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Precision Higgs Physics, QCD & Parton Showers [1/3]

PSI summer school - Zuoz, Aug 2024

Tentative outline of the lectures

- Lecture 1 (Monday): Collider physics & strong interactions
  - Introduction to LHC Physics & the Higgs sector
  - Exploring the Higgs boson with Quantum Chromodynamics (QCD)
- Lecture 2 (Thursday): Radiative corr^{ns} & Collider observables
  - Infrared & Collinear (IRC) safety and differential distributions
  - QCD beyond the standard perturbation theory
- Lecture 3 (Friday): Introduction to Parton Showers & Jets
  - Building a toy parton shower for Higgs production in gluon fusion
  - Modern event generators and hadronic jets

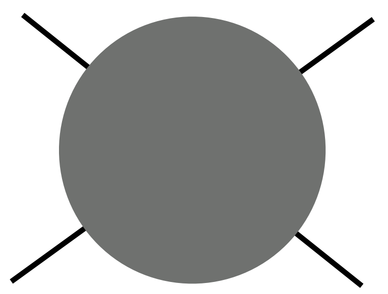
Disclaimer

Not enough time to cover in detail all the aspects that we'll touch upon. Please ask me for extra references if you are interested in reading more about a topic

Collider Physics & Strong interactions

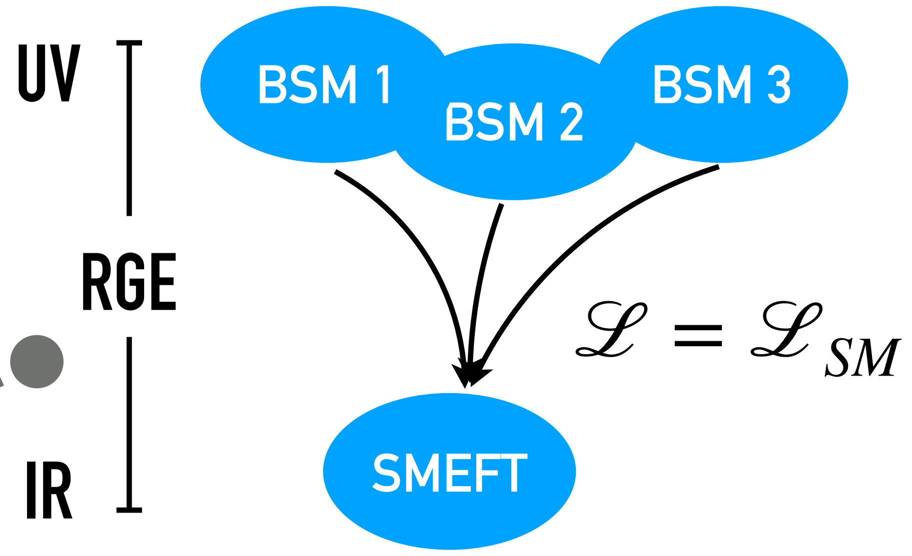
The incredibly diverse collider physics landscape

Formal developments



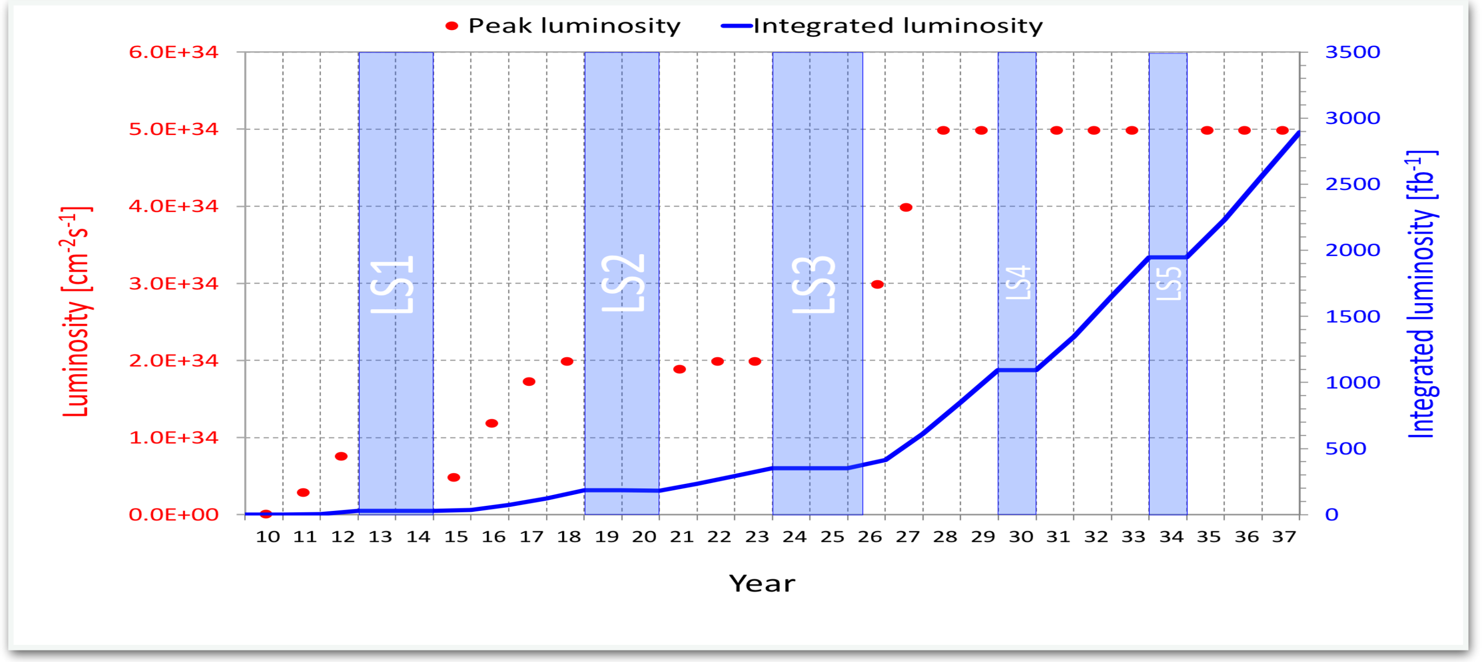
Analyticity
Crossing
Unitarity
$\Longrightarrow \quad S$

EFTs & New Physics models



$$\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{\Lambda^2}\sum_i C_i \mathscr{O}_i^{(6)} + \ldots$$

Swathes of experimental results



Theory developments

Collider pheno

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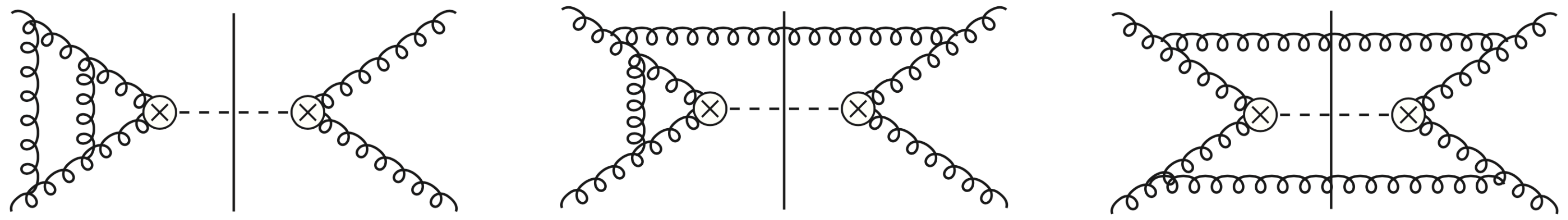
Perturbative methods



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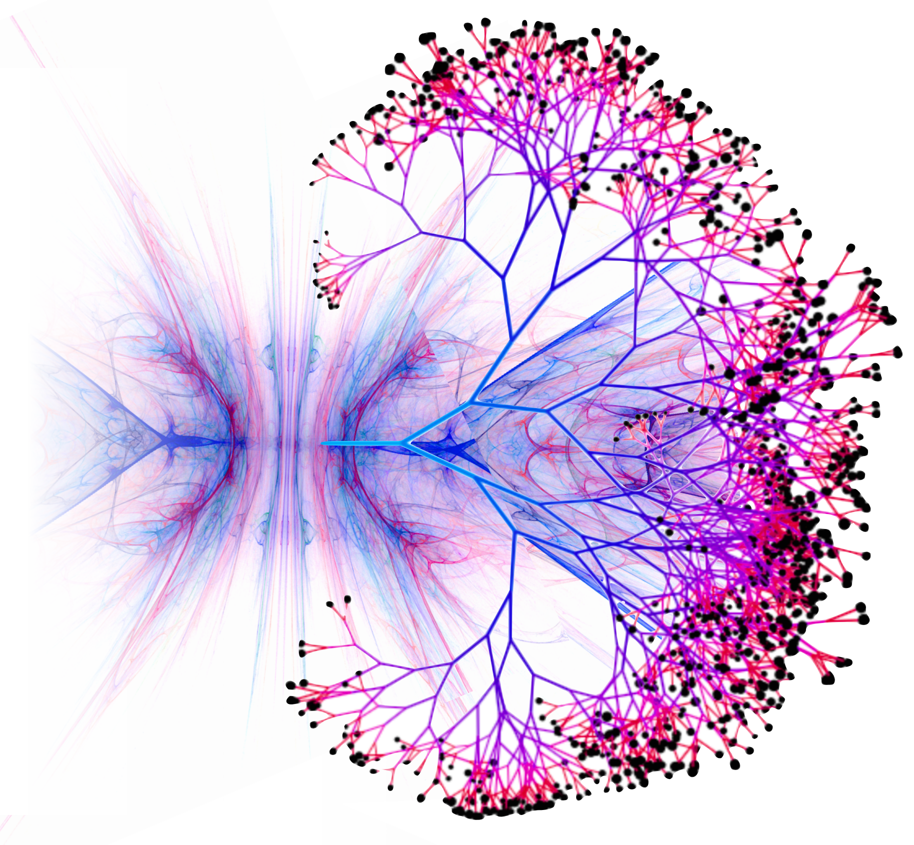
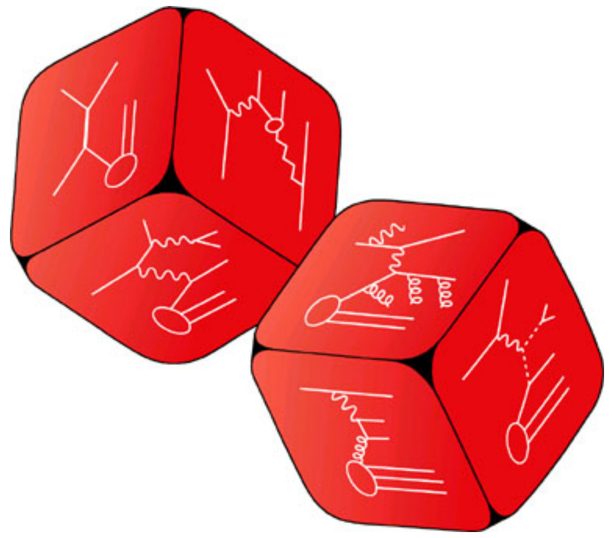
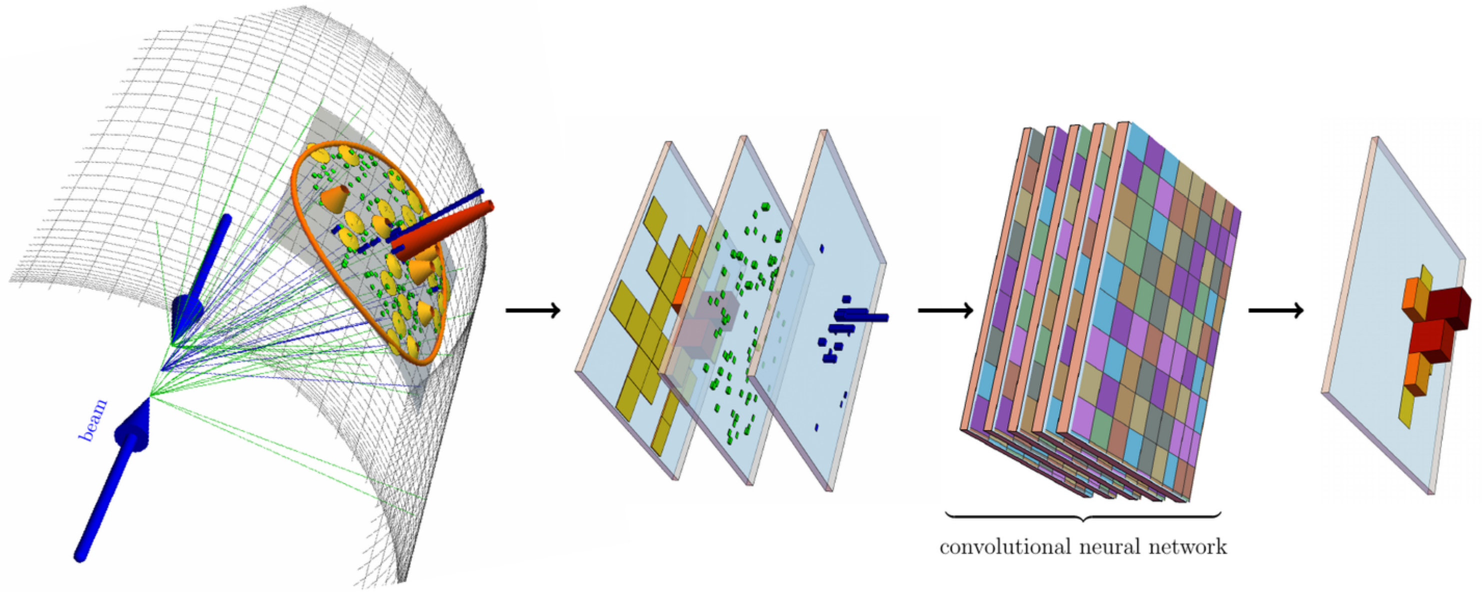
Non-perturbative QCD



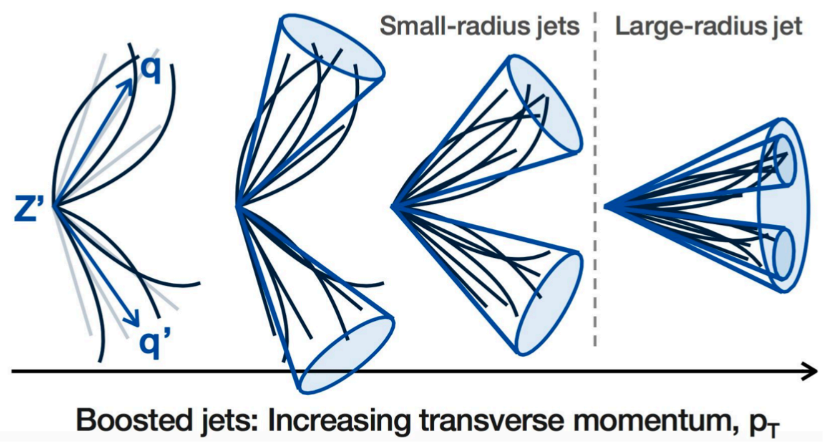
Event generators



Novel strategies (e.g. ML, new observables)



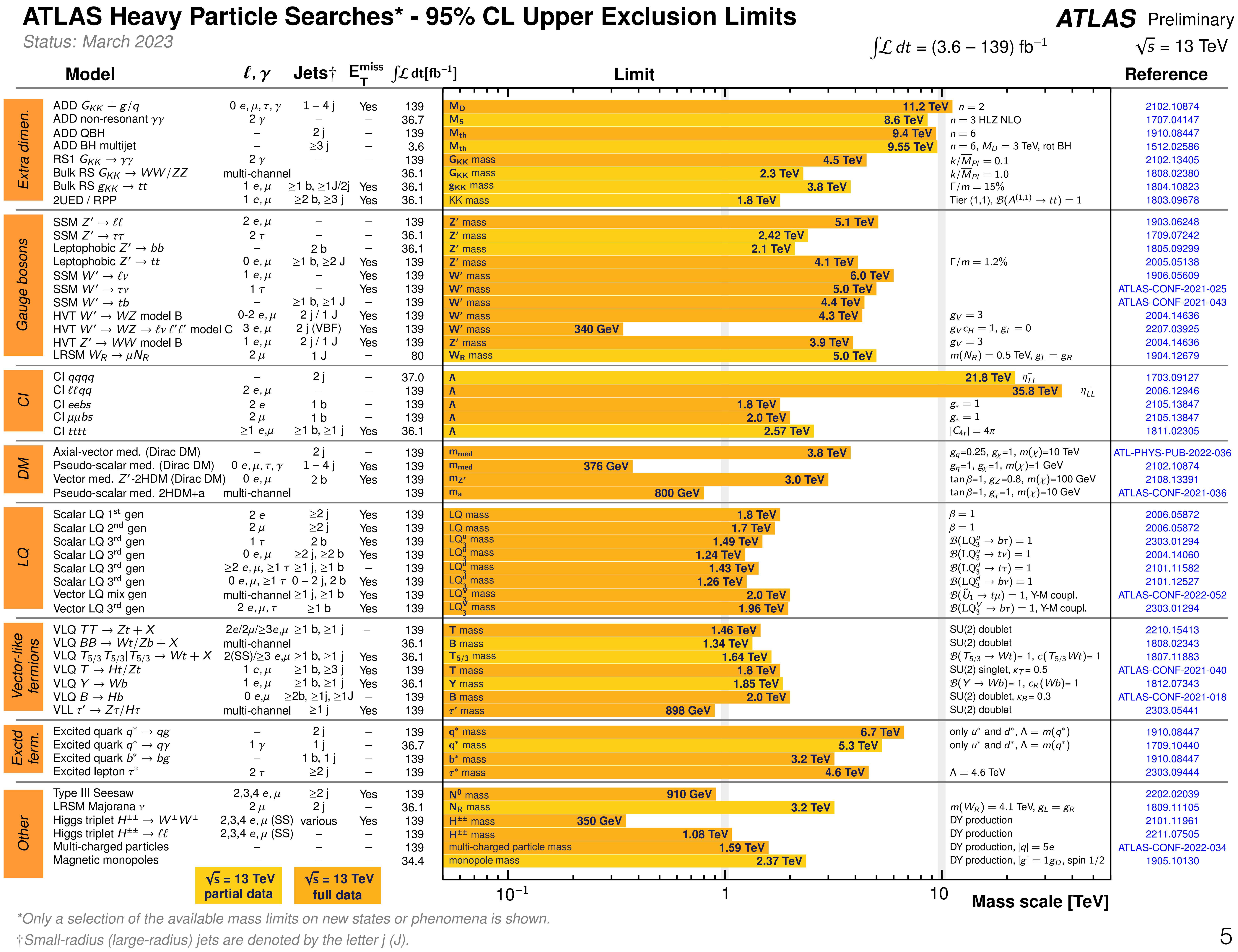
BSM searches & jets



The rich physics programme of the LHC

A central aspect is the search of new-physics (NP) phenomena, for which the LHC has set important constraints that forces us to think of some of the open questions from new angles…

e.g. scan for new phenomena (heavy new physics) within various models, with exclusion limits on NP scale



Source: *ATLAS physics results* (link)

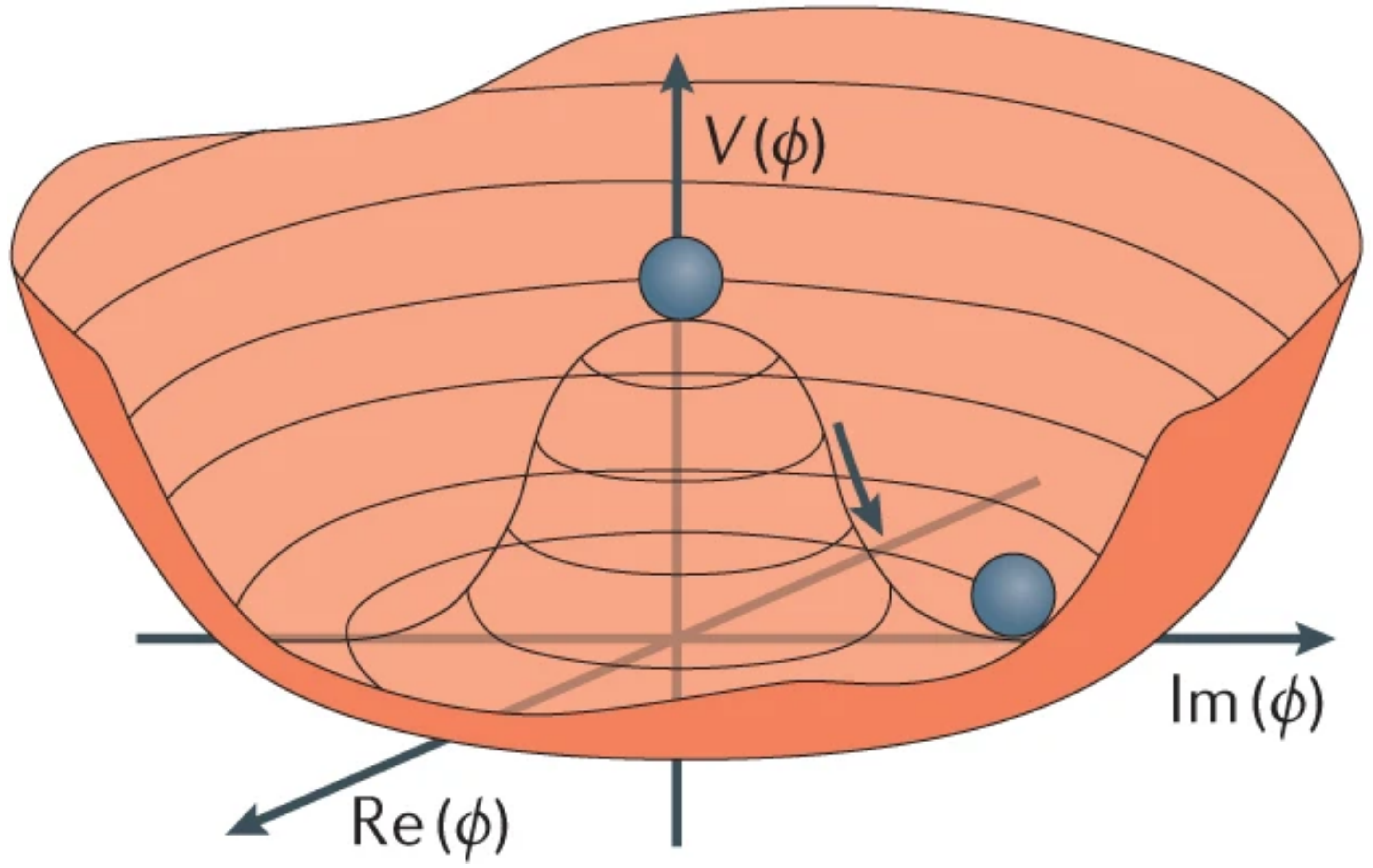
The rich physics programme of the LHC

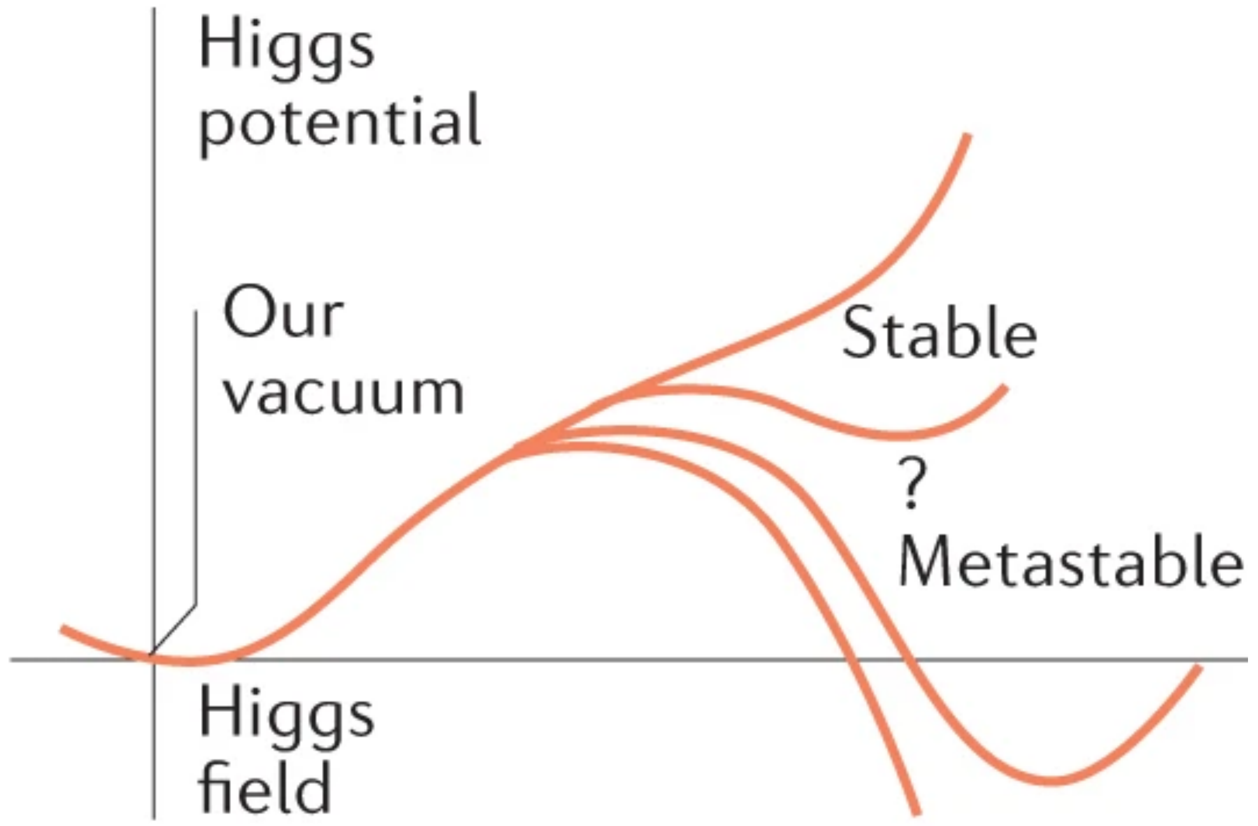
Figures from 1205.6497 & 2104.06821

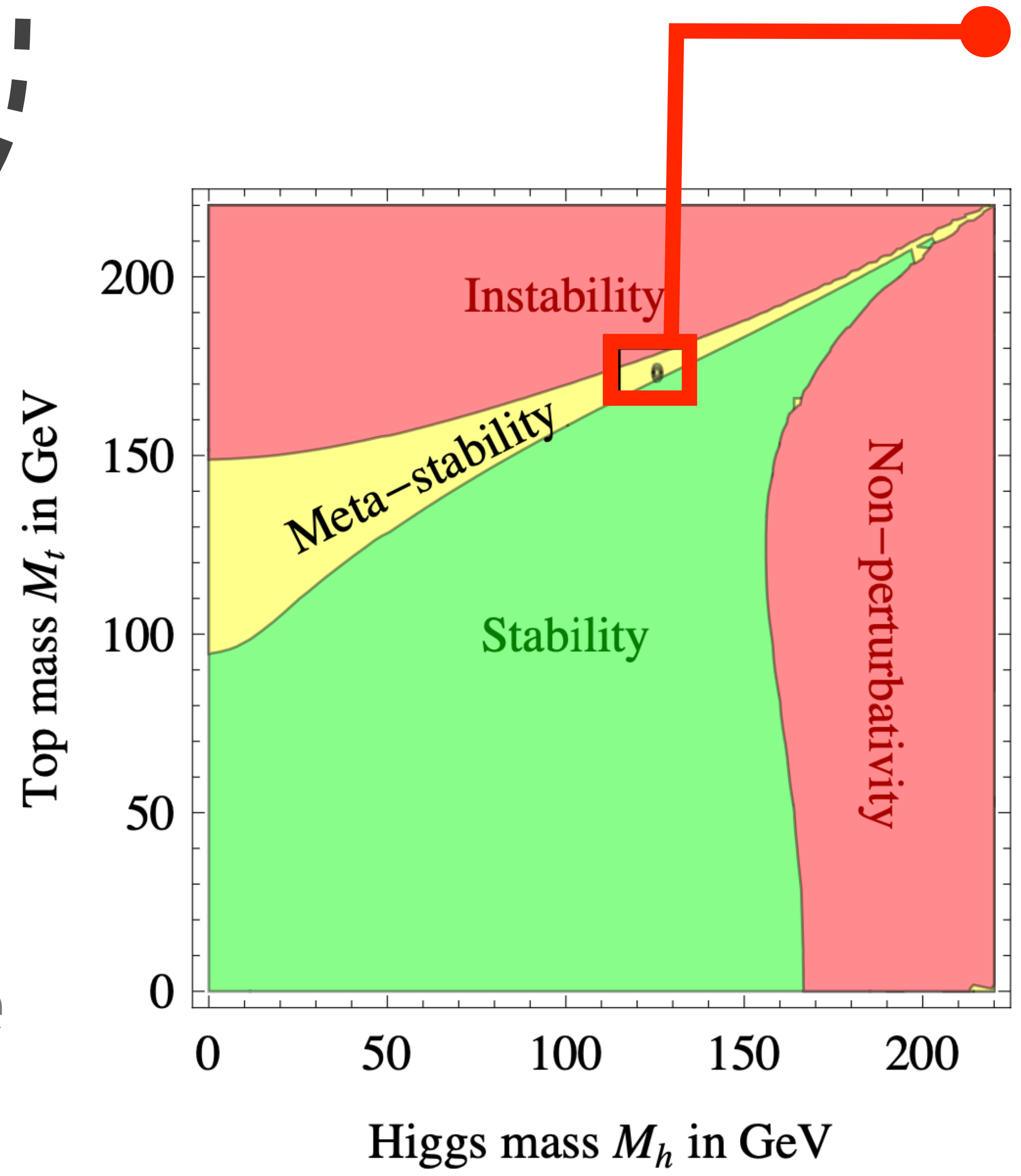
Another crucial element of the programme is precision physics. This provides an instrumental opportunity to a) study the fine structure of nature's QFTs, and b) exploit new creative ways of setting (indirect) constraints on NP models

