# Shot Noise Amplification and Suppression in High Brightness Electron Beams

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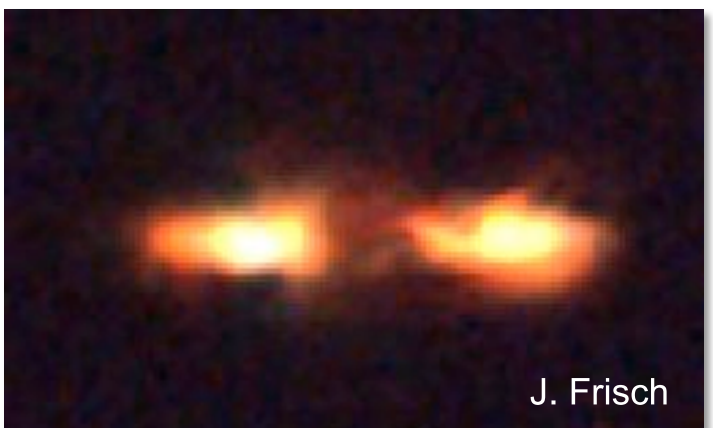
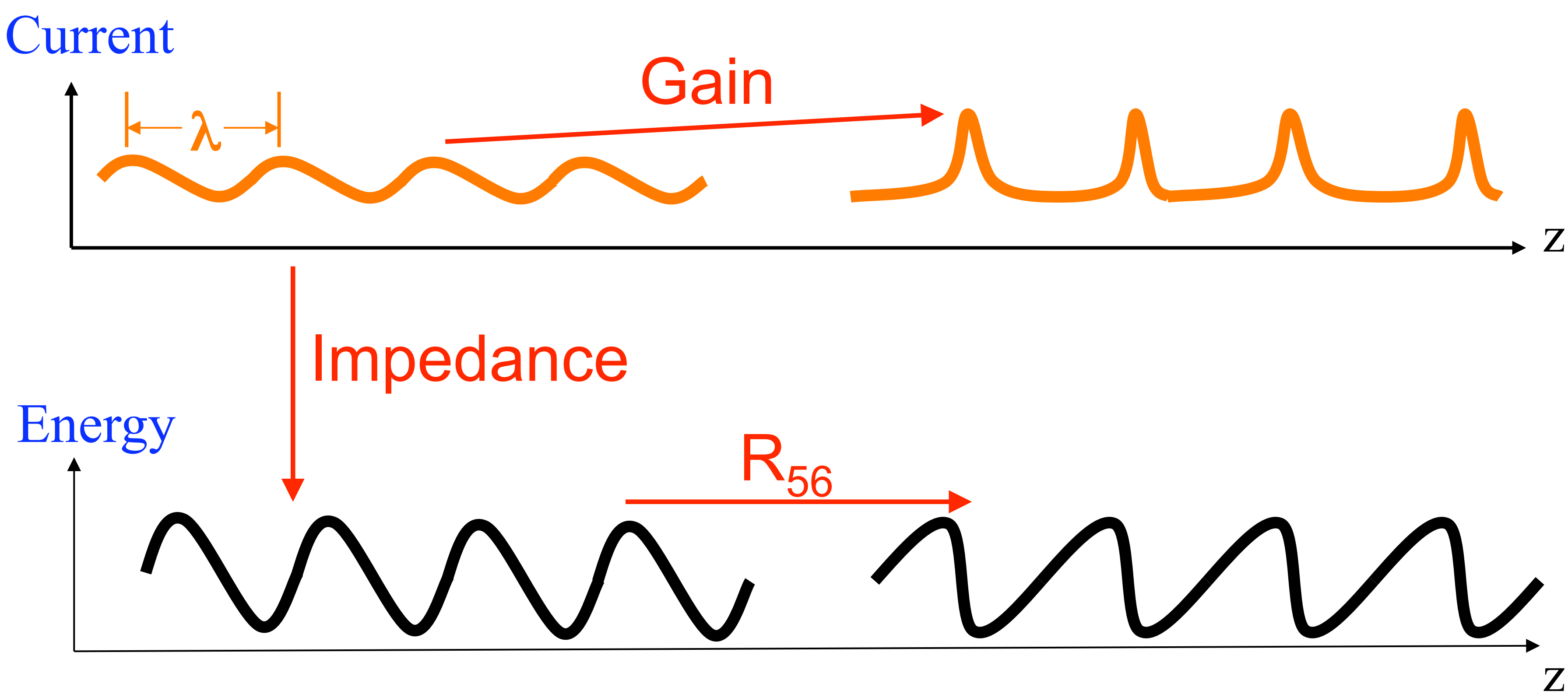
# Introduction

- Amplification:
  - Microbunching Instability (MBI)
  - 6D MBI model from shot noise
- Suppression:
  - Creating quiet electron beams (below shot noise)
  - General description of noise suppression



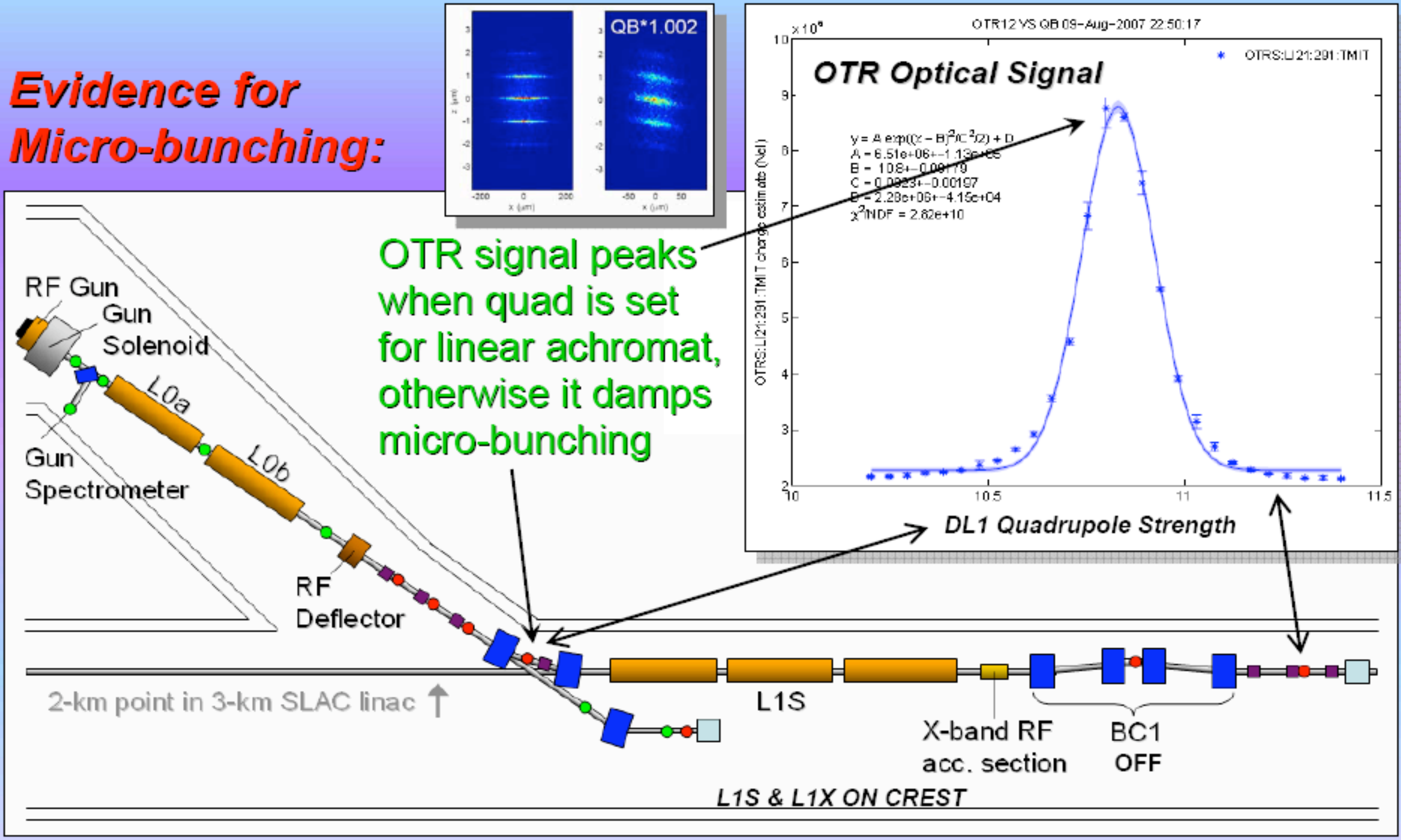

# Microbunching Instability

- Density Modulation → Energy Modulation → DENSITY Modulation



# **Unexpected Physics!** *Coherent OTR after 35-degree Bend, Even With No BC1*

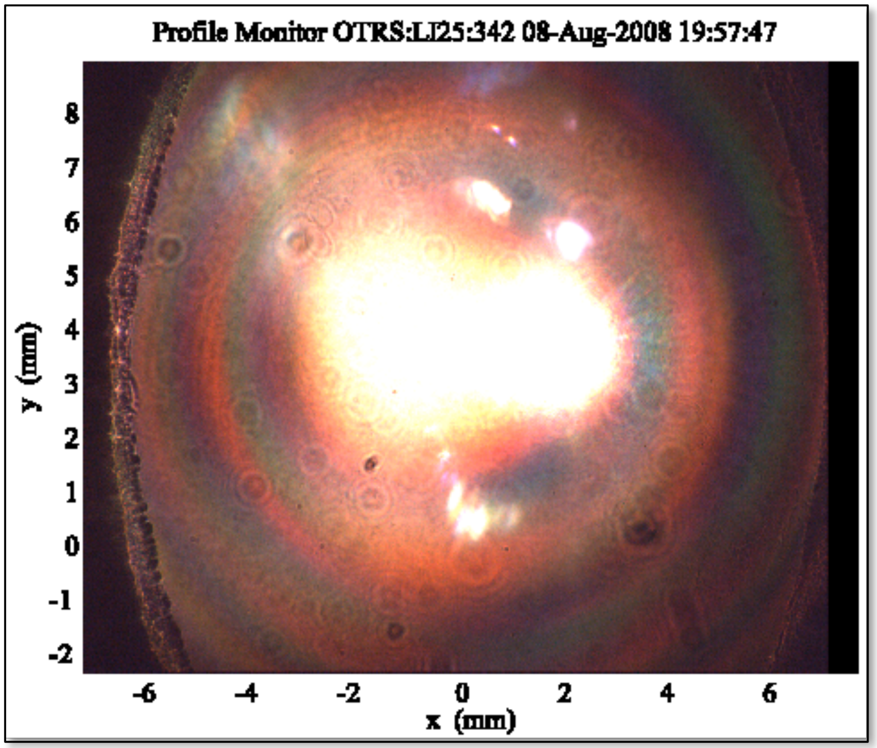
***Evidence for Micro-bunching:***

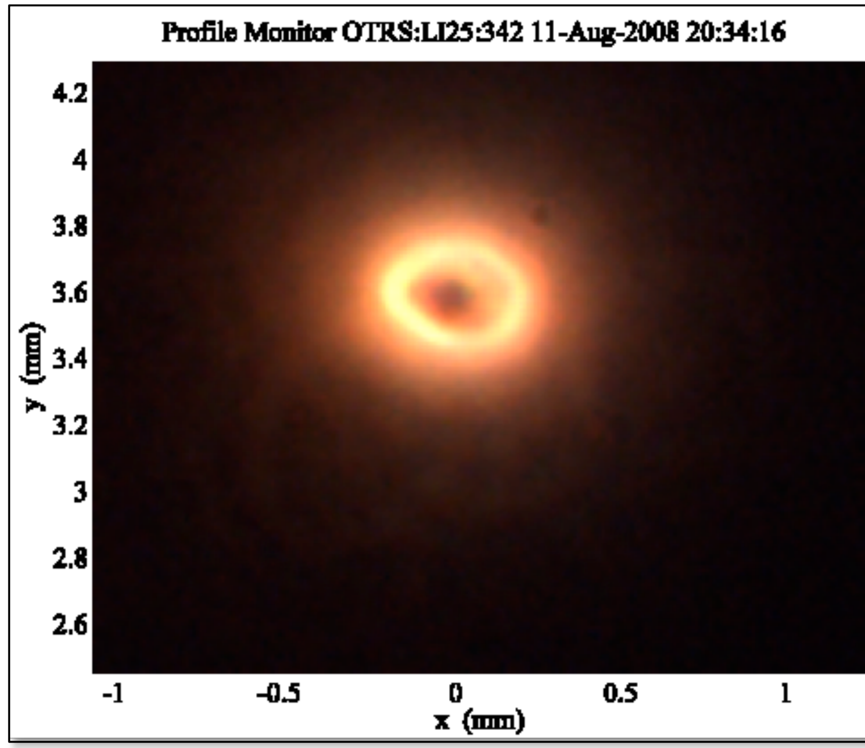



*R. Akre, et al., PRST-AB 11, 030703 (2008)*

- Bright coherent radiation incapacitates diagnostics

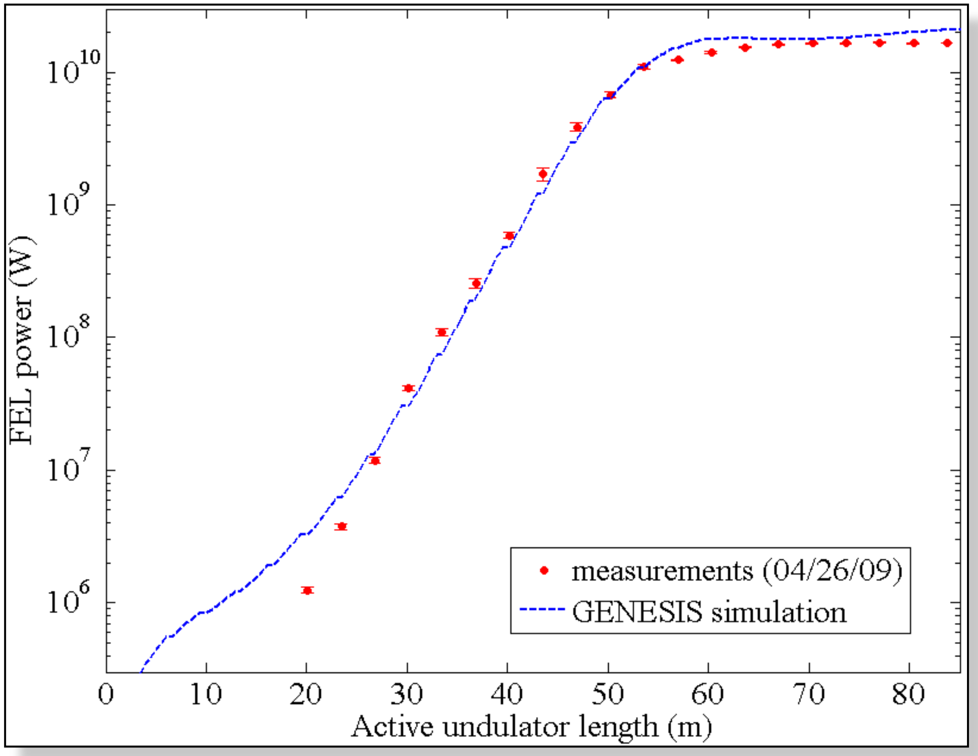
Coherent radiation on OTR screens at 4 GeV



