Finite element based computation of beam coupling impedances

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TECHNISCHE UNIVERSITÄT DARMSTADT

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Wakefield computation in the time domain

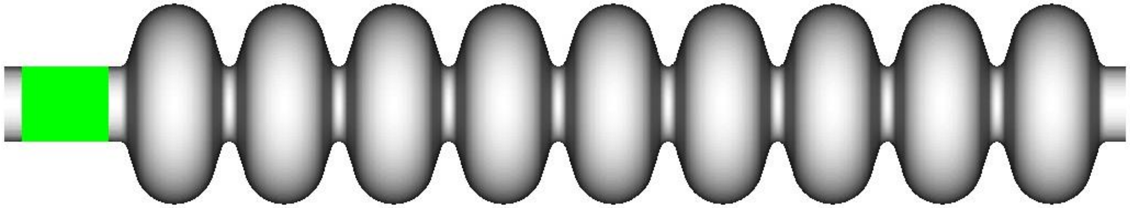
- The wakefield problem in the time domain

$$W_{\|}(r,s) = \frac{1}{Q}\int dz\, E_z\big(r,z,t(s,z)\big) \qquad t(s,z) = \frac{s+z}{c}$$

  - Solve time-dependent Maxwell's eqs. with beam current excitation
  - Get impedance by Fourier transform

$$Z_{\|}(r,\omega) = -\frac{1}{c}\frac{1}{\tilde{\lambda}(\omega)}\int ds W_{\|}(r,s)\exp\left(-i\frac{\omega}{c}s\right)$$

  - For short-range wakes: moving window / dispersion-free computation

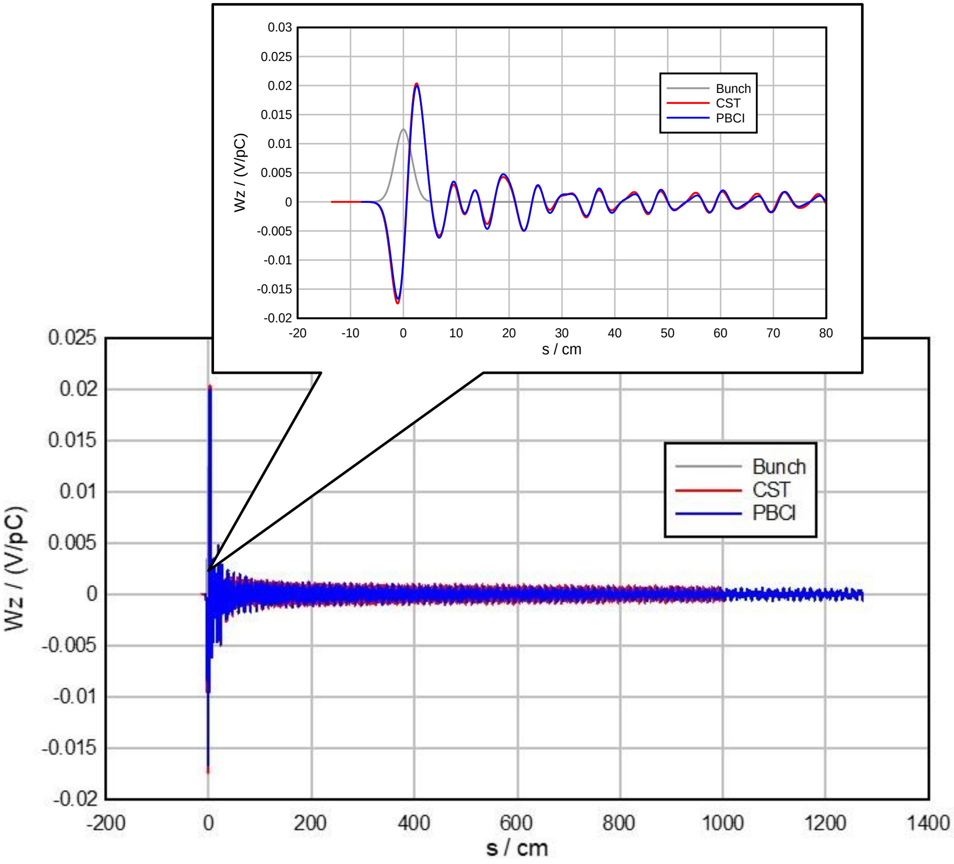


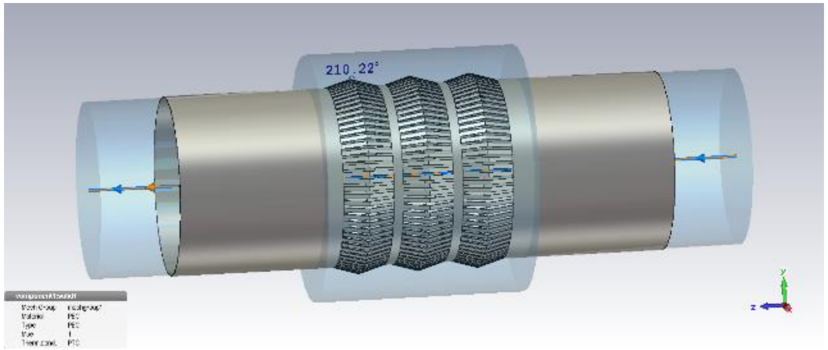
PBCI simulation: wakefield in a TESLA cavity

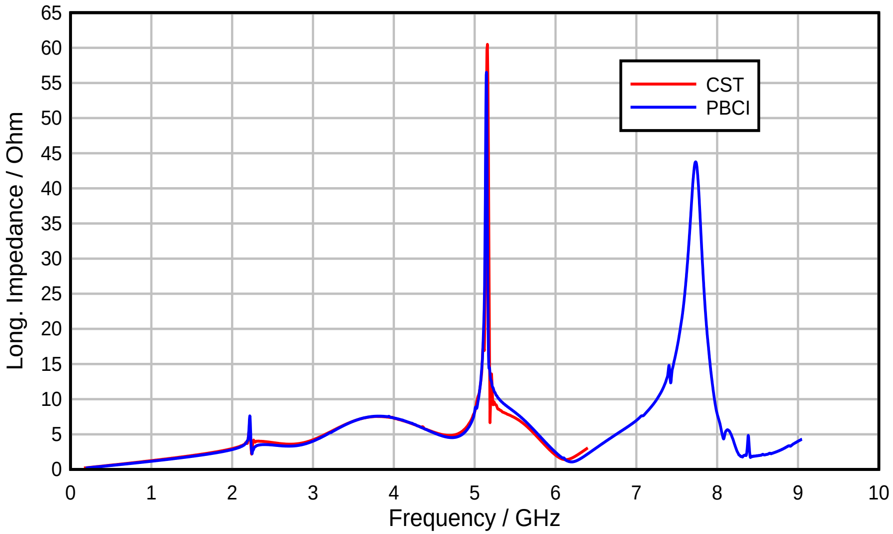
E. Gjonaj et al.,ICAP'06, MOM2IS02

Wakefield computation in the time domain

- LHC RF-Fingers







Contents

- A high order FE method for beam impedance computations
- Shielded CSR wakes
- Generalized S-Matrix formulation
- Lumped model cavity optimization
- Including external loads
- Domain decomposition approach

