Intrabeam scattering simulations in RF-Track

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Wednesday, 18th September 2024.

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PART I: INTRODUCTION

1. INTRODUCTION – 2. IMPLEMENTATION – 3. RESULTS – 4. CONCLUSIONS AND FURTHER WORK

Introduction - IBS

Intrabeam Scattering (IBS): Particle-to-particle elastic collisions by Coulomb interaction.

It has been **widely studied for circular accelerators**:

- It causes emittance growth → Huge impact in their performance.
- IBS is important in storage rings due to the amount of time in which the bunch stays in the accelerator → This is not the case for linacs, which explains why it has not been studied before for them.
- IBS has also been studied in damping rings, as IBS effect is increased when the energy is low. This is the case in a lot of linear lattices too, but until now, the effect was still neglectable due to the short travelling time.

Because of the above reasons, IBS **has not yet been fully studied for linacs**. However, the available technologies nowadays have changed, and several results in Free-Electron Laser light Sources (FELs) have shown that their performance can be limited due to IBS.

Introduction - Photoinjectors

- FELs have reached much more dense beams → IBS effect is expected to grow.
- IBS causes an increment on the Sliced Energy Spread (SES), a determinising factor in their performance.

The following facilities, which use similar guns and cathodes, have measured different values of SES:

Facility	Charge (pC)	Energy (MeV)	Distance Rf-gun and measurement (m)	SES found (keV)
SwissFEL [1]	200	150	110	**15**
European XFEL [2]	250	130	40	**6**
DESY PILTZ [3]	250	20	20	**2**

- The expected value was below 1 keV, which does not consider neither IBS nor microbunching instabilities (MBI).
  - **A simulation tool able to evaluate the contribution of IBS is crucial.**

- The difference between the results is remarkable, and it is thought to be partially attributed to the difference in distance between the Rf-gun and the measurement location.

[1] E. Prat et al., "Energy spread blowup by intrabeam scattering and microbunching at the SwissFEL injector", *Phys. Rev. Accel. Beams*, vol. 25, p. 104401, Oct. 2022.
[2] S. Tomin et al., "Accurate measurement of uncorrelated energy spread in electron beam", *Phys. Rev. Accel. Beams*, vol. 24, p. 064201, June 2021.
[3] H. Qian et al., "Slice energy spread measurement in the low energy photoinjector", *Phys. Rev. Accel. Beams*, vol. 25, p. 083401, Aug. 2022.

Introduction – Summary of the state of the art in IBS

Analytical descriptions

Assume Gaussian bunches, inadequate for FELs beam dynamics

- Piwiniski [4], Bjorken and Mtingwa [5], Bane [6].

Simulation codes

- For circular lattices.
  - Monte Carlo: SIRE [7] and MOCAC, specific for circular lattices. Can handle non-Gaussian Beams.
  - PDE evolution: Implementation in X-Suite [8], aimed for several turns of the rings and regarding computational time is faster than Monte Carlo approaches.
- For linear lattices
  - E.Gjonaj [9] is developing a Monte Carlo approach for IBS simulation in linacs and has applied it to European XFEL.

[4] A. Piwinski, “Excitation and damping of betratron oscillations and energy spread due to intra-beam scattering”, in *Proc. 9th Int. Conf. Part. Acc*, 1974.
[5] D. Bjorken, S.K. Mtingwa, “Intrabeam Scattering”, *Part. Acc*, vol. 13, pp. 115–143, 1983.
[6] K. Bane, “An Accurate, Simplified Model of Intrabeam Scattering”, SLAC-AP-141, May 2002.
[7] M. Martini, F. Antoniou, Y. Papaphilippou, “Intrabeam scattering”, *ICFA Beam Dyn. Newsl*. vol. 69, pp.38-59, 2016
[8] M.Zampetakis et al.,“Interplay of space charge, intrabeam scattering, and synchrotron radiation in the Compact Linear Collider damping rings”, *Phys. Rev. Acc. Beams*, vol. 27, p. 064403, June 2024.
[9] E. Gjonaj, “Intrabeam Scattering effects in the electron injector of the European XFEL” *in Proc. 40th Int. Free-Electron Laser Conf*., Trieste, Italy, Aug. 2022.

Introduction – RF-Track

RF-Track [10] is a tracking code aimed for the simulation of linear accelerators. It has been developed at CERN by A.Latina, who will introduce the code with more detail in his talk.

RF-Track can:

- Can simulate bunches of different species and charges.
- Space-charge effects, wakefields, beam-beam interactions, beam-loading and now, intrabeam scattering.
- Developed in C++ using parallel programming, can be used with an Octave or Python interface.
- Has been used in CLIC, AWAKE injector linac, the cooling and initial acceleration stage of the Muon Collider, and much more.

