CSR instability in rings, theory and modelling

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In collaboration with

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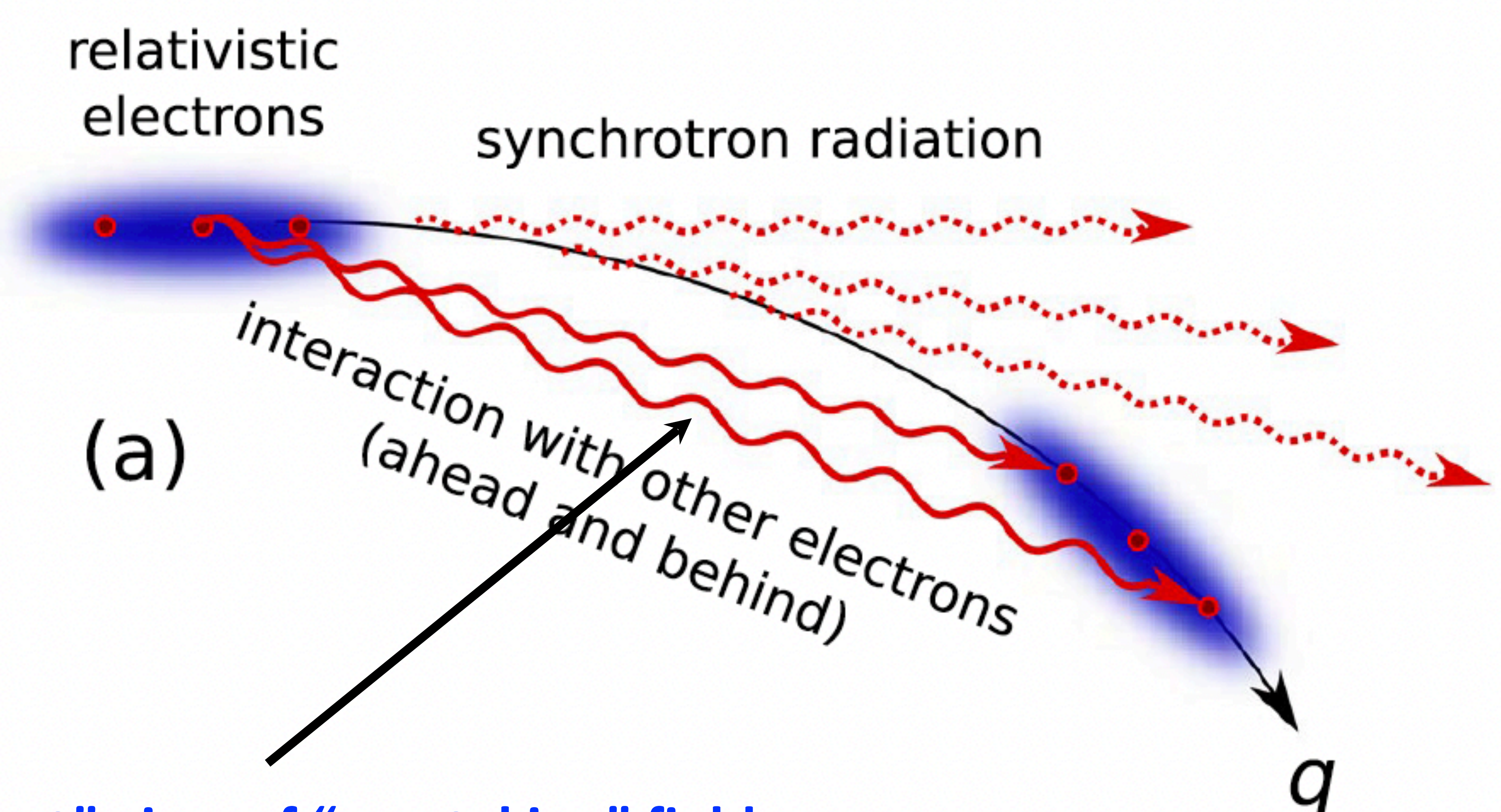
Outline

- Introduction
- CSR instability theories
- CSR impedance calculation
- CSR instability modelling
- Summary

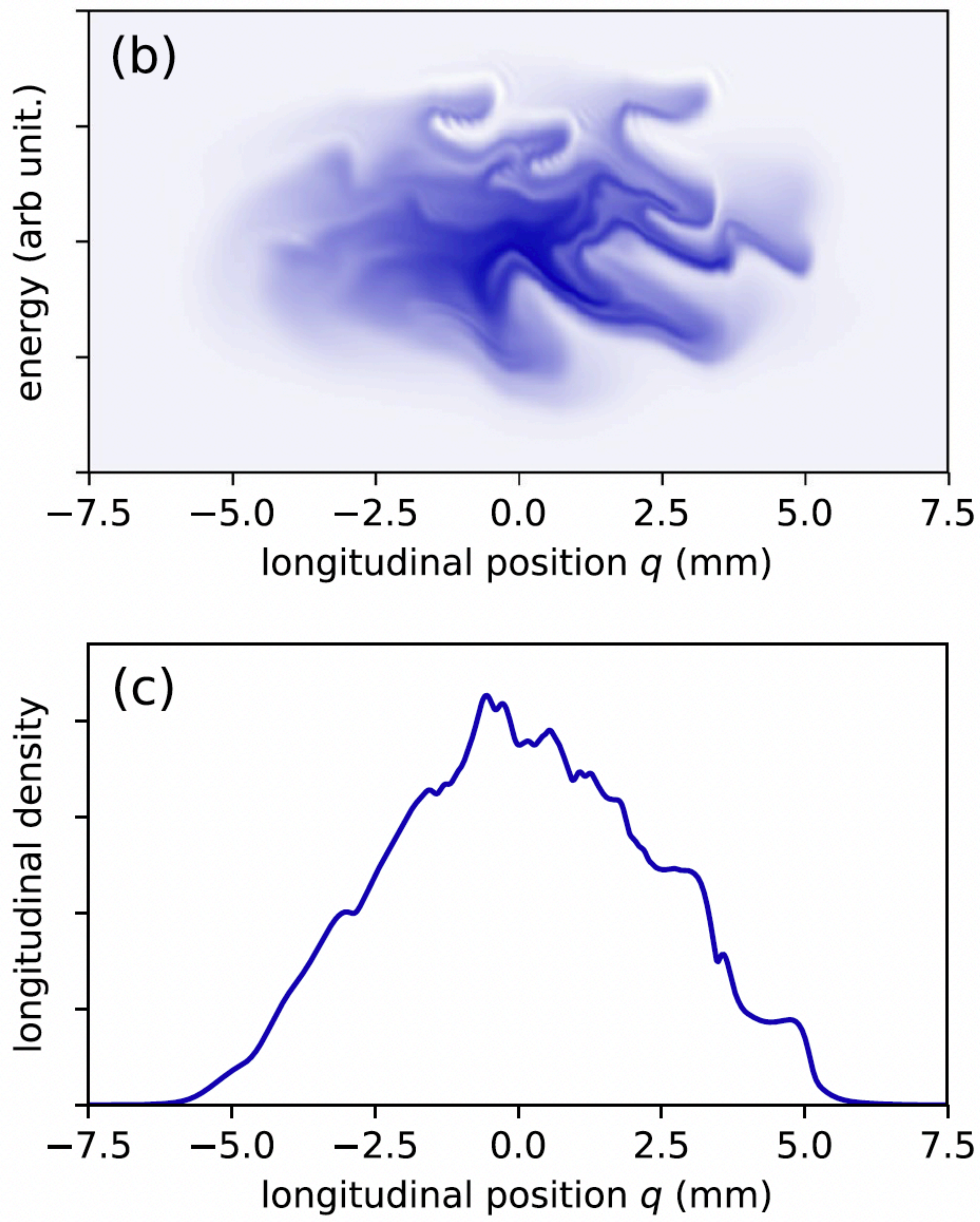
- This talk focuses on CSR instability in low-emittance electron storage rings
  - CSR is undesirable and may impose a low threshold on bunch current.
  - $\sigma_\perp \ll l_\perp \sim (\rho\sigma_z^2)^{1/3}$ and 1D CSR models are sufficient.
- The approach presented in this talk is detailed in the forthcoming publication:
  - S. Dastan, D. Zhou, T. Ishibashi, E. Karantzoulis, S. Di Mitri, R. Lindberg, “Coherent synchrotron radiation instability in low-emittance electron storage rings”.

Introduction

- Features of CSR
  - Most subtleties of CSR lie in the “overtaking” fields
  - Standard theories of wakefields do not apply to CSR
    - Panofsky-Wenzel theorem (need to be generalized)
    - Causality and resulting Hilbert-transform relation of impedance



“Lienard-Wiechert” view of “overtaking” fields



Microbunching driven by CSR [1]

[1] S. Bielawski et al., Scientific reports 9.1 (2019): 10391.

Introduction: Checklist for CSR instability in rings

- **Scaling laws and parameters [1]**
  - Scaling law with parallel-plates steady-state (PP-SS) CSR model: $I_{th} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z}{Z_0 h} \cdot 0.384$ → Global picture (Y. Cai, IPAC'11)
  - Critical wavenumber: $k_c = 3\gamma^3/(2\rho)$. Interaction distance of CSR: $z \gg 1/k_c$.
  - Wall shielding threshold: $k_w = \pi\sqrt{\rho/(2h)^3}$. When $\sigma_z \ll 1/k_w$, wall shielding is not crucial; when $\sigma_z \gg 1/k_w$, wall shielding becomes important.
  - NOTE: $\sigma_z \gg 1/k_w$ does not mean CSR is negligible. In theory, there always is a finite I_{th} for any $\sigma_z k_w$.
  - Critical CSR wavenumber: $k_{th} = 2\sqrt{\rho/h^3} \sim 2k_w$. CSR around k_{th} determines the threshold current.
  - Radiation formation length: $l_f = (24\rho^2\sigma_z)^{1/3}$. For long magnet $l_b \gtrsim l_f$, transient effects are negligible; for short magnet $l_b < l_f$, transient effects become significant.
  - Catch-up distance: $l_c = 2\sqrt{2\rho w}$ with w the distance from the beam orbit to the side wales and path difference $\Delta s = \frac{4}{3}\sqrt{2w^3/\rho}$. When $\Delta s \lesssim \sigma_z$, reflected CSR plays a role.
  - Slippage length: $l_s = \eta\sigma_\delta C$. Lumping the CSR impedance of distributed bends into one point is valid only when $l_s \ll \lambda_{CSR}$.

* CSR models
* Setup of simulations

[1] S. Dastan, D. Zhou, et al., to be published.

CSR instability theories

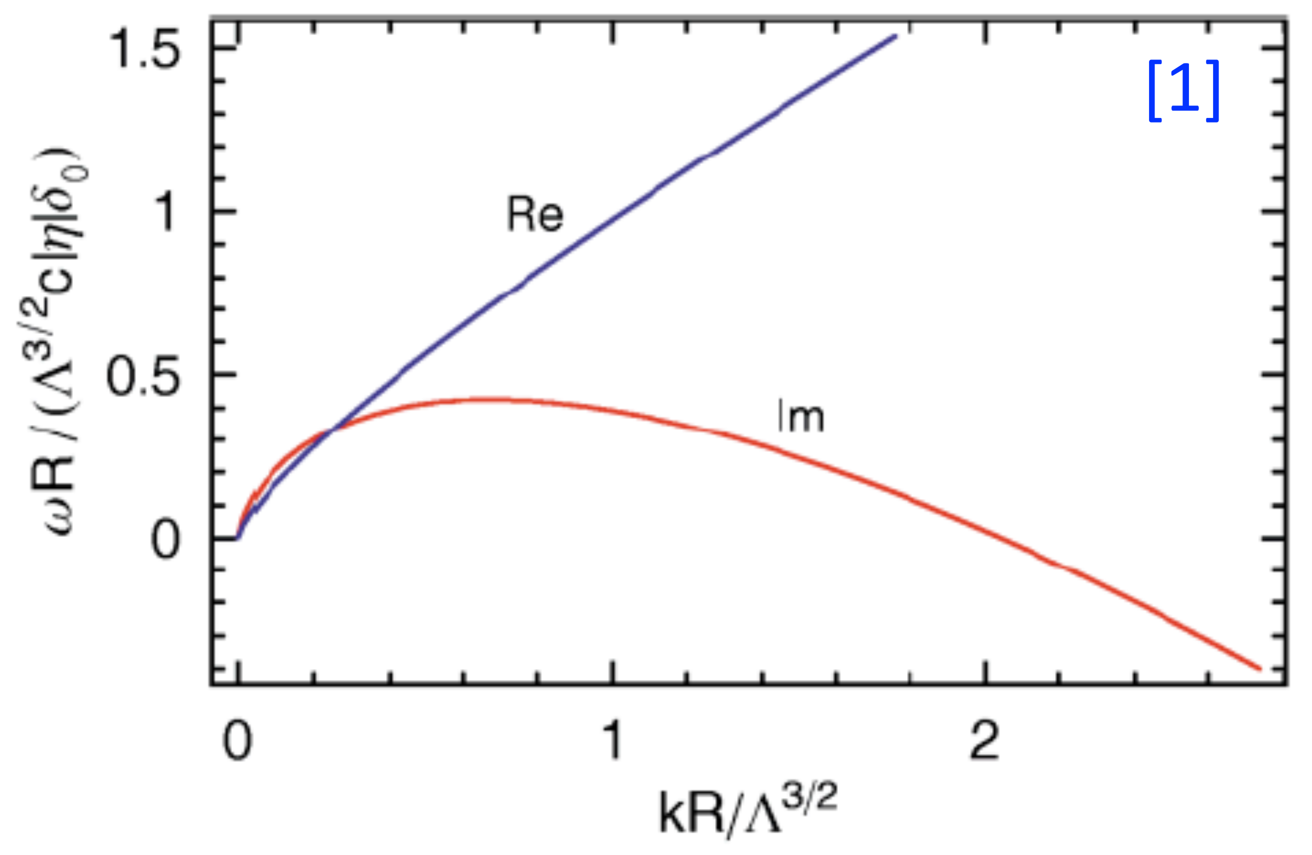
- Stupakov-Heifets (S-H) theory [1] on CSR instability
  - Coasting-beam approximation: $k\sigma_z \gg 1$.
  - S-H theory translated to bunch current threshold [2]:

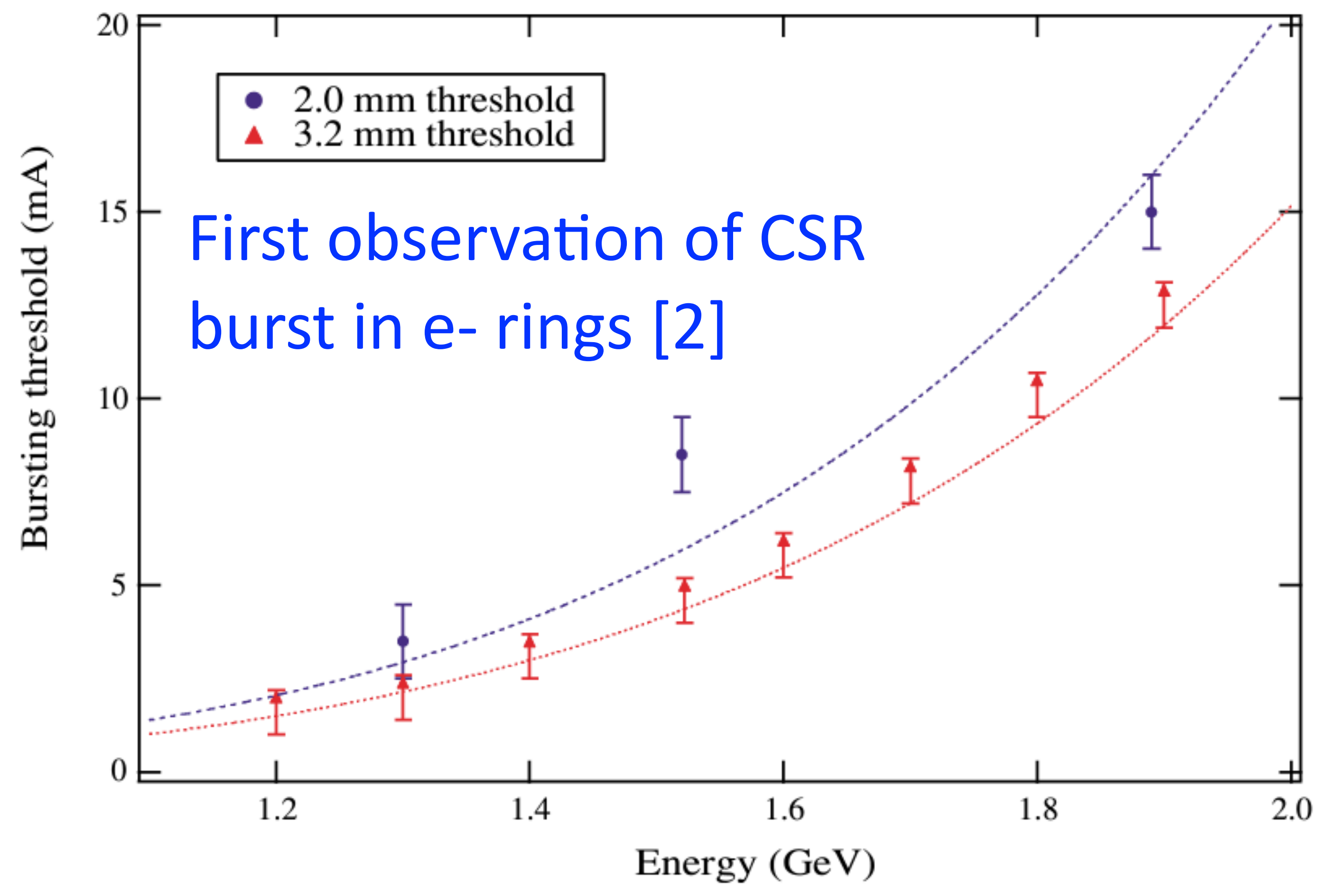
$$1 = \frac{i r_0 c Z(k)}{\gamma} \int \frac{d\delta \, (d\rho_0/d\delta)}{\omega + ck\eta\delta} \rightarrow I_b > \frac{\pi^{1/6}}{\sqrt{2}} \frac{ec}{r_0} \frac{\gamma}{\rho^{1/3}} \alpha_p \delta_0^2 \sigma_z \frac{1}{\lambda^{2/3}}$$

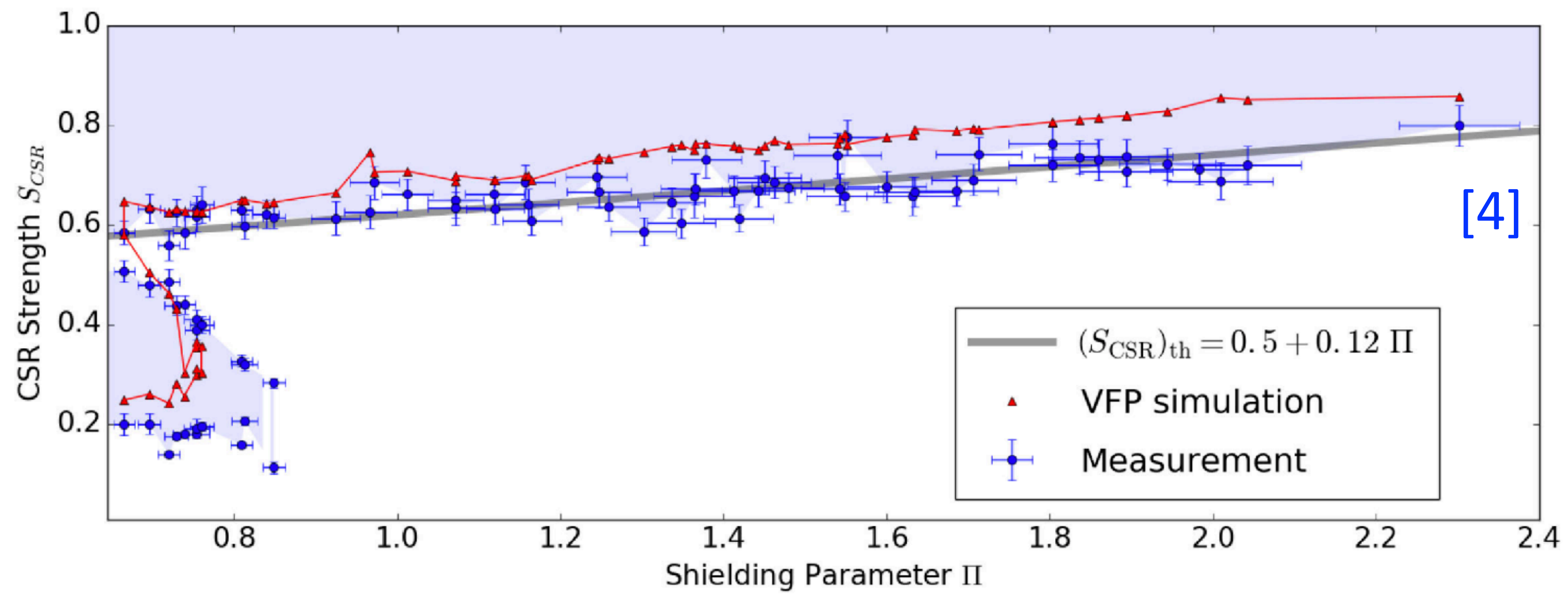
- Improvements on S-H theory
  - Simulation with parallel-plates steady-state (PP-SS) CSR model [3].

$$I_{th1} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z^{1/3}}{Z_0\rho^{1/3}} S_{th1} \qquad S_{th1} \approx 0.5 + 0.12\Pi \qquad \Pi \equiv \sigma_z\sqrt{\rho/h^3}$$

  - Validated by simulations and experiments [4]. Applicable when CSR dominates the instability.
  - Rectangular-chamber steady-state model CSR impedance model [5].







[1] G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002). [2] J. Byrd, et al., PRL 89, 22, Nov. 2002. [3] K.L. F. Bane, Y. Cai, and G. Stupakov, PRST-AB **13**, 104402 (2010).
[4] M. Brosi et al., PRAB 22, 020701 (2019). [5] Y. Cai, Phys. Rev. ST Accel. Beams **17**, 020702 (2014).

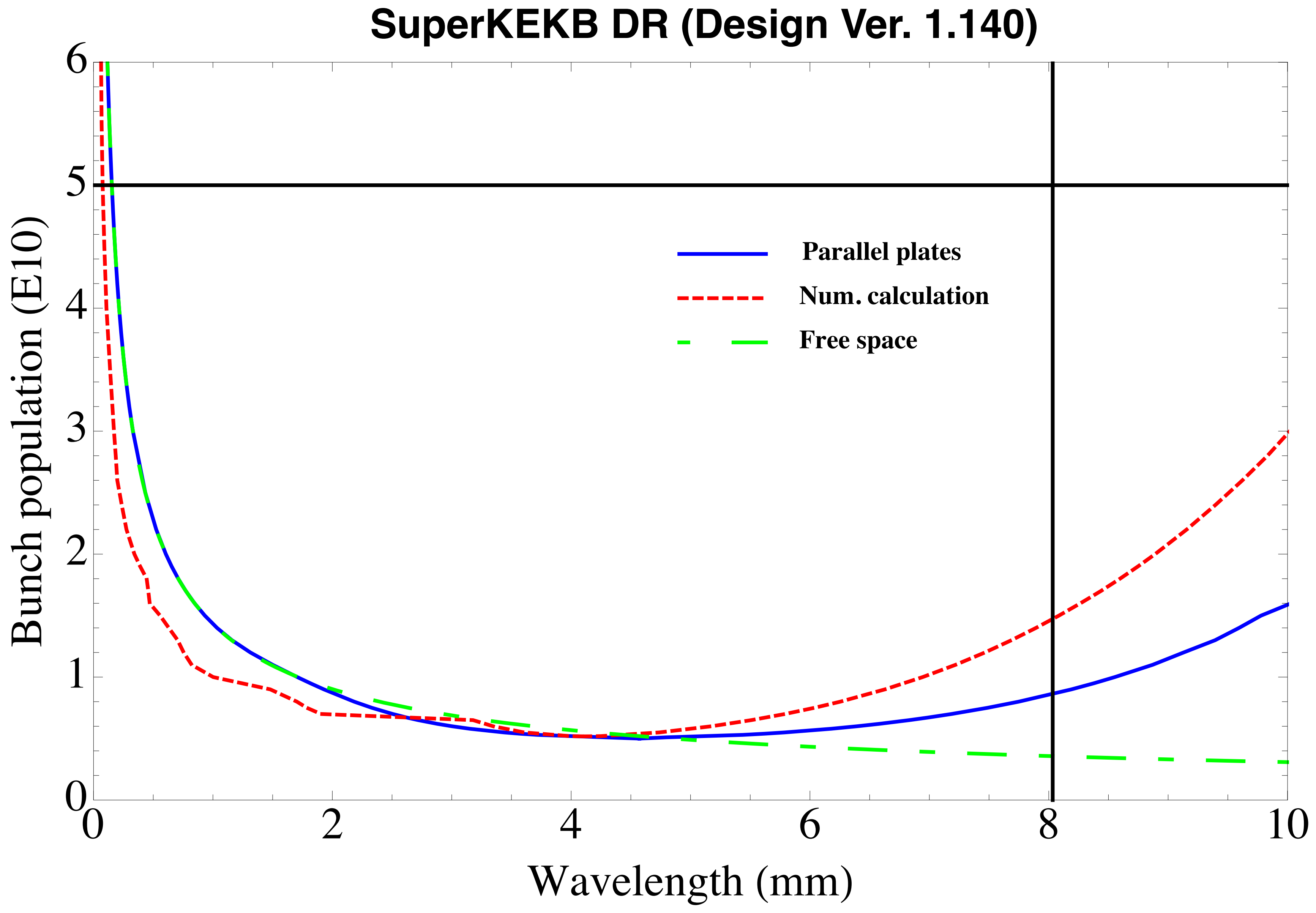
CSR instability theories

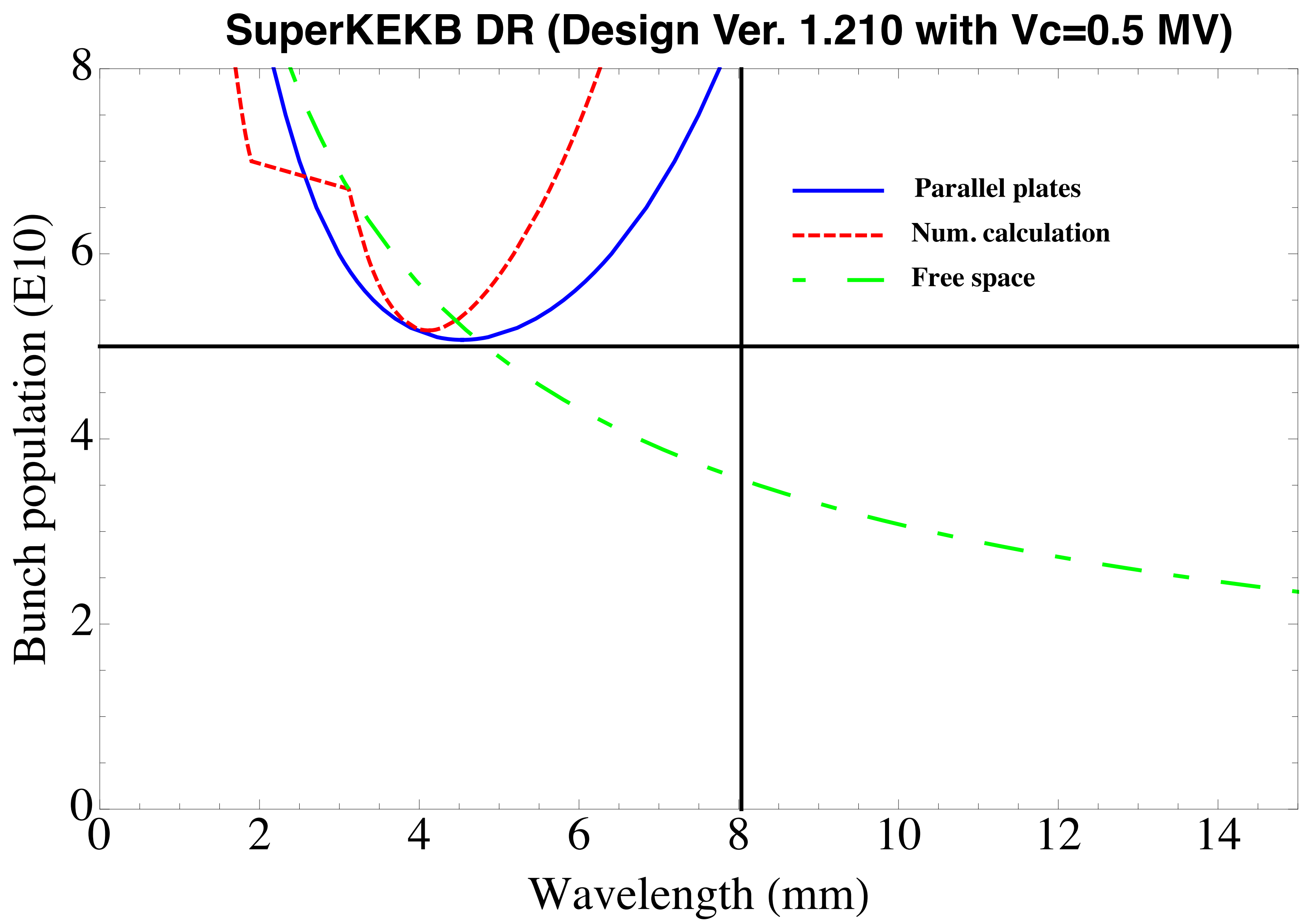
- Any given high-frequency broadband impedance
  - Solve the dispersion relation numerically [1]:

$$-if(I_b)\frac{Z(k)}{k}G(A) = 1 \qquad A = \Omega/(ck\eta\sigma_p)$$

$$G(A) = \int_{-\infty}^{\infty} dp \frac{pe^{-p^2/2}}{A+p} \qquad f(I_b) = \frac{I_b}{2\pi(E/e)\eta\sigma_p^2\sigma_z}$$

  - $Z(k)$ can be obtained by analytical or numerical methods.
  - Low-frequency part of $Z(k)$ mainly affected by chamber shielding; high-frequency part mainly affected by transient effects.
  - For SuperKEKB damping ring, the design had to be changed due to strong CSR instability





[1] D. Zhou et al., IPAC'10, Kyoto, Japan.

