Comparative study of microbunching instability at the FERMI FEL

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Motivations and contents

1. The design of MBI-sensitive FELs, such as the FERMI upgrade plan, asks for a comprehensive, accurate, and self-consistent modelling of the instability, for **fast optimization of the machine parameters**.
2. Since Huang's note in 2002, **several approximated IBS models** have been proposed. Though in rough agreement with measurements and simulations, some formulas contain inaccuracies, implicit forms, and have never been systematically compared.
3. A validated MBI semi-analytical model including IBS, suitable for **multi-stage compression**, is presented. **Two formalisms (HK, BK) are compared**.

Outline

- Instability and cures
- Models
  - Huang-Kim
  - Bosch-Kleman
  - IBS and other features
- Comparison
- FERMI-U
- Conclusions

Credits

Acknowledgements

- Giovanni Perosa (now Uppsala Univ.) – *theory and modelling*
- Cheng-Ying Tsai (HUST), Alexander Brynes (Elettra, now STFC) – *simulation techniques*
- FERMI team – *measurements and discussions*

References for modelling

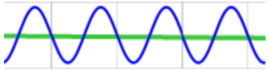
- Z. Huang K.-J- Kim, Phys. Rev. Spec. Top.-Accel. Beams 5 074401 (2002)
- R. A. Bosch, J. Wu, Phys. Rev. Spec. Top. Accel. Beams 11, 090702 (2008).
- Z. Huang, Technical Note No. LCLS-TN-02-8, also SLAC-TN-05-026 (2002)
- S. Di Mitri, G. Perosa et al., New J. Phys. 22 (2020) 083053
- G. Perosa, S. Di Mitri, Sci. Rep. (2021) 11:7895
- M. Venturini, NIM A 599 (2009) 140–145.

Instability

Peak at $\frac{r_b k}{\gamma} \simeq 1$

$\lambda_f \sim 1 - 10\ \mu m$

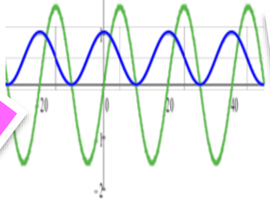
density mod.



LSC

linac

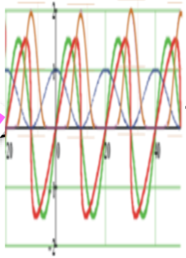
energy mod.



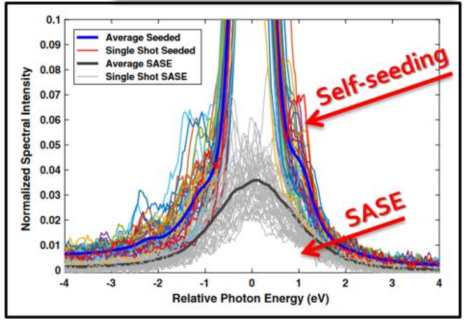
R_{56}, CSR

magnetic compressor

compression & amplification

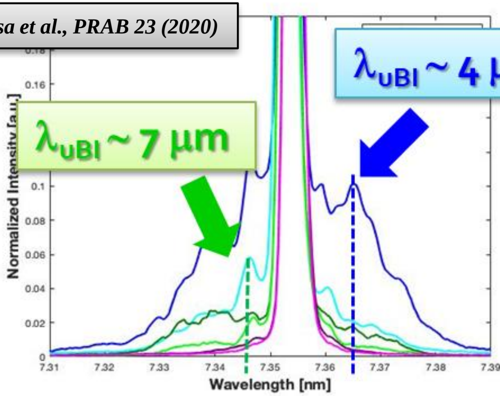


G. Marcus et al., PRAB 22 (2019)



$k_{FEL} = hk_{seed} \pm mk_{ubi}$

G. Perosa et al., PRAB 23 (2020)

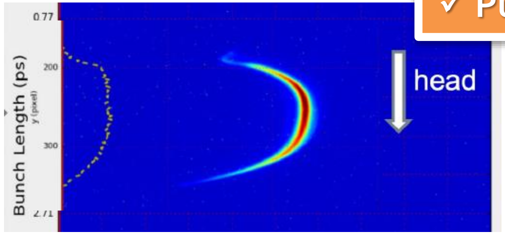


@ SASE: saturation power reduced by $\sigma_{\delta,f} \geq \rho$.

@ seeded FELs: spectral pedestal, sidebands.

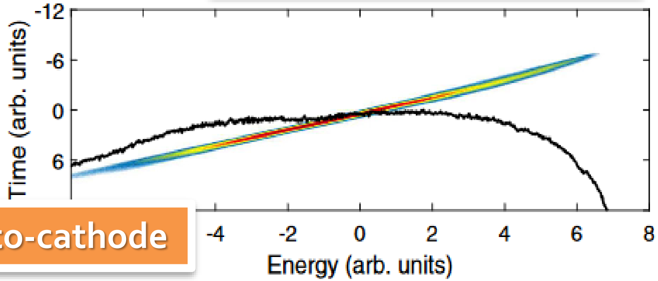
Cures

SXFEL, Courtesy C. Feng



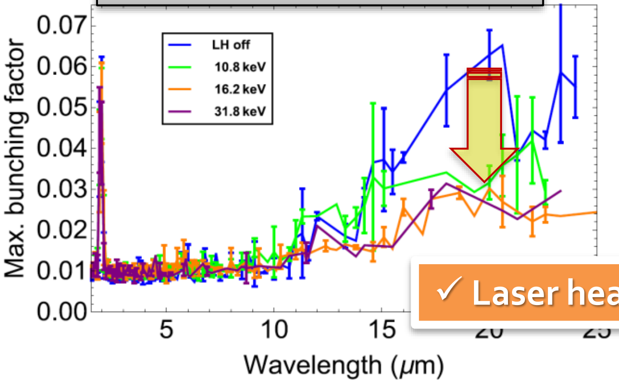
✓ Pulse stretcher vs. stacking

SwissFEL, S. Bettoni et al., PRAB 23 (2020)