This work:

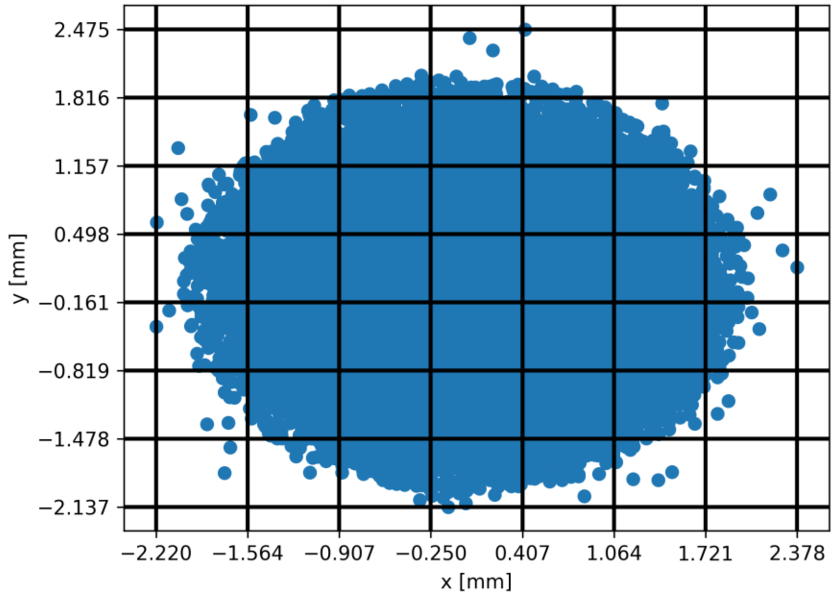
1. Describes the implementation of Intrabeam scattering in RF-Track, which follows a novel hybrid-kinetic Monte Carlo approach.
2. Shows the results obtained for two test cases:
   - One of them shows SES growth, with the beam characterization in European XFEL.
   - A second one showing emittance growth in a case in which the IBS effect was high.

[10] A. Latina, "RF-Track reference manual", June 2020, doi: 10.5281/zenodo.4580369.

PART II: IMPLEMENTATION

1. INTRODUCTION – **2. IMPLEMENTATION** – 3. RESULTS – 4. CONCLUSIONS AND FURTHER WORK

Implementation – Mesh creation



The algorithm starts by creating a 3D grid using the bunch coordinates, and obtains 3 different meshes:

- Mesh 1: Average **number of particles** per cell.
- Mesh 2: Average **velocity** (3d vector) per cell.
- Mesh 3: Standard deviations of velocity (3d vectors), also known as **temperature**, per cell.

Then, the algorithm iterates particle by particle, repeating the steps that will be described as follows.

ALGORITHM STEPS

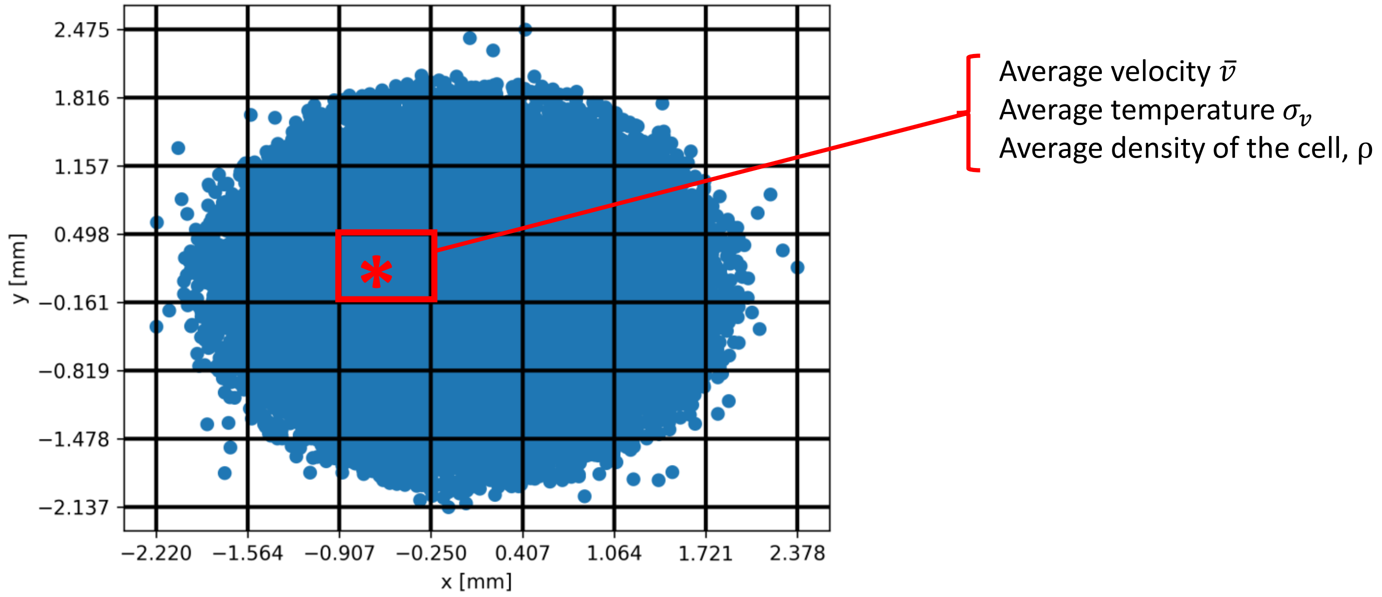
- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. Computation of IBS parameters.
3. Collision.

- Kick calculation and update.

Implementation – Average particle interp.



For a real particle p (* in the figure), by **interpolating the mesh**, the average velocity $\bar{v}$ and temperature σ_v are obtained.

Then, a new velocity $\overrightarrow{q_v}$, is randomly extracted from **a normal distribution**:

$$\overrightarrow{q_v} \rightarrow N\ (\bar{v}\ , \sigma_v) \quad (1)$$

Note: The density in the cell is also stored, and will be used later.

IBS parameters will be computed in the target ($\overrightarrow{q_v}$) **rest frame**. Therefore, 3D Lorentz boost is performed.

Note II: From now on, all the magnitudes that appear in the expressions, refer to $\overrightarrow{q_v}$ rest frame.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. **Average particle interpolation.**
2. Computation of IBS parameters.
3. Collision.