FE method for beam impedance

- The frequency domain problem

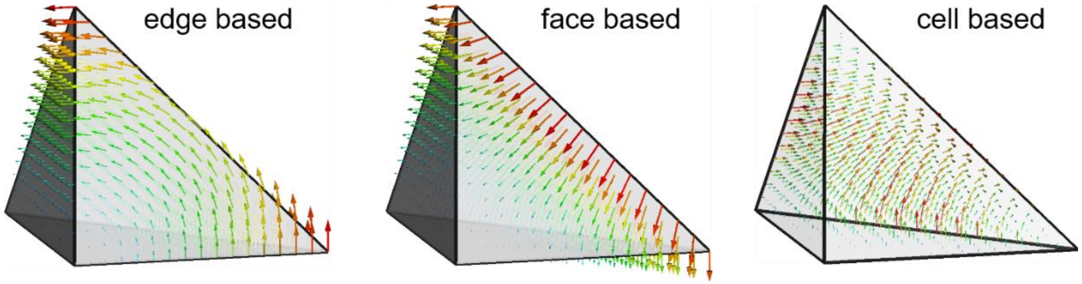
$$\nabla \times \mu^{-1} \nabla \times E - k_0^2 \varepsilon E = -jk_0 Z_0 J_s \qquad J_s(x, y, z, \omega) = \delta(x - x_0)\delta(y - y_0) e^{-i\frac{\omega}{v}z}$$

- Weak formulation: find $E \in H(curl)^*$ such that:

$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h = -jk_0 Z_0 \int dV\, J_s \cdot v_h$$

$$+ \underbrace{\oint_S dS\ n \cdot [v_h \times \mu^{-1} \nabla \times E]}_{\text{boundary term}}$$

boundary term



FE method for beam impedance

- Treatment of boundary surfaces

$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon \, E \cdot v_h =$$

$$-jk_0 Z_0 \int dV \, J_s \cdot v_h + \underbrace{\int_{S_{SIBC}} \boldsymbol{dS\, n \cdot [v_h \times \mu^{-1} \nabla \times E]}}_{\text{resistive wall}} + \underbrace{\int_{S_{WG}} dS n \cdot [v_h \times \mu^{-1} \nabla \times E]}_{\text{in \& outgoing pipes}}$$

resistive wall **in & outgoing pipes**

- Surface impedance

$$\oint_{S_{SIBC}} dS \;\, n \cdot [v_h \times \mu^{-1} \nabla \times E] = \cdots = j\omega \boldsymbol{Y_S(\omega)} \oint_{S_{SIBC}} dS \;\, v_h \cdot [n \times n \times E]$$

- Simple modification of the system matrix on SIBC surfaces
- No fitting of the surface impedance function or ADE/convolution is needed

FE method for beam impedance

- Treatment of boundary surfaces

$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h =$$

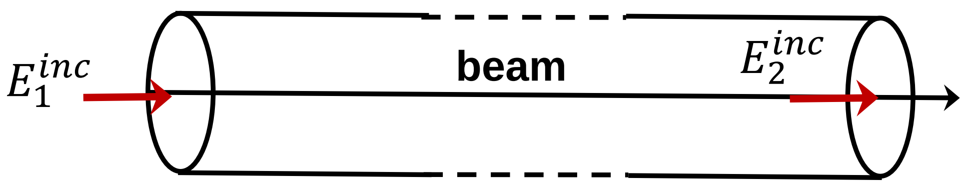
$$-jk_0 Z_0 \int dV\, J_s \cdot v_h + \underbrace{\int_{S_{SIBC}} dS\, n \cdot [v_h \times \mu^{-1} \nabla \times E]}_{\text{resistive wall}} + \underbrace{\int_{S_{WG}} \boldsymbol{dSn} \cdot [\boldsymbol{v_h} \times \boldsymbol{\mu^{-1}} \boldsymbol{\nabla} \times \boldsymbol{E}]}_{\text{in \& outgoing pipes}}$$

- Beam pipes

$$n \times \nabla \times E = n \times \nabla \times \boldsymbol{E^{inc}} + \sum_m a_m^{TE} \gamma_m^{TE} \boldsymbol{e_m^{TE}} + \sum_m a_m^{TM} \frac{-k_0^2}{\gamma_m^{TM}} \boldsymbol{e_m^{TM}}$$

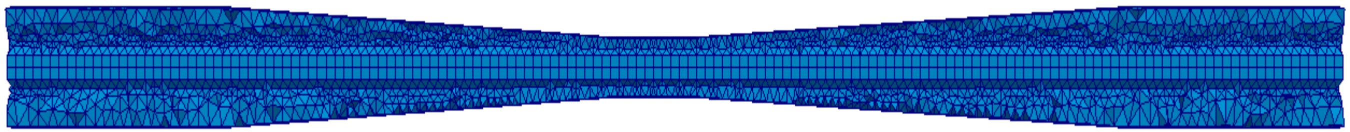
$$a_m^{TE} = \int_{S_{WG}} dS e_m^{TE} \cdot [E - E^{inc}]$$

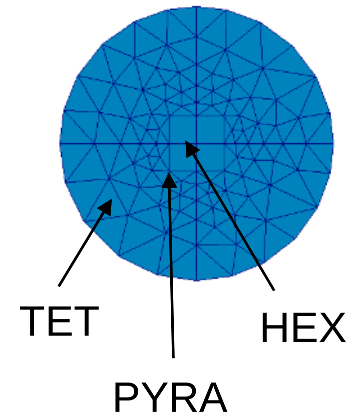
$$a_m^{TM} = \int_{S_{WG}} dS e_m^{TM} \cdot [E - E^{inc}]$$

