# Modelling of Wakefields from Sub-Relativistic Beams


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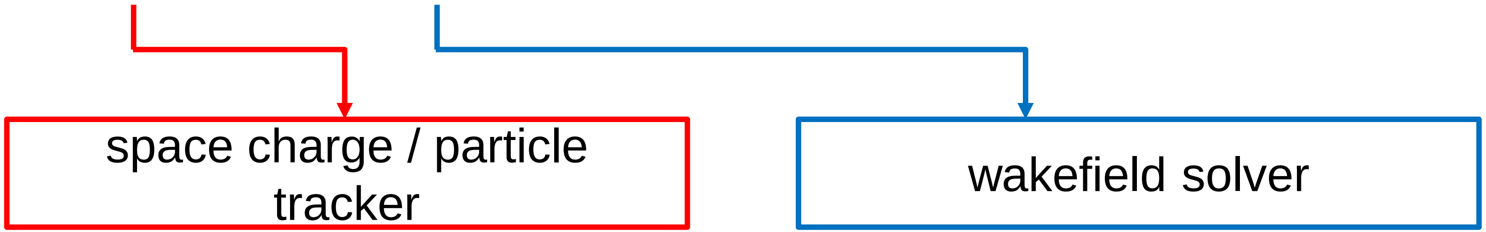
# Outline

- Beam dynamics: Solvers @TEMF
- Scattered Field Formulation for Coupled Space Charge and Wakefield Calculations
- Results for traveling wave gun @SwissFEL

# Task

- **Solve Maxwell's eqs. + Eq. of Motion:**
  - # Particles, Geometry, multi-scale → ~~Full EM-Particle in cell~~




**space charge / particle tracker**

- Poisson eq. in Lorentz frame
- Free-space assumption
- − No transient fields

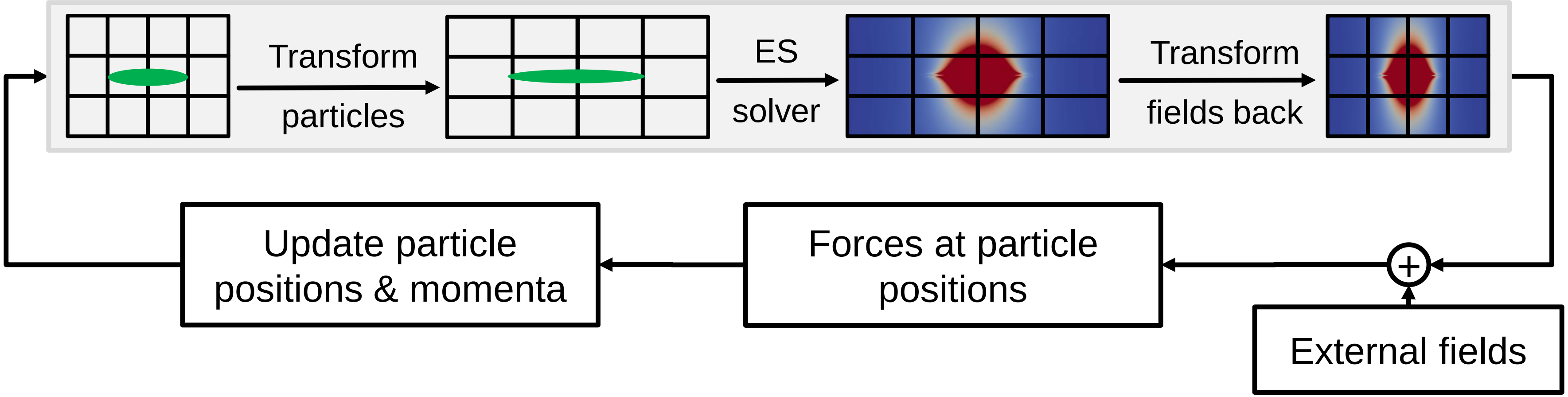
**wakefield solver**

- EM wave eq.
- Particles => current
- − No intermediate feedback

Solver @TEMF: REPTIL | PBCI

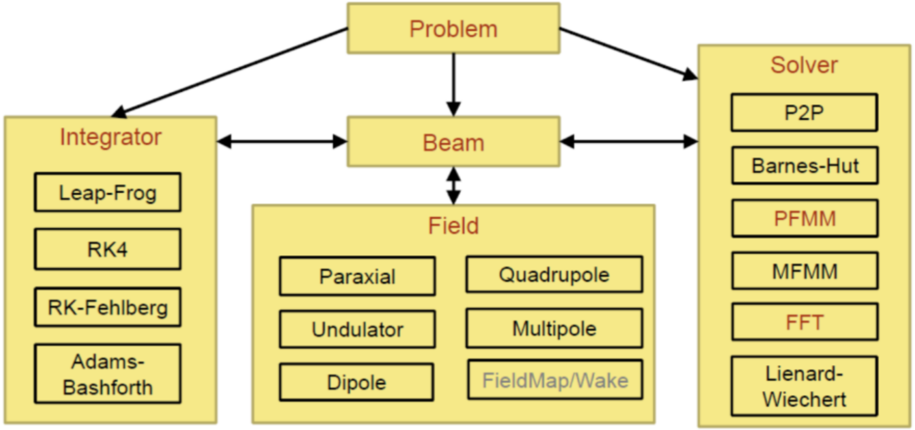
# Particle Tracking in REPTIL

- **Solve Space Charge field + Eqs. of motion**
  - Assume particle cloud in free space + nearly-uniform movement
  - Electrostatic field solver in particle's rest frame