- Kick calculation and update.

Implementation – IBS parameters (I)

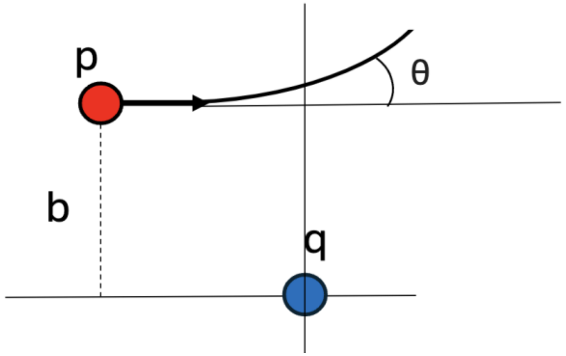
Maximum impact parameter, b_{max}

The impact parameter is **the perpendicular distance** between the particles colliding.

- Maximum impact parameter → Minimum scattering angle, a key parameter.

- Considering that most of the movement of the particle is in the longitudinal axis, Z, the maximum impact parameter is calculated on the transversal plane, XY.

Note: This magnitude is not affected by the change of frame, as it is purely transversal → Calculated in the lab frame.



In the state of the art, b_{max} has been calculated in different ways:

- Maximum transversal mesh size → Creates a dependence on the meshing.
- Maximum between σ_x and σ_y , or combinations of them → These magnitudes are very sensitive to tails.

This work calculates b_{max} locally.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. **Computation of IBS parameters.**
3. Collision.

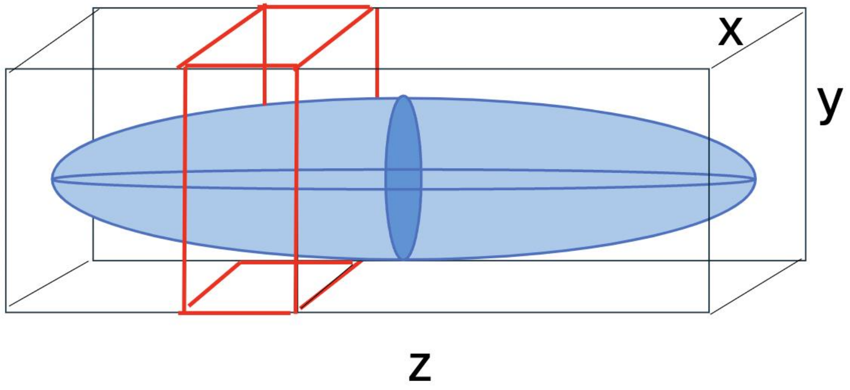
- Kick calculation and update.

Implementation – IBS parameters (II)

Maximum impact parameter, b_{max}

The maximum impact parameter **is determined locally** as follows:

- From the 3D mesh, a 2D transversal mesh is created for each value of z.
- Interpolating the z position of the particle p, a 2D mesh is obtained, and from there we get the following quantities:
  - Local variance in X and Y coordinates: $VAR_X(z), VAR_Y(z)$.
  - Local covariance in X and Y coordinates, $COV_{X,Y}(z)$, added to consider x-y coupling.
  - Then, the **maximum eigenvalue** of the covariance matrix can be calculated, which will be b_{max}



Reduction of the sensitivity to tails (important in FELs, observed problems of convergence when it was not taken into account in our tests) **by defining the impact parameter as a local quantity.**

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. **Computation of IBS parameters.**
3. Collision.

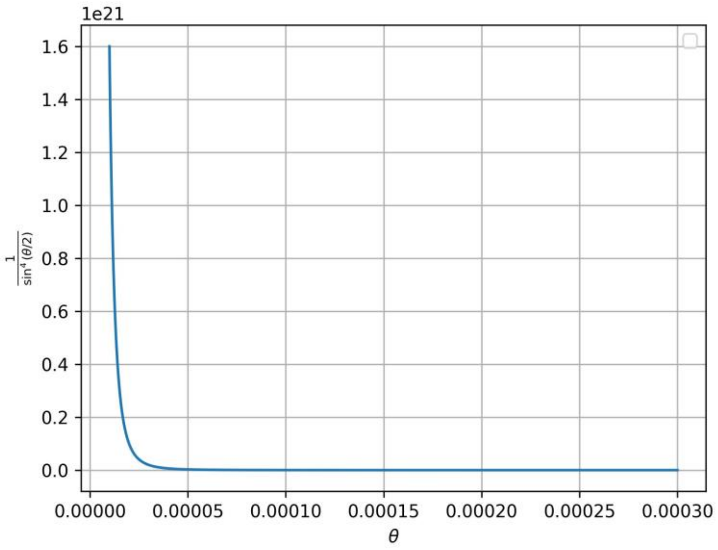
- Kick calculation and update.

Implementation – IBS parameters (III)

Differential Rutherford Cross Section

As any Coulomb scattering process, the cross section of IBS is determined by Rutherford cross section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{q^2}{4\pi\varepsilon_0}\right)^2 \left(\frac{1}{4E_{Kin}}\right)^2 \frac{1}{\sin(\theta/2)^4} \quad (2)$$



A **minimum angle** is needed both to avoid divergence, θ_{min}.

- **The determination of this angle is crucial**, as it changes complety the total cross section.