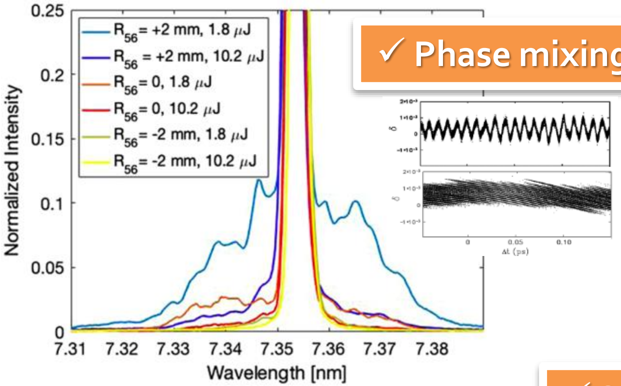
✓ Slow-response photo-cathode

FERMI, A. Brynes et al., Sci. Rep. 10 (2020)



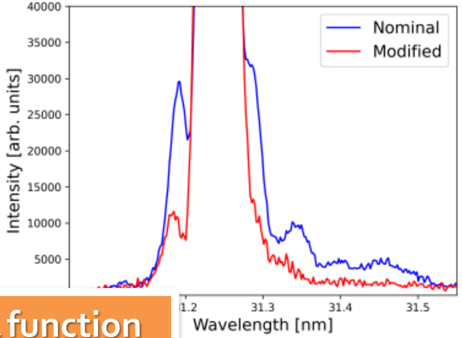
✓ Laser heater

FERMI, Di Mitri, Spampinati, PRL (2014)



✓ Phase mixing

FERMI, A. Brynes et al., PRAB (2024)



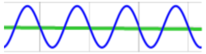
✓ *Curl-H$_x$* function

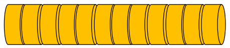
Impedances

S. Di Mitri, CAS 2018 Vol.I

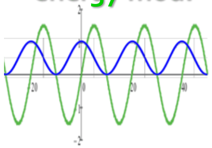
compression & amplification

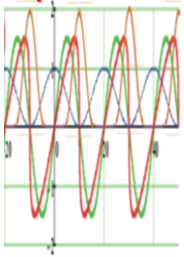
density mod.





energy mod.





Longitudinal Space Charge:
effective 3-D for round beam, energy-dependent, shielding

$$Z_{LSC}(k) = \frac{iZ_0}{\pi k r_b^2}\left[1 - 2I_1\left(\frac{kr_b}{\gamma}\right)K_1\left(\frac{kr_b}{\gamma}\right)\right]$$

RF geometric impedance:
1-D, short bunches

$$Z_{RF}(k) \approx \frac{iZ_0}{\pi k a^2} .$$

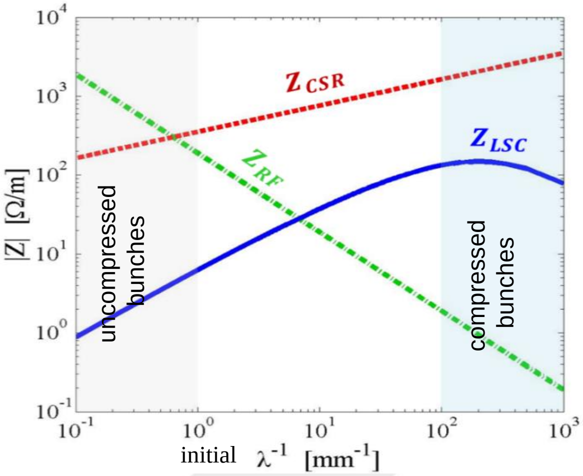
Coherent Synchrotron Radiation:
1-D, steady-state

$$Z_{CSR}(k) = \frac{Z_0 k^{1/3}}{\pi R^{2/3}}(0.41 + i0.23)$$

Coherent Edge Radiation:
1-D, steady-state

$$\int Z_{CER}(k) = \frac{Z_0}{2\pi}\ln\left(\frac{\min(L_d, \lambda\gamma^2/2\pi)}{\rho^{2/3}\lambda^{1/3}}\right)$$

a=10mm, R=5m, r_b=0.4mm, E=0.3GeV



Z. Huang, K.-J. Kim, PRST-AB 5, 074401 (2002)

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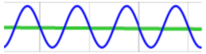
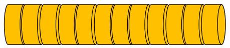
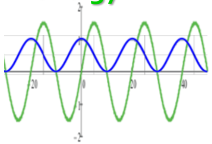
Huang–Kim model

density mod.

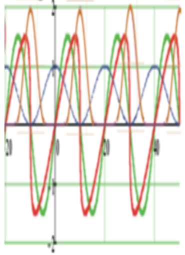




energy mod.



compression & amplification



- Integral linearized Vlasov eq., solved by iteration at 2nd order in kernel.
- We keep low gain terms in.