# Gain Length Measurement

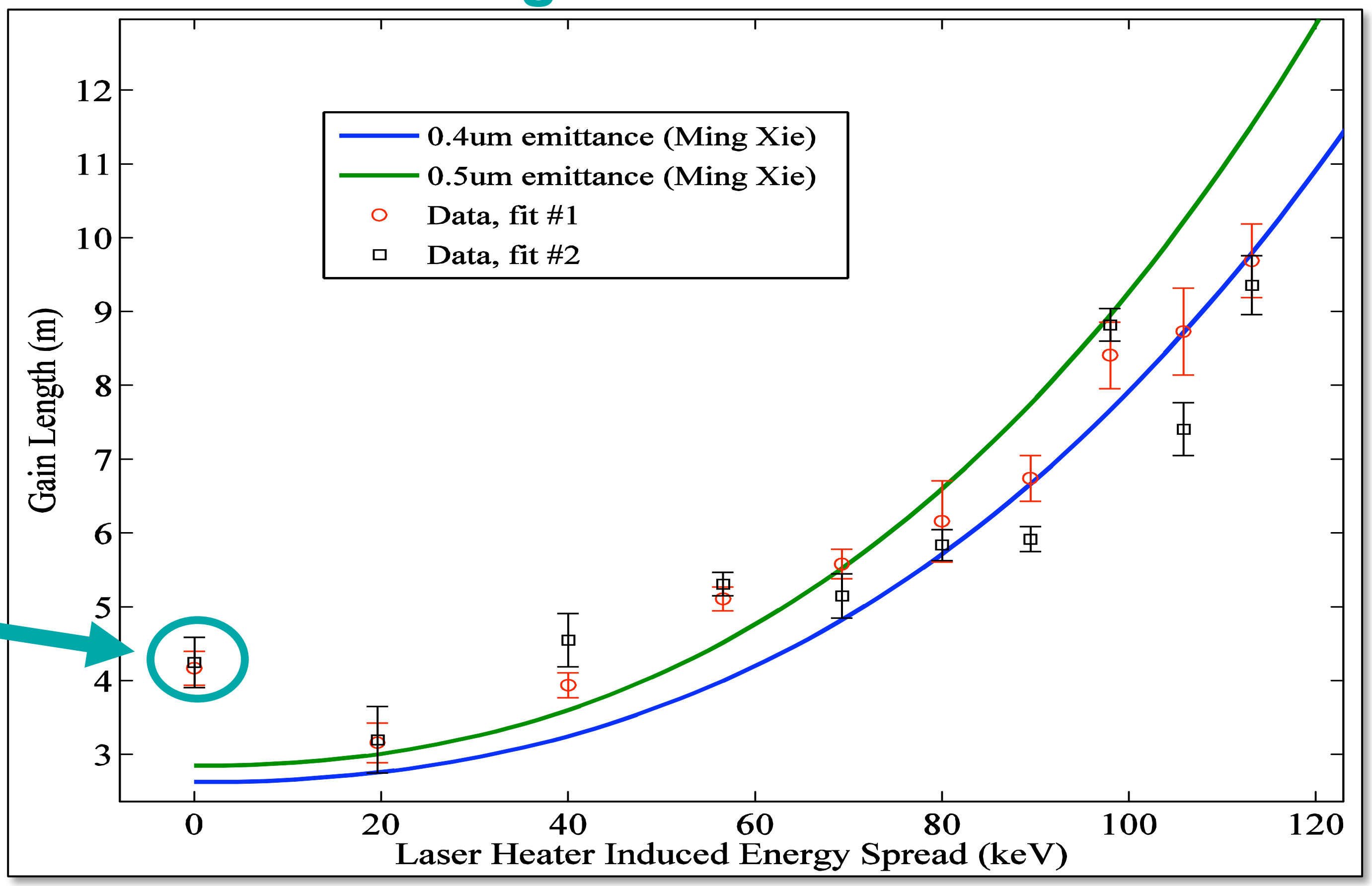
- Instability may affect FEL performance
  - Laser heater suppresses instability
  - Use gain length as FEL metric



# MBI at LCLS

- Instability DOES affect FEL performance

## Gain Length vs. Laser Heater

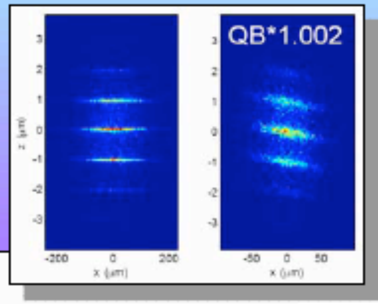


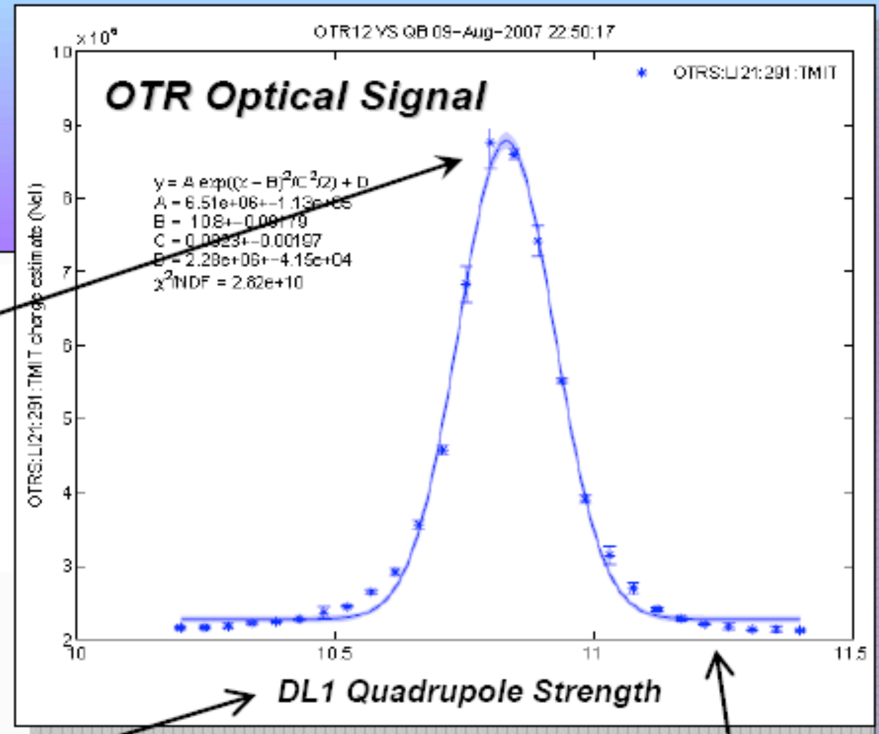

MBI suppresses FEL with no laser heater

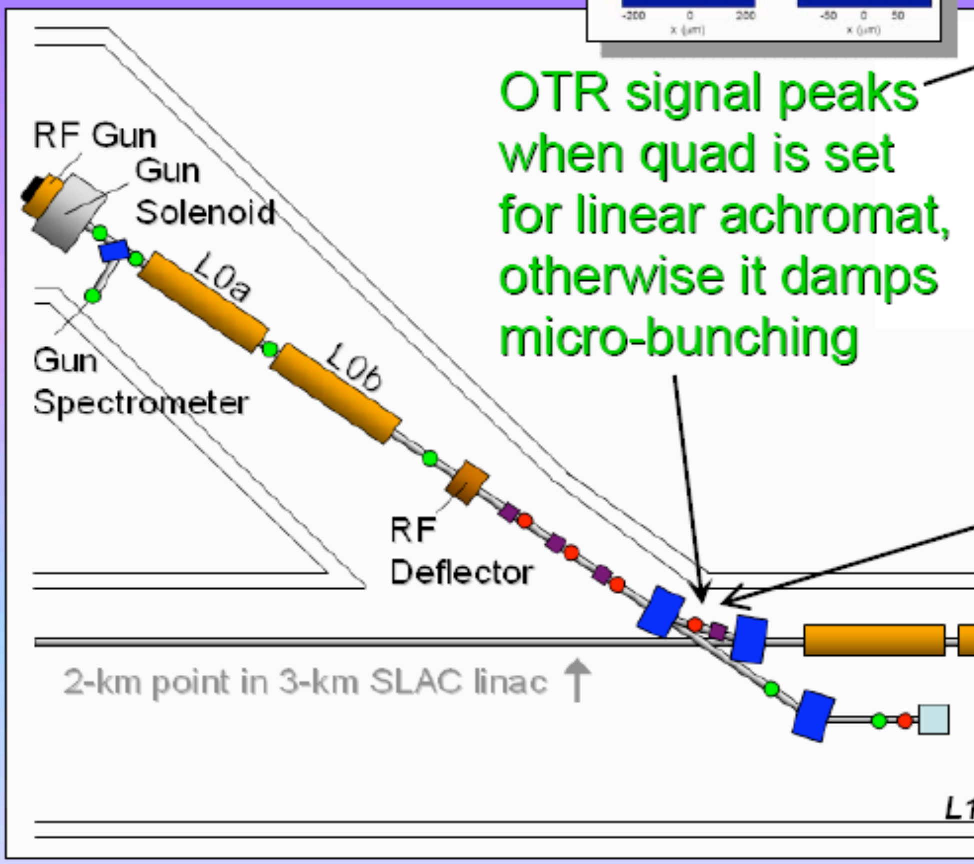
**Parameters: 1.5Å, 250pC, 3kA, Compression factor = 90**

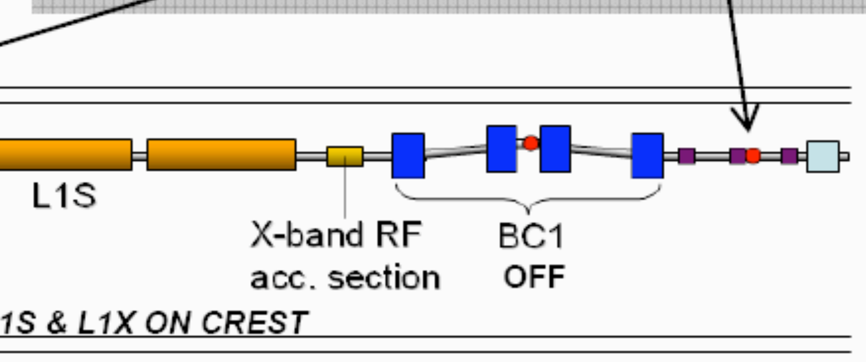
# **Unexpected Physics!** *Coherent OTR after 35-degree Bend, Even With No BC1*

***Evidence for Micro-bunching:***



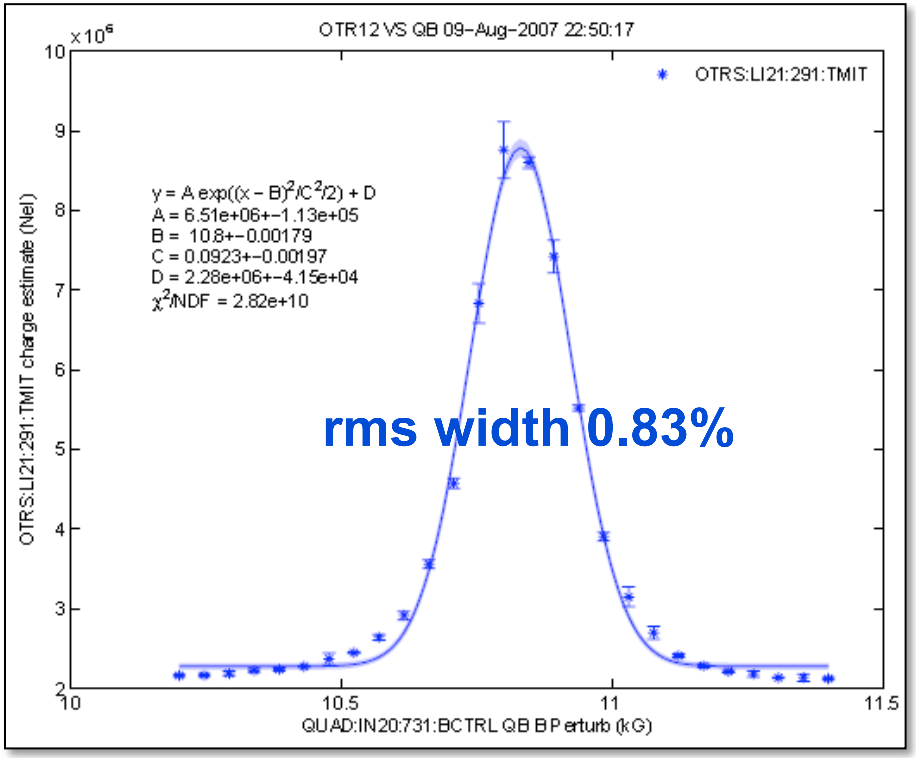



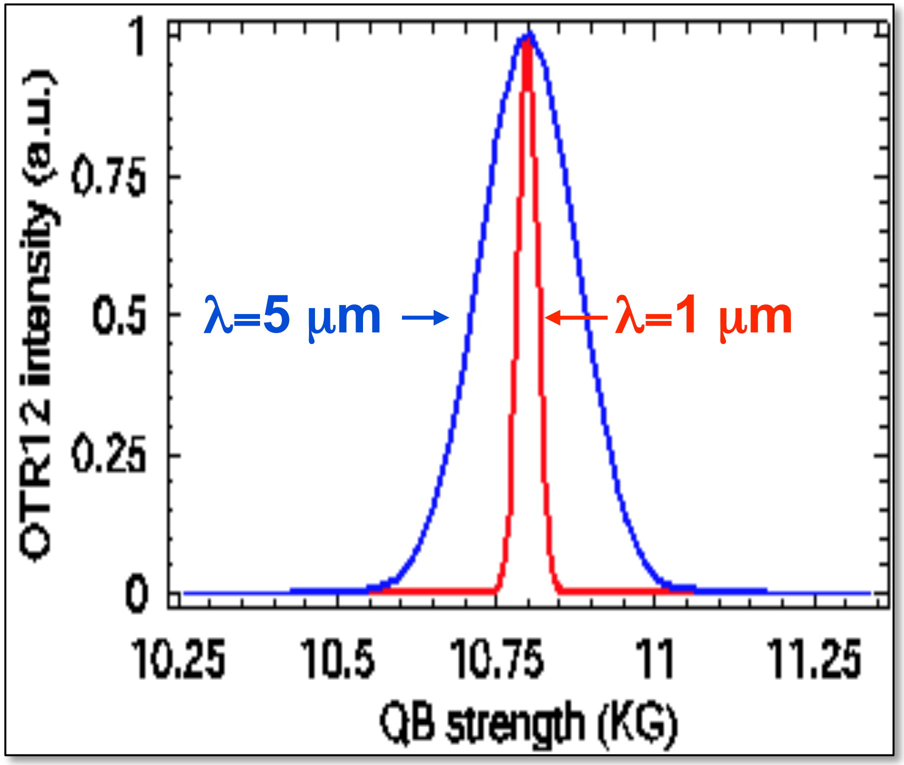

*R. Akre, et al., PRST-AB 11, 030703 (2008)*

# MBI at LCLS



- Uniform E-field: $\langle |b(k)|^2 \rangle \propto e^{-k^2\sigma^2 R_{51}^2 - k^2 \sigma'^2 R_{52}^2}$
- QB curve has ~1% width