Integrating the above expression, the **total cross section** is obtained:

$$\sigma = \int_0^{2\pi} \int_{\theta min}^{\pi} \frac{d\sigma}{d\Omega} \sin(\theta)\, d\theta d\varphi = \left[2\pi \left(\frac{q^2}{4\pi\varepsilon_0}\right)^2 \left(\frac{1}{4E_{Kin}}\right)^2 \frac{-2}{\sin(\theta/2)^2} \right]_{\theta min}^{\pi} \quad (3)$$

Note: The kinetic energy refers to the one at $\overrightarrow{q_v}$ rest frame.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. **Computation of IBS parameters.**
3. Collision.

- Kick calculation and update.

Implementation – IBS parameters (IV)

Minimum scattering angle, $\boldsymbol{\theta_{min}}$

Having calculated the maximum impact parameter, the minimum scattering can be determined:

$$\theta_{min} = 2\, arctan\left(\frac{q^2}{4\pi\varepsilon_0}\frac{1}{2E_{kin}b_{max}}\right) \qquad (4)$$

With θ_{min} determined, the total cross section σ can be calculated with Eq.(3).

Mean free path, $\boldsymbol{\lambda}$

The mean free path is the average distance that particles do between collisions:

$$\lambda = \frac{1}{\rho\sigma} \qquad (5)$$

Being ρ the local interpolated density and σ the total cross section, both in $\overrightarrow{q_v}$ frame.

Step distance, $\boldsymbol{dS}$

The time step dt is an **input parameter** that determines each how much time a kick is computed.
For our code it is convenient to translate it to an integration distance, which is done as follows.

$$dS = \frac{dt\ \beta c}{\gamma} \qquad (6)$$

Being β the relativistic velocity of the particle, time dilation is taken into account with γ, the relativistic factor of $\overrightarrow{q_v}$.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. **Computation of IBS parameters.**
3. Collision.

- Kick calculation and update.

Implementation – IBS parameters (V)

Number of collisions

The average number of collisions expected in a kick can be determined with the mean free path defined in Eq.(5) and the integration distance in Eq.(6).

$$N = \frac{dS}{\lambda}$$

Case A: Less than one collision: $dS < \lambda$

Then, dS^* (that will be later used) is dS as in Eq.(6)

Case B: Many collisions: $dS > \lambda$

Then, dS^* (that will be later used) is λ as in Eq.(5)

Having defined and determined these key concepts, we can define the collision computation. To sum up:

- Maximum impact parameter b_{max} → Defines the minimum scattering angle, θ_{min}.
- Mean free path λ and integration distance, dS → Define the number of collisions N.
- Differential and total cross section.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. **Computation of IBS parameters.**
3. Collision.

- Kick calculation and update.

Implementation – Collision

Two angles are extracted from random distributions:

- $\phi \rightarrow \mathrm{U}(0, 2\pi)$.
- θ from the CDF method, taking as PDF the differential cross section in Eq.(3) multiplied by $sin\theta$ and restricting the angles to be larger than θ_{min}.

Then, **a new momentum** $\overrightarrow{p'}$ is defined as follows:

$$\overrightarrow{p'} = (|\overrightarrow{p_0}|cos\theta, |\overrightarrow{p_0}|sin\theta cos\varphi, |\overrightarrow{p_0}|sin\theta sin\varphi) \quad (7)$$

A 3D rotation is used to align the collision's outcoming direction with the momentum of the particle.

RF-Track tracks macroparticles → Not all particles in a macroparticle are expected to scatter.

- A **weight** w that accounts for the number of particles that will scatter, is introduced:

$$w = 1 - exp\left(-\frac{dS *}{\lambda}\right) \quad (8)$$

Where $dS *$, is rather dS or λ, as we had previously discussed.

Finally, the new momentum of the macroparticle $\overrightarrow{p_f}$ is the following:

$$\overrightarrow{p_f} = \overrightarrow{p'}w + (1 - w)\overrightarrow{p_0} \quad (9)$$

THIS PROCESS IS REPEATED AS MANY TIMES AS COLLISIONS ARE IN THE KICK

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. Computation of IBS parameters.
3. **Collision.**

- Kick calculation and update.

Implementation – Kick calc. and update

Once the change of momentum is computed in $\overrightarrow{q_v}$ rest frame, the force is calculated back in lab frame as follows:

$$\vec{F} = \frac{\overrightarrow{\Delta p}}{dt}$$

This force is computed for all the macroparticles.

Then, a Nx3 force matrix is applied to the kick (with N being the number of macroparticles), and the described process is repeated in steps of dt.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. Computation of IBS parameters.
3. **Collision.**

- Kick calculation and update.

Implementation – Sum up

To sum up, in the previous slides, we have described the implementation of our novel hybrid-kinetic Montecarlo algorithm for Intrabeam Scattering in RF-Track.

- **Monte-Carlo**:
  1. The collision angles are taken from random distributions.
  2. The velocity $\overrightarrow{q_v}$ also has a random component coming from the local temperature.
- **Kinetic**: The algorithm is integrated in a tracking code. To include IBS in the tracking, it computes changes of force and updates them after each kick.
- **Hybrid**: At the beginning of each kick updates the 3D meshes.

ALGORITHM STEPS

- Mesh creation.

- Iteration over particles:

1. Average particle interpolation.
2. Computation of IBS parameters.
3. .Collision.

- Kick calculation and update.

PART III: PRELIMINARY RESULTS IN RF-TRACK

1. INTRODUCTION – 2. IMPLEMENTATION – **3. RESULTS** – 4. CONCLUSIONS AND FURTHER WORK

1. SES growth in the European XFEL.
   - Obtained results and convergence tests.
   - Comparison with previous simulations.
   - Comparison against analytical expressions.