Bunching factor:
1-D, shot noise

$$|b_0(\lambda)|^2 = \frac{2e}{I_0}\Delta\nu = \frac{2ec}{I_0}\frac{\Delta\lambda}{\lambda^2} \approx \frac{2ec}{I_0\lambda}$$

Energy modulation:
energy-dependent, s-integral

$$|\Delta\gamma(k)| = b(k)\frac{I_0}{I_A}\int_0^s d\tau\,\frac{4\pi Z(k;s)}{Z_0}$$

Bunching factor:

$$b[k(s);s] = b_0[k(s);s] + \int_0^s d\tau\,K(\tau,s)\,b[k(\tau);\tau]$$

where

$$K(\tau,s) = ik(s)R_{56}(\tau\to s)\frac{I(\tau)Z[k(\tau);\tau]}{\gamma I_A}e^{-k_0^2U^2(s,\tau)\sigma_\delta^2/2}\times$$
$$\exp\left[-\frac{k^2(s)\epsilon_0\beta_0}{2}\left(V(s,\tau)-\frac{\alpha_0}{\beta_0}W(s,\tau)\right)^2-\frac{k^2(s)\epsilon_0}{2\beta_0}W^2(s,\tau)\right]$$

is specialized to a *4-dipoles chicane* (most of CSR driven in the 4th dipole).

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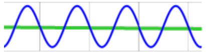
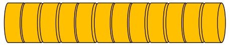
Bosch–Kleman model

R. Bosch et al., PRST-AB 5, 090702 (2008)

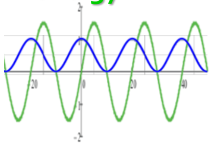
- Matrix model for modulations in the 2-D long. ph. sp.
- Low gain terms included by def.

density mod.

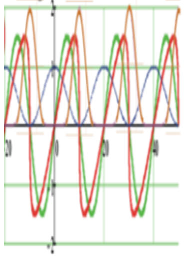




energy mod.



compression & amplification



Bunching factor:
1-D, shot noise

$$|b_0(\lambda)|^2 = \frac{2e}{I_0}\Delta\nu = \frac{2ec}{I_0}\frac{\Delta\lambda}{\lambda^2} \approx \frac{2ec}{I_0\lambda}$$

Energy modulation:
energy-dependent, s-integral

$$S(s;\lambda) = \frac{1}{E(s)}\begin{pmatrix} E(s) & 0 \\ -Z_{int}(\lambda)s & E(0) \end{pmatrix}$$

Bunching factor:

$$T_{II} = \frac{\Delta I/I_{out}}{\Delta I/I_{in}} = F_3 - \frac{iF_1F_2C_1^2eZ_{LSC}I_0k_0R_{56}}{E_1}$$

where

$$F_1(\lambda) = \int d\delta f(\delta)\cos[k(\lambda)CR_{56}\,\delta/E]$$

$$G_1(\lambda) = \int d\delta f(\delta)\sin[k(\lambda)CR_{56}\,\delta/E]$$

are for *generic dispersive terms*.

CERTIFIED MANAGEMENT SYSTEM CQY CERTIQUALITY UNI EN ISO 9001:2015 UNI ISO 45001:2018

Longitudinal Landau damping

- **Intrinsic energy spread, IBS, LH:** added in qudrature step-wise along the line. Exponential suppression of bunching.

Huang–Kim

$$exp\left[-\frac{1}{2}\left(CkR_{56}\sigma_{\delta,i}\right)^2\right]$$

Bosch–Kleman

$$\mathrm{D}(\lambda)=\begin{pmatrix} F_1(\lambda) & ikR_{56}CF(\lambda) \\ i\,CG_1(\lambda)/E & CF_1(\lambda)-kR_{56}C^2G_1(\lambda)/E \end{pmatrix}$$

- **Laser heater:** round beam, Gaussian distributions, *arbitrary electrons-laser overlap.*

$$\Delta\gamma_{LH}=\sqrt{\frac{P_L}{P_0}\frac{KL_u}{\gamma\sigma_r}}[JJ_{01}(K)]\xi_1(\sigma_r^2,\sigma_{LH}^2)$$

$$(\mathrm{F},\mathrm{G})_{\mathrm{LH}}(\lambda)=exp\left[-\frac{1}{2}\left(CkR_{56}\sigma_{\delta,i}\right)\right]\xi_{0,1}(\sigma_r^2,\sigma_{LH}^2)$$

where $\xi_{0,1}(\sigma_r^2,\sigma_{LH}^2)=\int R\,dRexp\left(-\frac{R^2}{2}\right)J_{0,1}\left(kR_{56}\delta_{LH}exp\left(-\frac{R^2\sigma_r^2}{4\sigma_{r,LH}^2}\right)\right)$

Transverse Landau damping

- Dispersive motion coupled to betatron emittance: $\mathbf{H_x \varepsilon_x}$.

Huang–Kim

$$exp\left[-\left(k\theta_b\sqrt{\varepsilon_x\beta_x}\right)^2\right]$$

and related quantities for 1- and 2-stage amplification through a 4-dipole chicane.

*The **damping** appears to be dominated by the derivative of dispersion, i.e., **outer dipoles**.*

Bosch–Kleman

$$exp\left[-\left(k\theta_b\sqrt{\varepsilon_x\beta_x}\right)^2\right],\ exp\left[-\left(kL\theta_b\sqrt{\varepsilon_x/\beta_x}\right)^2\right]$$

and related quantities for uncompressed and compressed wavelengths through a 4-dipole chicane

*The damping is distributed along the chicane, associated to **both dispersion** and its **first derivative**.*

Gain & final energy spread

- Gain = ratio of final and initial bunching:
  - defined at any point along the line;
  - linear regime, including low gain terms + 2nd *order correction.*

$$G(k_i) \cong \left| \frac{\Delta\hat{\rho}_f / \hat{\rho}_f}{\Delta\hat{\rho}_i / \hat{\rho}_i} \right| \approx |(1 - iR_{56}\Delta p(k_0;0))| e^{-\frac{1}{2}C^2k^2R_{56}^2\sigma_\delta^2} \cong \frac{4\pi}{Z_0}\frac{I_0}{I_A} Ck_i |R_{56}| \left| \int_0^L \frac{Z_{LSC}(k_i;s)}{\gamma(s)} ds \right| \exp\left[-\frac{1}{2}\left(Ck_i R_{56}\sigma_{\delta,u} \right)^2 \right].$$

DEF.

