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Comparison of Magnetic Field Maps by Direct Measurement and Reconstruction Using Boundary Element Methods

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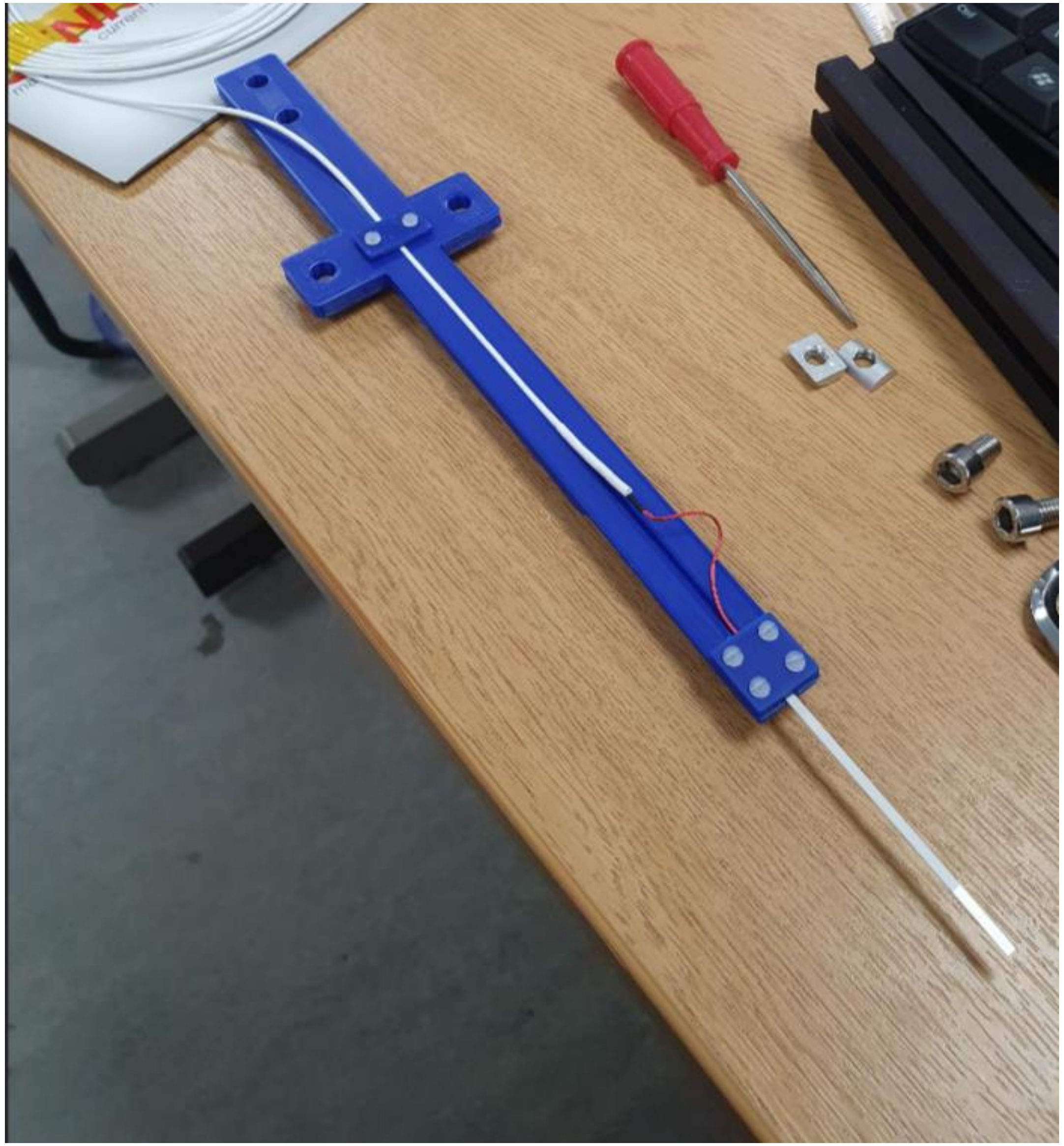
Introduction

- Hall sensor measurements are used to measure the magnetic field vectors at a point.
- **3-axis probes** used to measure all three field components.
- Probe mounted to precision **3-axis motion stage.**
- Can map out full shape and strength of magnetic field with a 3D volume.
- Precise but **slow**!



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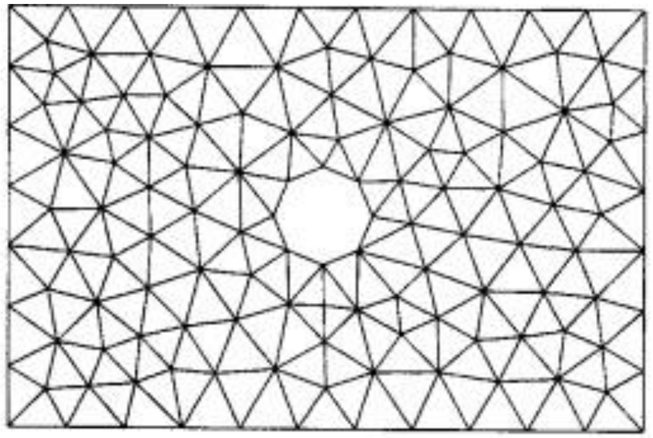
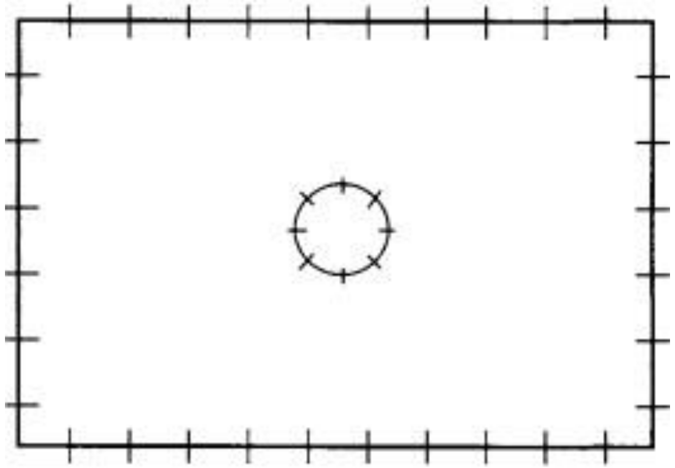
Boundary Element Methods

- Boundary Element Methods (**BEM**) provide an alternative method for determining the 3D magnetic field vectors in a volume.
- Only requires measurement of field vectors on **volume surface**.
- Number of points to measure **scales with square** of volume dimensions for BEM.
- Number of measurements scales with cube of dimensions for direct mapping.
- Significant **time reduction** in measurements for BEM over large volumes.



(FEM Discretization: 228 Elements)



(BEM Discretization: 44 Elements)

Image from Boundary Element Method for Elasticity Problems, 2010, https://api.semanticscholar.org/CorpusID:28633673}

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Theory (Briefly!)

- **Magnetic field** can be expressed in terms of **magnetic scalar potential**.

$$\vec{B}(\vec{r}) = -\mu_0 \nabla \varphi_m$$

- Divergence of magnetic field is 0.

$$\nabla . \vec{B}(\vec{r}) = 0$$

- Magnetic scalar potential is a **solution to Laplace's equation**.

$$\Delta \varphi_m = 0$$

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Representation Formula

- Magnetic scalar potential at any point in a domain Ω can be evaluated from the **representation formula for the Laplace equation**:

Single-layer potential

Double-layer potential

$$\varphi_m(\vec{r}) = \left(\tilde{V}\partial_{\vec{n}}\varphi_m\right)(\vec{r}) - (W\varphi_m)(\vec{r})$$

The scalar potential inside the domain Ω.

Derivative of the scalar potential normal to the domain boundary – the **Neumann data.**

The scalar potential on the domain boundary – the **Dirichlet data.**

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The Neumann Data

- The **Neumann data** is given by the derivative of the scalar potential normal to the domain boundary.

$$g_N = \partial_{\vec{n}} \varphi_m$$

- Magnetic field related to derivative of scalar potential.

$$\vec{B} = -\mu_0 \nabla \varphi_m$$

$$B_n = -\mu_0 \partial_{\vec{n}} \varphi_m$$

Measure with 3-axis Hall sensor!

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The Dirichlet Data

- The **Dirichlet data** can be evaluated from the known Neumann data using a Neumann to Dirichlet map:

$$Du = (\left(\frac{1}{2}\right) ID - K')g_N$$

Hypersingular potential operator.

Identity potential operator.

Adjoint double layer potential operator.

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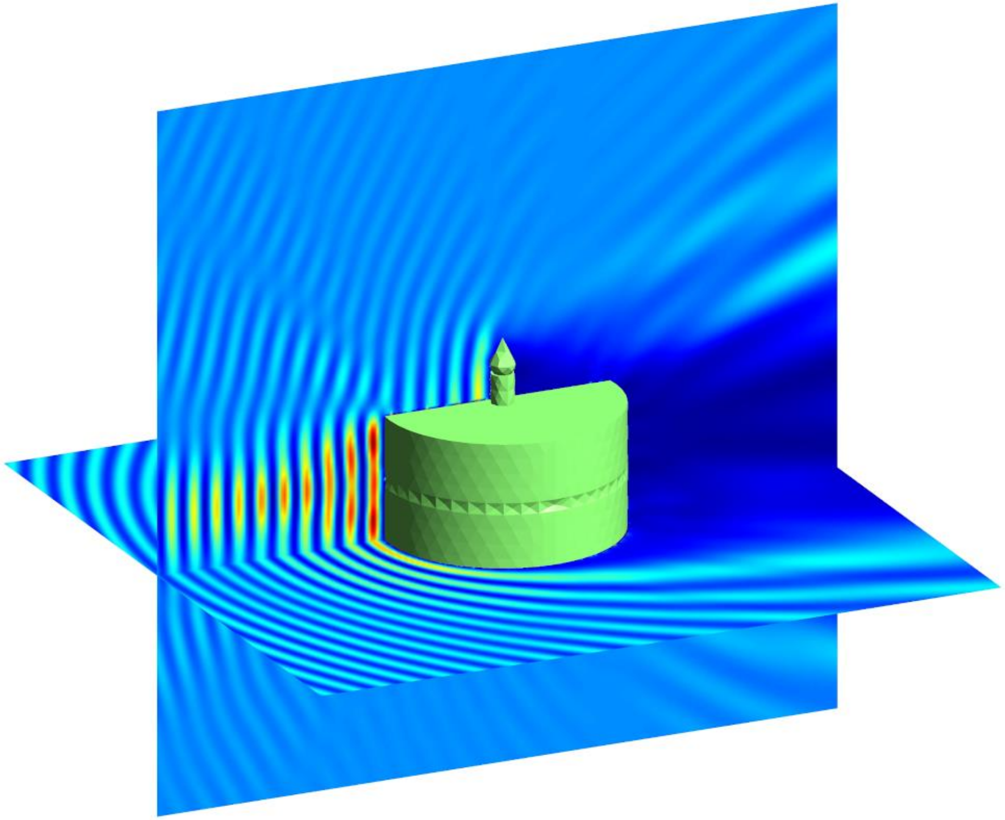
Bempp*

- An open-source Python package for solving boundary element problems.
- Can be used to solve electrostatic, acoustic and electromagnetic problems.
- Pre-built definitions of required potentials and operators to solve the boundary integral equations.
- Can be used to solve Laplace problems with Neumann boundary conditions.



* T. Betcke & M. W. Scroggs. Bempp-cl: A fast Python based just-in-time compiling boundary element library, *Journal of Open Source Software* 6(59) (2021) 2879. [doi.org/10.21105/joss.02879]

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Daresbury Magnet Laboratory

- 3 axis motion controller mounted on synthetic granite bench.
- Mclennan PM1000 motor controllers and absolute encoders < 5µm precision.
- Senis 3MH6 Teslameter with type C tri-axial Hall sensor.
- DC accuracy < 0.01 %.
- DC resolution < 1 ppm.
- 1000 readings averaged per point, 1 kHz sample rate.

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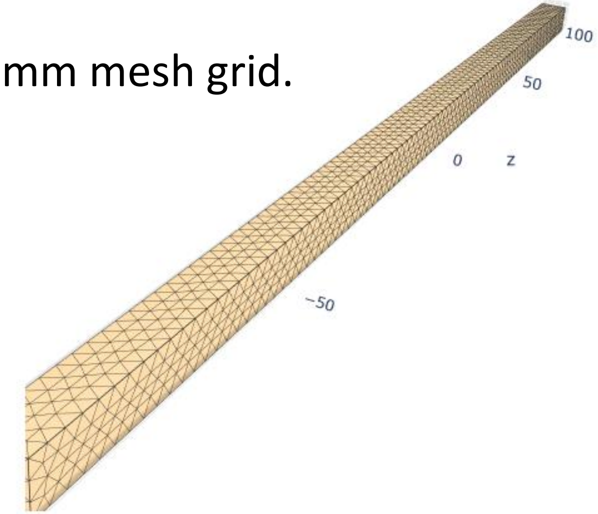
ZEPTO-DLS Quadrupole

- ZEro Power Tuneable Optics.
- Tuneable permanent magnet quadrupole built and installed on Diamond Light Source.
- Measurements were performed before install using Senis type C Hall sensor and 3MH6 teslameter.
- Including 3D field map at high gradient.
- Can BEM be used to reconstruct fields inside the measurement volume?

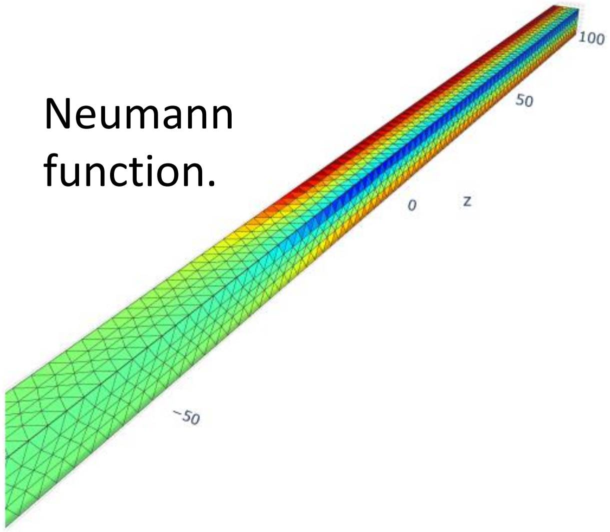
3D Grid

2 mm mesh grid.

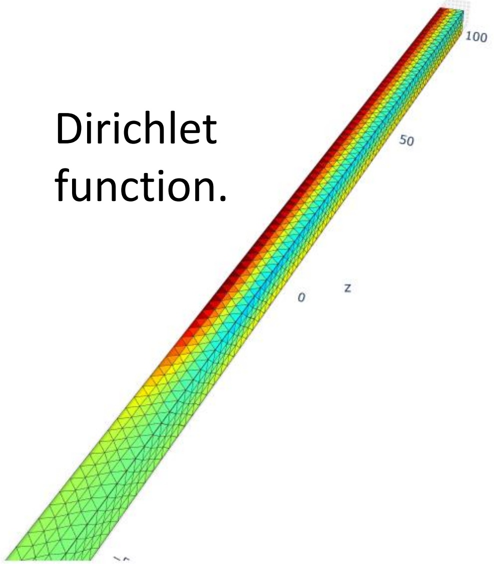


- Measurement over 6x6x190 mm^3 volume.
- From outside magnet, into part way through bore.
- Cubic radial basis function used to interpolate fields on nodes.
- 1 mm step size in x, y directions.
- 5 mm step size in z direction.
- 1911 points total.
- 738 points on boundary.
- ~ 1 hour to measure volume.
- ~ 25 minutes estimated to measure boundary only.

Neumann function.



Dirichlet function.