2. Emittance growth in a transfer line.
   - Obtained results and convergence tests.
   - Effect on IBS of the charge and energy variation.

Results – SES growth in the European XFEL (I)

To test the effect of IBS in FELS, the case of the European XFEL was studied. The beam is characterized by the following quantities:

Magnitude	Value
Charge $[pC]$	250
Kinetic energy, $[MeV]$	130
Transversal emittance ϵ_T, $[mm.mrad]$	0.547
RMS of long. coordinate σ_Z, $[mm]$	1.255
RMS of relative mom.spread, $\frac{\sigma_P}{P}$	0.02693 %

A tracking simulation including **IBS of the 19m of only drift space** in the injector was performed.

- European XFEL injector section has 40m, so the simulation is not complete.
- The simulation does only include IBS, and not other sources which also contribute to the SES growth such as space-charge or MBI effects.

The initial phase space was provided by E.Gjonaj (Tech. University of Darmstadt), who also performed similar tests but including space-charge [9] in its algorithm.

Results – SES growth in the European XFEL (II)

1) Obtained results and convergence tests.

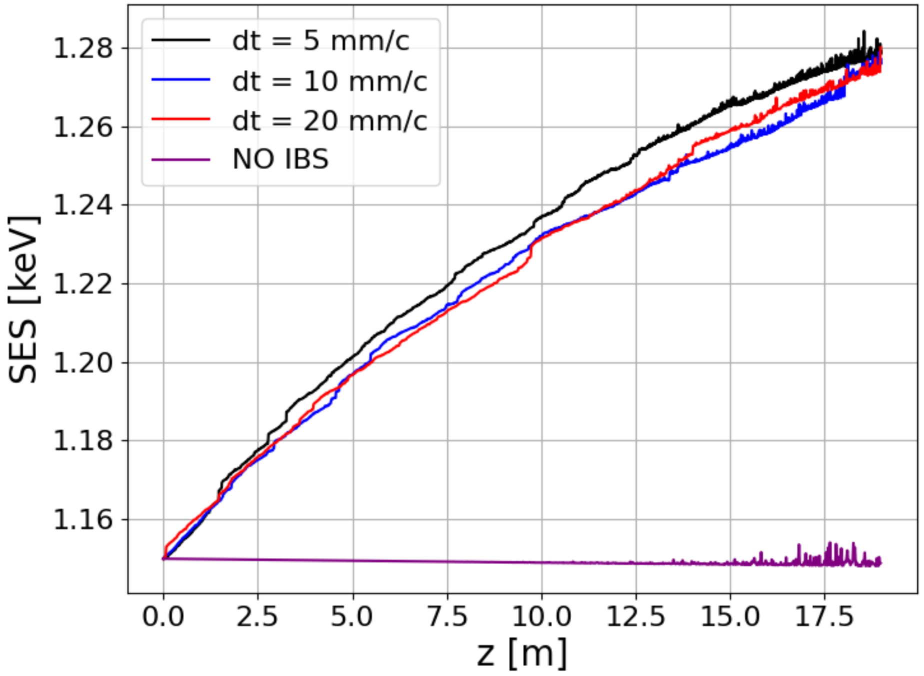


Figure 1: RF-Track tracking simulation of the injector drift space of European XFEL with RF-Track with different time steps.

- The same result was achieved by **using different time steps**, which proves the stability of the method.
- There are certain differences in the tracking attributed to the randomness of the Monte Carlo method.
- There is an **increment of around 0.13 keV in 19m of drift caused by only IBS**.
- Following the analytic formulation [6], the SES growth is not linear with the distance, so this result should be carefully translated to longer injectors.
- **The increment due to IBS is too low to explain the SES growth found in FELs itself but should be considered as it is not neglectable and may affect the performance of the machines.**

Results – SES growth in the European XFEL (III)

2) Comparison with previous simulations.

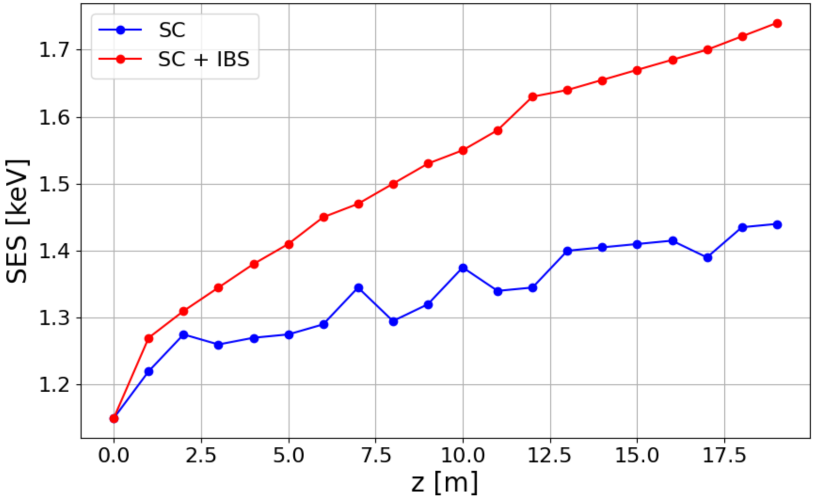


Figure 2: Tracking simulation of the injector drift space of European XFEL with Reptil, from E.Gjonaj [9]. In blue, only space-charge effects. In red, both space-charge and IBS.

The results found can be compared with the ones obtained by E.Gjonaj [9], also along the 19m-long drift of European XFEL.