LINEAR REGIME

we include 2nd order correction (frequency mixing)

HIGH GAIN APPROX.

we include low gain terms

$$|\Delta\gamma(\lambda)|^2 = \left| G(\lambda)\, b_0(\lambda)\, Z_{\rm LSC}^{\rm int}(\lambda) \right|^2 \Rightarrow \sigma_\gamma^2 = \int |\Delta\gamma(\lambda)|^2 = \frac{2ec}{I_0} \int d\lambda \frac{|G(\lambda) Z_{int}(\lambda)|^2}{\lambda^2}$$

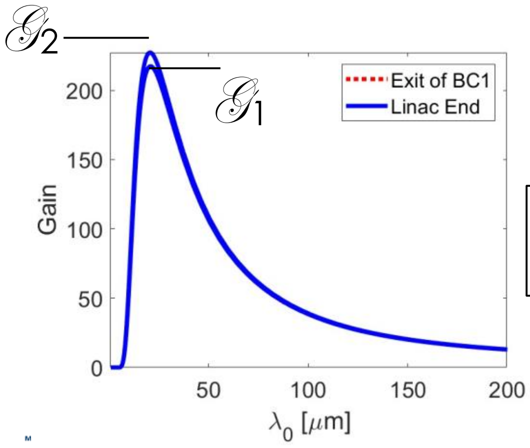
+ intrinsic^2 + IBS^2

Venturini's 2nd order gain correction

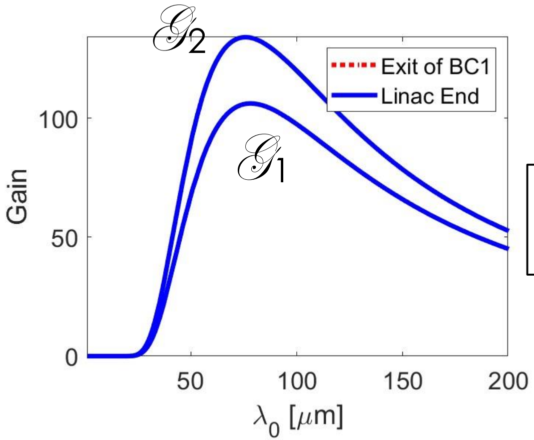
- k_1 and k_2 wave numbers mix to generate modulations at k_1+k_2 and k_1-k_2.
- If $CR_{56}\sigma_\delta|k_1 - k_2| \approx 1$, modulation at **$k_1$–$k_2$ survives to damping**, although k_1 and k_2 would individually be damped.

$$\mathcal{G}(k_n) = \mathcal{G}^{(1)}(k_n)\left[1 + \frac{2\gamma_*}{\pi n_b r_b}\left(\frac{IL_s}{I_A r_b \gamma\gamma_*}\right)^2 (R_{56}Ck_n)^2 \frac{\xi_n^2 \mathcal{F}(\xi_n)}{[1-\xi_n K_1(\xi_n)]^2}\right]^{1/2} \propto C^2 R_{56}^2$$

M. Venturini, NIM A 599 (2009) 140



$\mathbf{I_0 = 65A, CF = 10,}$
$\boldsymbol{\theta = 85}$**mrad**

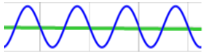


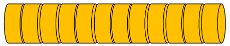
$\mathbf{I_0 = 65A, CF = 10,}$
$\boldsymbol{\theta = 170}$**mrad**

Intrabeam scattering

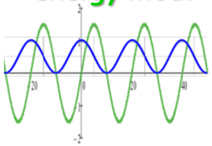
Z. Huang, LCLS-TN-02-8 (2002)

density mod.

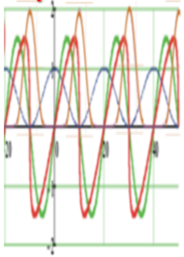




energy mod.



compression & amplification



$$\frac{1}{\sigma_\delta}\frac{d\sigma_\delta}{ds}\bigg|_{disp} = \frac{r_e^2 N_e [\log]}{8\gamma^2 \epsilon_n \sigma_x \sigma_z \sigma_\delta^3}\sigma_H$$

→ Added in quadrature to σ_δ, step-wise along the line

$$[\log] = \log\left(\frac{b_{max}}{b_{min}}\right) \approx \log\left(\frac{\theta_{max}}{\theta_{min}}\right) \approx \ln\left(\frac{\epsilon_n q_{max}}{2\sqrt{2}r_e}\right)$$

Piwinski, Bane Raubenheimer

Sets the **maximum scattering angle**, **discards** single scattering effects (cutoff).

Cutoff *time scale ≡ the time the bunch takes to travel along the section.*

Impact of IBS on the gain

G. Perosa, S. Di Mitri, Sci. Rep. 11:7895 (2011)

- Since the cutoff depends on the beam energy, its effect has to be properly integrated along accelerating sections.