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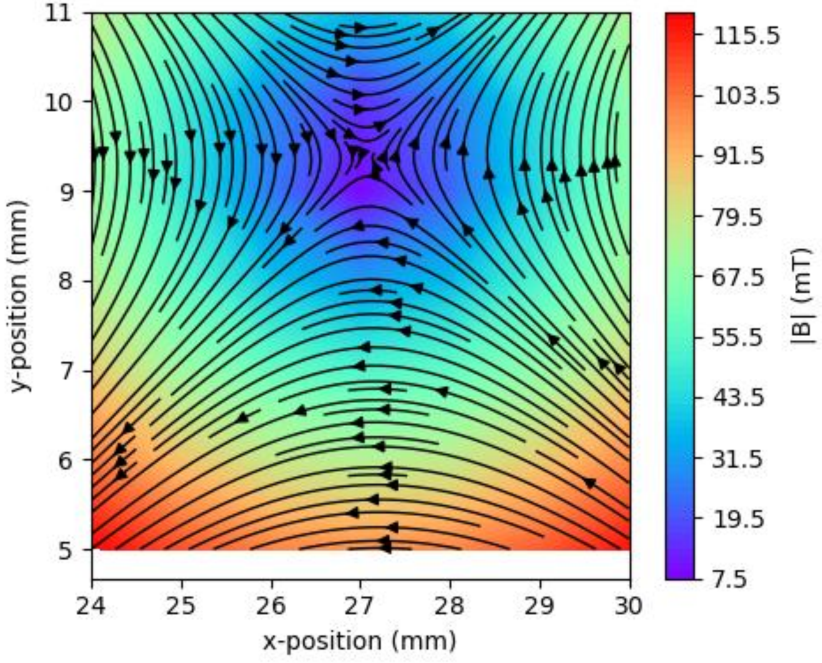
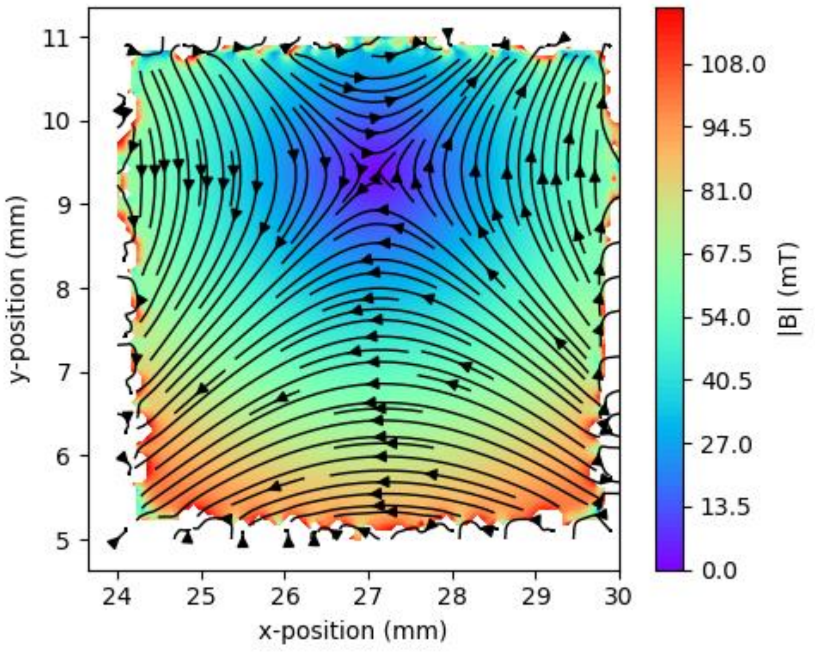
Fields in a Plane

Z axis coordinate = 80 mm

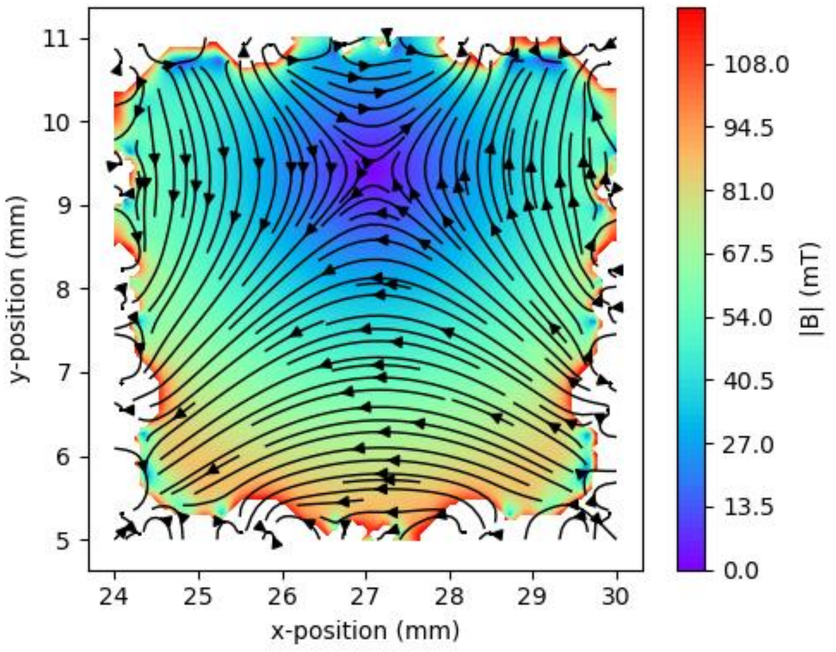
Direct Measurement



BEM – 1 mm mesh



BEM – 2 mm mesh



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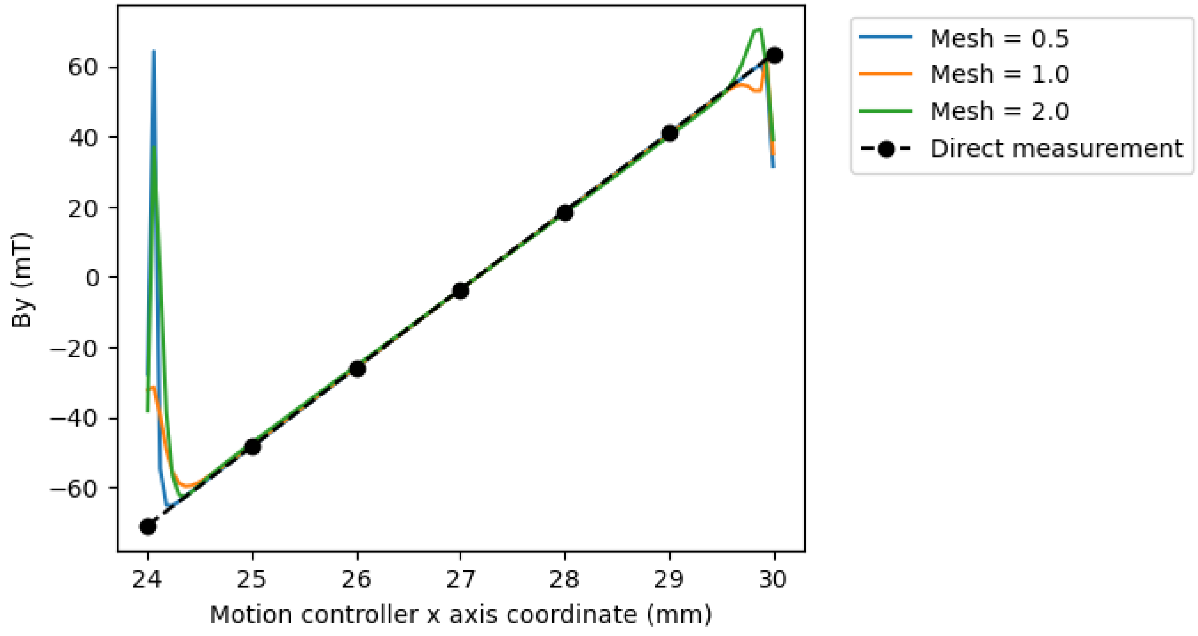
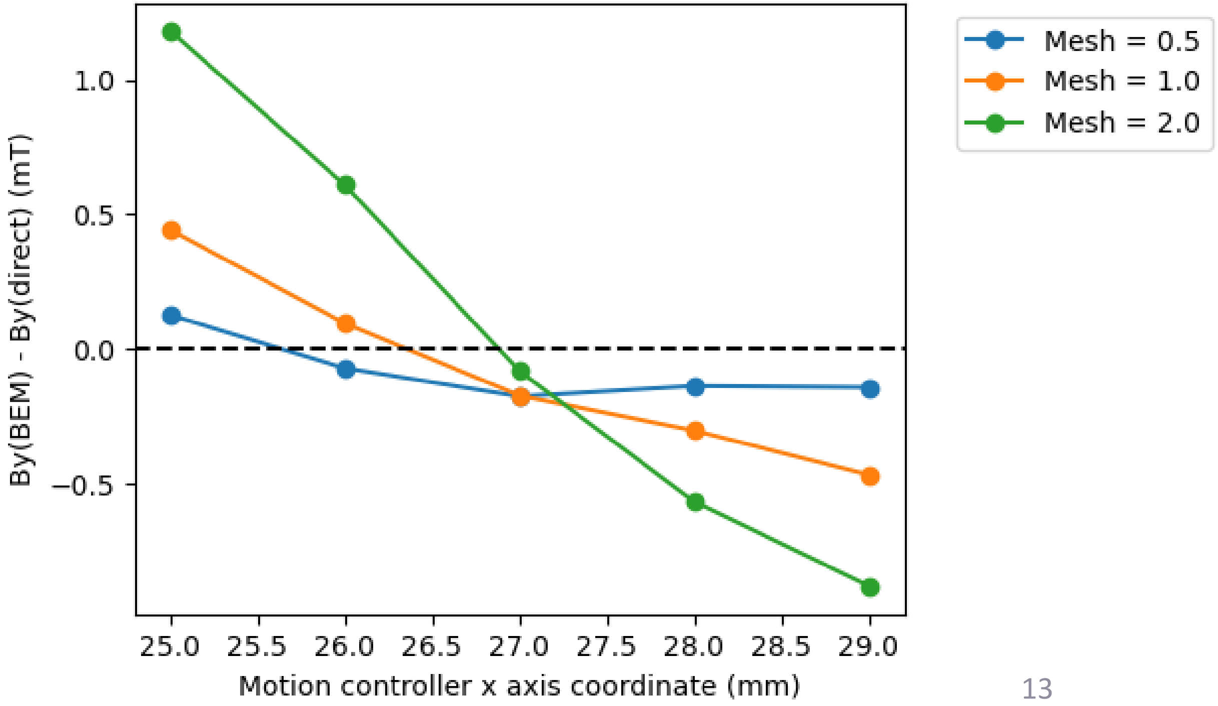
Fields on a line

- Plot of By field component vs motion controller *x* axis.
- *y* = 8 mm, *z* = 65 mm.
- Good agreement between BEM and directly measured fields within one measurement step size of boundary.
- Smaller mesh size = smaller differences between BEM and direct fields.
- Smaller differences between BEM and direct fields near centre of measurement range.





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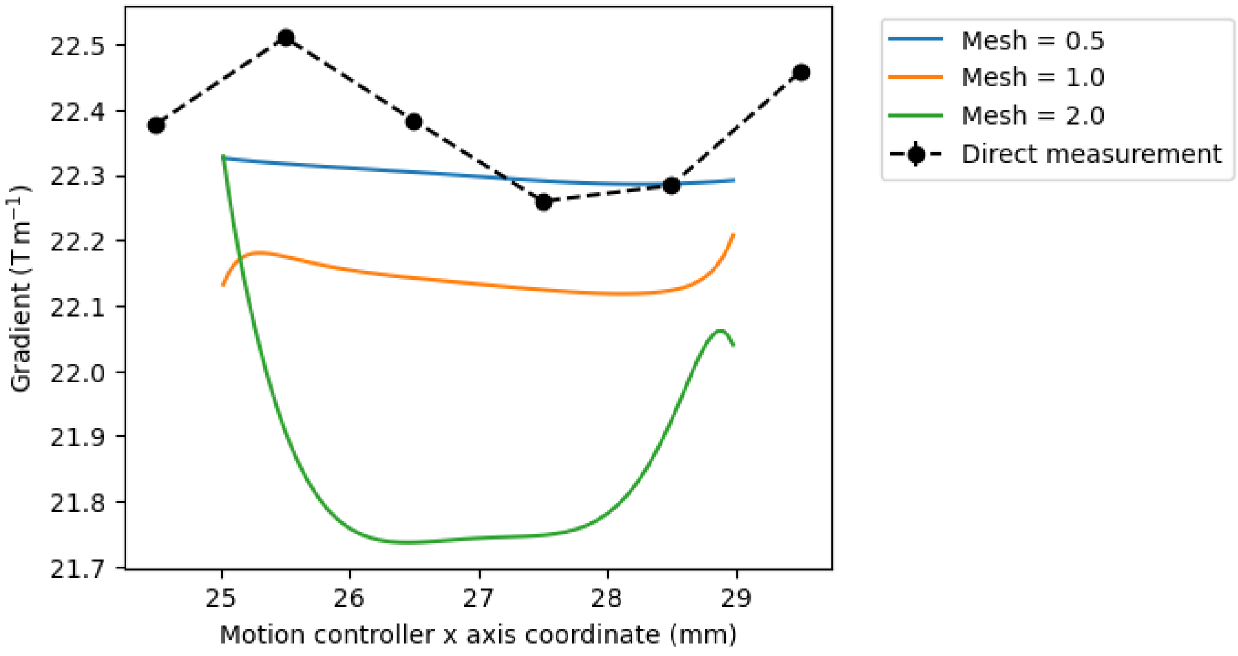
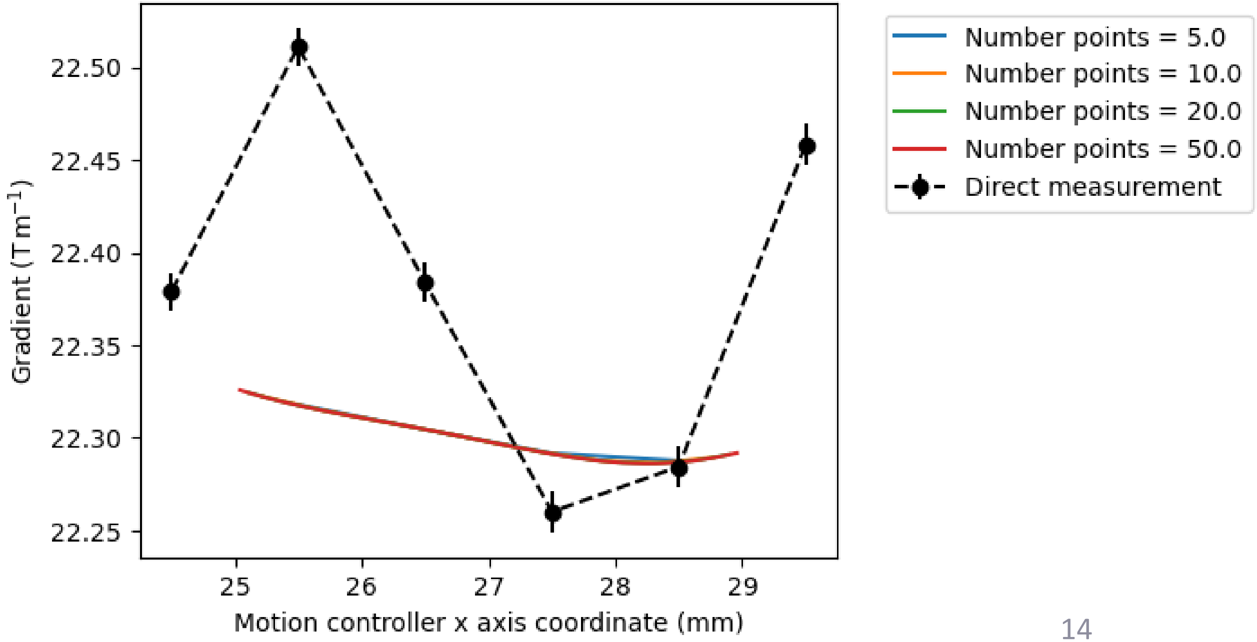
Field gradient

- Magnitude of calculated gradient increases with reducing mesh size.
- Gradients calculated using BEM vary more smoothly than gradients calculated by numerical differentiation of point measurements.
- Gradients diverge for measurements within one mesh size of boundary.
- Gradient independent of discretization along axis.