In his paper, he shows the results obtained with his implementation in the tracking code Reptil (Relativistic Particle Tracker for Injectors and Linacs).

In his case, he studies the effect **together of IBS with space-charge:**

- In blue with dots, the simulations of only space-charge, with Astra.
- In red, the simulations of space-charge + IBS.

While we found a SES increment of 0.13 keV, E.Gjonaj finds one of almost 0.15 keV, showing similar values.

- He finds an increment in the SES final value of 1.74 -1.44= 0.3 keV due to IBS, but as the initial SES was 0.15, the final SES induced by IBS is 0.3 – 0.15 = 0.15 keV.
- The above comparison assumes completely addictive effects between IBS and SC.

Results – SES growth in the European XFEL (IV)

3) Comparison against analytical expressions.

The results were compared with the analytical formulation of Bane [6], which assumes Gaussian bunches and high energies.

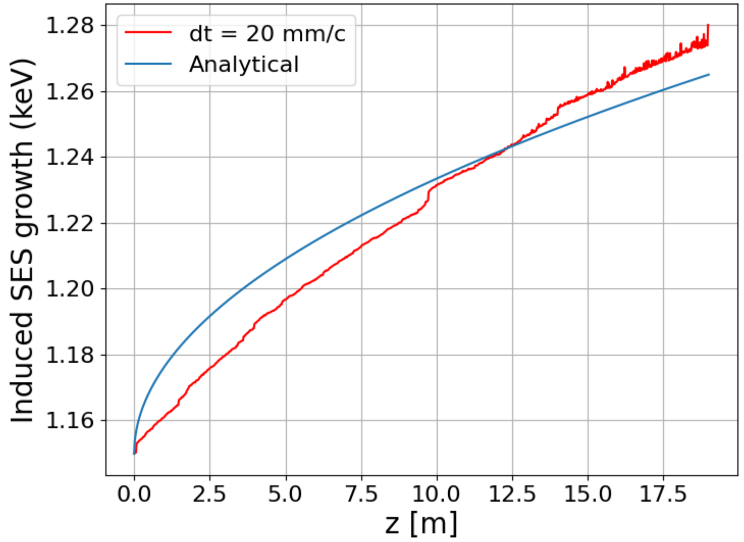


Figure 3: Tracking simulation of the injector drift space of European XFEL with RF-Track against the SES provided by the analytic expression of Bane [6].

- **The tracking results are similar to the analytical predictions**, even that some differences can be observed. The final value is similar, which indicates that the Gaussian approximation may not be that inaccurate for the simulation of IBS in a FEL whose beam is characterized in a similar way to the one of European XFEL.

- The bunch is highly non-Gaussian in the longitudinal plane, and similar to a round beam in the transversal space

Results – Emittance growth in a transfer line (I)

Another case in which the **effect of IBS was expected** was studied. To maximize the effect of IBS, we searched for a low-energy and high-density beam:

- High bunch population → We took 5.5nC, one of the largest charges for injectors in the state of the art, which has been proposed for FCC-ee.
- Low kinetic energy, emittances and bunch length.

Magnitude	Value
Charge $[pC]$	5500
Kinetic energy, $[MeV]$	12.5
Transversal emittance ϵ_T, $[mm.mrad]$	0.25
RMS of long. coordinate σ_Z, $[mm]$	0.1
RMS of relative mom.spread, $\frac{\sigma_P}{P}$	0.1 %

A 0.5m long drift was studied, showing high impact results in such little distance, as it will be shown in the following slide.

Results – Emittance growth in a transfer line (II)

4) Obtained results and convergence tests.

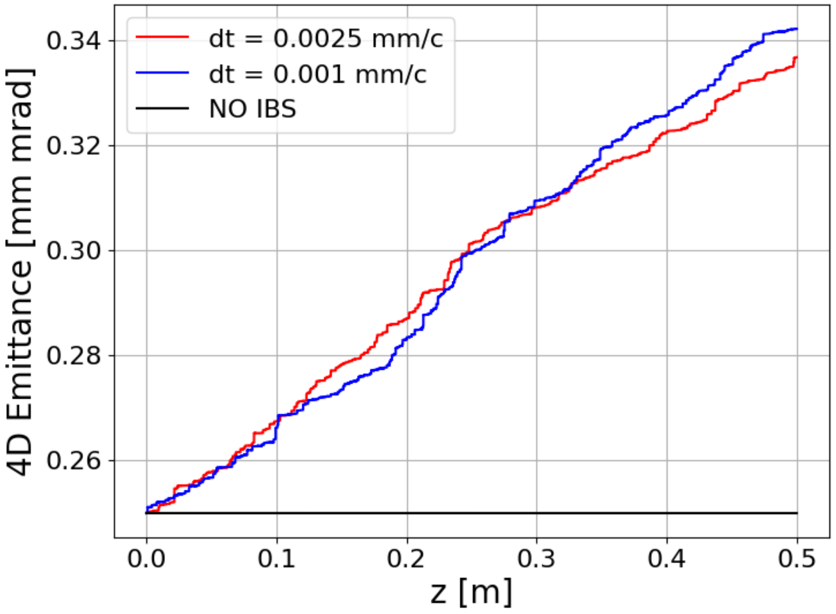


Figure 4: Tracking simulation along a 0.5m-long drift analysing the 4D-emittance growth.