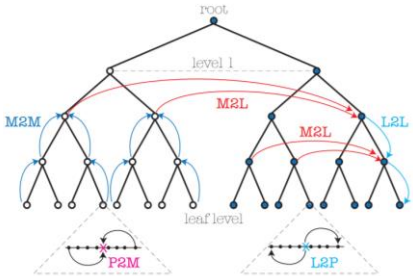


# Particle Tracking in REPTIL

- **Relativistic Particle Tracker for Injectors and Linacs (REPTIL)**
  - Nx6D time domain, multi-node & multi-thread
  - Space Charge Field solvers: Grid-based (e.g. 3D-**FFT**) or non-grid (**FMM**, LW)
  - Time integrators (adaptive, symplectic, …)
  - Fieldmaps, optimization engine




$$\varphi(x) = \int G(x - x')\varrho(x')d^D x$$



# Wakefield Simulation in PBCI

- **Solve Maxwell's wave equations in time domain**

$$\frac{d}{dt} E = \varepsilon^{-1} \operatorname{curl} H - \varepsilon^{-1} J$$
$$\frac{d}{dt} H = -\mu^{-1} \operatorname{curl} E$$

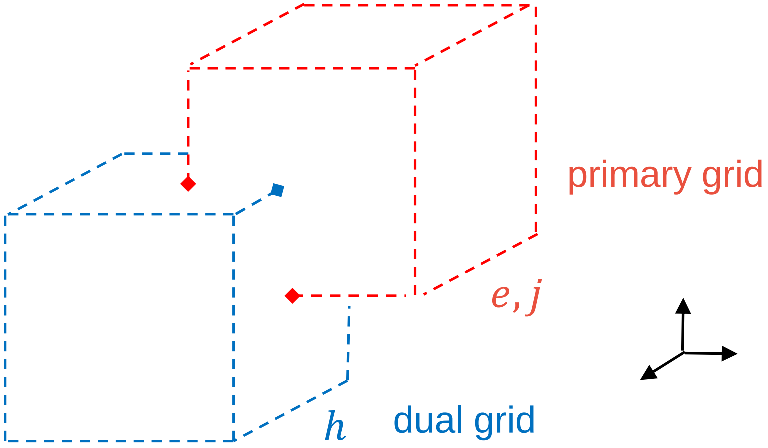
Discretization → FIT / FDTD

current through grid faces

voltages over grid edges

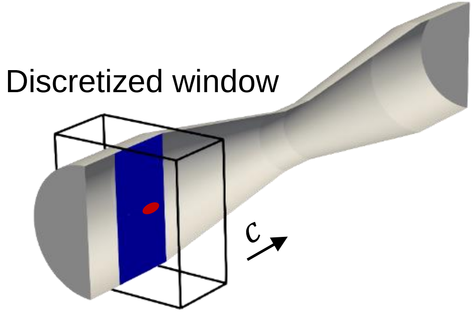
$$\frac{d}{dt}\begin{pmatrix} h \\ e \end{pmatrix} = \begin{pmatrix} 0 & -M_\mu^{-1} C \\ M_\varepsilon^{-1} C^T & 0 \end{pmatrix}\begin{pmatrix} h \\ e \end{pmatrix} - \begin{pmatrix} 0 \\ M_\varepsilon^{-1} j \end{pmatrix}$$

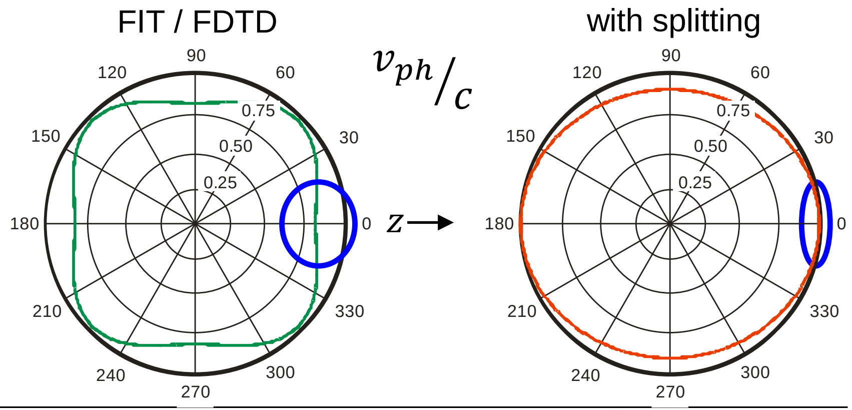
- Wakefield codes: particle beam is
  1) ultra-relativistic ($v_z = c$) and
  2) rigid ($v_z =$ const.)
  