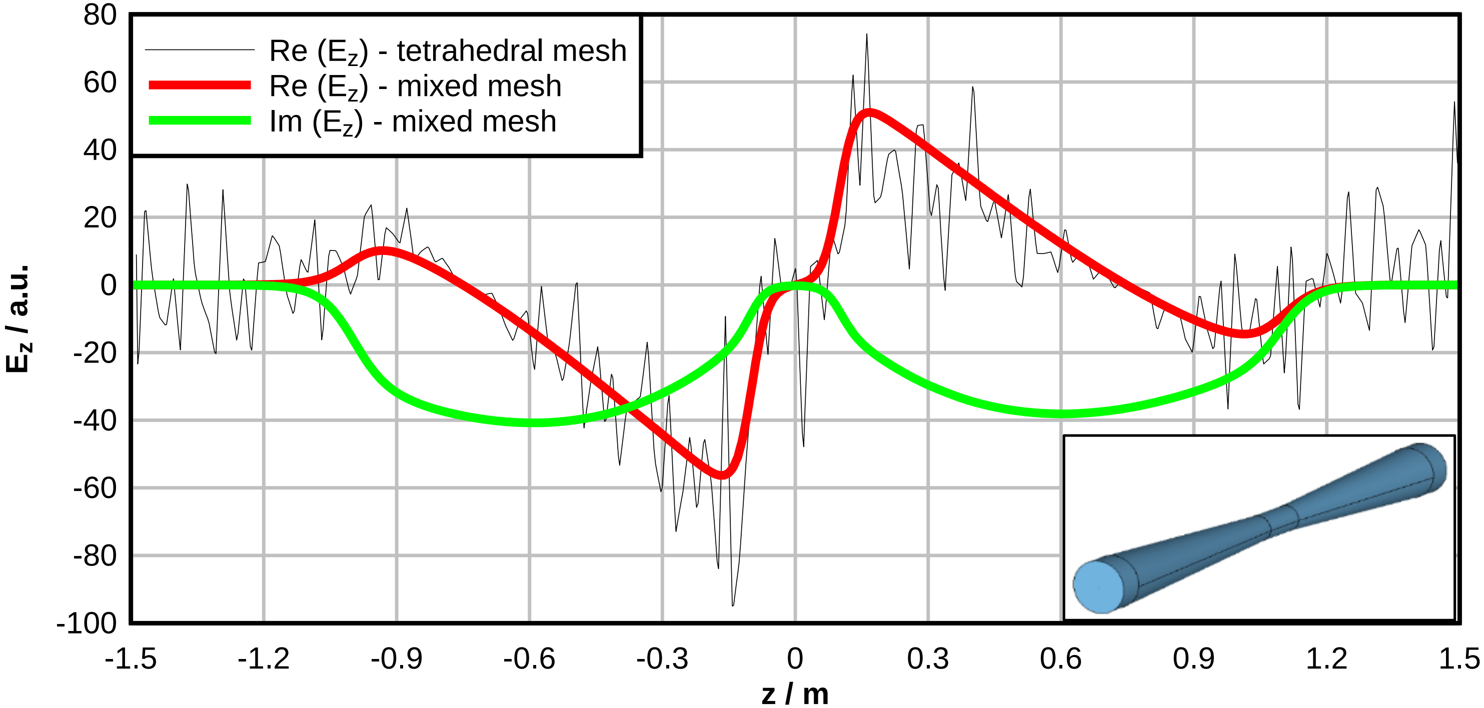


FE method for beam impedance

- Hybrid meshes – collimator example



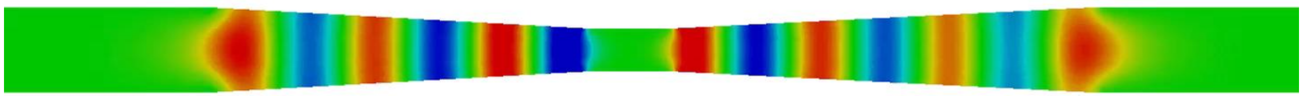




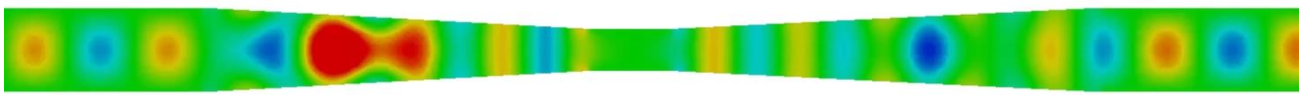
Longitudinal wakefield on axis at 100MHz

FE method for beam impedance

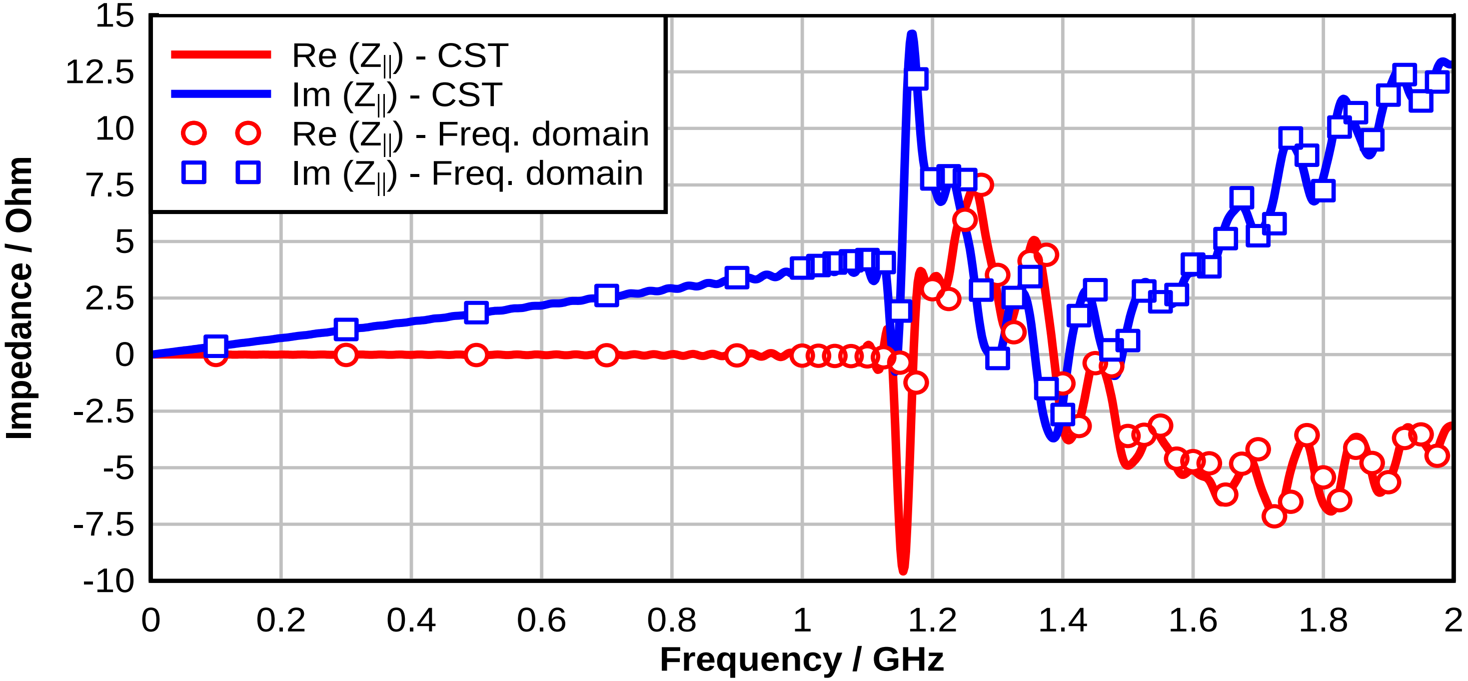
- Hybrid meshes – collimator example



E_z – 1GHz



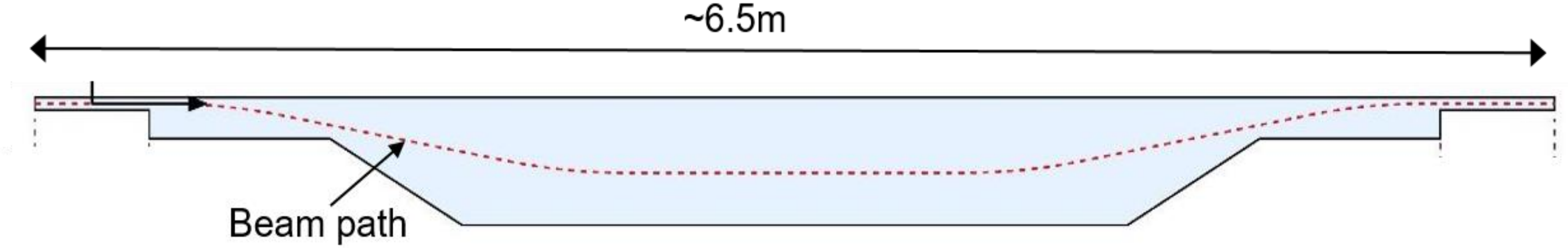
E_z – 1.5GHz

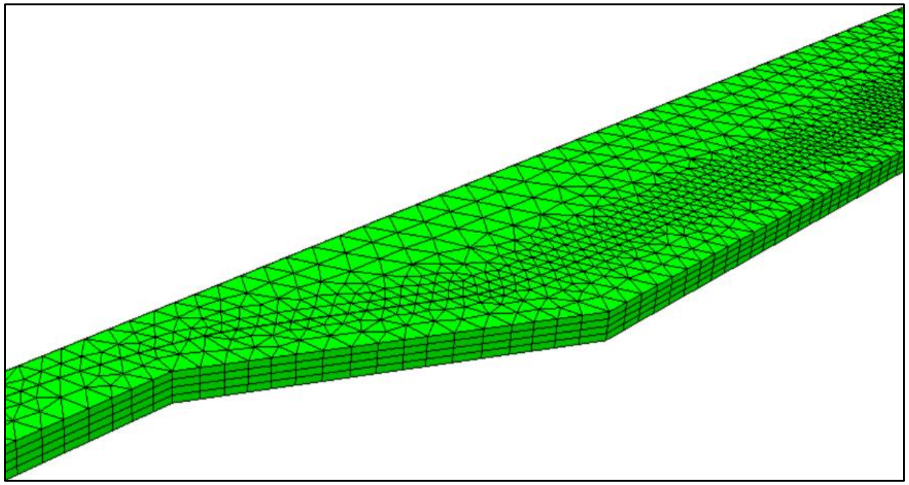


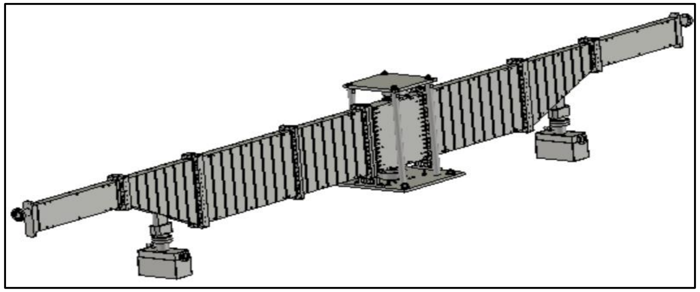
Comparison with CST

Shielded CSR wakes

- DESY-EuXFEL bunch compressor (BC1)