CSR instability theories

- Any given high-frequency broadband impedance
  - Simplify the dispersion relation with threshold condition Im$[\Omega]$=0 [1]

$$G(A) = \int_{-\infty}^{\infty} dp \frac{p e^{-p^2/2}}{A+p} = \sqrt{2\pi} + i\pi A e^{-\frac{A^2}{2}} \left[\text{sgn}[\text{Im}[A]] + i\,\text{erfi}\left[\frac{A}{\sqrt{2}}\right]\right]$$

$$\frac{G_i(A_{th})}{G_r(A_{th})} = \frac{Z_r(k)}{Z_i(k)}$$

$$\frac{I_{th}}{2\pi(E/e)\eta\sigma_\delta^2\sigma_z} = \frac{kZ_r}{G_i(A_{th})(Z_r^2+Z_i^2)}$$

→ Roughly, $I_{\text{th}} \propto 1/(Z_{\|}/n)$

$$G_r(A_r) = \sqrt{2\pi} - \pi A_r e^{-A_r^2/2} \text{erfi}[A_r/\sqrt{2}]$$

$$G_i(A_r) = \text{sgn}[\eta]\pi A_r e^{-A_r^2/2}$$

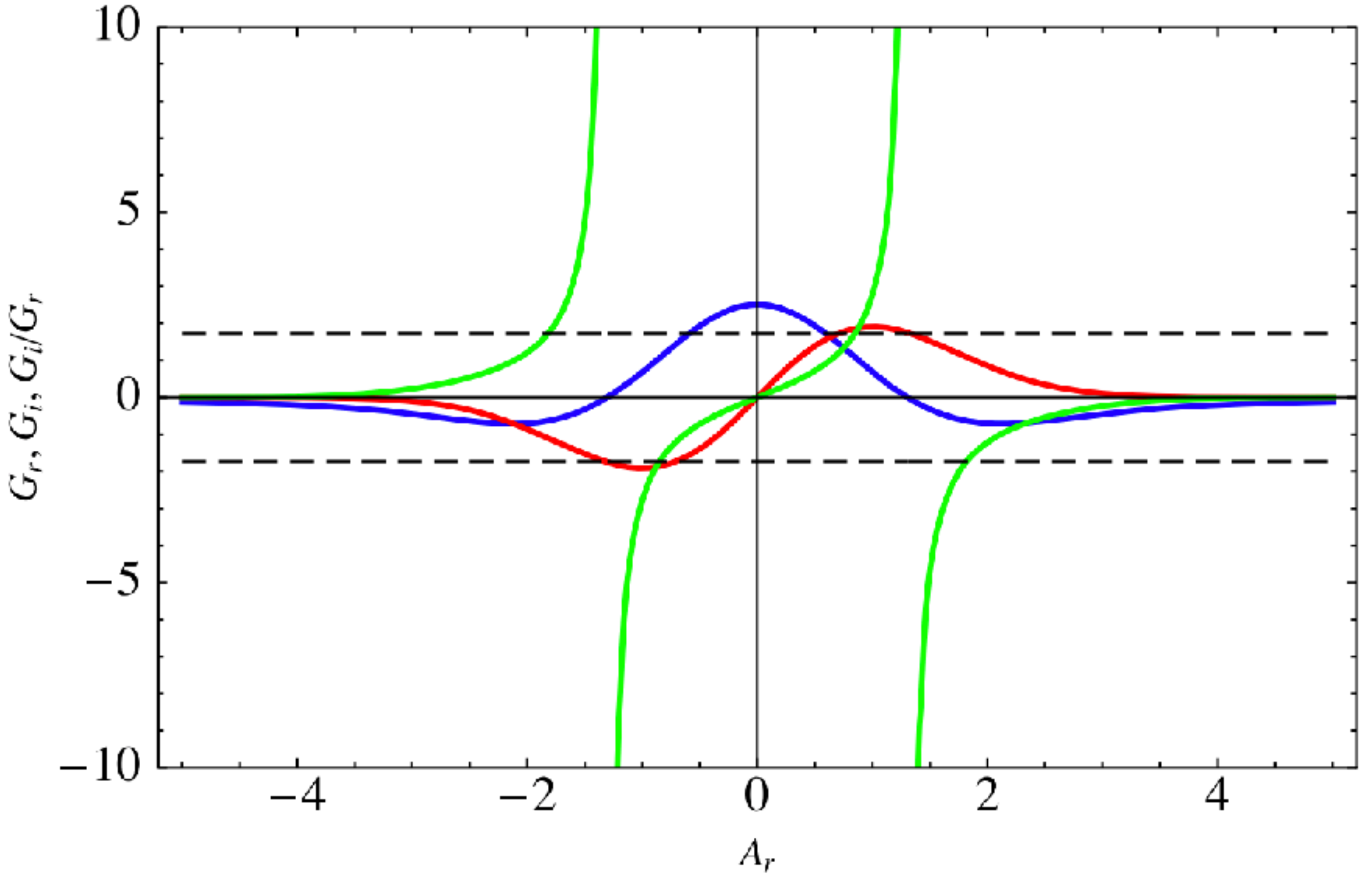


FIG. 13. The functions $G_r(A_r)$ (blue line), $G_i(A_r)$ (red line), and $G_i(A_r)/G_r(A_r)$ (green line) with real A_r and positive η. The horizontal dashed lines indicate $G_i/G_r = \pm\sqrt{3}$.

[1] A. Blednykh, D. Zhou, et al., Phys. Rev. Accel. Beams **26**, 051002 (2023).

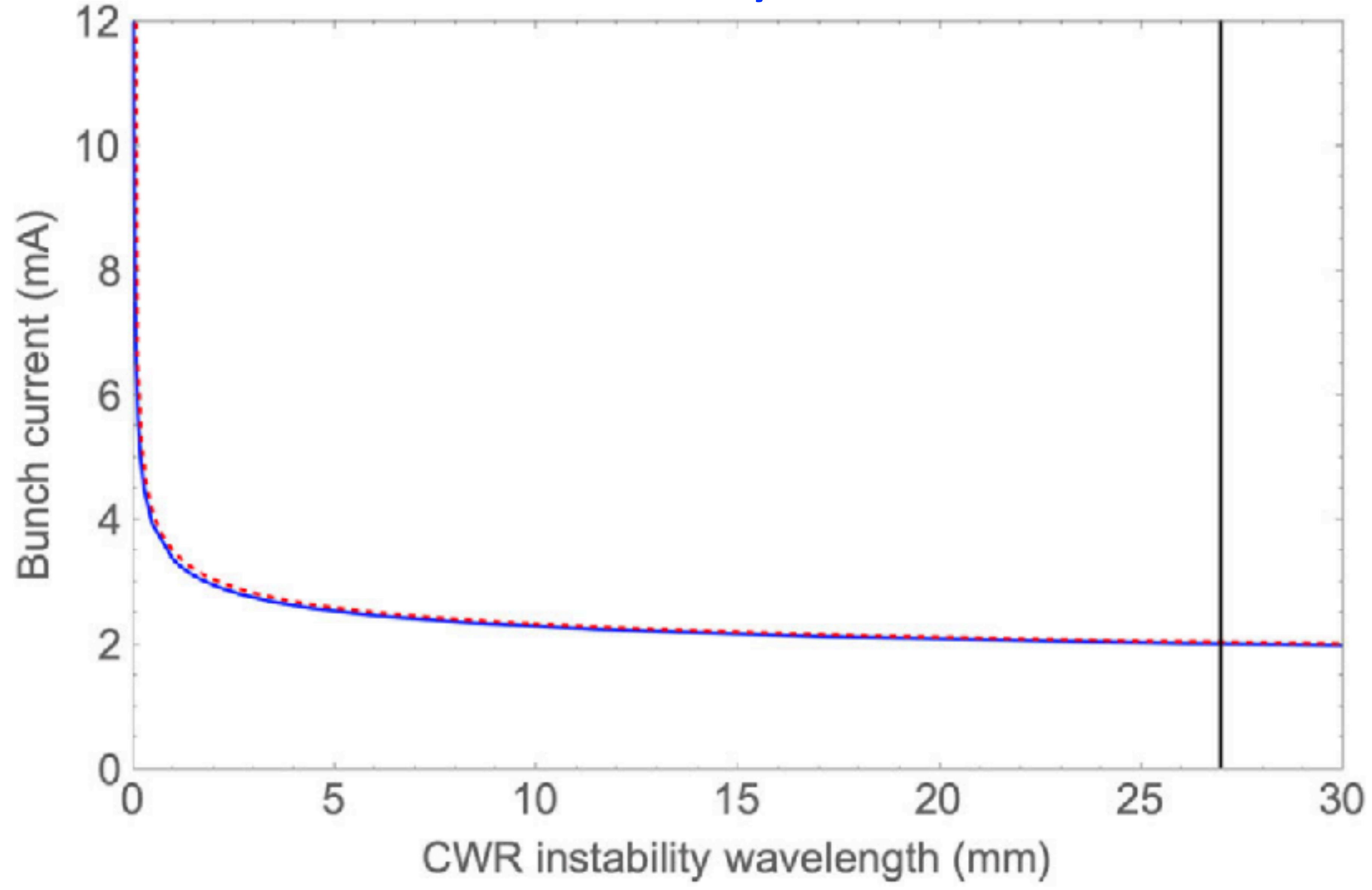
CSR instability theories

- Any given high-frequency broadband impedance
  - **Application-1**: Scaling law of Coherent Wiggler Radiation (CWR) instability in damping rings (only valid for positive η):

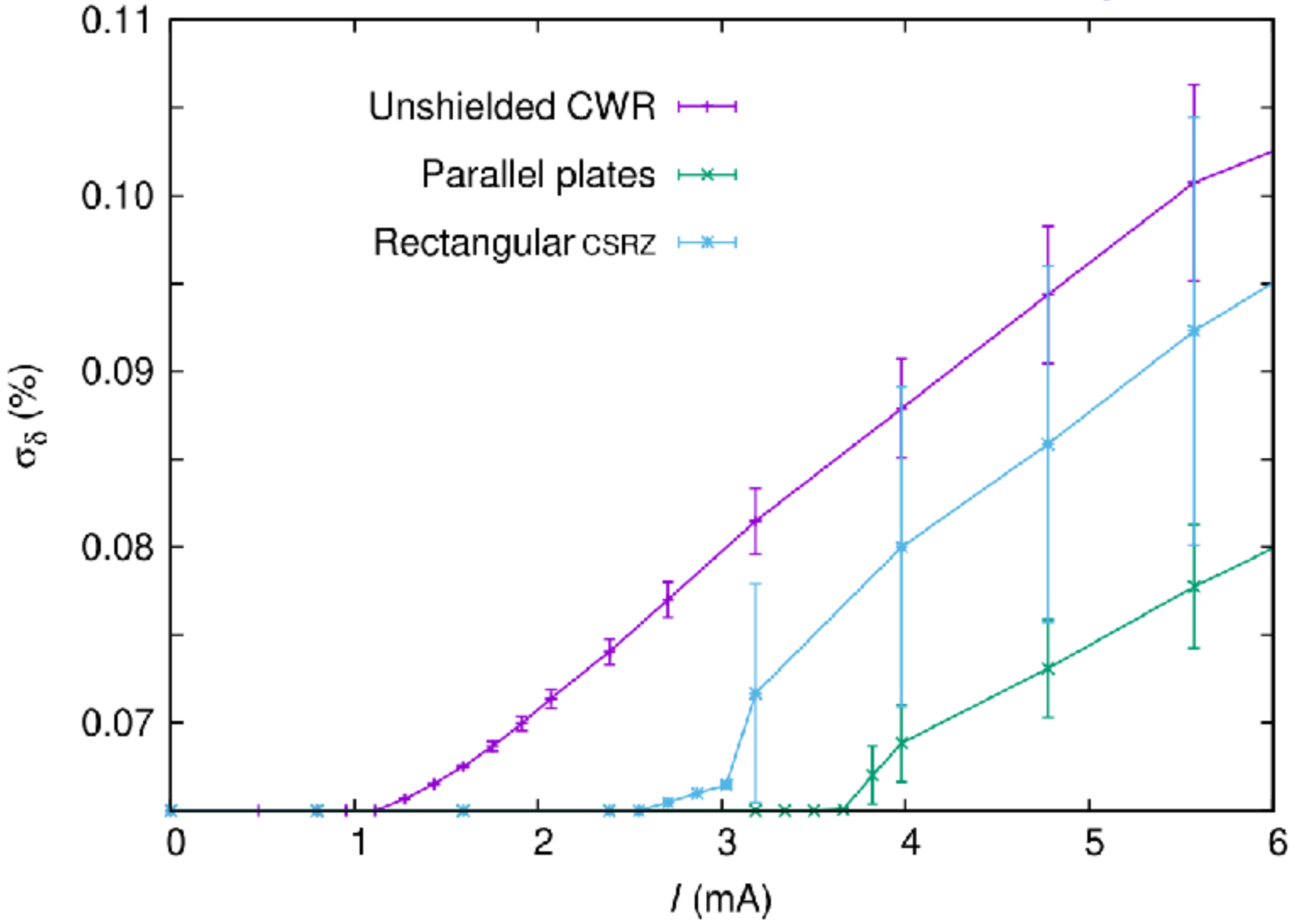
$$I_b^{th}(\lambda) \approx \frac{8\pi\sqrt{2\pi}(E/e)\eta\sigma_p^2\sigma_z}{LZ_0\theta_0^2 \ln \frac{2k_w\lambda}{\pi\theta_0^2}}$$

  - This scaling law well explains the simulated CWR instability in the ring cooler for EIC ($\sigma_{z0} = 48$ mm) [1].

Theory



VFP simulation (R. Lindberg)



[1] A. Blednykh, D. Zhou, et al., Phys. Rev. Accel. Beams **26**, 051002 (2023).

CSR instability theories

- Any given high-frequency broadband impedance
  - **Application-2**: Scaling law for PP-SS [1]:

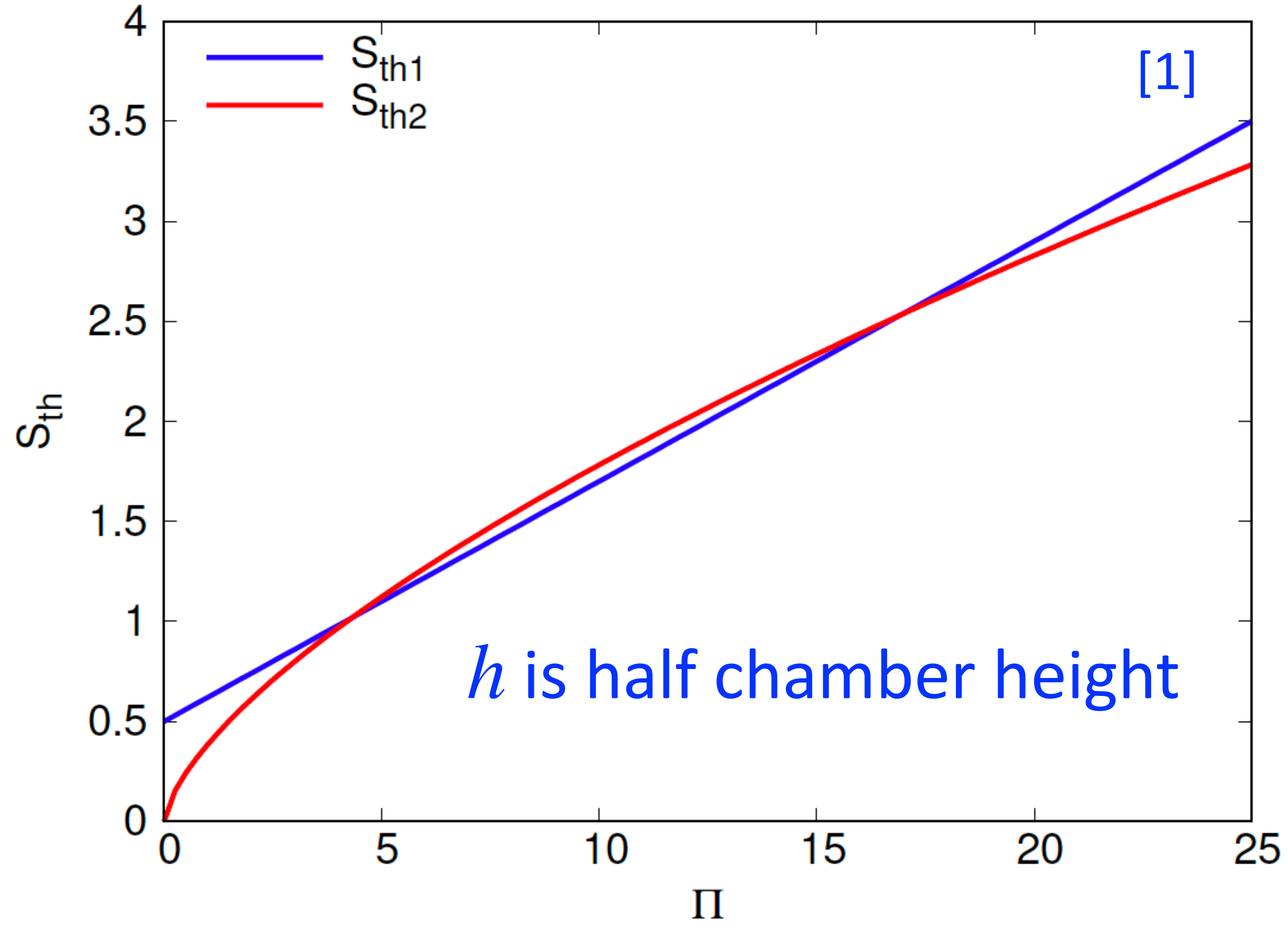
$$I_{th2} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z^{1/3}}{Z_0\rho^{1/3}}S_{th2} \approx \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z}{Z_0 h}\cdot 0.384$$

