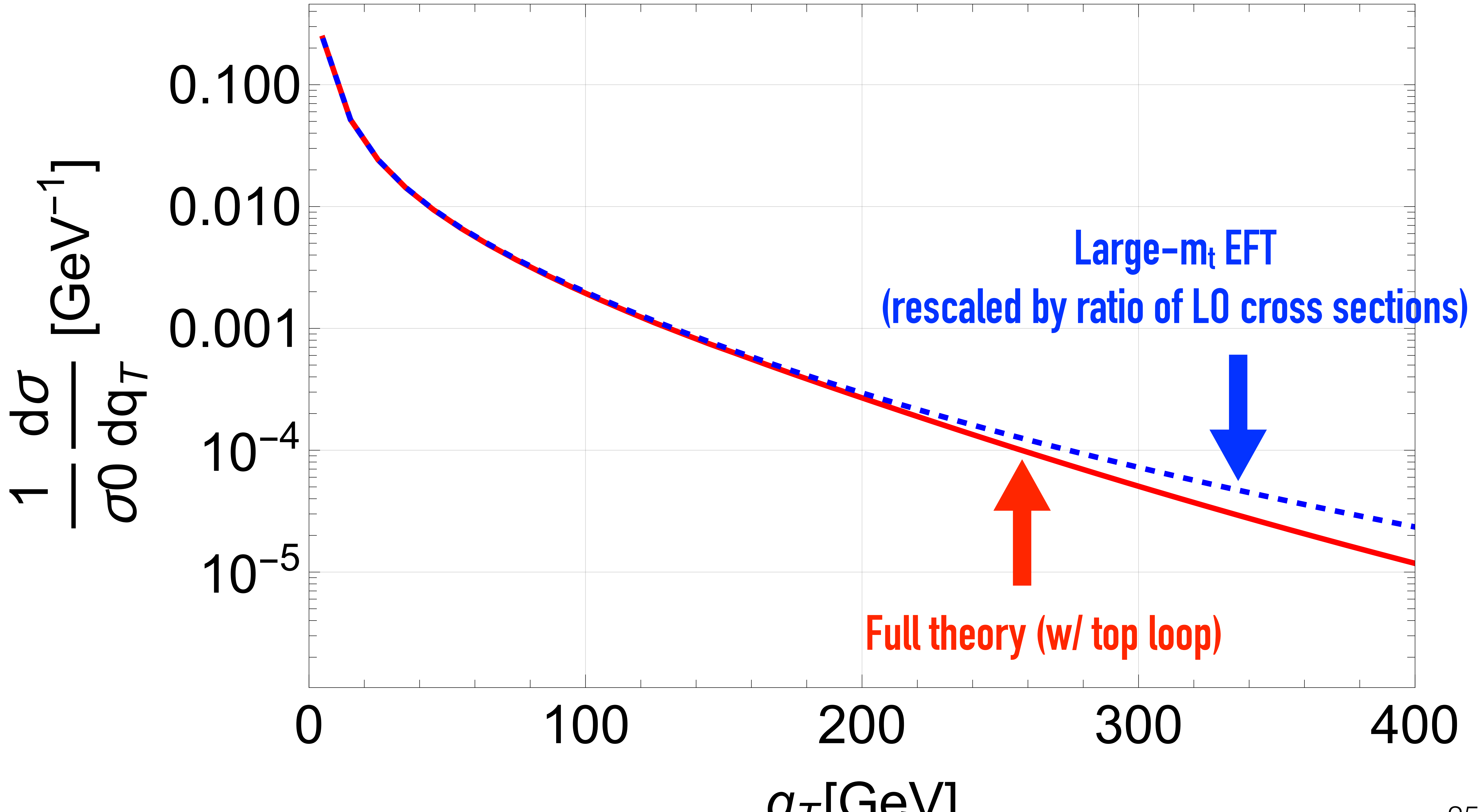
Predictions here obtained with the H1Jet public code

Parton luminosity (as in total XS)

$$\frac{d\sigma}{dq_T} = \frac{q_T}{8\pi}\int_{-\eta_{\max}}^{\eta_{\max}} d\eta \sum_{ij}\left[\frac{|M_{ij}|^2(\hat{s},\hat{t},\hat{u},\mu_R)}{E_h\,\hat{s}^{3/2}}\mathcal{L}_{ij}\left(\frac{\hat{s}}{s},\mu_F\right)\right]_{\hat{s}=\left(q_T\cosh\eta+\sqrt{m_h^2+q_T^2\cosh^2\eta}\right)^2}$$

$$\hat{s} = (p_1+p_2)^2$$
$$\hat{t} = (p_1-k)^2$$
$$\hat{u} = (p_2-k)^2$$

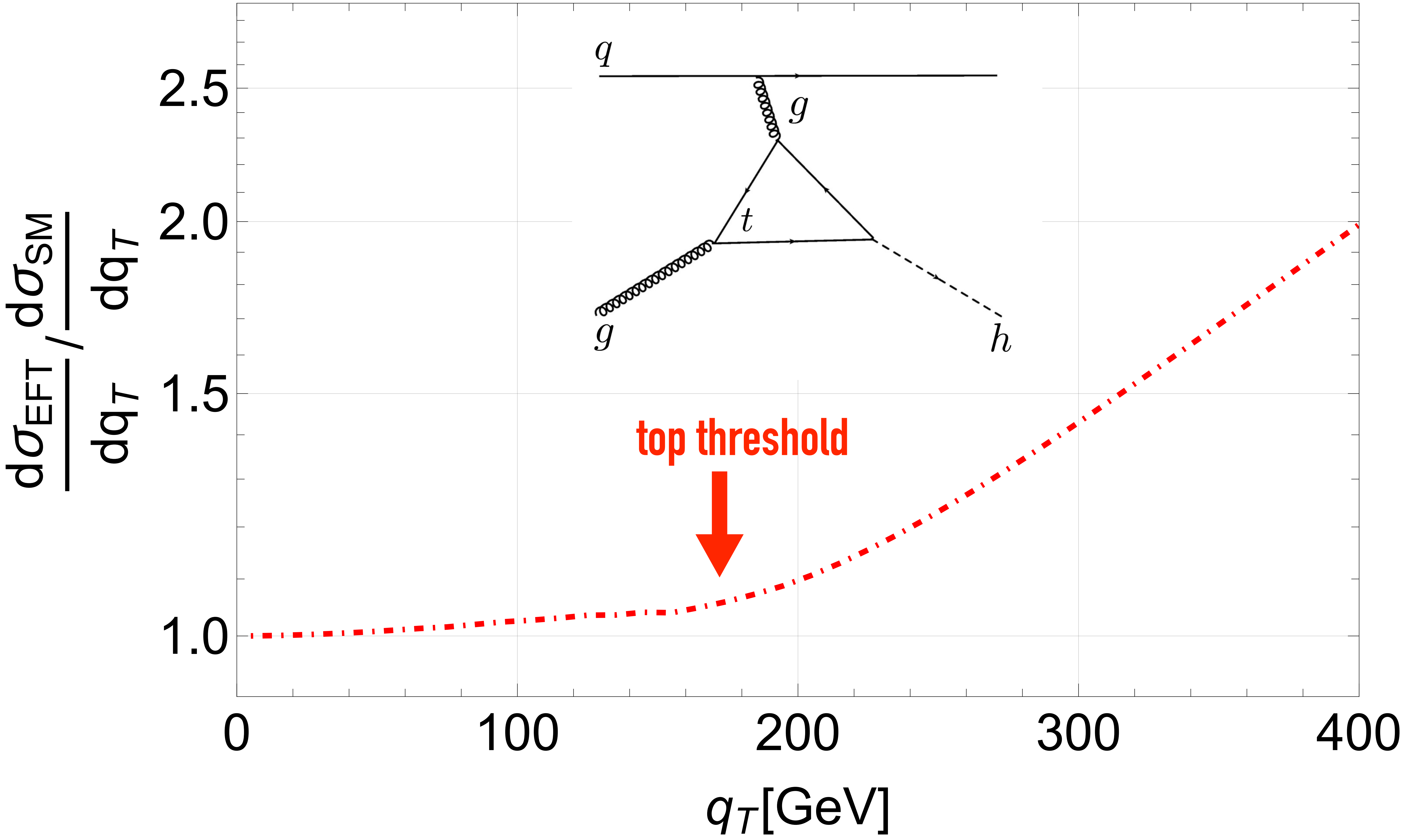
$$\eta_{\max} = \ln\left(\frac{s-m_h^2}{2\sqrt{s}q_T}+\sqrt{\left(\frac{s-m_h^2}{2\sqrt{s}q_T}\right)^2-1}\right)$$