  → prescribed current density $j$
  
  → kicks (per component)



# Wakefield Simulation in PBCI

- **Wakefield solver Parallel Beam Cavity Interaction (PBCI)**
  - Especially for short relativistic bunches, long transients
  - 3D time domain, **boundary conformal** FIT / FDTD Maxwell EM-wave solver, multi-node & multi-thread
  - **Moving window**, dispersion-free along z (**operator splitting**), PML, SIBC, conducting material, indirect integration



# Scattered Field Formulation

- **Idea: Separate field contributions**

"Incident field"
- Mxw. with beam current

$$\frac{d}{dt} E = \varepsilon^{-1} \operatorname{curl} H - \varepsilon^{-1} J$$
$$\frac{d}{dt} H = -\mu^{-1} \operatorname{curl} E$$

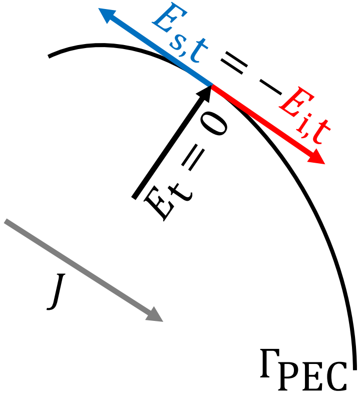
BC: $E_t = 0$ on $\Gamma_{PEC}$

"Scattered field"
- homogeneous Mxw.
- modified boundary conditions

$E = E_s + E_i$

Particle space charge solver REPTIL

Wakefield solver PBCI (new BC)




$$E_s(0) = E(0) - E_i(0)$$

Set "incident field" to the space charge field of the particles in free-space

# Scattered Field Formulation in FIT

- **Realization in FIT - staircase:**

$$e = e_{\mathrm{s}} + e_{\mathrm{i}}$$

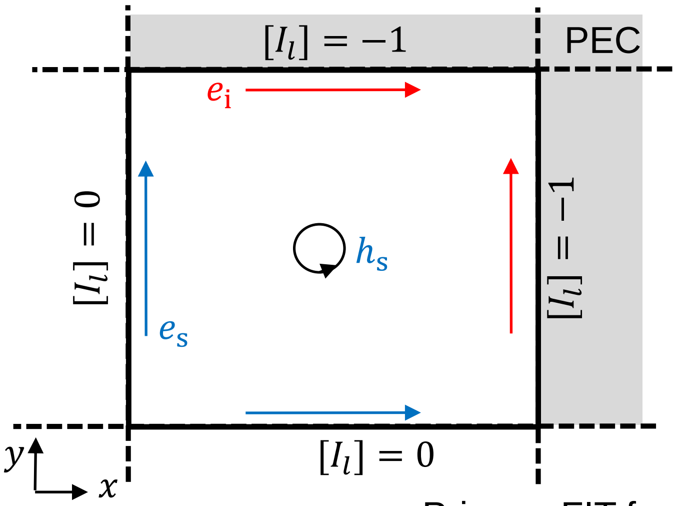
  - Modification of Faraday's law at PEC boundary

$$\frac{d}{dt}\begin{pmatrix} h_{\mathrm{s}} \\ e_{\mathrm{s}} \end{pmatrix} = \begin{pmatrix} 0 & -M_{\mu}^{-1}C \\ M_{\varepsilon}^{-1}C^{T} & 0 \end{pmatrix}\begin{pmatrix} h_{\mathrm{s}} \\ e_{\mathrm{s}} \end{pmatrix} - \begin{pmatrix} M_{\mu}^{-1}\, j_{\mathrm{mag}} \\ 0 \end{pmatrix}$$

  - Equivalent magnetic current at the boundary

$$j_{\mathrm{mag}} = C\, I_{l}\, e_{\mathrm{i}}$$

  - With local interpolation matrix $I_l = \begin{cases} -1 & \text{if edge in PEC} \\ 0 & \text{else} \end{cases}$
  - Rest of FIT- operators remain the same



Primary FIT face

# Scattered Field Formulation in FIT II

- **Realization in FIT – conforming boundaries:**
  - Modification of Faraday's law at PEC boundary

$$\frac{d}{dt}\begin{pmatrix} h_s \\ e_s \end{pmatrix} = \begin{pmatrix} 0 & -M_\mu^{-1}C \\ M_\varepsilon^{-1}C^T & 0 \end{pmatrix}\begin{pmatrix} h_s \\ e_s \end{pmatrix} - \begin{pmatrix} M_\mu^{-1} j_{mag} \\ 0 \end{pmatrix}$$