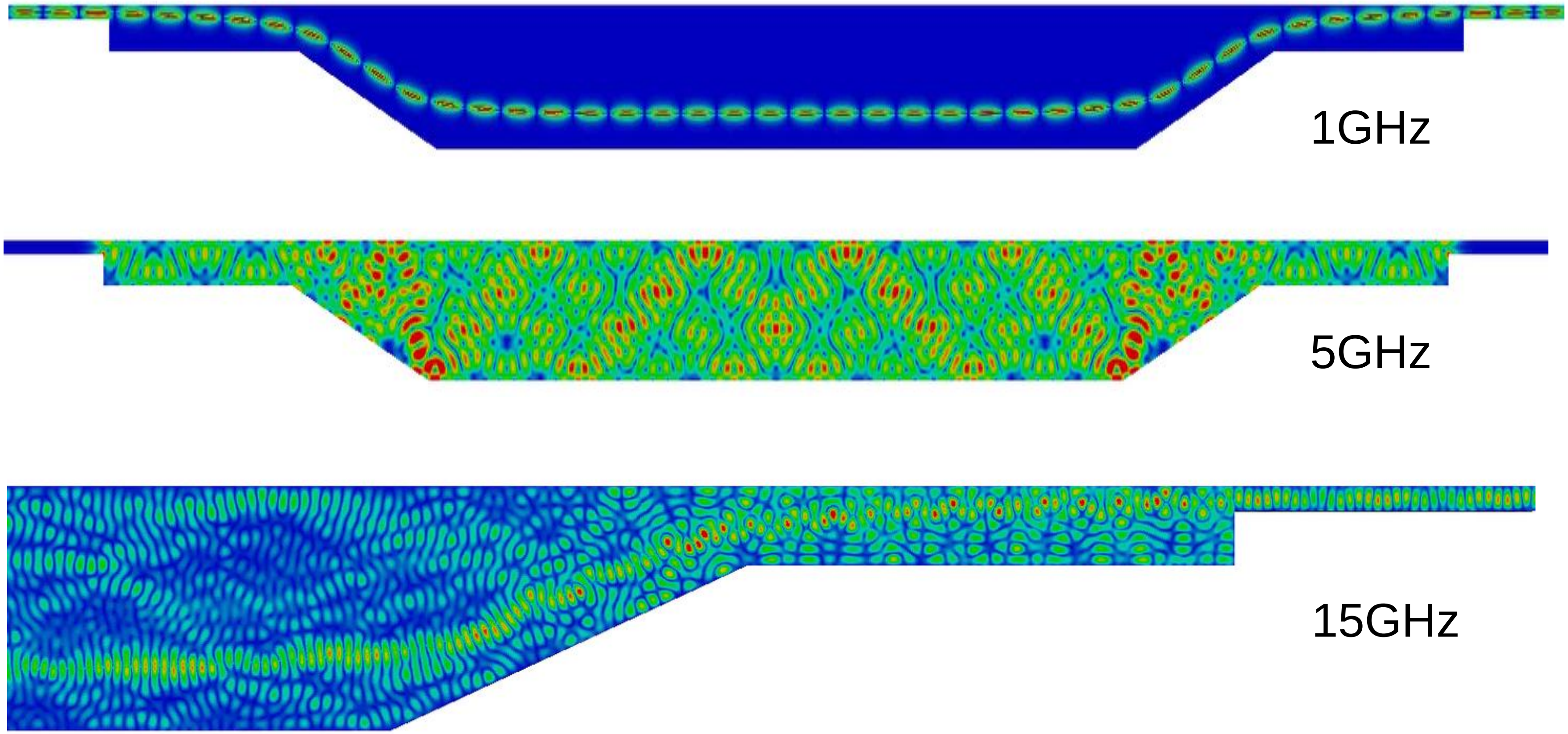




Prismatic element mesh:
$\Delta \approx 5$mm
600k cells
4^{th} order FEM

Shielded CSR wakes

- DESY-EuXFEL bunch compressor (BC1)

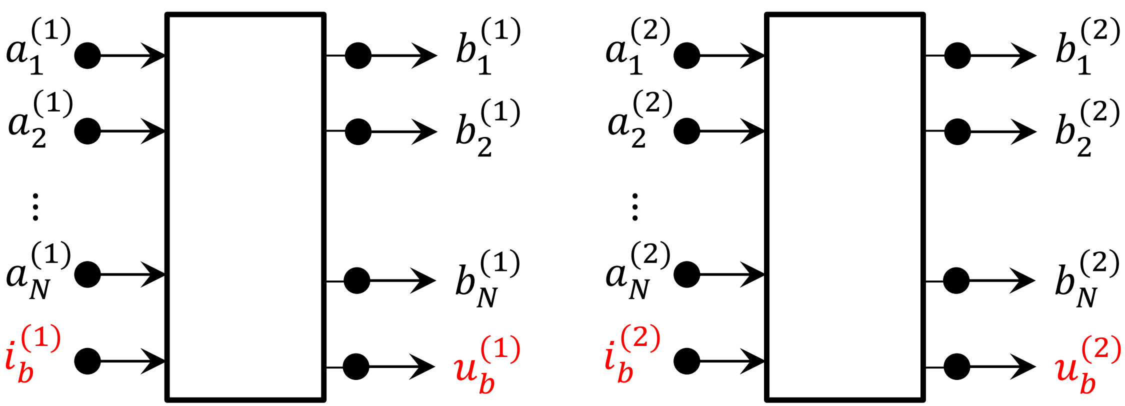


Generalized S-Matrix

- Scattering matrix with beam:

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \tilde{S} & \tilde{k} \\ \tilde{h} & Z_b \end{pmatrix} \begin{pmatrix} a_m \\ i_b \end{pmatrix} = \begin{pmatrix} b_m \\ u_b \end{pmatrix}$$

  - Concatenation of cascaded structures:



Matching conditions:

$$b_i^{(n)} = a_i^{(n+1)}$$

$$i_b^{(n)} = i_b^{(n-1)} e^{ik_0 L_{n-1}}$$

$$\sum_n u_b^{(n)} = u_b^{tot}$$

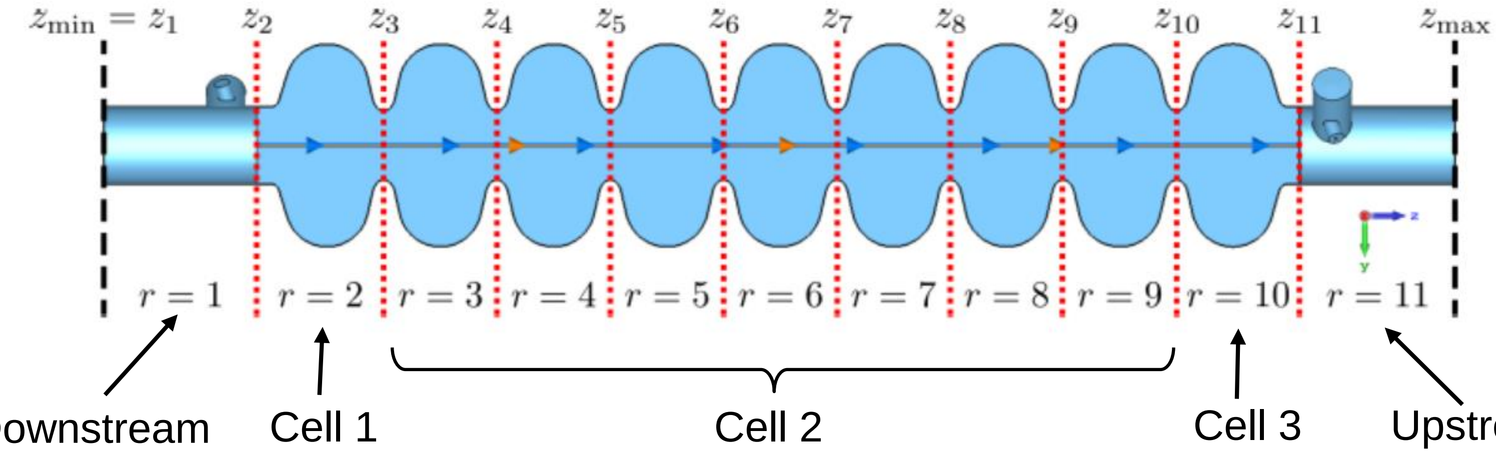
  - Total scattering matrix:

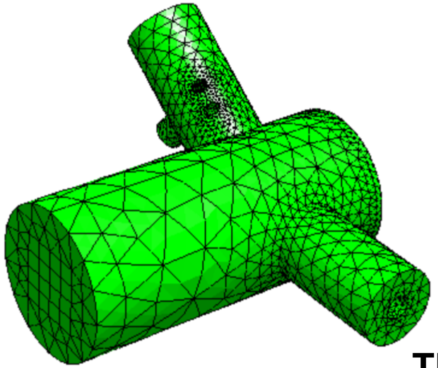
$$\begin{pmatrix} \tilde{S}^{tot} & \tilde{k} \\ \tilde{h} & Z_b^{tot} \end{pmatrix} \begin{pmatrix} a_m \\ i_b \end{pmatrix} = \begin{pmatrix} b_m \\ u_b^{tot} \end{pmatrix}$$

Flisgen; Gjonaj, et al.: PRAB (2020)

Generalized S-Matrix

- Tesla 1.3GHz cavity

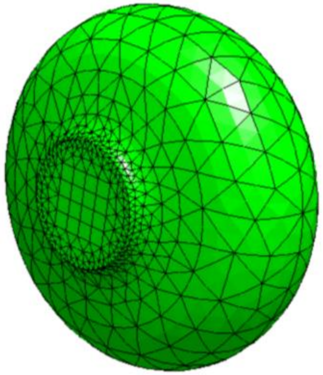




15 TE-Modes
($\omega_{max} = 8.2$Ghz)

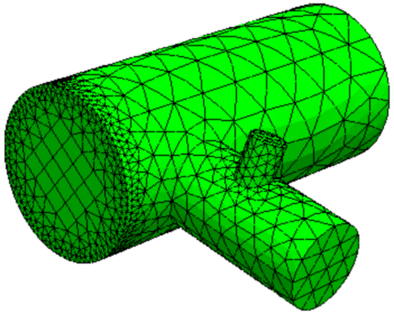
15 TM-Modes
($\omega_{max} = 10.6$Ghz)

TEM, …



15 TE-Modes
($\omega_{max} = 8.2Ghz$)

15 TM-Modes
($\omega_{max} = 10.6$Ghz)



Generalized S-Matrix

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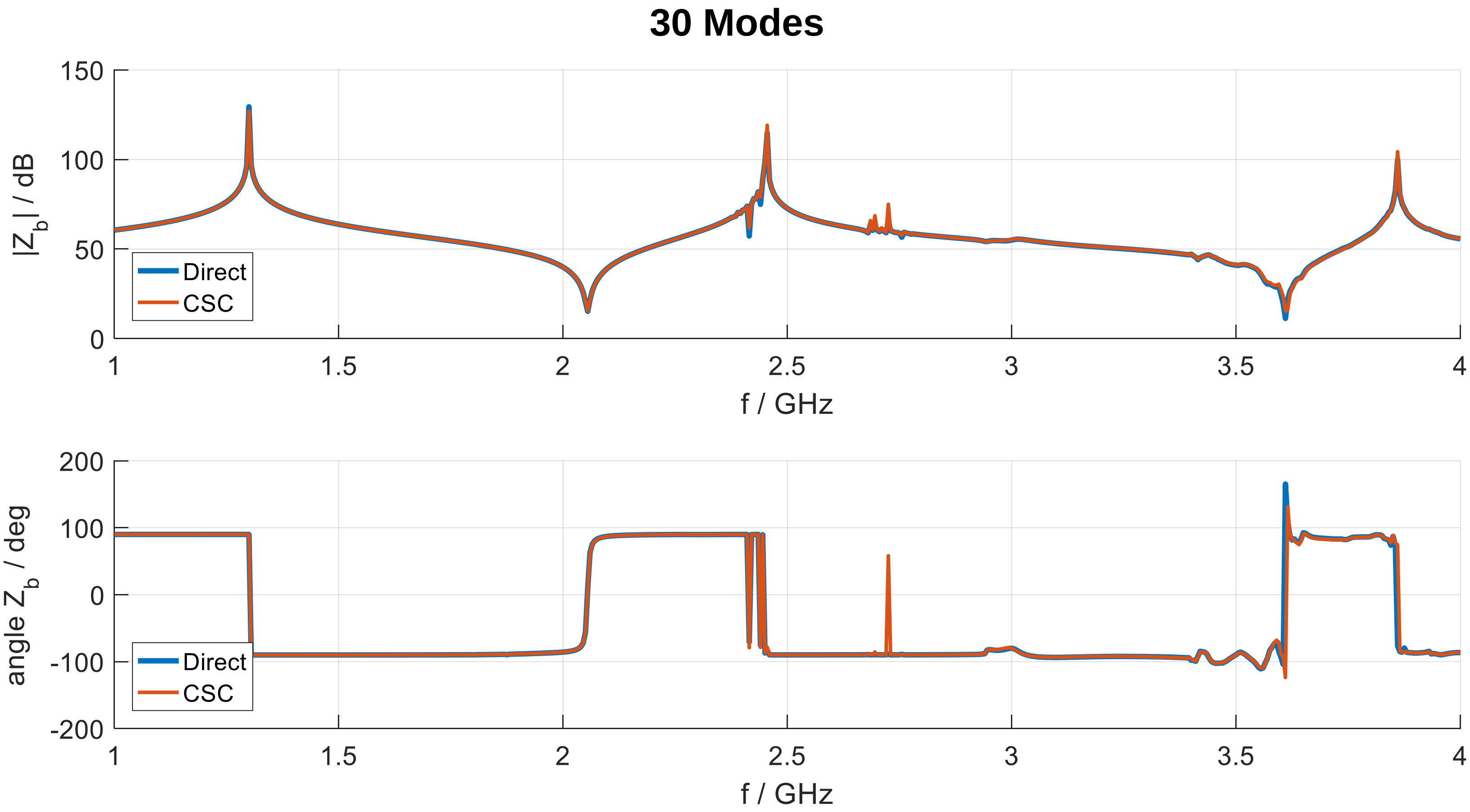
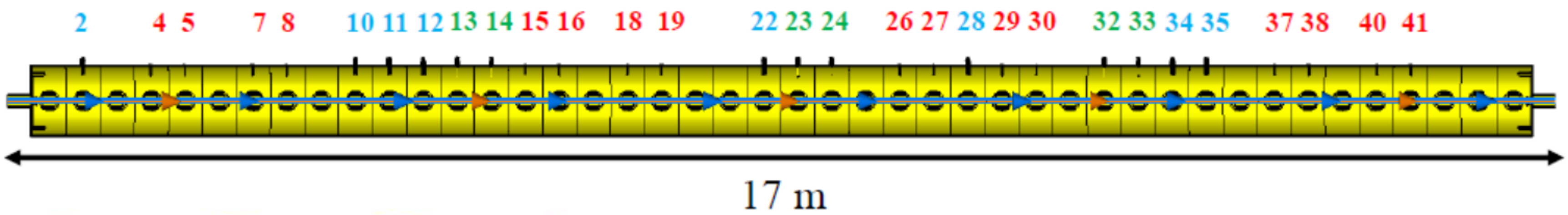
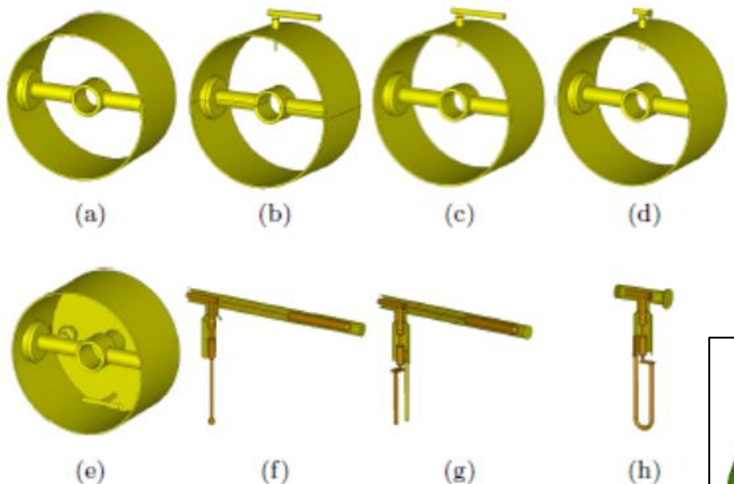
- Tesla 1.3GHz cavity