# 3D vs. 1D



## ❑ Transverse Models

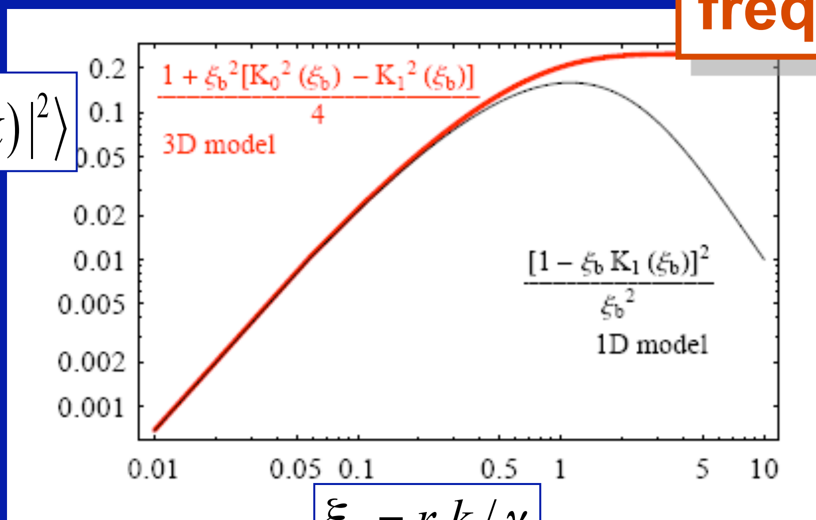
### 3D *vs.* 1D model of shot noise

On-axis longitudinal field fluctu

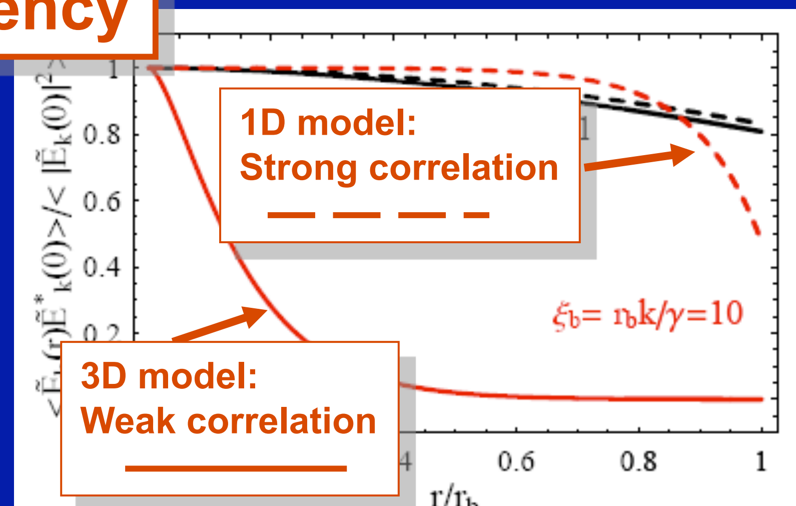
**High frequency**

Radial correlations of longitudinal E-field

$\langle |\tilde{E}_z(k)|^2 \rangle$




$\xi_b = r_b k / \gamma$




**1D model: Strong correlation**

**3D model: Weak correlation**

From M. Venturini

$$k \equiv 2\pi / \lambda$$

# MBI 6D Model

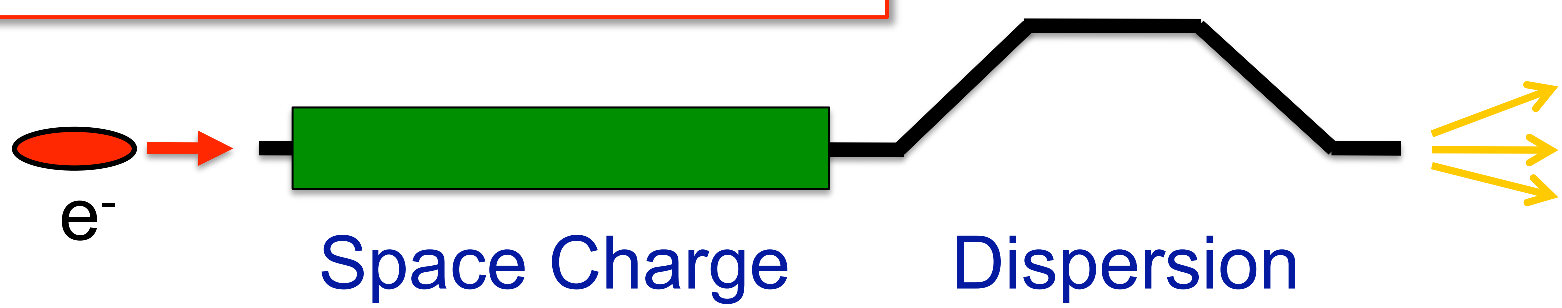
- Model Microbunching Instability (MBI)
- Radiation from beam:

$$\left(\frac{d^2 I}{d\omega d\Omega}\right)_{\text{tot}} = \left(\frac{d^2 I}{d\omega d\Omega}\right)_1 |b(\vec{k})|^2$$

$$\left(\frac{d^2 I}{d\omega d\Omega}\right)_1 \propto \frac{\gamma^4(\theta_x^2 + \theta_y^2)}{\left[1 + \gamma^2(\theta_x^2 + \theta_y^2)\right]^2}$$

$$\boxed{b(\vec{k}) = \frac{1}{N} \sum_j \exp\left[-i\tilde{K} X_f\right]}$$

$$\tilde{K} \equiv [k\theta_x \ 0 \ k\theta_y \ 0 \ k \ 0]$$



# MBI 6D Model



- Model Microbunching Instability (MBI)
- Calculate bunching factor from shot noise:

$$\boxed{b(\vec{k}) = \frac{1}{N}\sum_j \exp\left[-i\tilde{K}X_f\right]}$$

- Klimontovich density distribution

$$\boxed{\rho(\vec{X}) = \sum_j^N \delta(x-x_j)\delta(y-y_j)\delta(z-z_j)\delta(x'-x'_j)\delta(y'-y'_j)\delta(p-p_j)}$$

# MBI 6D Model



- Separate out space charge contribution:

$$b(\vec{k}) = \frac{1}{N}\sum_j \exp\left[-i\underbrace{\tilde{K}X_j(L)}\right]\exp\left[-i\underbrace{k\delta_j(L)}\right]$$

$$X_j(L) \approx R_{0\to L}X_{0j}$$

$$\frac{d\delta_j(s)}{ds} = \frac{R^{(56)}_{s\to L}eE(X_j(s))}{\gamma_s mc^2}$$

# MBI 6D Model



- Our goal: Bunching factor squared :

$$\left\langle \left| b(\vec{k}) \right|^2 \right\rangle = \frac{1}{N} \left\langle \sum_j^N \sum_l^N e^{-i\tilde{K}(X_j(L)-X_l(L))} e^{-ik(\sum_{i\neq j}\delta_{ji} - \sum_{i\neq j}\delta_{li})} \right\rangle$$

$$\delta_j = \sum_{i\neq j}^N \delta_{j,i} = \sum_{i\neq j}^N \frac{e}{mc^2}\frac{e}{4\pi\epsilon_0}\int ds \frac{R^{(56)}_{s\to L}}{\gamma_s}\frac{\partial}{\partial z_i}\frac{1}{|X_j(s)-X_i(s)|}$$

- From X(0), can find <b(k)$^2$>
- Split into incoherent and coherent terms:

$$j = l\,, \quad j \neq l$$

# MBI 6D Model

- Split into coherent and incoherent terms:

$$j = l\,, \quad j \neq l$$

- Incoherent:

$$\left\langle \left| b(\vec{k}) \right|^2 \right\rangle_{SN} = \frac{1}{N} \sum_{j}^{N} 1 = 1$$

- Coherent:

$$\left\langle \left| b(\vec{k}) \right|^2 \right\rangle_{C} \equiv N \int dX_{01} ... dX_{0N} \Psi(X_{01}) ... \Psi(X_{0N})$$
$$e^{-i\tilde{K}(R_{0\to L} X_{01} - R_{0\to L} X_{02})} e^{-ik(\sum_{i\neq 1} \delta_{1i} - \sum_{i\neq 2} \delta_{2i})}$$

# MBI 6D Model



- Coherent terms:

$$\left\langle \left| b(\vec{k}) \right|^2 \right\rangle_C \approx N \int dX_{01} \int dX_{02} \Psi(X_{01}) \Psi(X_{02}) e^{-i\tilde{K}(R_{0\to L} X_{01} - R_{0\to L} X_{02})} \left[ 1 + \Gamma_1 + \Gamma_2 \right]$$

$$\Gamma_1 \approx -ik(\delta_{1,2} - \delta_{2,1})$$

$$\Gamma_2 \approx N k^2 \int dX_{0i} \Psi(X_{0i}) \delta_{1,i} \delta_{2,i}$$

**Large** **Small**

# MBI 6D Model



- MBI term ($\Gamma_2$):
  - Gaussian initial distribution, $\Psi(X_0)$
  - Nasty, but can be solved

$$\left\langle \left| b(\vec{k}) \right|^2 \right\rangle_{(\delta^2)} = \left( \frac{I}{I_A} \frac{2k^2}{\sigma^2 \sigma'^2} \right)^2 \frac{e^{-k^2 R_{56}^2 p_0^2}}{9(2\pi)^4} \int ds_1 \int ds_2 \frac{R^{(56)}_{s_1 \to L}}{\gamma_1^3} \frac{R^{(56)}_{s_2 \to L}}{\gamma_2^3} \int \frac{r_1 dr_1 d\theta_1}{J_1} \int \frac{r_2 dr_2 d\theta_2}{J_2} K_0\left(\frac{r_1 k}{\gamma_1}\right) K_0\left(\frac{r_2 k}{\gamma_2}\right) e^{-ikR^{(51)}_{s \to L}(r_2 \cos\theta_2 - r_1 \cos\theta_1)} G_2$$

$$G_2 \equiv \int dx_1' dy_1' \int dx_2' dy_2' e^{-i\tilde{K}\left[R_{s_2 \to L} Y_2 - R_{s_1 \to L} Y_1\right]} e^{-ikR^{(52)}_{s \to L}(x_2' - x_1')} \exp \frac{1}{3} \left[ \tilde{Y}_1 \tilde{R}_{s_1 \to 0} U^{-1} R_{s_2 \to 0} Y_2 - \left( \tilde{Y}_1 \tilde{R}_{s_1 \to 0} - \tilde{Y}_2 \tilde{R}_{s_2 \to 0} \right) U^{-1} \left( R_{s_1 \to 0} Y_1 - R_{s_2 \to 0} Y_2 \right) \right]$$

Gaussian integral over transverse angles, Y

# MBI 6D Model



- For short impedance section in high frequency limit:

$$\left\langle |b(\vec{k})|^2 \right\rangle_{\delta^2} \approx \frac{4}{3}\left[\frac{I}{I_A\gamma}\frac{R_{56}L}{\sigma^2}\right]^2 \frac{e^{\left[-\sigma'^2 k^2 \left(R_2^2+\theta_y^2 R_{34}^2\right)-k^2 R_{56}^2\sigma_p^2\right]}}{[\gamma^2 R_q^2+1]^2}$$

$$R_1 \equiv R_{51} + \theta_x R_{11}$$
$$R_2 \equiv R_{52} + \theta_x R_{12}$$

$$R_q \equiv \sqrt{R_1^2 + \theta_y^2 R_{33}^2}$$

- Compare to uniform E-field model:

$$\left\langle |b(\vec{k})|^2 \right\rangle_C = \left[\frac{I}{I_A\gamma}\frac{R_{56}L}{\sigma^2}\right]^2 e^{-k^2\left[R_{56}^2\sigma_p^2+\sigma^2(R_1^2+R_{33}^2\theta_y^2)+\sigma'^2(R_2^2+R_{34}^2\theta_y^2)\right]}$$

# MBI 6D Model

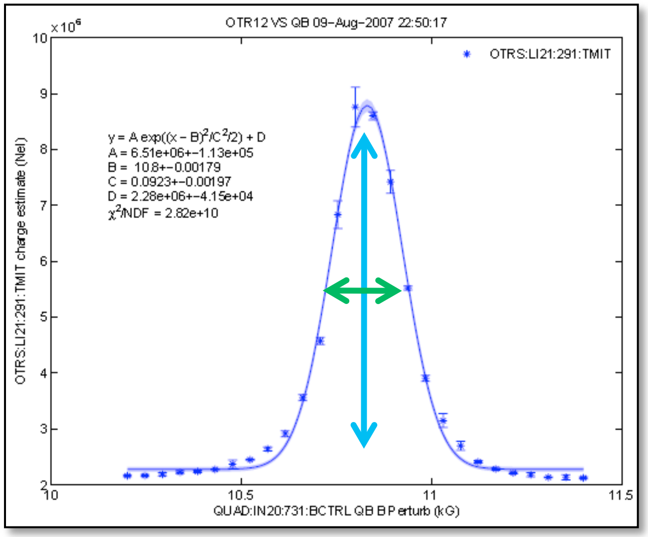
- Any upshots from model?
- LCLS predictions:
  - Weak (relatively) Lorentzian suppression
  - 3D regime has no $\gamma_0$ dependence
  - Can we observe either?

$$\left\langle |b(\vec{k})|^2 \right\rangle_{\delta^2} \approx \frac{4}{3}\left[\frac{I}{I_A\gamma}\frac{R_{56}L}{\sigma^2}\right]^2 \frac{e^{\left[-\sigma'^2 k^2\left(R_2^2+\theta_y^2 R_{34}^2\right)-k^2 R_{56}^2\sigma_p^2\right]}}{\left[\gamma^2 R_q^2+1\right]^2}$$