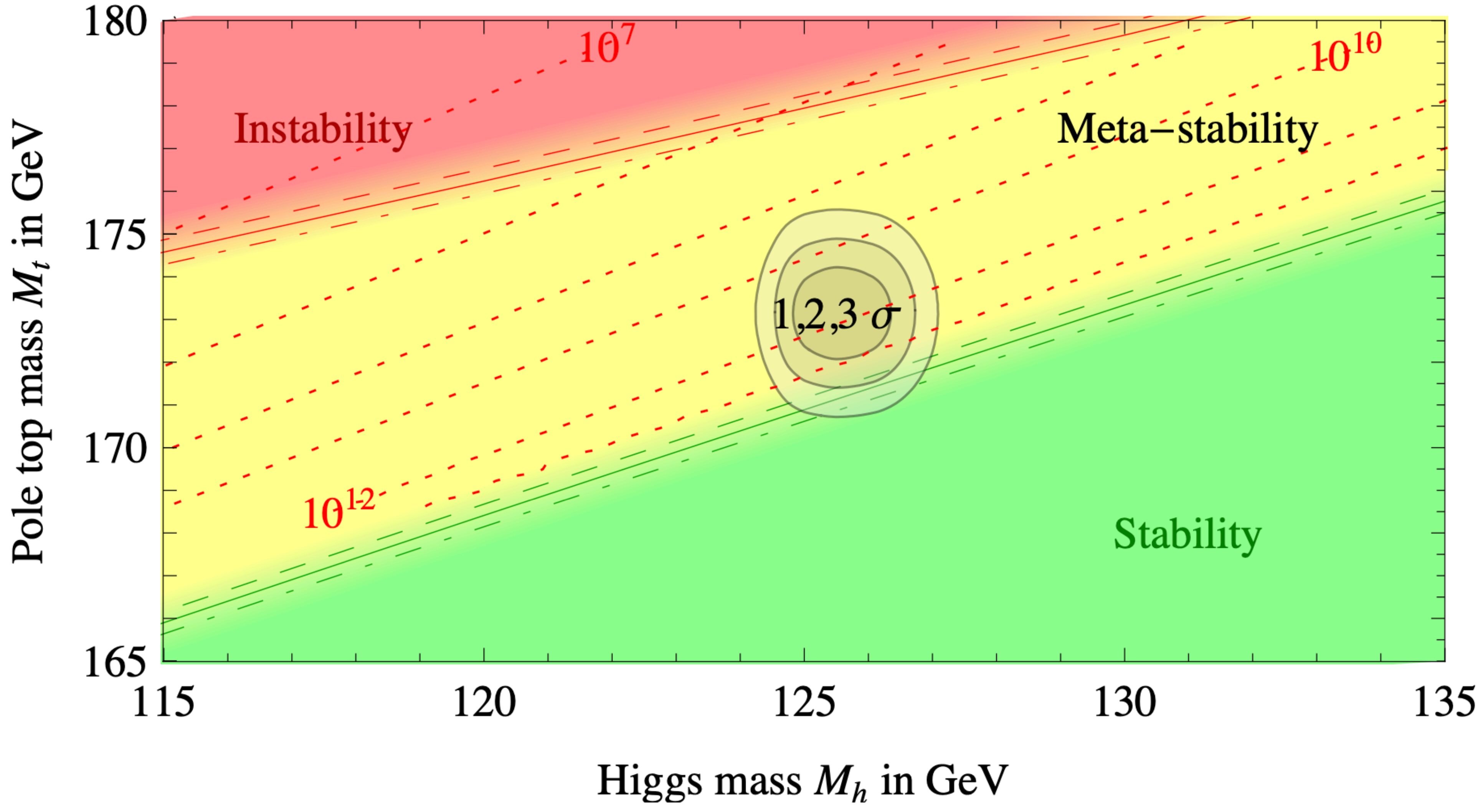
$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$

e.g. precise measurements of the top and Higgs mass has direct implications on the stability of the vacuum of our universe









What makes the Higgs boson special?

- The Higgs boson plays a central role in this programme. Only fundamental (?) scalar observed so far
  - ➡ Mass of scalar particles not protected by symmetry arguments (e.g. like for gauge bosons), and can be as large as the theory cutoff (Planck scale M_{Planck}?)

e.g. analogy with the pion (scalar, lightest hadron):
π mass is determined by the hadronic scale of the theory Λ (~ 300 MeV).
Why is the Higgs boson mass so much smaller than M_{Planck}?

$$m_\pi \sim 130 - 140\,\text{MeV} \sim \Lambda$$

$$m_h \sim 125\,\text{GeV} \ll M_{\text{Planck}}\,(??)$$

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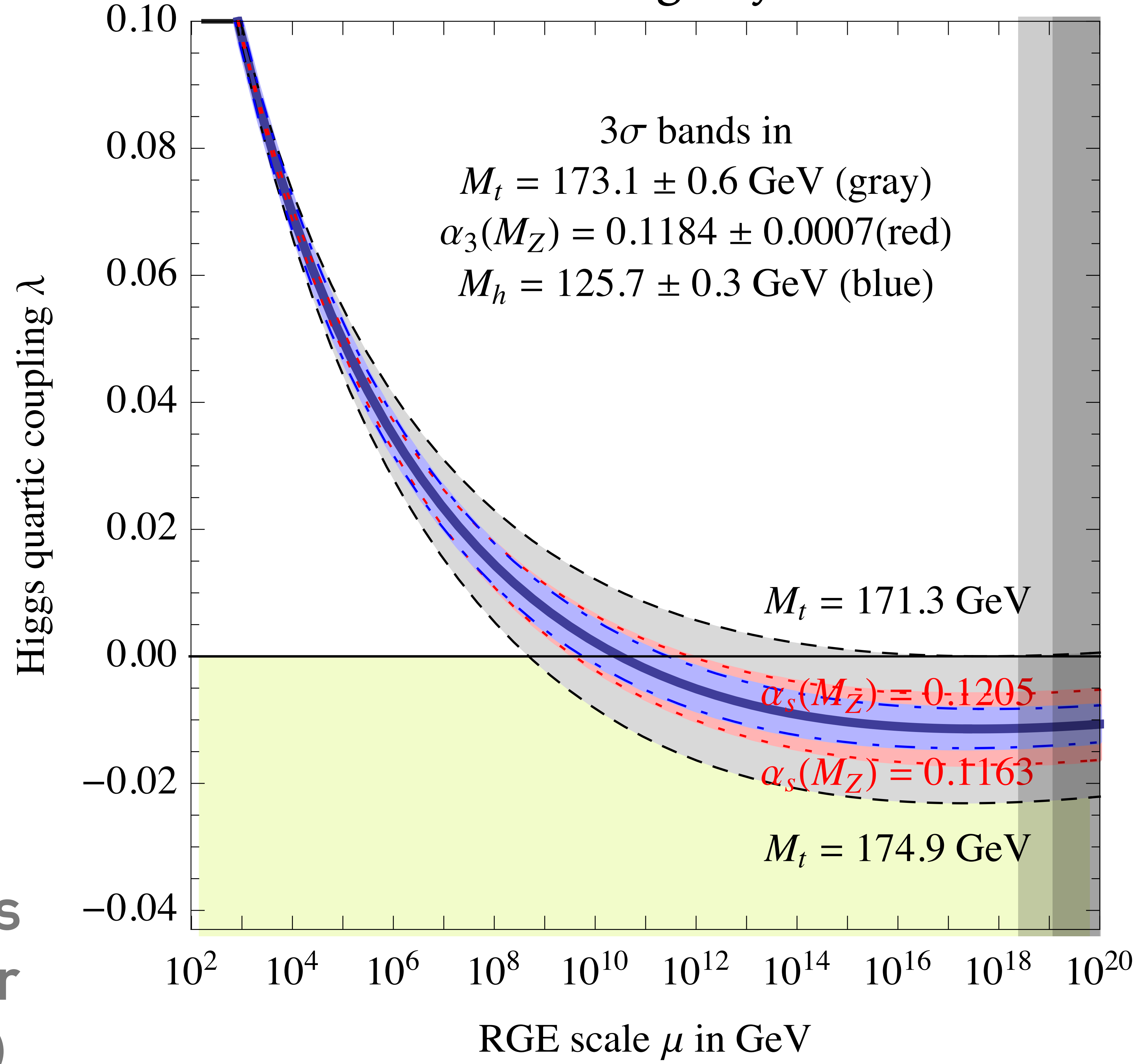
- ➡ Evolution of SM Higgs' quartic interaction $\lambda(\mu)$ crosses zero at high energies, i.e. the H potential has other minima at large Higgs field and our vacuum will decay w/ very long lifetime (meta-stability)

e.g. a somewhat similar behaviour is observed in (scalar) condensates of pairs of electrons in a superconducting material. In this case the scalar field (Cooper pairs) is just a low-energy manifestation of a more fundamental theory (BCS)

Figure from 1205.6497



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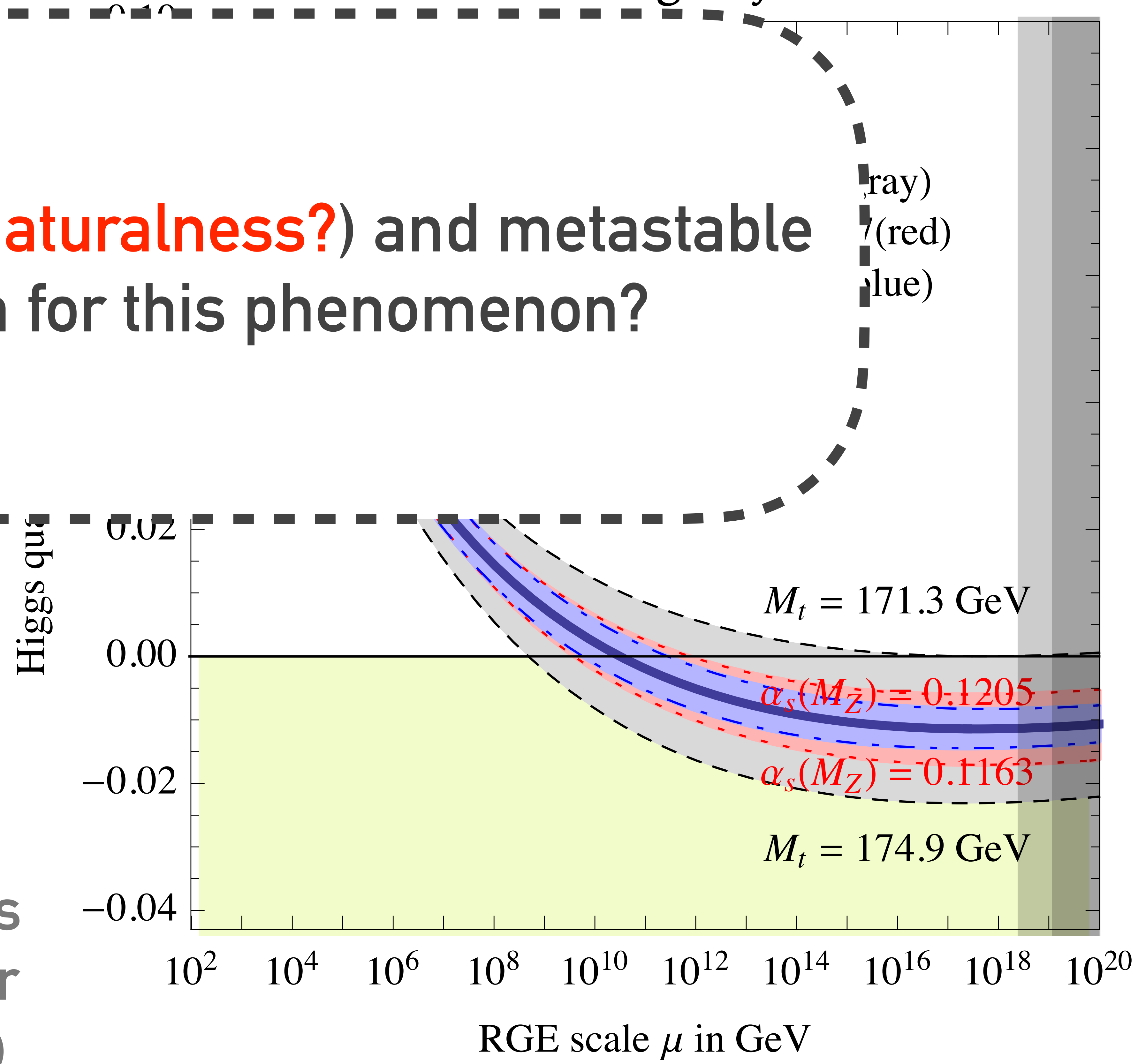
e.g. analogy with the pion (scalar, lightest hadron):

π mass i

$m_\pi \sim 1$

Figure from 1205.6497



The Higgs boson indicates that we are living in a fine-tuned (naturalness?) and metastable universe (criticality?). Is there a microscopic explanation for this phenomenon?

  - ➡ Evolution of SM Higgs' quartic interaction $\lambda(\mu)$ crosses zero at high energies, i.e. the H potential has other minima at large Higgs field and our vacuum will decay w/ very long lifetime (meta-stability)

e.g. a somewhat similar behaviour is observed in (scalar) condensates of pairs of electrons in a superconducting material. In this case the scalar field (Cooper pairs) is just a low-energy manifestation of a more fundamental theory (BCS)

The Higgs boson's possible connection to (some) big open questions

Higgs

Naturalness: Is the Higgs boson a (composite) condensate? Are there heavy top partners that could stabilise the Higgs mass (e.g. SUSY)?

The flavour problem: Why is there such a strong hierarchy between Yukawa couplings (fermion masses)? Are there flavour non-diagonal Yukawa couplings?

Cosmological constant: The Higgs potential would lead to a large cosmological constant. Why is this much smaller in nature?

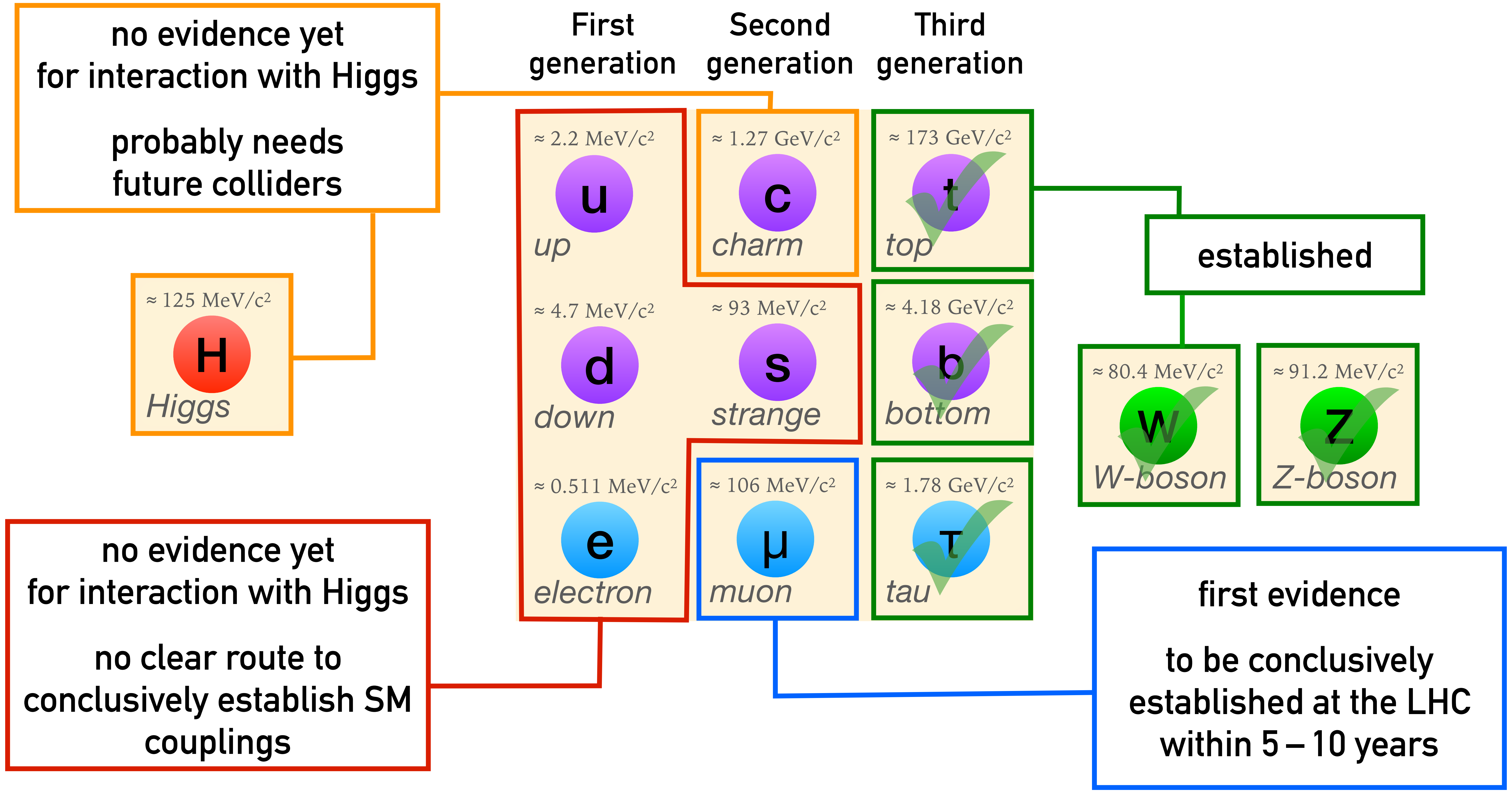
…

EW baryogenesis: Deviations from the SM in the Higgs potential could unveil whether the EWSB was a first order phase transition.

Dark Matter (DM): The Higgs boson could be part of a larger Higgs sector (Higgs portal). Invisible Higgs decays can be used to set bounds on certain DM models.

Is it the SM Higgs boson? e.g. interaction with SM particles

- Interaction with EW bosons & 3rd generation fermions (Yukawa interactions) established to be SM like
  - ➡ first exploration of some of 2nd generation Yukawa interactions ongoing



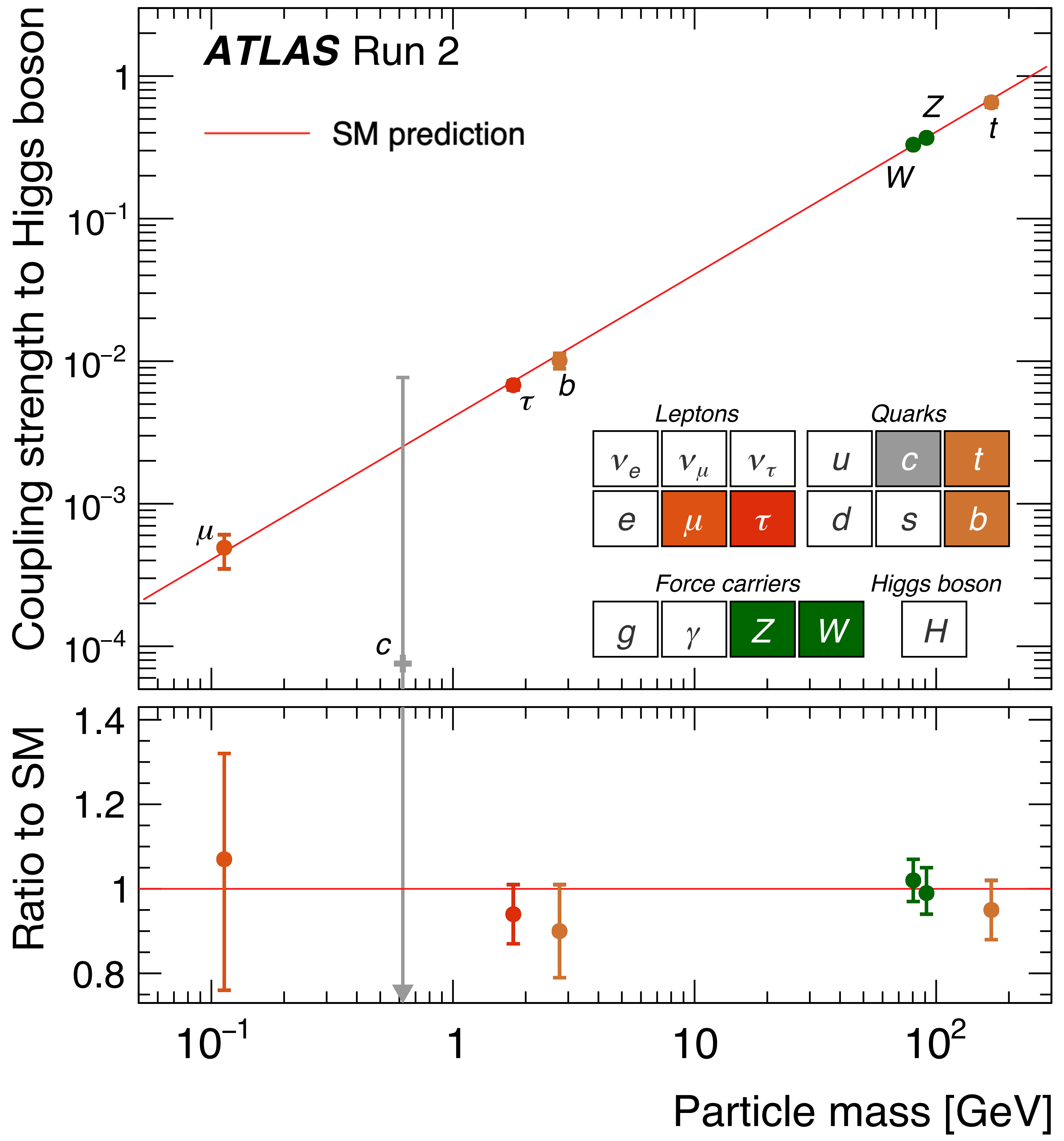
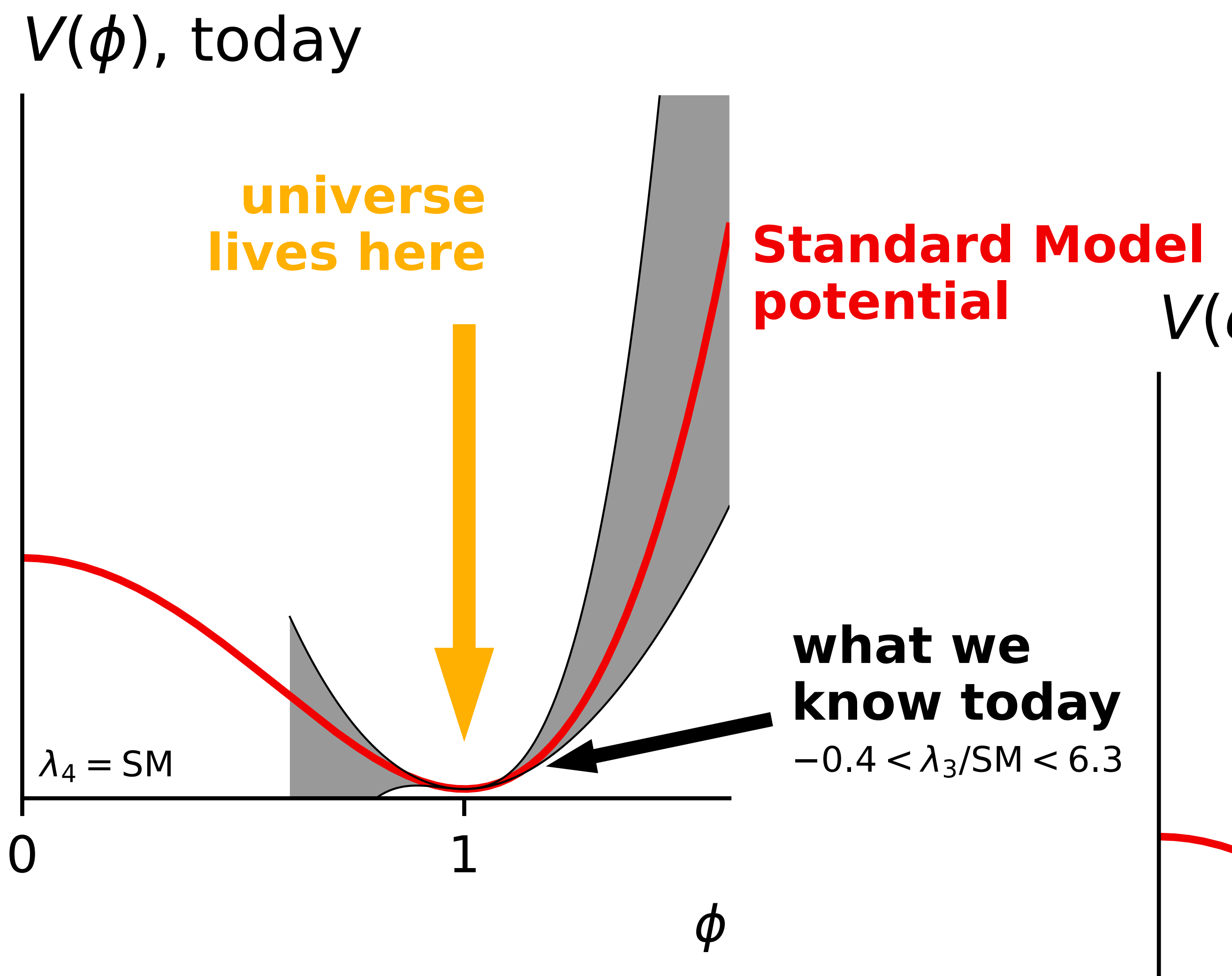