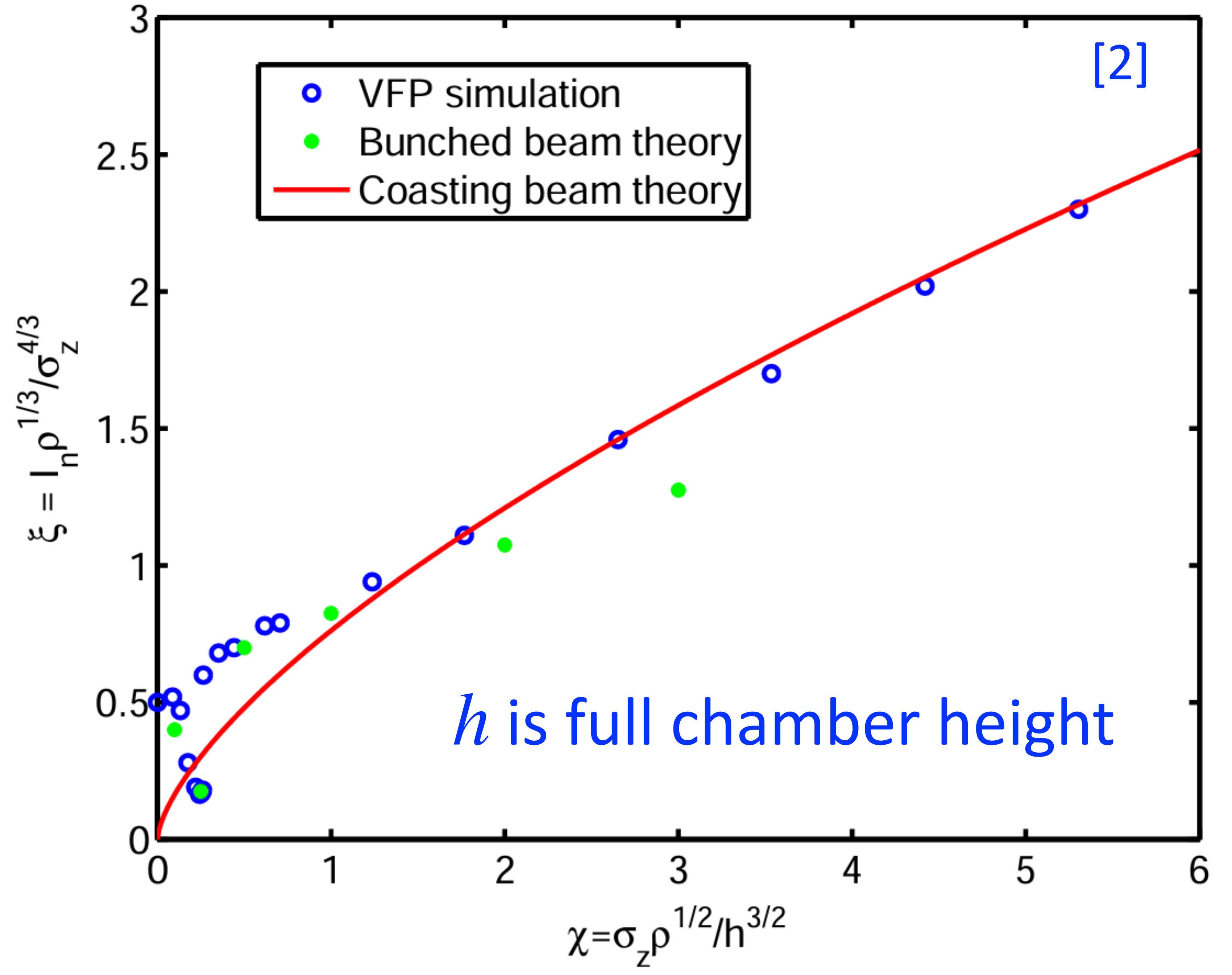
$$S_{th2} \approx 0.384\Pi^{2/3}$$

$$\Pi \equiv \sigma_z\sqrt{\rho/h^3}$$

$$S_{th1} \approx 0.5 + 0.12\Pi$$

  - This scaling law (first found by Y. Cai [2]) is valid when $\Pi \gg 0.5$.
  - It suggests CSR threshold is proportional to $\gamma\eta\sigma_\delta^2\sigma_z/h$, but independent of ρ [2].
  - The linear scaling law of S_{th1} is an approximation of S_{th2}.





[1] S. Dastan, D. Zhou, et al., to be published. [2] Y. Cai, IPAC2011, FRXAA01.

CSR instability theories

- Any given high-frequency broadband impedance
  - **Application-3**: Scaling law of Resistive Wall (RW) instability [1]:

$$Z_{\parallel}^{RW}(k) = \frac{f_Y Z_0 L_{RW}}{\pi h \left(2\sqrt{iZ_0\sigma_c/k} - ihk\right)}$$

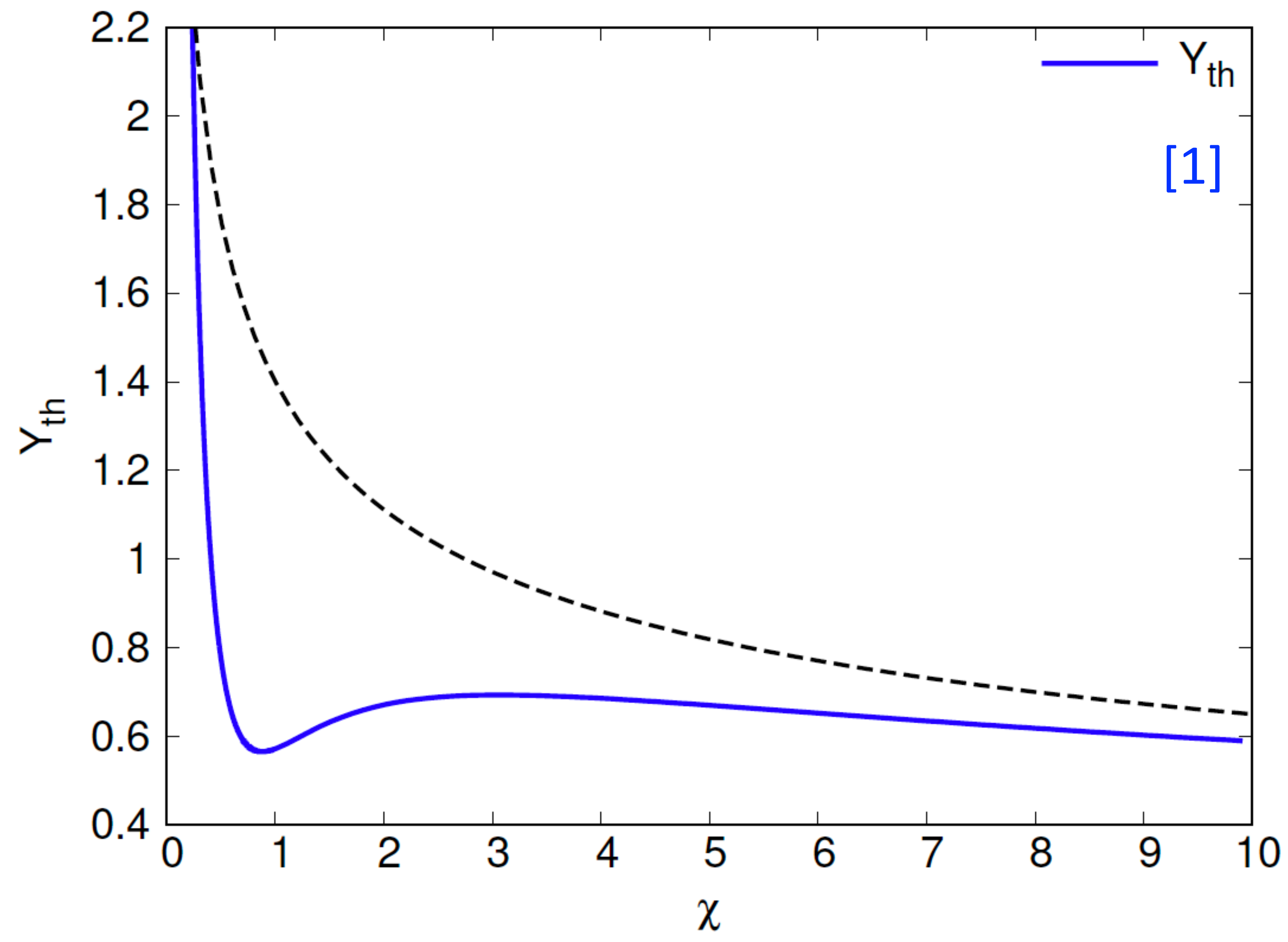
$$I_{th} = \frac{2\pi^2 (E/e)\eta\sigma_\delta^2\sigma_z(2Z_0\sigma_c h)^{2/3}}{f_Y Z_0 L_{RW}}\mathrm{Min}[Y_{th}(\chi)]$$

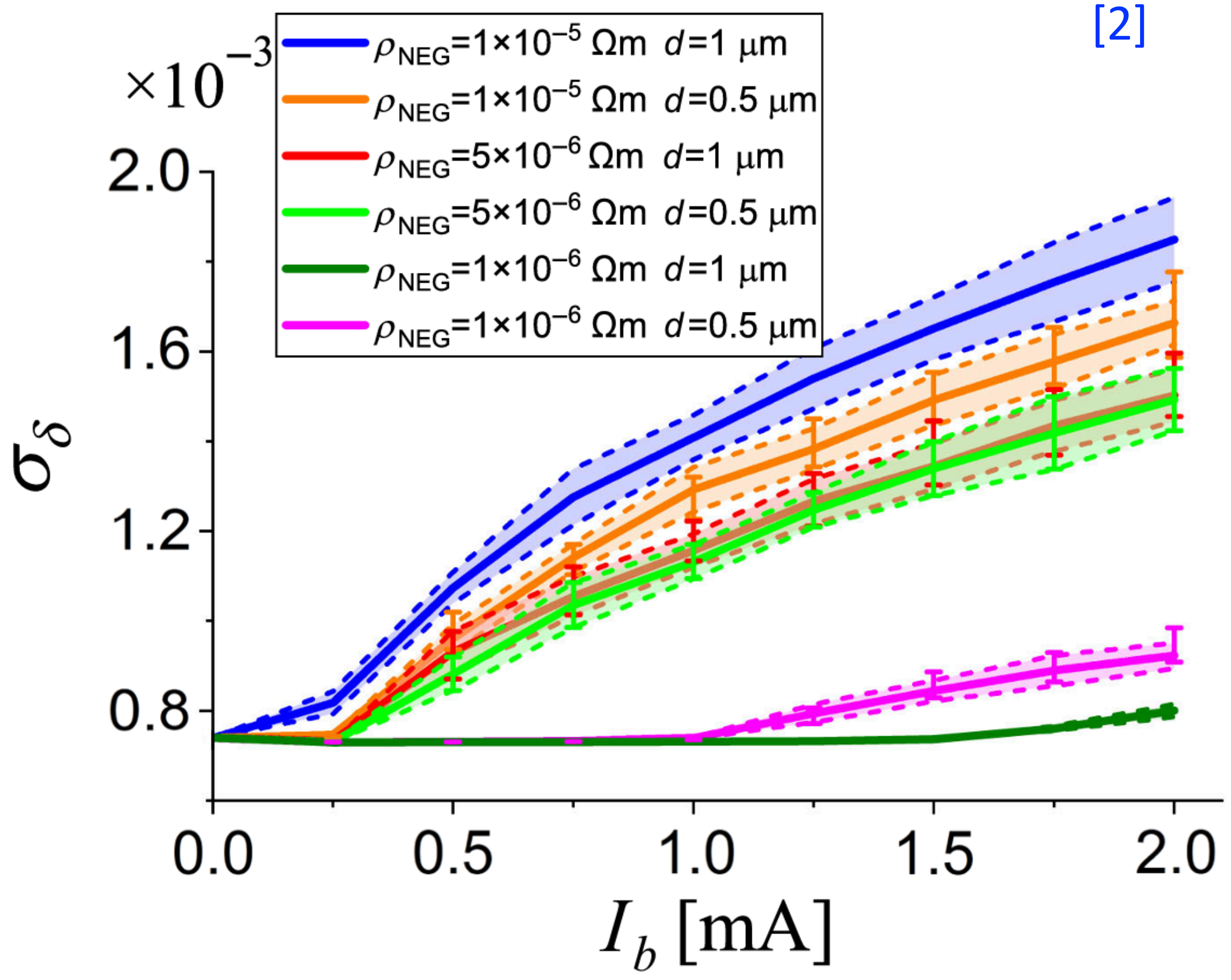
$$Y_{th}(\chi) = \frac{1}{G_i(A_{th})\chi^{1/3}} \qquad \chi = \frac{\sqrt{2Z_0\sigma_c}}{hk^{3/2}}$$

  - $\mathrm{Min}[Y_{th}(\chi)] \approx 0.566$. The scaling law is valid when

$$\Pi_{RW} \equiv \sigma_z\left(\frac{Z_0\sigma_c}{h^2}\right)^{1/3} \gg 0.73$$

  - RW instability has been investigated in HALF by W. Li et al., “Terahertz scale microbunching instability …” [2]





[1] S. Dastan, D. Zhou, et al., to be published. [2] W. Li, T. He, Z. Bai, PRAB 27, 034401 (2024)

CSR impedance calculation

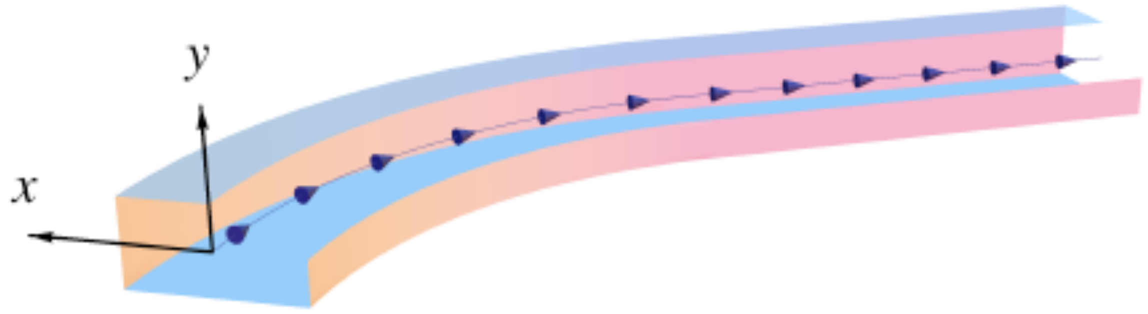
- CSR impedance calculation for rings
  - G. Stupakov, T. Agoh et al. developed the method of calculating CSR impedance using parabolic equation (PE).
  - The CSRZ code following this line to solve PE:

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp - \frac{1}{\epsilon_0}\nabla_\perp \rho_0 + 2k^2\left(\frac{x}{R(s)} - \frac{1}{2\gamma^2}\right)\vec{E}_\perp\right]$$

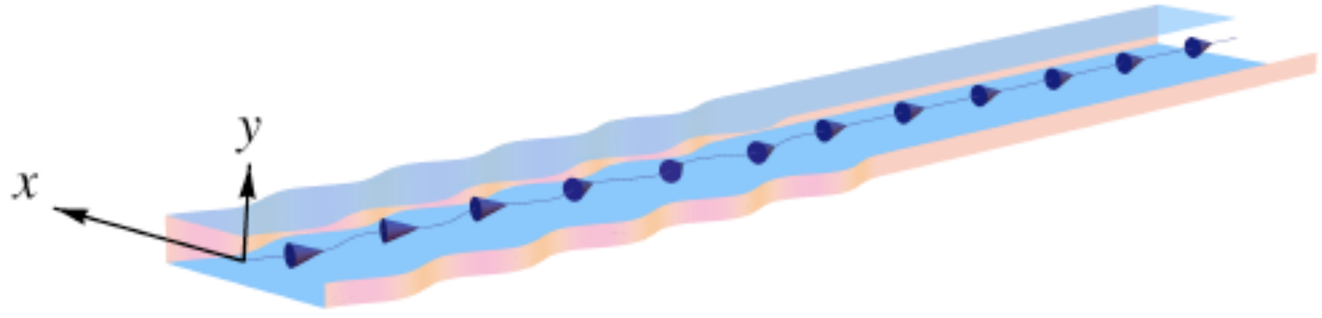
$$E_s = \frac{i}{k}\left(\nabla_\perp \cdot \vec{E}_\perp - \mu_0 c J_s\right) \quad Z(k) = -\frac{1}{q}\int_0^\infty E_s(x_c, y_c)ds$$

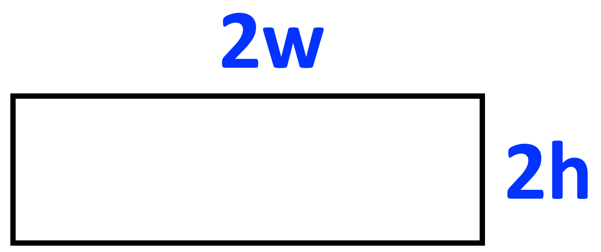
  - CSRZ takes into account: Arbitrary curvature of beam orbit $R(s)$ (CSR), finite beam energy γ (space charge effects, SC), and resistive wall (RW). The total impedance is not a simple sum of Z_{CSR} + Z_{SC} + Z_{RW}, but includes their interaction.
  - CSRZ assumes uniform rectangular chamber referring to the beam orbit.
  - See [1] for an overview, [2] for details of CSRZ code, [3,4,5] for recent applications.

A single bend



A wiggler with "wiggling" chamber





[1] D. Zhou et al., "An Alternative 1D Model for CSR with Chamber Shielding", in Proceedings of IPAC'12, New Orleans, Louisiana, USA.

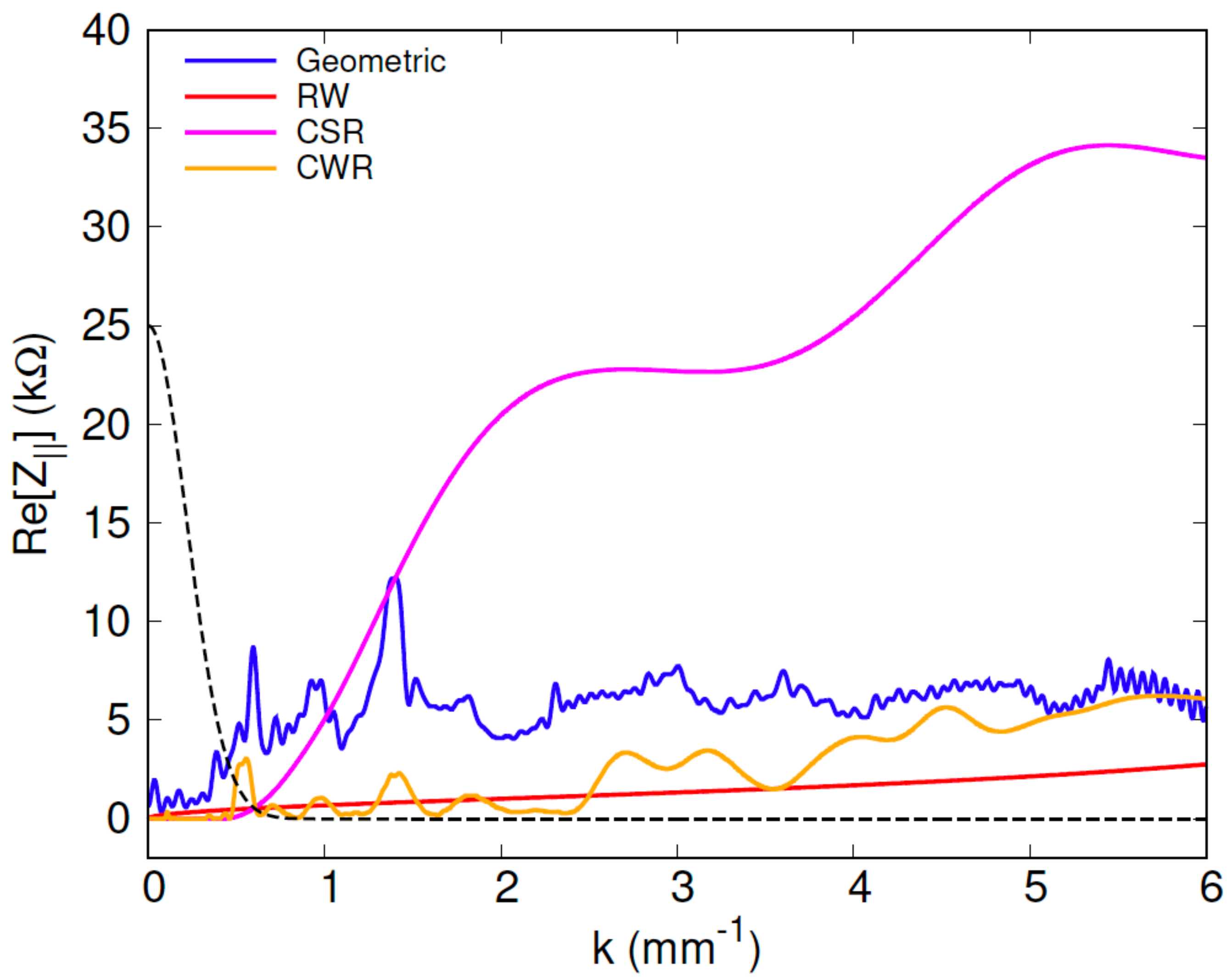
[2] D. Zhou, Coherent Synchrotron Radiation and Microwave Instability in Electron Storage Rings, Ph.D. thesis, SOKENDAI and KEK, 2011.

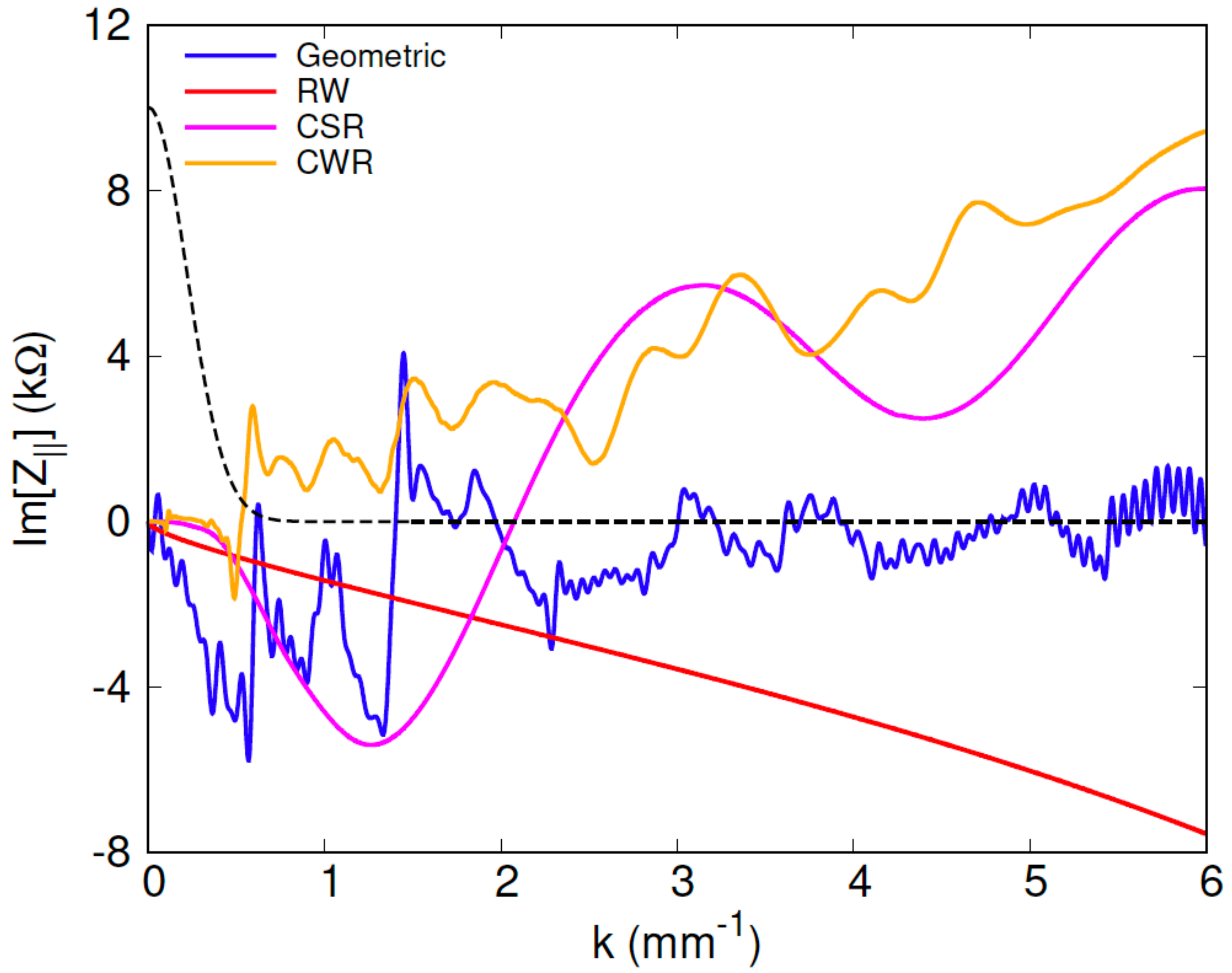
[3] G. Stupakov and D. Zhou, PRAB 19, 044402 (2016). [4] A. Gamelin, et al., NIM-A 999 (2021): 165191. [5] L. Carver et al., PRAB 26, 044402 (2023).