Large-q_T regime: validity of heavy-top EFT

Even when rescaled by the ratio of LO cross sections, the EFT approximation breaks down around the top threshold, due to sensitivity to particle content in the loop.

This is an important observation: the tail of the q_T distribution is sensitive to potential heavy new physics in the loop!

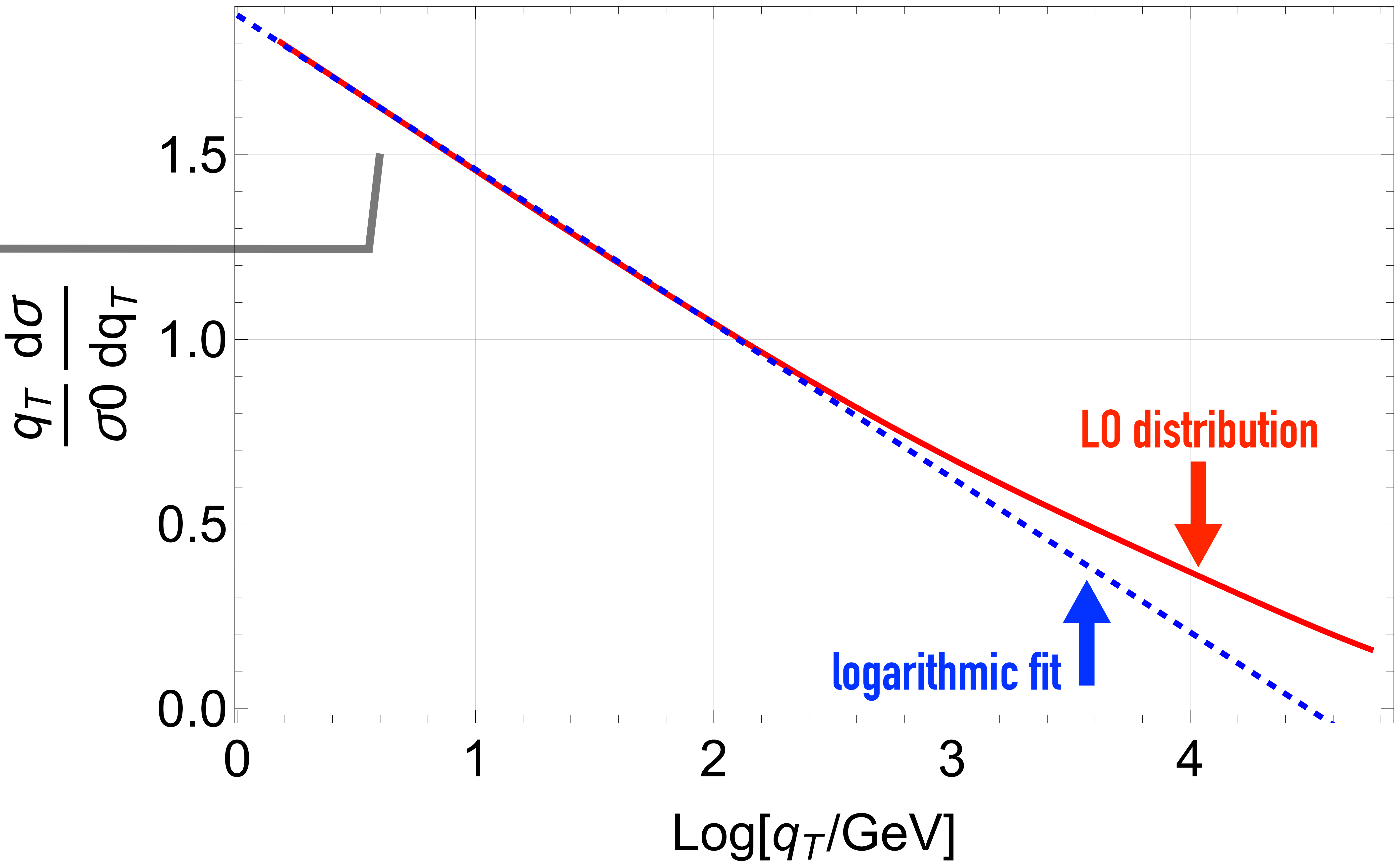


Low-q_T regime: logarithmic divergences

- The divergent structure of gauge theories in the IRC regime leaves behind a logarithmic sensitivity to large hierarchies of scales

→ Physical enhancement of configurations characterised by soft and/or collinear radiation

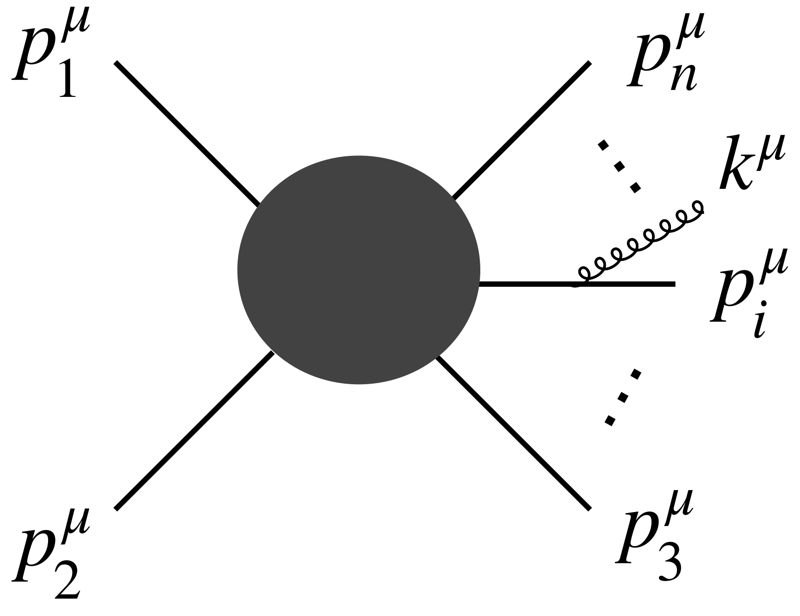
e.g. transverse momentum spectrum: LO distribution diverges as a logarithm of q_T in the small-q_T limit! Can we predict the structure of the divergence?