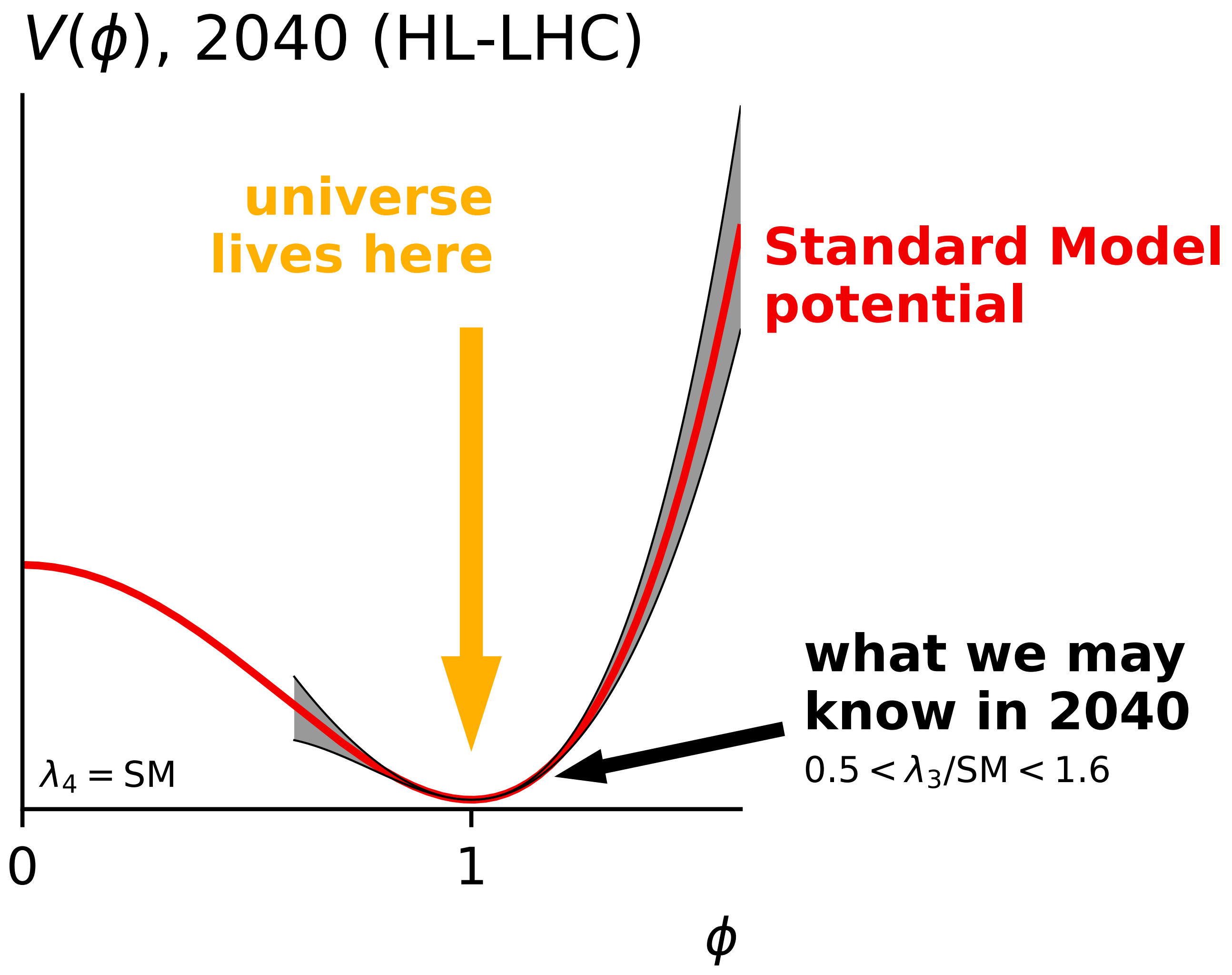
Figure from The Higgs boson turns 10 2207.00478

Is it the SM Higgs boson? e.g. the potential

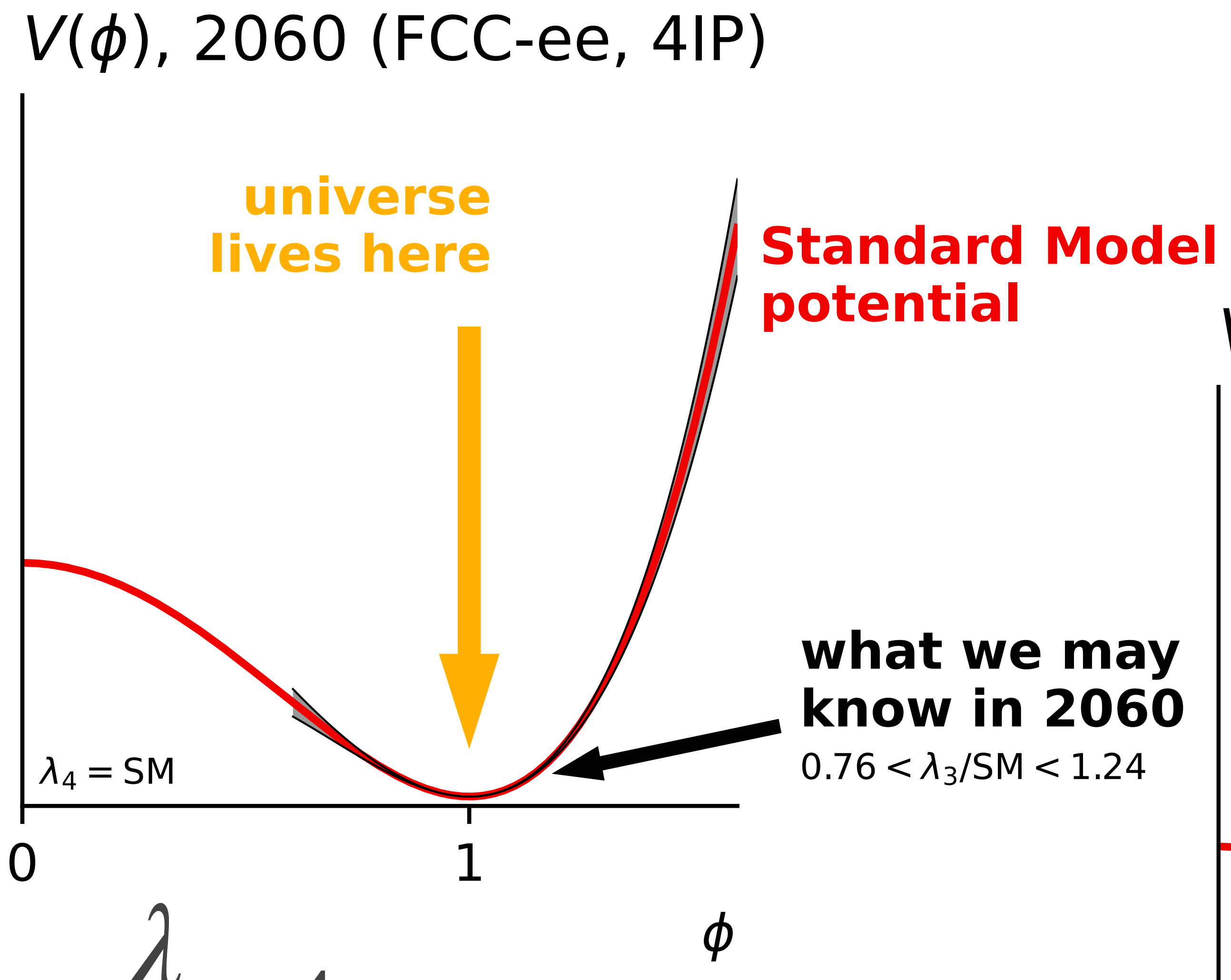
Figures from G. Salam's talk at ICFA 2023

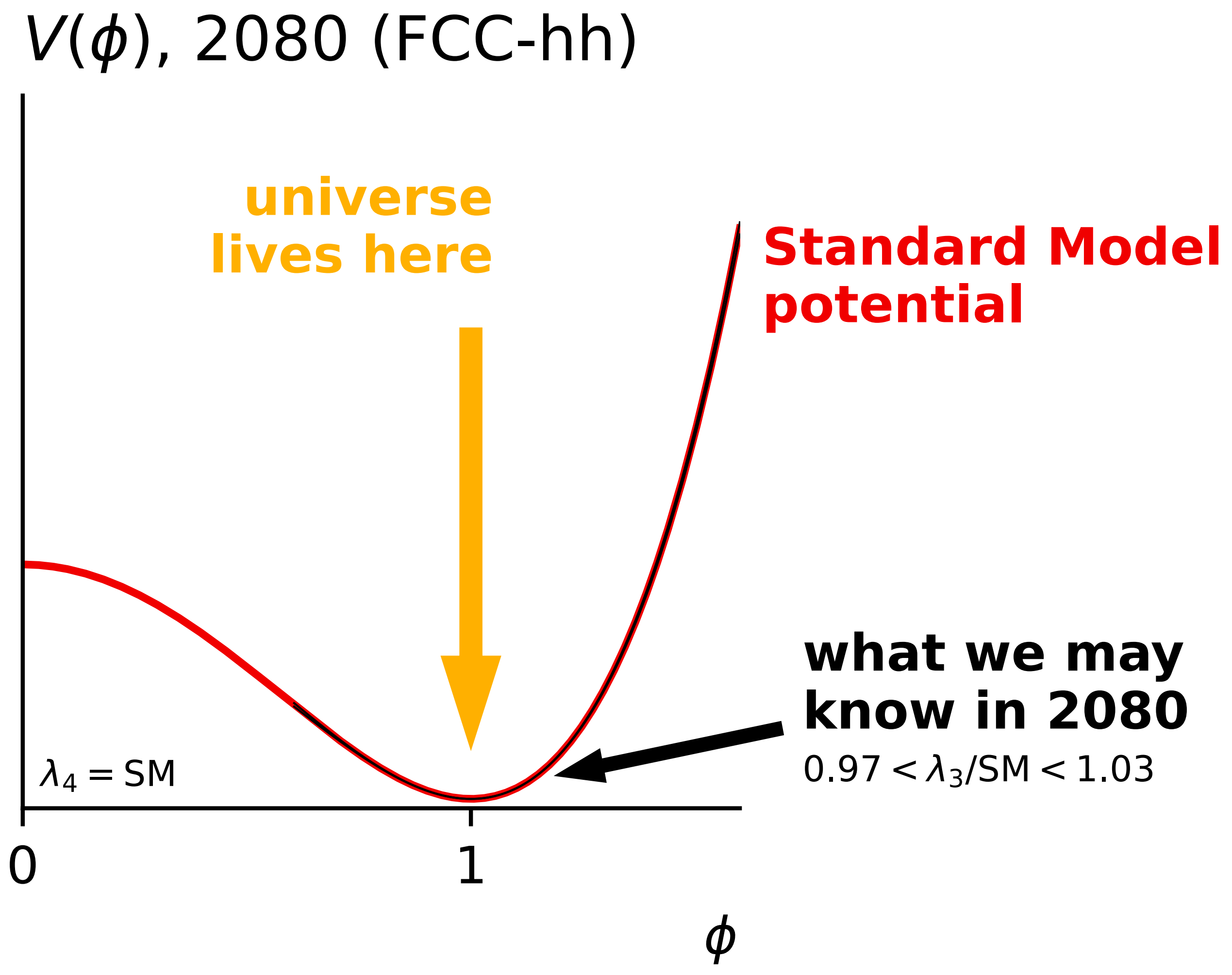
e.g. Display of current vs. future constraints on Higgs trilinear coupling (with $\lambda_4 = \mathrm{SM}$)





The exploration of the Higgs sector is a mandatory step on the path towards the next discovery in particle physics!





$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4 \overset{\text{EWSB}}{\supset} \underbrace{\lambda v^2}_{\frac{m_h^2}{2}} h^2 + \underbrace{\lambda v}_{\lambda_3} h^3 + \underbrace{\frac{\lambda}{4}}_{\lambda_4} h^4$$

Future colliders necessary for <u>stringent</u> constraints & direct measurement. Present LHC data shows $\lambda_3 \lesssim 6 \times \mathrm{SM}$

At present not clear how to measure this

Goal of these lectures

Snapshots from: Physics Letters B 716 (2012) 1–29 (ATLAS)
Physics Letters B 716 (2012) 30–61 (CMS)

- The goal of these lectures is to explore the main concepts used in the theoretical description of collider events. We will take a learn-by-doing approach, using the Higgs boson as a concrete example

e.g. A few snapshots of the Higgs observation papers:

Samples of MC events used to represent signal and background are fully simulated using GEANT4 [103]. The simulations include pileup interactions matching the distribution of the number of such interactions observed in data. The description of the Higgs boson signal is obtained from MC simulation using, for most of the decay modes and production processes, the next-to-leading-order (NLO) matrix-element generator POWHEG [104,105], interfaced with PYTHIA 6.4 [106]. For the dominant gluon–gluon fusion process, the transverse momentum spectrum of the Higgs boson in the 7 TeV MC samples is reweighted to the next-to-next-to-leading-logarithmic (NNLL) + NLO distribution computed with HQT [71,72,107] and FEHIPRO [108,109], except in the $H \to ZZ$ analysis, where the effect is marginal. The agreement of the p_T spectrum in the simulation at 8 TeV with the NNLL + NLO distribution makes reweighting unnecessary. The improved agreement is due to a modification in the POWHEG setup recommended in Ref. [102]. The simulation of associated-production signal samples uses PYTHIA and all signal samples for $H \to bb$ are made using POWHEG interfaced to HERWIG++ [110]. Samples used for background studies are generated with PYTHIA, POWHEG, and MADGRAPH [111], and the normalisations are obtained from the best available NNLO or NLO calculations. The uncertainty in the signal cross section related to the choice of parton distribution functions is determined with the PDF4LHC prescription [96–100].

? ? ?

An exclusive category of events containing two jets improves the sensitivity to VBF. The other nine categories are defined by the presence or not of converted photons, η of the selected photons, and p_{Tt}, the component[3] of the diphoton p_T that is orthogonal to the axis defined by the difference between the two photon momenta [99,100].

Jets are reconstructed [101] using the anti-k_t algorithm [102] with radius parameter $R = 0.4$. At least two jets with $|\eta| < 4.5$ and $p_T > 25$ GeV are required in the 2-jet selection. In the analysis of the 8 TeV data, the p_T threshold is raised to 30 GeV for jets with $2.5 < |\eta| < 4.5$.

? ?

The event generators used to model signal and background processes in samples of Monte Carlo (MC) simulated events are listed in Table 1. The normalisations of the generated samples are obtained from the state of the art calculations described above. Several different programs are used to generate the hard-scattering processes. To generate parton showers and their hadronisation, and to simulate the underlying event [66–68], PYTHIA6 [69] (for 7 TeV samples and 8 TeV samples produced with MadGraph [70,71] or AcerMC) or PYTHIA8 [72] (for other 8 TeV samples) are used. Alternatively, HERWIG [73] or SHERPA [74] are used to generate and hadronise parton showers, with the HERWIG underlying event simulation performed using JIMMY [75].

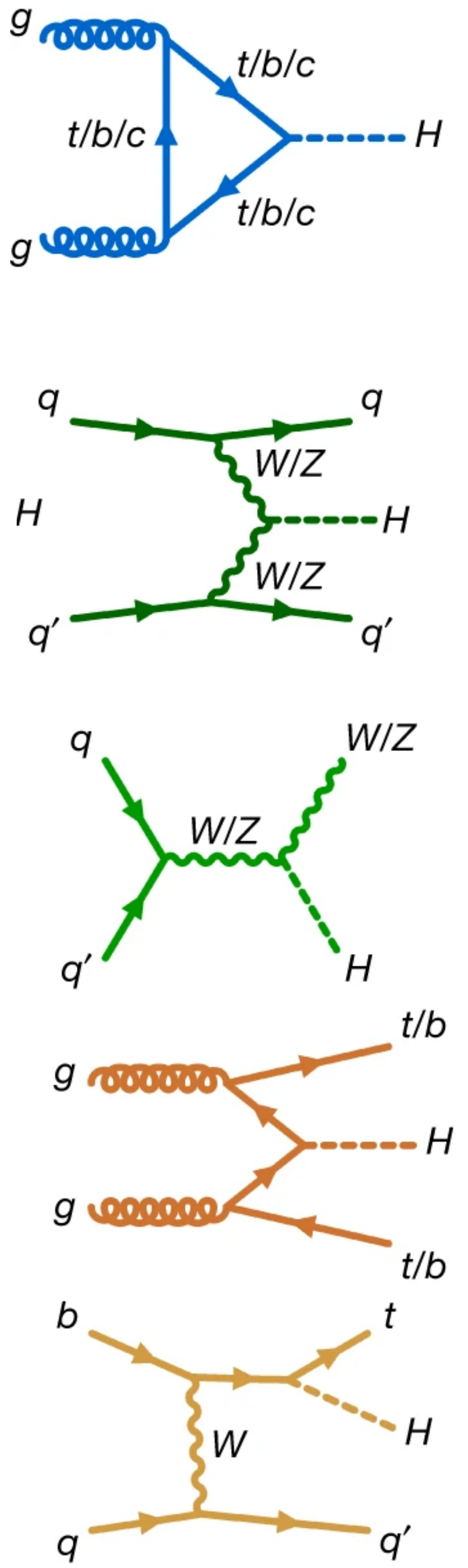
? ? ?

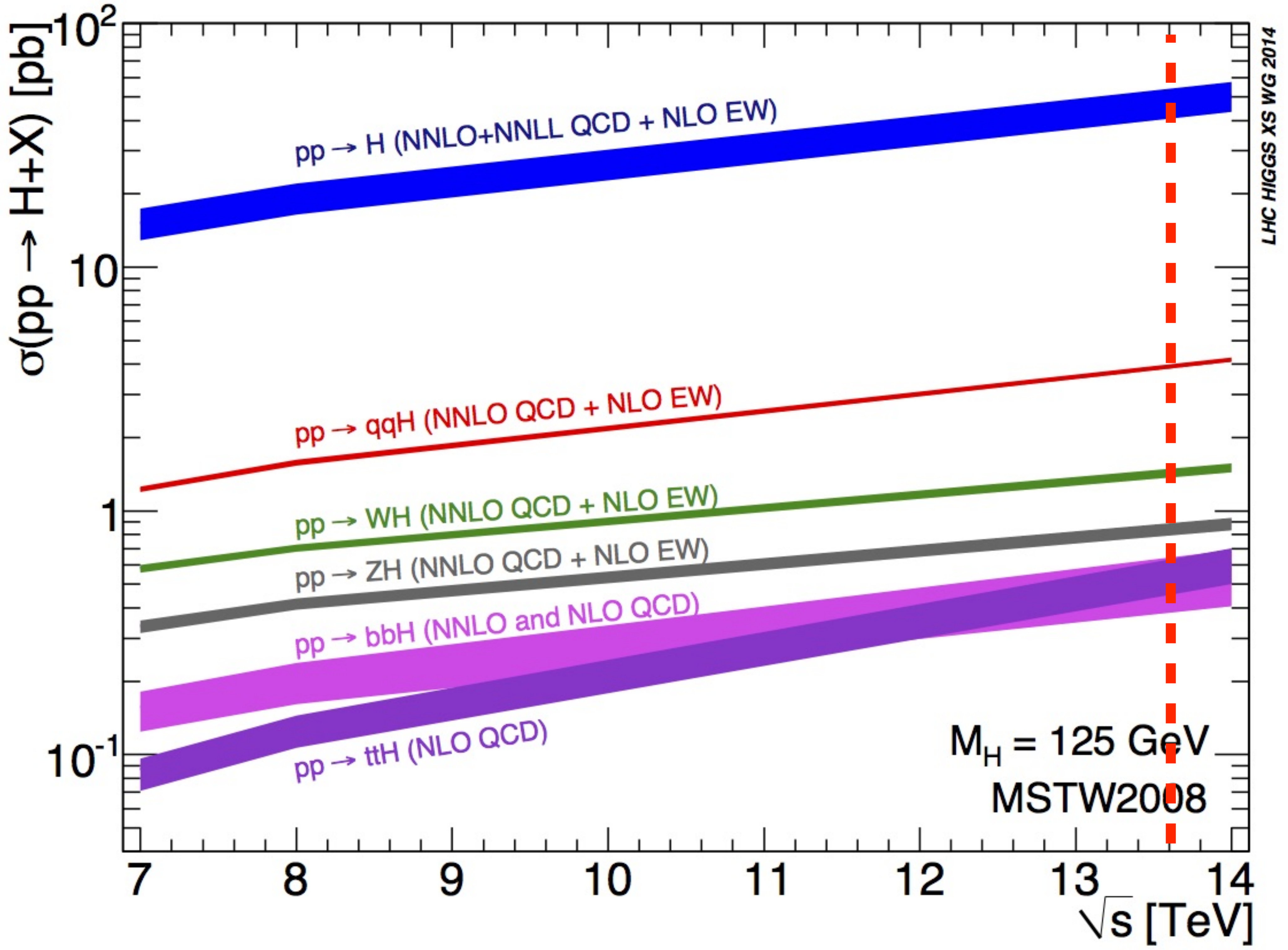
How to study the Higgs boson at the LHC

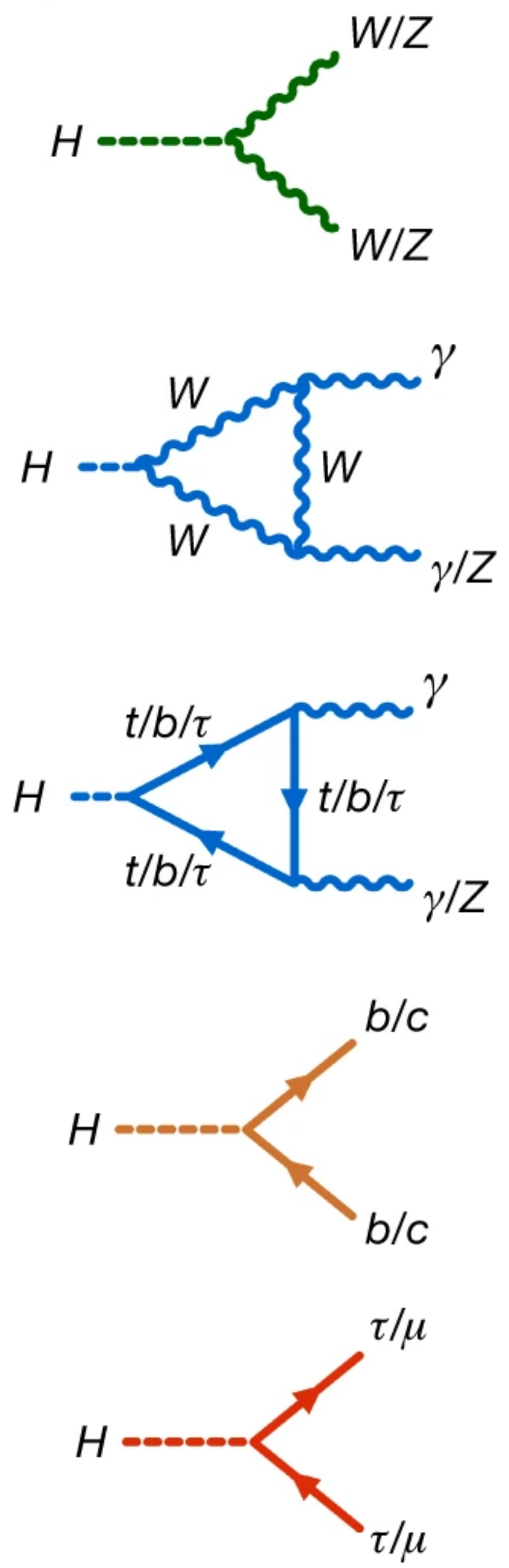
Feynman diagrams from 2207.00092

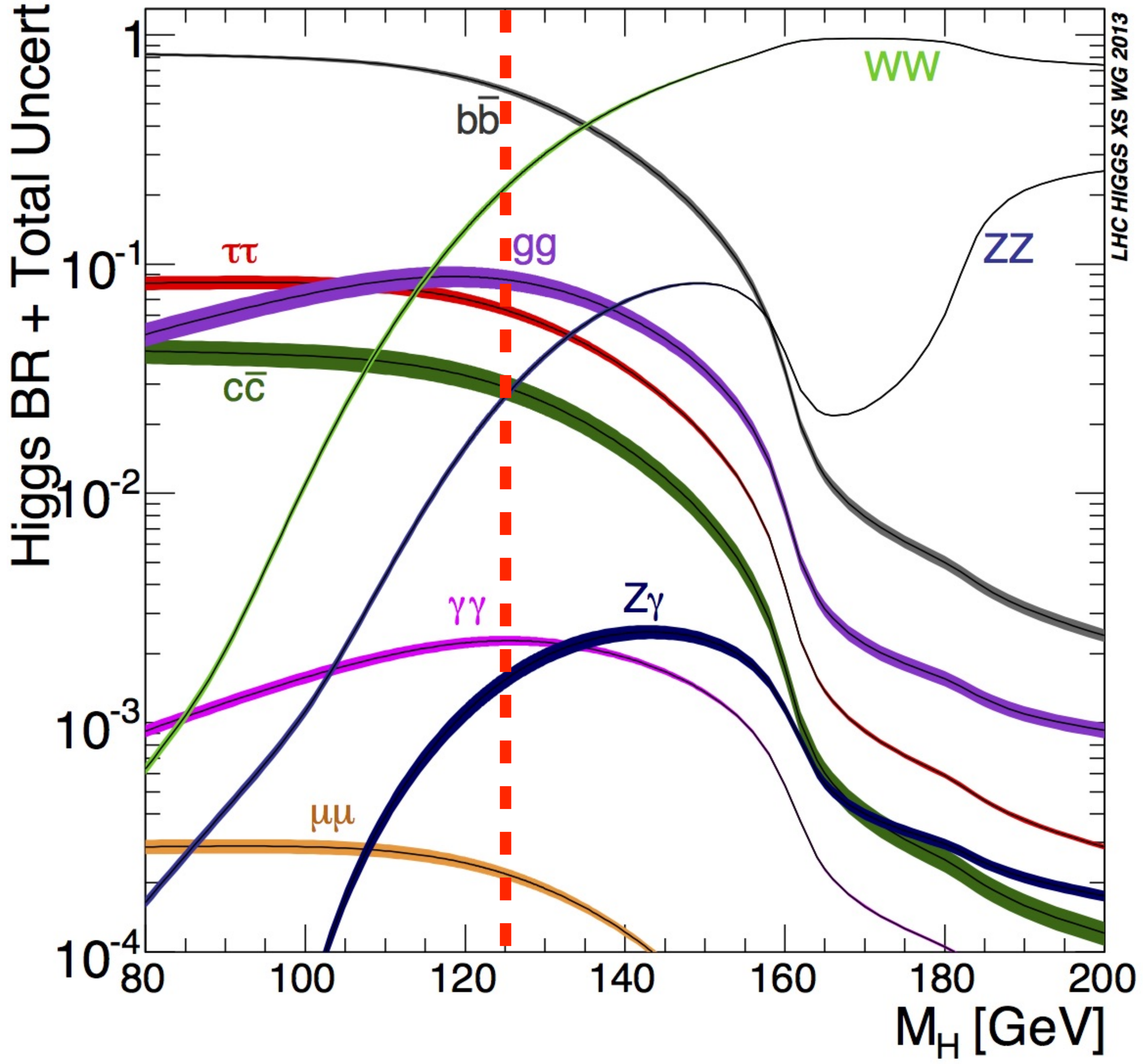
- Exploring the Higgs sector at the LHC demands an accurate control over hadronic events
  - ➡ Precise prediction for Higgs production and decay modes, as well as for background processes

    e.g. total prod^n cross section and branching ratios:









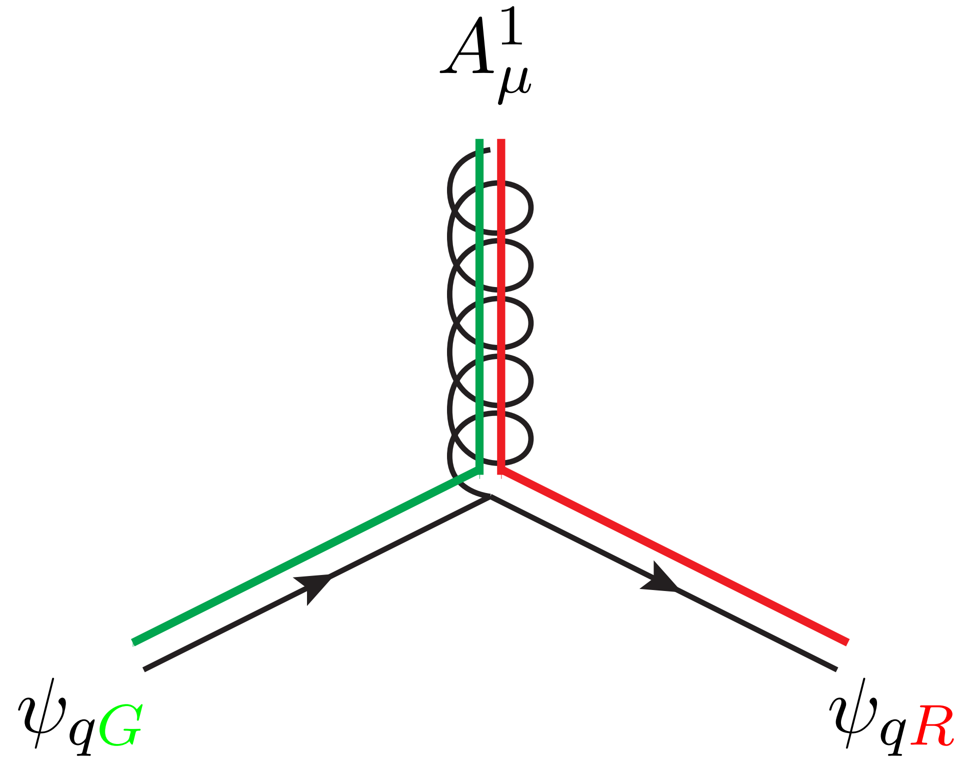
LHC events are shaped by strong interactions (QCD)

Vertex from Introduction to QCD 1207.2389

- **General principles:** SU(3)$_{\text{colour}}$ gauge invariance, Poincaré invariance (also causality, unitarity)

Quarks (fermions): Fundamental 3 (quark), $\bar{3}$ (anti-quark) representation of SU(3)$_{\text{colour}}$ (3 colour configurations)

Gluons (bosons): Adjoint representation of SU(3)$_{\text{colour}}$: $3 \oplus \bar{3} = 8 \oplus 1$ (8 colour configurations)



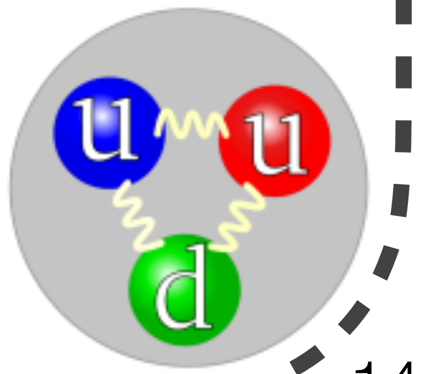
$$\propto \bar{\psi}_{qR} \, \lambda^1 \, \psi_{qG} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

6 types (3 families):

1^{st}	2^{nd}	3^{rd}	Q/e
$\underbrace{u}_{m_u \simeq 0}$	$\underbrace{c}_{m_c \simeq 1.3\,\text{GeV}}$	$\underbrace{t}_{m_t \simeq 172.7\,\text{GeV}}$	$+\frac{2}{3}$
$\underbrace{d}_{m_d \simeq 0}$	$\underbrace{s}_{m_s \simeq 0}$	$\underbrace{b}_{m_b \simeq 4.2\,\text{GeV}}$	$-\frac{1}{3}$

Confinement

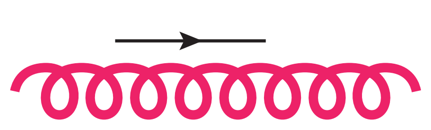
A key property is **confinement**: quarks & gluons not observed as free particles. They bind (Appendix) into colour-singlet hadrons:
mesons: equal no. of quarks & anti-quarks (usually 1 pair),
baryons: odd number of quarks (usually 3)
(+ a sea of gluons and additional $q\bar{q}$ pairs)



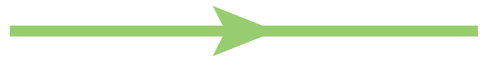
The QCD Lagrangian

Feynman rules adapted from
Introduction to QCD and loop calculations G.Heinrich (TUM)

$$\delta^{ab}\Delta_{\mu\nu}$$

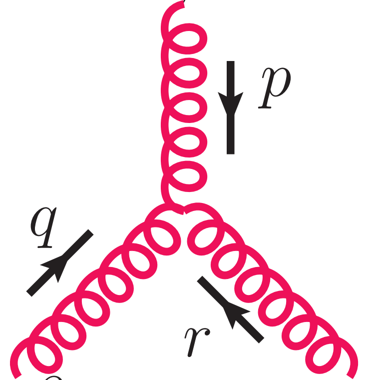


$$i\delta^{ij}\frac{\gamma_\mu p^\mu + m}{p^2 - m^2}$$

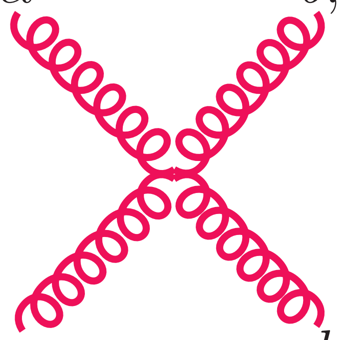


$$-i\, g_s\, (t^a)_{ij}\gamma^\mu$$

$$-i\, g_s \quad f^{abc} \quad V_{\alpha\beta\gamma}(p,q,r)^{\ddagger}$$



$$-i\, g_s^2 \begin{bmatrix} +f^{xac}\, f^{xbd}\, (g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\beta\gamma}) \\ +f^{xad}\, f^{xcb}\, (g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\beta}g_{\gamma\delta}) \\ +f^{xab}\, f^{xdc}\, (g_{\alpha\delta}g_{\beta\gamma} - g_{\alpha\gamma}g_{\beta\delta}) \end{bmatrix}$$



Non abelian gauge theory

Unobserved, CP violating term, strong experimental bounds on θ (neutron electric dipole moment)

$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \sum_{\mathrm{q}} \bar{\psi}_q(i\gamma_\mu D^\mu - m_q)\psi_q + \theta\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$(D^\mu)_{ab} = \delta_{ab}\partial^\mu + ig_s\, t^c_{ab}A^{c,\mu} \qquad F^c_{\mu\nu} = \partial_\mu A^c_\nu - \partial_\nu A^c_\mu - g_s f^{abc}A^b_\mu A^c_\nu$$

$^{\ddagger}V_{\alpha\beta\gamma}(p,q,r) = (p-q)_\gamma g_{\alpha\beta} + (q-r)_\alpha g_{\beta\gamma} + (r-p)_\beta g_{\alpha\gamma}$

The QCD Lagrangian

Feynman rules adapted from
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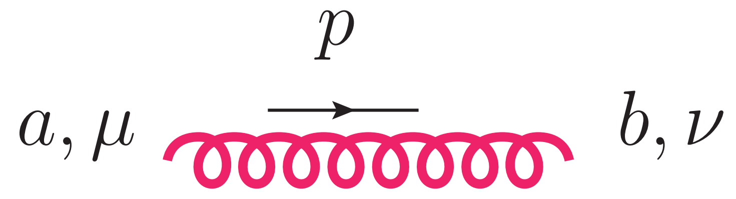
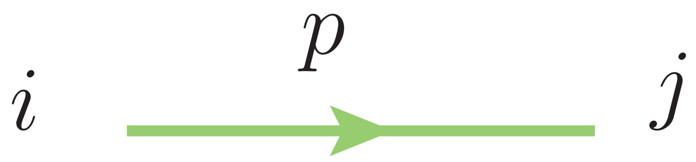
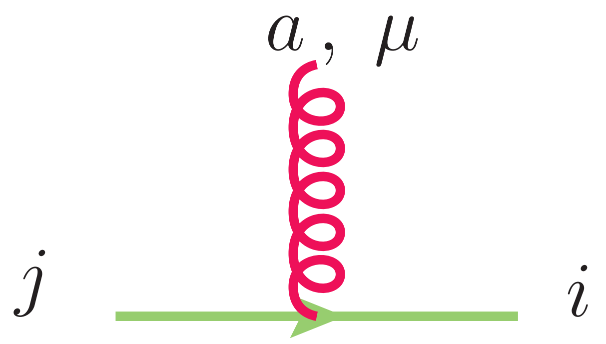
$\delta^{ab}\Delta_{\mu\nu}$



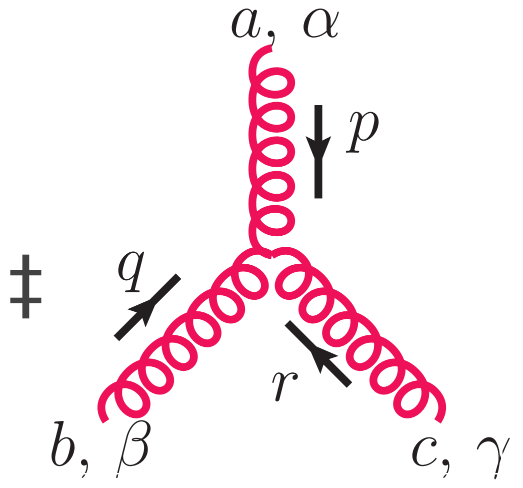
$i\delta^{ij}\dfrac{\gamma_\mu p^\mu + m}{p^2 - m^2}$



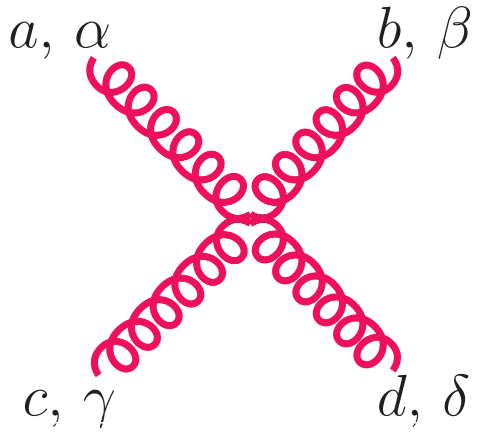
$-i\, g_s\, (t^a)_{ij}\gamma^\mu$



$-i\, g_s \quad f^{abc} \quad V_{\alpha\beta\gamma}(p,q,r)^{\ddagger}$



$$-i\, g_s^2 \begin{bmatrix} +f^{xac}\, f^{xbd}\, (g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\beta\gamma}) \\ +f^{xad}\, f^{xcb}\, (g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\beta}g_{\gamma\delta}) \\ +f^{xab}\, f^{xdc}\, (g_{\alpha\delta}g_{\beta\gamma} - g_{\alpha\gamma}g_{\beta\delta}) \end{bmatrix}$$



Non abelian gauge theory

$^{\ddagger}V_{\alpha\beta\gamma}(p,q,r) = (p-q)_\gamma g_{\alpha\beta} + (q-r)_\alpha g_{\beta\gamma} + (r-p)_\beta g_{\alpha\gamma}$

Unobserved, CP violating term, strong experimental bounds on θ (neutron electric dipole moment)

$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \sum_{\mathrm{q}} \bar{\psi}_q(i\gamma_\mu D^\mu - m_q)\psi_q + \theta\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$(D^\mu)_{ab} = \delta_{ab}\partial^\mu + ig_s\, t^c_{ab}A^{c,\mu} \qquad F^c_{\mu\nu} = \partial_\mu A^c_\nu - \partial_\nu A^c_\mu - g_s f^{abc}A^b_\mu A^c_\nu$$

Basic colour algebra we'll use later

The <u>emission</u> of a gluon of colour c from a parton $i \in \{q,g\}$ is associated with a colour charge operator $\mathbf{T}_i$

$$\left(\mathbf{T}_i\right)^c_{ab} \equiv if_{acb}, \quad i = g; \quad \left(\mathbf{T}_i\right)^c_{ab} \equiv t^c_{ab}, \quad i = q$$

$$\mathrm{Tr}(t^a t^b) = \underbrace{\mathrm{T_R}}_{=1/2}\ \delta^{ab}, \quad \sum_{\mathrm{c,b}} t^c_{ab}t^c_{bd} = \underbrace{\mathrm{C_F}}_{=4/3}\ \delta_{ad}, \quad \sum_{\mathrm{b,c}} f_{abc}f_{dbc} = \underbrace{\mathrm{C_A}}_{=3}\ \delta_{ad}$$

Representation of SU(3) generators in terms of the Gell-Mann matrices (cf. Appendix)

QCD for high-energy scattering

- Any observable (e.g. Green functions, S matrix from LSZ) can be obtained from the functional integral

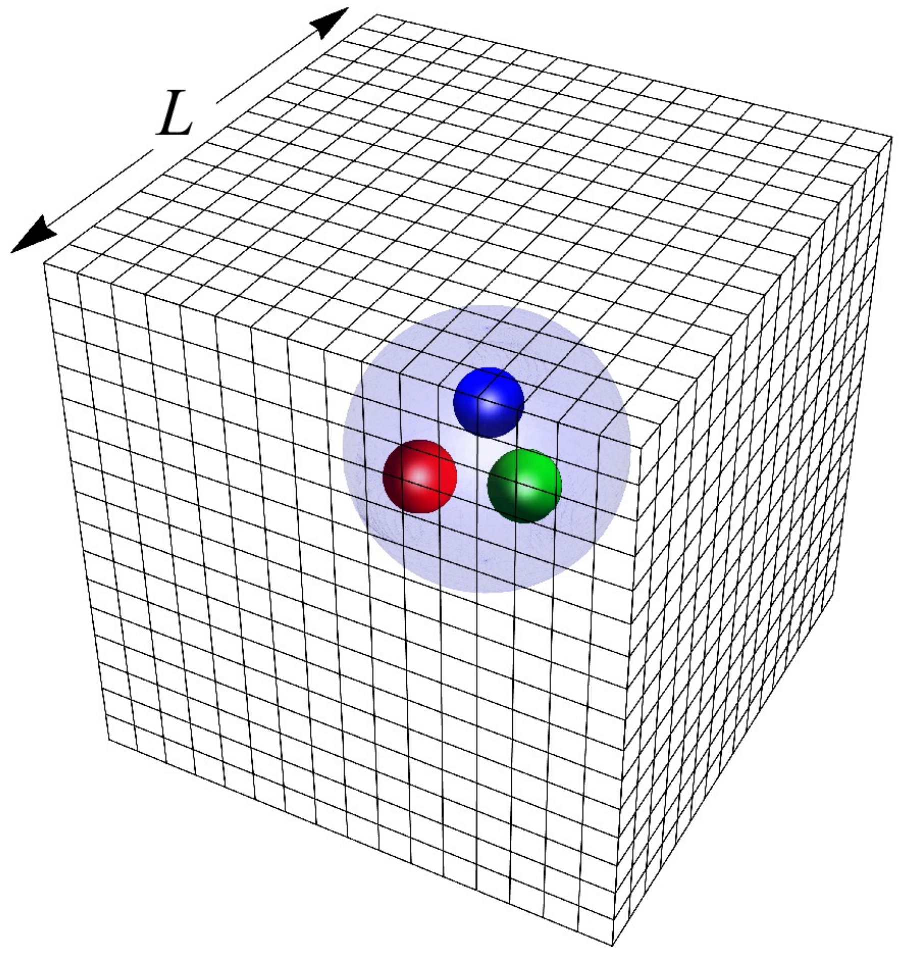
Functional of the QCD fields (e.g. $G = F_{\mu\nu}F^{\mu\nu}\bar{\psi}\psi$)

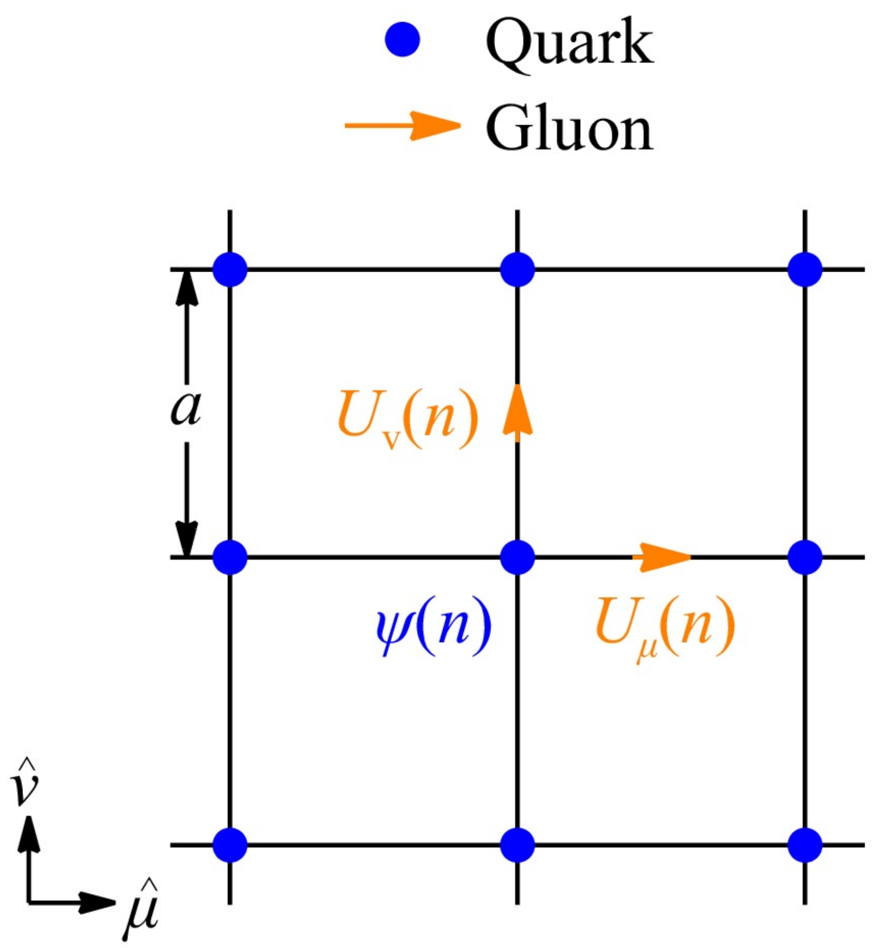
$$\langle 0 | TG[A,\psi,\bar{\psi}] | 0 \rangle = \mathcal{N} \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi} e^{i\int d^4x \mathcal{L}(x)} G[A,\psi,\bar{\psi}]$$

Time ordering

Vacuum

NB: including gauge fixing and Faddeev-Popov Lagrangians necessary to keep formulation well defined

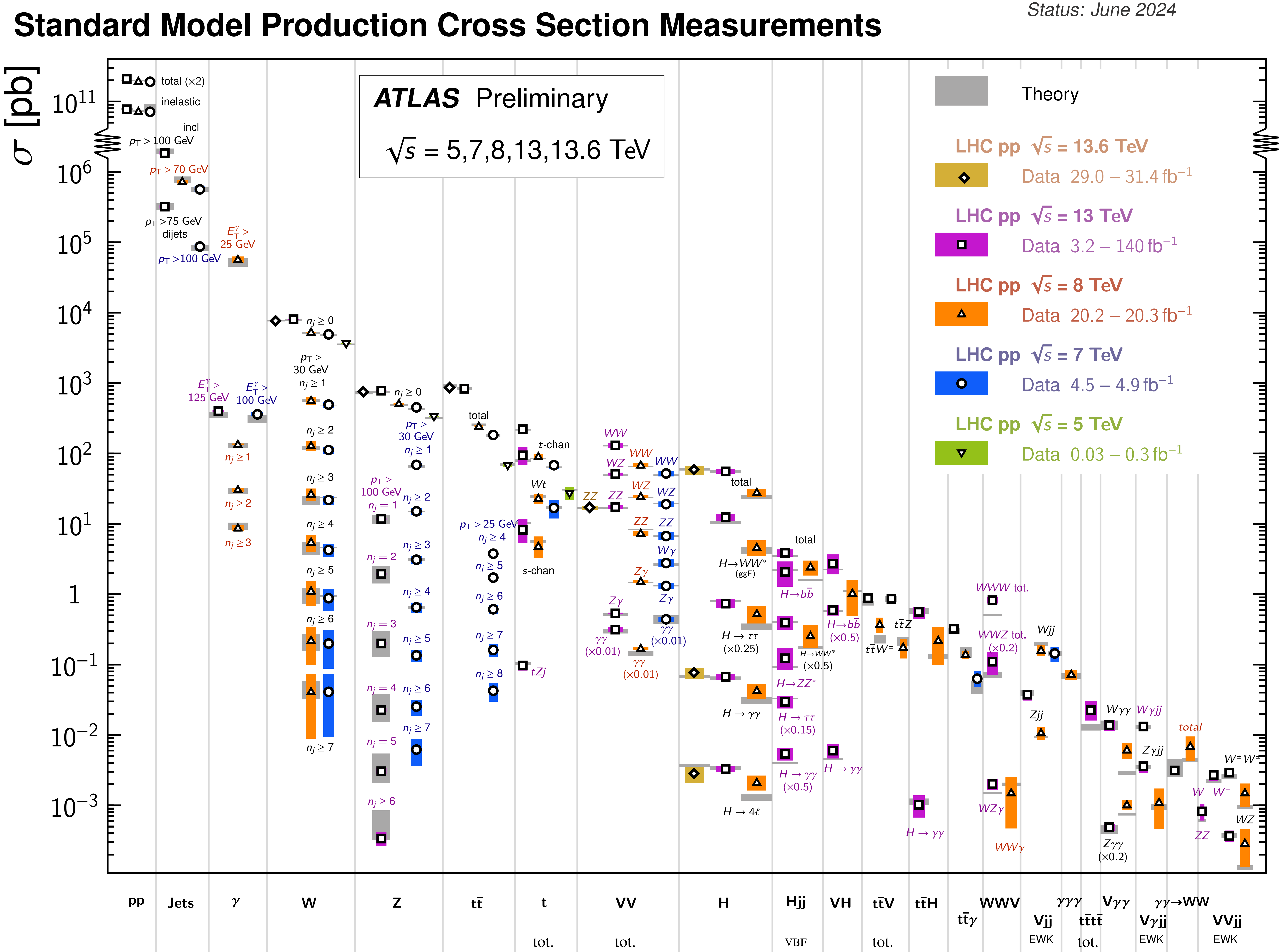




- → Contains full information about the theory, but extremely hard to solve exactly. An exception is given by lattice methods, although describing a scattering process is unfeasible at present (Minkowskian problem, enormous lattice size required)

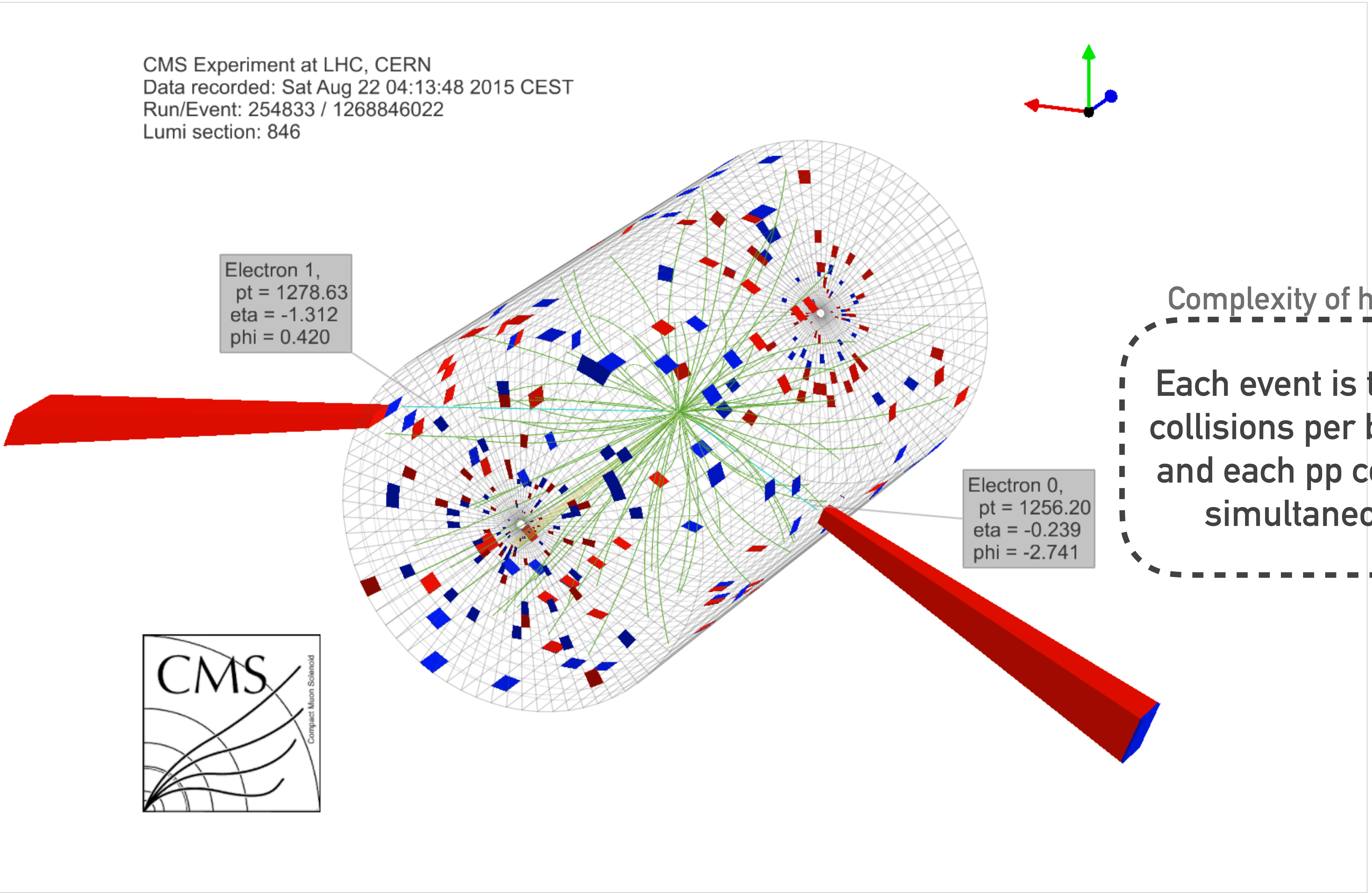
- → In practice we resort to perturbative methods, i.e. solve integral for the free theory (simple!) and then account for interacting Lagrangian as perturbations around the free-theory solution

Great success of perturbation theory at the LHC



Remarkable agreement with experimental data for broad range of reactions (multiple orders of magnitude in cross section units)!