- Again, convergence for different time steps was proved, demonstrating the stability of the tool.
- The emittance grows approximately 0.08 mm (a 32%) in 0.5m-long drift due to IBS.
- This simulation suggests that IBS effects in the emittance in cases where the charge is high (such as in FCC injectors) or when the energy and emittances are low, should be carefully studied.

Results – Emittance growth in a transfer line (III)

5) Effect on IBS of the charge and energy variation.

To have a better understanding of IBS, we have studied this same case varying both the charge and the kinetic energy.

As expected (also from analytical expressions), the results show a big sensitivity of IBS effect under these parameters:

- IBS highly increases with the bunch population → Decreases the mean free path, causing more collisions.
- IBS decreases with the bunch energy → The cross section is reduced.

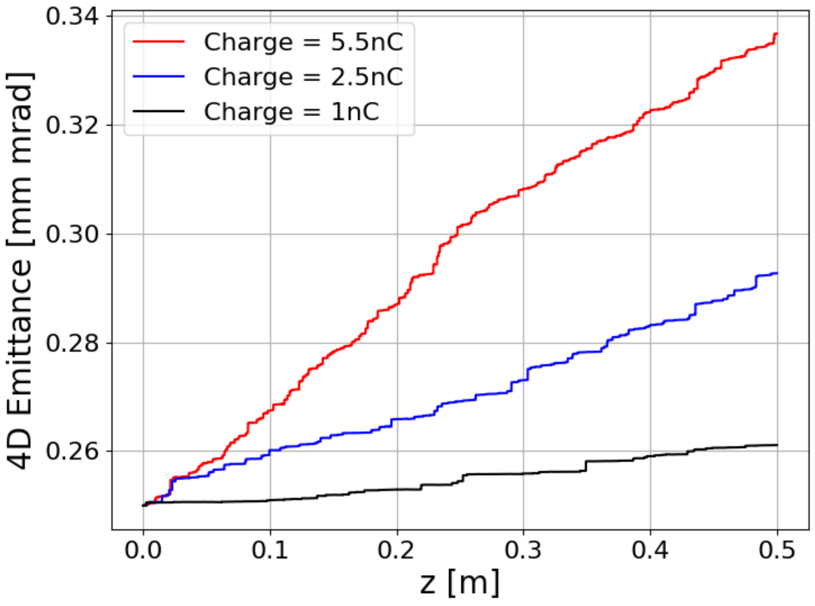


Figure 5: Sensitivity test of the emittance growth with the bunch population.

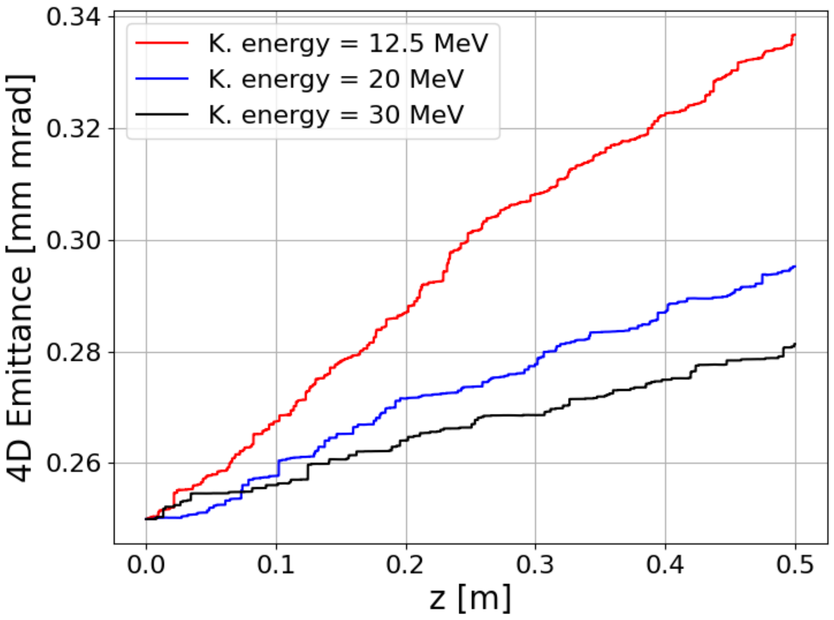


Figure 6: Sensitivity test of the emittance growth with the bunch kinetic energy.

PART IV: CONCLUSIONS AND FURTHER WORK

1. INTRODUCTION – 2. IMPLEMENTATION – 3. RESULTS – **4. CONCLUSIONS AND FURTHER WORK**

Conclusions

A novel hybrid-kinetic Monte Carlo approach has been used in the implementation of Intrabeam Scattering (IBS) in RF-Track.

The simulations have shown a growth on SES attributed only to IBS.

- This growth was of 0.13 keV for a 19m-long drift with the same bunch characteristics as the ones in European XFEL.
- This growth is not high enough by itself to explain the SES growth seen at FELs.
- The growth is very close to the predictions of Bane’s analytical method, which assumes Gaussian beams. This indicates that the non-Gaussian behaviour is not determinant.
- The simulations show similar values to the ones obtained by E.Gjonaj.
- IBS should be considered in FELs simulations, as its effect is not neglectable, and growths with the distance.

The simulations have shown a big impact of IBS in the emittance (32% of grow in a 0.5m-long drift) when the bunch charge was high and the beam was dense.

- The results indicate that IBS should be studied with detail in the mentioned cases.
- The sensitivity of IBS with the bunch charge and the bunch energy has been also analysed, matching the expectations.

The results were stable when different input parameters were used, proving the stability of the algorithm.