Recall soft/collinear factorisation of QCD squared amplitudes

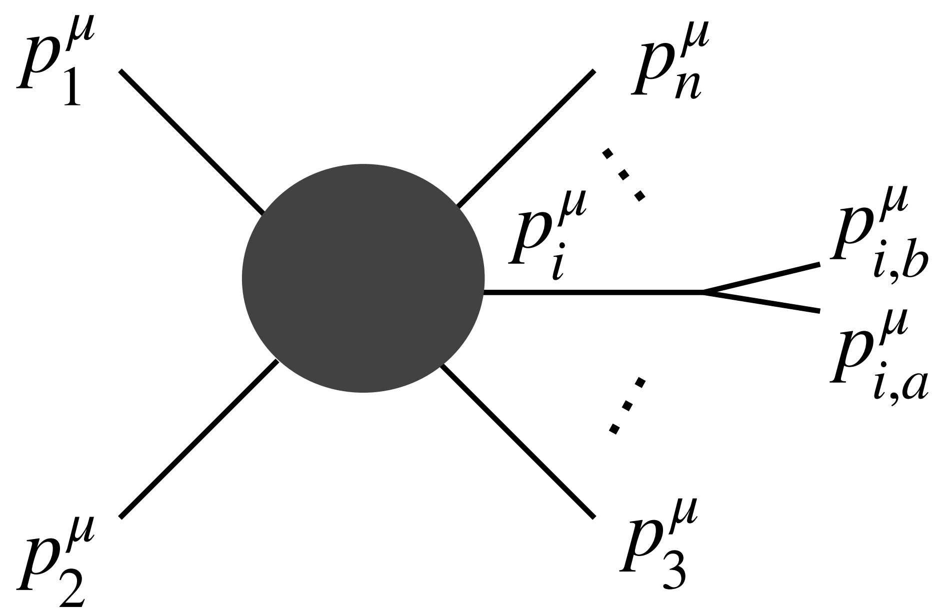
- In the logarithmic limits, QCD squared amplitudes factorise into lower-point squared amplitudes and universal singular kernels (shown below at the lowest perturbative order)

Soft factorisation



$$\mathscr{A}_{n+1}\mathscr{A}^\dagger_{n+1} \underset{k^\mu \to 0}{\simeq} -4\pi\,\mu^{2\epsilon}\alpha_s \sum_{i,j=1}^{n} \frac{p_i \cdot p_j}{p_i \cdot k\, p_j \cdot k} \mathscr{A}_n (\mathbf{T}_i \cdot \mathbf{T}_j) \mathscr{A}^\dagger_n$$

Collinear factorisation‡



$$\mathscr{A}_{n+1}\mathscr{A}^\dagger_{n+1} \underset{\theta_{ab} \to 0}{\simeq} \frac{8\pi}{s_{ab}}\,\mu^{2\epsilon}\alpha_s\, P_{ai}(z,\epsilon)\, \mathscr{A}_n \mathscr{A}^\dagger_n$$

- This factorisation allows for a systematic control of the logarithmic IRC divergences

‡ Here we neglect spin correlations between the hard squared amplitude and the splitting kernel

Calculating the leading divergence of the q_T spectrum

- The dominant singularity in the NLO q_T spectrum arises when the radiation is simultaneously soft and collinear to the beam

Soft factorisation (previous slide), together with colour conservation $\mathbf{T}_1 \cdot \mathbf{T}_2 = -\mathbf{T}_1^2 = -C_A = -3$ predicts (set $\epsilon = 0$)

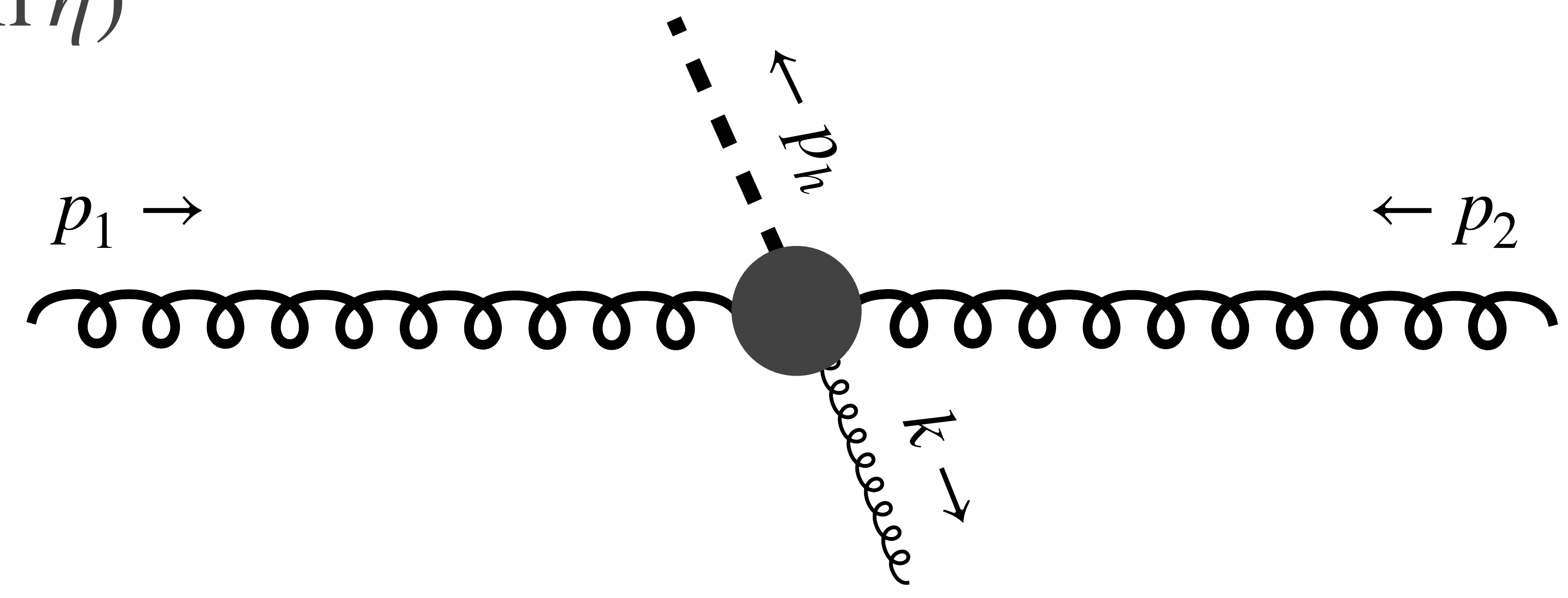
$$|\mathscr{A}_{g(p_1)g(p_2)\to g(k)h(p_h)}|^2 \underset{k^\mu \to 0}{\simeq} 4\pi\,\alpha_s C_A \frac{p_1\cdot p_2}{p_1\cdot k\, p_2\cdot k}|\mathscr{A}_{g(p_1)g(p_2)\to h(p_h)}|^2$$