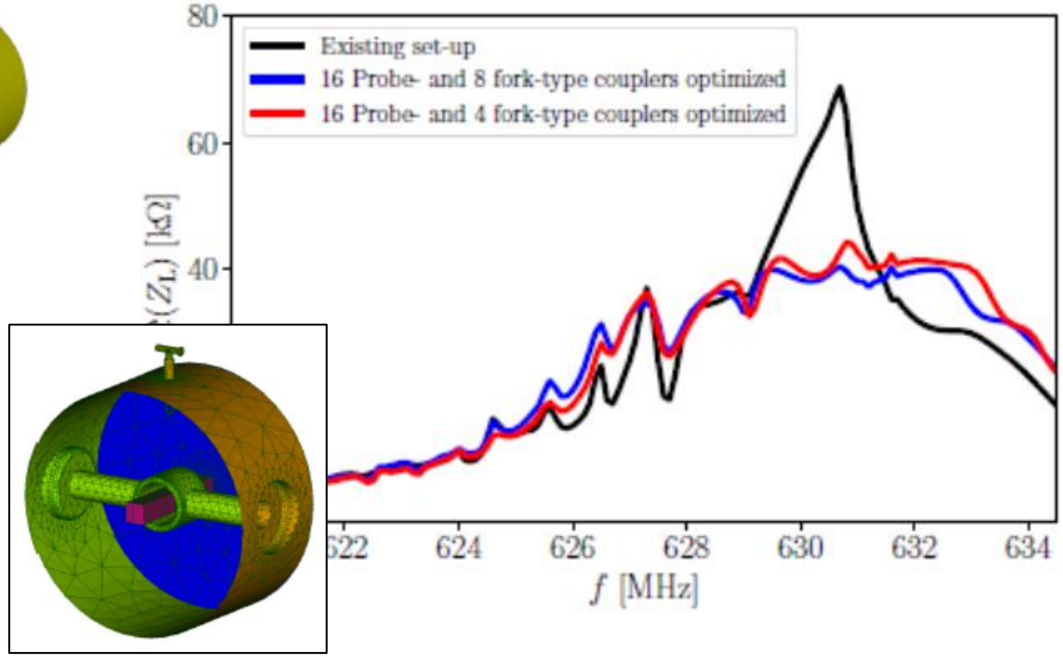


Lumped model cavity optimization

- SPS 44-cell 200MHz TW cavities



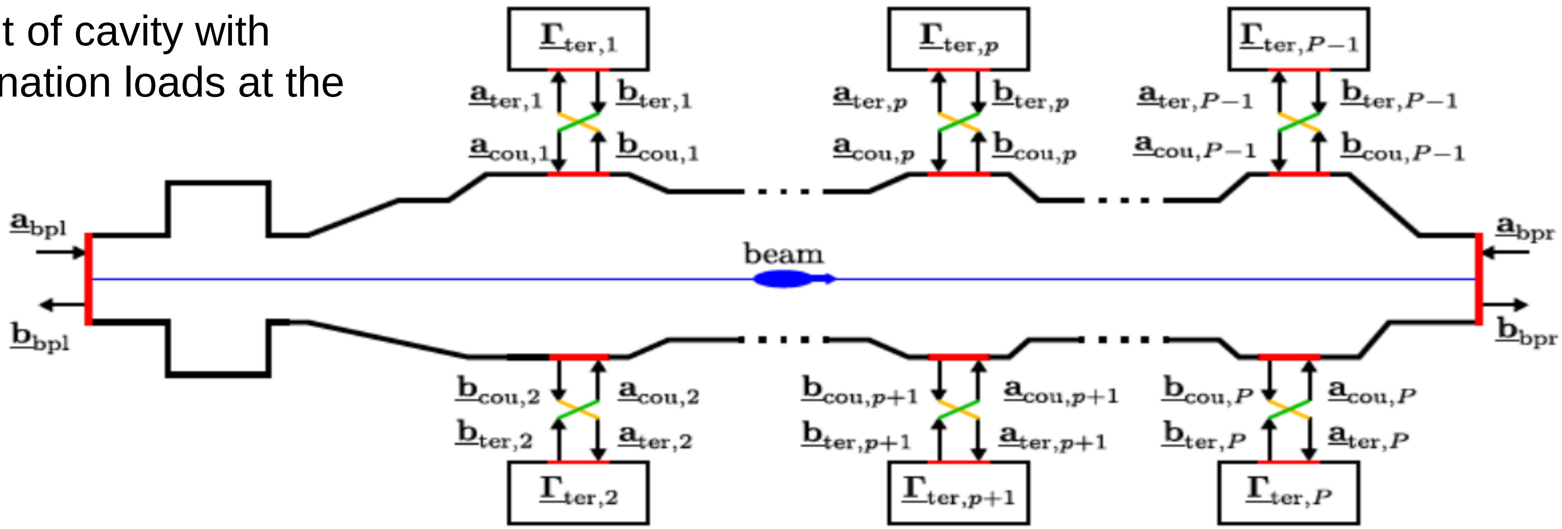




Zadeh, Gjonaj, et. al, PRAB 2022

Including termination loads

General layout of cavity with external termination loads at the coupler ports



$$
\begin{cases}
\begin{pmatrix} \tilde{S} & \tilde{k} \\ \tilde{h} & Z_b \end{pmatrix} \begin{pmatrix} a_m \\ i_b \end{pmatrix} = \begin{pmatrix} b_m \\ u_b \end{pmatrix} \\
a_{cou} = b_{ter} \\
a_{ter} = b_{cou} \\
R a_{ter} = b_{ter}
\end{cases}
$$