**No $\gamma_0$ dependence** **Lorentzian Suppression**

# MBI at LCLS

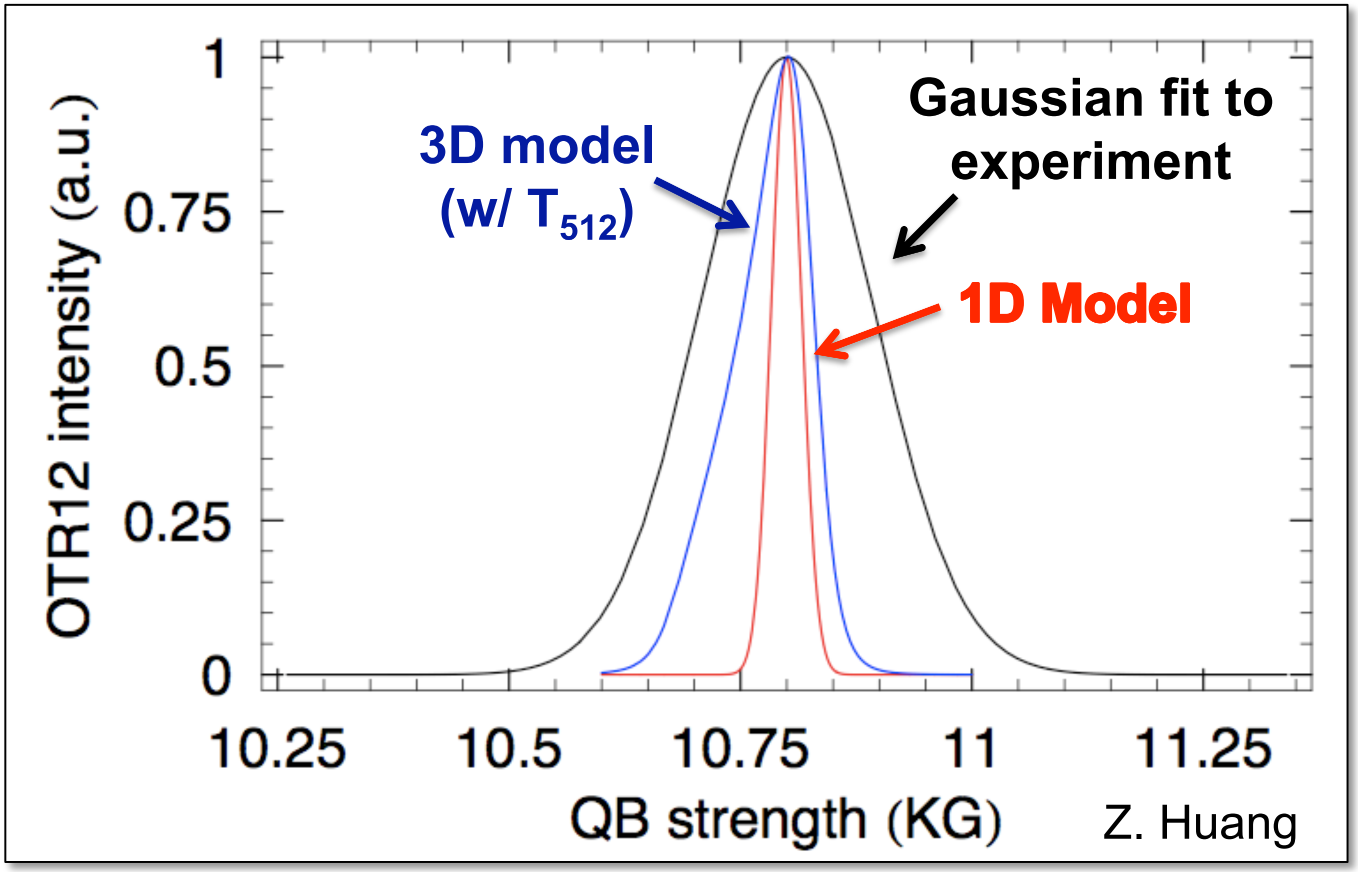
- Any upshots from model?
- LCLS predictions:
  - Weak (relatively) Lorentzian suppression
  - 3D regime has no $\gamma$ dependence
  - Can we observe either? QB Curve




**Gain → Height**

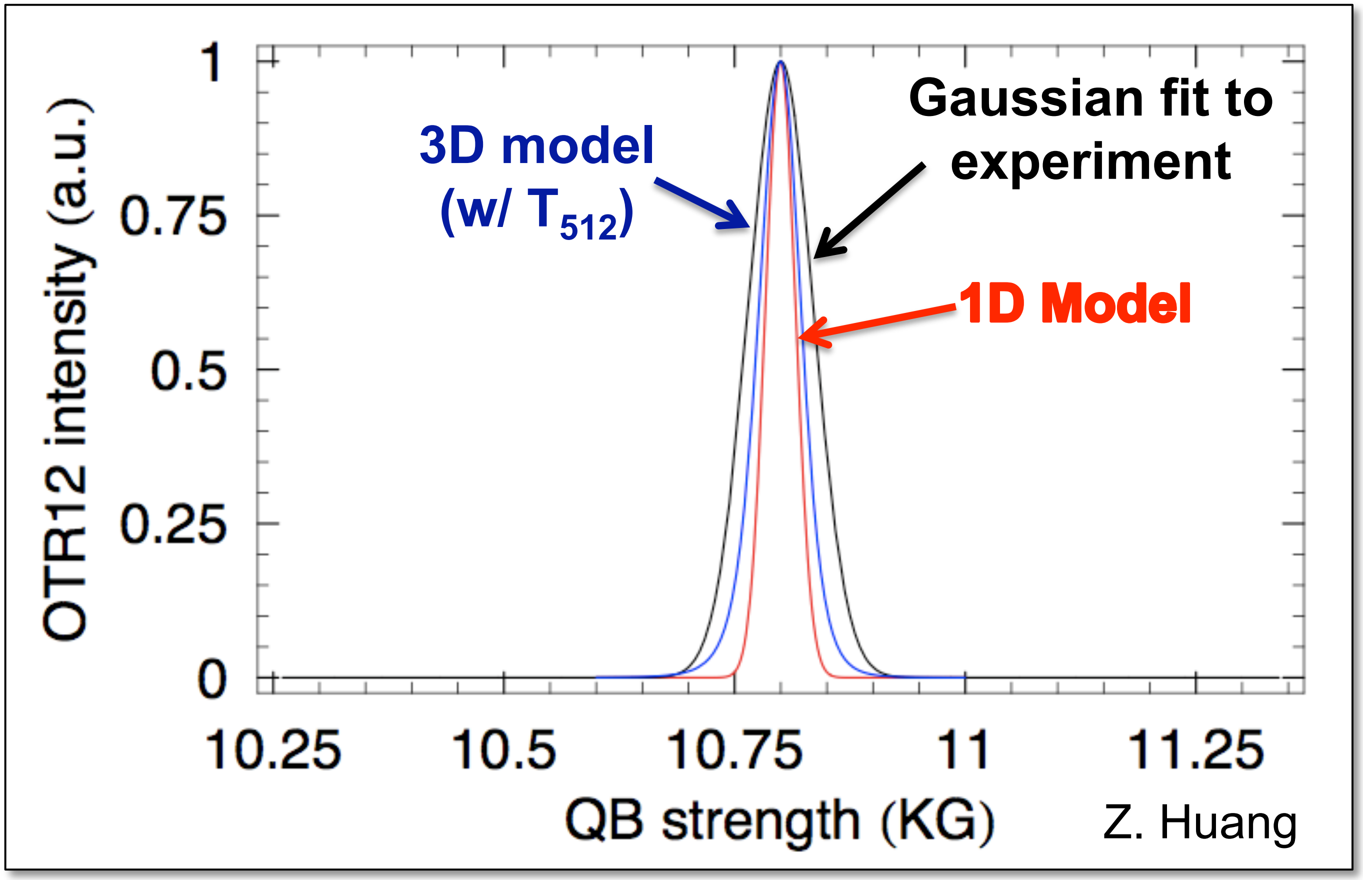
**Suppression → Width**

- LCLS QB curve (2007):



# MBI at LCLS

- LCLS QB curve (2008):



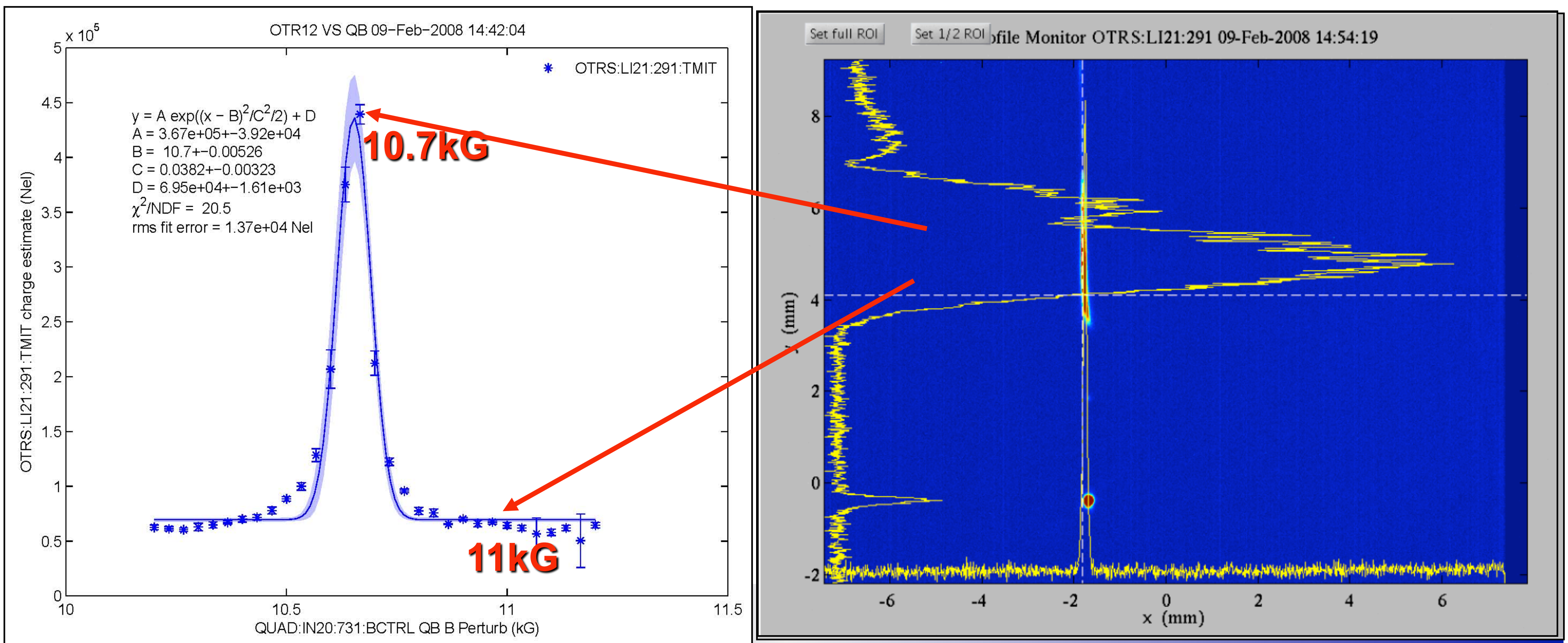
# Spectral Gain



- Spectral OTR data in optical range
- 2 images with BC1 off, 250pC (D. Dowell et al., 2008)

No COTR (QB = 11 kG, nonzero R51&R52 after DL1 suppress $\mu$-bunching)

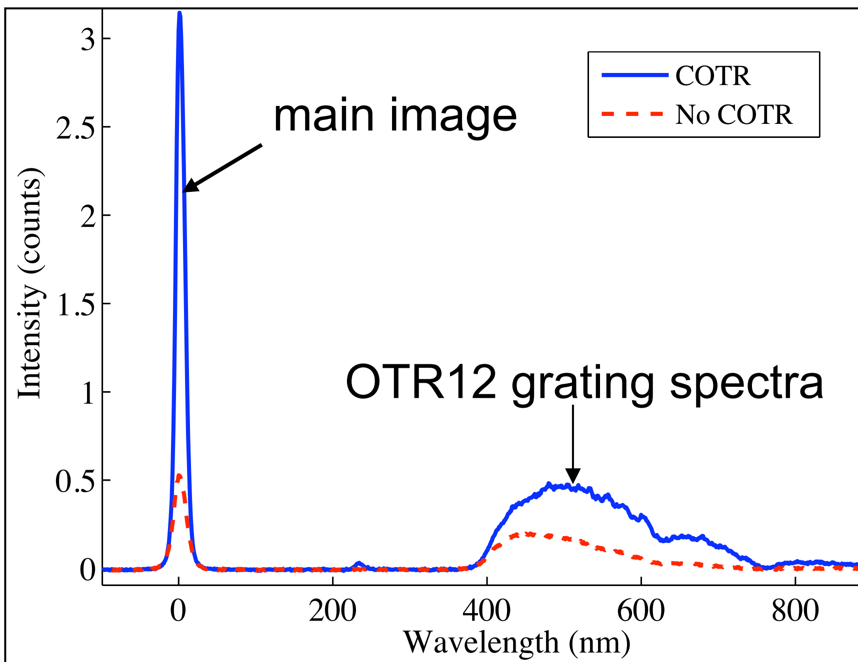
COTR (QB = 10.7 kG, DL1 is linear achromat, $\mu$-bunching enhances signal)

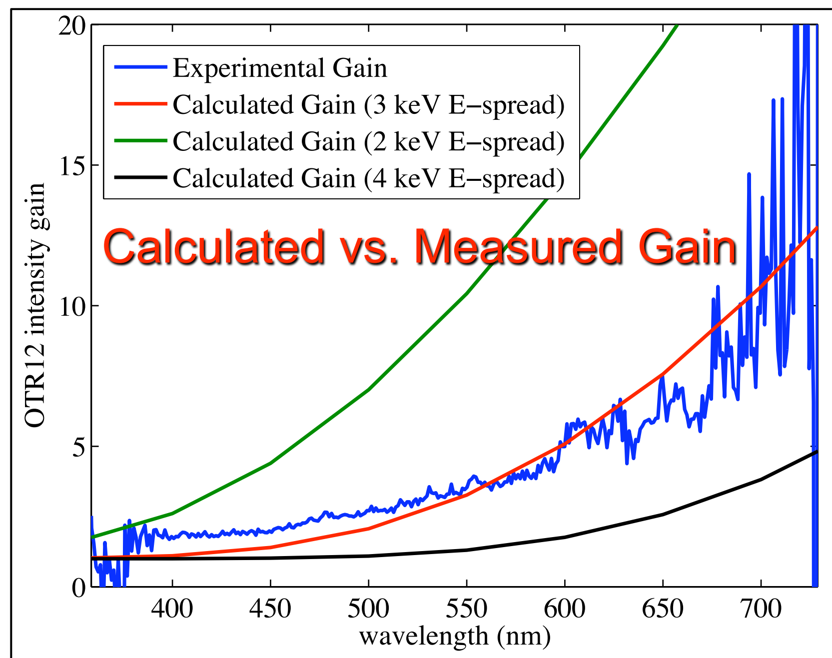


# Spectral Gain



- Spectral data in optical range
  - Exp. intensity gain from ratio of COTR to No COTR spectra
  - Calculated intensity gain from 40 A peak current (BC1 off), 1 μm emittance ➔ 3 keV slice energy spread



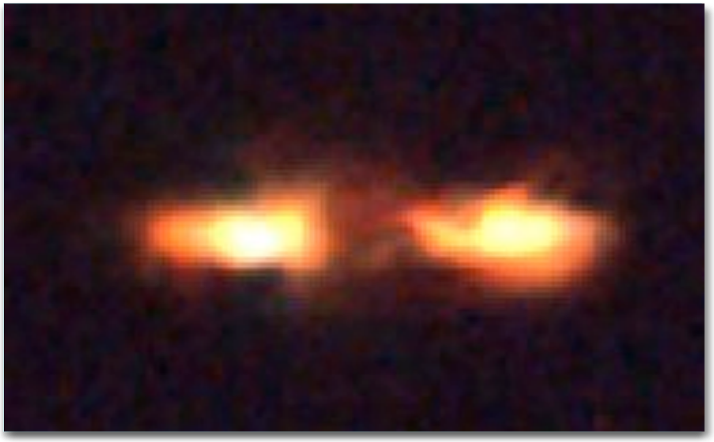



# Shot Noise Suppression

- Amplification Summary:
  - 6D model calculates MBI for arbitrary accelerator motion
  - No solid experimental confirmation
  - Will compare with Impact simulations soon
  - Further experiments?