➡ We can use a simple kinematic parametrisation to calculate the resulting phase space integral

$$p_1^\mu = \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \qquad k^\mu = \alpha_k\, p_1^\mu + \beta_k\, p_2^\mu + k_\perp^\mu = k_t\,(\cosh\eta, \cos\phi, \sin\phi, \sinh\eta)$$

$$p_2^\mu = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1) \qquad k_t = |k_\perp| \qquad |\eta| = \frac{1}{2}\left|\ln\frac{\alpha_k}{\beta_k}\right| \leq \ln\frac{\sqrt{\hat{s}}\simeq m_h}{k_t} \equiv \eta_{\max}$$

$|\eta|$: emission's rapidity



$$\frac{d\sigma^{\rm DL}}{dq_T} = \sigma_0 \frac{\alpha_s}{2\pi} 4\,C_A \int_0^{\sqrt{\hat{s}}} \frac{dk_t}{k_t} \int_{-\eta_{\max}}^{\eta_{\max}} d\eta \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\delta(q_T - k_t) = \sigma_0 \frac{\alpha_s}{2\pi}\, 8\, C_A \frac{\ln\frac{m_h}{q_T}}{q_T}$$

Structure of logarithmic divergences

- The pattern of the logarithmic divergences can be predicted using soft and collinear factorisation
  - ➡ It's convenient to work with the cumulative distribution ($L \equiv \ln \frac{m_h}{q_T}$)

Cumulative distribution=XS for producing a Higgs boson with transverse momentum < q_T

$$\sigma(q_T) = \sigma - \int_{q_T} dq'_T \frac{d\sigma}{dq'_T}$$

$$\mathcal{O}(\alpha_s): \quad \alpha_s L^2 \quad \alpha_s L \quad \alpha_s$$

$$\mathcal{O}(\alpha_s^2): \quad \alpha_s^2 L^4 \quad \alpha_s^2 L^3 \quad \alpha_s^2 L^2 \quad \alpha_s^2 L \quad \alpha_s^2$$

$$\mathcal{O}(\alpha_3^2): \quad \alpha_s^3 L^6 \quad \alpha_s^3 L^5 \quad \alpha_s^3 L^4 \quad \alpha_s^3 L^3 \quad \alpha_s^3 L^2 \quad \alpha_s^3 L \quad \alpha_s^3$$

$$\ldots$$

$$\mathcal{O}(\alpha_n^2): \quad \alpha_s^n L^{2n} \quad \alpha_s^n L^{2n-1} \quad \alpha_s^n L^{2n-2} \quad \cdots \quad \alpha_s^n L \quad \alpha_s^n$$

- ➡ Problem: when $q_T \ll m_h$ (e.g. $L^2 \sim 1/\alpha_s$) we face a breakdown of the perturbative expansion!

Rescuing the predictive power of perturbative QCD: resummation

- Convergence can be recast by reorganising the perturbative series across orders: resummation
  - For instance, if we are interested in resumming the double-logarithmic (DL) tower of terms

Cumulative distribution=XS for producing a Higgs boson with transverse momentum < q_T

$$\sigma(q_T) = \sigma - \int_{q_T} dq'_T \frac{d\sigma}{dq'_T}$$

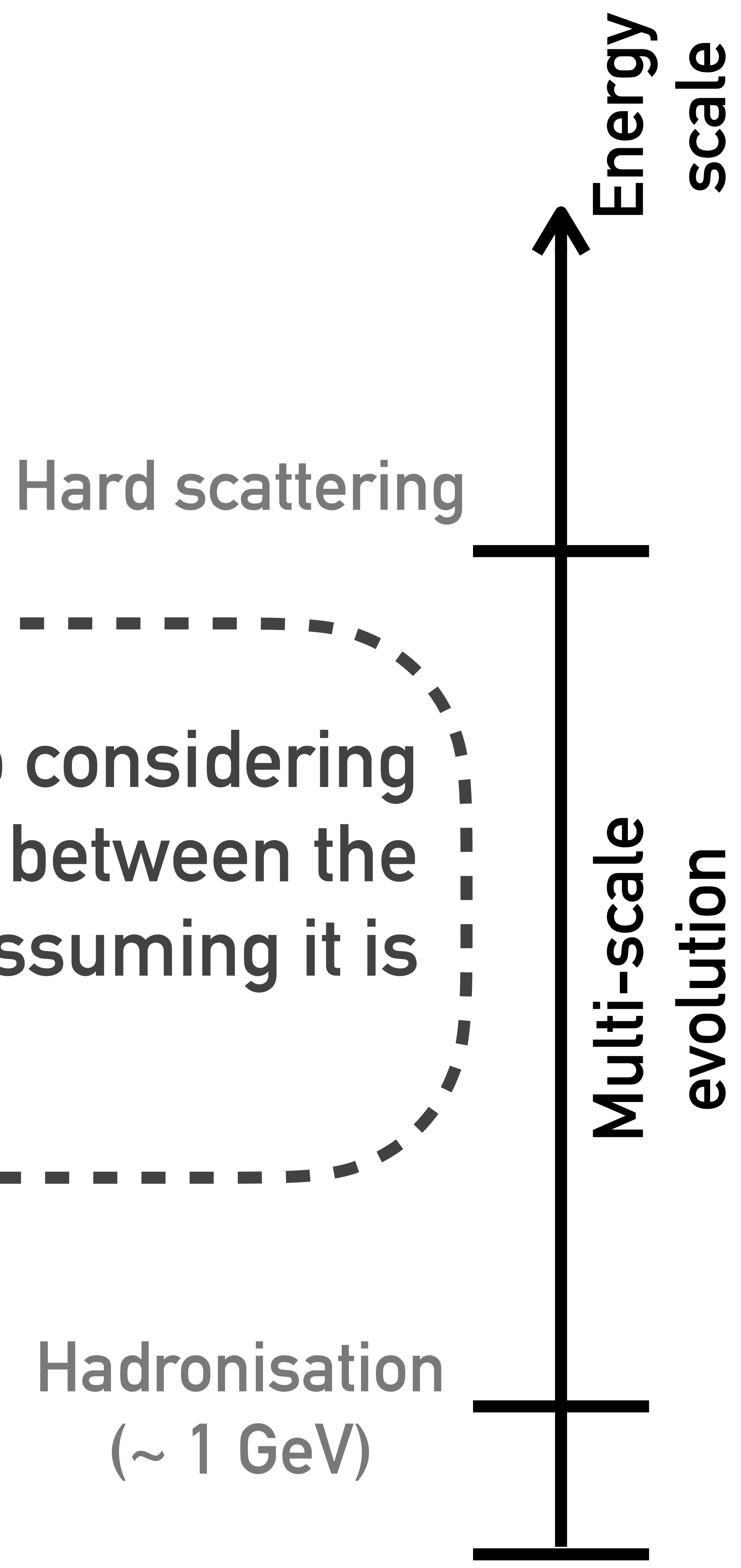
$$\begin{array}{llllllll}
\mathcal{O}(\alpha_s): & \alpha_s L^2 & \alpha_s L & \alpha_s & & & & \\
 & + & & & & & & \\
\mathcal{O}(\alpha_s^2): & \alpha_s^2 L^4 & \alpha_s^2 L^3 & \alpha_s^2 L^2 & \alpha_s^2 L & \alpha_s^2 & & \\
 & + & & & & & & \\
\mathcal{O}(\alpha_3^2): & \alpha_s^3 L^6 & \alpha_s^3 L^5 & \alpha_s^3 L^4 & \alpha_s^3 L^3 & \alpha_s^3 L^2 & \alpha_s^3 L & \alpha_s^3 \\
 & + & & & & & & \\
 & \dots & & \dots & & & & \\
\mathcal{O}(\alpha_n^2): & \alpha_s^n L^{2n} & \alpha_s^n L^{2n-1} & \alpha_s^n L^{2n-2} & \cdots & \alpha_s^n L & \alpha_s^n &
\end{array}$$