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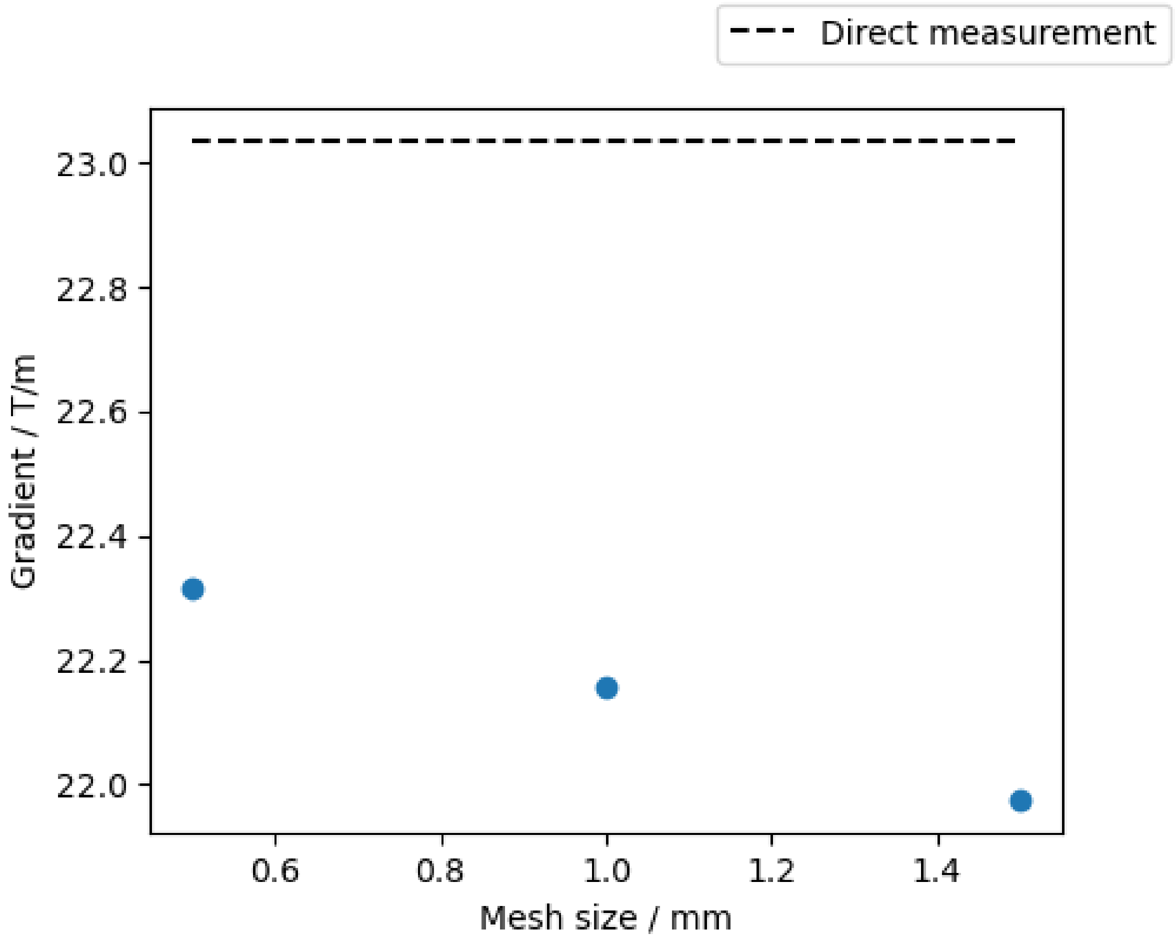
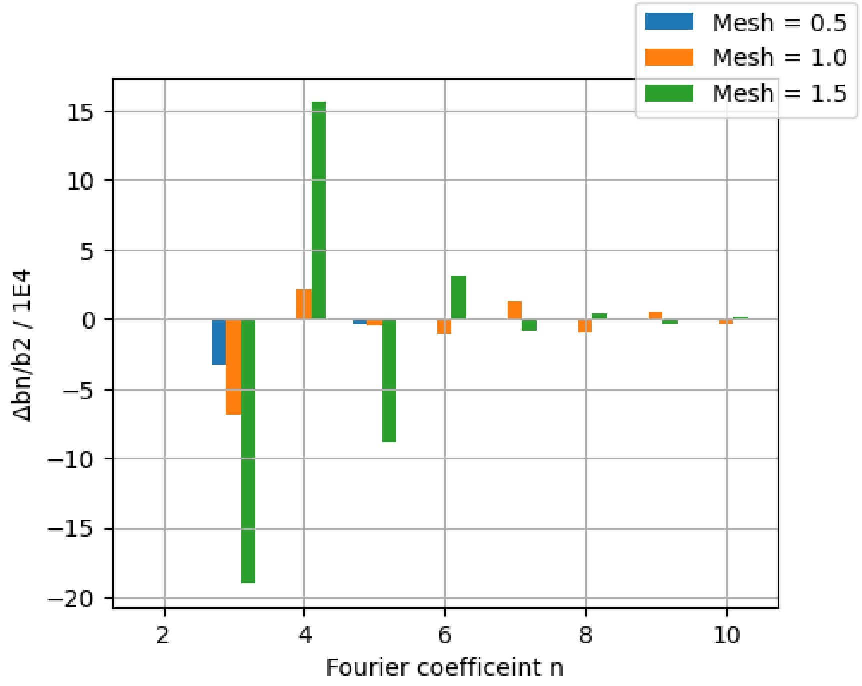
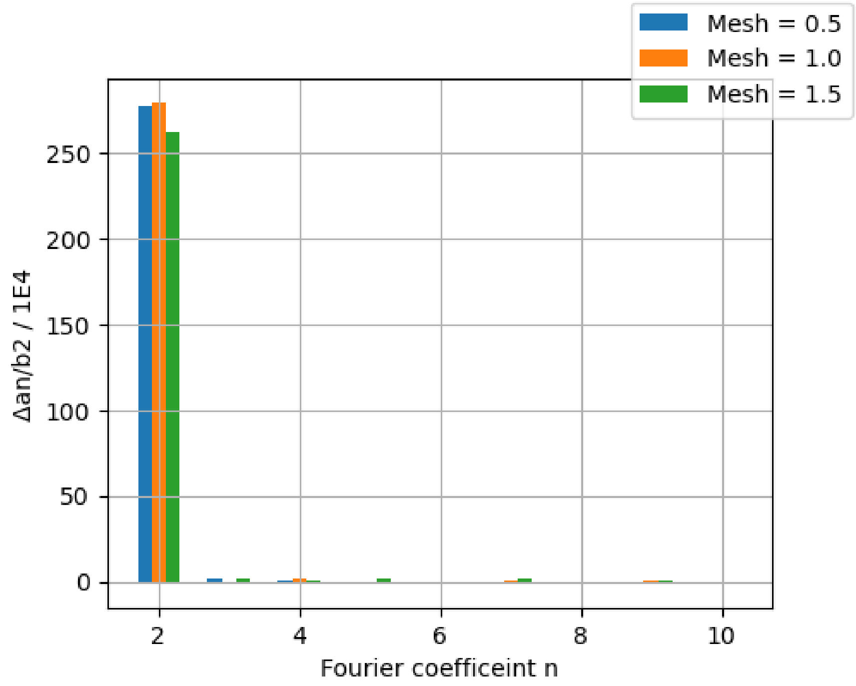
Multipoles

- Multipoles evaluated on 1 mm radius in centre of measurement volume.

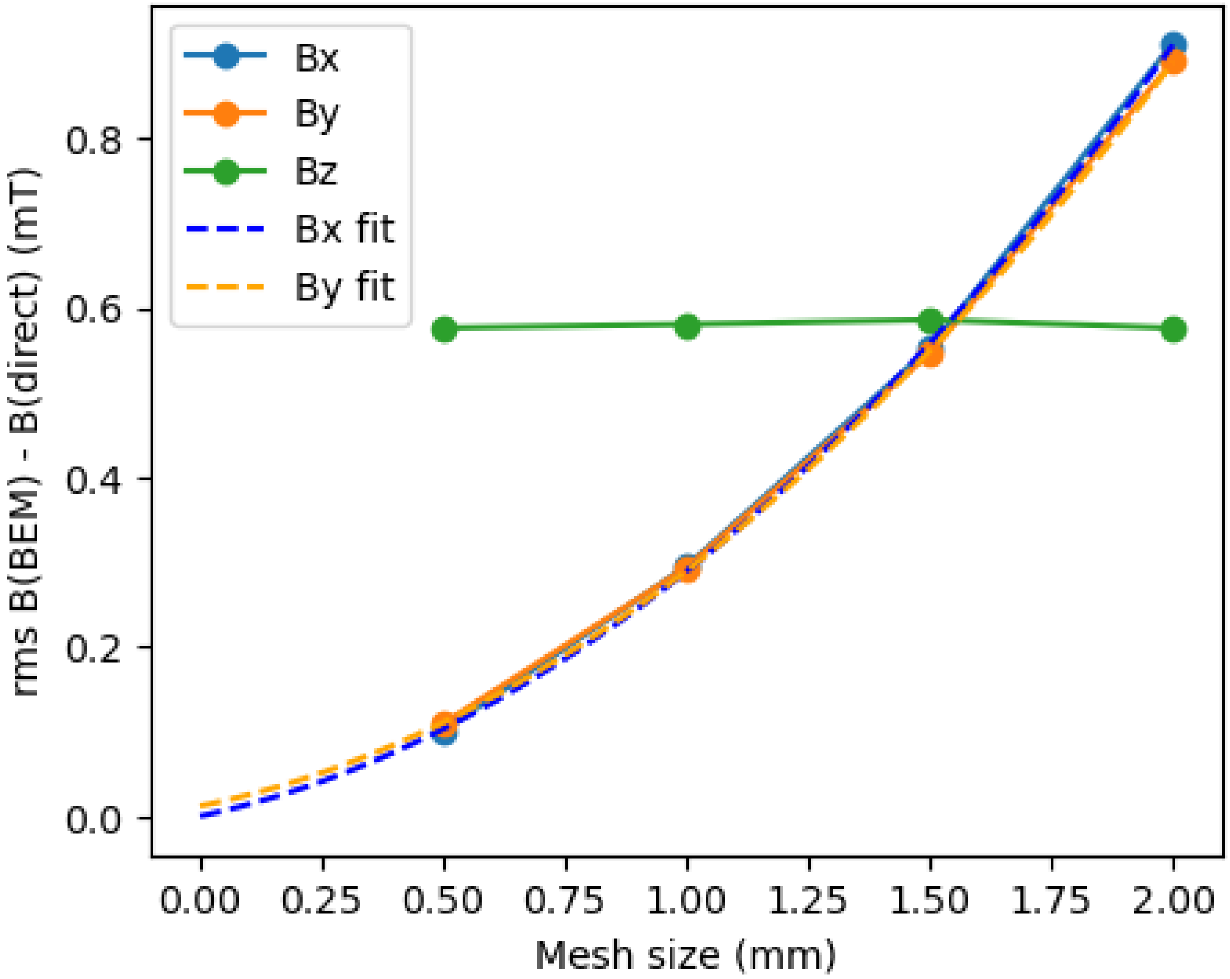






rms Field Error

- Root mean square (rms) difference between directly measured and BEM field components plotted as function of mesh size within one measurement step size of boundary (925 points).
- Predicted field rms field error with ~0 mesh size (quadratic fit):
  - Bx = 5E-4 mT
  - By = 1E-2 mT
- Uncertainty on direct measurements: ± 6E-3 mT.



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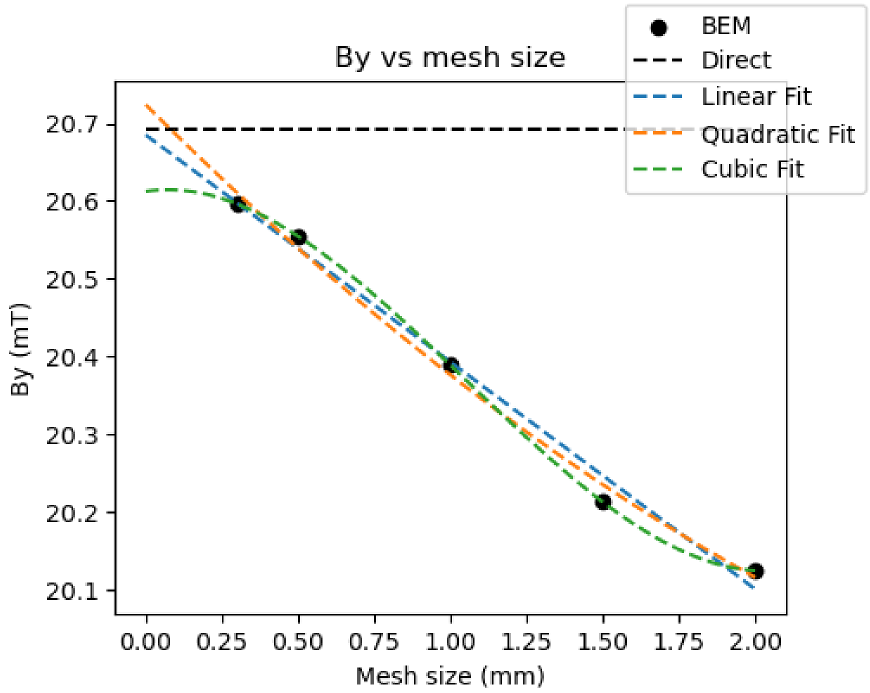
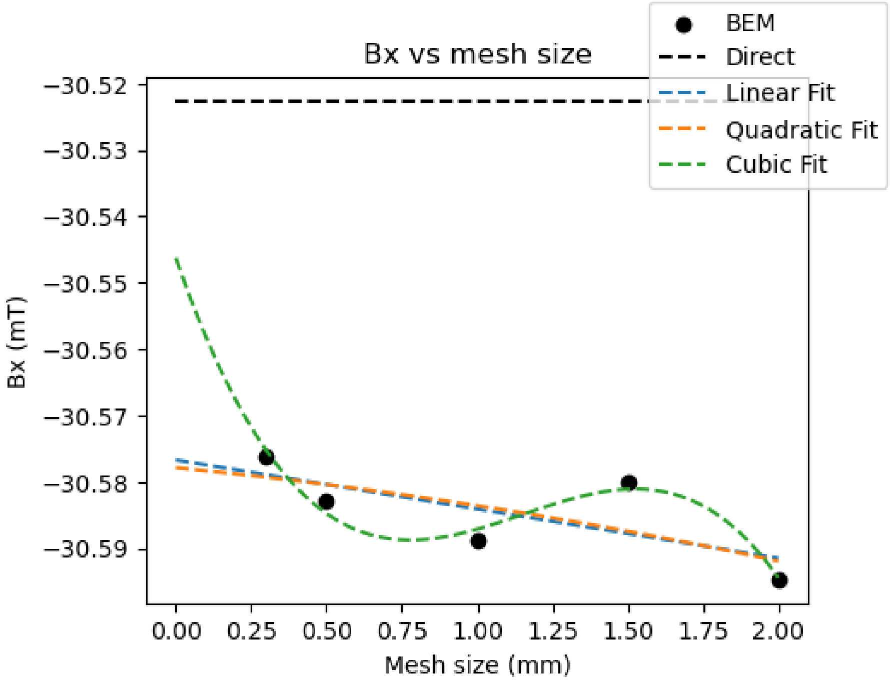
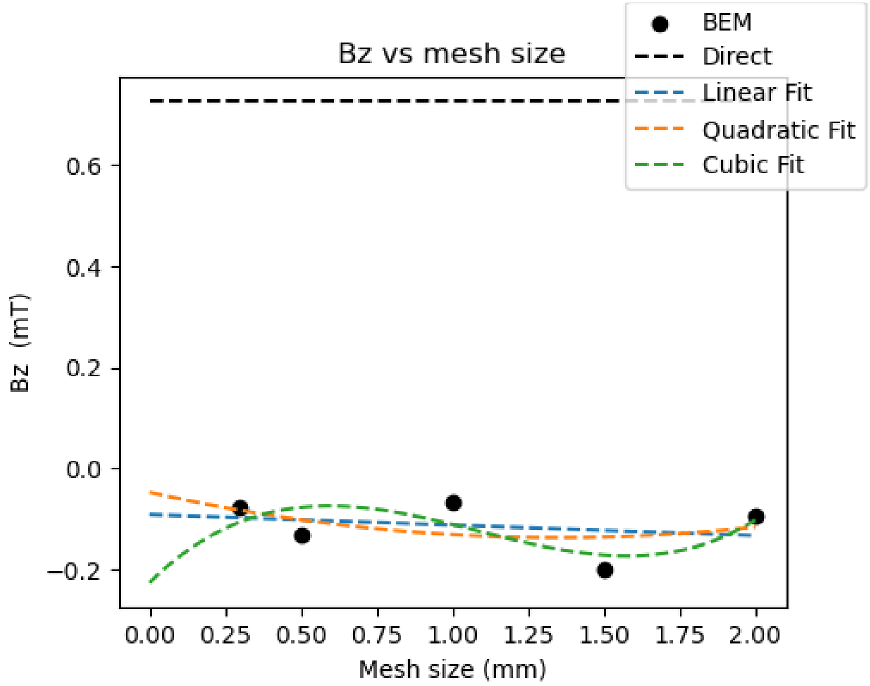
Extrapolate field at a point

- Fit field components to mesh size.
- Extrapolate to ~0 mesh size.
- Can difference between BEM and directly measured fields be reduced?
- Sample coordinate (x,y,z) = (28, 8, 65).