CONSTANT Clog

$$\sigma^2_{\delta,IBS} \cong \frac{4}{3}\frac{k(\bar{\gamma})}{G/m_e c^2}\left(\frac{\gamma_f^{3/2}-\gamma_0^{3/2}}{\gamma_f^2}\right), \quad \gamma_f \approx \gamma_0$$

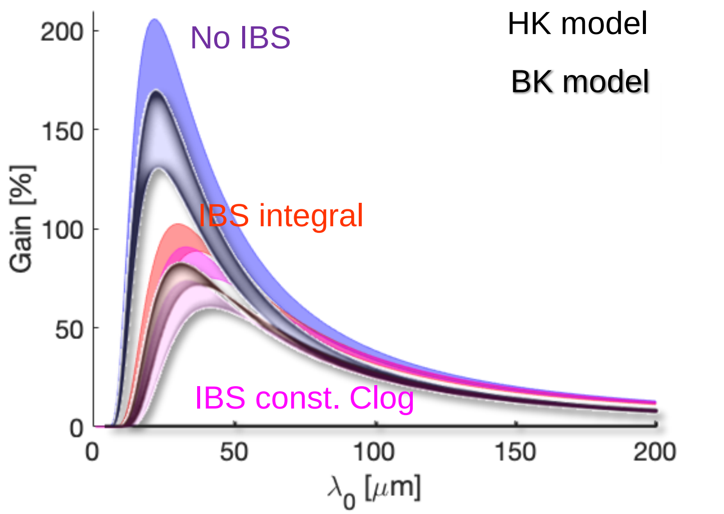
INTEGRAL, approx.

$$\sigma^2_{\delta,IBS} \cong \frac{1}{G/m_e c^2}\frac{\int_{\gamma_0}^{\gamma_f} d\gamma' k(\gamma')}{\gamma_f^2} + \mathcal{O}(\delta)$$

CLOSED FORM, approx.

$$\sigma^2_{\delta,IBS} \cong \frac{A}{G/m_e c^2}\left(\frac{a+\ln(\gamma_f)}{\sqrt{\gamma_f}} - \frac{a+\ln(\gamma_0)}{\sqrt{\gamma_0}}\right) + \mathcal{O}(\delta)$$

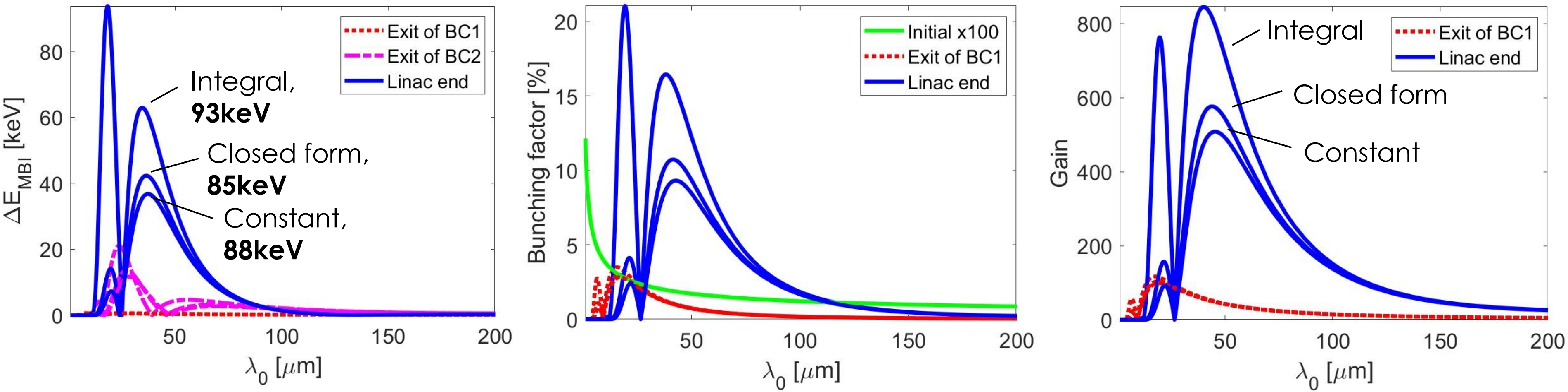
FERMI linac, C1=10



- IBS reduces the peak gain by ~50%.
- Integral form predicts a gain ~15% larger than the analytical form.
- HK-model predicts a peak gain ~20% larger than BK-model.