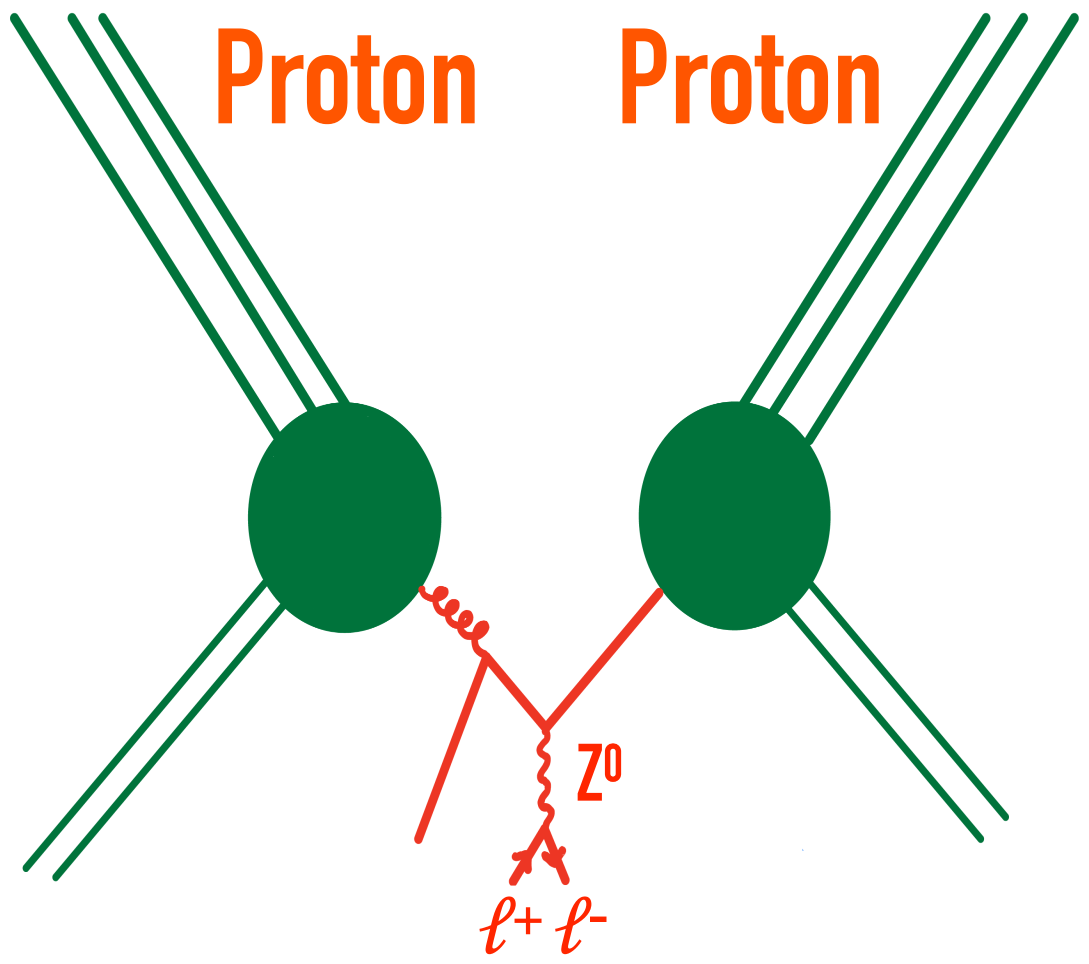
$$\sigma(q_T) \to \sigma^{\mathrm{DL}}(q_T) \equiv \sigma_0 \left(1 + \sum_{n=1}^{\infty} c_n \alpha_s^n L^{2n} \right)$$

We now proceed to calculate $\dfrac{d\sigma^{\mathrm{DL}}}{dq_T}$

Physical interpretation of resummation

Summing the perturbative series corresponds to considering any number of (soft and/or collinear) emissions between the hard scale and the scale of the measurement (assuming it is in the perturbative regime)





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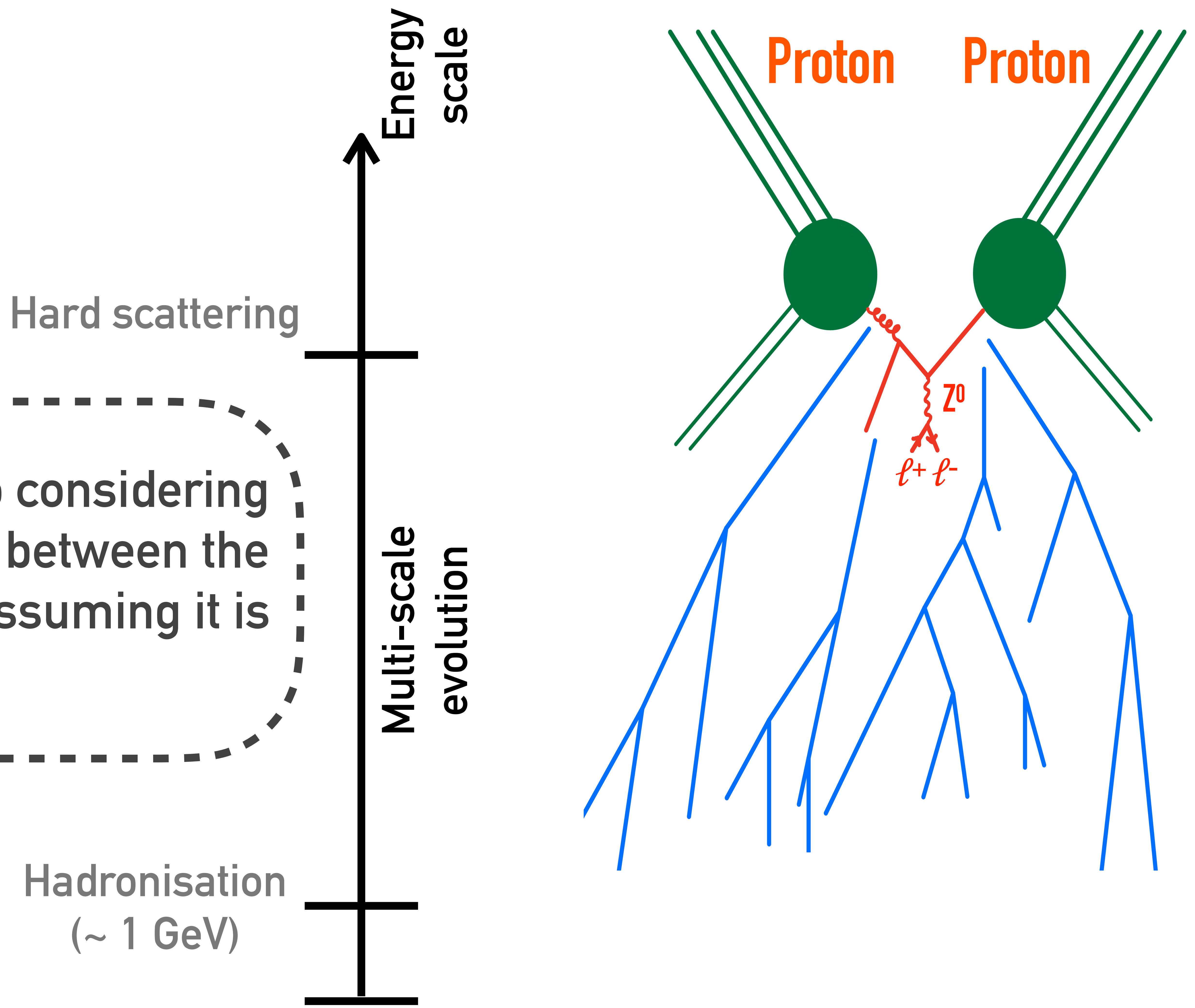