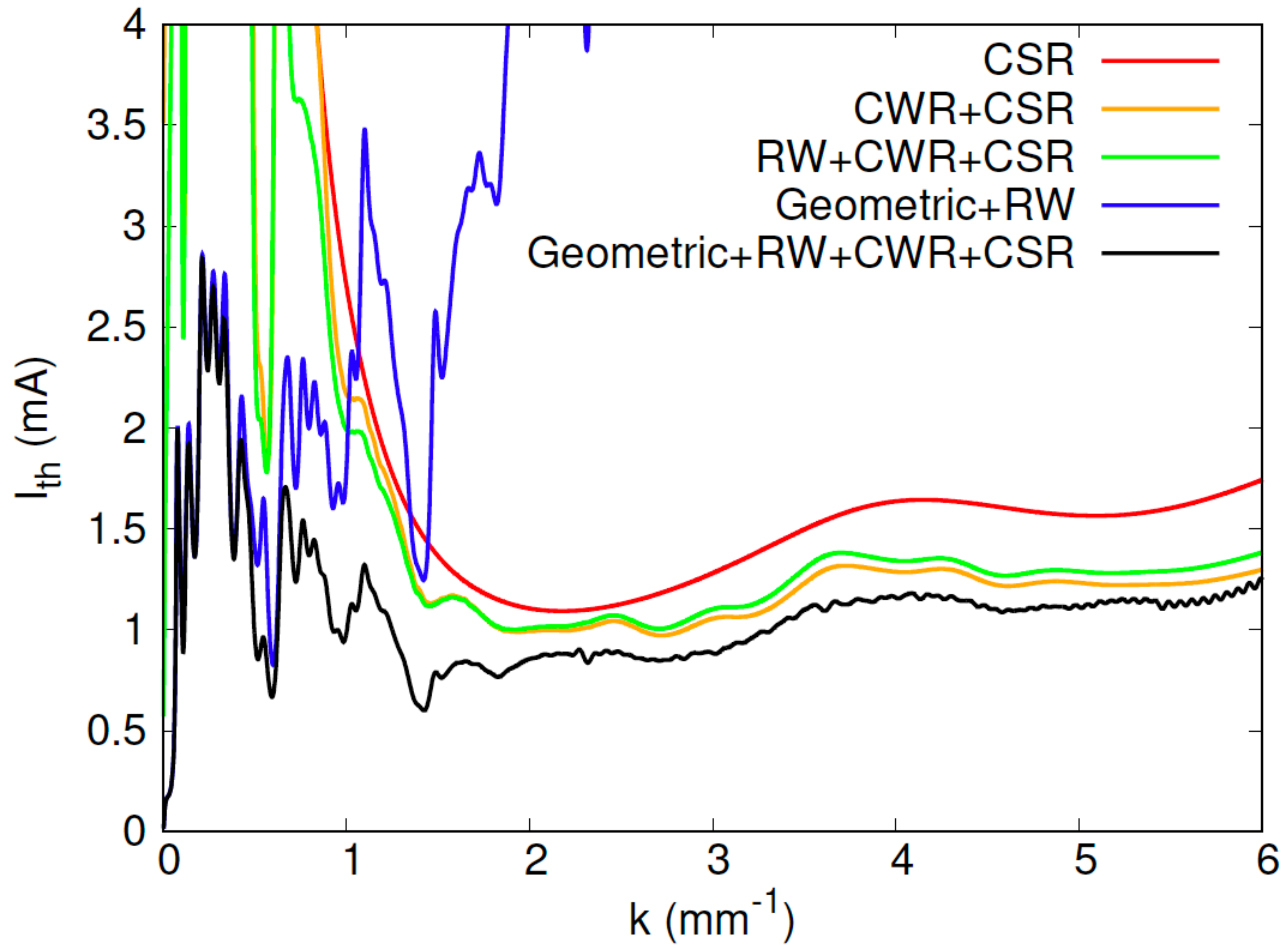
CSR instability modelling

$h = 45$ mm, $\rho = 74.7$ m

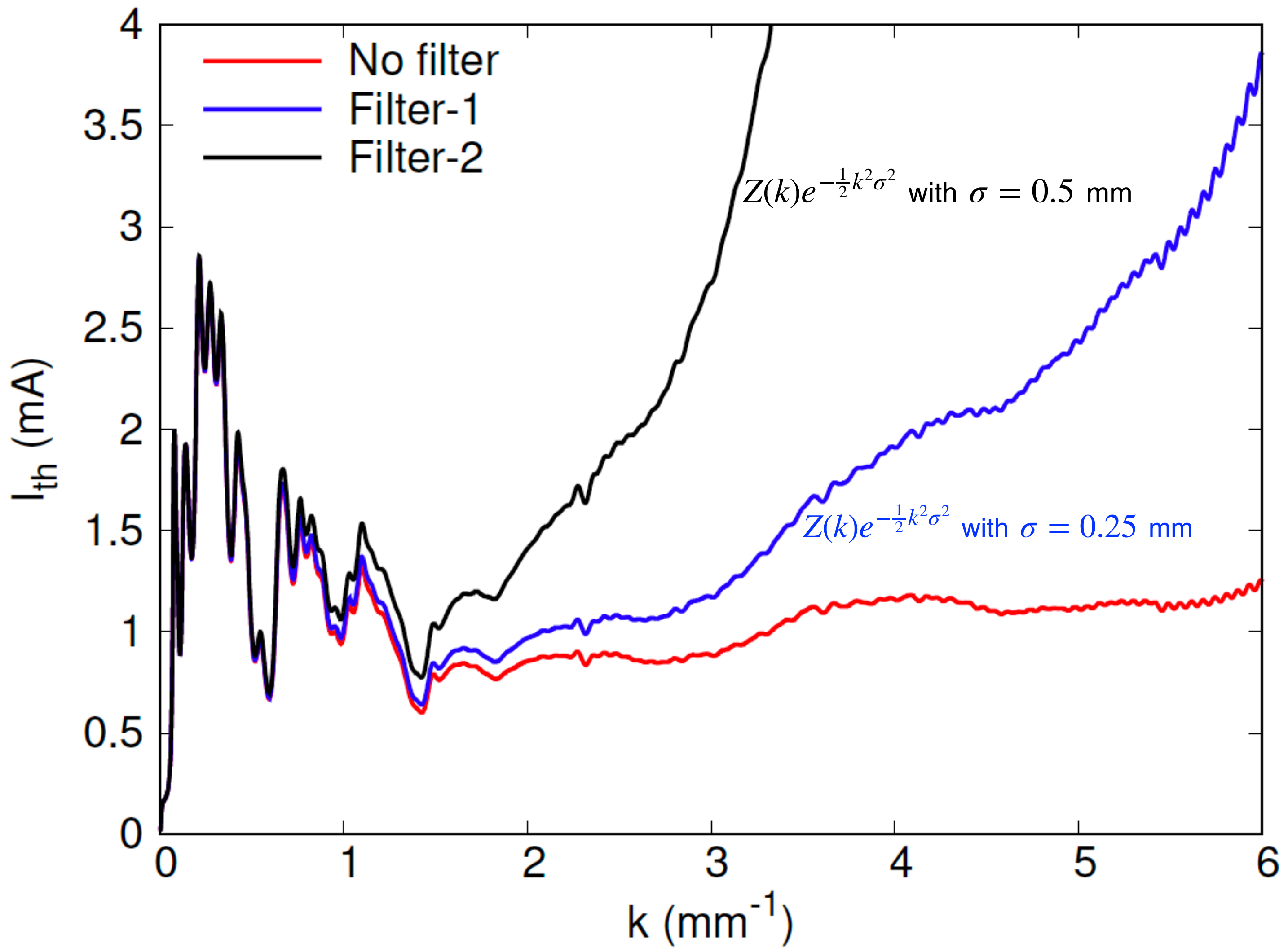
- Example-1: SuperKEKB LER [1]
  - Impedance modeling of CSR/CWR by CSRZ, RW by IW2D, and geometric wakes by GdfidL, CST, and ECHO3D [2].
  - Instability analysis to determine $I_{th}(k)$.
  - Choosing important parameters: maximum k_{max} for impedance model, minimum mesh size $\Delta z \ll 2\pi/k_{max}$.
    - k_{max}=6 mm^{-1} $\rightarrow$ f_{max}=286 GHz.
  - Note: Be careful in choosing filtering function (for pseudo-Green function wake) to damp high-frequency impedances.







CSR is important source for MWI in SuperKEKB LER

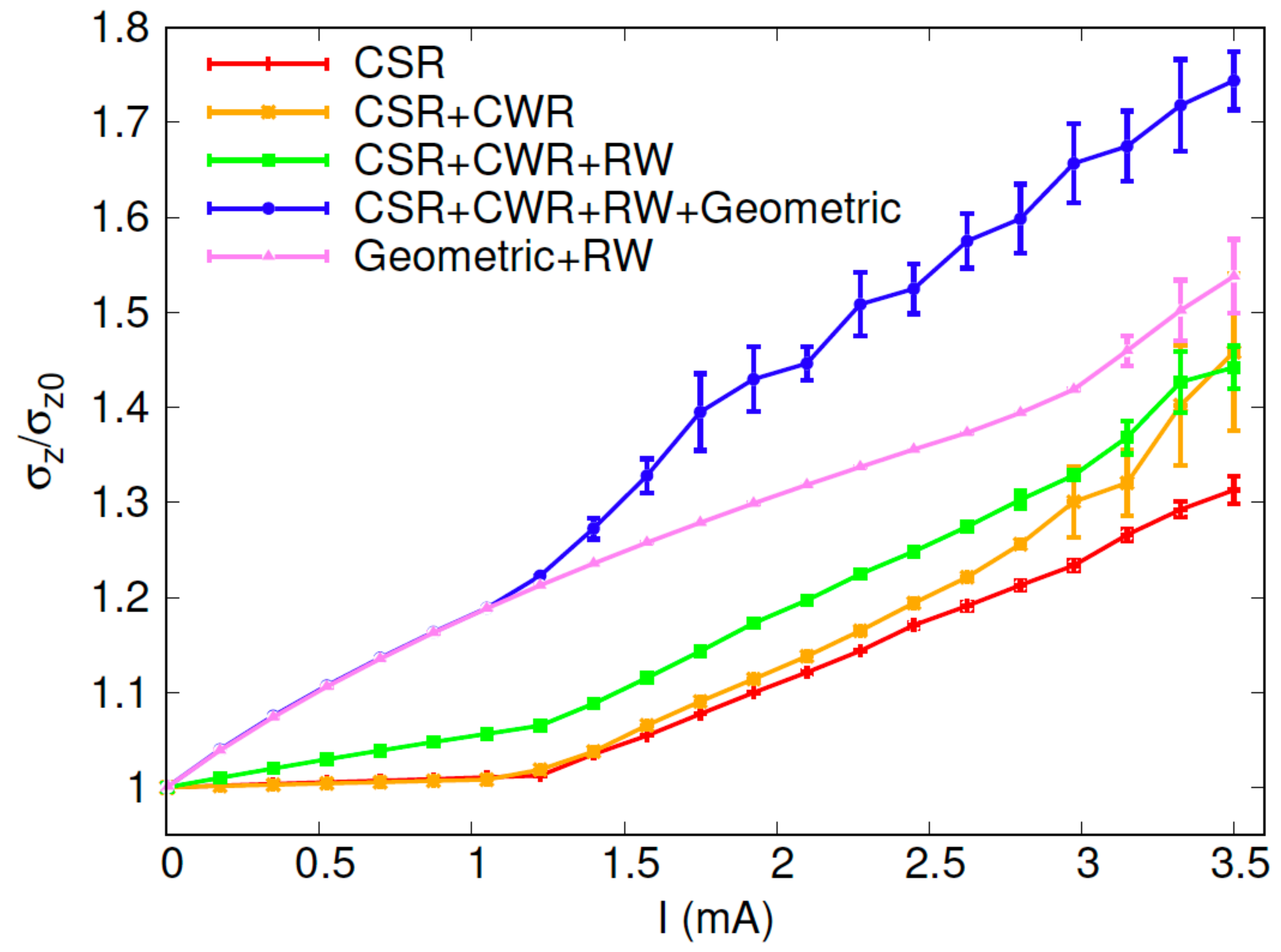


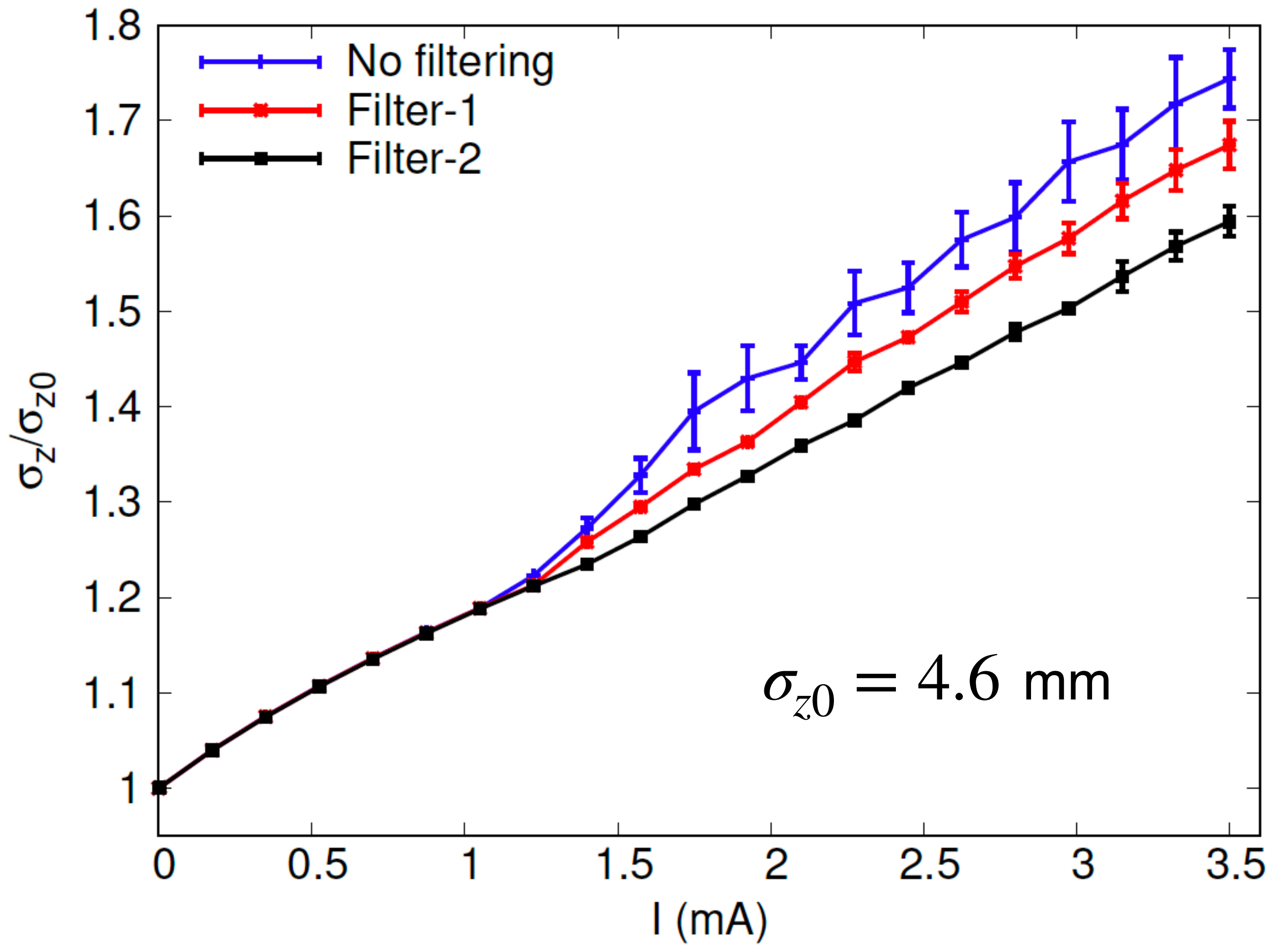
Filtered impedance models underestimate MWI threshold

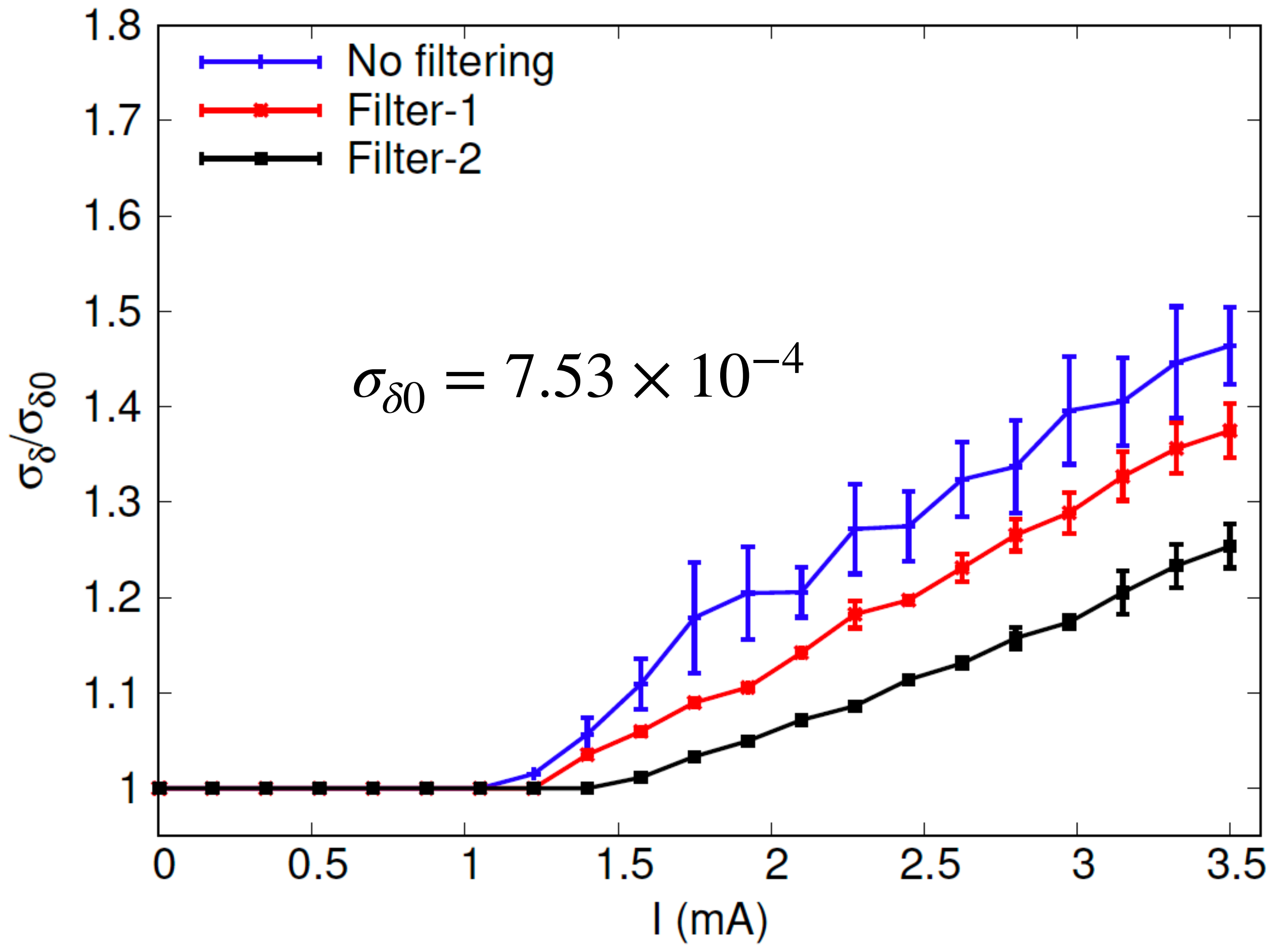
[1] S. Dastan, D. Zhou, et al., to be published. [2] T. Ishibashi et al 2024 JINST 19 P02013.