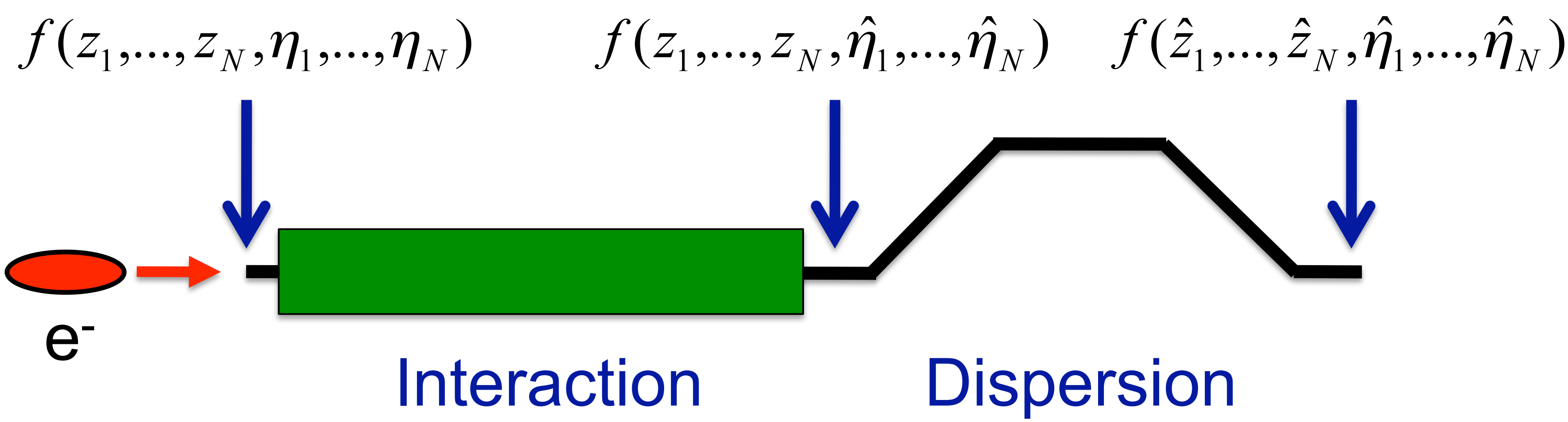
# Introduction

- Amplification:
  - Microbunching Instability (MBI)
  - 6D MBI model from shot noise
- Suppression:
  - Creating quiet electron beams (below shot noise)
  - General description of noise suppression



## Shot Noise suppression

- Proposed by Gover and Litvinenko
- Suppress MBI, improve seeding, …?
- Ignore transverse coordinates (1D model)
- Arbitrary interaction, $h(\zeta = z_1 - z_2)$



# Shot Noise suppression

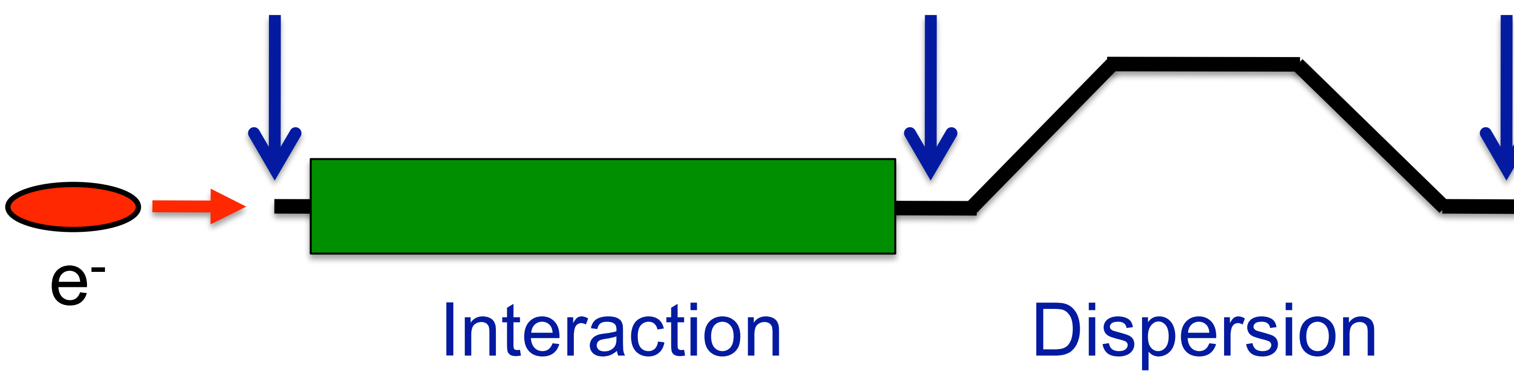
$$\langle F(k)\rangle \approx 1 + n_0 e^{-k^2 R_{56}^2 \sigma_\eta^2} \left\{ \int_{-\infty}^{\infty} d\zeta e^{ik\zeta} \left[\Gamma_1(\zeta) + \Gamma_2(\zeta)\right] \right\}$$

particle density

$$\Gamma_1(\zeta) \approx ikR_{56}\left[h(\zeta) - h(-\zeta)\right]$$

$$\Gamma_2(\zeta) \approx n_0 k^2 R_{56}^2 \int_{-\infty}^{\infty} d\tau \left[h(-\tau+\zeta)h(-\tau) - h(-\tau)^2\right]$$

$$f(z_1,\ldots,z_N,\eta_1,\ldots,\eta_N) \qquad f(z_1,\ldots,z_N,\hat{\eta}_1,\ldots,\hat{\eta}_N) \qquad f(\hat{z}_1,\ldots,\hat{z}_N,\hat{\eta}_1,\ldots,\hat{\eta}_N)$$



# Shot Noise Suppression

- Taking Fourier transform we find

$$\langle F(k)\rangle \approx 1 - 2n_0 k R_{56}\mathrm{Im}\,[\tilde{h}(k)]\cancel{e^{-k^2R_{56}^2\sigma_\eta^2}} + n_0^2k^2R_{56}^2|\tilde{h}(k)|^2\cancel{e^{-k^2R_{56}^2\sigma_\eta^2}}$$

- And for imaginary FT{h}

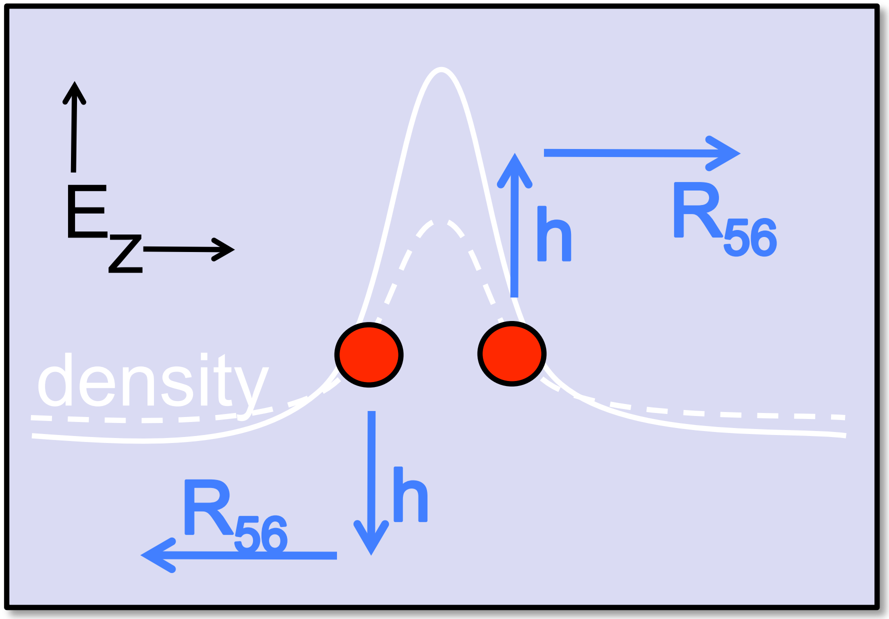
$$\boxed{\langle F(k)\rangle \approx (1-\Upsilon)^2} \qquad \boxed{\Upsilon \equiv n_0 k R_{56}\mathrm{Im}\,[\tilde{h}(k)]}$$

- Noise is suppressed!
- For step function, FT{h} ~ A/k, noise suppressed at all freq: $\Upsilon = n_0 R_{56} A_u$

# Shot Noise Suppression



- Physical picture: why imaginary FT?




**Space Charge**

# Shot Noise Suppression



## ❑ Undulator Case: helical undulator

$$h_u(\zeta) = \begin{cases} -A_u\left(1 - \frac{\zeta}{N_u\lambda_0}\right)\cos k_0\zeta & 0 < \zeta < N_u\lambda_0 \\ 0 & \text{otherwise} \end{cases}$$

$$A_u \equiv 2\pi\frac{e^2K^2N_u\lambda_u^2}{S\gamma^3mc^2\lambda_0} = 4\pi\frac{r_eL_u}{S\gamma}\frac{K^2}{1+K^2}$$

**With undulator strength, $A_u$, periods, $N_u$, and resonant wavelength, $\lambda_0$**

# Shot Noise Suppression

- Undulator Case: FT of interaction

$$\tilde{h}_u(k) = -iA_u N_u \lambda_0 \left[ \frac{m}{(m^2-1)\alpha} - i\frac{(1+m^2)(1-e^{im\alpha})}{(m^2-1)^2\alpha^2} \right]$$

$$m \equiv k/k_0\ , \quad \alpha \equiv 2\pi N_u$$

**High Frequency Limit:**

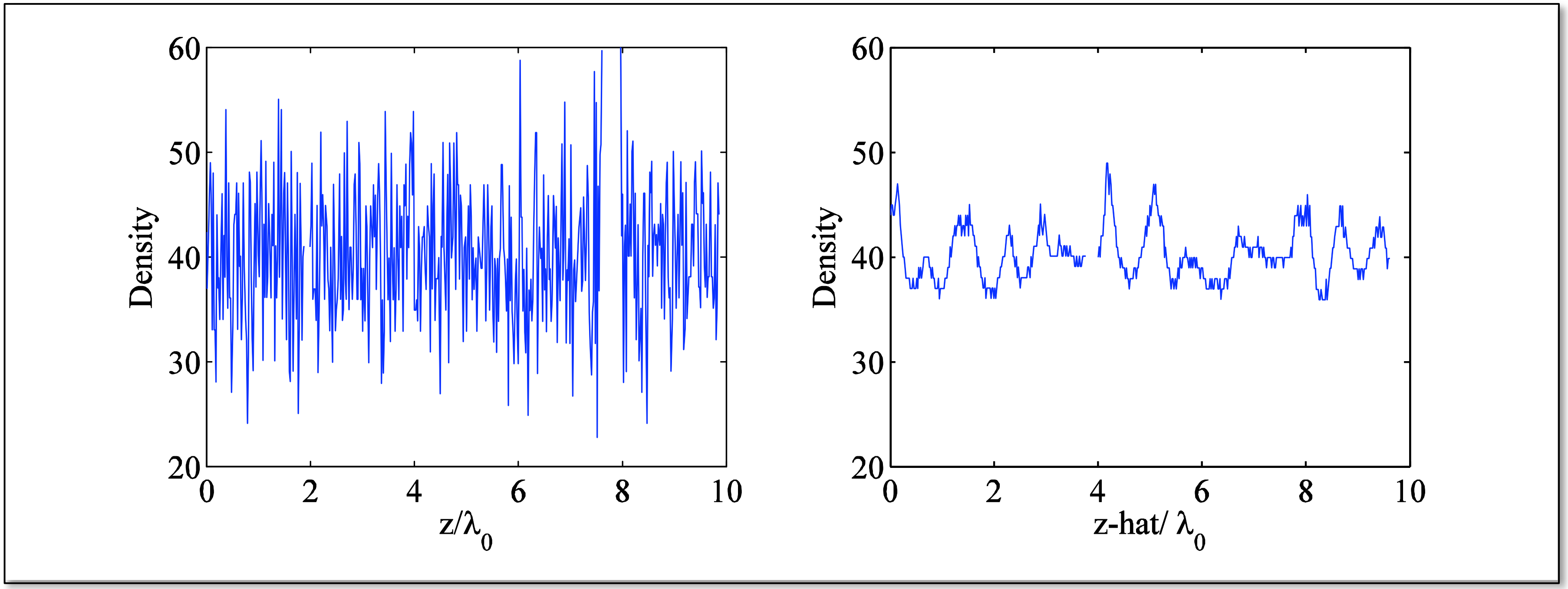
$$\tilde{h}_u(k) \approx -i\frac{A_u}{k}$$

$$\Upsilon_u = -A_u n_0 R_{56}$$

# Shot Noise Suppression

- Simulation illustrates undulator case
  - 1D code with interaction and dispersion