DL resummation of Higgs q_T

Mathematica code available at this URL

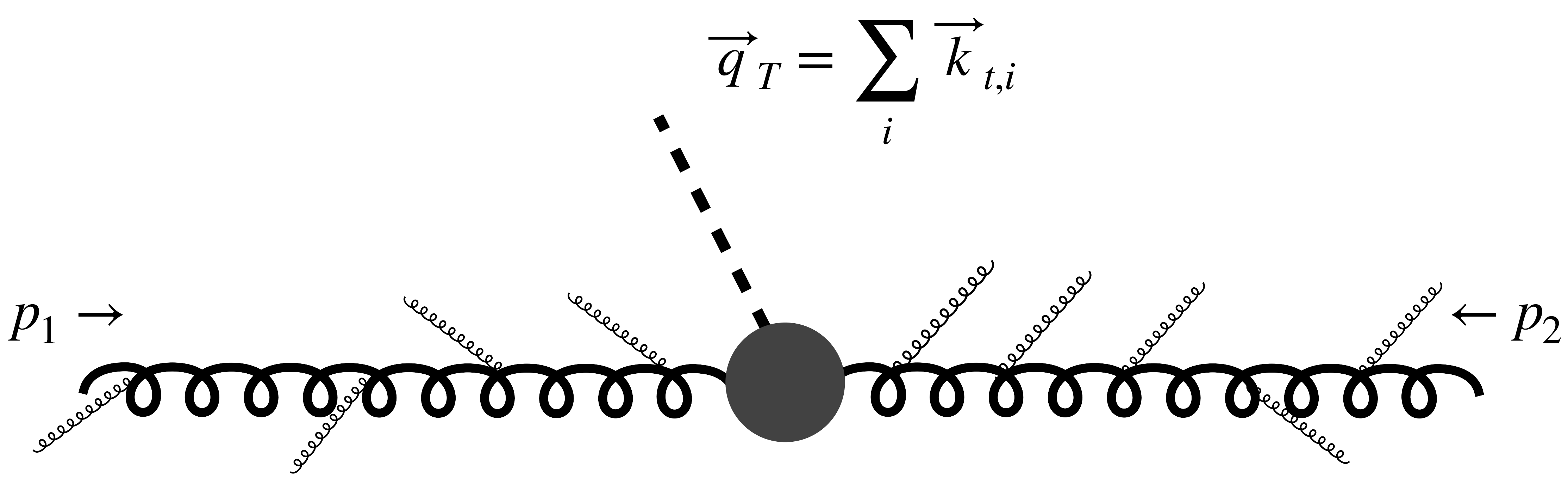
The multi-particle squared amplitude at tree-level

- The dominant (DL) divergence arises in the presence of any number of gluons simultaneously soft and collinear to the incoming partons

  - At DL order we can then use the following approximation of the real emission squared amplitude (independent emissions picture)

$$|\mathscr{A}_{g(p_1)g(p_2)\to h(p_h)+X}|^2 \simeq |\mathscr{A}_{g(p_1)g(p_2)\to h(p_h)}|^2 \underbrace{\sum_{n=0}^{\infty}\left(\prod_{i=1}^{n}|\mathscr{M}_{sc}(k_i)|^2\right)}_{1+\dots}$$

$$|\mathscr{M}_{sc}(k_i)|^2 = 4\pi\alpha_s C_A \frac{p_1\cdot p_2}{p_1\cdot k_i\, p_2\cdot k_i}$$



This formula describes any number of soft/collinear independent emissions

Factorisation of the measurement

- In order to resum this series to all orders, we need to address the space integration
  - multi-particle phase space is now constrained by the measurement

Phase space $[dk] = k_t\, dk_t\, d\eta \frac{d\phi}{(2\pi)^3}$

$$\frac{d\sigma^{\mathrm{DL,\,reals}}}{d^2\vec{q}_T} = \sigma_0 \sum_{n=0}^{\infty} \frac{1}{n!} \int \left(\prod_{i=1}^{n} [dk_i]\, |\mathcal{M}_{\mathrm{sc}}(k_i)|^2 \right) \delta^2(\vec{q}_T - \sum_{i=1}^{n} \vec{k}_{t,i})$$

Combinatorial factor for n identical gluons

  - We can recast the measurement as (Fourier transform)

$$\delta^2(\vec{q}_T - \sum_{i=1}^{n} \vec{k}_{t,i}) = \int \frac{d^2\vec{b}}{(2\pi)^2} e^{-\vec{b}\cdot\vec{q}_T} \prod_{i=1}^{n} e^{\vec{b}\cdot\vec{k}_{t,i}}$$

We achieve the factorisation of the observable's phase space. $\vec{b}$ is the impact parameter, small q_T corresponds to large $b \equiv |\vec{b}|$

DL resummation

- We are still missing the virtual corrections. At DL order, the finite parts of the virtual diagrams are not relevant, and we only need the IRC singularities to remove the divergences of the reals
  - A simple prescription is to subtract, for each emission, a virtual term with the exact same weight (unitarity)
  - We can now resum the DL series, noticing that it's just a Taylor series of an exponential function