$R = diag\{\Gamma_{ter,1}, \Gamma_{ter,2}, \dots\}$ termination reflection matrix

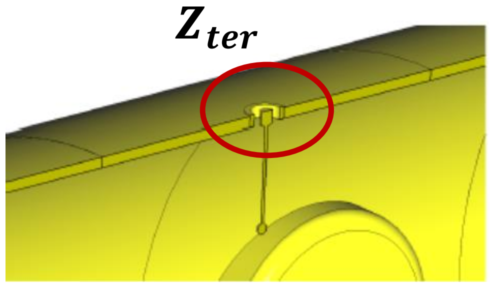
$$\tilde{G}_{mod} \begin{pmatrix} a_m \\ i_b \end{pmatrix} = \begin{pmatrix} b_m \\ u_b \end{pmatrix}$$

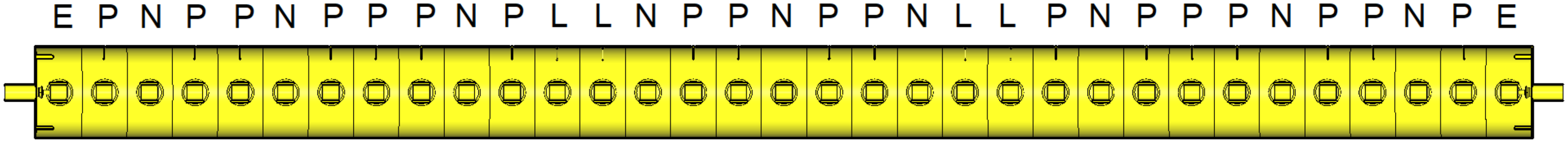
scattering matrix with termination + beam impedance