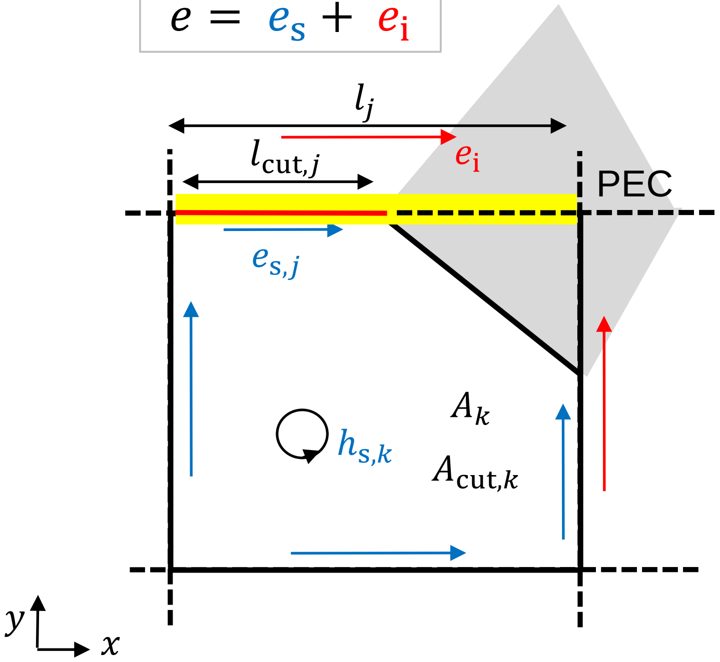
  - Restriction of incident field to conformal lengths / areas

$$e_j = e_{s,j} + \frac{l_{cut,j}}{l_j} e_{i,j} \qquad b_k = b_{s,k} + \frac{A_{cut,k}}{A_k} b_{i,k}$$

$$\Rightarrow j_{mag} = C\, I_L\, e_i + I_A\, C\, e_i$$

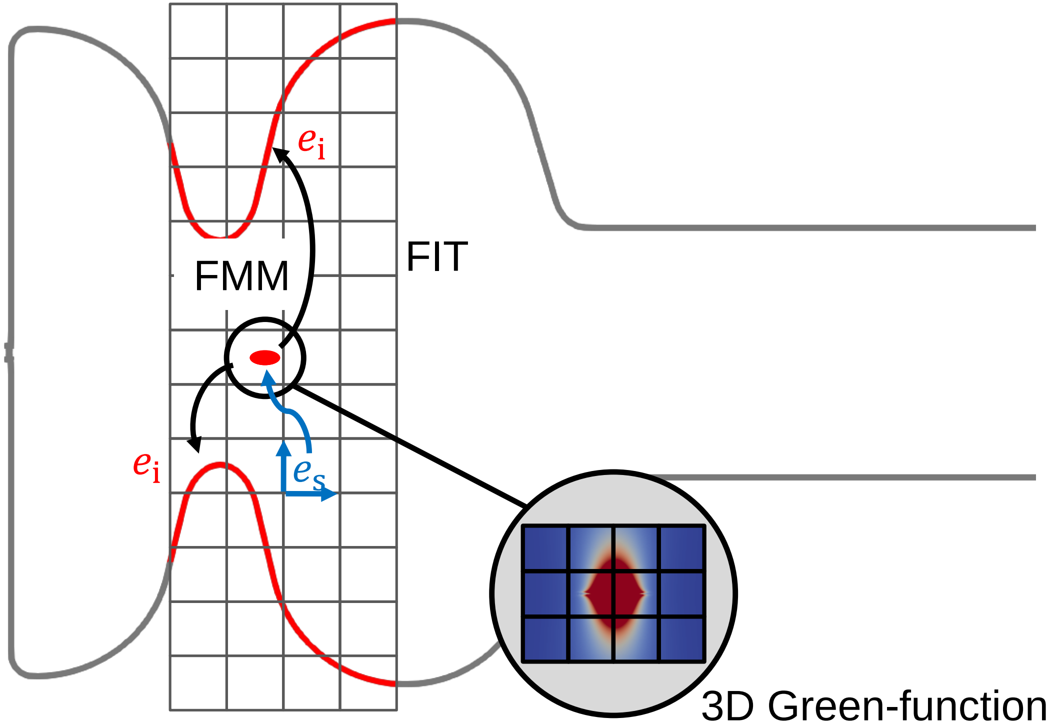
  - Rest of FIT- operators remain the same