Future work

Simulations in a circular lattice:

- Very recently we have performed an IBS implementation for circular lattices.
- The next step is to verify the code with circular lattices, where the bibliography is much more extense.
  - Testing Gaussian beams with the analytical formulations of Bjorken and Mtingwa.
  - Testing non-Gaussian beams with SIRE, which also uses a Monte Carlo approach.
  - Testing a tracking example against X-Suite, which does not follow a Monte Carlo approach.

Simulations in a linear lattice:

- Collaboration with SwissFEL to perform simulations of their facility, and to whoever is open ☺
- Study the effect of IBS in future collider's injectors, such as in FCC (high charge) or the initial acceleration of the muon collider (low energy).

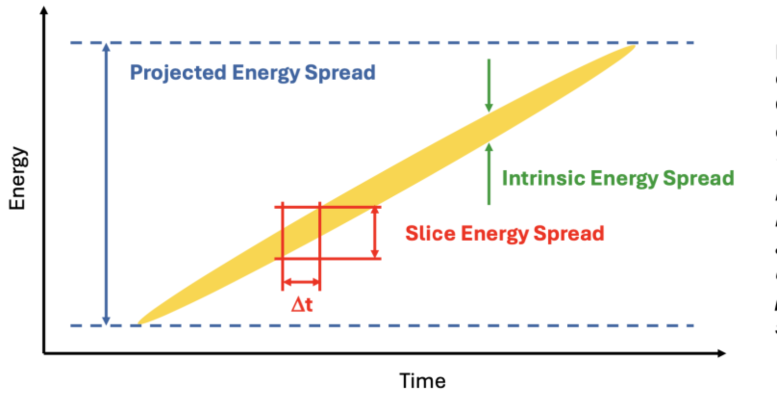
Work in the code: We would like to allow the code to have reliable bigger time steps.

Thank you for your time

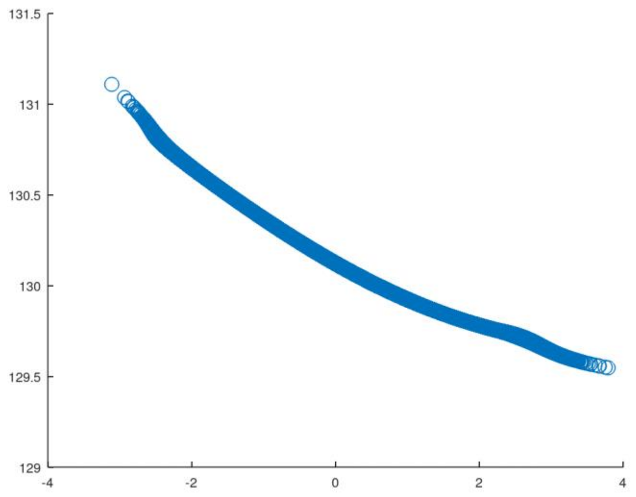
The authors of this work would like to thank E.Gjonaj for his help, and to Y. Papaphilipou, T.G. Lucas and P. Craievich for their useful and valuable discussions.

SES Calculation

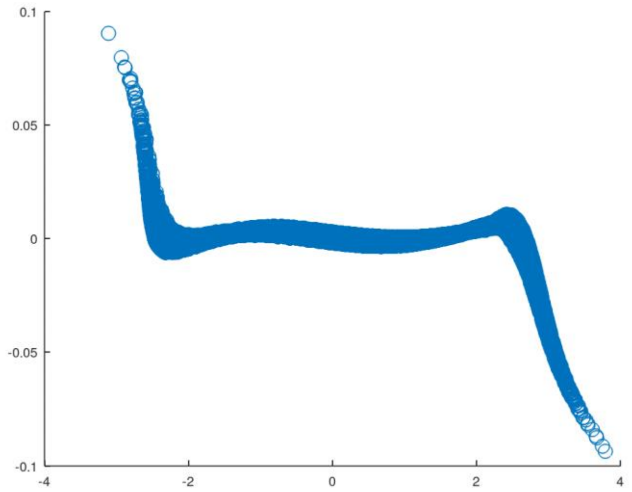
- We have first removed the correlated part. So, being Z and E our phase-space variables, we have done a polynomial fit to remove the correlation, and later we have selected the 10% of centered values to calculate their dispersion.
- What we have considered in this work as SES, was defined yesterday as Intrinsec Energy Spread by Sven Reiche (we could say we are taking the central intrinsec energy spread), which is **smaller** tan what PSI considers as SES.



From yesterday's slides of S.Reiche



Before removing corelation



After removing corelation

Analytical formulation

Bane [6] proposes an analytical model to account for IBS applicable to **high energies and Gaussian beams**, also applicable for circular lattices:

$$\frac{1}{T_p} = \frac{1}{\sigma_p}\frac{d\sigma_p}{dt} \,, \qquad \frac{1}{T_p} \approx \frac{r_0^2 cN(\log)}{16\gamma^3\epsilon_x^{3/4}\epsilon_y^{3/4}\sigma_s\sigma_p^3}\left\langle \sigma_H \, g(a/b) \, (\beta_x\beta_y)^{-1/4} \right\rangle$$

So the SES grows as sqrt(s), being s the distance.

Assuming:
- Round beams
- Stable beta and alpha function along the lattice.

We can reduce Bane's expression to the following [1]:

$$\sigma_{\gamma,\mathrm{IBS}} = \left(\frac{2r_e^2 N_e}{\varepsilon_n \sigma \sigma_s} z\right)^{\frac{1}{2}}$$

From where we took the analytical graph in Figure 3.