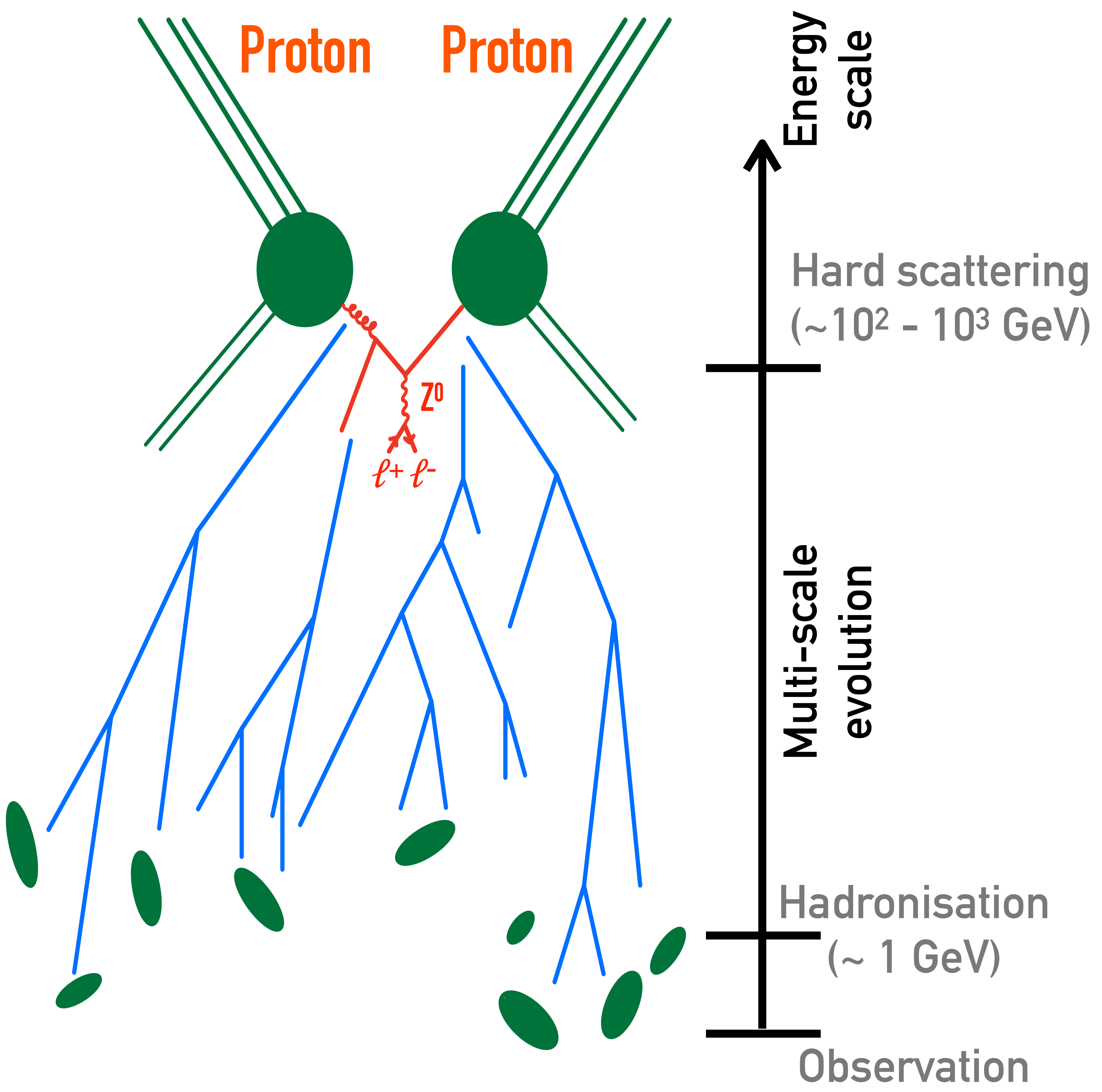
A realistic LHC event: e.g. dilepton (Drell-Yan) production



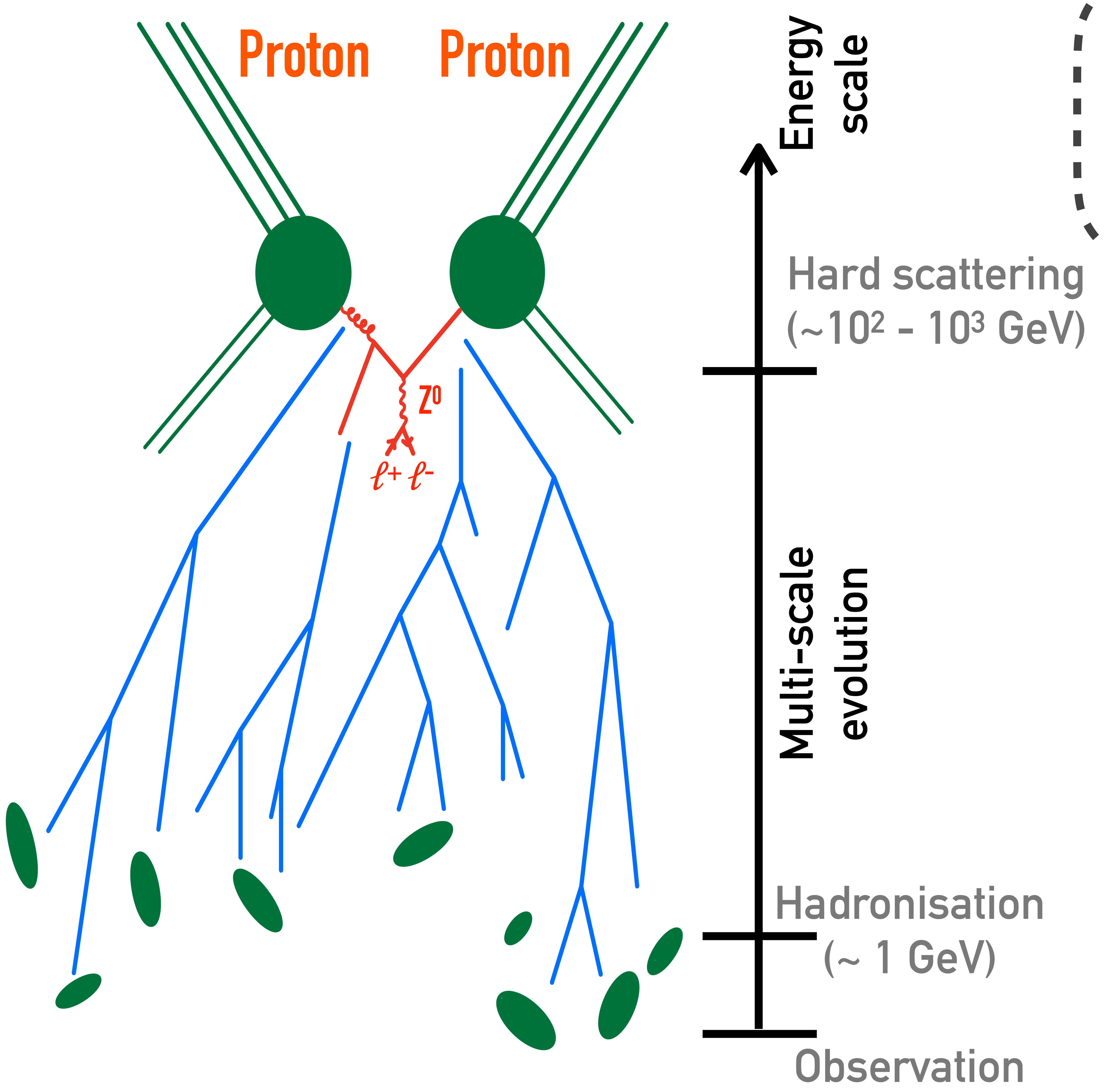
Complexity of hadronic scattering

Each event is the result of multiple pp collisions per bunch crossing (pile up), and each pp collision involves several simultaneous scatterings (MPI)

A simplified structure of a LHC event



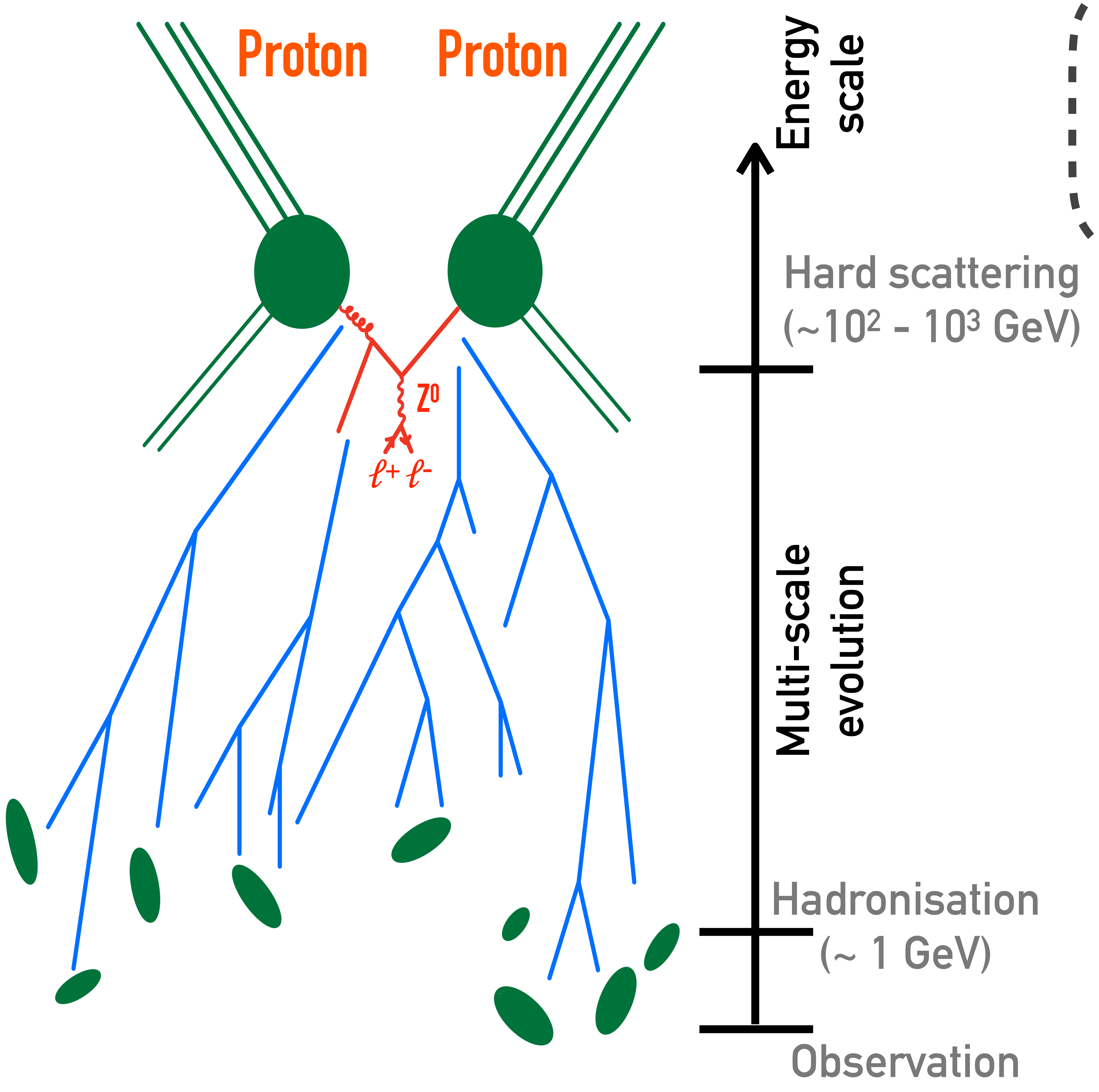
A simplified structure of a LHC event



A “spherical-cow” approximation (which however captures most of the physics!)


Credits: Keenan Crane

A simplified structure of a LHC event



A "spherical-cow" approximation (which however captures most of the physics!)


Credits: Keenan Crane

- Hard scattering (2 hardest partons): large momentum transfer, where new physics may be hiding

- Multi-scale evol.^{n}: copious emission of (mainly) strongly interacting particles. System evolves towards lower energies

  ➡ Connects observation/measurement to hard event

- QCD phase transition (non-perturbative): partons are combined into the colour-singlet hadrons eventually observed in the detector

Asymptotic freedom ⊕ factorisation → perturbative QCD

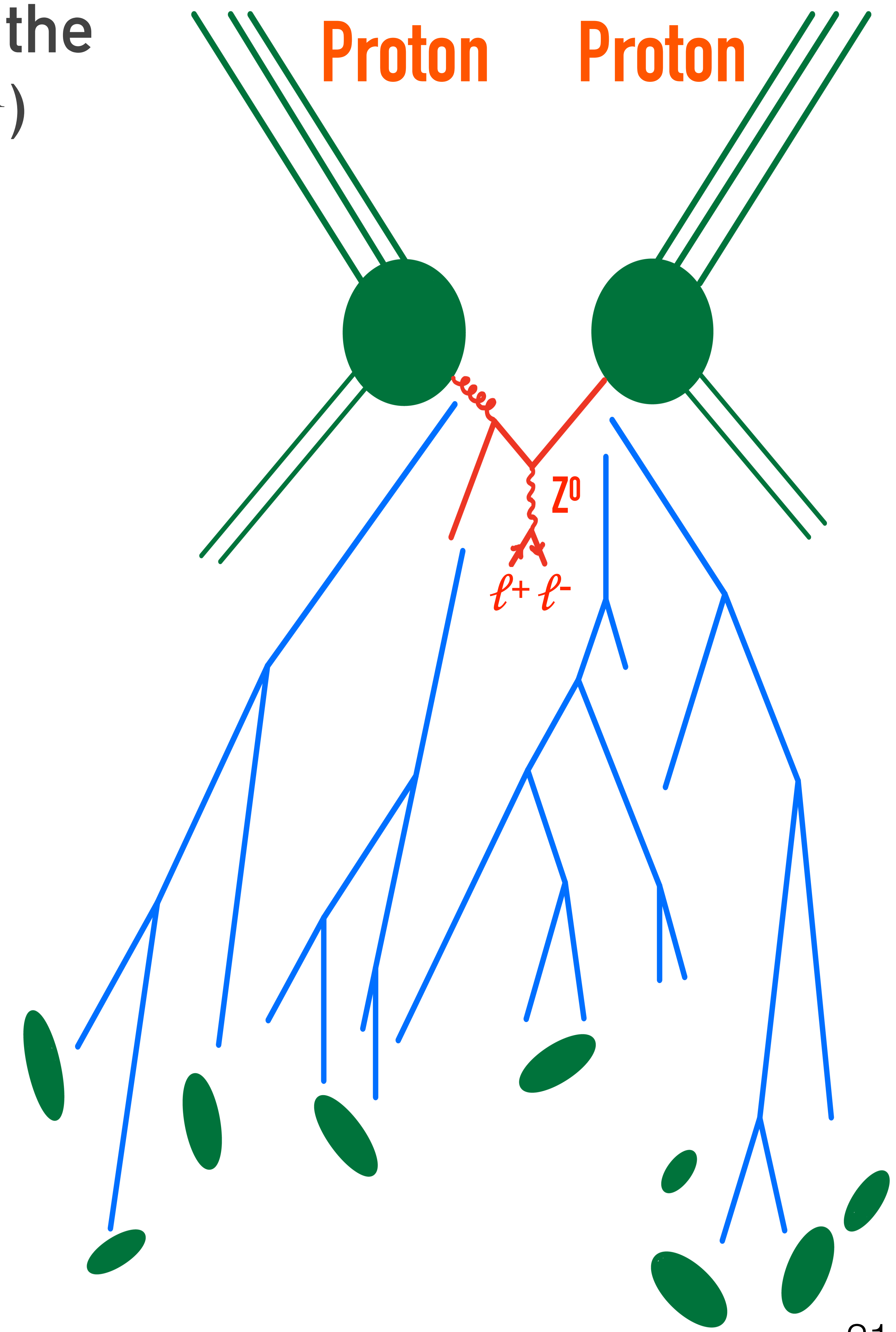
- Key observation 1: separation (factorisation) of dynamics taking place at different time scales‡
  - The "hard" scattering happens on shorter time scales ($\tau \sim 10^{-2}\,\mathrm{GeV}^{-1}$) than the interactions within the proton or among the final-state hadrons ($\tau \sim 1\,\mathrm{GeV}^{-1}$)

Dynamics at long time scales

$$d\sigma_{pp\to X} = \sum_{ij} \int_0^1 dx_1\, dx_2\, f_i(x_1, \mu_F) f_j(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F) + \mathcal{O}\left(\frac{\Lambda^p}{m_X^p}\right)$$

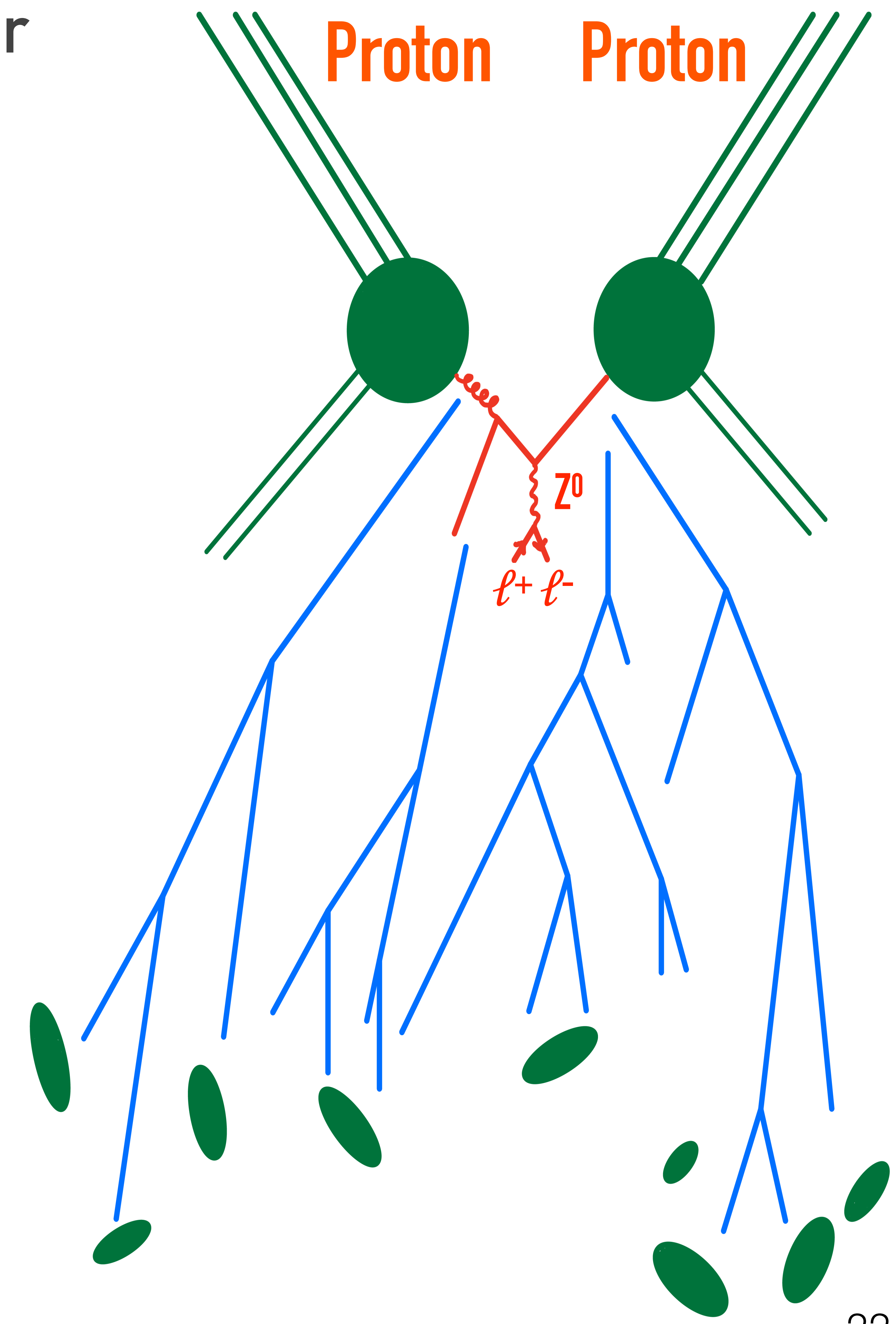
Dynamics at short time scales



‡ Actually proven to all order only for very simple quantities, e.g. Drell-Yan total cross section

Asymptotic freedom ⊕ factorisation → perturbative QCD

- Key observation 2: strong interactions become weakly coupled at high energies (asymptotic freedom)
  - Allows for a perturbative approximation of the primary scattering as a power series in α_s (truncated when the desired precision is reached)



$$d\sigma_{pp\to X} = \sum_{ij} \int_0^1 dx_1\, dx_2\, f_i(x_1,\mu_F) f_j(x_2,\mu_F)$$

$$\times \underbrace{d\hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F)}_{\sum_n \alpha_s^{n+n_B}(\mu_R)\, d\hat{\sigma}^{(n)}_{ij\to X}} + \mathcal{O}\left(\frac{\Lambda^p}{m_X^p}\right)$$

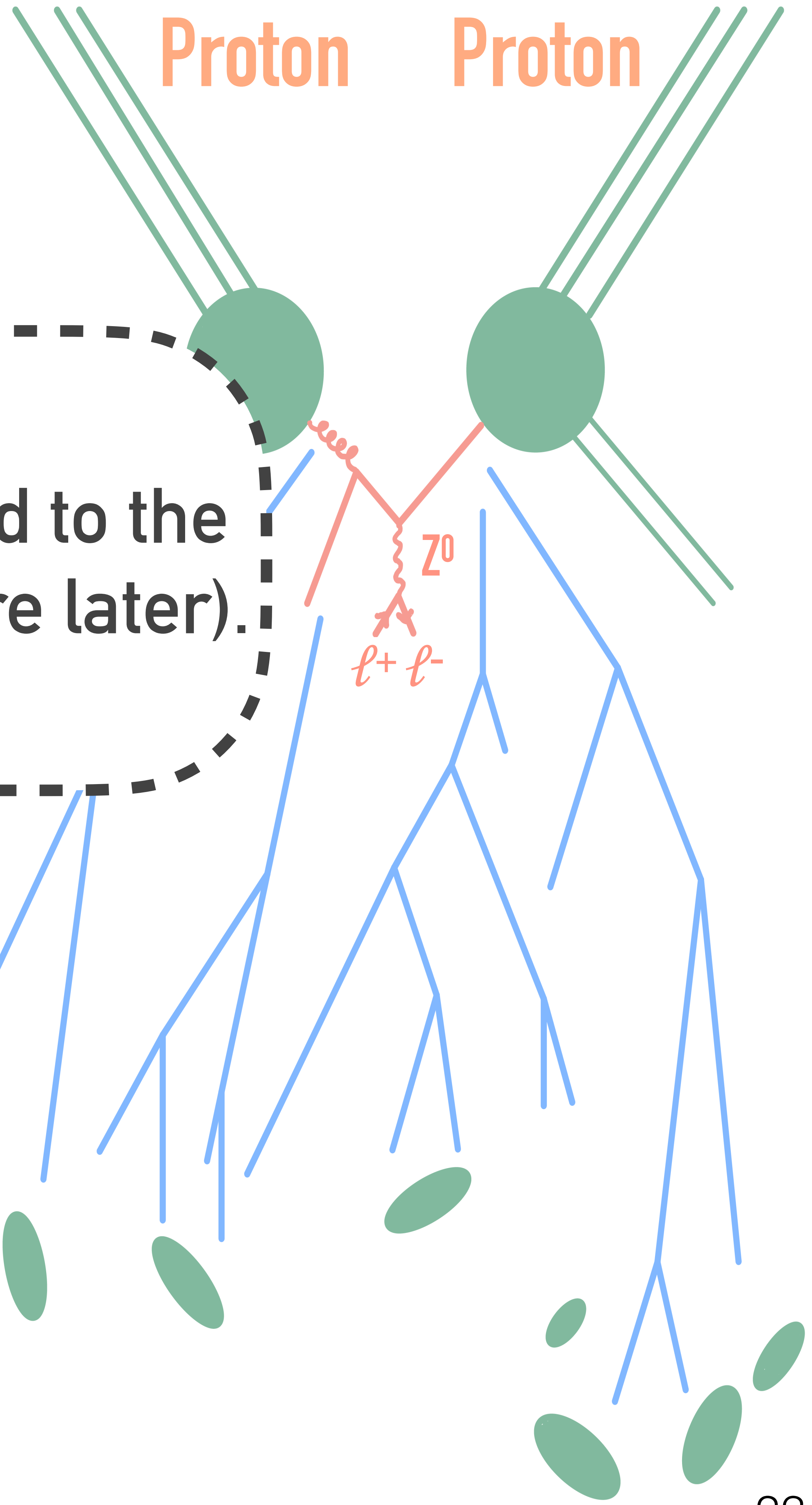
Asymptotic freedom ⊕ factorisation → perturbative QCD

- Key observation 2: strong interactions become weakly coupled at high energies (asymptotic freedom)
  - Allows for a perturbative approximation of the primary scattering as a power series in α_s (truncated when the desired precision is reached)

$$d\sigma_{pp\to X} =$$

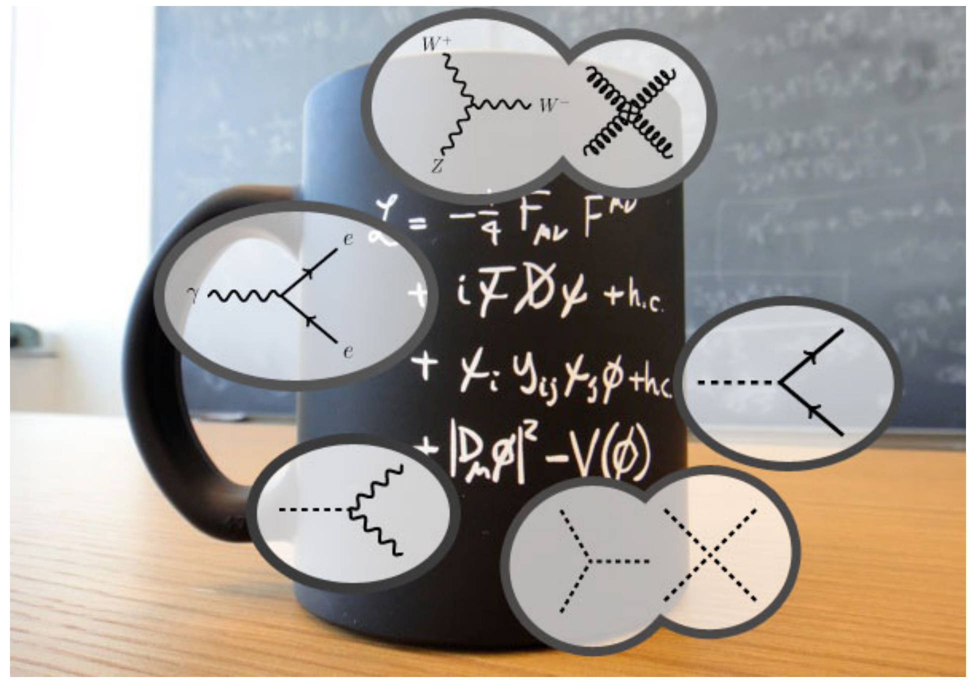
$$\underbrace{\times\, d\sigma_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F)}_{\sum_n \alpha_s^{n+n_B}(\mu_R)\, d\hat{\sigma}^{(n)}_{ij\to X}} + \mathcal{O}\left(\frac{\quad}{m_X^p}\right)$$

The properties of asymptotic freedom and factorisation are intimately tied to the divergences structure of gauge theories in different kinematic limits (more later).