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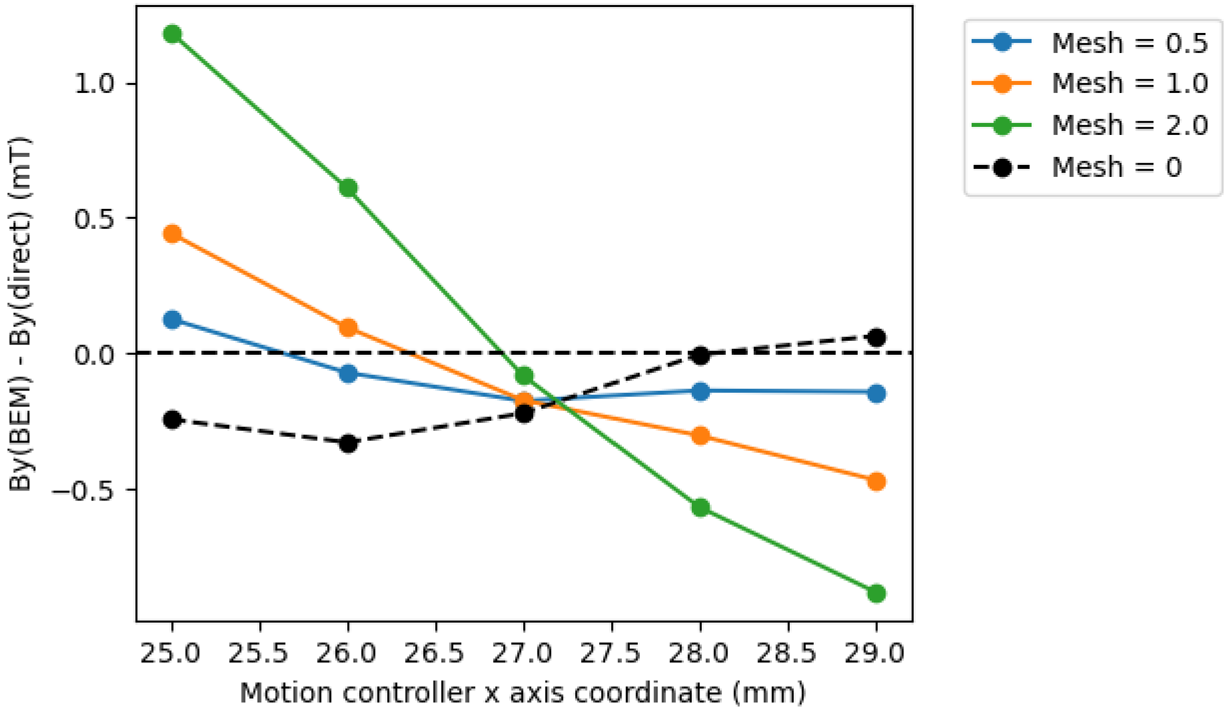
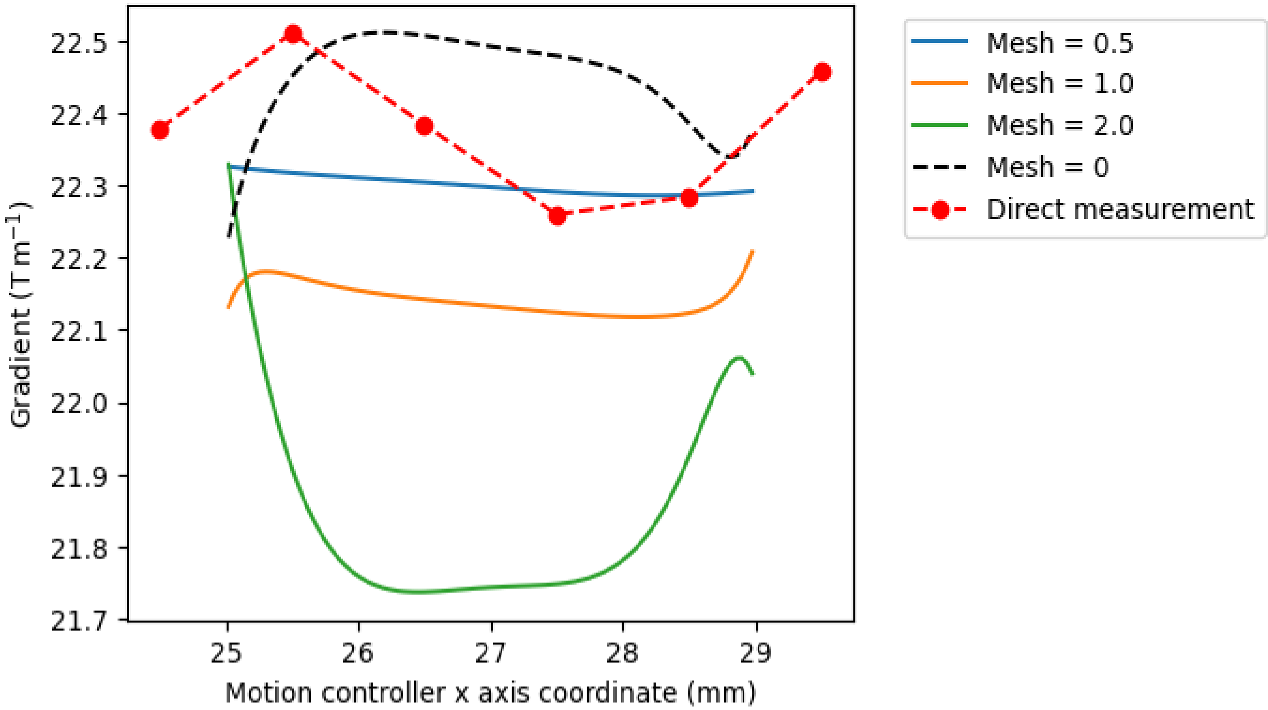
Extrapolate fields on a line

- Linear fit to 0 mesh size.





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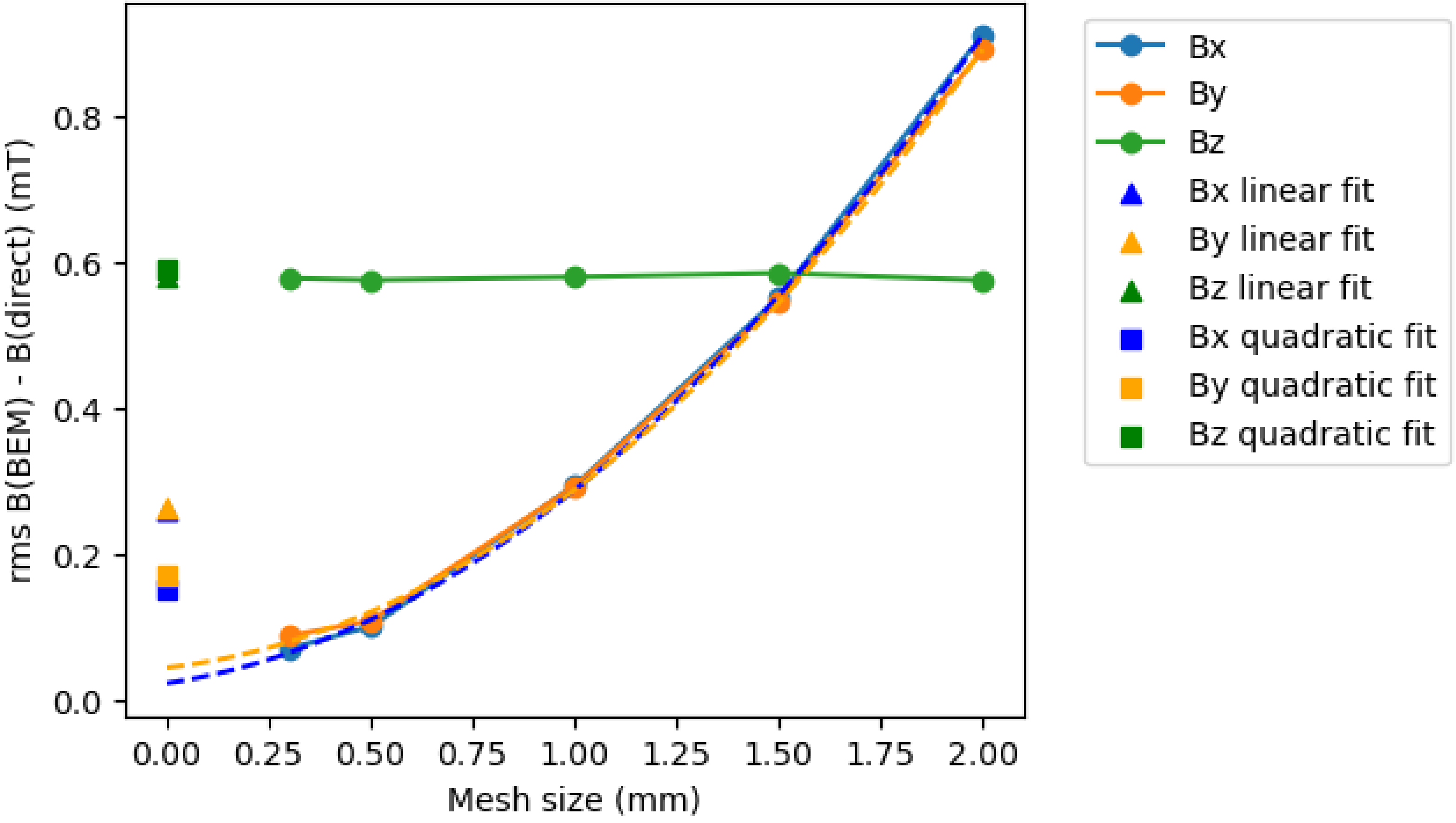
rms Field Error

- Root mean square (rms) difference between directly measured and BEM field components plotted as function of mesh size within one measurement step size of boundary (925 points).
- At each point, field extrapolated to 0 mesh.



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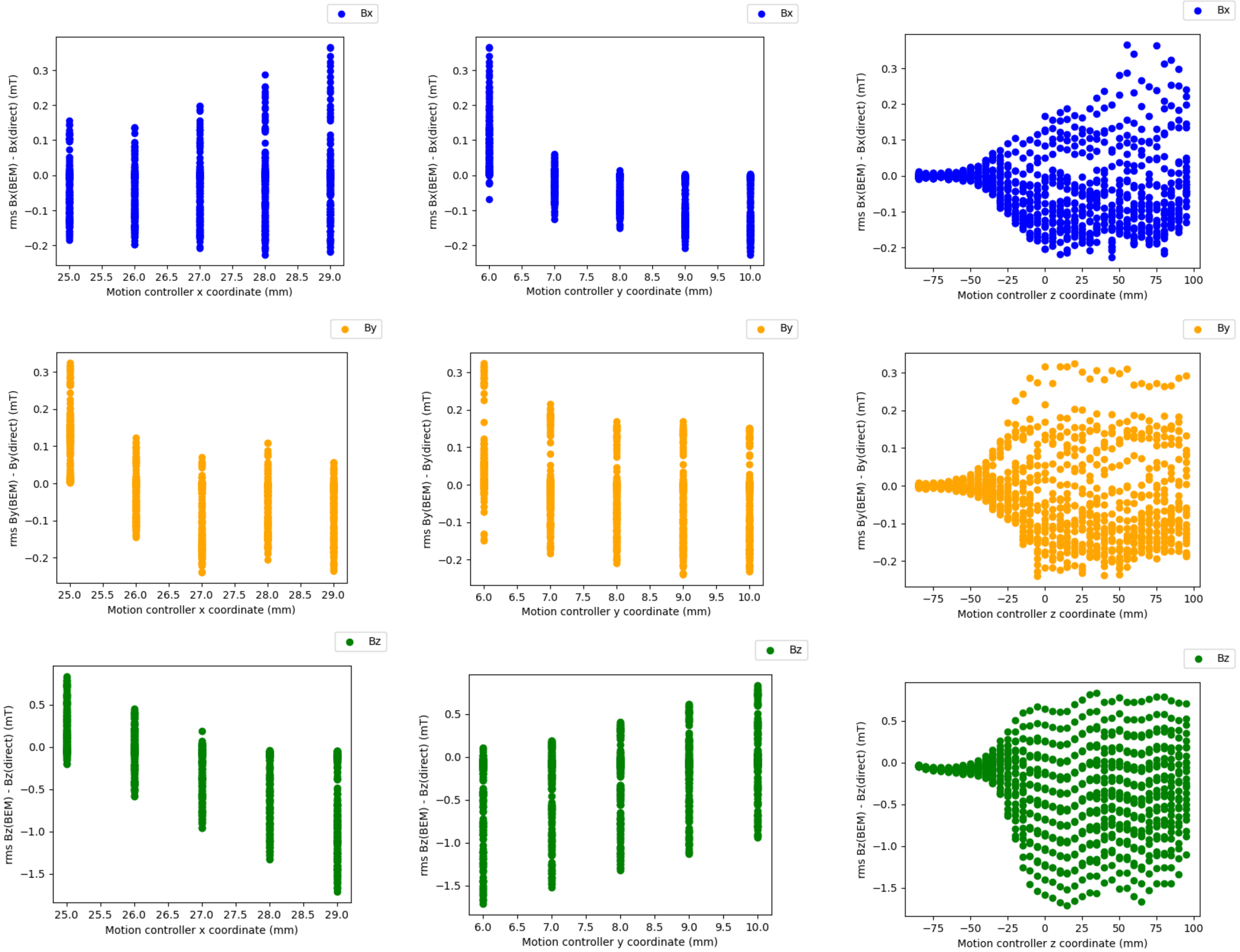
- Differences between BEM and direct fields as function of position.
- Can some fitting function be applied to correct for differences?