Virtual corrections

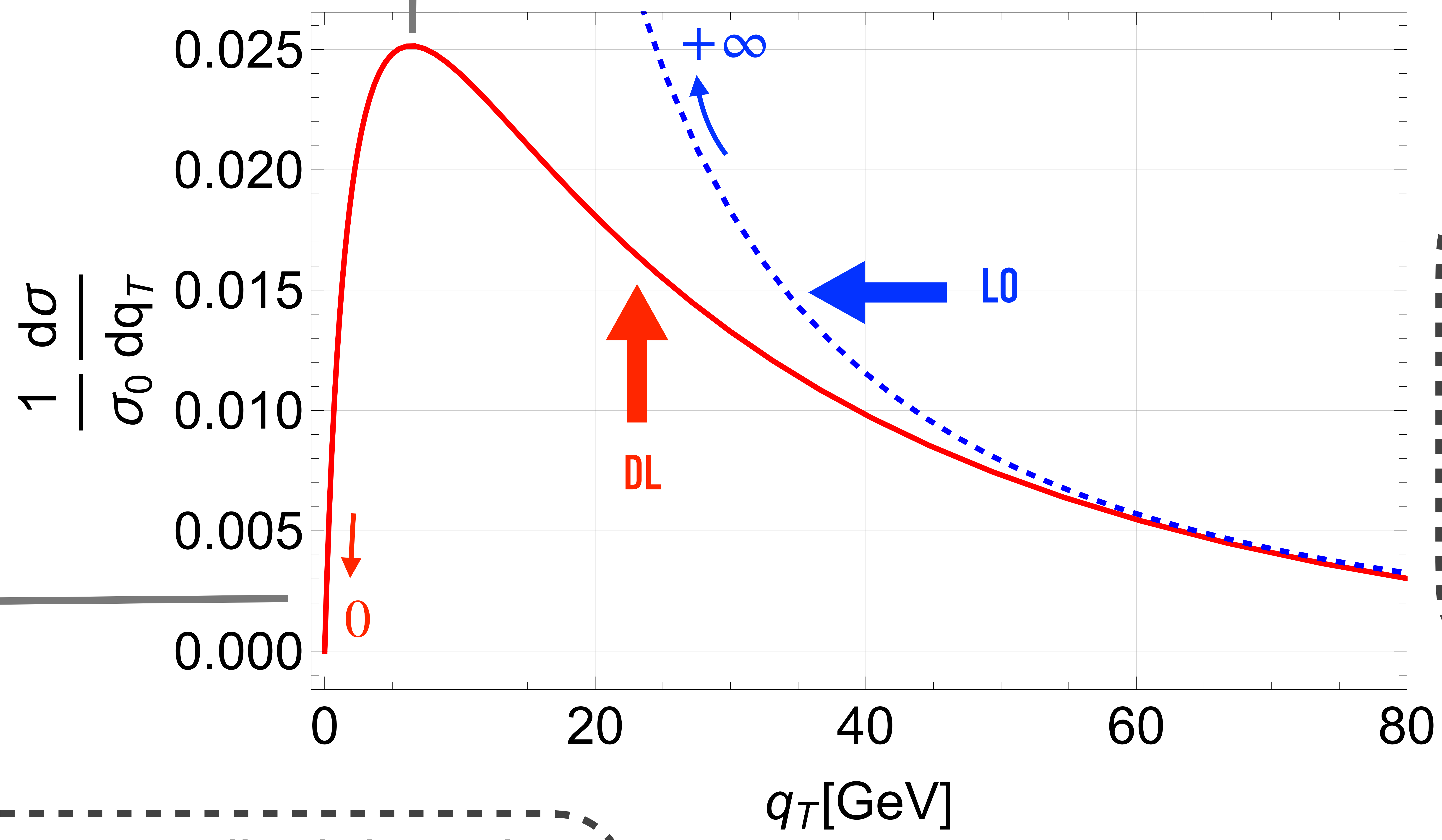
$$\frac{d\sigma^{\mathrm{DL}}}{d^2\vec{q}_T} = \sigma_0 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{q}_T} \sum_{n=0}^{\infty} \frac{1}{n!} \int \left(\prod_{i=1}^{n} [dk_i] \, |\mathcal{M}_{\mathrm{sc}}(k_i)|^2 \, (e^{i\vec{b}\cdot\vec{k}_{t,i}} - 1) \right) \xrightarrow{\text{azimuthal integration}} \frac{d\sigma^{\mathrm{DL}}}{dq_T} = \sigma_0 q_T \int_0^{+\infty} db \, b \, J_0(bq_T) \, e^{-R(b)}$$

$$R(b) = \int [dk] \, |\mathcal{M}_{\mathrm{sc}}(k)|^2 \, (1 - e^{i\vec{b}\cdot\vec{k}_t}) \underbrace{\simeq}_{\frac{1}{b} \ll m_h} \frac{\alpha_s}{2\pi} 4C_A \ln^2 \frac{b_0}{b\, m_h}, \quad b_0 = 2e^{-\gamma_E}$$

- Resummation restores the predictive power of perturbation theory, albeit with a different perturbative expansion that captures towers of terms at all orders in α_s

The Higgs q_T distribution in DL approximation

Sudakov peak: indication of DL exponential (Sudakov) suppression in the regime $\vec{q}_T \simeq k_{t,1} \gg k_{t,2} \gg \cdots \gg k_{t,n}$



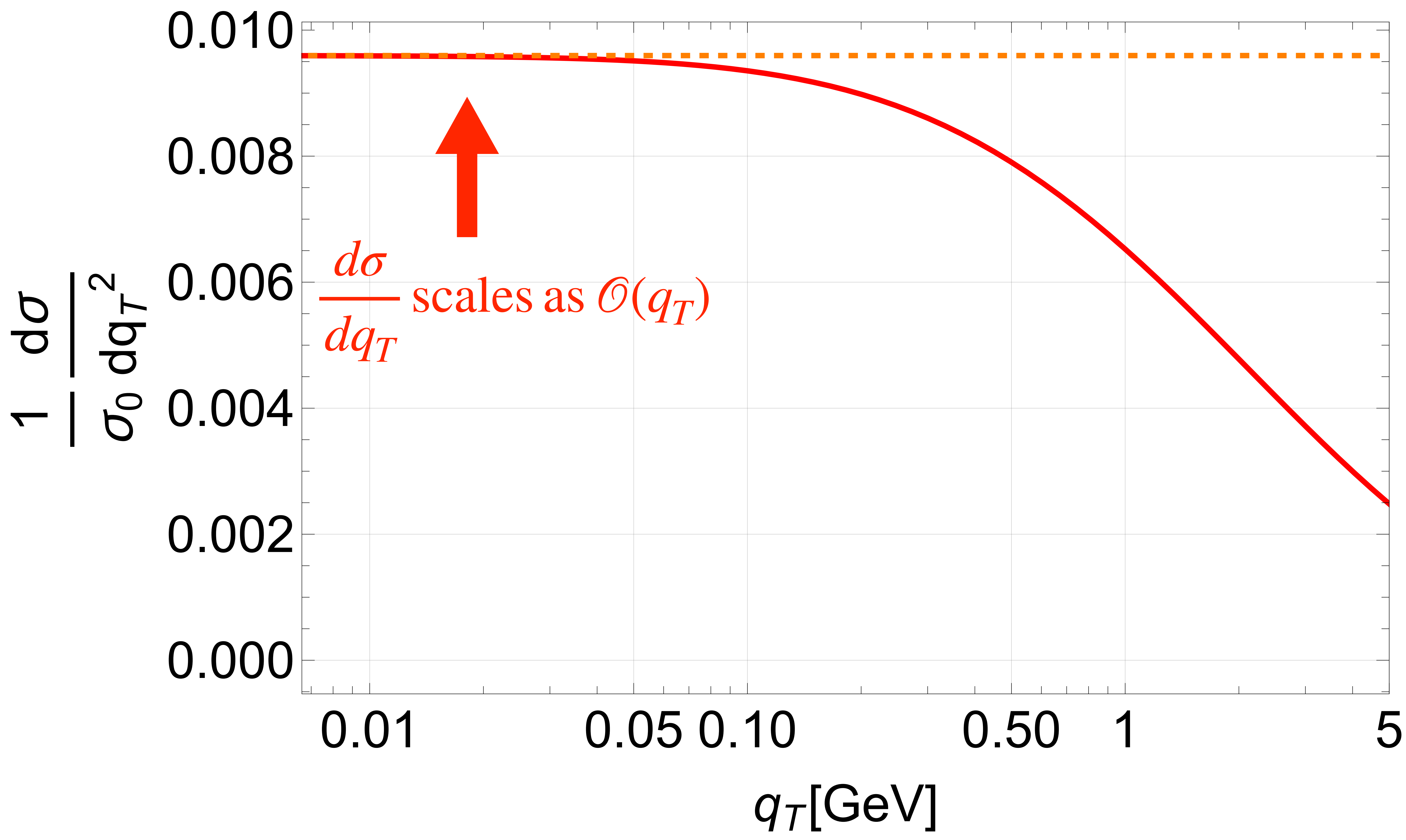
At high q_T we want to combine the standard perturbative expansion at fixed order to achieve a reliable prediction across the whole spectrum (matching)