Comparison of IBS models

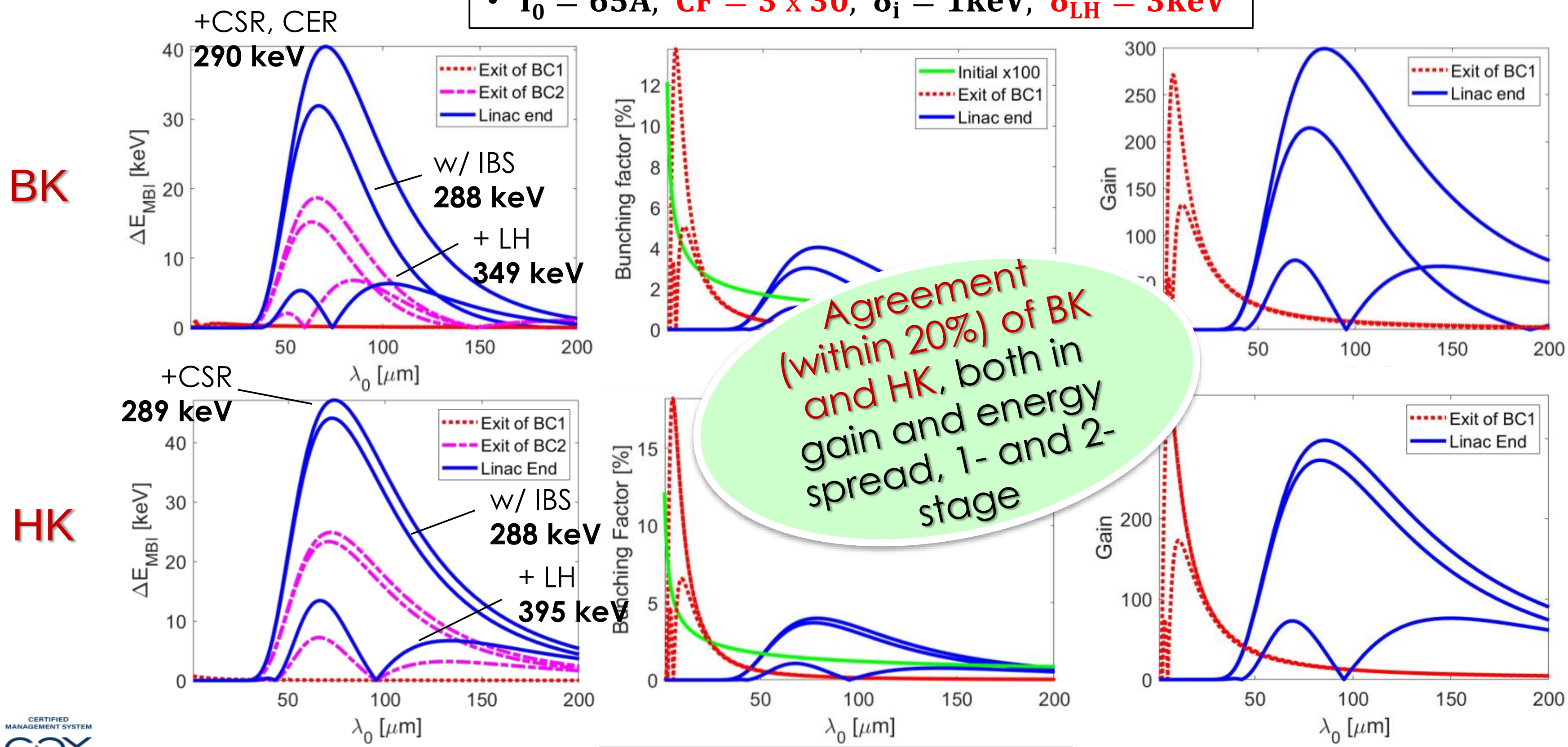
- $\mathbf{I_0 = 65A}$, $\mathbf{CF = 5\times3}$, $\mathbf{\delta_i = 1keV}$, $\mathbf{\delta_{LH} = 3keV}$



- All IBS methods well agree in one-stage compression.
- Integral IBS form predicts peak gain up to 40% larger in 2-stage, but comparable final energy spread → *conservative model*

Comparison of BK and HK models

- $\mathbf{I_0 = 65A}$, $\mathbf{CF = 3 \times 30}$, $\mathbf{\delta_i = 1keV}$, $\mathbf{\delta_{LH} = 3keV}$