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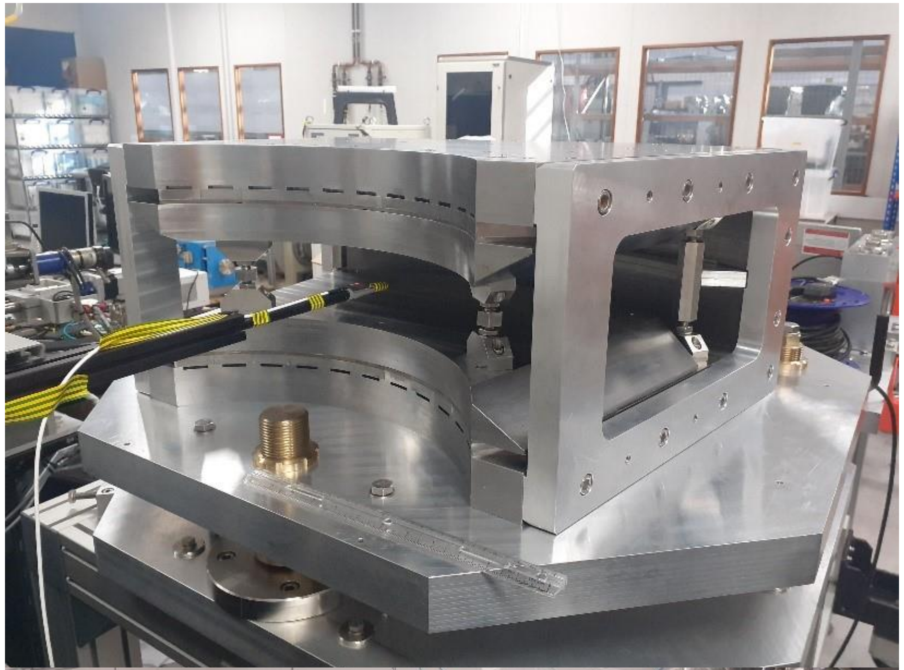
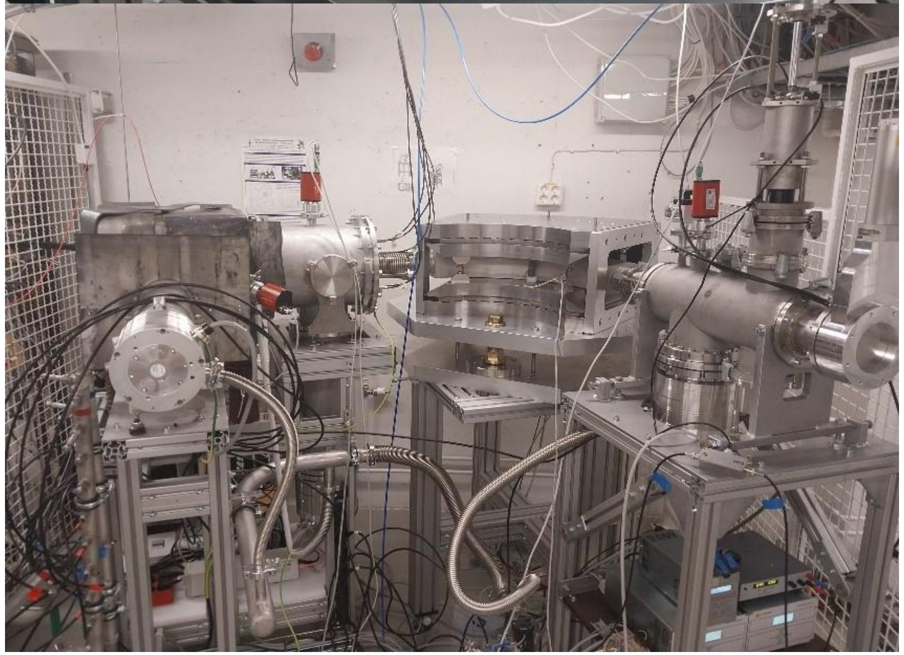
ECRIS Dipole

- Tuneable permanent magnet dipole designed for use on an Electron Cyclotron Resonance Ion Source (ECRIS).
- Tuneable strength for charge state selection.
- Novel carbon free accelerator for Ion Beam Analysis.
- Built and measured at Daresbury Laboratory.
- Magnet integrated into low energy beam line of a prototype PM ion source at University of Jyväskylä.
- First experiments in May 2024 with argon ion beams showed production of IBA-relevant charge states and beam currents.





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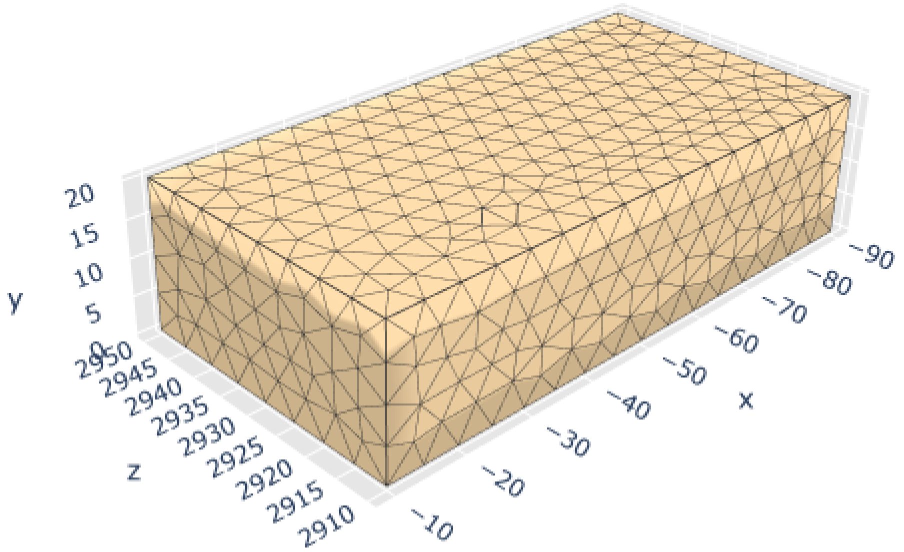
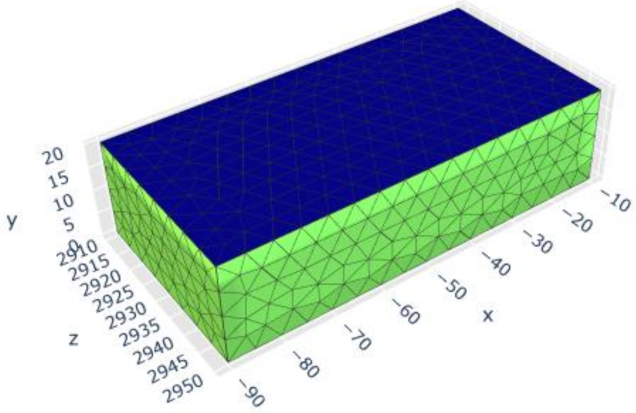
3D Grid

- Measurement over 80x20x40 mm^3 volume.
- Measurements inside magnet.
- 2 mm step size in x, y, z directions.
- 9471 points total.
- 2802 points on boundary.
- ~ 5.3 hours to measure volume.
- ~ 1.6 hours estimated to measure boundary only.

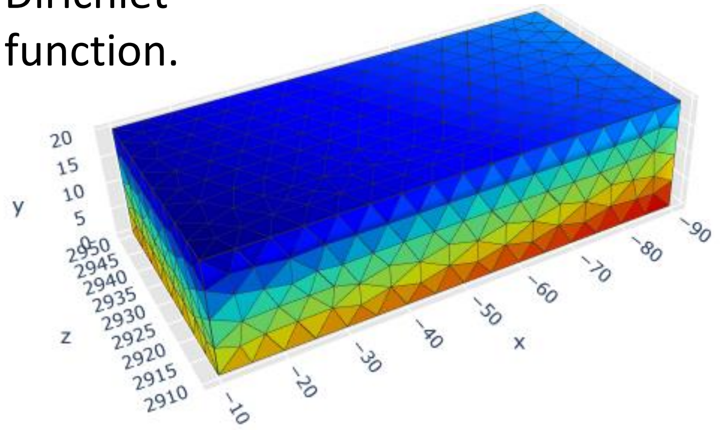
5 mm mesh grid.



Neumann function.

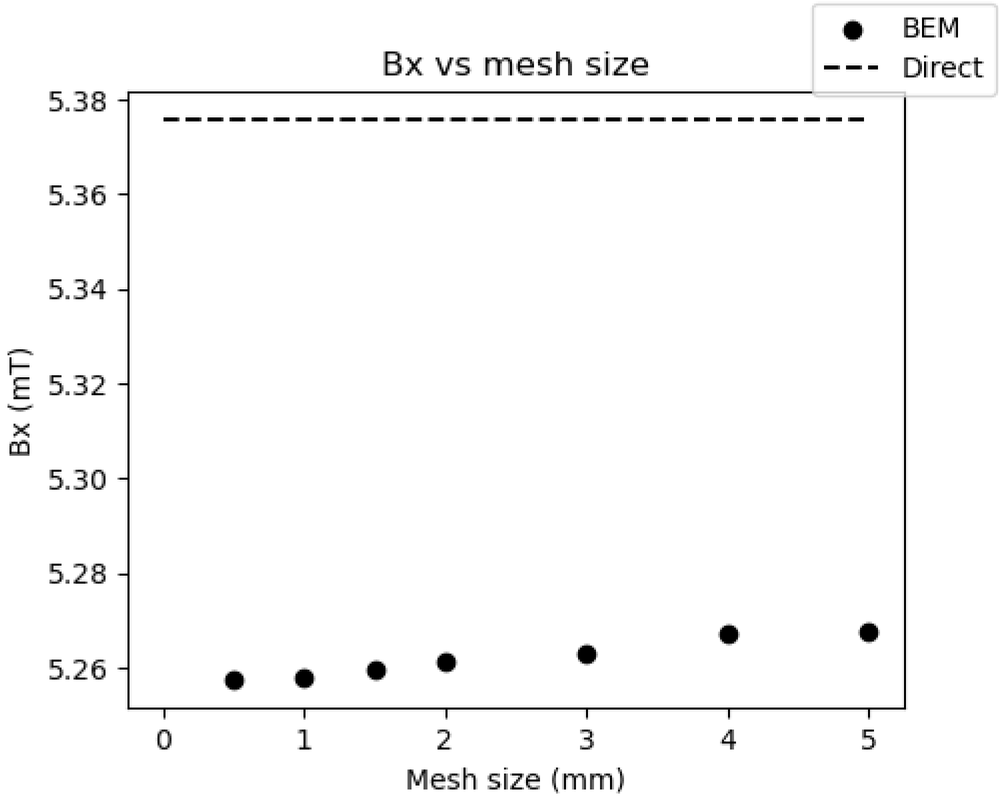


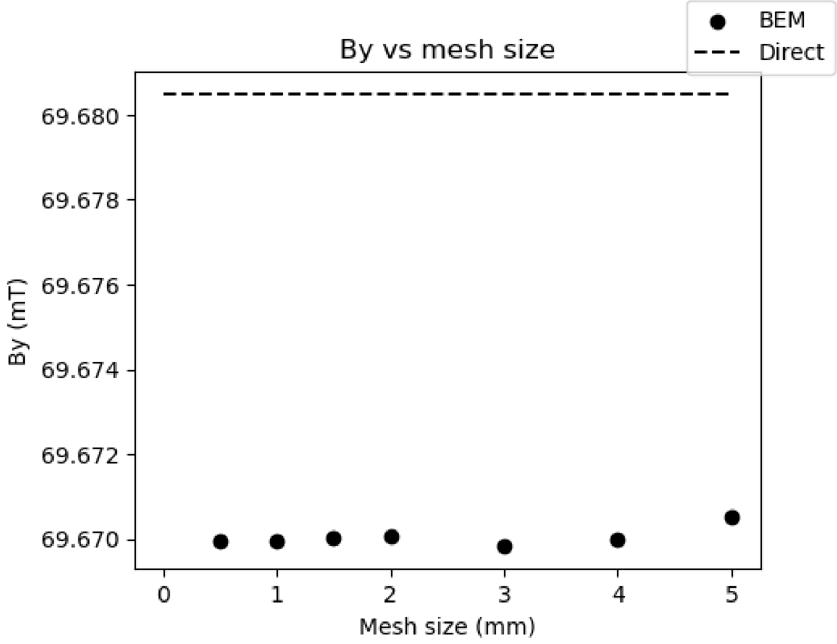
Dirichlet function.

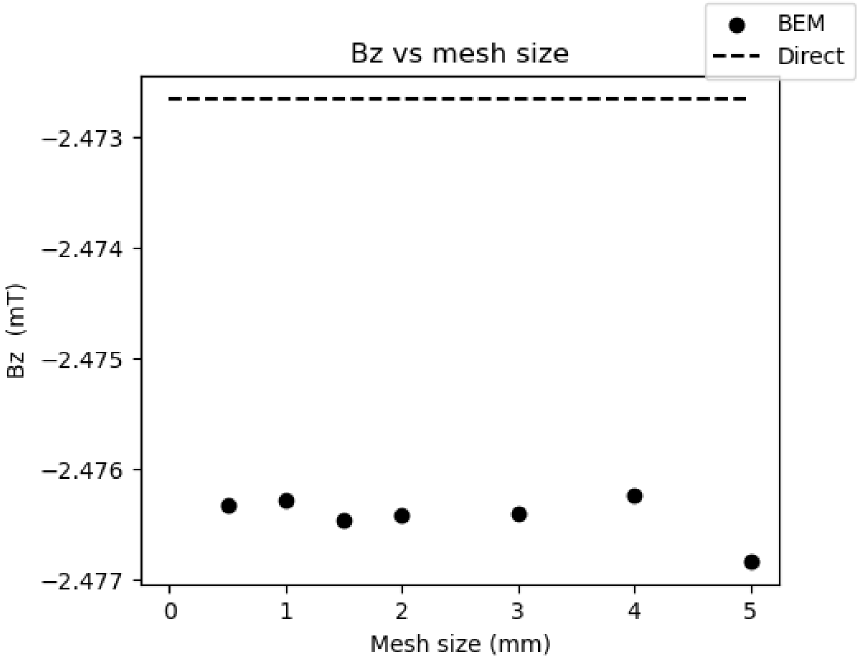


Fields at a point

- (x, y, z) = (-50, 10, 2940)