Scaling at very small q_T is instead ruled by kinematic cancellations

The Higgs q_T distribution in DL approximation

Power (linear) scaling at very small q_T is a consequence of kinematic cancellations

$$\vec{q}_T = \sum \vec{k}_{t,i} \simeq 0$$



Elements of higher order resummation for Higgs q_T

- The DL resummation we just performed can be extended to include additional towers of logarithmic corrections. The counting can be defined at the level of the logarithm of the cumulative distribution, i.e.

$$\ln \sigma(q_T) \sim \alpha_s^n L^{n+1}\,(\mathrm{LL}) + \alpha_s^n L^{n}\,(\mathrm{NLL}) + \alpha_s^n L^{n-1}\,(\mathrm{NNLL}) + \alpha_s^n L^{n-2}\,(\mathrm{N^3LL}) + \cdots$$

Current state of the art, also with elements of N^4LL

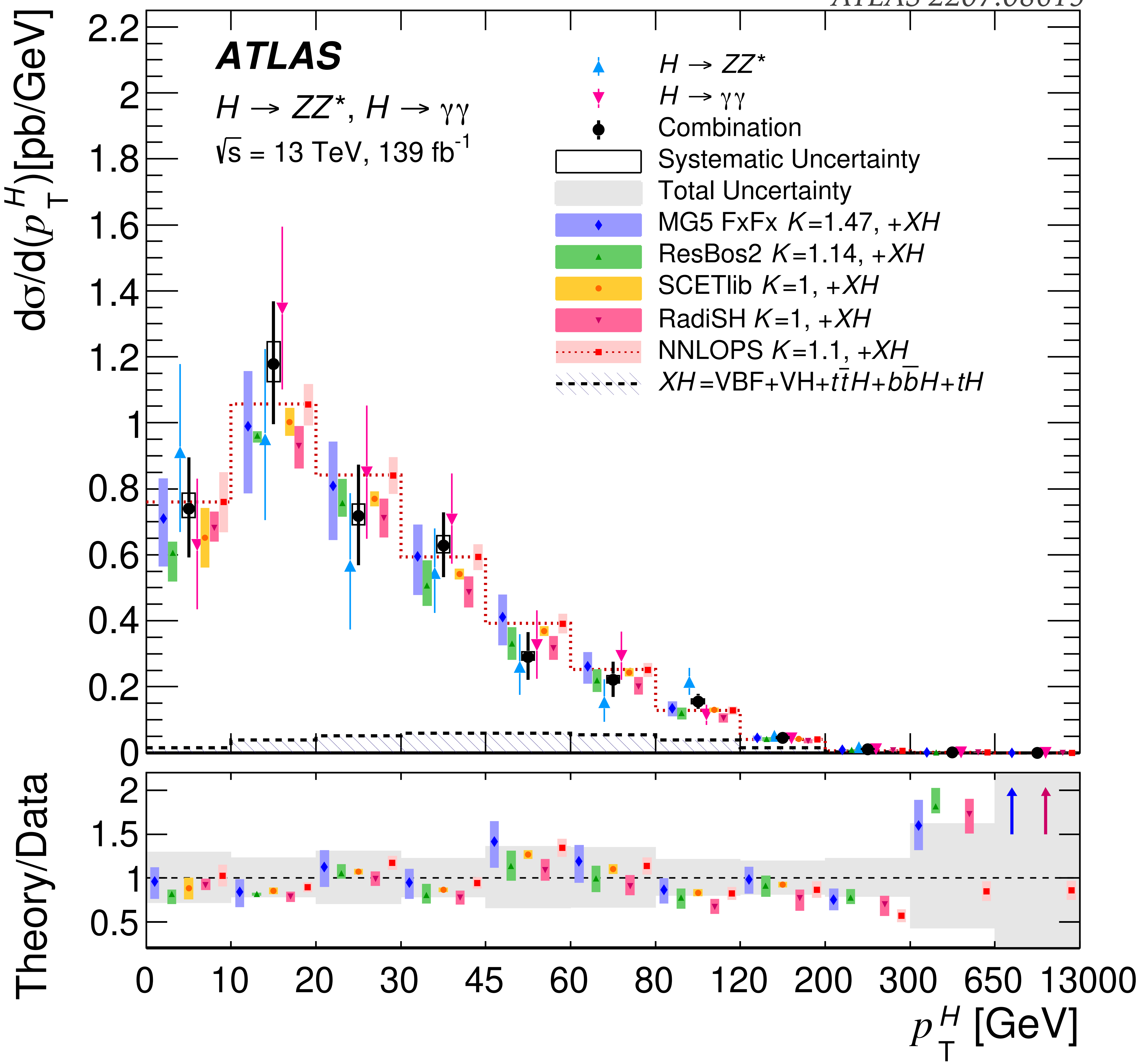
NB: DL only corresponds to the n=1 term of the LL series!

- Going beyond DL entails several aspects (hard-collinear limit, running coupling effects, DGLAP evolution of PDFs, higher-order corrections to splitting kernels, …)
- Field very mature, with many different formulations of the resummation (e.g. QCD, SCET, b-space/q_T space, …)
- Rich phenomenology at the LHC

Comparison to LHC experimental data

Good description of data for different decay modes (state of the art is N³LL⊕NNLO). Experimental precision expected to reach the ~5% level (or better) at HL-LHC!

ATLAS 2207.08615



Other examples of resummations in Higgs physics

- Large logarithms appear whenever a collider observable is sensitive to a hierarchy of scales (in the previous case $q_T \ll m_h$), different types of resummations can be formulated for different problems. E.g.

  - Higgs total XS & rapidity distribution: sensitive to threshold logarithms $\ln(1 - m_h^2/\hat{s})$
  - Light-quark mass effects (e.g. bottom quark) in Higgs XS and q_T distribution: sensitive to logarithms $\ln(m_q/m_h)\,,\ln(m_q/q_T)$, with $m_q \ll q_T\,,m_h$
  - Jet veto resummation when imposing a veto on additional jets produced with the Higgs boson: sensitive to logarithms $\ln(p_t^{\rm veto}/m_h)$
  - … Many other examples for specific collider observables