$Y_u$=1, $N_u$=1

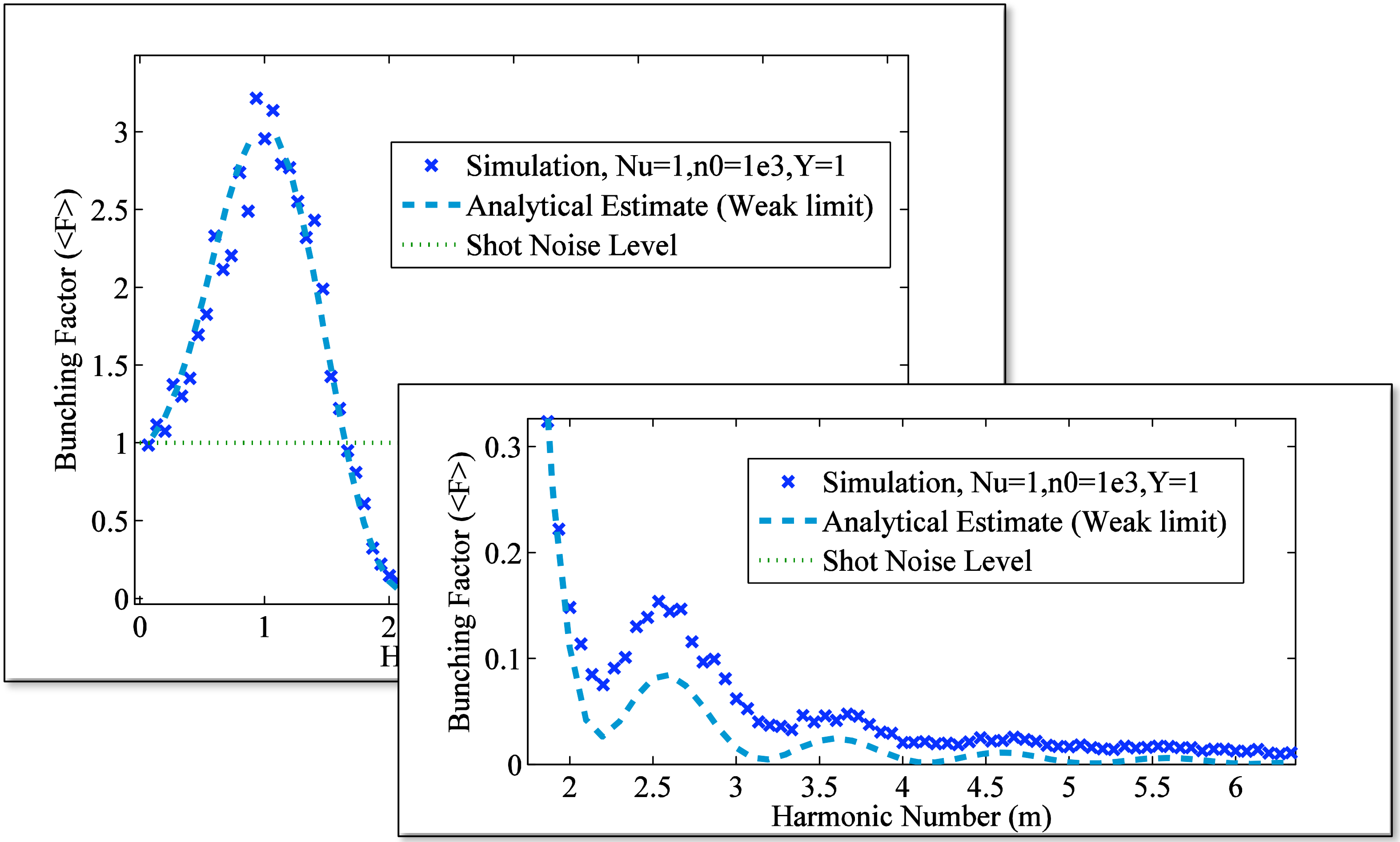



**Initial Distribution** **Final Distribution**

# Shot Noise Suppression



- ❑ Simulation illustrates undulator case



# Shot Noise Suppression

- Need better approximation

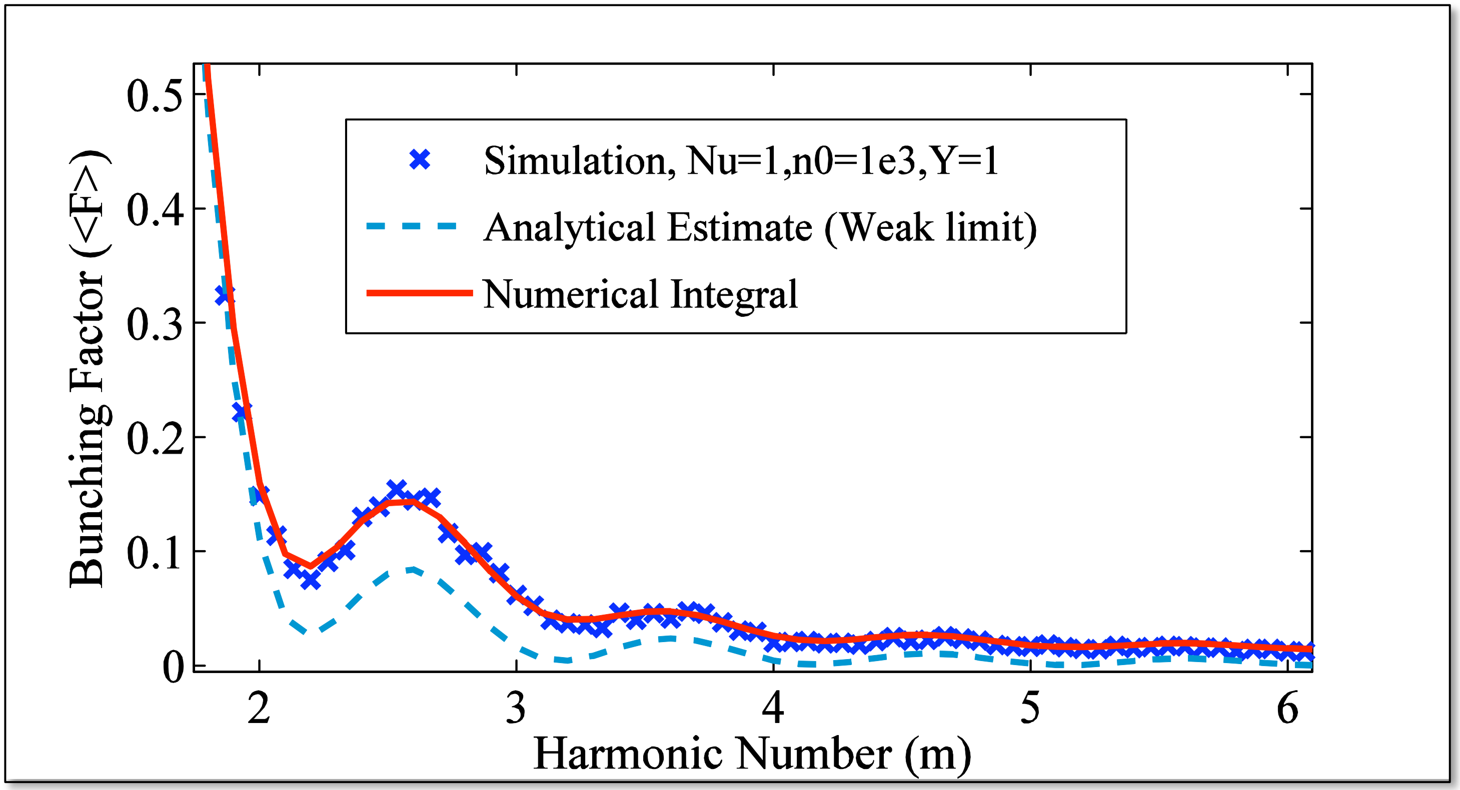
$$\langle F(k)\rangle_C \approx n_0 e^{-k^2 R_{56}^2 \sigma_\eta^2} \int_{-\infty}^{\infty} d\zeta e^{ik\zeta} \left[1+\Gamma_1(\zeta)\right] \left[1+\frac{1}{N}\Gamma_2(\zeta)\right]^{N-2}$$

$$\langle F(k)\rangle_C = n_0 e^{-k^2 R_{56}^2 \sigma_\eta^2} \int d\zeta e^{ik\zeta} e^{\Gamma_2(\zeta)} [1+\Gamma_1(\zeta)]$$

$$\langle F(k)\rangle_C = n_0 e^{-k^2 R_{56}^2 \sigma_\eta^2} \int d\zeta \left[\left(e^{\Gamma_2(\zeta)} - e^{\bar{\Gamma}_2}\right)\cos(k\zeta) + e^{\Gamma_2(\zeta)}\Gamma_1(\zeta)\sin(k\zeta)\right]$$

# Shot Noise Suppression

- ❑ Simulation illustrates undulator case



# Shot Noise Suppression

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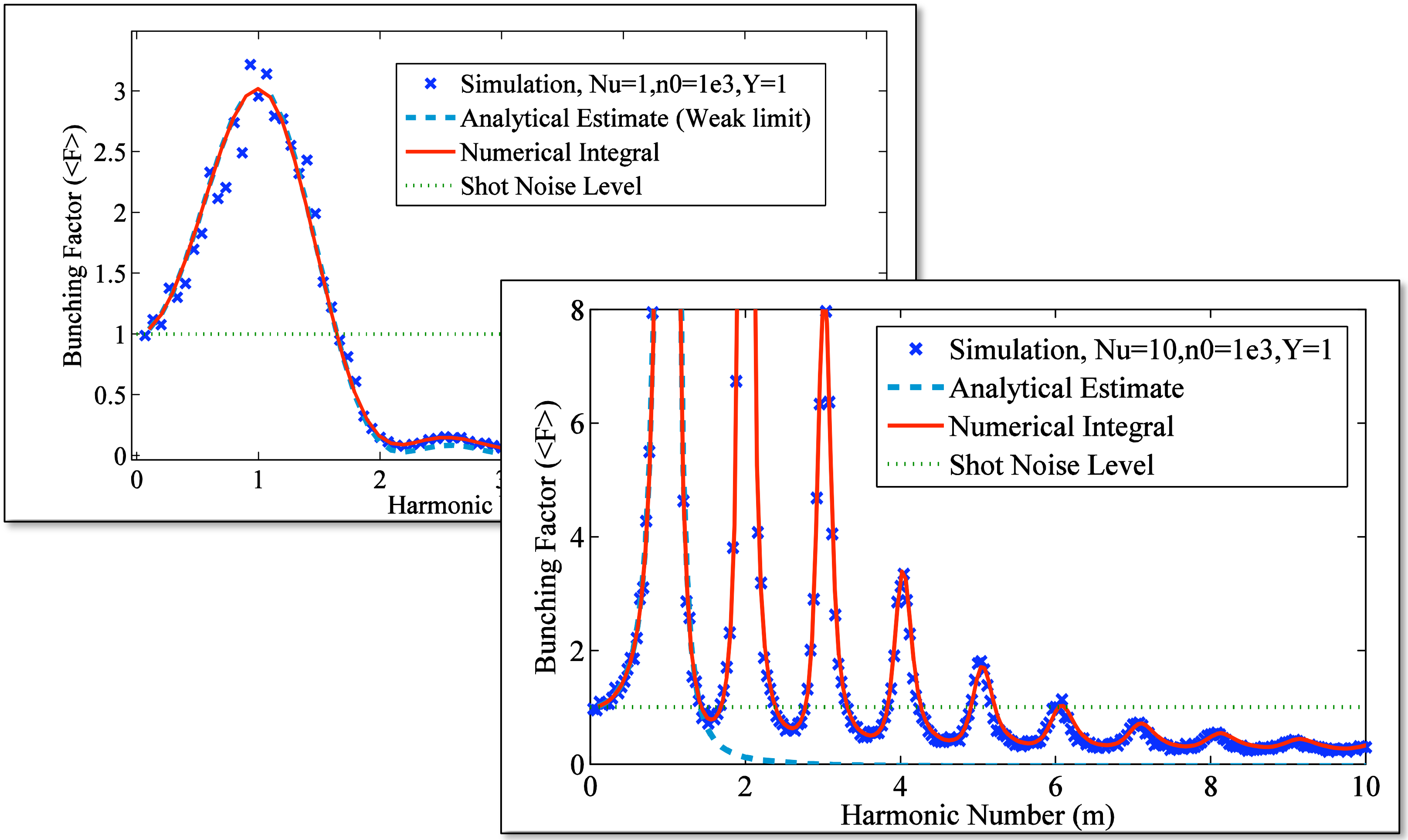
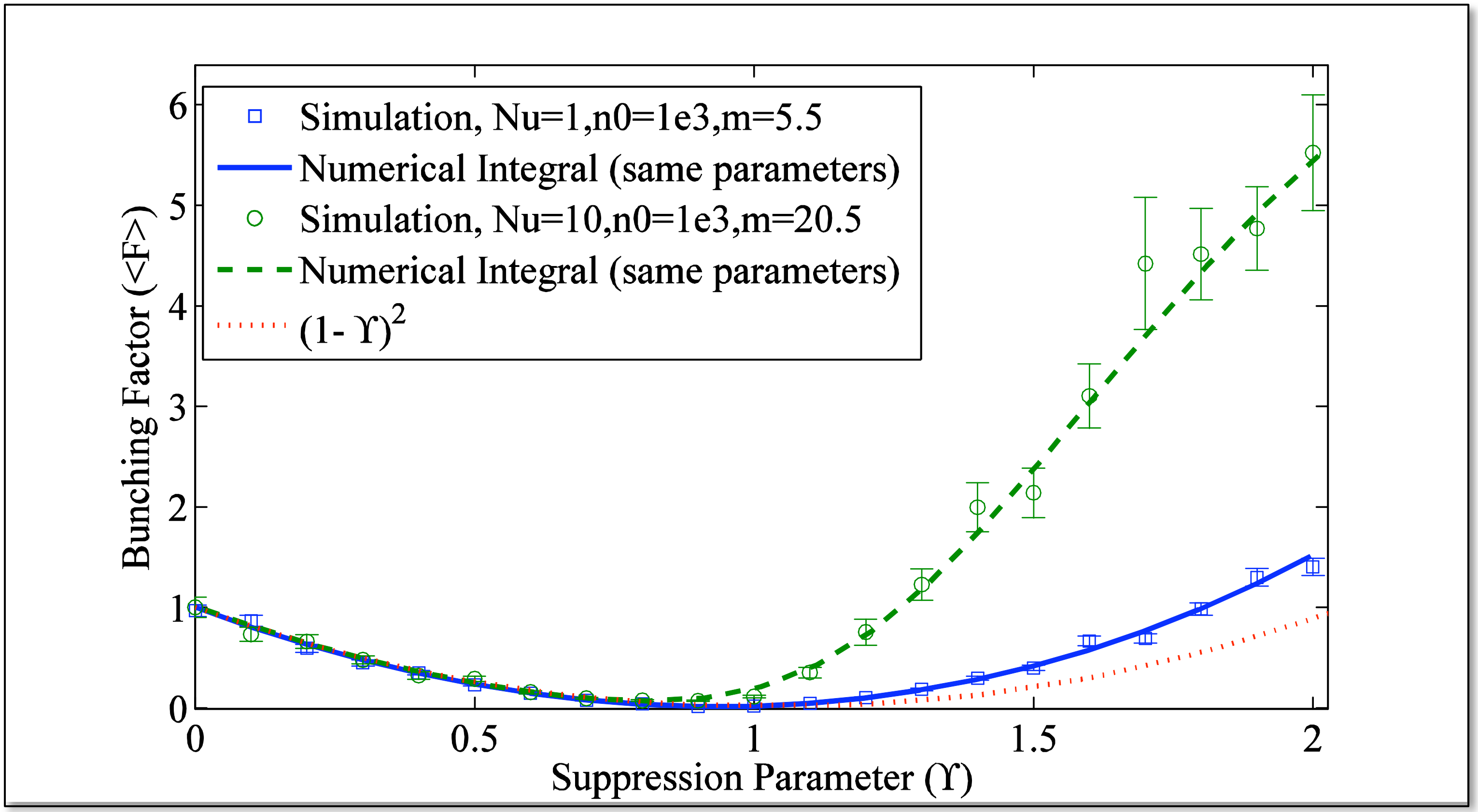