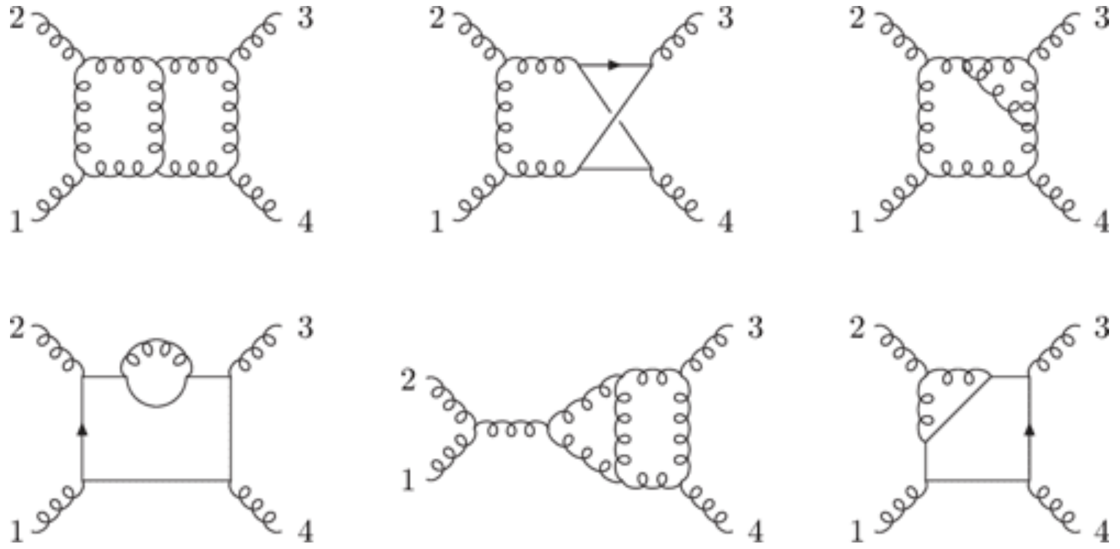


The perturbative-QCDer's workflow

Feynman rules



Scattering amplitudes



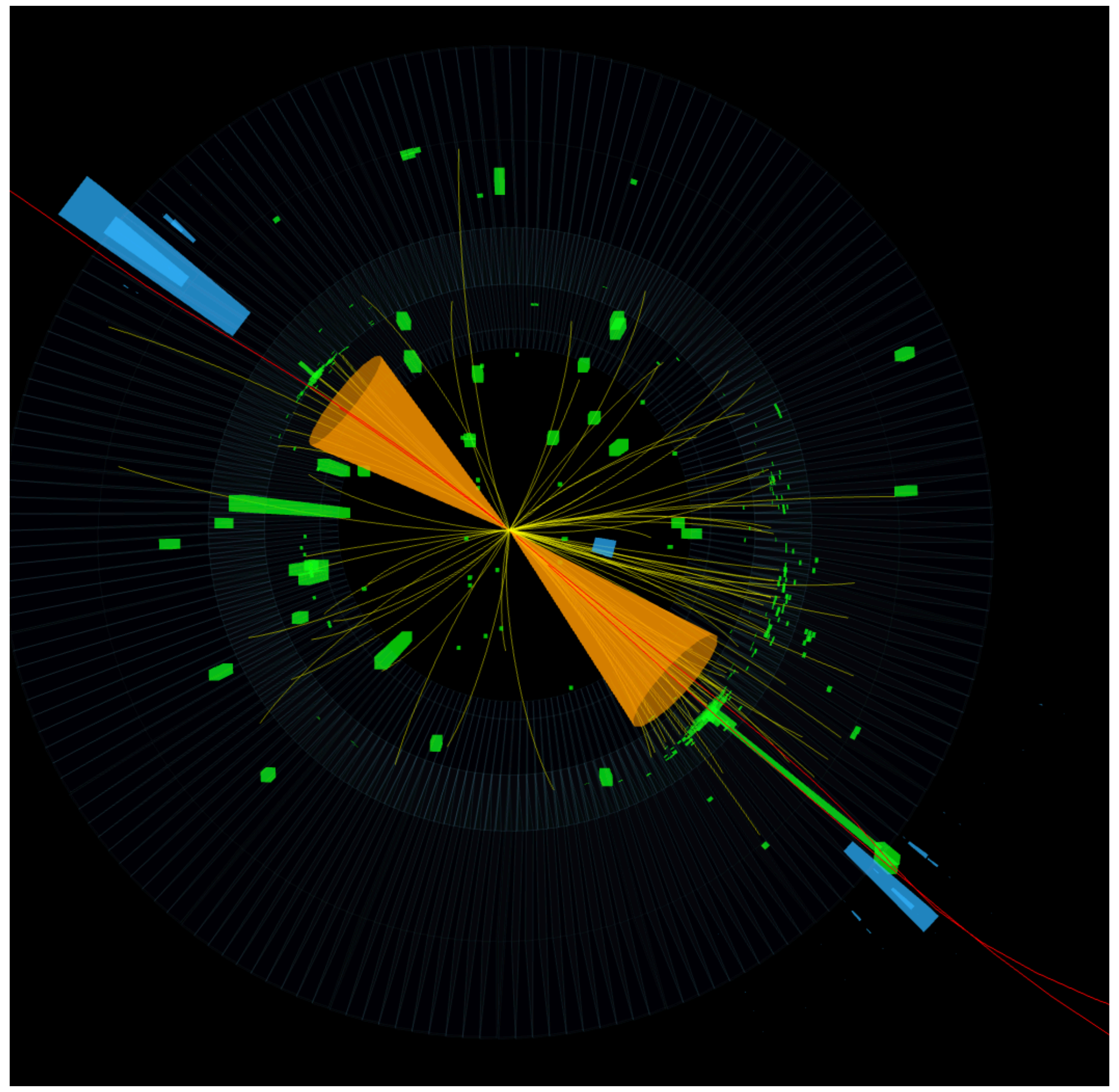
Cross sections

$$d\hat{\sigma}_{2\to n} = \frac{1}{F}\int \langle |\mathscr{A}|^2 \rangle d\Phi_n \mathcal{O}(\Phi_n)$$

$$d\sigma_{2\to n} = \int f_1(x_1) f_2(x_2) d\hat{\sigma} dx_1 dx_2$$

Event rates

$$N_{\text{events}} = \mathscr{L} \times \sigma$$

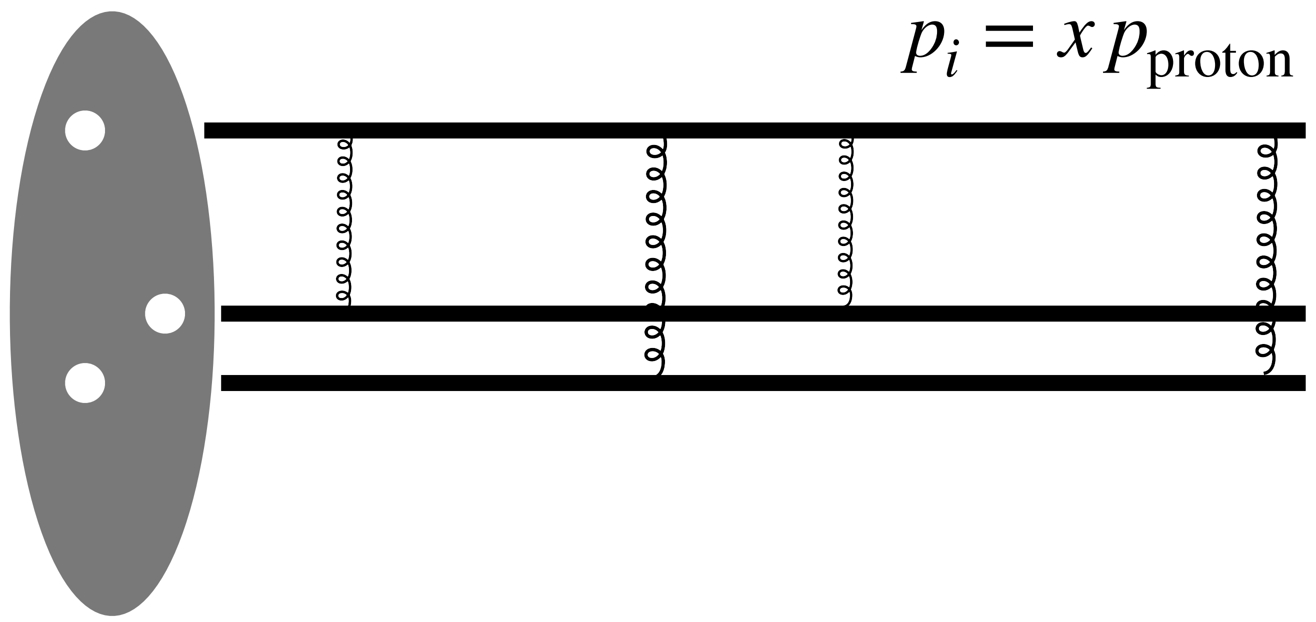


Ingredients of the master formula: parton distribution functions

- PDF $f_i(x, \mu)$ encodes the distribution of partons of flavour i and longitudinal momentum x within the proton probed at a scale μ. Composition of the proton evolves with the scale μ (QCD improved parton model).

➡ Heuristic interpretation in factorisation theorem: resolve partons in the proton at resolution scale μ_F

$$d\sigma_{pp\to X} = \sum_{ij} \int_0^1 dx_1\, dx_2\, f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times d\hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F) + \mathcal{O}\left(\frac{\Lambda^p}{m_X^p}\right)$$



In collinear factorisation partons fly in exactly the same direction as the proton, and share its longitudinal momentum. Transverse d.o.f.s are neglected as part of Λ^p/m_X^p corrections (higher twist).

DGLAP evolution equation

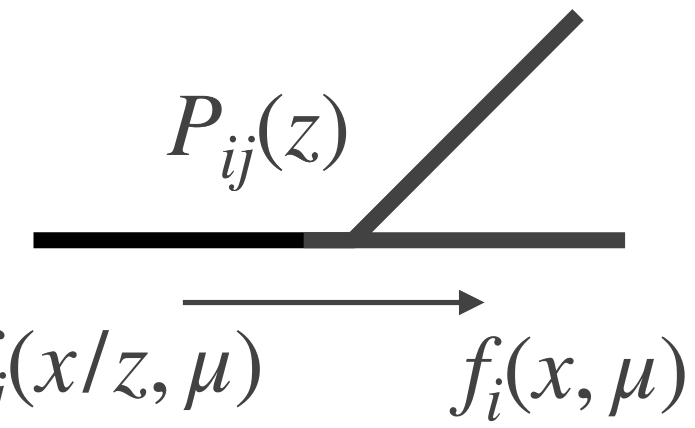
- Key property: although PDFs are intrinsically non-perturbative objects, their evolution with the scale at which the proton is resolved is perturbative! The evolution is governed by the DGLAP equation

The scale evolution of a flavour interplays with that of other flavours via the anomalous dimension $\hat{P}_{ij}$

Splitting functions

$$\hat{P}_{ij}(z, \alpha_s(\mu)) = \hat{P}^{(0)}_{ij}(z) + \frac{\alpha_s(\mu)}{2\pi}\hat{P}^{(1)}_{ij}(z) + \mathcal{O}(\alpha_s^2)$$

$$\frac{d}{d \ln \mu^2} f_i(x, \mu) = \frac{\alpha_s(\mu)}{2\pi} \int_x^1 \frac{dz}{z} \hat{P}_{ij}(z, \alpha_s(\mu)) f_j\left(\frac{x}{z}, \mu\right) \equiv \frac{\alpha_s(\mu)}{2\pi} \hat{P}_{ij}(x, \alpha_s(\mu)) \otimes f_j\left(x, \mu\right)$$



$$\hat{P}^{(0)}_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right), \quad b_0 = \frac{11C_A - 4T_R n_f}{6}$$

$$\hat{P}^{(0)}_{gg}(z) = 2\,C_A\left(\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right) + \delta(1-z) b_0$$

$$\hat{P}^{(0)}_{gq}(z) = T_R(z^2 + (1-z)^2), \qquad \hat{P}^{(0)}_{qg}(z) = C_F\frac{1+(1-z)^2}{z}$$

"Plus" distributions regularise the soft singularities due to the radiation of a gluon ($z \to 1$). Notice the enhancement at $z \to 0$, which drives the growth of the gluon PDF with the scale μ in this regime.

Currently, the complete NNLO splitting functions ($\hat{P}^{(2)}(z)$), and many of the N3LO terms ($\hat{P}^{(3)}(z)$) are known.

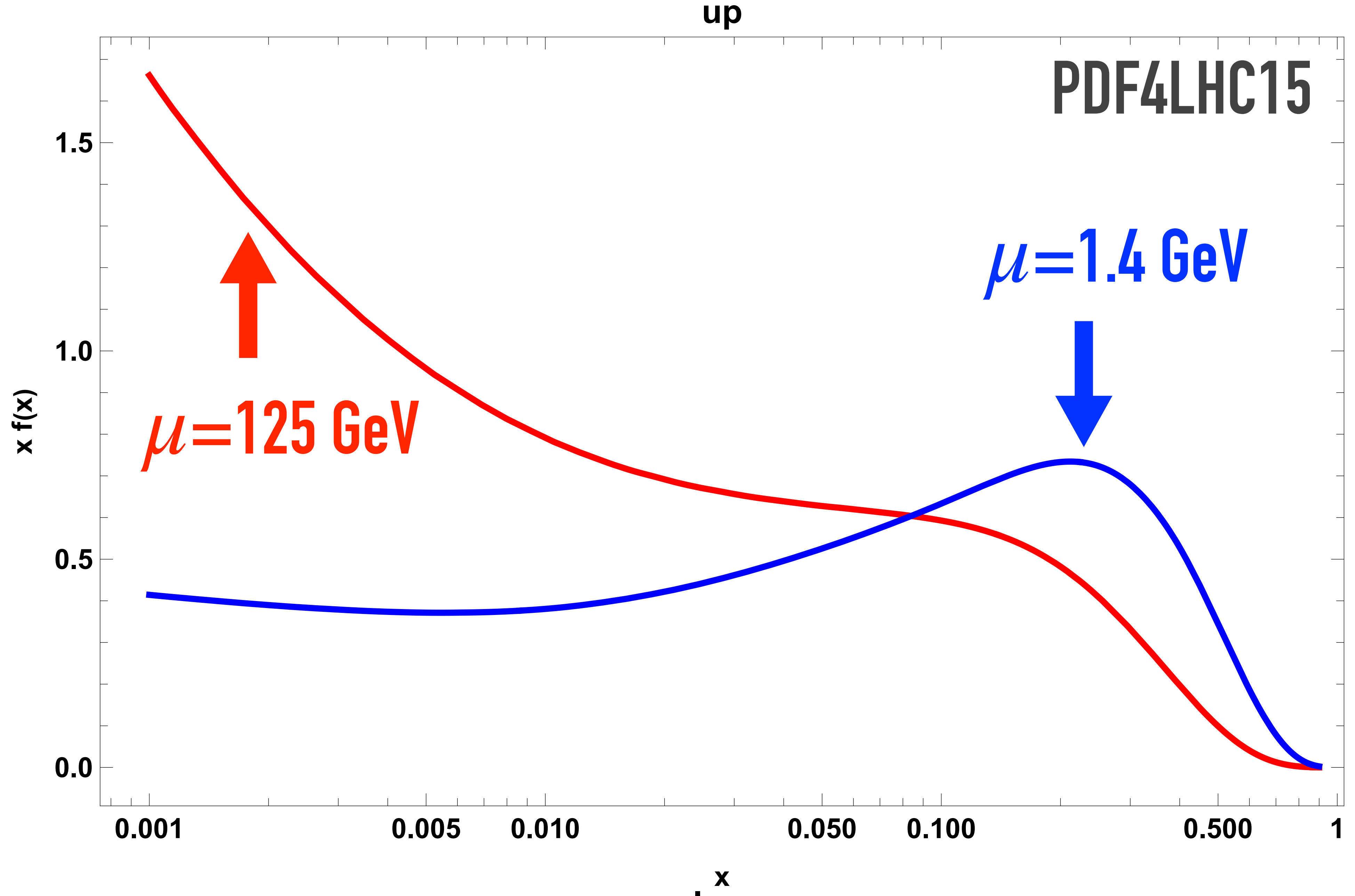
DGLAP evolution equation

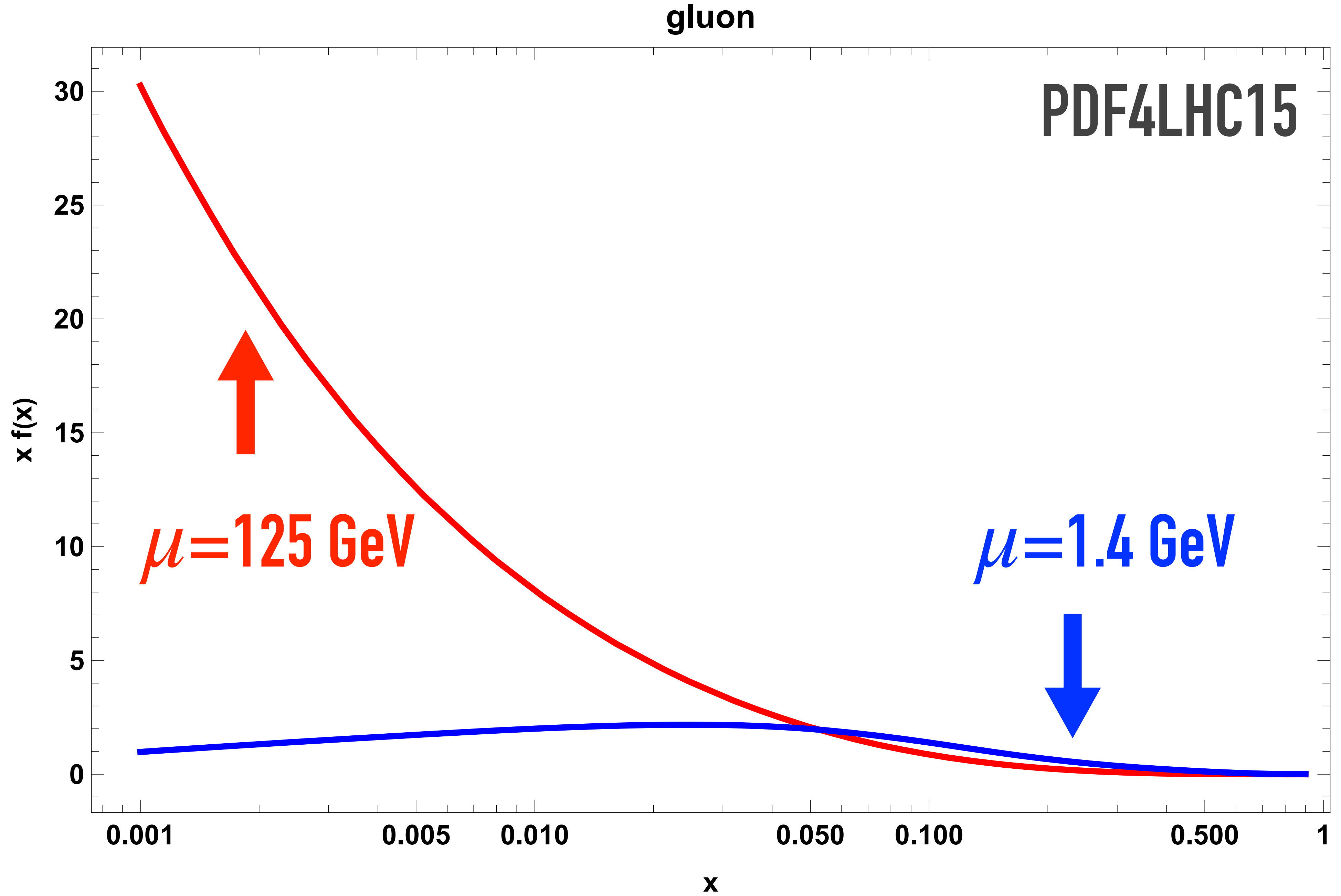
Figures made with ManeParse public code (see also Apfel)

Mathematica code available at this URL

DGLAP evolution towards larger scales depletes the distributions at larger x to fuel the growth of the distributions at smaller x.