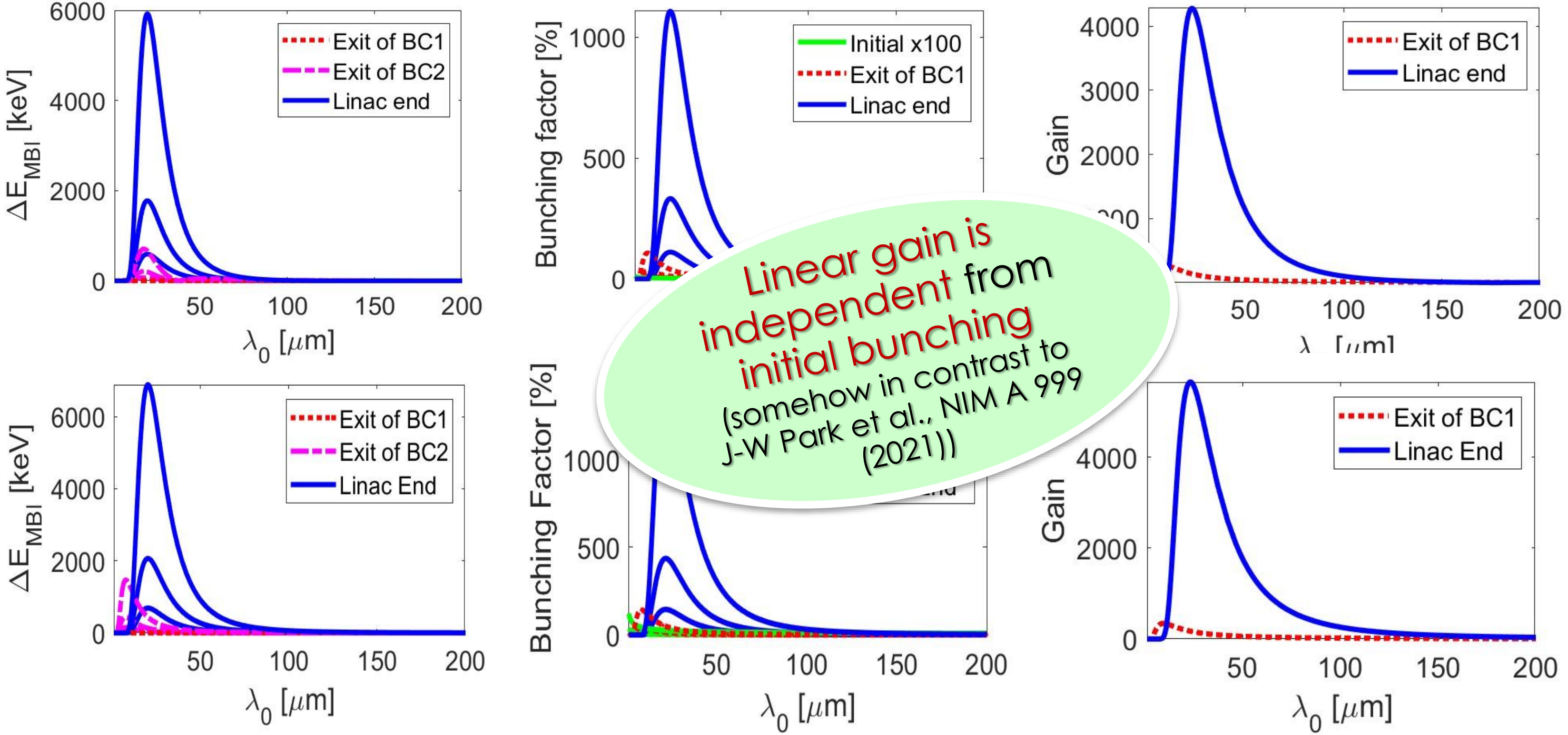


Validation: initial bunching

- $\mathbf{I_0 = 65A,\ CF = 5 \times 3,\ \delta_i = 1keV,\ \delta_{LH} = 0,\ b0 = 1\times, 3\times, 10\times}$

BK

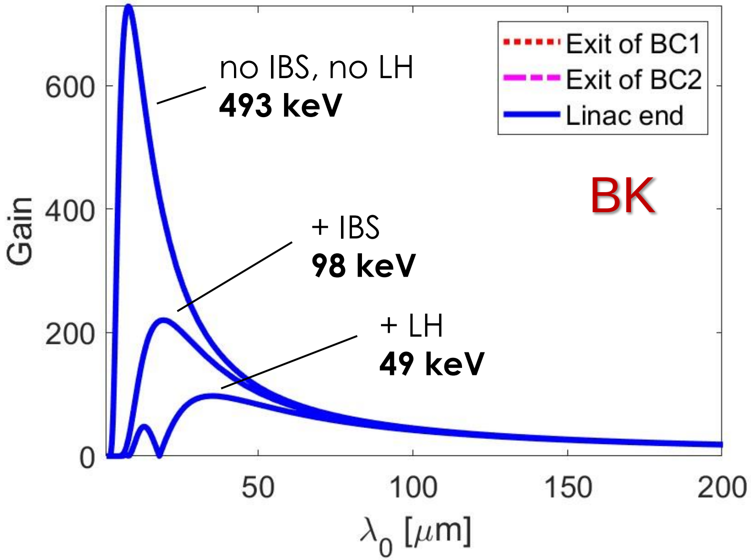
HK

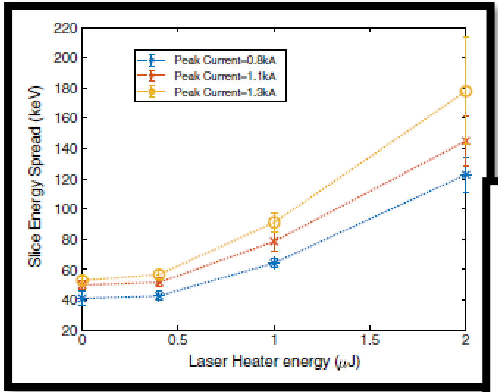


Linear gain is independent from initial bunching
(somehow in contrast to J-W Park et al., NIM A 999 (2021))