$$e = e_s + e_i$$




Primary FIT face

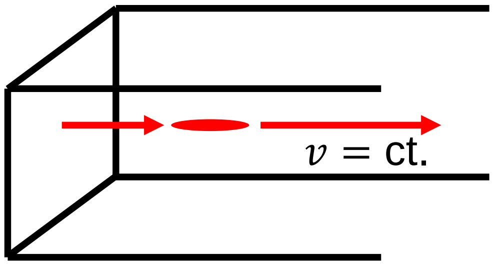
# Coupling: PBCI + REPTIL



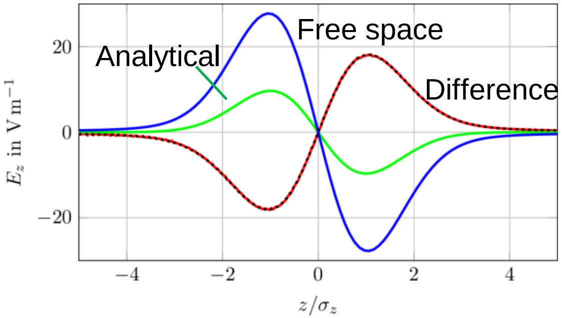

- Mesh-free, fast evaluation of space-charge farfield on boundary: FMM
- Solvers independent (grid, time step, optimization, …)
  - Arbitrary geometry
  - Arbitrary beam dynamics

# Validation: Space Charge Impedance

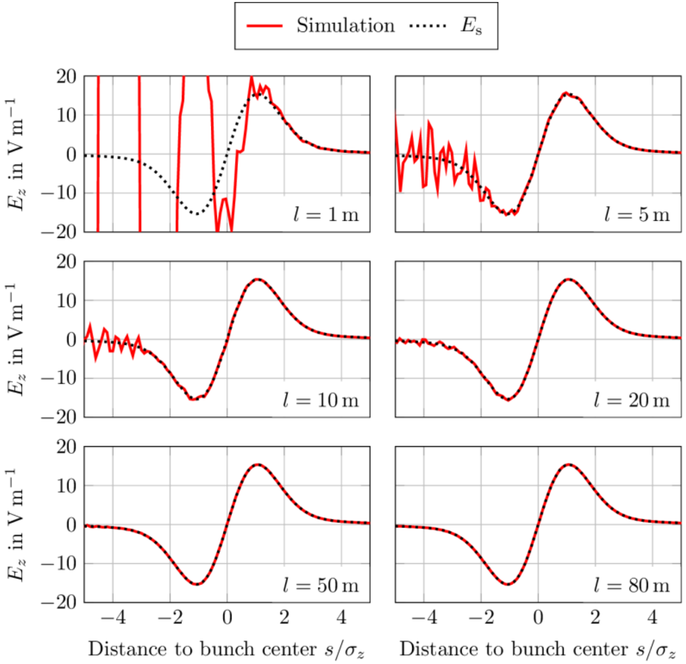
- **Space charge impedance in a uniform beam pipe**



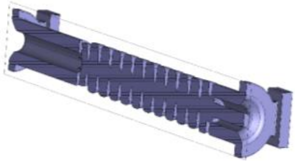

Analytical estimation for stationary state:




$$E_z(z) = \frac{-Q}{2\pi\varepsilon_0\gamma^2}\Lambda\frac{d\lambda(z)}{dz}$$

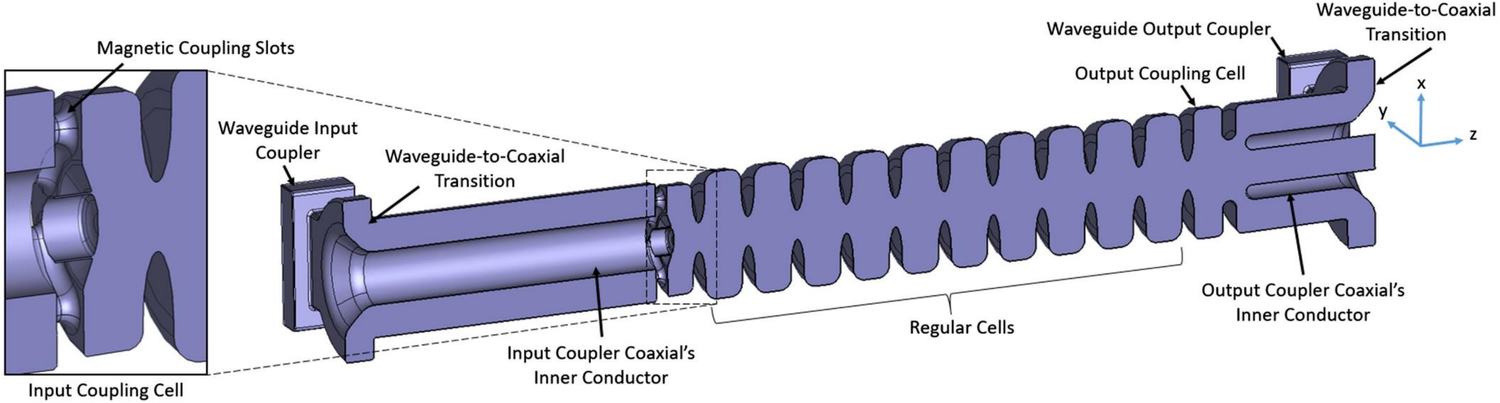


# Traveling Wave Gun Model



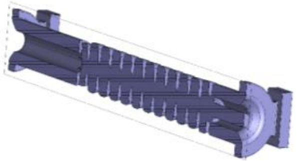
- 12-cell TW gun under design at SwissFEL (Lucas)
- Narrow, long geometry: 5mm iris radius, ~22cm acceleration path length

| | |
|---|---|
| Bunch: | |
| Charge | 0.2nC |
| Length | ~0.5mm |
| Size | ~1mm |
| Energy | 13MeV at gun exit |




