Effective Field Theories

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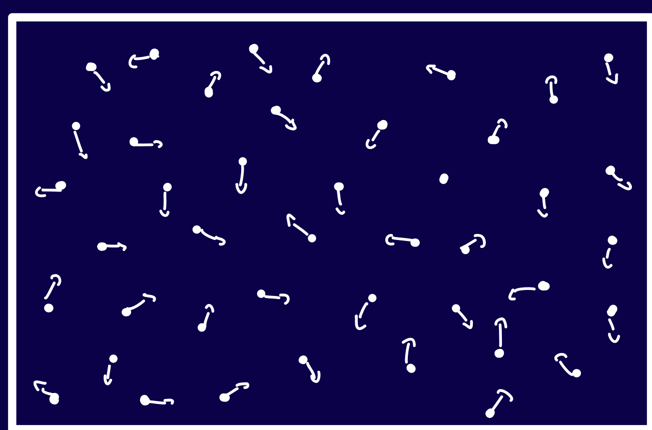
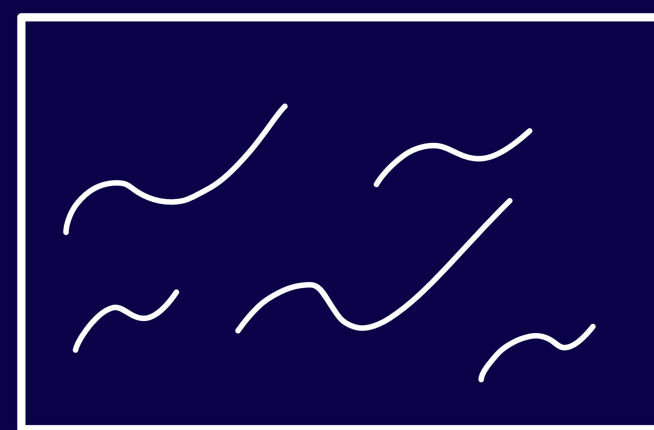
$$\vec{x}_i(t), \vec{v}_i(t)$$

$$i = 1, \dots, 10^{28}$$



$$\rho(\vec{x},t),\ T(\vec{x},t),\ P(\vec{x},t)$$



Effective Theory: Model only the parts we are interested in

Examples

Effective Theory	Underlying theory
Thermodynamics	Kinetic Theory
$F_G = mg$	$F_G = G\frac{mM}{r^2}$
$F_G = G\frac{mM}{r^2}$	$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$
Pizza	Core Theory (SM + Gravity)
Core Theory	

Properties of effective theories

- Vocabulary can be very different
- Limited domain of applicability
- Exist in their own right
- Multiple Realizability
- Easier to use than full theory

Modern way of thinking:

Every theory ever built is an effective theory.

Effective Field Theories

Kenneth Wilson:

"Split vibrating fields into fast (ϕ_F) and slow (ϕ_s) vibrations."

$$\int \mathcal{D}\phi \, e^{iS_{uv}[\phi]} \underset{\substack{\uparrow \\ \text{math}}}{=} \int \mathcal{D}\phi_s \, \mathcal{D}\phi_F \, e^{iS_{uv}[\phi_s, \phi_F]} \underset{\substack{\uparrow \\ \text{physics}}}{=} \int \mathcal{D}\phi_s \, e^{iS_{eff}[\phi_L]}$$

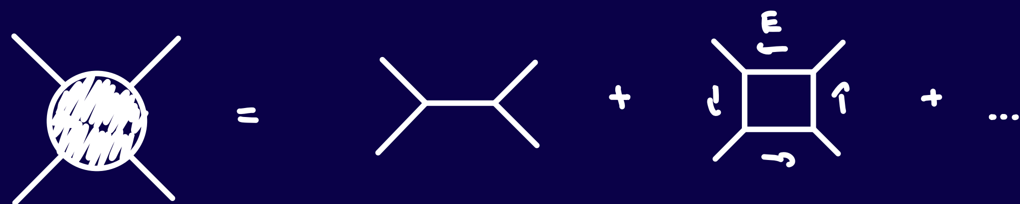
- integral over fast vibrations gives action : $\int \mathcal{D}\phi_s \, e^{iS[\phi_L, \phi_s]} = e^{iS_{eff}[\phi]}$
- $S_{eff}[\phi]$ action for a <u>local</u> field theory : the EFT !
- keep track only of slow vibrations
  - $\rightarrow$ ϕ_F interactions become point interactions of ϕ_s

top-down

Regularization

e.g. $\mathcal{L}[\phi] = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2 + c_3\phi^3 + \ldots$

compute $e^{iS} = e^{i\int d^4x\,\mathcal{L}}$:

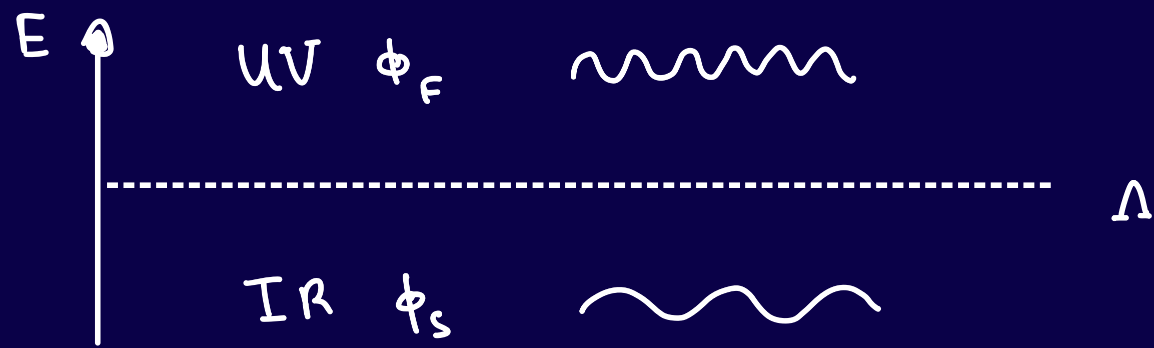


Loops give Feynman integrals : $\int_0^\infty dE \ldots =$ 💩

More honest approach : $\int_0^\Lambda dE \ldots = \text{finite}$

(Practical calculations are done in $d = 4 - 2\epsilon$ dimensions)

The Renormalization Group



But then: $m = m(\Lambda)$, $C_i = C_i(\Lambda)$

$$\mathcal{L}_{eff}[\phi_s] = \frac{1}{2}\partial_\mu \phi_s \partial^\mu \phi_s - \frac{m^2}{2}\phi_s^2 + C_4 \phi^4 + C_6 \phi^6 + \dots$$

Power Counting

Units: $\hbar = 1, \quad c = 1$

$[\partial] = [m] = [\phi] = E$, thus we have:

$$\underbrace{\frac{m^2}{2}\Lambda^2\phi^2}_{\text{relevant}} + \underbrace{C_4\,\phi^4}_{\text{marginal}} + \underbrace{\frac{C_6}{\Lambda^2}\phi^6 + \frac{C_8}{\Lambda^4}\phi^8}_{\text{irrelevant}} + \dots$$

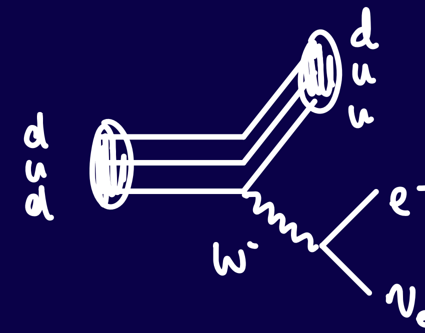
$\longrightarrow$ need only a few terms

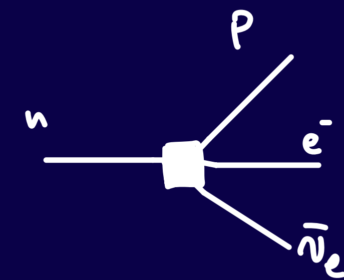
the LEFT

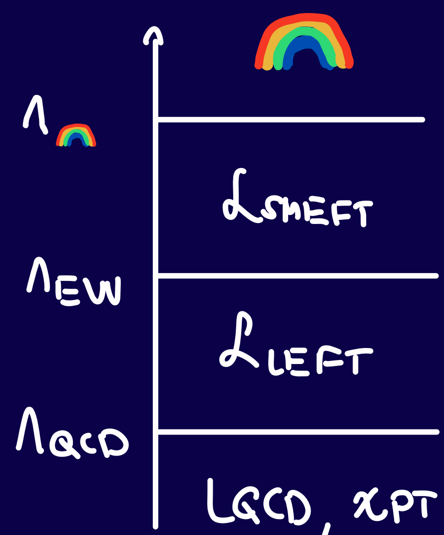
Valid for $E \ll \Lambda_{EW}$.