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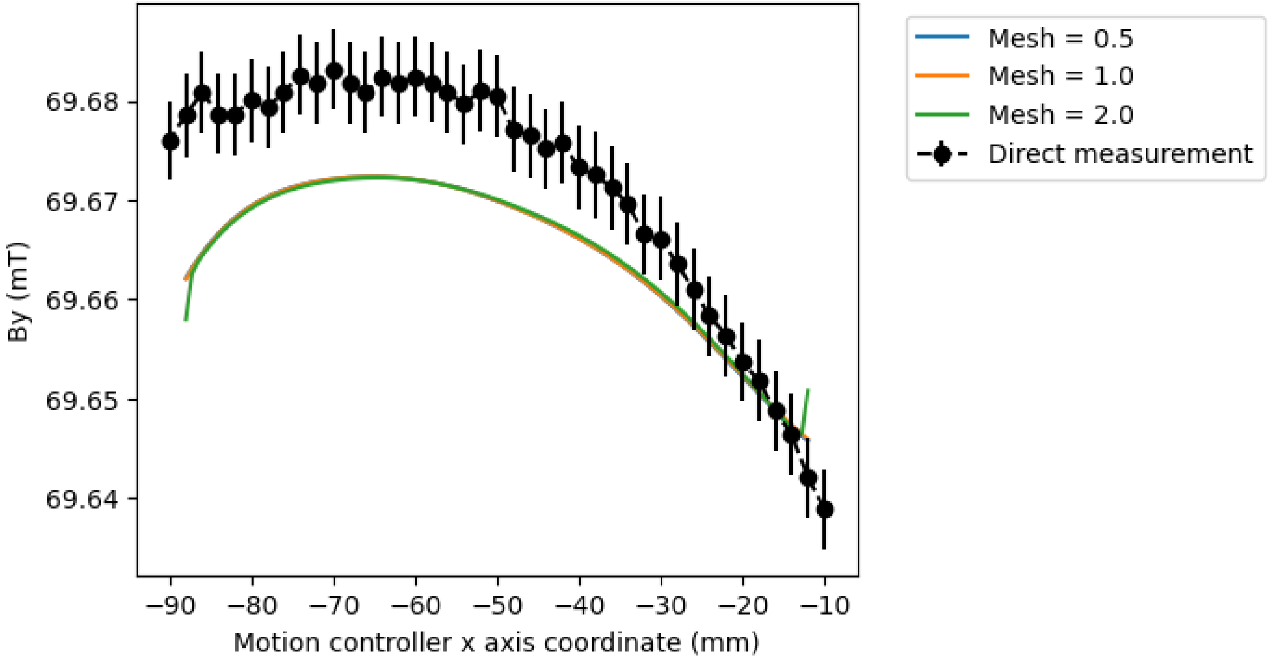
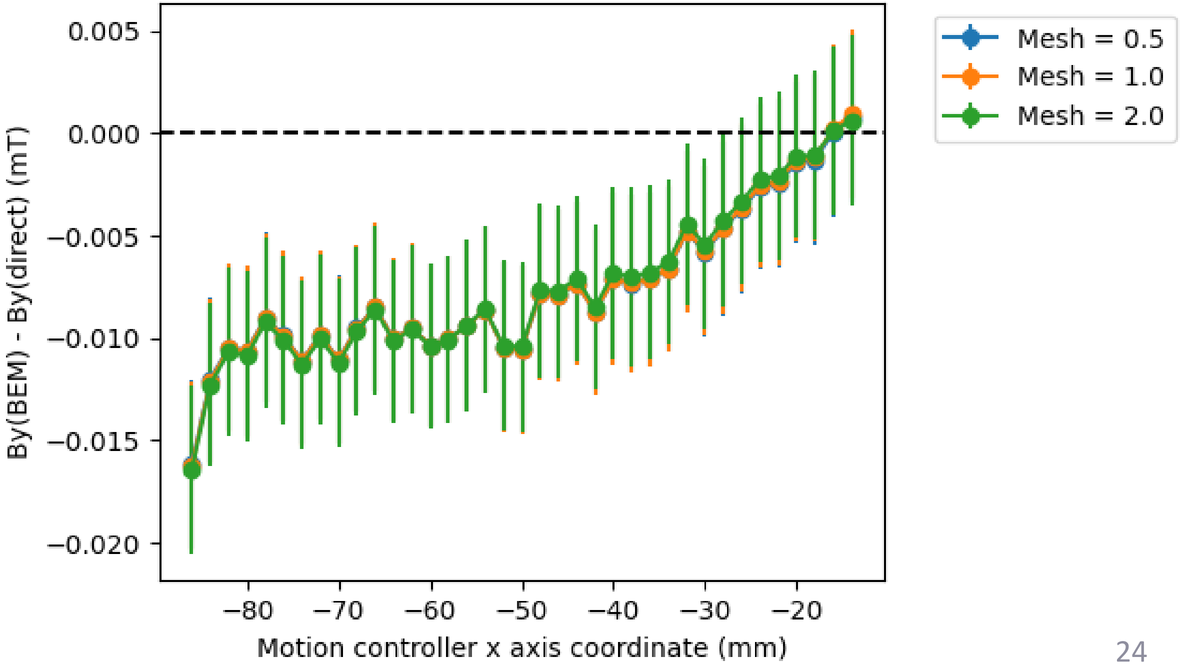
Fields on a line

- Plot of By field component vs motion controller *x* axis.
- *y* = 10 mm,
  z = 2940 mm.
- Standard deviation on direct measurement = 4 µT.





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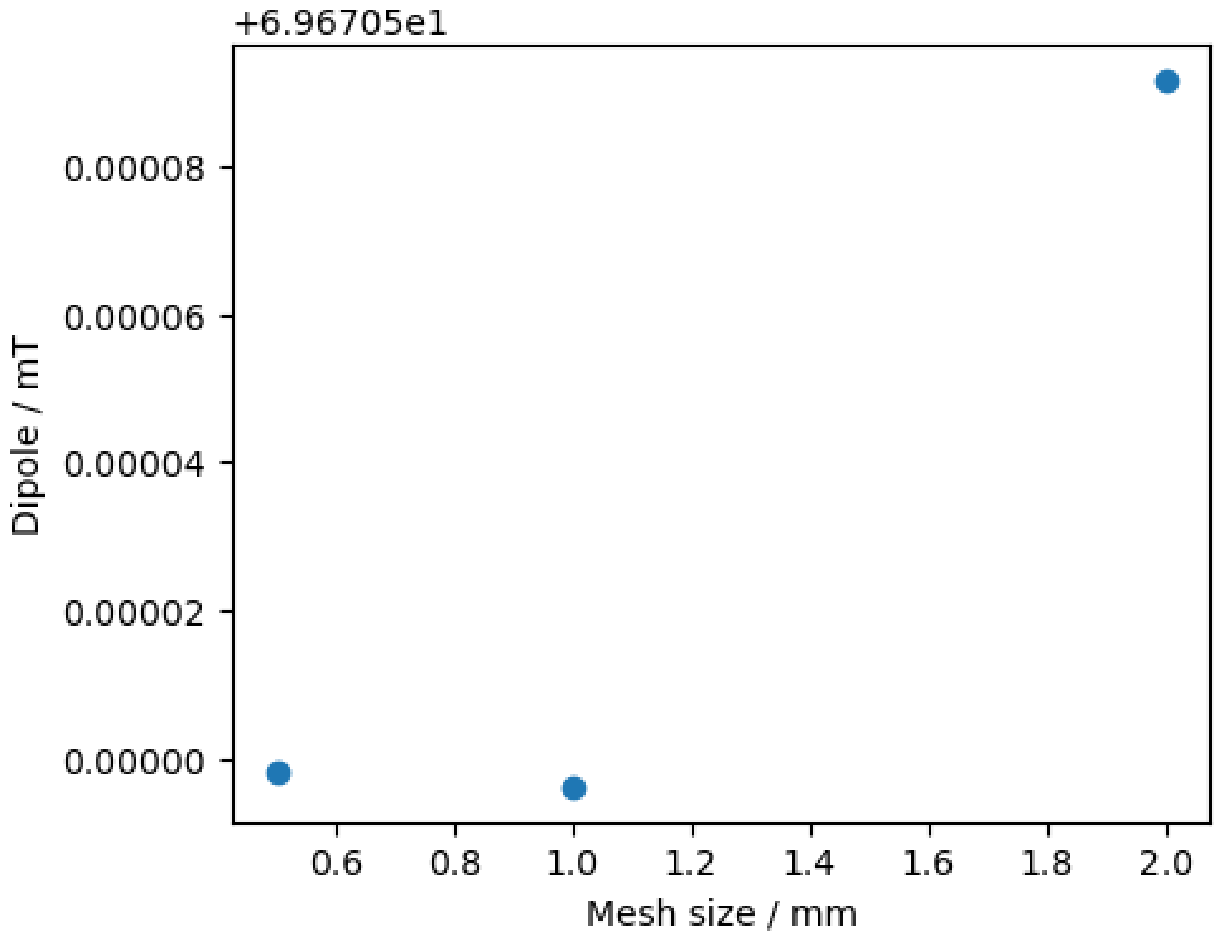
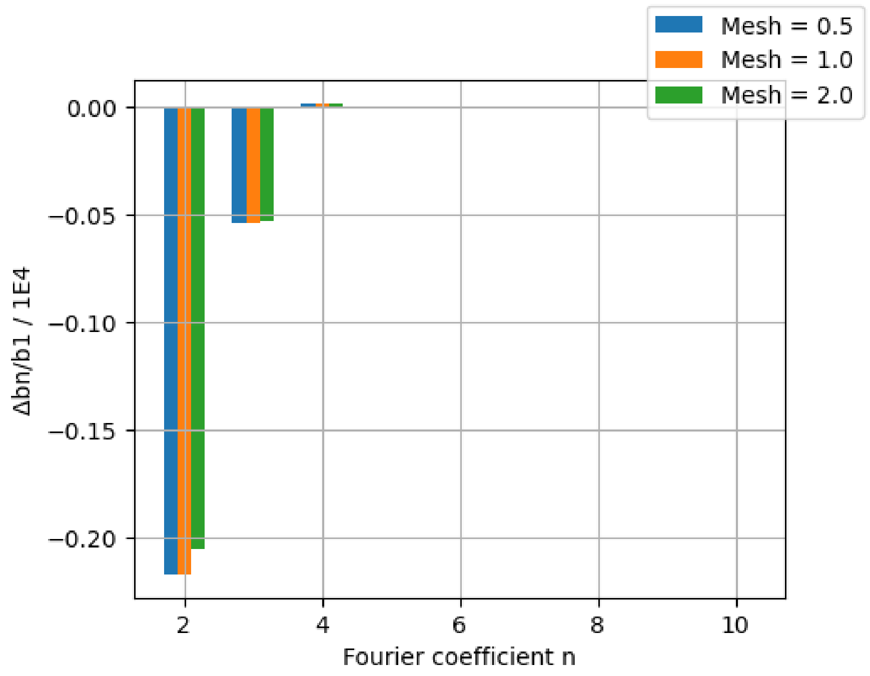
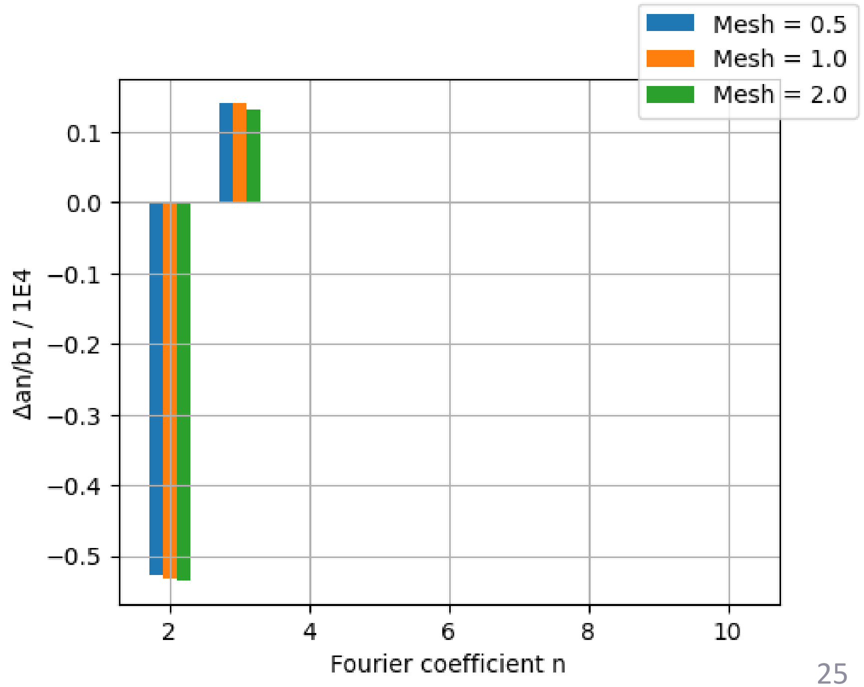
Multipoles

- Multipoles evaluated on 5 mm radius in centre of measurement volume.

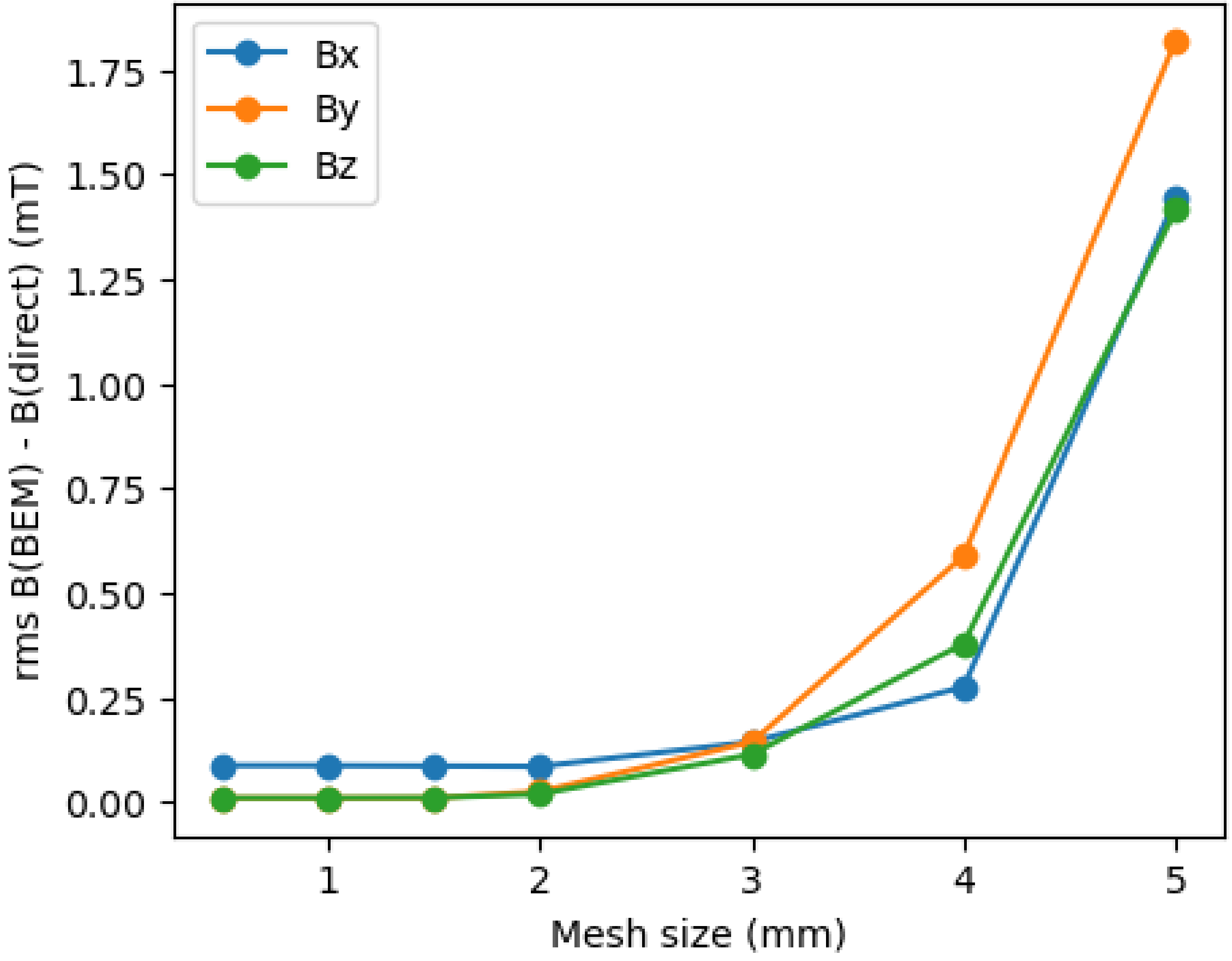






rms Field Error

- Root mean square (rms) difference between directly measured and BEM field components plotted as function of mesh size within one measurement step size of boundary (6669 points).
- At 0.5 mm mesh:
  - Bx = 0.088 mT
  - By = 0.011 mT
  - Bz = 0.011 mT
- Std on direct measurements = 0.004 mT.