Lucas 2023, DOI: 10.1103/physrevaccelbeams.26.103401

# Traveling Wave Gun Model



- 12-cell TW gun under design at SwissFEL (Lucas)
- Narrow, long geometry: 5mm iris radius, ~22cm acceleration path length
- Video: fields build up over time

| Bunch: | |
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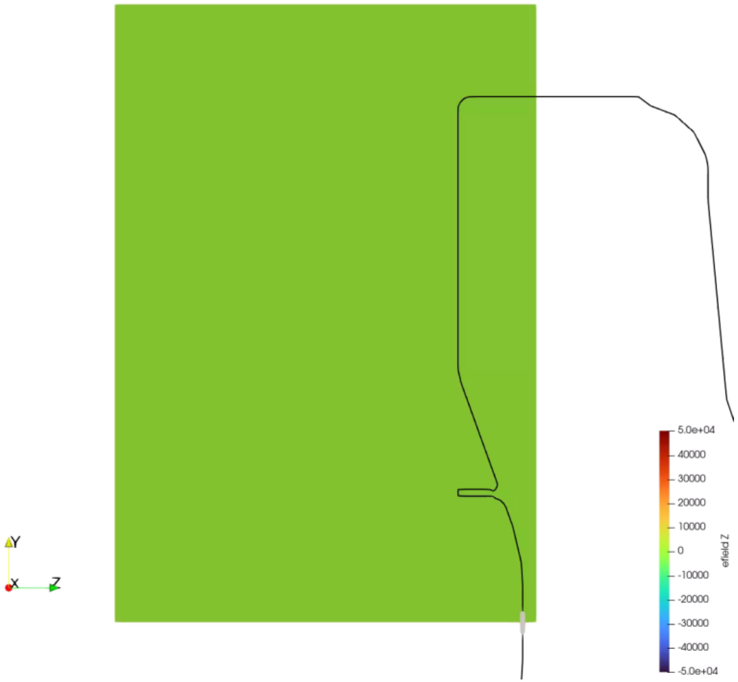
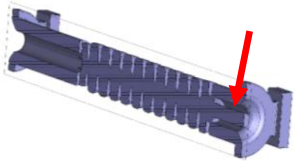
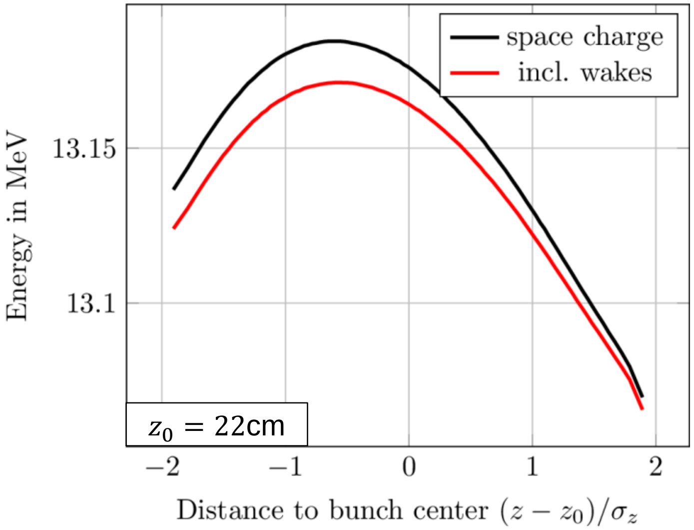


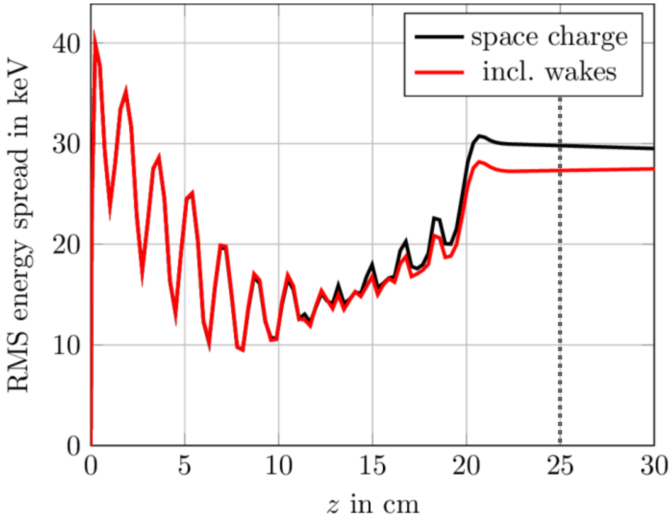

cmp. IPAC'24, DOI: 10.18429/JACoW-IPAC2024-WEPR71

# Energy Chirp

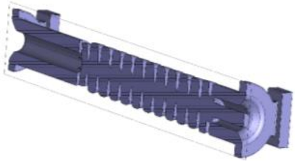


- Wakefields reduce energy chirp in gun
  - Wakes reach tail first
- ~10% RMS energy spread reduction at end of gun