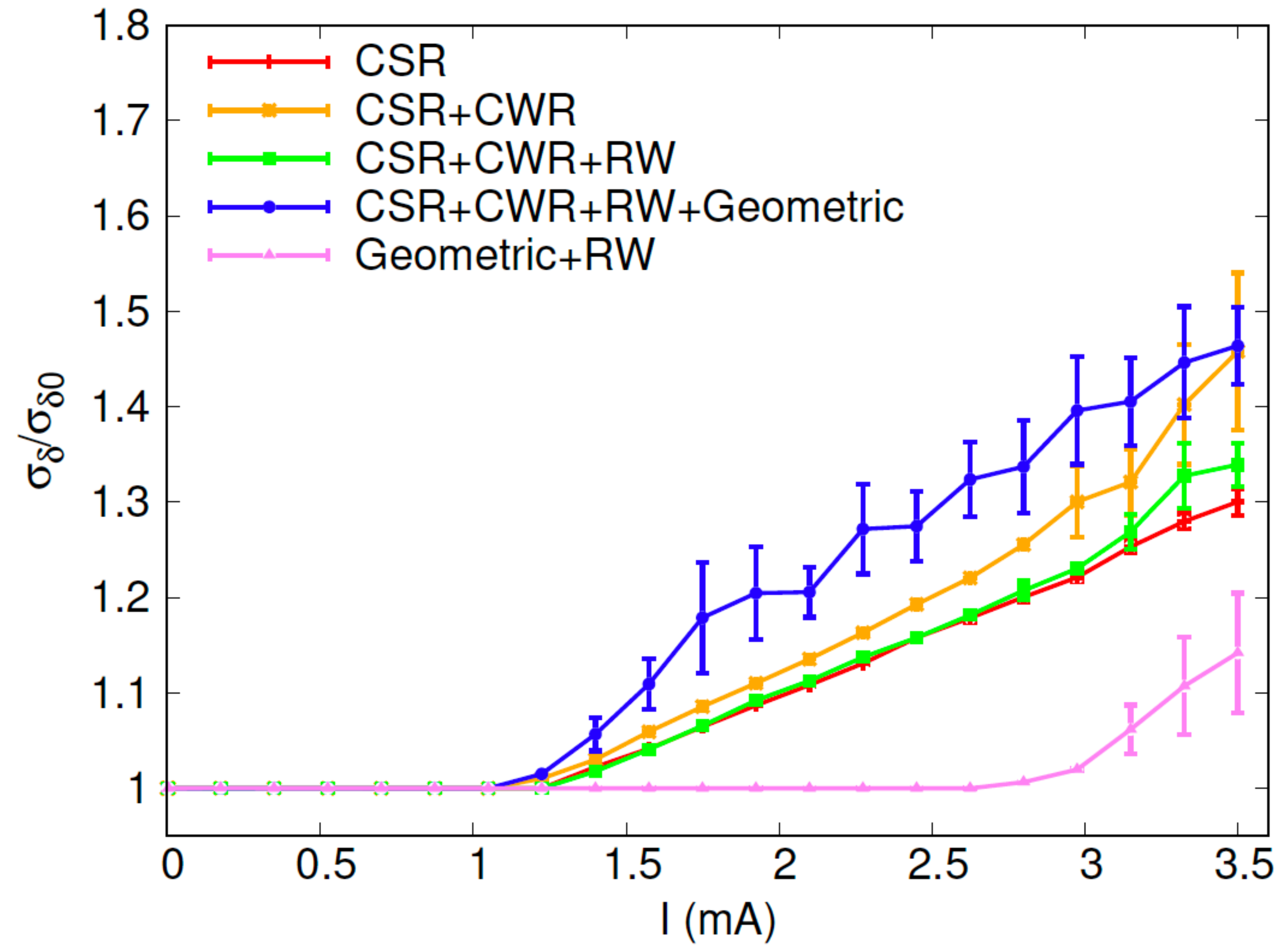
CSR instability modelling

- Example-1: SuperKEKB LER [1]
  - Run VFP simulations
    - Different combinations of impedance sources: CSR sets MWI threshold
    - Different filtering functions for impedance model
  - Check consistency between theories and simulations
    - “Numerical arts”: Interpolation, smoothing histogram, mesh size, number of wake kicks per turn, mesh boundaries, cutoff of impedance beyond k_{max}, …





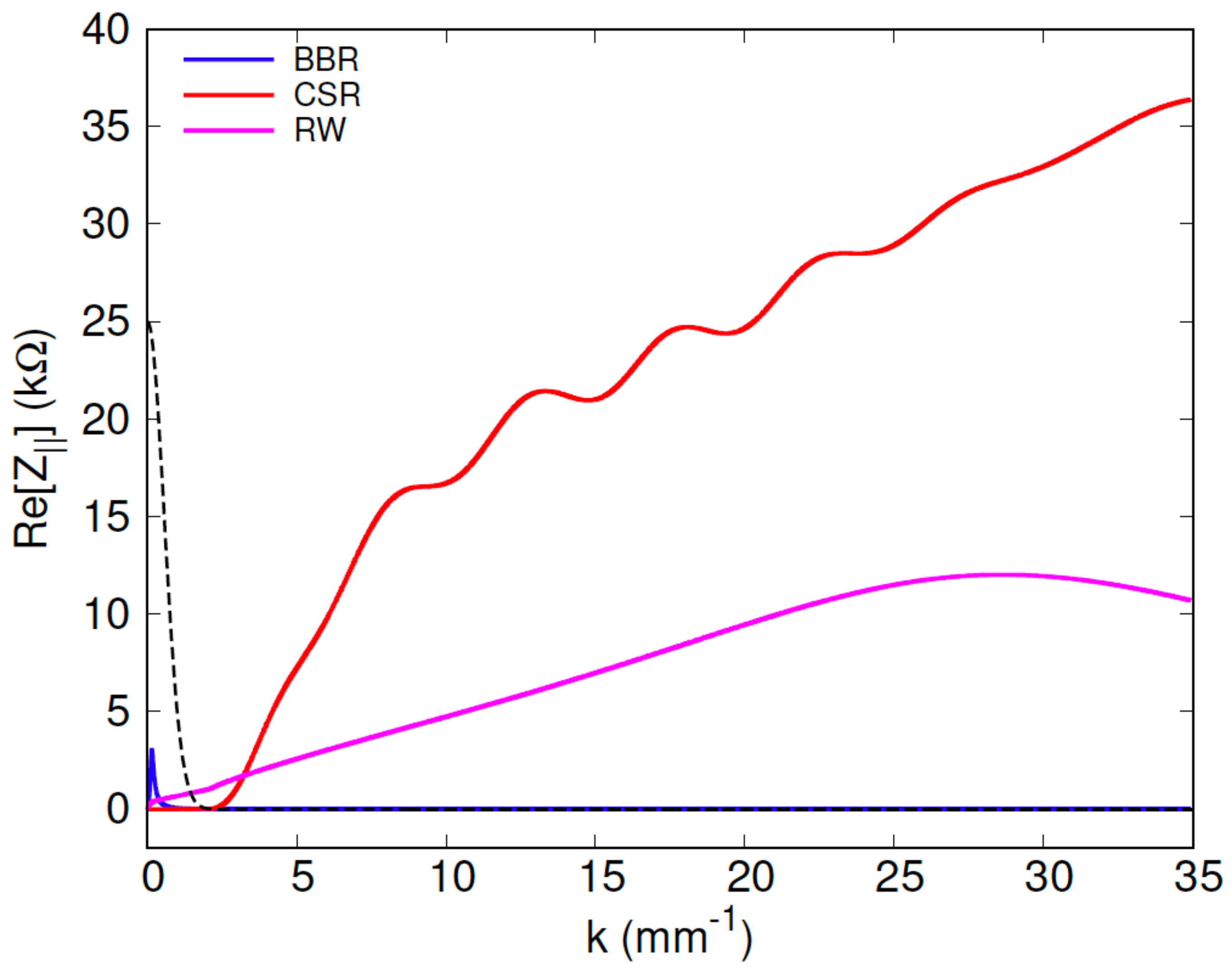


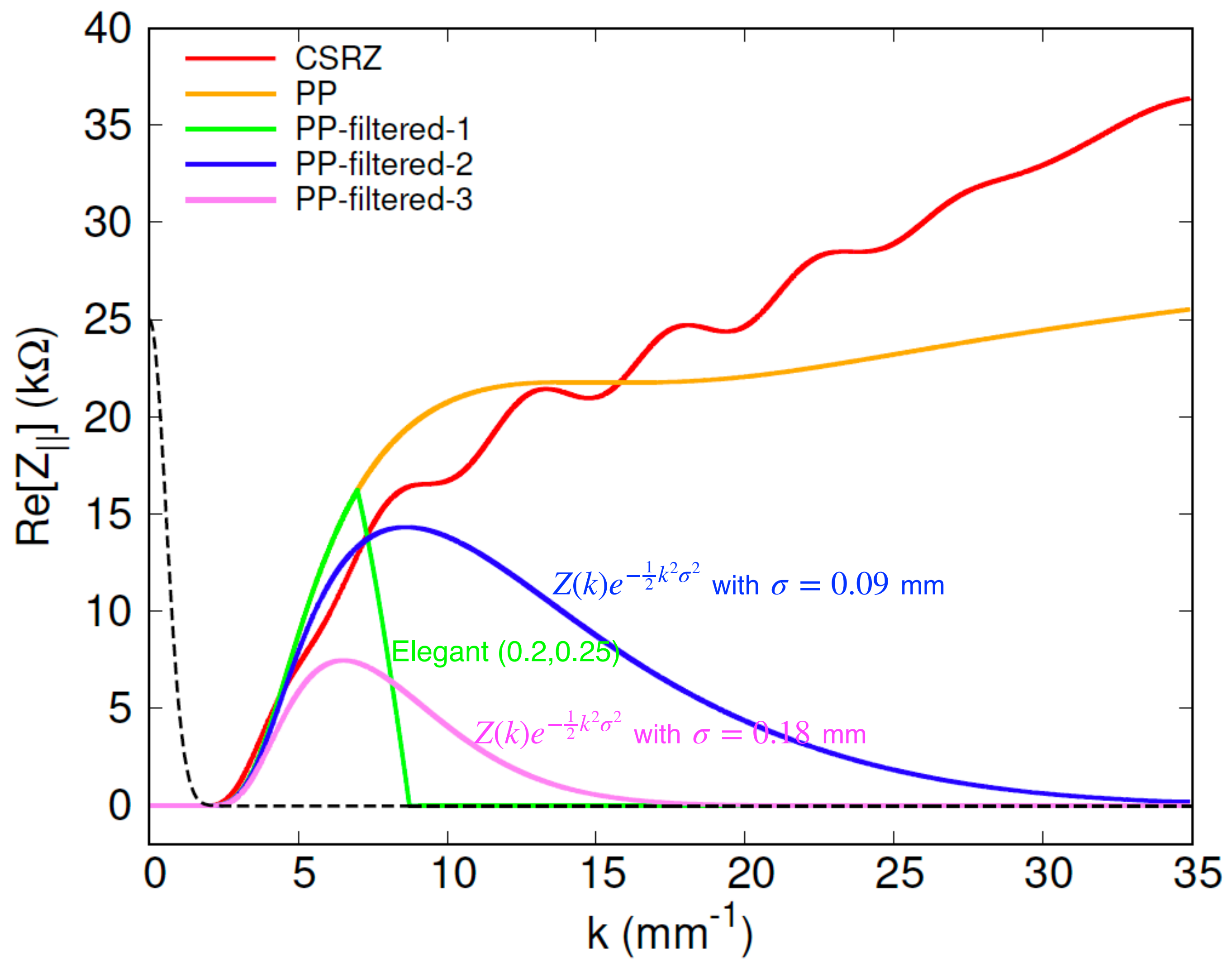


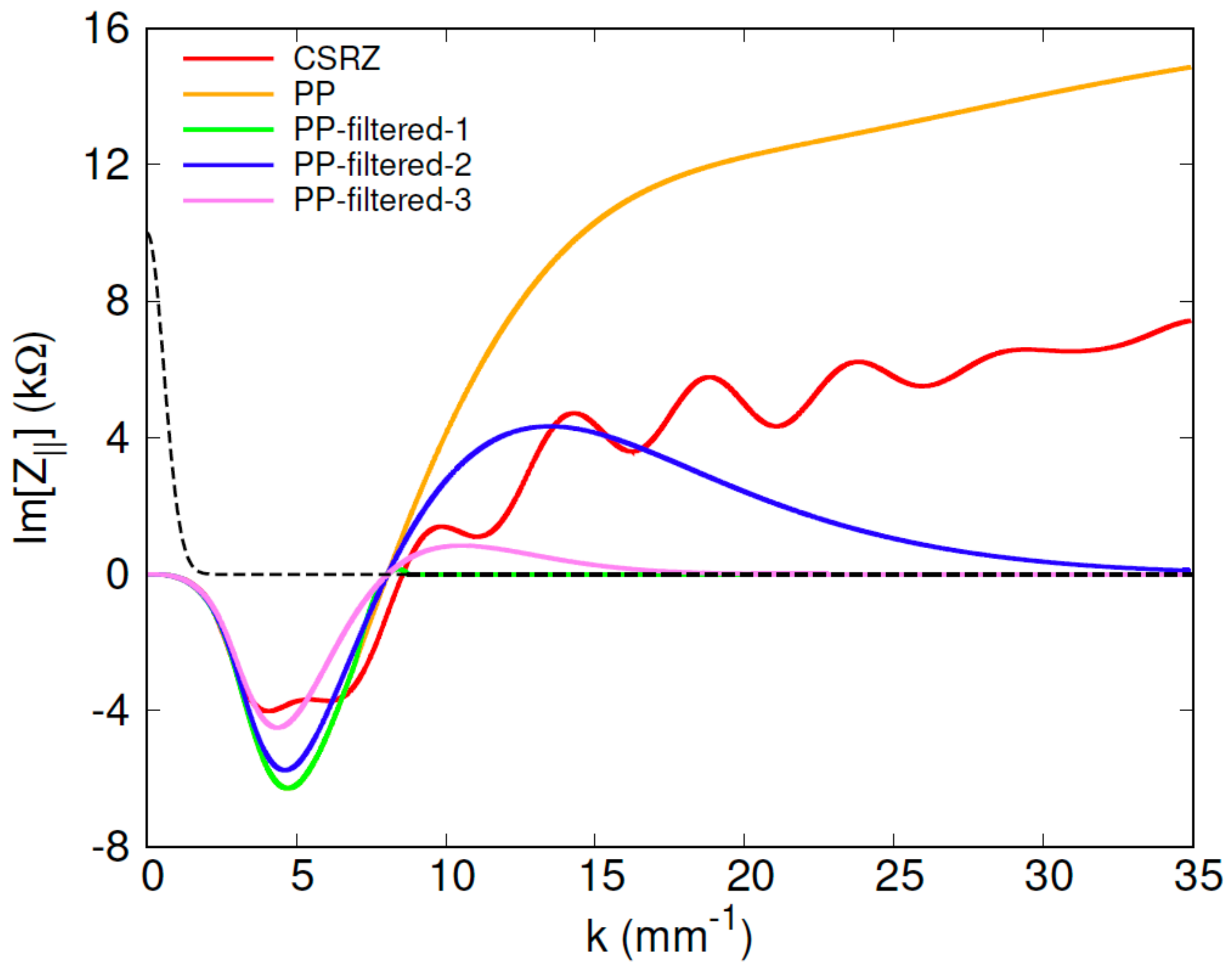
[1] S. Dastan, D. Zhou, et al., to be published.