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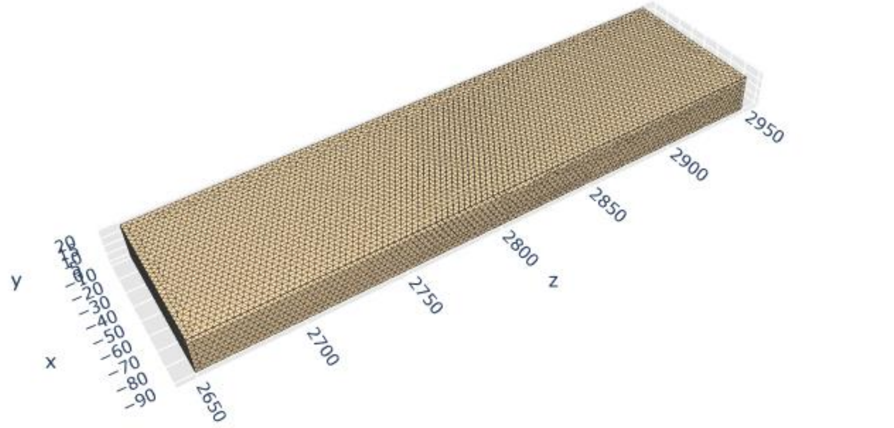
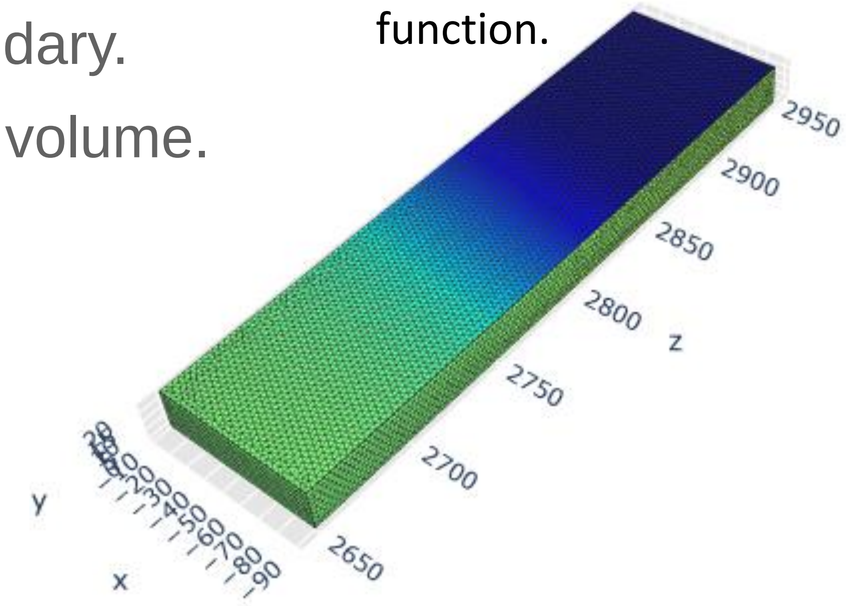
Full range

- Measurement over 80x20x300 mm^3 volume.
- Measurements from inside to outside magnet.
- 2 mm step size in x, y, z directions.
- 68,101 points total.
- 15,802 points on boundary.
- ~1.6 days to measure volume.
- ~ 0.4 days to measure boundary.

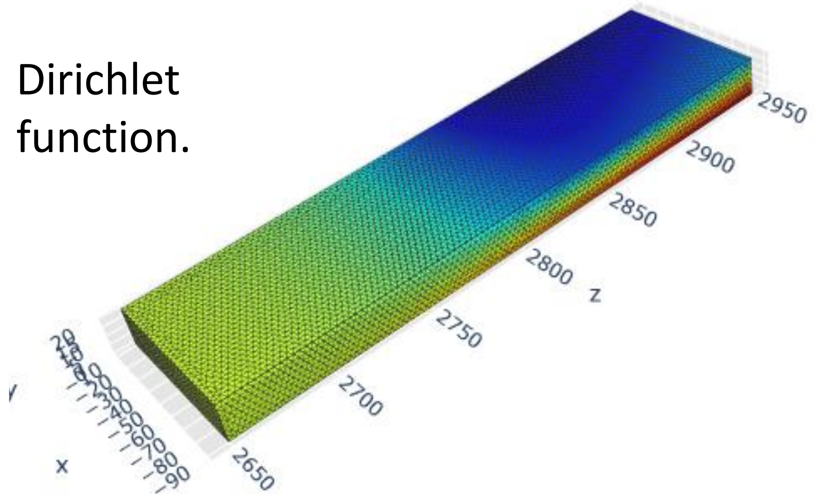
3 mm mesh grid.



Neumann function.



Dirichlet function.



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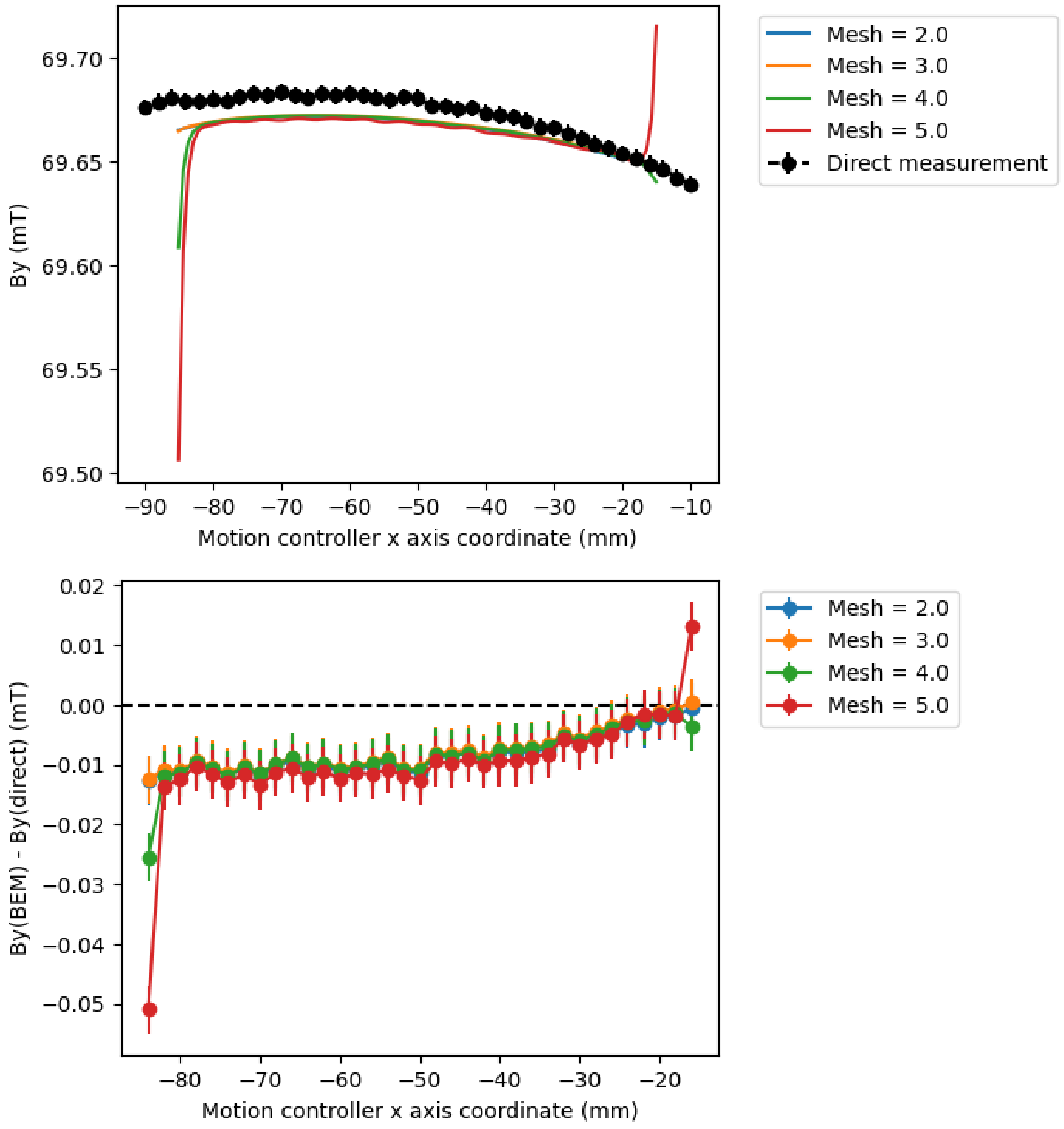
Fields vs x axis

- y = 10 mm
- z = 2940 mm
- Little difference between mesh sizes in centre of volume.
- Smaller mesh size shows better agreement over larger range.



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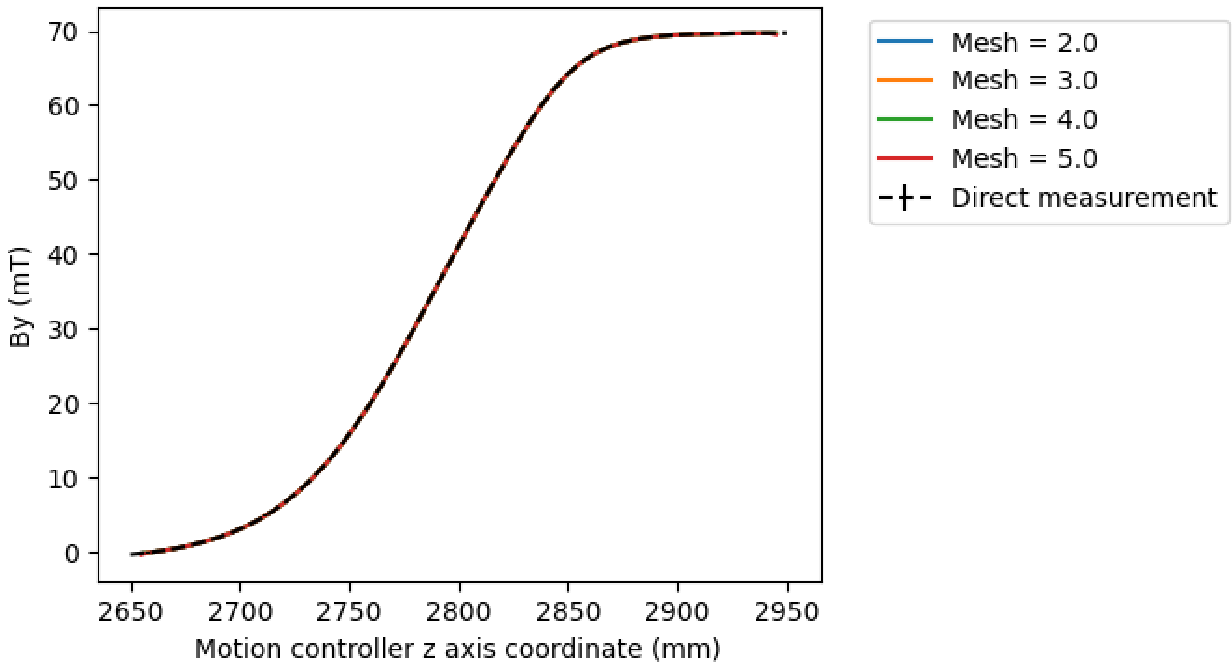
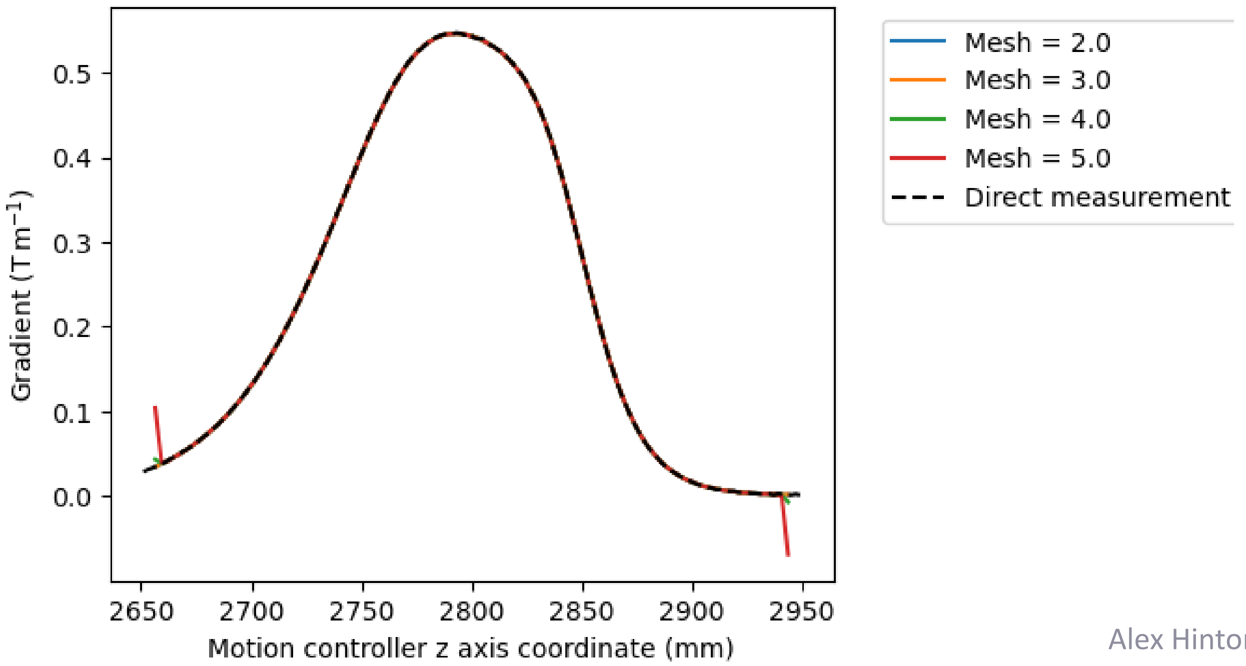
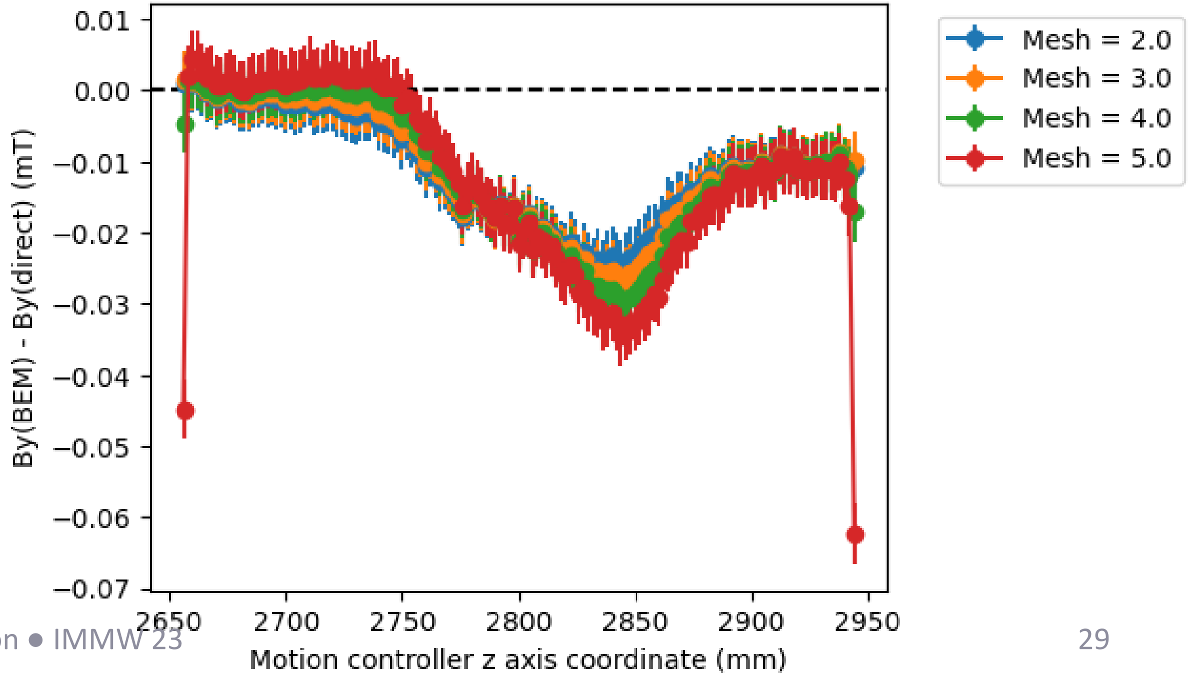
Fields vs z axis

- x = -50 mm
- y = 10 mm
- Consistency in longitudinal field gradient dBy/dz.