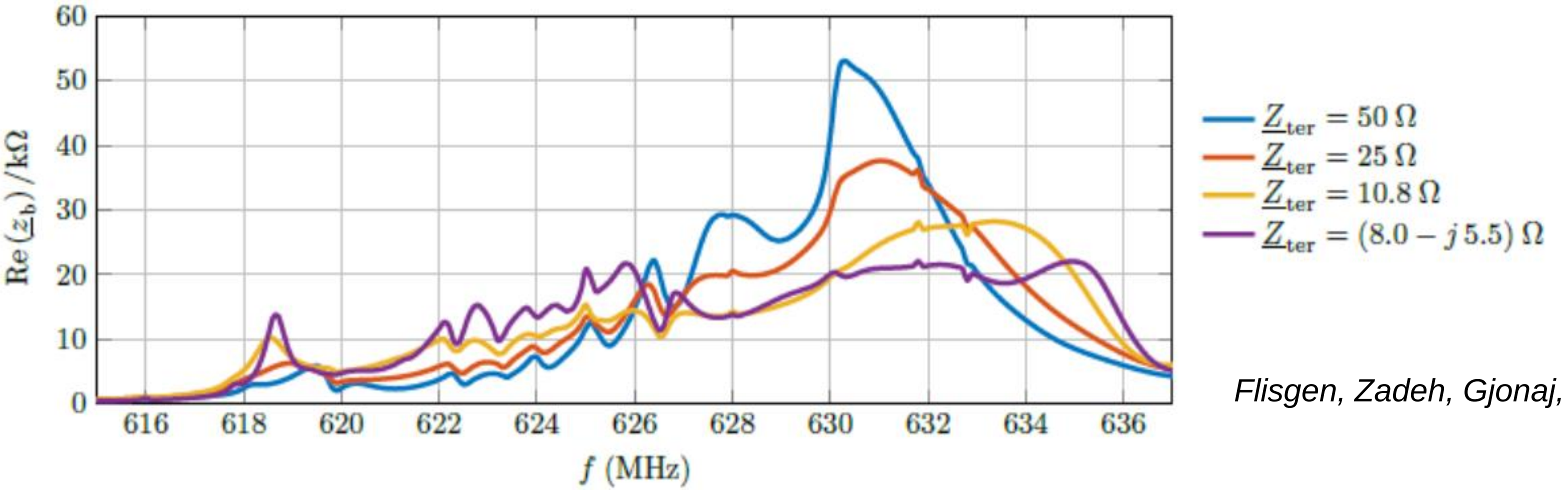
Flisgen, Zadeh, Gjonaj, PRAB 2023

Including termination loads

- SPS 33-cell 200 MHz TW-structure



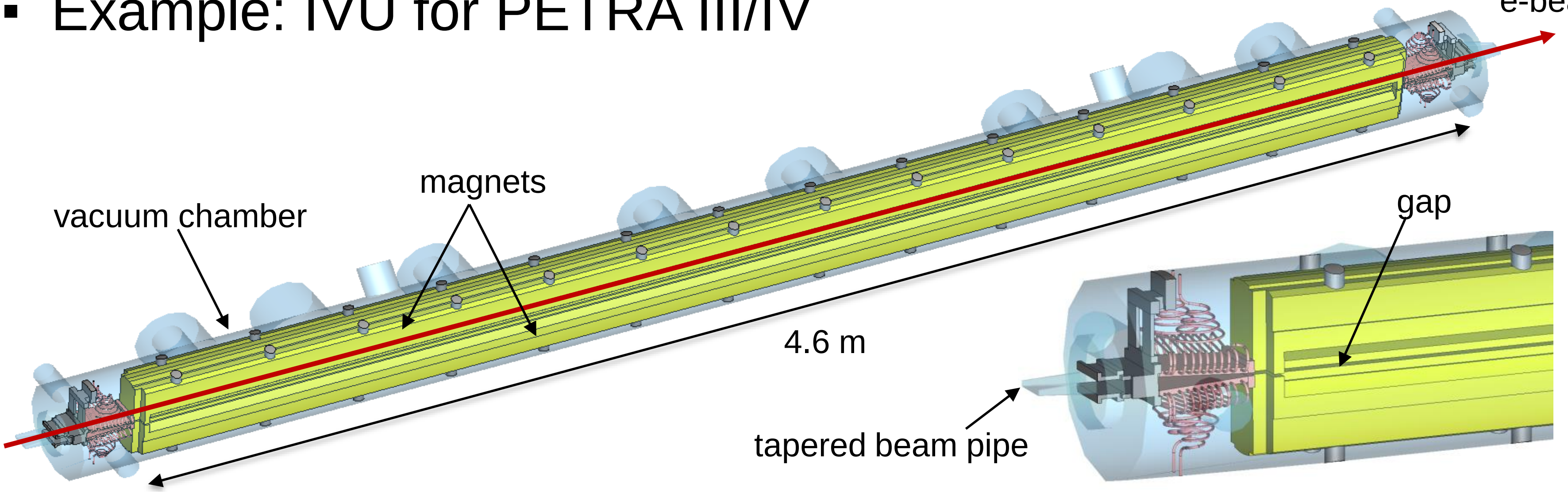




Flisgen, Zadeh, Gjonaj, PRAB 2023

Domain decomposition

- Example: IVU for PETRA III/IV

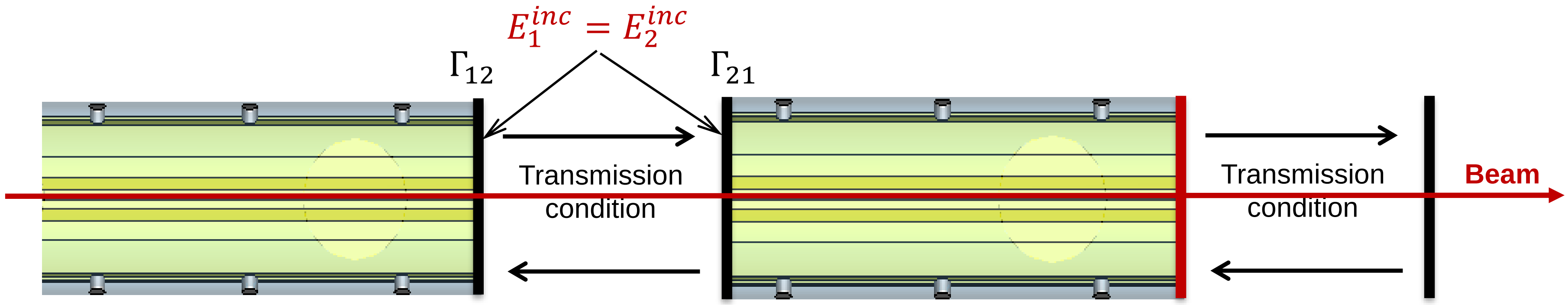


Order	# DoFs	Order	# DoFs
0	0.9e6	3	19e6
1	2e6	4	38e6
2	7e6	5	

- Multiple trapped modes at 0.1-1GHz
- High-Q dominated by wall losses
- Local heat spots / surface fields
- ~1500GB RAM
- ~ weeks of CPU-time

Domain decomposition

- Decompose geometry by suitable boundary conditions



$$F_{12}^{\text{out}} = \underbrace{\mathcal{S}(n_1 \times n_1 \times E_1{}')}_{\text{absorbing operator}} + n_1 \times (\nabla \times E_1{}') = \underbrace{\mathcal{S}(n_2 \times n_2 \times E_2{}') - n_2 \times (\nabla \times E_2{}')}_{\text{one-way wave operator}} = F_{21}^{\text{in}} \quad \text{on} \quad \Gamma_{12}$$

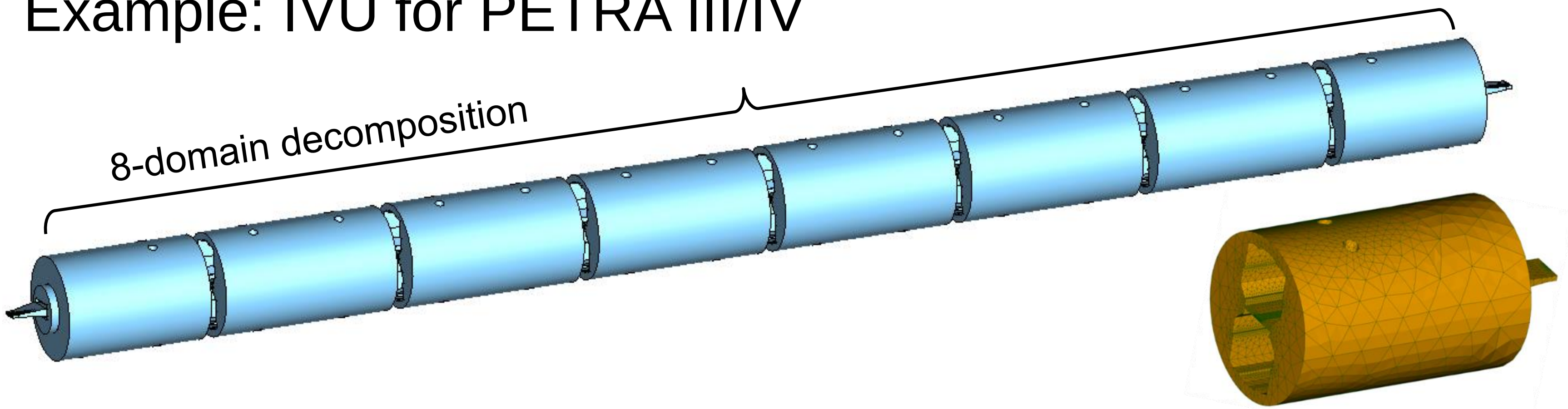
with iteration prescription: $\begin{cases} F_{21}^{\text{in}}(n+1) = F_{12}^{\text{out}}(n) \\ F_{12}^{\text{in}}(n+1) = F_{21}^{\text{out}}(n) \end{cases}$ (using fixed-point or a Krylov iteration)

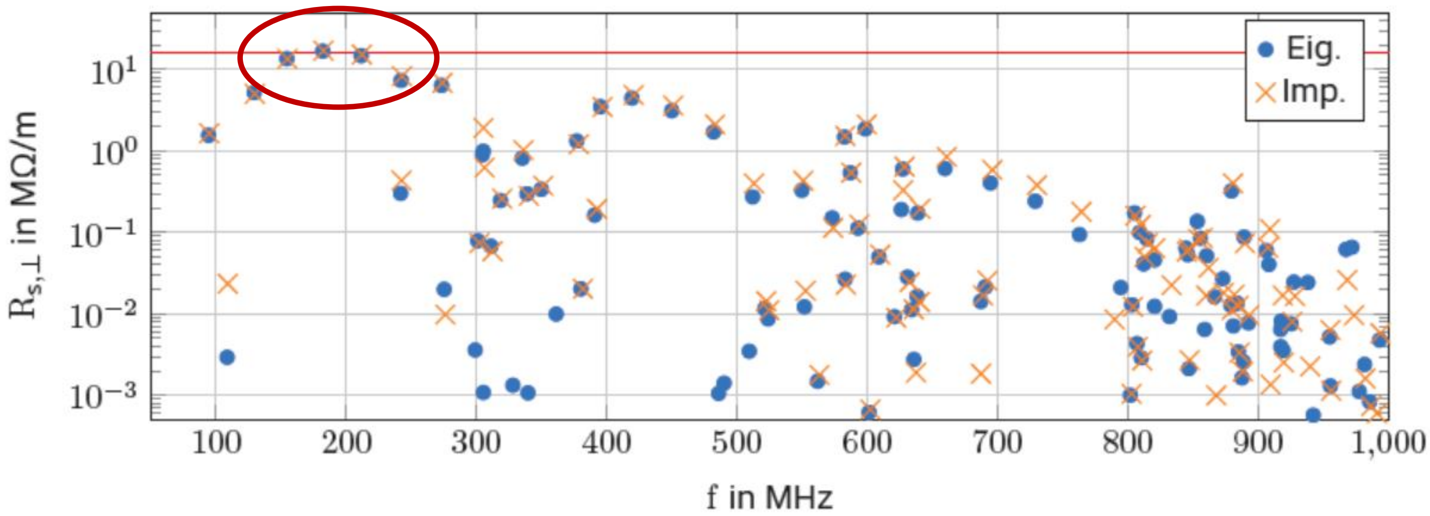
Apply TC using waveguide port operator*: $F_{12}^{\text{out}} = P(E_1 - E_1^{inc}) + n_1 \times \left(\nabla \times (E_1 - E_1^{inc})\right)$

**Gjonaj, et al.: IPAC'24, THPC62*

Domain decomposition

- Example: IVU for PETRA III/IV





Transverse shunt impedances

Summary & Conclusions

- The Finite-Element-Frequency-Domain approach
  - Fills the gap for some important wakefield / impedance problems
    - Complicated chamber geometries
    - Resonant structures → long-range wakefields
    - Resistive, rough surfaces, dispersive materials, waveguide openings
    - Curved beam trajectories and CSR
  - Allows lumped parameter modeling
    - Concatenation of large structures by generalized S-Matrix
    - Fast cavity impedance optimization including termination loads
    - Quasi-periodic THz structures
  - Limitation: huge size of 3D discrete problems for ultra-high frequencies
    - Parallel iterative solvers by non-overlapping DDM