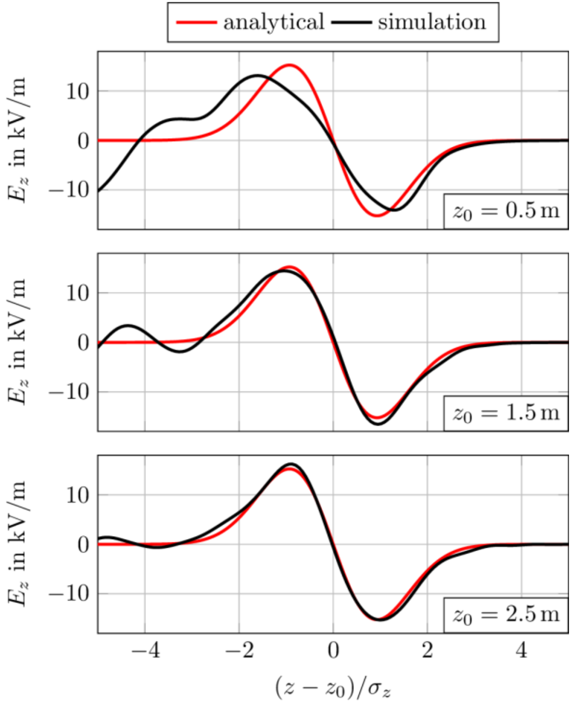
# Full Injector Line Simulation



- Field in beam pipe approaches space charge impedance field

  → Weak coupling of wakefields to beam pipe and downstream sections

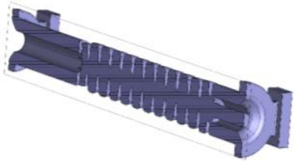
  → Include wakefields up to first accelerating section, continue with space charge solver only




Space Charge Impedance:

$$E_z(z) = \frac{-Q}{2\pi\varepsilon_0\gamma^2}\Lambda\frac{d\lambda(z)}{dz}$$

# Full Injector Line Simulation



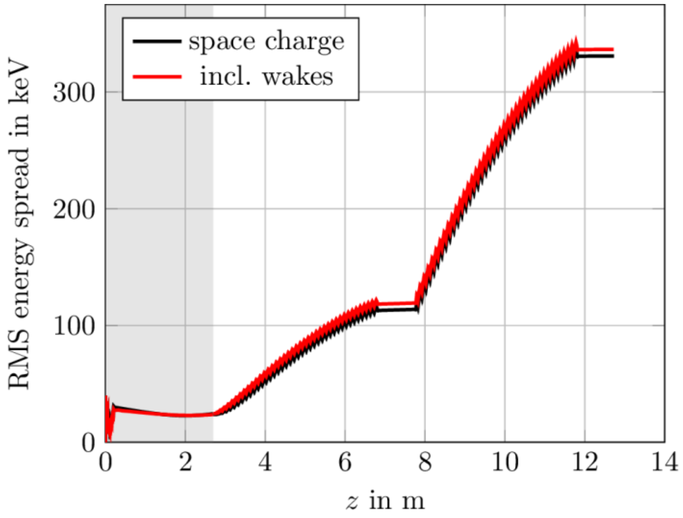
- Field in beam pipe approaches space charge impedance field

  → Weak coupling of wakefields to beam pipe and downstream sections

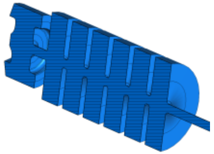
  → Include wakefields up to first accelerating section, continue with space charge solver only

- Difference in RMS energy spread:

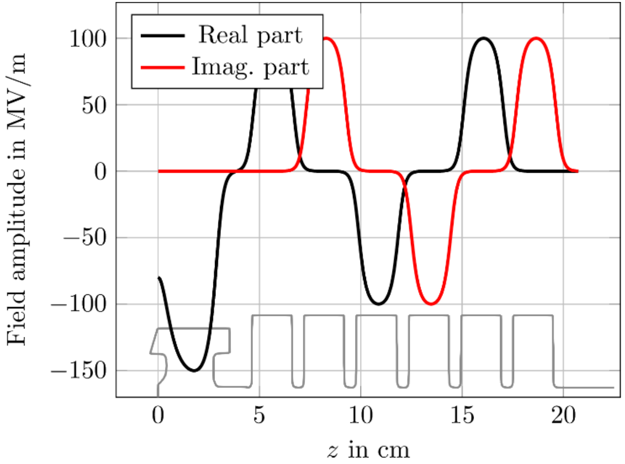
  5.5keV (simulated), 7.1keV (analytical)