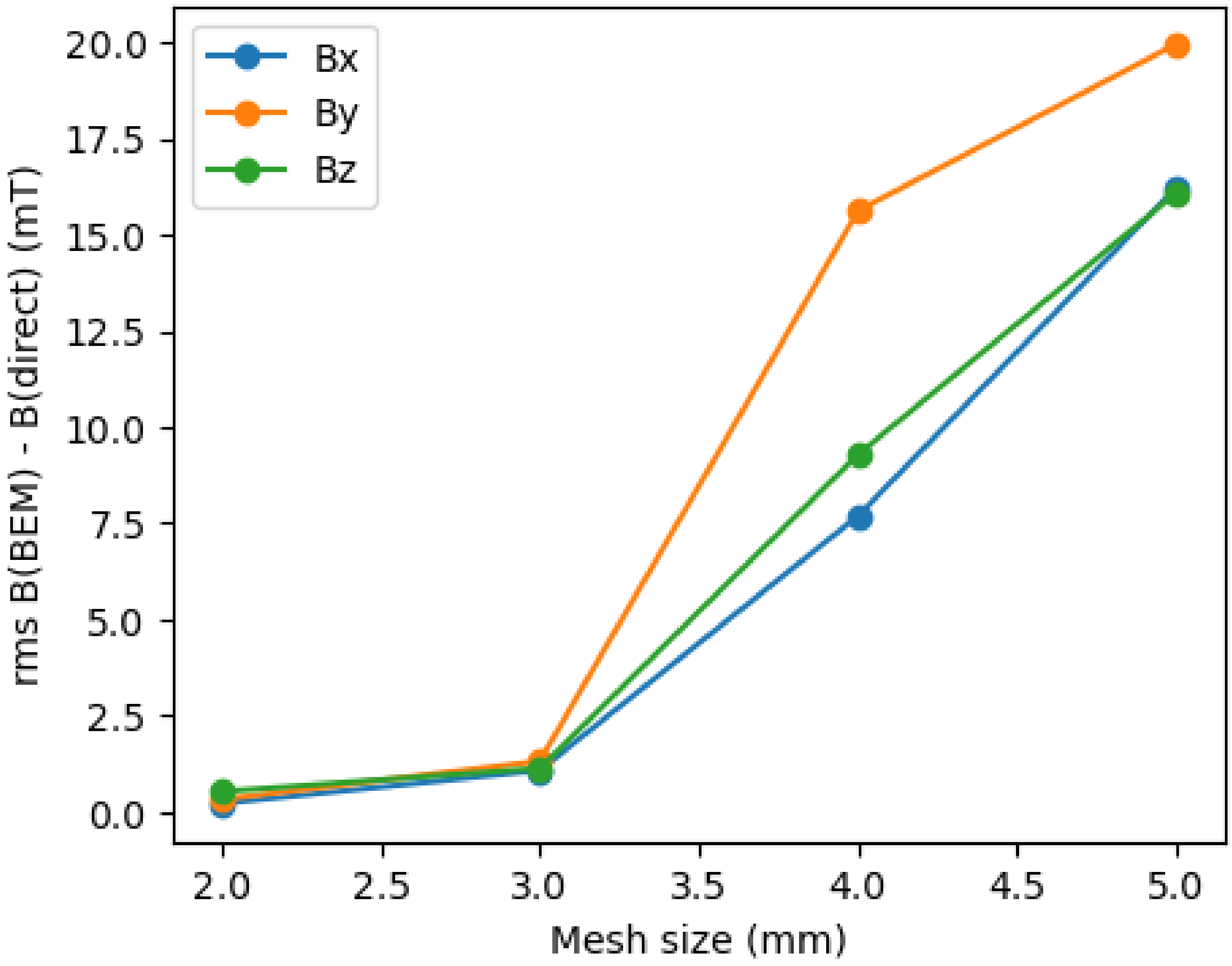






rms Field Error

- rms difference between directly measured and BEM fields over 52299 points within 1 measurement step size of boundary.
- rms differences @ 2 mm mesh:
    - Bx = 0.218 mT
    - By = 0.324 mT
    - Bz = 0.533 mT



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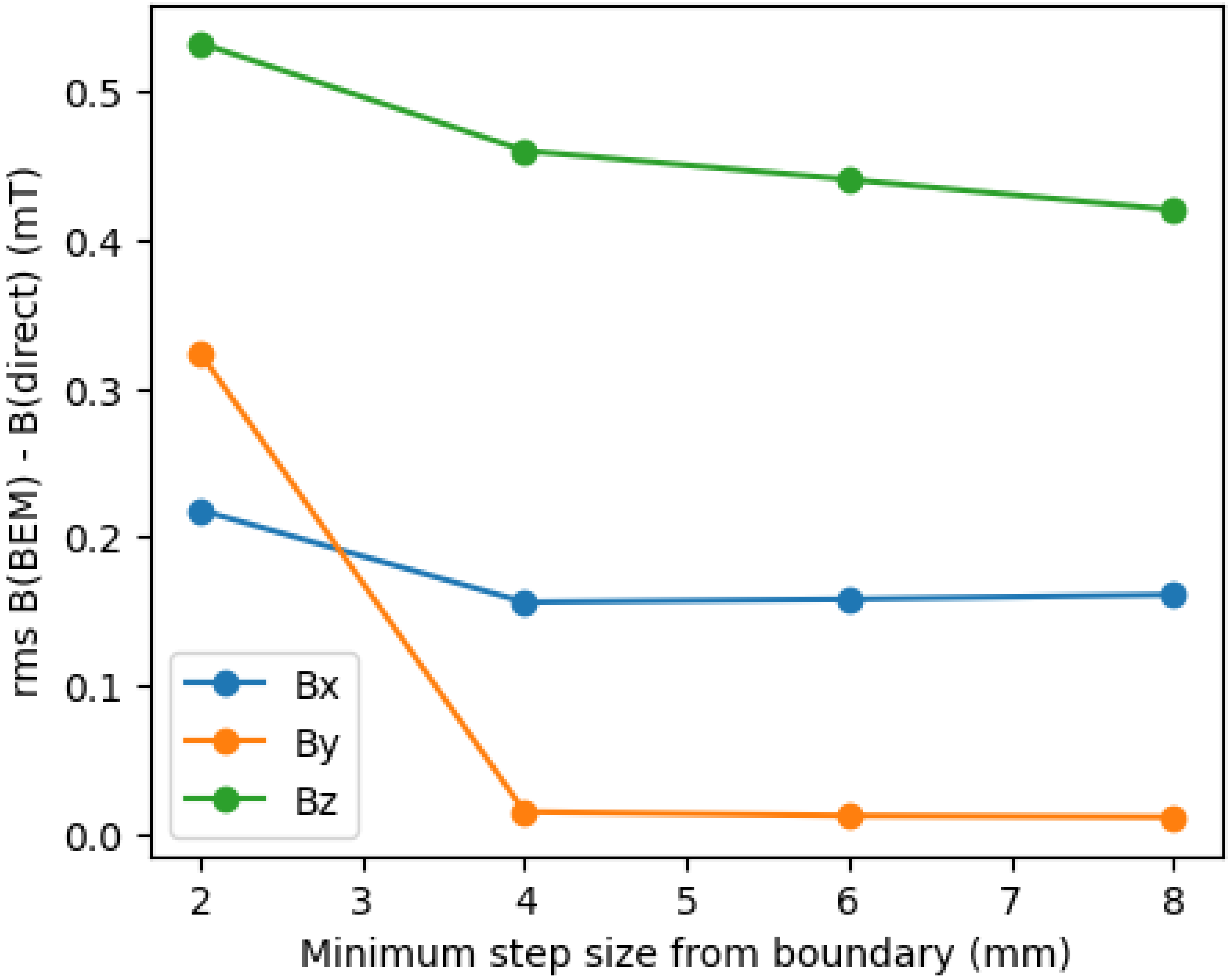
rms Field Error

- 2 mm mesh.
- rms field errors lower for fields taken closer to centre of measurement volume.



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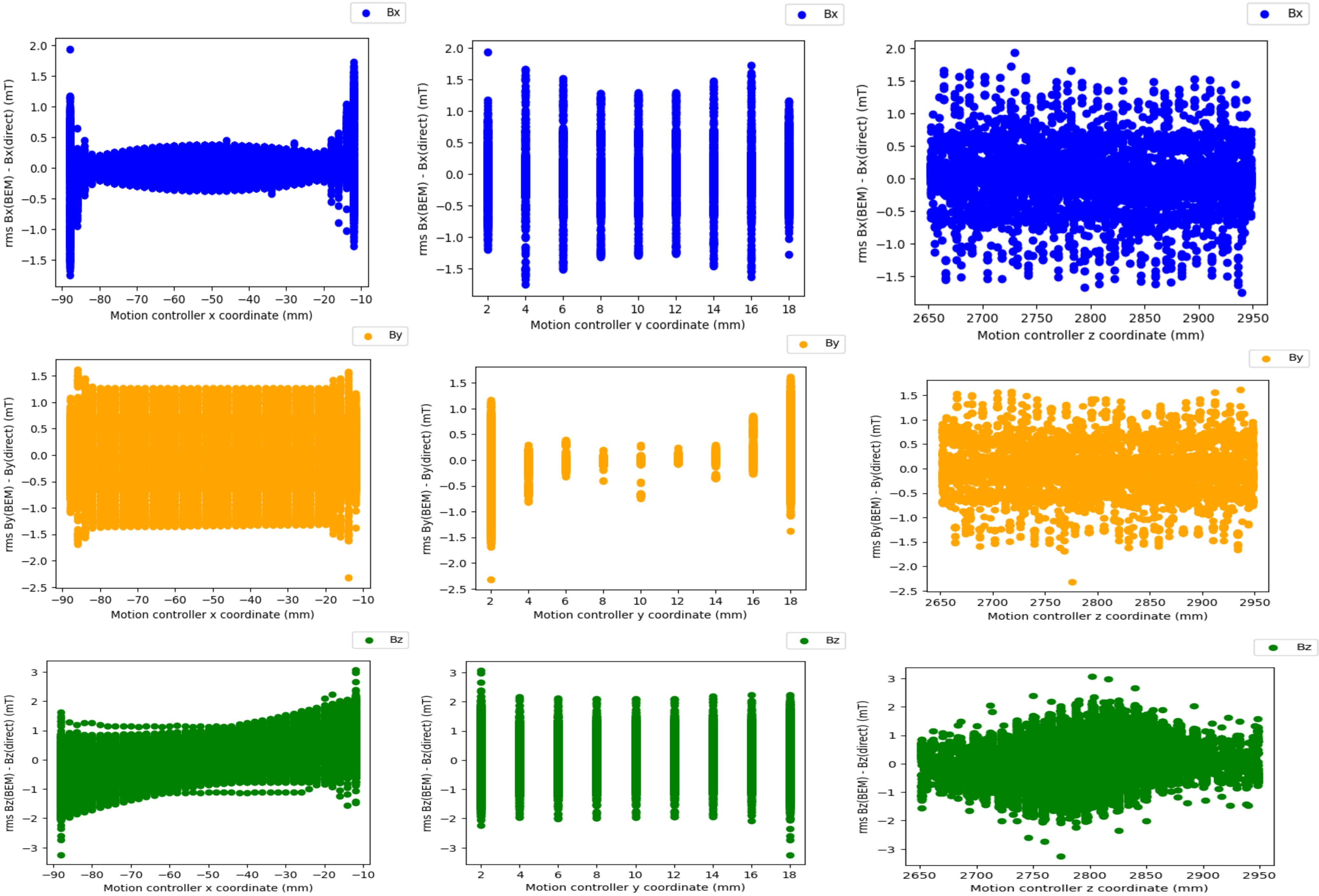
Bx
rms Bx(BEM) - Bx(direct) (mT)
Motion controller x coordinate (mm)
Bx
rms Bx(BEM) - Bx(direct) (mT)
Motion controller y coordinate (mm)
Bx
rms Bx(BEM) - Bx(direct) (mT)
Motion controller z coordinate (mm)
By
rms By(BEM) - By(direct) (mT)
Motion controller x coordinate (mm)
By
rms By(BEM) - By(direct) (mT)
Motion controller y coordinate (mm)
By
rms By(BEM) - By(direct) (mT)
Motion controller z coordinate (mm)
Bz
rms Bz(BEM) - Bz(direct) (mT)
Motion controller x coordinate (mm)
Bz
rms Bz(BEM) - Bz(direct) (mT)
Motion controller y coordinate (mm)
Bz
rms Bz(BEM) - Bz(direct) (mT)
Motion controller z coordinate (mm)

Evaluation

Advantages of BEM	Disadvantages of BEM
Significant reduction in measurement time for large volumes.	Requires good alignment of motion control and Hall sensor axes.
Physics based field calculations.	Mesh dependent results.
Continuous evaluation of fields.	Dense matrix inversions – large computing power required for large volumes with dense meshes.
Measurements designed for BEM.	Some verification of accuracy required.
	Dependent on accuracy of boundary field measurements.

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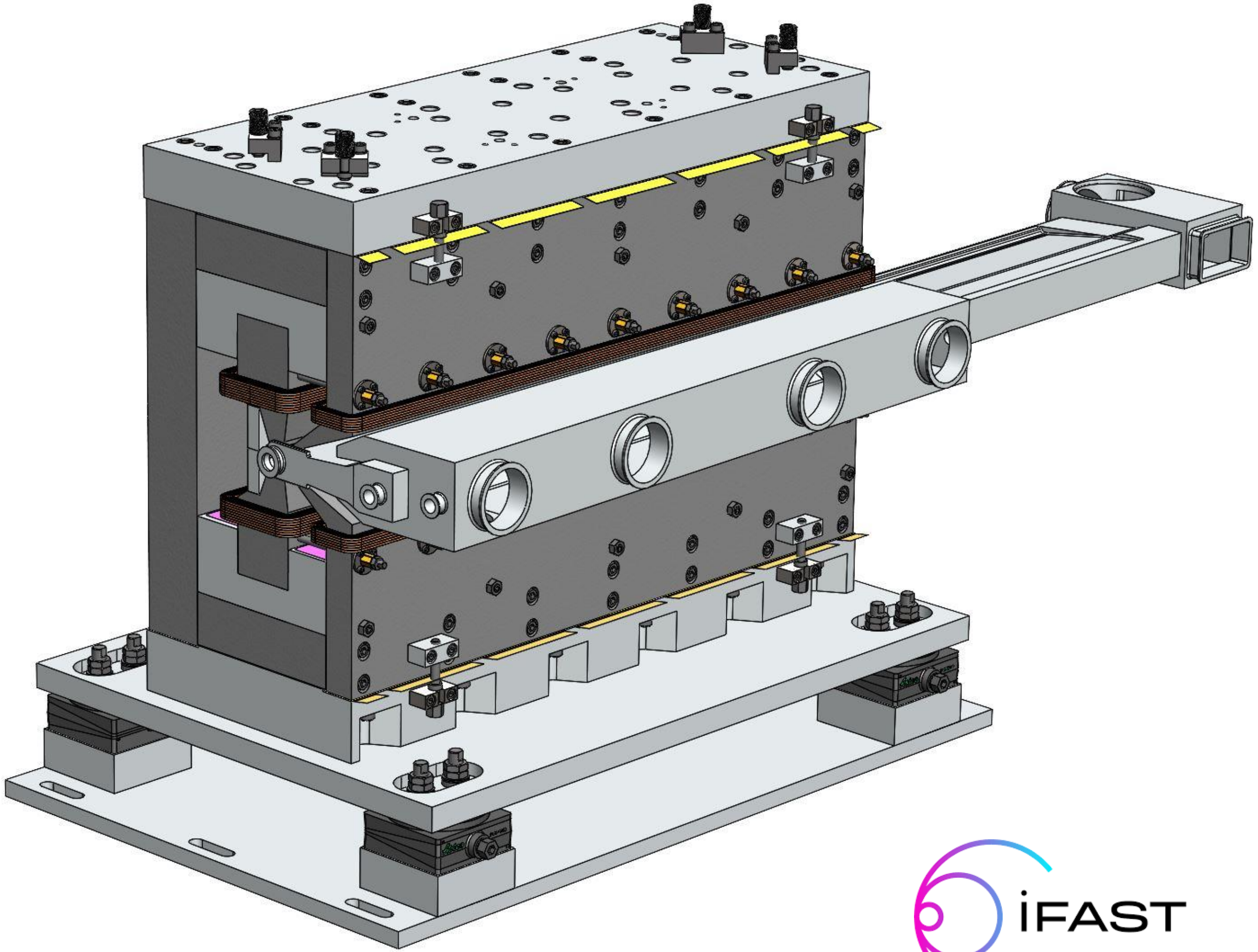
Conclusions and Future Work

- BEM provides a time-saving alternative method to determining 3D field vectors in a volume to direct point-by-point scanning with a Hall sensor.
- Upcoming measurement projects @ DL:
    - ZEPTO-DLS remeasure.
    - HEPTO prototype DQ.
- How to improve accuracy of results?



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Thank You

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