Gluon PDF becomes substantial. Its growth is driven by $q \to qg$ and $g \to gg$ splittings (enhancement of corresponding splitting functions)





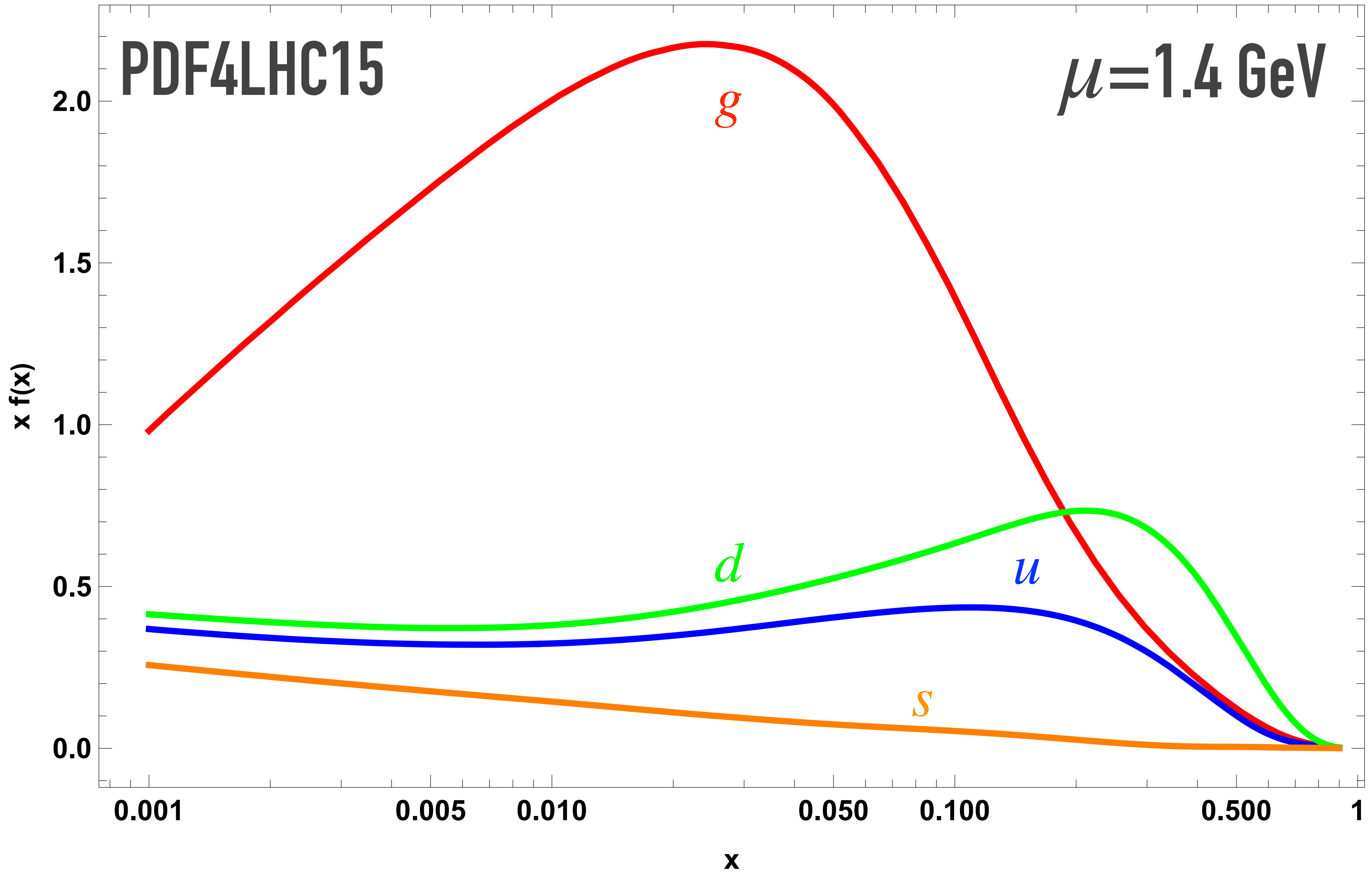
Composition of the proton

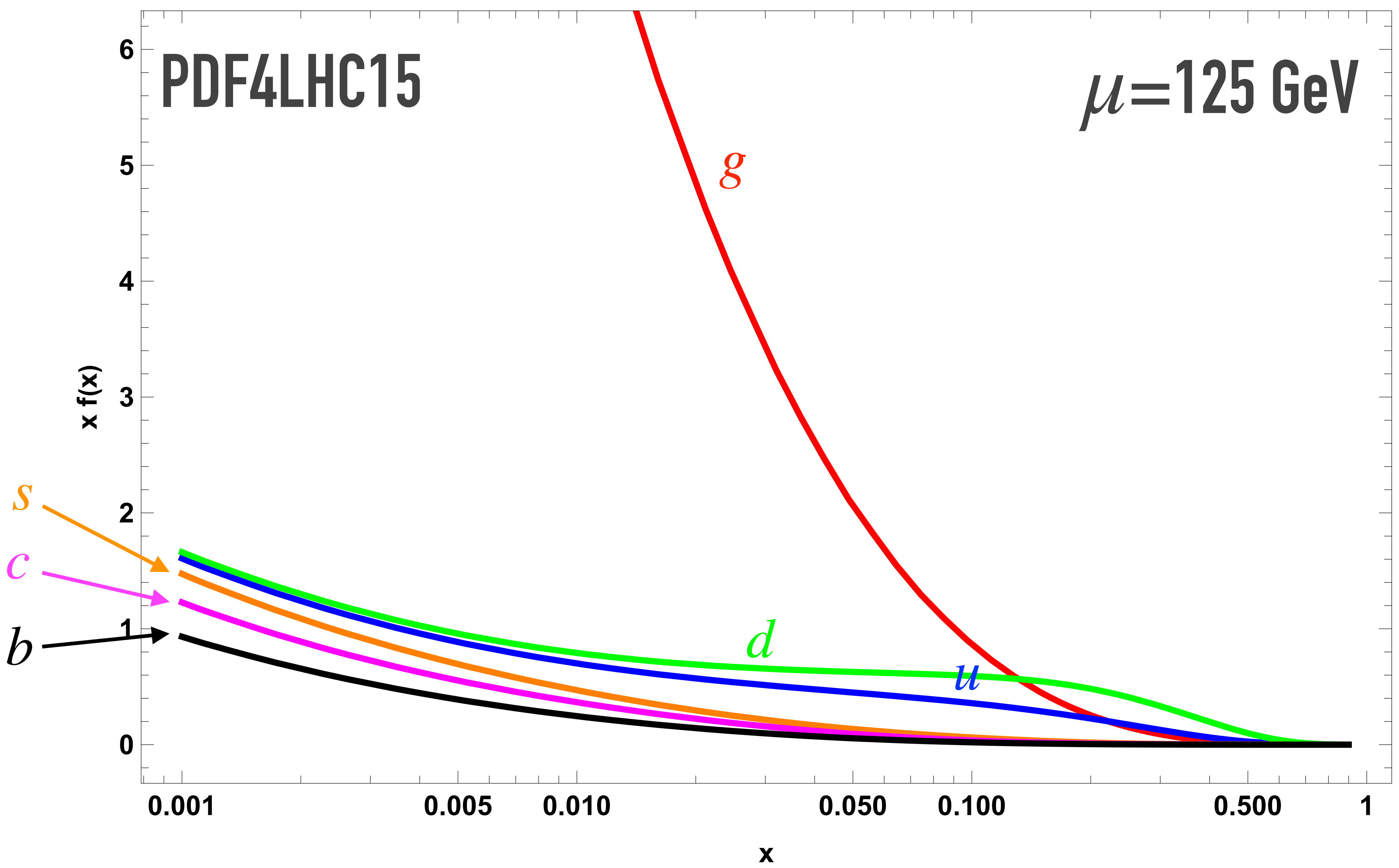
Figures made with ManeParse *public code (see also* Apfel*)*

DGLAP evolution determines the composition of the proton at perturbative scales given a fit of the parton densities at small (~non-perturbative) scales.

The growth of the gluon PDFs has a substantial impact on LHC phenomenology (e.g. Higgs, $t\bar{t}$, jets,…).

Heavier flavours (e.g. c, b) are produced dynamically via gluon splitting. Ongoing debate as to whether there is an "intrinsic" component in the proton (e.g. intrinsic charm)





Momentum sum rule:

$$\int_0^1 dx\, x \left(\sum_{i \in q,\bar{q}} f_i(x,\mu) + f_g(x,\mu) \right) = 1$$

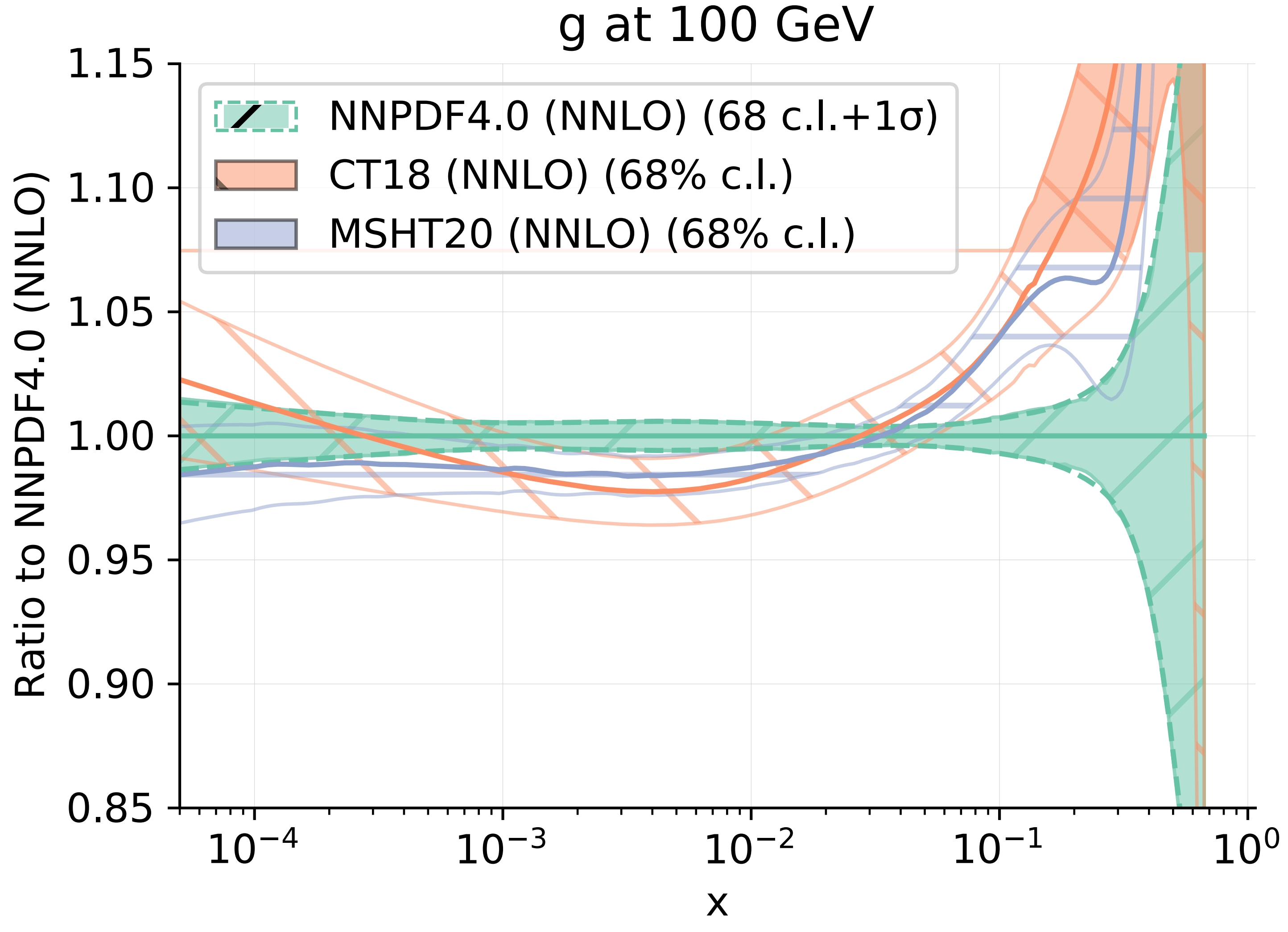
Gluons carry roughly 50% of the proton's momentum at $\mu = m_h$

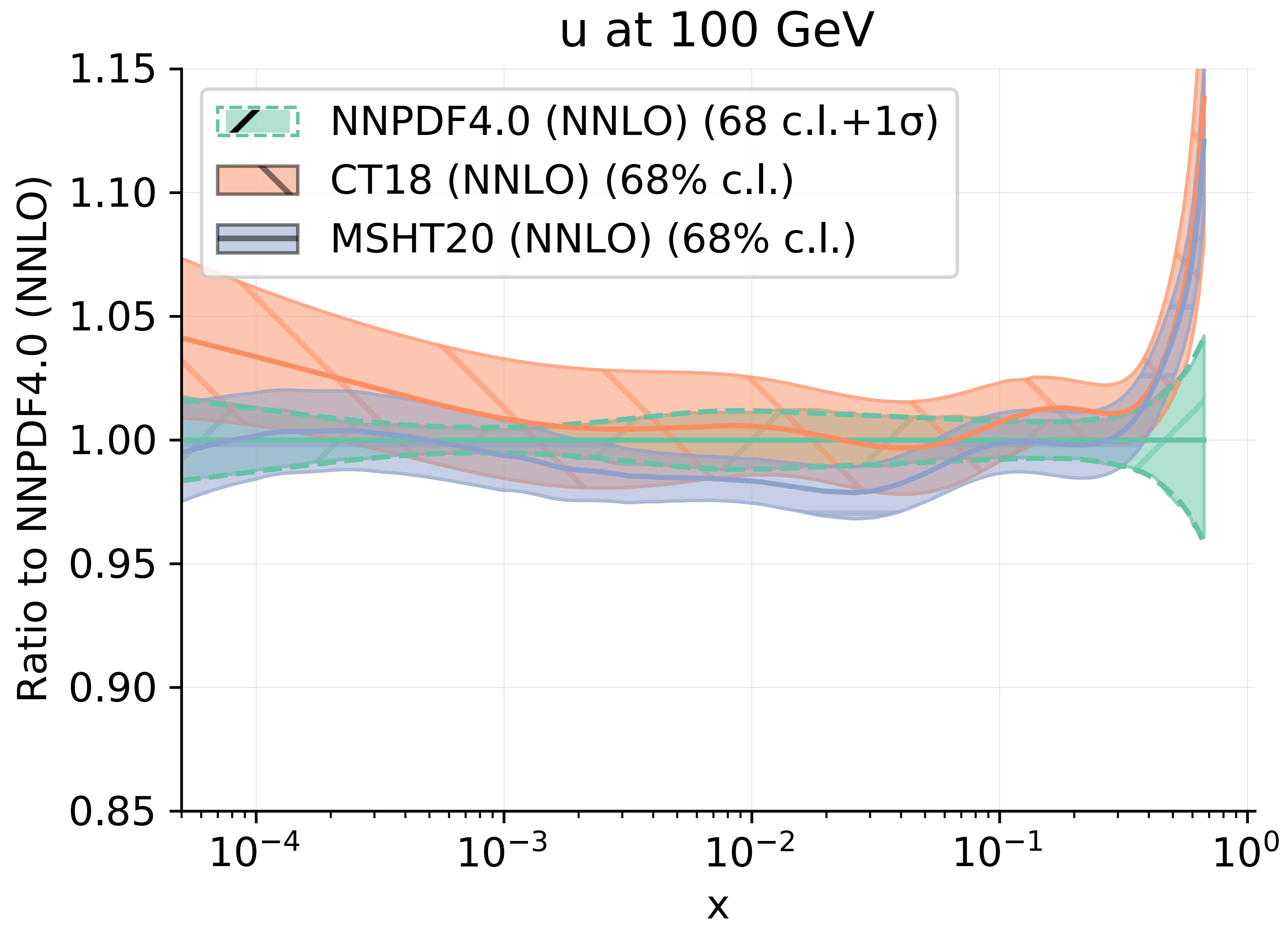
Current status of global PDF determinations

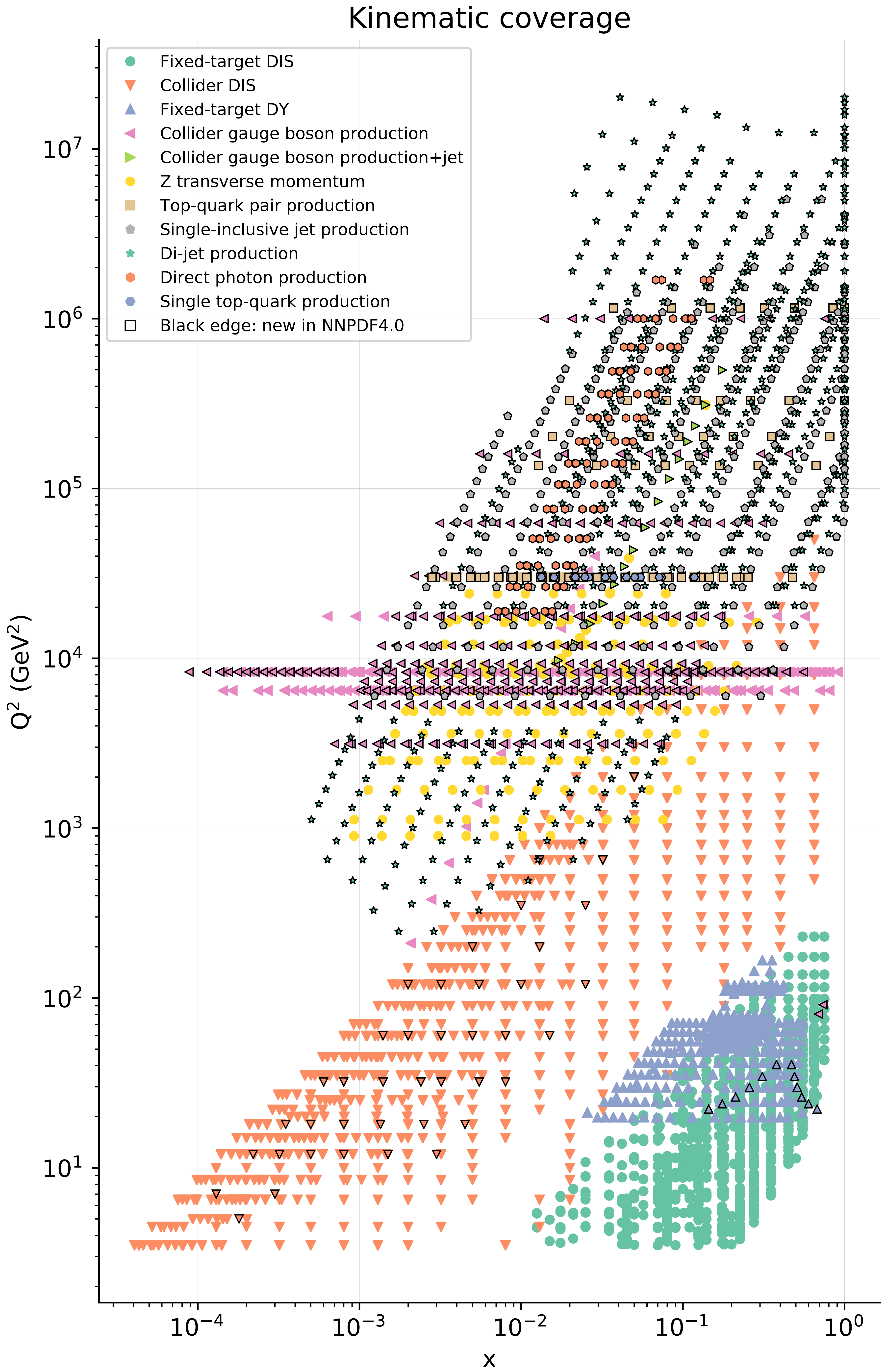
- Many determinations for LHC. Modern global fits reach few-% precision for $x \in [10^{-3}, 0.1]$, although estimate of PDF uncertainties is currently an open problem (fit/theory uncertainties)

  ➡ State of the art sets are extracted with NNLO (DGLAP and QCD predictions for $\hat{\sigma}$), and a lot of data. First steps towards N³LO sets are being taken

e.g. Comparison of PDFs (g & u) between different fitting methodologies (neural networks, hessian, ...) and parametric settings (m_c, α_s, ...)







Figures from The path to proton structures at 1% accuracy 2109.02653

Ingredients of the master formula: the partonic cross section

- Encodes the actual perturbative part of the high-energy partonic scattering

$$d\sigma_{pp\to X} = \sum_{ij} \int_0^1 dx_1\, dx_2\, f_i(x_1,\mu_F) f_j(x_2,\mu_F) \times \underbrace{d\hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F)}_{\sum_n \alpha_s^{n+n_B}(\mu_R)\, d\hat{\sigma}^{(n)}_{ij\to X}} + \mathcal{O}\left(\frac{\Lambda^p}{m_X^p}\right)$$

LO (TH error ~ 50%) | NLO (TH error ~ 20%) | NNLO (TH error ~ 5-10%) | N^{3}LO (TH error ~ few %)

$$d\hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F) = \alpha_s^{n_B}(\mu_R)\left(d\hat{\sigma}^{(0)}_{ij\to X} + \alpha_s(\mu_R)\, d\hat{\sigma}^{(1)}_{ij\to X} + \alpha_s^2(\mu_R)\, d\hat{\sigma}^{(2)}_{ij\to X} + \alpha_s^3(\mu_R)\, d\hat{\sigma}^{(3)}_{ij\to X} + \ldots\right)$$

LO (Born) cross section, $n_B > 0$ for QCD mediated processes (e.g. $gg \to h, t\bar{t}$, jets)

The dependence on the unphysical scales (μ_R, μ_F) will always be of higher orders w.r.t. the perturbative accuracy reached (gives a handle to estimate the size of missing corrections)

Computing the partonic cross section

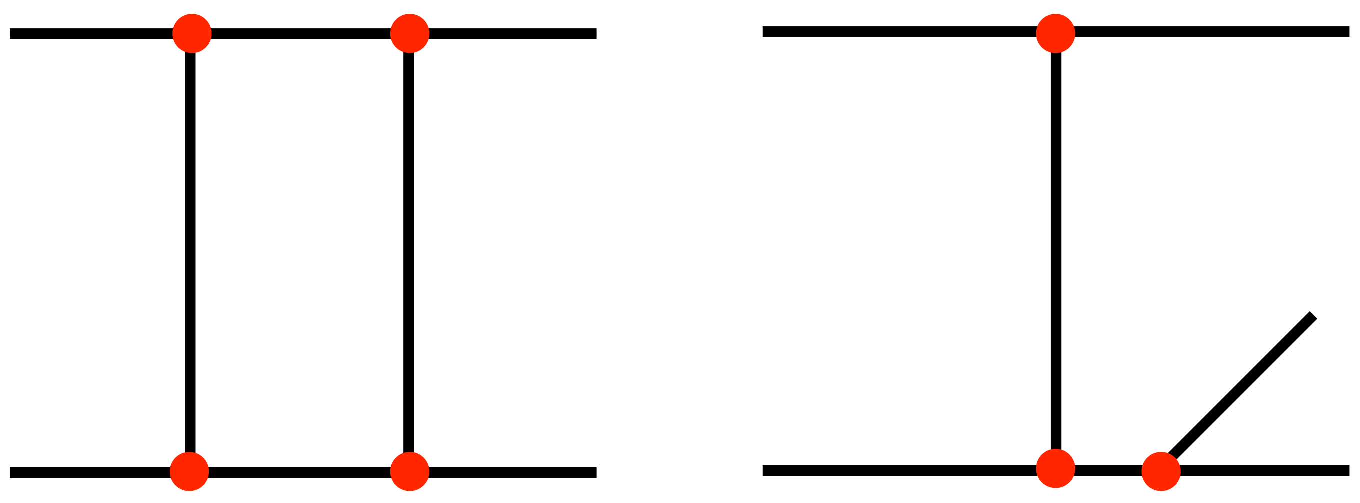
m body phase space (for all m contributing to a given perturbative order)

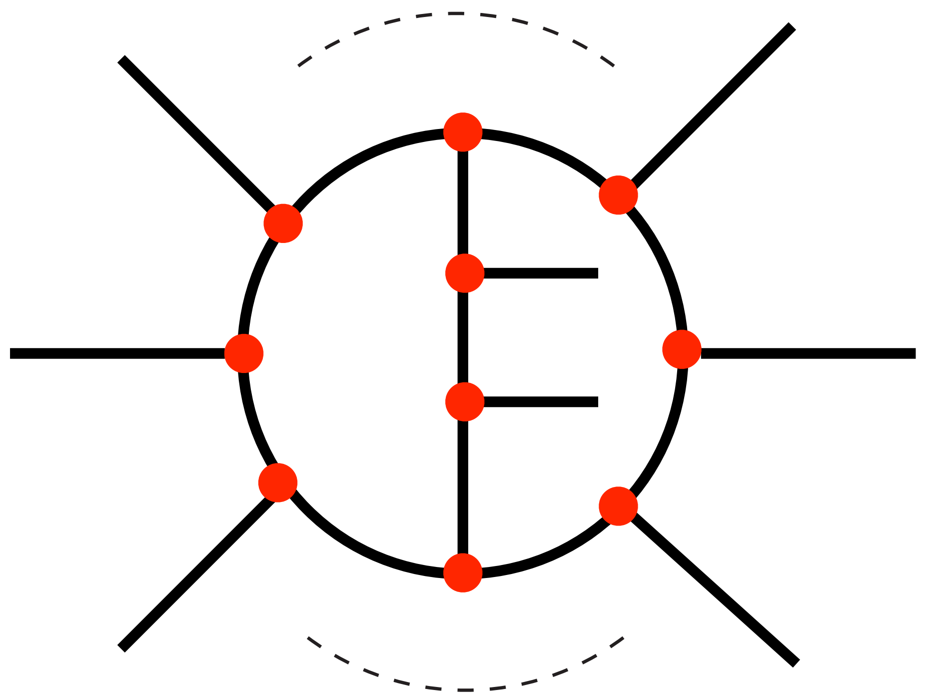
$$d\hat{\sigma}^{(n)}_{ij\to X} = \frac{1}{F}\sum_{m}\int d\Phi_m \,\langle\, |\mathscr{A}_{2\to m}|^2 \rangle \,\mathscr{O}(\Phi_m)$$

Observable's measurement

Flux factor

Squared amplitude (averaged over initial-state spin & colour)



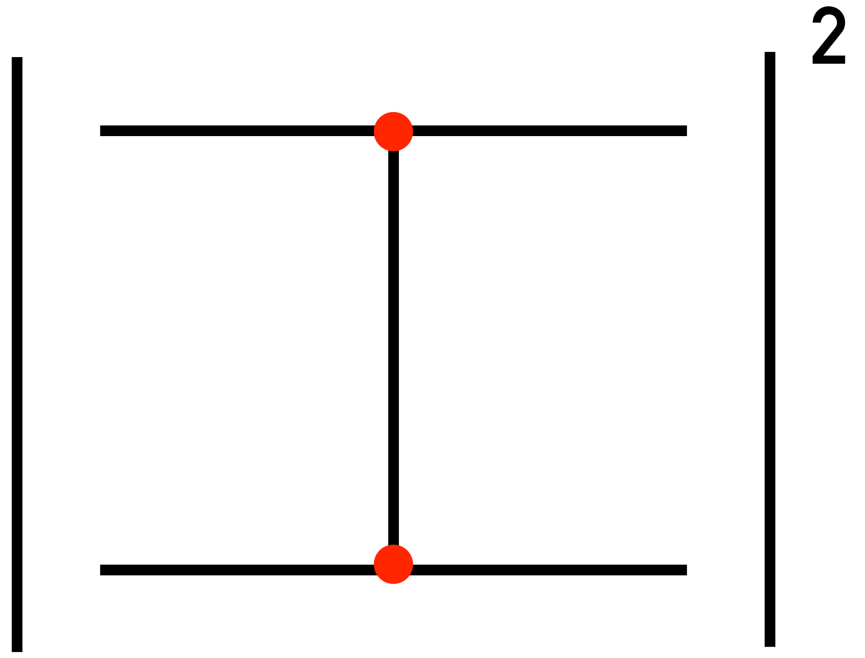


Computation of phase space integrals entails delicate cancellations of singularities (subtractions) between real and virtual corrections. Significant computational challenge for state of the art calculations

Computation of scattering amplitudes at higher loops: entails VERY large expressions (algebraic complexity) & spaces of special functions (analytic complexity)

The squared amplitude (e.g. NLO for $2 \to 2$ partonic process)

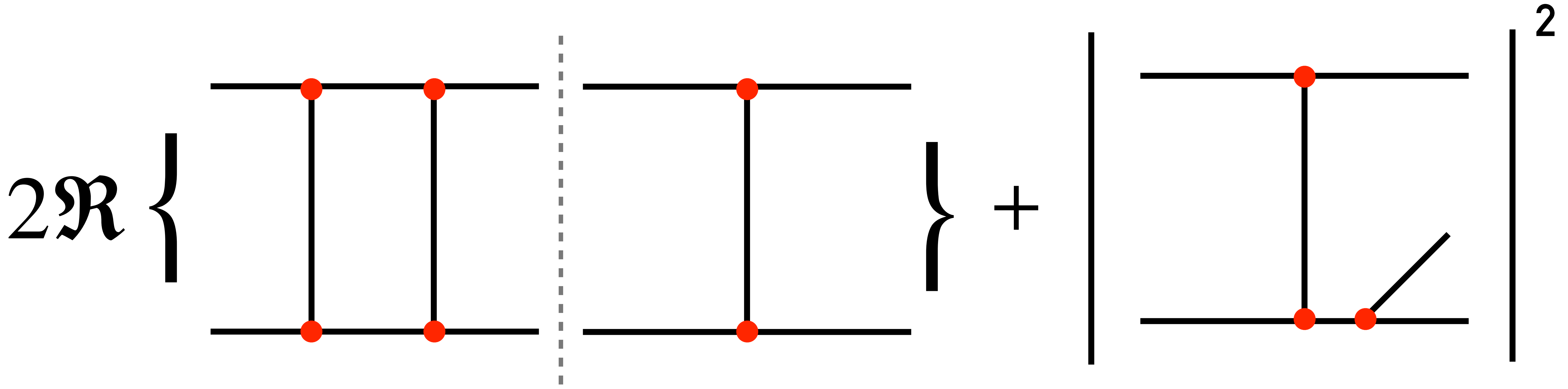
LO‡
(only tree-level diagrams)



$\sim \mathcal{O}(g_s^4) \sim \mathcal{O}(\alpha_s^2)$

NLO‡
1) Add a virtual loop to the LO process and expand the squared norm

2) Add a real emission to the LO process



$\sim \mathcal{O}(\alpha_s^3)$

$$|\mathscr{A}^{(0)} + \alpha_s \mathscr{A}^{(1)}|^2 = |\mathscr{A}^{(0)}|^2 + \alpha_s\, 2\Re \mathscr{A}^{(0)} (\mathscr{A}^{(1)})^\dagger + \dots$$

Which type of diagrams enter a NNLO calculation for this reaction?