Integrated out DOF: t, h, W, Z

$$W = \int \mathcal{D}A \mathcal{D}\varphi \exp\left\{ i \int d^4x \left[\mathcal{L}_{QCD+QED} + L_i O_i \right] \right\}$$







the SMEFT

$$W = \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\phi\, \exp\left\{ i\int d^4x \left[-\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}_i\, i\not{D}\,\psi_i + \bar{\psi}_L^i V_{ij}\,\phi\,\psi_R^j - |D_\mu\phi|^2 - V(\phi) + C_i O_i \right] \right\}$$

$$= \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\phi\, \exp\left\{ i\int d^4x \left[\mathcal{L}_{SM} + C_i O_i \right] \right\}$$

$$\sum_d \frac{C_i^{(d)}}{\Lambda^{d-4}}$$

bottom-up

The Core Theory

$$W = \underbrace{\int \mathcal{D}g\, \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\phi\, \exp\Big\{ i \int}_{\text{QM}} \underbrace{d^4x \sqrt{-g}}_{\text{spacetime}} \Big[\underbrace{\frac{2}{\kappa} R}_{\text{gravity}} \underbrace{- \frac{1}{4}\left(F^a_{\mu\nu}\right)^2}_{\text{other forces}} + \underbrace{\bar{\psi}_i\, i \not{D}\, \psi_i + \bar{\psi}_i V_{ij}\, \phi\, \psi^j_R}_{\text{matter}} \underbrace{- |D_\mu \phi|^2 - V(\phi)}_{\text{Higgs}} + \underbrace{C_i O_i}_{\text{effective operators}} \Big\}$$

$C_i O_i$ can include effective Gravity operators

$$\Lambda + R + \frac{c_1}{\Lambda^2} R^2 + \dots \qquad\qquad R \sim \partial \Gamma \sim \partial^2 g \sim p^2$$

On earth, linear term $\sim 10^{50}$ times bigger

Formal problems of $R + R^2$ theory disappear in EFT.

Issues with loop expansion

Problem (1): large logs

$\mathcal{L}_{\text{Full}}$ with (m, M)

large

Loop expansion : $1 + g\log\left(\frac{p}{M}\right) + g^2\log\left(\frac{p}{M}\right)^2 + \ldots$

But in EFT: only $\log\left(\frac{p}{m}\right)$

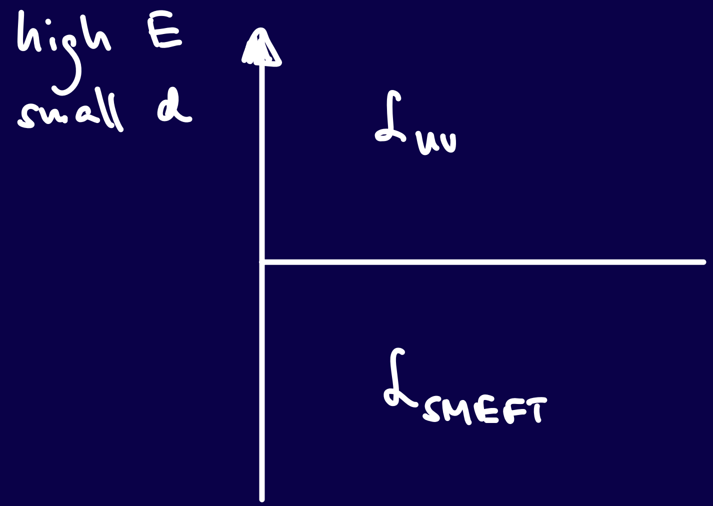
Problem (2): Strong interactions

$1 + g \cdot \log + g^2\log^2 + \ldots$ bad because $g > 1$

Can still construct a perturbative EFT !

$\longrightarrow$ Chiral Perturbation Theory, or χPT

Patchwork laws?



Reduction & unification means $\mathcal{L}_{SMEFT} \subset \mathcal{L}_{UV}$

But what if $\mathcal{L}_{SMEFT} \not\subset \mathcal{L}_{UV}$?

$\longrightarrow$ patchwork laws

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