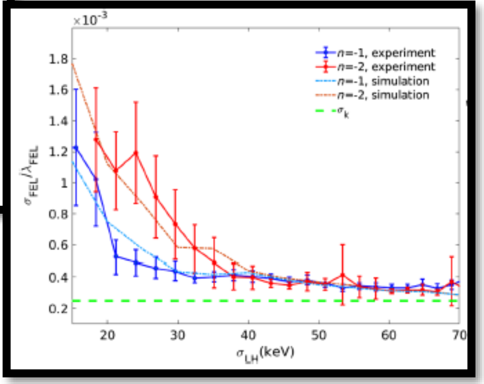
BK vs. HK: *FERMI BC1 only*

- $I_0 = 65A$, $CF = 10$, $\delta_i = 1keV$, $\delta_{LH} = 3keV$





G. Penco, G. Perosa et al., PRAB 23, 120704 (2020)

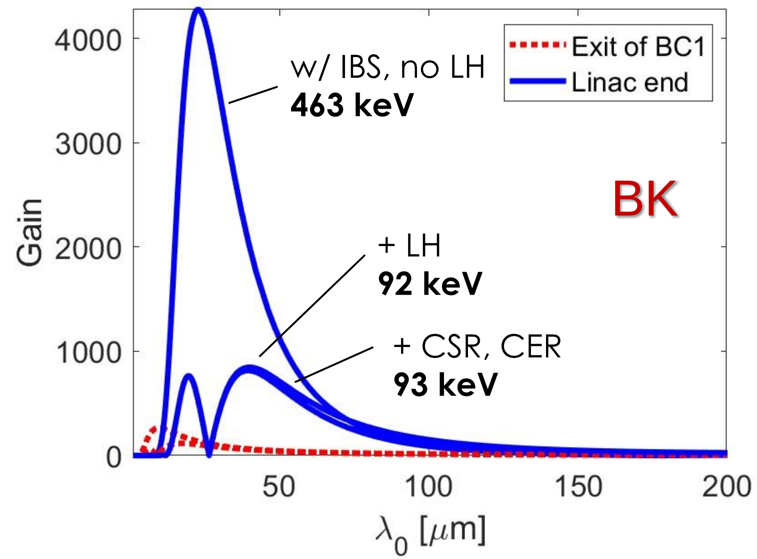


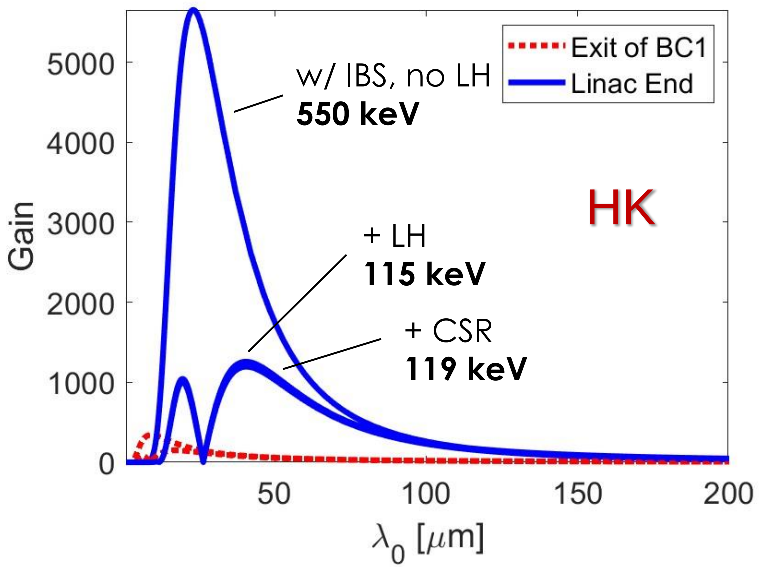
N. Mirian et al., PRAB 24, 080702 (2021)

- *"Cold-beam"* observations confirmed (exp.2020)
- Strong MBI in FERMI EEHG (exp.2019) has to be attributed to initial modulations above shot noise level - *cathode surface disruption?*

BK vs. HK: *FERMI BC1+BC2*

- $I_0 = 65A$, $CF = 4 \times 2.5$, $\delta_i = 1keV$, $\delta_{LH} = 3keV$





- BC1+BC2 apparently still compatible with lasing (energy spread), ***BUT***….
- …peak gain 10-times larger than in BC1-only,
- …final energy spread >> intrinsic one (120 vs. 30 keV) → *strong residual MBI*

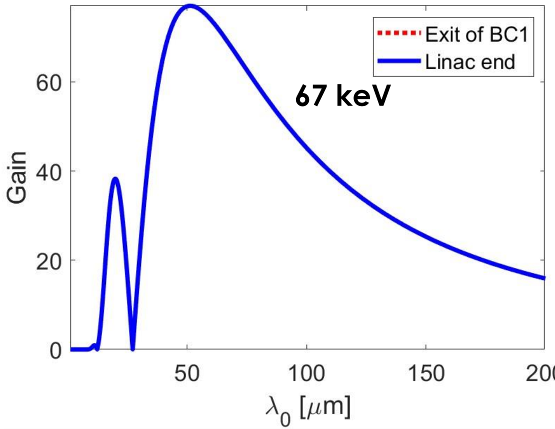
Elettra Sincrotrone Trieste

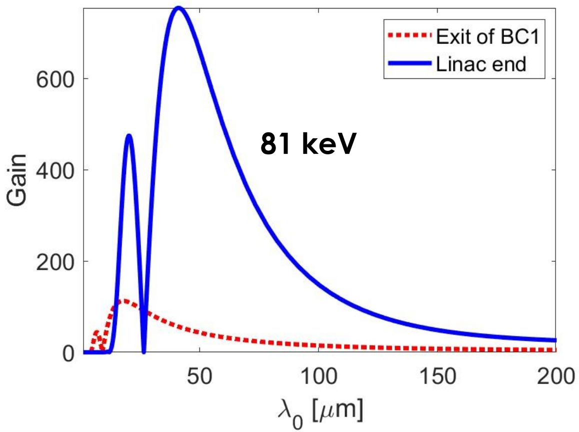
FERMI-U, *BC1 vs. BC1+BC2*

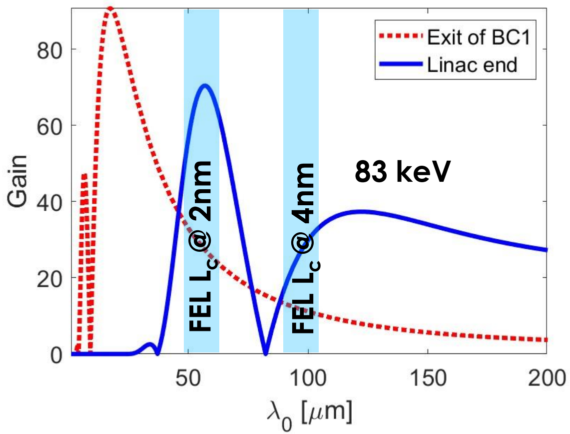
See also S. Di Mitri et al., NJP 22, 083053 (2022)

- $I_0 = 65A$, $CF\_tot = 15$, $\delta_i = 1keV$, $\delta_{LH} = 3keV$

BC1-only → **BC1+BC2** → **BC1+BC2 optimized**







- The model allows fast optimization of the compression scheme, e.g., lower initial current, stronger phase mixing in BC2.
- 2-stage can turn to be helpful for I> 1kA, C>>1, and w/ strong long. wakes

Conclusions

- Both HK and BK theory is suitable for modelling MBI in multi-stage compression.
- Reasonable agreement in a large variety of configurations, range of parameters, and implemented features.
  - Discrepancy tends to exceed 30% for peak gain at $\lambda_0 \leq 1\ \mu m$ (numerical issue?), or for large gain in 2-stage (amplification).
- Average optics along linac sections and definition of RMS bunch length add uncertainty to the prediction.
  - The models need gauging vs. numerical or experimental result.
- Still, both they capture the relevant physics and allow for fast optimization of machine parameters and layout.
  - *Next step is validation of 3-compressors, spreader, undulator line.*