‡ We use representative diagrams, the actual number of Feynman diagrams explodes with the perturbative order

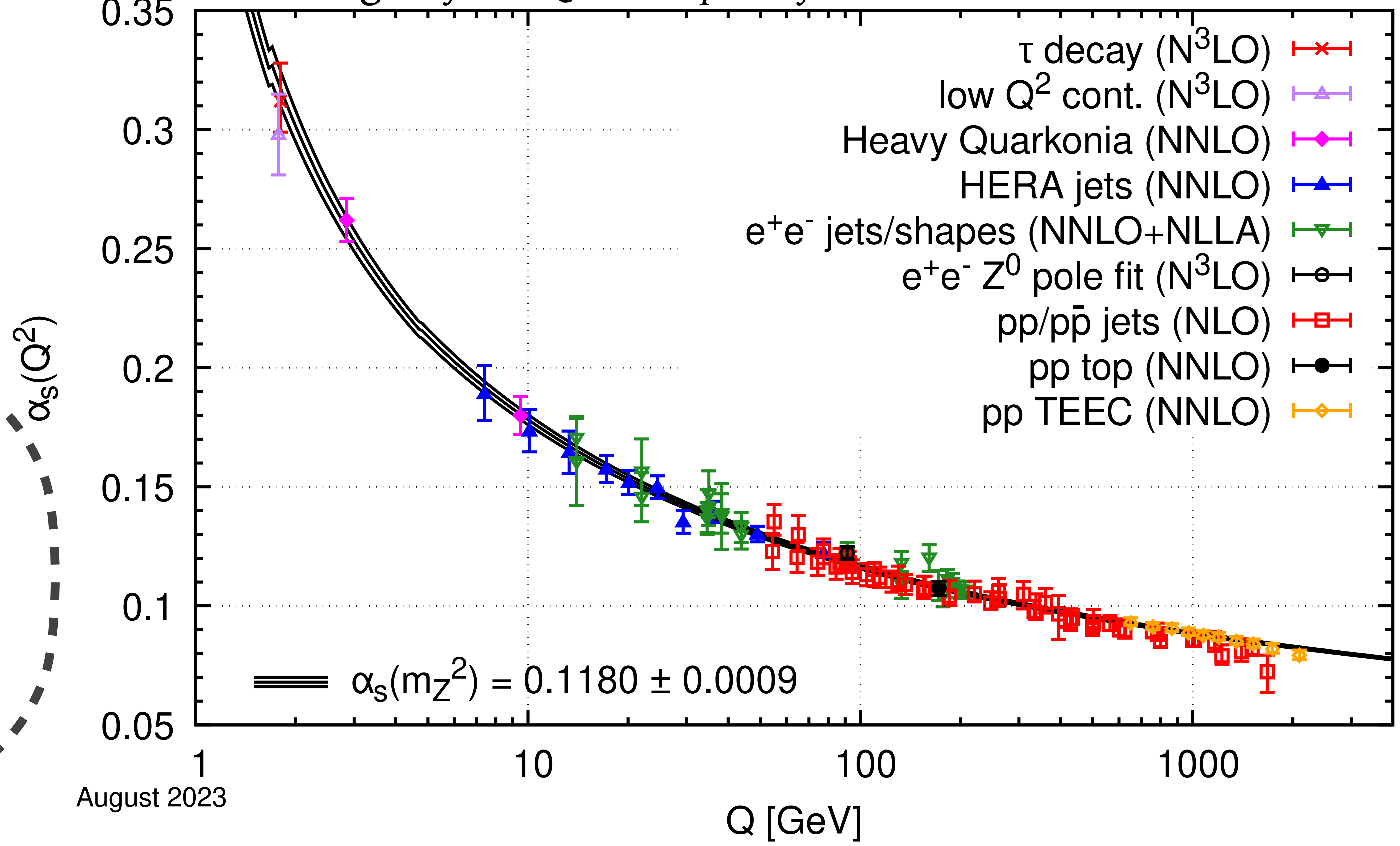
Ingredients of the master formula: the strong coupling constant

- The size of α_s determines how many perturbative orders are needed to reach the desired precision
  - ➡ As for PDFs, the coupling "runs" with the energy scale (renormalisation group equation)

$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} = \beta(\alpha_s(\mu)) \overset{\ddagger}{=} -\beta_0\,\alpha_s^2(\mu) + \mathcal{O}(\alpha_s^3)\,, \quad \beta_0 = (11C_A - 4T_R n_f)/(12\pi) > 0$$

Figure from QCD chapter of Particle Data Book 2312.14015

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1+\alpha_s(\mu_0)\beta_0\ln\frac{\mu^2}{\mu_0^2}} = \frac{1}{\beta_0\,\ln\frac{\mu^2}{\Lambda^2}}$$



Coupling becomes small at large scales (asymptotic freedom) and has a logarithmic divergence at small scales (breakdown of pQCD due to confinement) parametrised by theory IR cutoff Λ

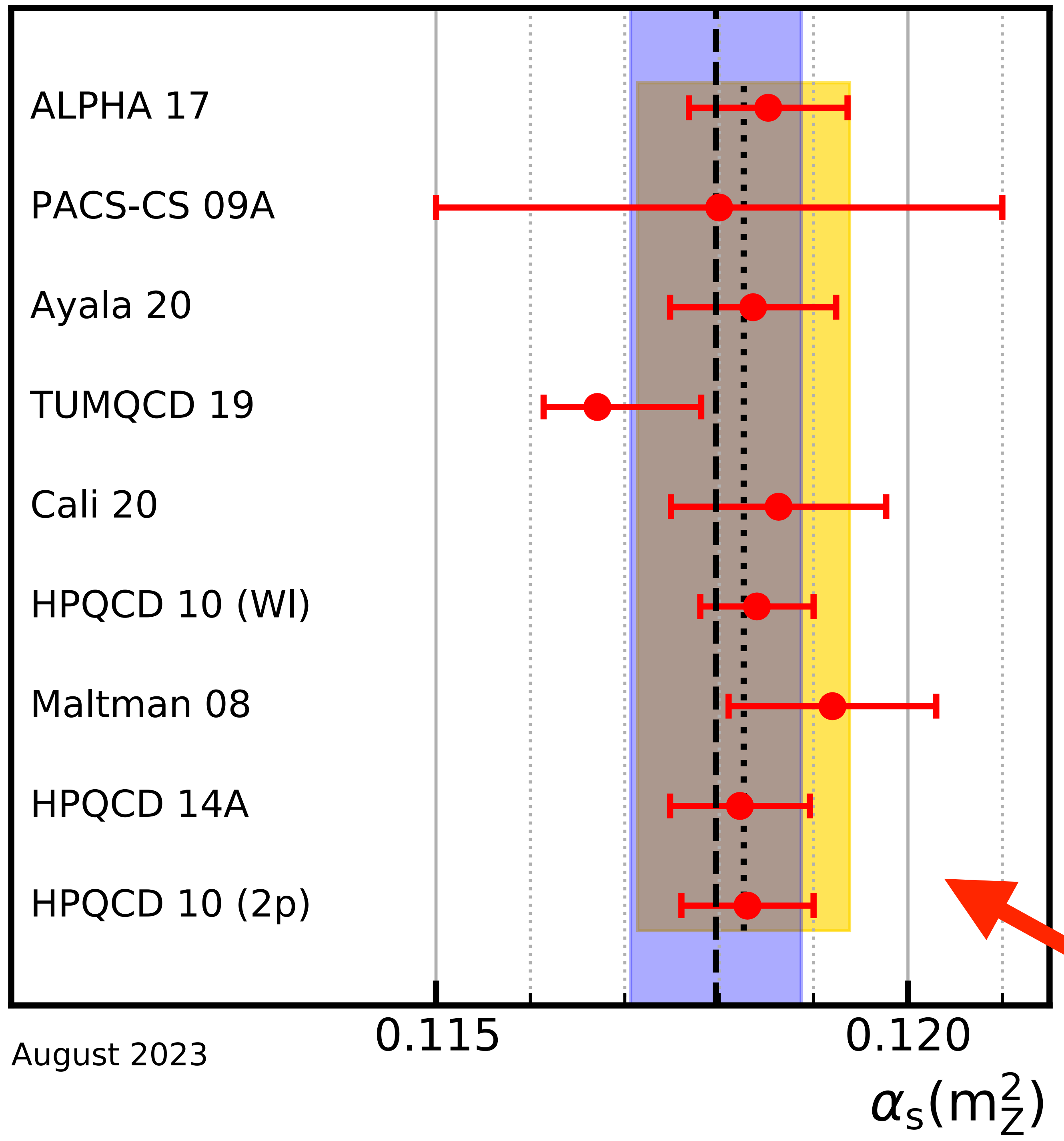
‡ $\beta(\alpha_s)$ currently known to 5 loops!

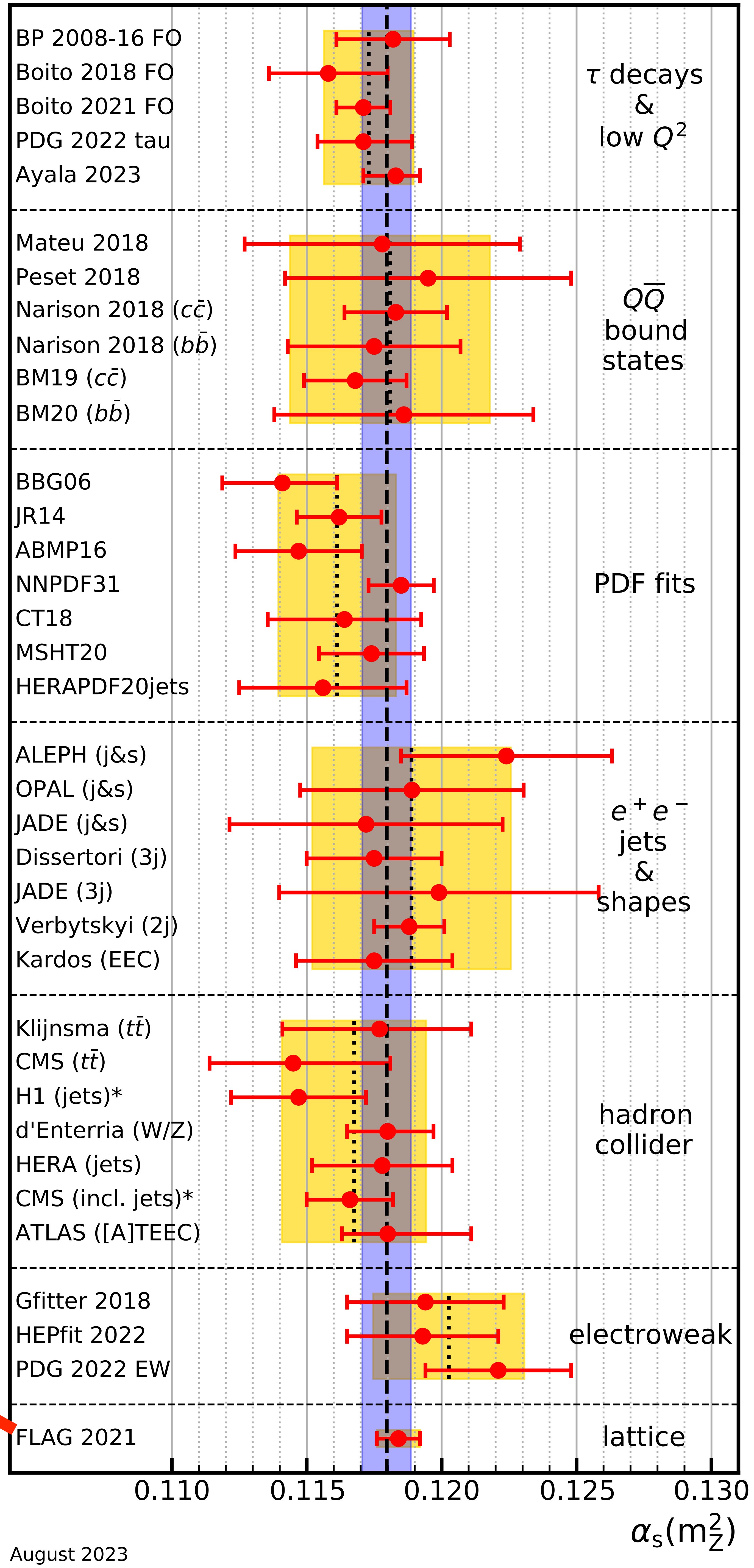
Precise determinations of the strong coupling constant

- As for PDFs, we can predict the evolution between scales but not the absolute value of the coupling, which must be extracted from data
  - Several extractions from different experiments/observables/methods
  - Ongoing debate about uncertainties in many fits (e.g. hadronization corr^{ns})
  - First-principle computation possible with lattice QCD (many observables), uncert. reliably at and below % level
  - Optimistic perspective to reach higher accuracy (few permille) from future lattice extractions and future colliders (e.g. FCC-ee)

2023 World Average:

$$\alpha_s(m_Z) = 0.1180 \pm 0.0009\,(0.76\%)$$





Figures from QCD chapter of Particle Data Book 2312.14015

Ingredients of the master formula: power corrections

- Encode physics at hadronic scale due to either hadronization or dynamics within the protons (e.g. intrinsic transverse momentum, multiple parton scatterings). No general first-principle approach to control them at present (analytic methods in simple cases, otherwise Monte Carlo models)

$$d\sigma_{pp\to X} = \sum_{ij}\int_0^1 dx_1\, dx_2\, f_i(x_1,\mu_F) f_j(x_2,\mu_F) \times d\hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu_R, \mu_F) + \mathcal{O}\left(\frac{\Lambda^p}{m_X^p}\right)$$

➡ Value of parameter p is observable dependent & it is crucial for precision physics programme

e.g. if m_X~100 GeV a very rough estimate suggests

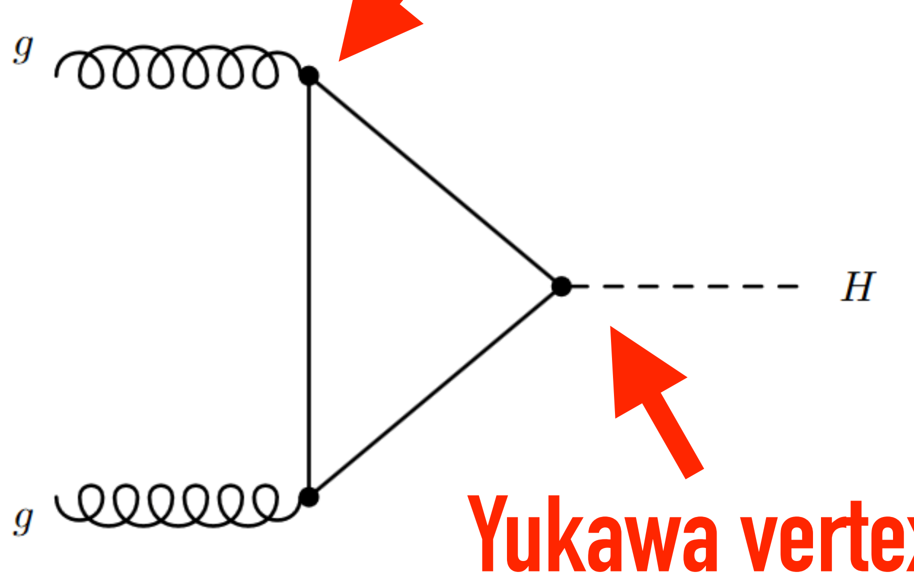
$$\frac{\Lambda}{m_X} \sim \mathcal{O}(1\%)\,, \quad \left(\frac{\Lambda}{m_X}\right)^2 \sim \mathcal{O}(0.1\%)$$

Some examples: p=2 for the Drell-Yan (Higgs) total cross section and related inclusive q_T distribution ☺; p=1 for most jet observables 😑

Let's put all this into practice:
The Higgs total cross section (ggF)

The leading order (LO) cross section (effectively a one loop calculation)

- After averaging over colour & spin states, the partonic XS reads $\hat{\sigma}_0 = A_{gg}\,\delta(1-z)$



$$A_{gg} = \frac{\alpha_s(\mu_R)^2}{\pi}\frac{1}{256v^2}\left|\sum_{q\in\text{loop}}\tau_q(1+(1-\tau_q)f(\tau_q))\right|^2, \quad z = \frac{m_h^2}{\hat{s}}$$

$$f(\tau_q) = -\frac{1}{4}\left(\ln\frac{1+\sqrt{1-\tau_q}}{1-\sqrt{1-\tau_q}} - i\pi\right)^2 \qquad \text{if } \tau_q < 1$$

$$\tau_q = 4m_q^2/m_h^2$$

$$f(\tau_q) = \arcsin^2\left(\sqrt{1/\tau_q}\right) \qquad \text{if } \tau_q \geq 1$$

- Total cross section is simply given by

$$\sigma_0 = \int_0^1 dx_1 dx_2 f_g(x_2,\mu_F) f_g(x_1,\mu_F)\, m_h^2\, A_{gg}\,\delta(\hat{s}-m_h^2) = m_h^2\, A_{gg}\,\mathscr{L}_{gg}\left(\frac{m_h^2}{s}\right)$$

$$\mathscr{L}_{ij}(\tau) = \int_\tau^1 \frac{dx}{x} f_i(x,\mu_F) f_j\left(\frac{\tau}{x},\mu_F\right)$$

$\hat{s} = x_1 x_2 s$

Parton (gluon) luminosity

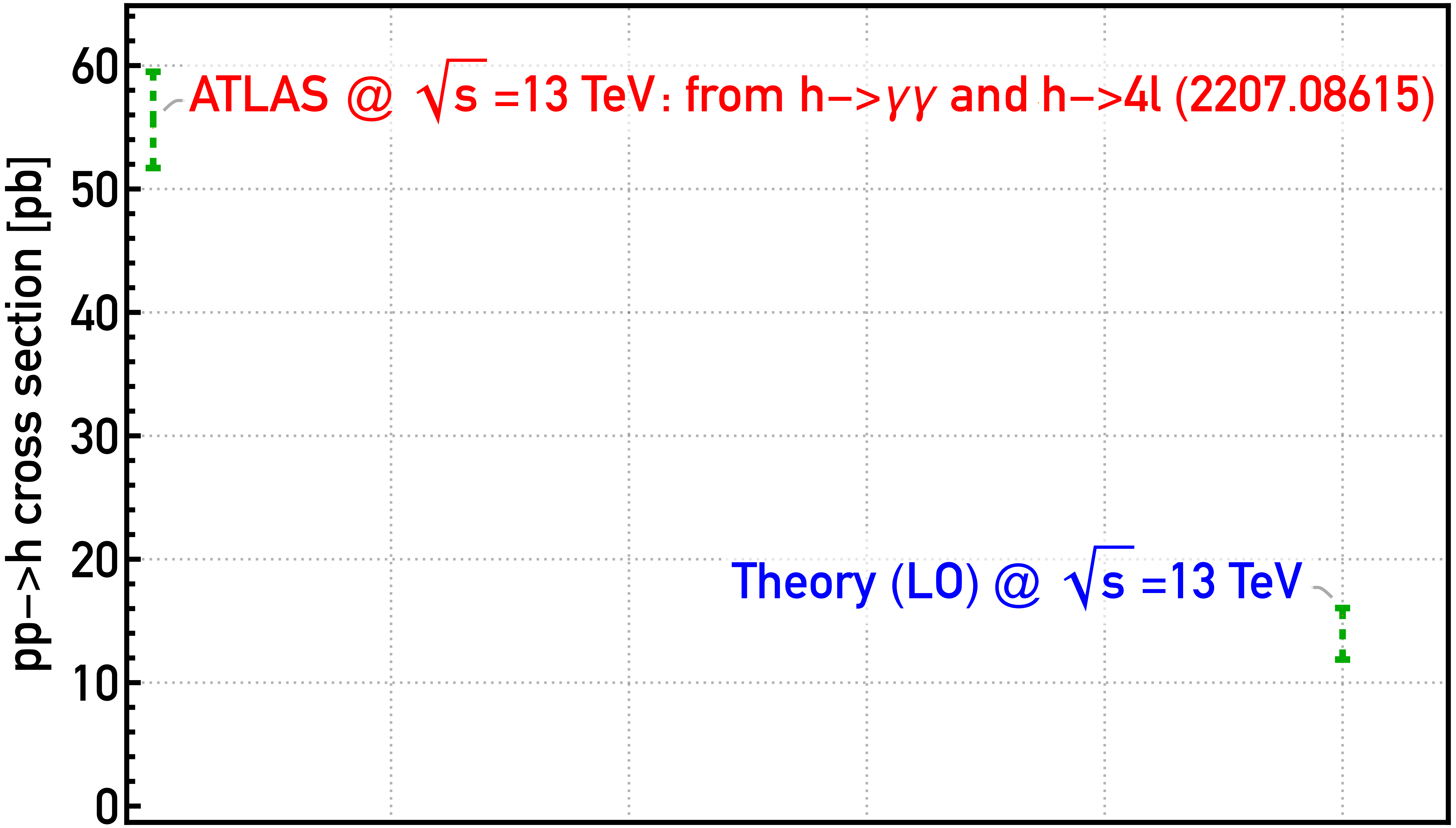
The LO cross section vs. experiment

Predictions here obtained with the ggHiggs public code

- However, comparison to data reveals a large discrepancy. Possible explanations:
  - It may be a sign of new physics!
  - Is the theory prediction sufficiently accurate & reliable? What is the theory uncertainty of our calculation?

Central obstacle in modern collider physics: getting theoretical calculations to be sufficiently accurate & reliable (e.g. in our case it involves quantum corrections to $gg \to h$ & other production modes).

Let's consider next-to-leading-order (NLO) corrections next …



Appendix

The generators of the SU(3) (colour) algebra

- The (traceless and Hermitian) Gell-Mann matrices span the SU(3) Lie algebra $[t^a, t^b] = if^{abc}t^c$, with $t^a_{ij} = \dfrac{\lambda^a_{ij}}{2}$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

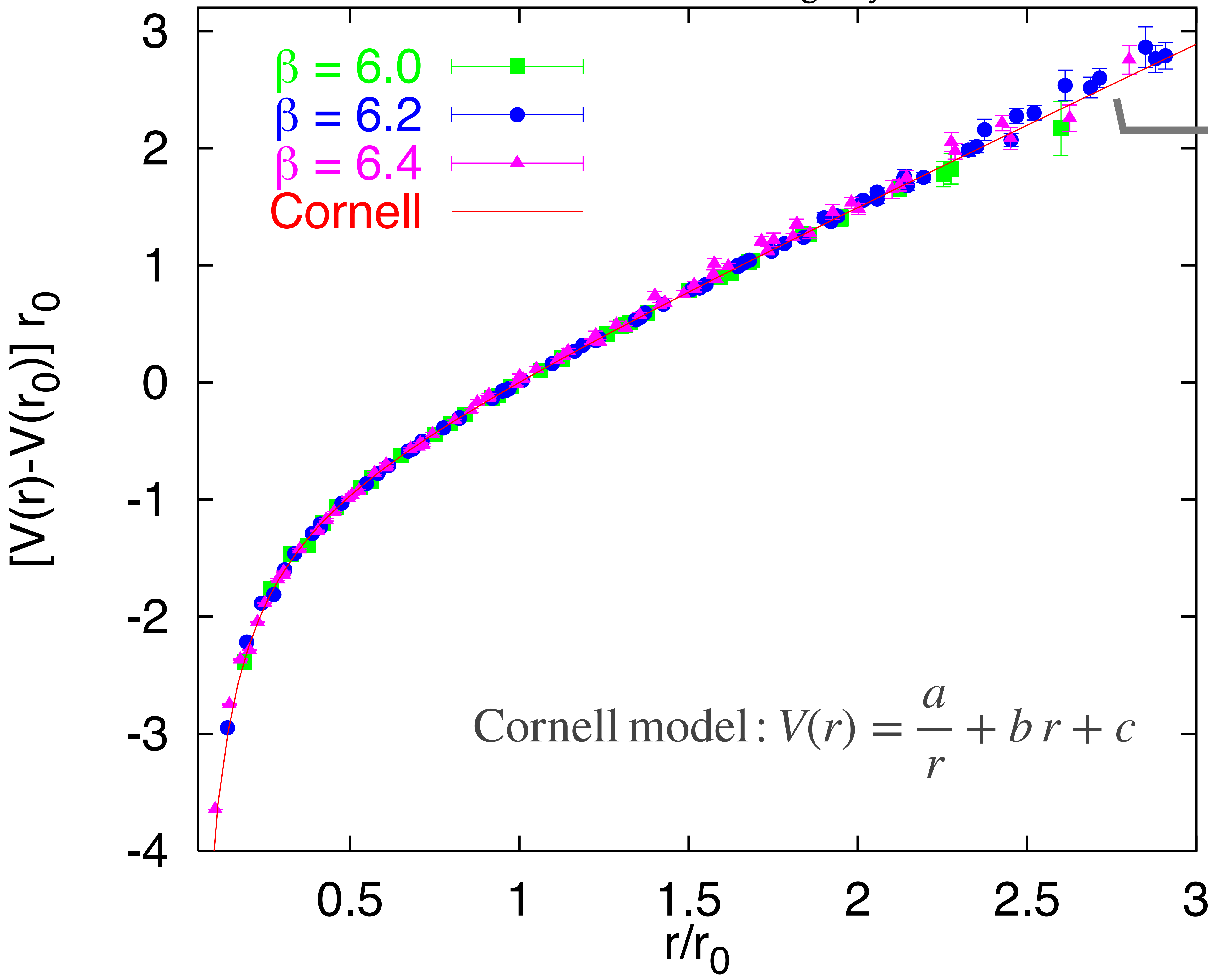
- The non-zero structure constants can be obtained from the commutation relation

$$f^{123} = 1 \qquad f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2} \qquad f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

The static quark-antiquark potential

- Computation of the QCD potential can be carried out with Lattice techniques (Wilson loop)

Figure from G. Bali 0001312



$$\text{Cornell model}: V(r) = \frac{a}{r} + b\,r + c$$

linear attractive behaviour at long distances indicates confinement

Some phenomenological models for non-perturbative QCD (e.g. used in event generators) are based on this paradigm