# Shot Noise Suppression

- Check $(1-\Upsilon)^2$ scaling



# Shot Noise Suppression



- What is distribution at particle level?
  - take limit of no energy spread

$$\boxed{\Delta z_k^{\text{new}} = \Delta z_k + R_{56}(\Delta E_{k+1} - \Delta E_k)}$$

$$\boxed{\Delta E_k = \sum_{i=1}^{N} h(z_k - z_i)\,, \qquad \Delta E_{k+1} = \sum_{i=1}^{N} h(z_{k+1} - z_i)}$$

$$\boxed{\Delta E_{k+1} - \Delta E_k = \left[h(\Delta z_k) - h(-\Delta z_k)\right] + \Delta z_k \underbrace{\sum_{i\neq k}^{N} h'(z_k - z_i)}_{-n_0[h(0^+) - h(0^-)]}}$$

for step function: $n_0 A \Leftarrow \boxed{-n_0[h(0^+) - h(0^-)]}$

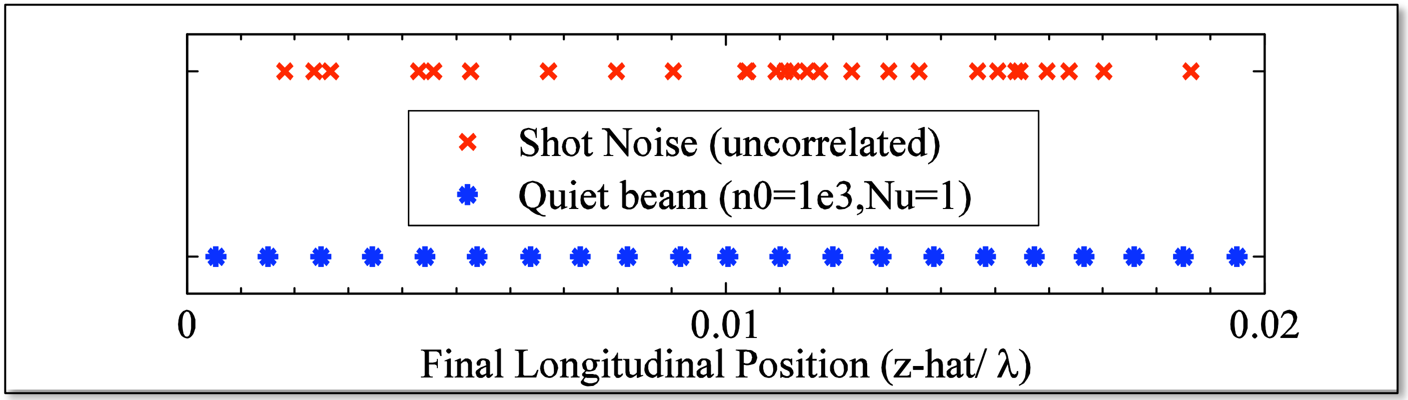
# Shot Noise Suppression



- What does particle level look like?

$$\Delta z_k^{\text{new}} = R_{56}A + (1 - R_{56}n_0A)\Delta z_k$$

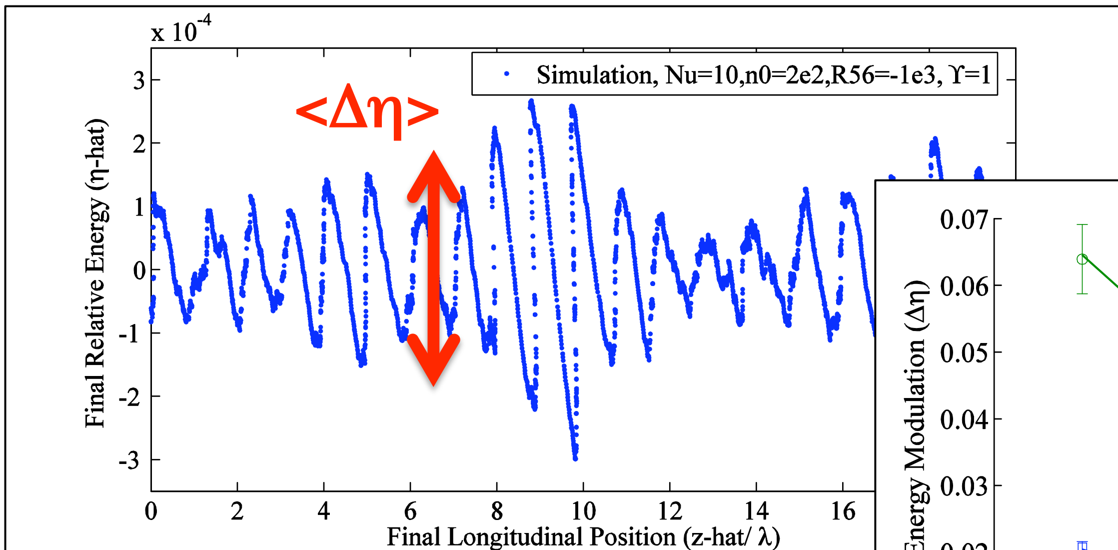
$$\Upsilon = R_{56}n_0A = 1 \quad \Rightarrow \quad \Delta z_k^{\text{new}} = \frac{1}{n_0}$$

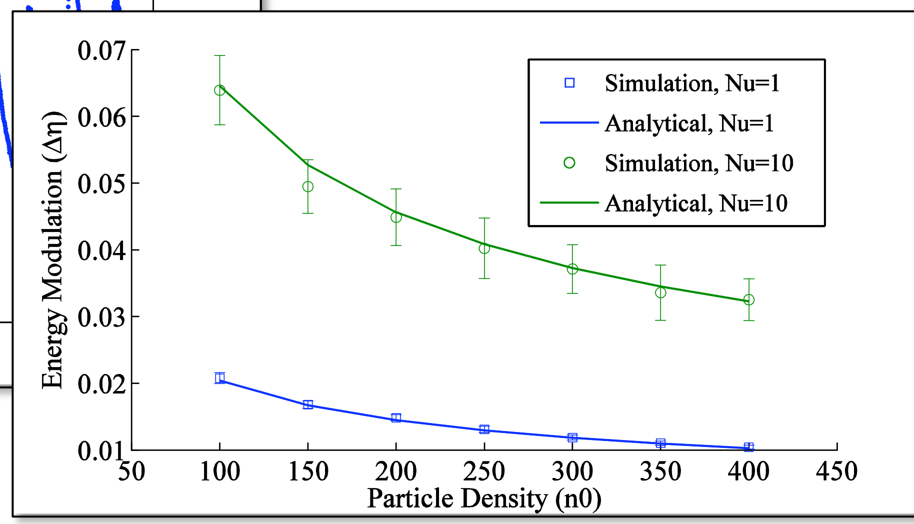



- Crystalline beam!

- Energy spread and modulation

$$\langle h_u^2(\zeta)\rangle \approx \frac{A_u^2}{6} \quad \Rightarrow \quad \langle \Delta\eta \rangle \approx -\sqrt{\frac{N_u\lambda_0}{6n_0}\frac{1}{R_{56}}}$$



# Shot Noise Suppression



- Energy spread washes out suppression:

$$\boxed{e^{-k^2 R_{56}^2 \sigma_\eta^2}} \Rightarrow kR_{56}\sigma_\eta \ll 1$$

- Decreasing $R_{56}$, decrease $\lambda_{min}$…
- But increases energy spread

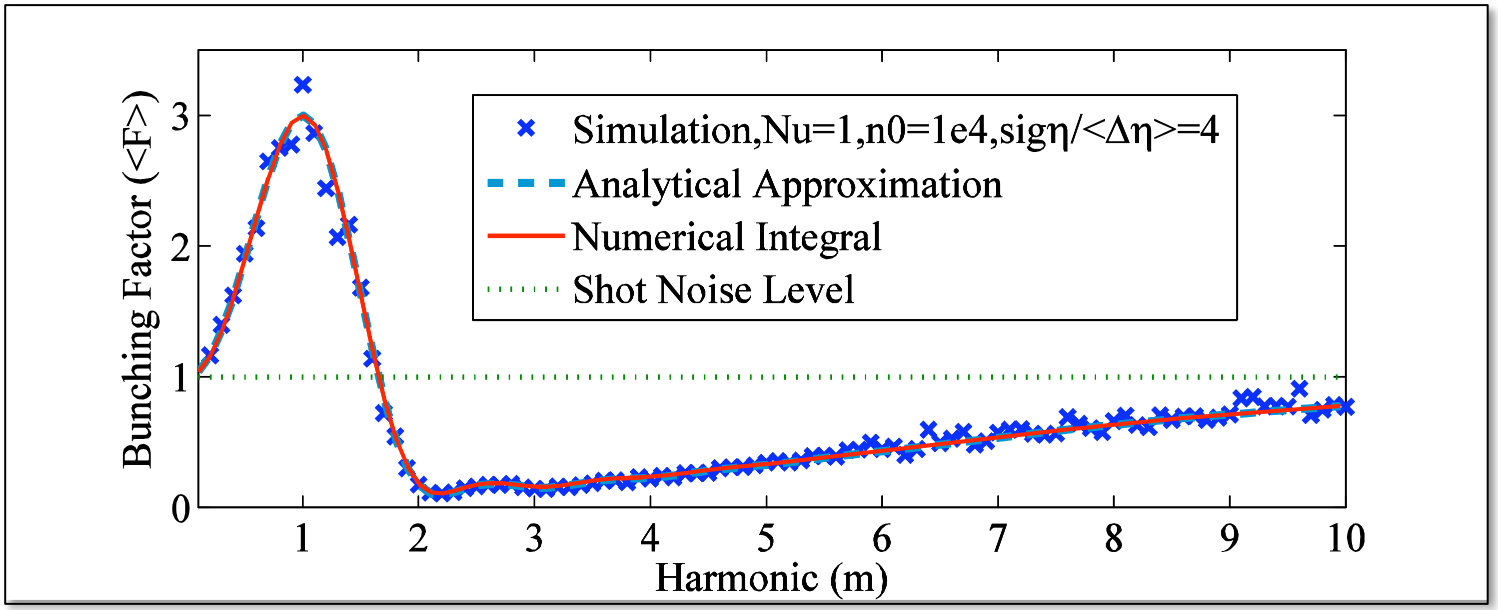
$$\boxed{\langle \Delta\eta \rangle \approx -\sqrt{\frac{N_u \lambda_0}{6 n_0}} \frac{1}{R_{56}}}$$

# Shot Noise Suppression

- Sets lower limit on suppression wavelength:

$$\lambda_{\min} = 2\pi\sqrt{\frac{N_u\lambda_0}{6n_0}\frac{\sigma_\eta}{\langle\Delta\eta\rangle}}$$

**Modulation can be smaller than energy spread**



# Shot Noise Suppression

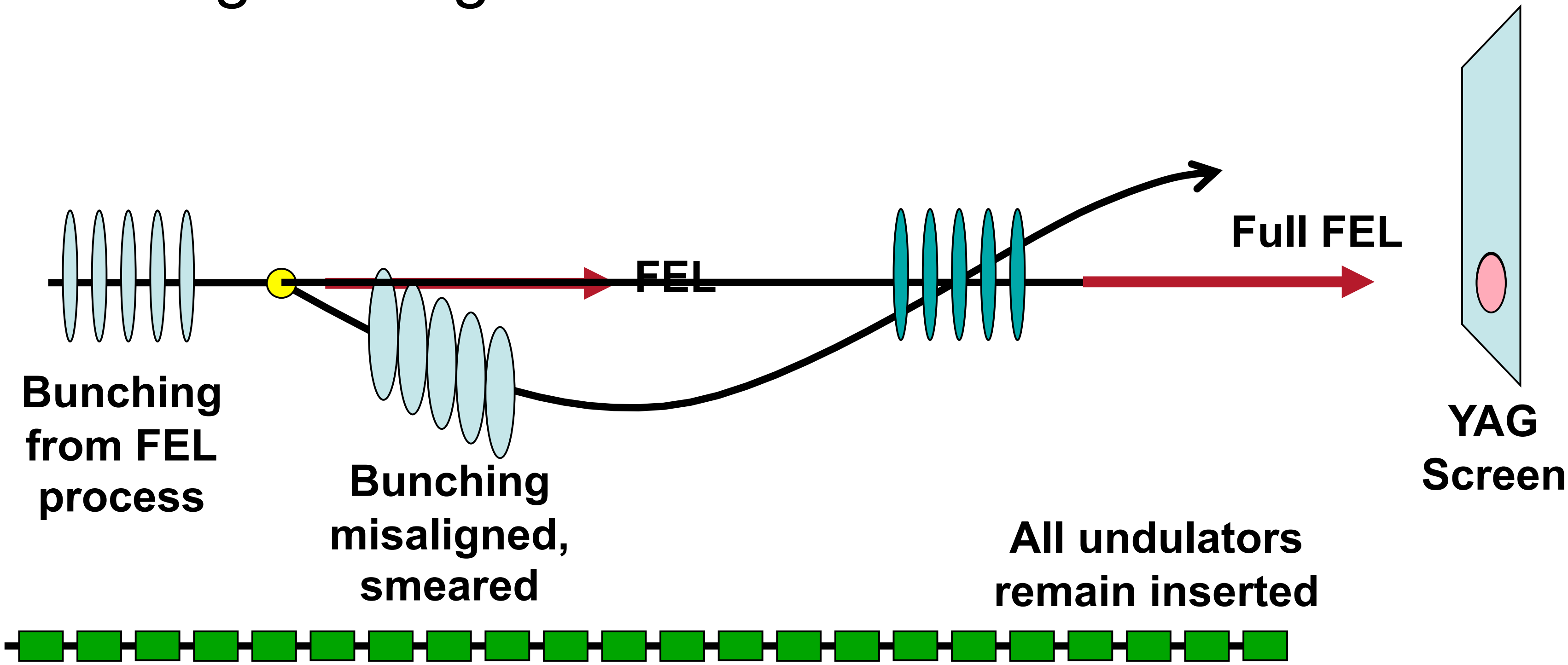


- Noise Suppression Summary:
  - Step function interactions suppress shot noise at wide bandwidths
  - In cold beam limit, produces crystalline beam
- Experimental test?
  - NLCTA space charge case:
    - For 10m section, 100 MeV, 20 A, 1mm radius
      - ➔ 2mm $R_{56}$ to suppress shot noise
    - But need to study true 3D system first…

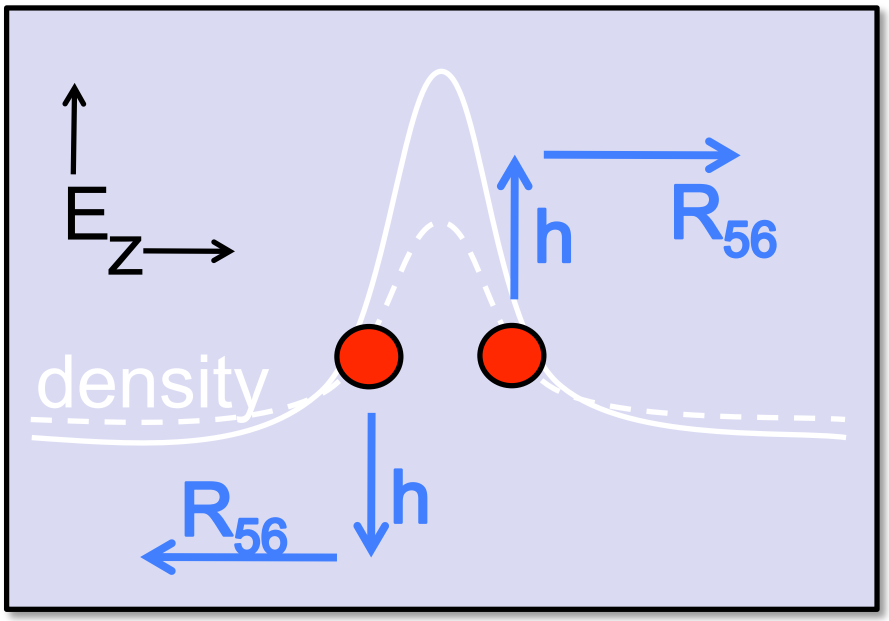
Thanks!