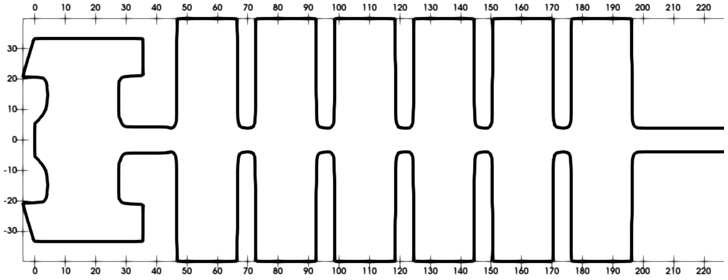


# Quasi-Traveling Wave Gun Model

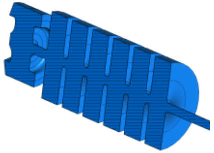


- Motivation: Natsui, Yoshida (KEK, IPAC'14)
- 7 Side-Coupled standing wave cavities (alternating field phase)
- Bending of acc. Field -> RF-focusing, no solenoid required
- High bunch charge
- Narrow, long geometry: 3.5mm iris radius, ~20cm acceleration path length



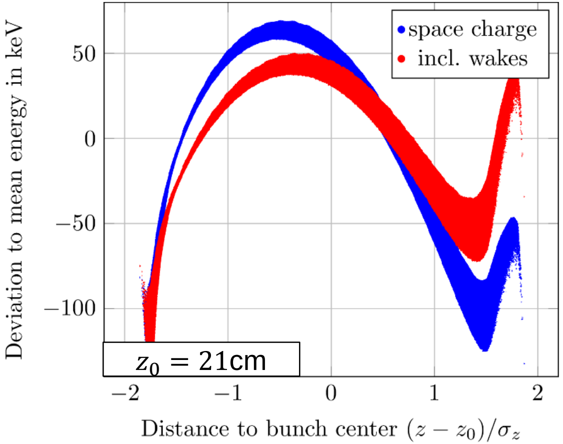
# Quasi-Traveling Wave Gun Model

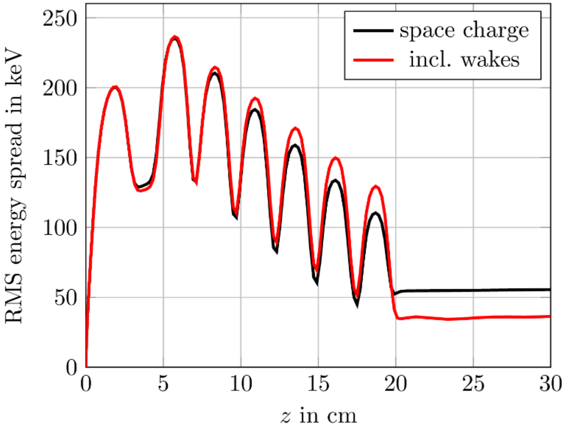


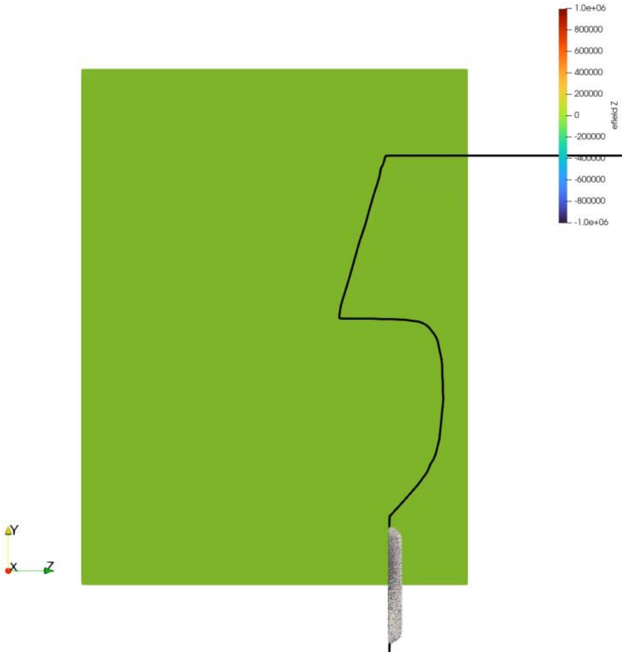
Bunch:

| | |
|---|---|
| Charge | 5nC |
| Length | ~0.5mm |
| Size | ~1mm |
| Energy | 13.3MeV at gun exit |

~30% RMS energy spread reduction at end of gun



# Summary

- Coupled Simulations:
  - Space Charge Solver REPTIL
  - Wakefield Solver PBCI
  - Scattered Field Formulation
- Electron Gun:
  - Effect of wakes on energy chirp
  - Limited coupling to downstream section
- Upcoming: simulation of bunch compressor