# Gain Length Measurement

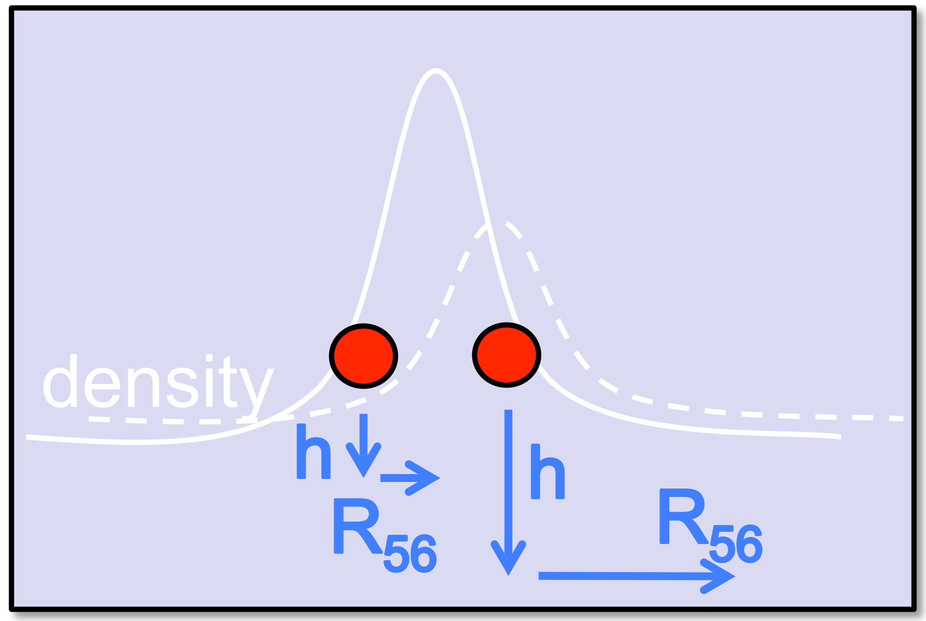
- Instability may affect FEL performance
  - Laser heater suppresses instability
  - Use gain length as FEL metric



- Physical picture: why imaginary FT?




**Space Charge**




**Undulator**

# Microbunching Gain Scaling

- At high frequency ($\sigma/\bar{\lambda}\gamma >> 1$), longitudinal space charge (LSC) field has no $\gamma$ dependence

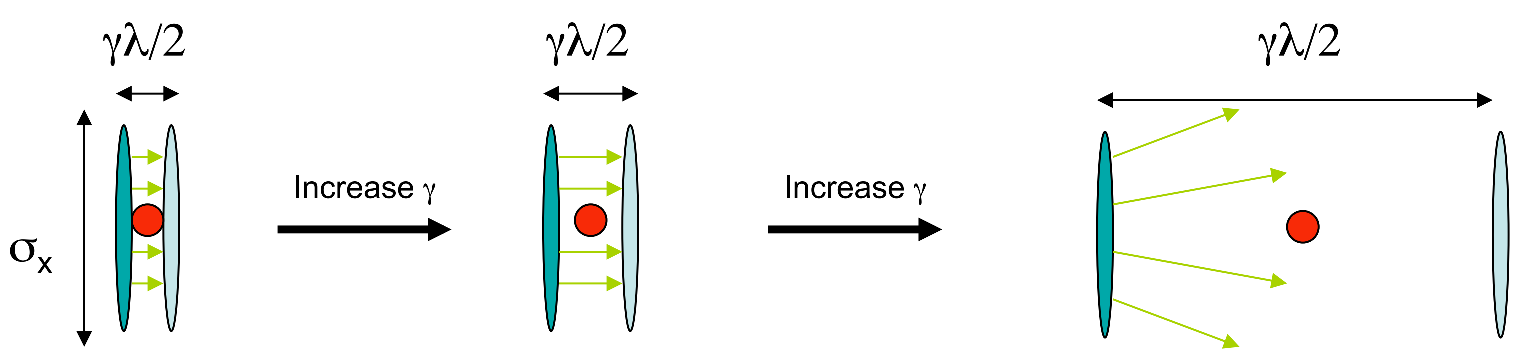
  ➔ LSC field proportional to electron volume density

  ➔ impedance inversely proportional to transverse beam area ($\sigma^{-2}$)

  ➔ bunching dominated by smallest beam, not lowest energy

High frequency/Pancake beam

Low frequency/Pencil beam




Fat beam E-field independent of $\gamma\lambda$

Pencil beam E-field scales as $\gamma^{-2}$

# Gain Calculation



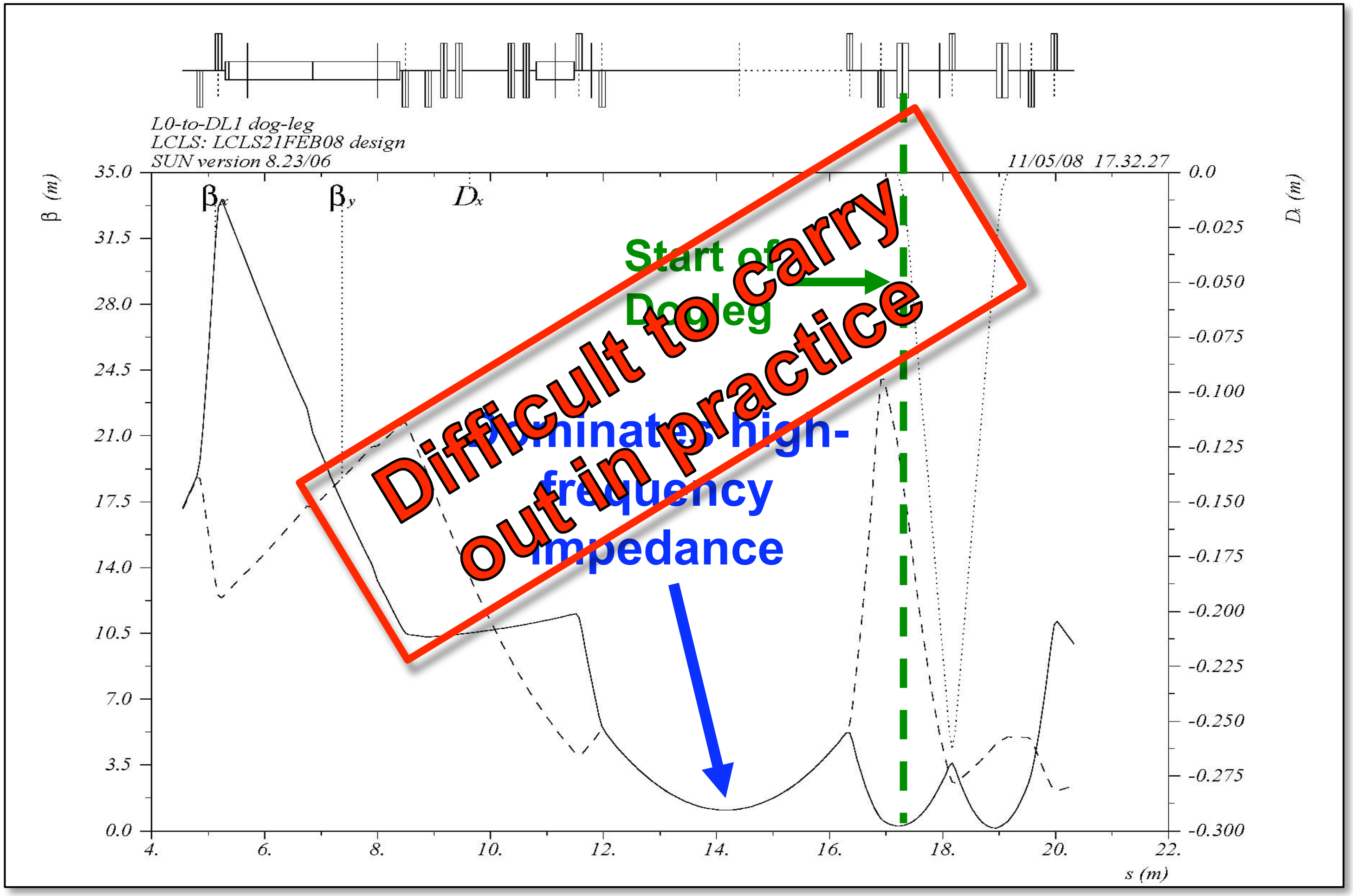
- Drift space 3D gain calculation:

$$\left\langle |b_k|^2 \right\rangle \propto \left(\frac{I}{I_A}\right)^2 \frac{R_{56}^2}{\varepsilon^2} \underbrace{\int_{-L/\beta}^{L/\beta}\int_{-L/\beta}^{L/\beta} \frac{\mathrm{d}s_1 \mathrm{d}s_2}{4(1+s_1)^2(1+s_2)^2 - (1+s_1 s_2)^2}}_{\text{Numerical factor gives } \beta \text{ dependence}} \mathrm{e}^{-R_{56}^2 k^2 \sigma_\delta^2}$$

(ε is normalized emittance, β is value at waist, L = 2m)

- Nominal case: β = 1.2m
  - Widen waist to β = 5 ➔ gain decreases by factor of 7
  - Narrow waist to β = 1/3 ➔ gain increases by 80%
  - Suggests changing waist size should change results!

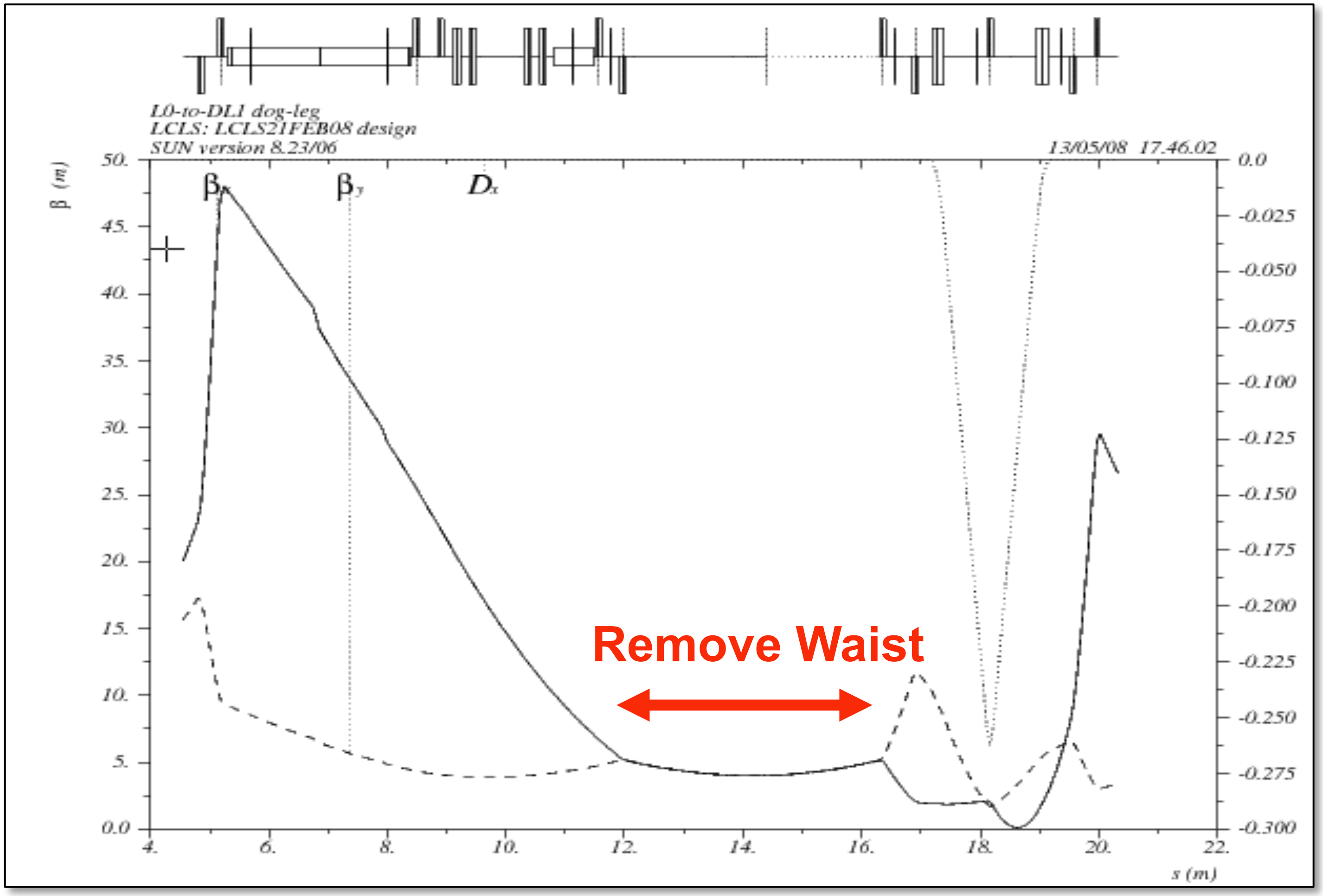
## Standard Lattice



# Big Waist Matching



## Settings to suppress microbunching



# Small Waist Matching



## Settings to amplify microbunching