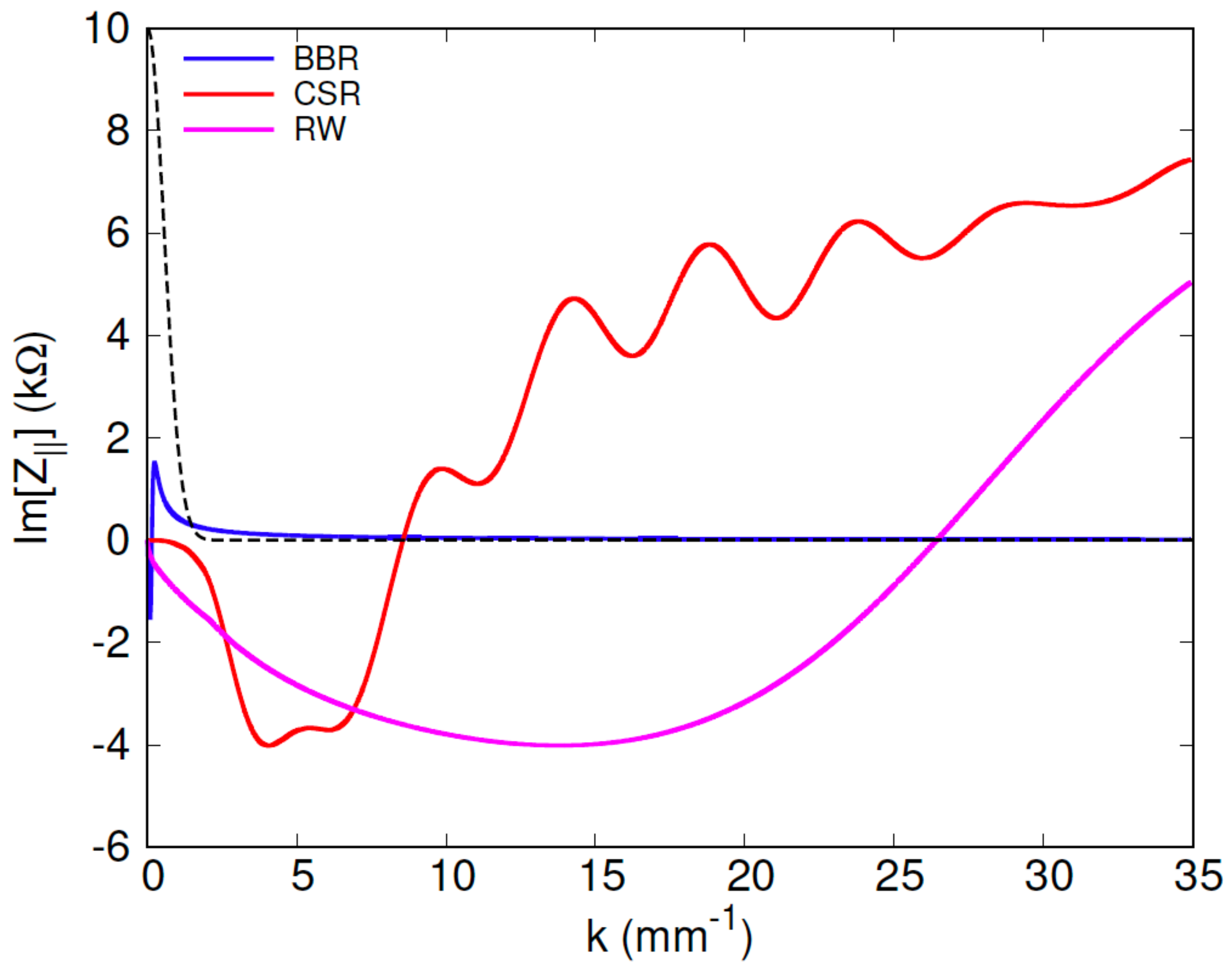
CSR instability modelling

- Example-2: Elettra 2.0 [1]
  - Impedance modeling of CSR by CSRZ, RW by IW2D, and geometric wakes by broadband resonator (BBR).
    - Alternative CSR models: CSRZ, PP, filtered PP
    - $h = 7.5$ mm, $\bar{\rho} = 7.8$ m
    - BBR: Q=1, f_r=7 GHz, $R_s \approx 3000\ \Omega$ (|Im[$Z_{||}$]/n| = 0.5 Ω [2])





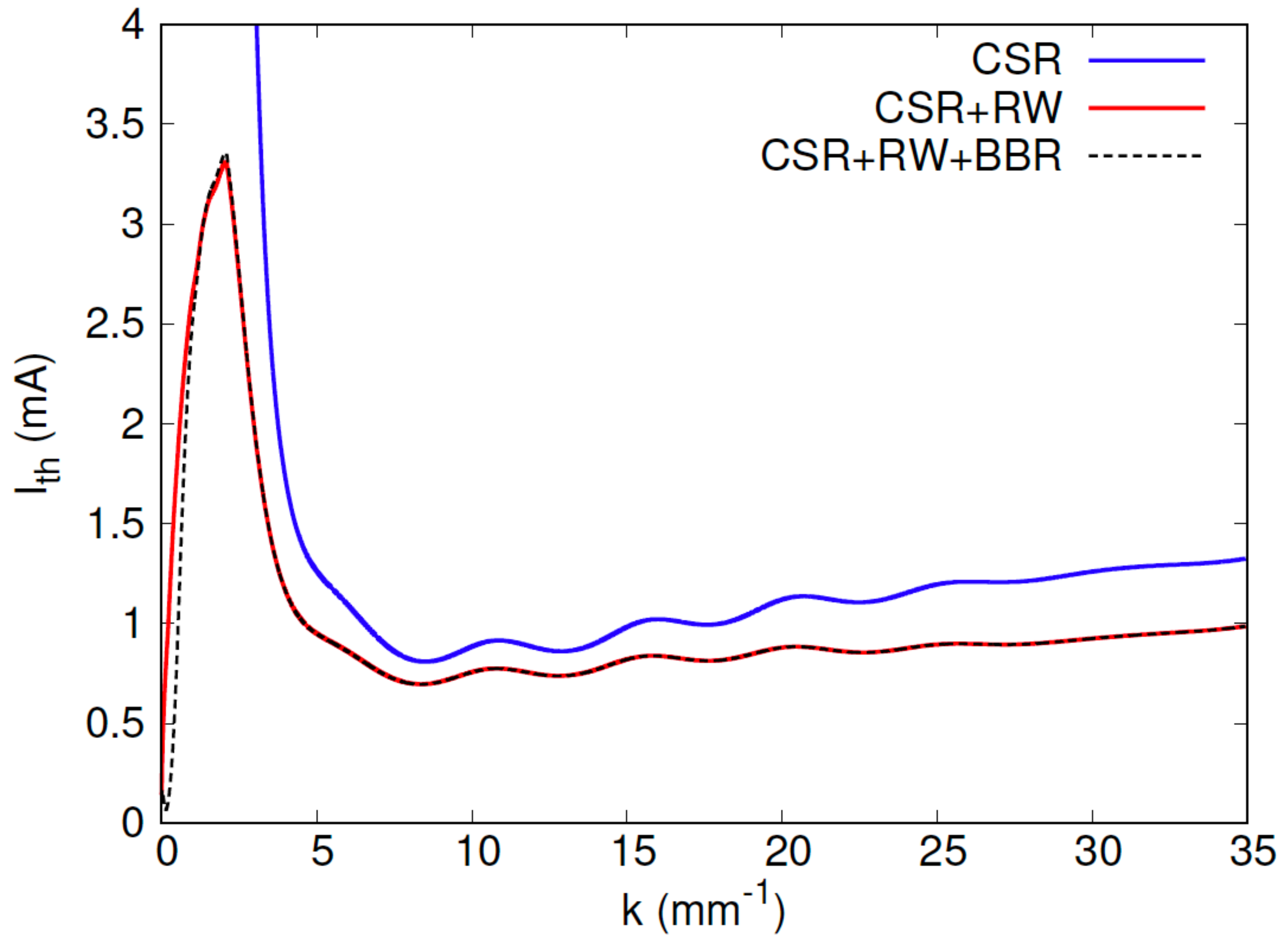




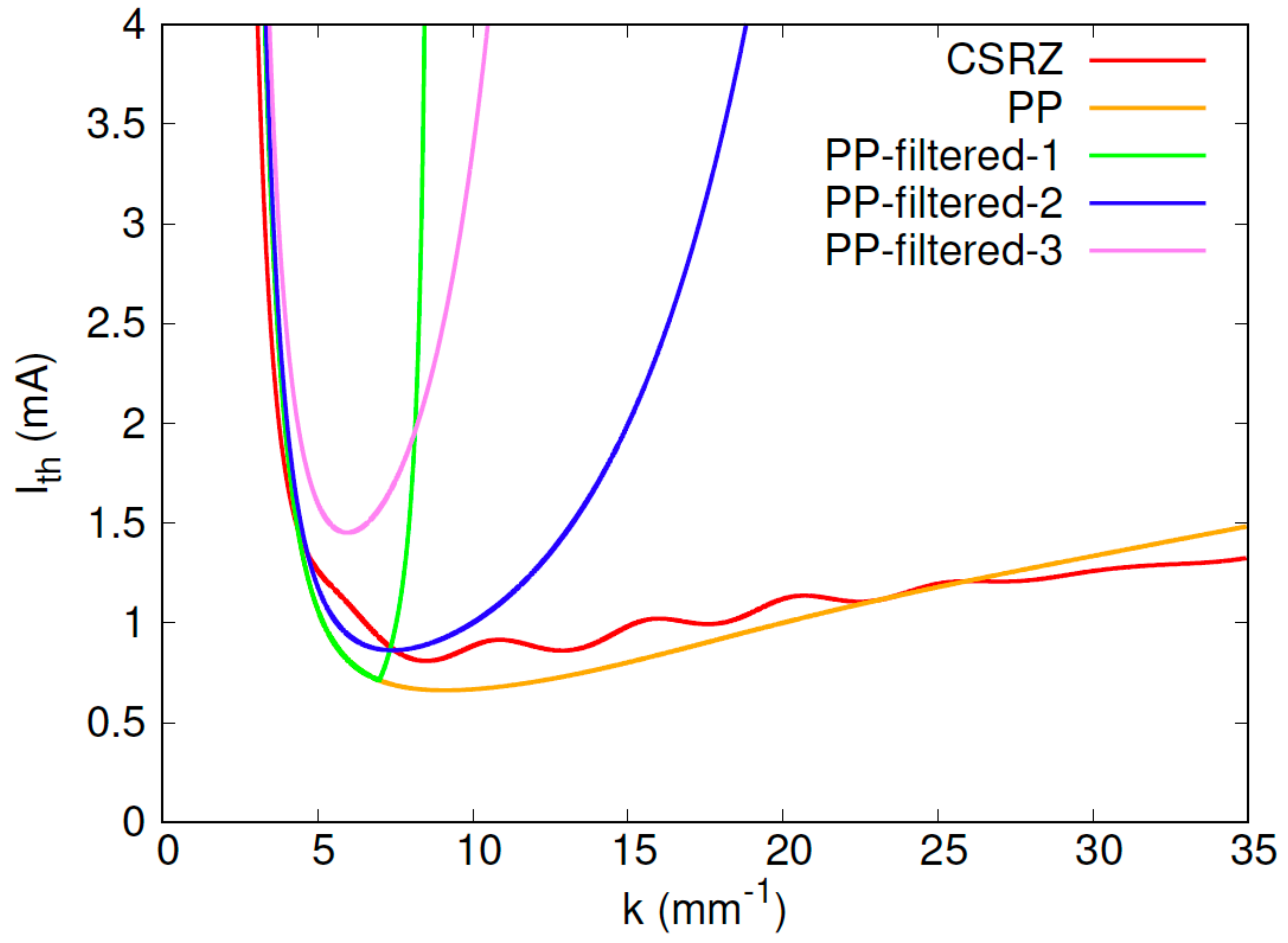
[1] S. Dastan, D. Zhou, et al., to be published. [2] Elettra 2.0 TDR, Part three: Machine and Infrastructure.

CSR instability modelling

- Example-2: Elettra 2.0 [1]
  - Instability analysis to determine $I_{th}(k)$.
    - σ_{z0}=1.8 mm without harmonic cavity (3HC) and σ_{z0}=4.5 mm with 3HC
  - Choosing important parameters: maximum k_{max} for impedance model, minimum mesh size $\Delta z \ll 2\pi/k_{max}$.
    - k_{max}=35 mm^{-1} $\rightarrow$ f_{max}=1.67 THz.



CSR and RW re important source for MWI in Elettra 2.0



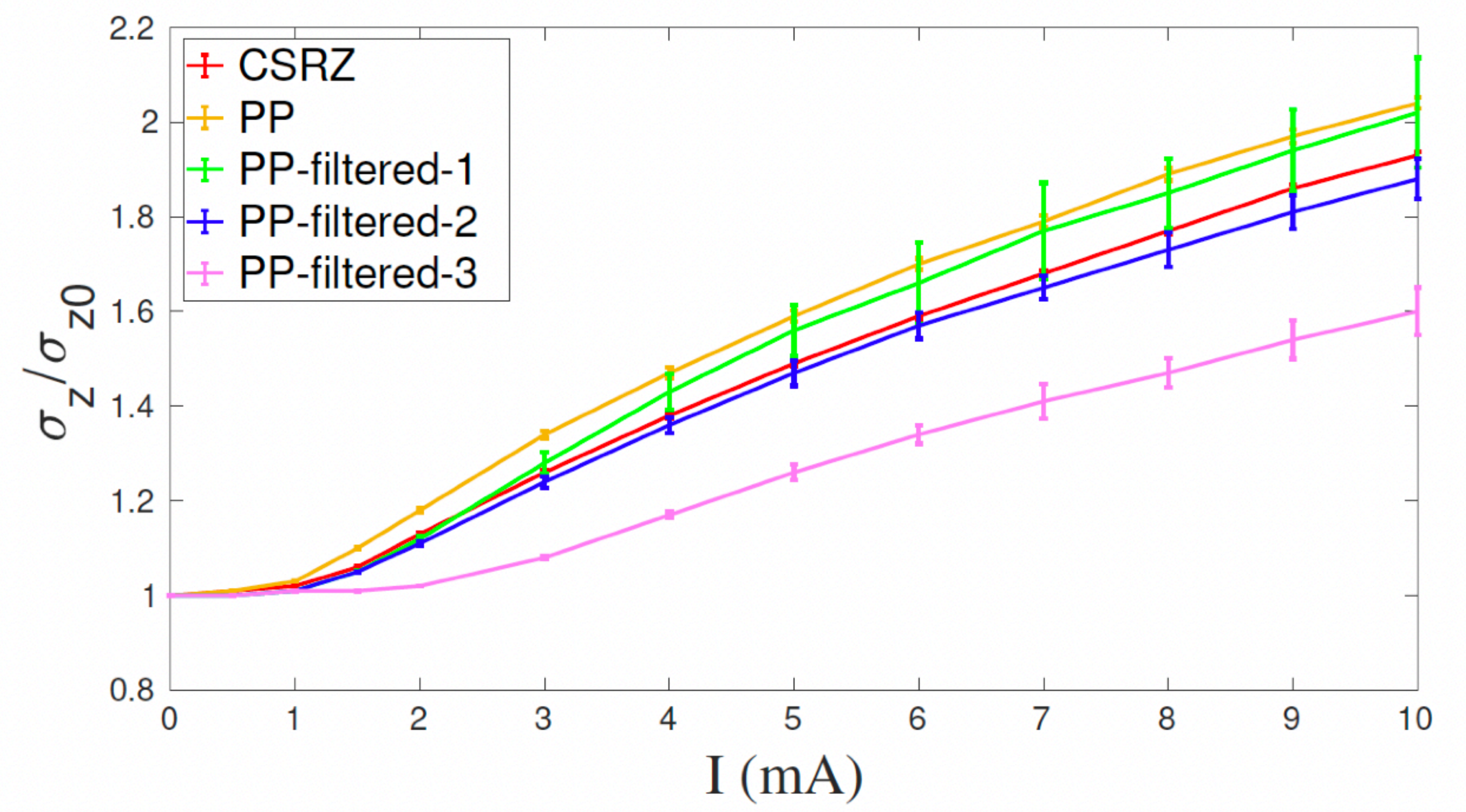
Filtered impedance models predicts higher MWI threshold

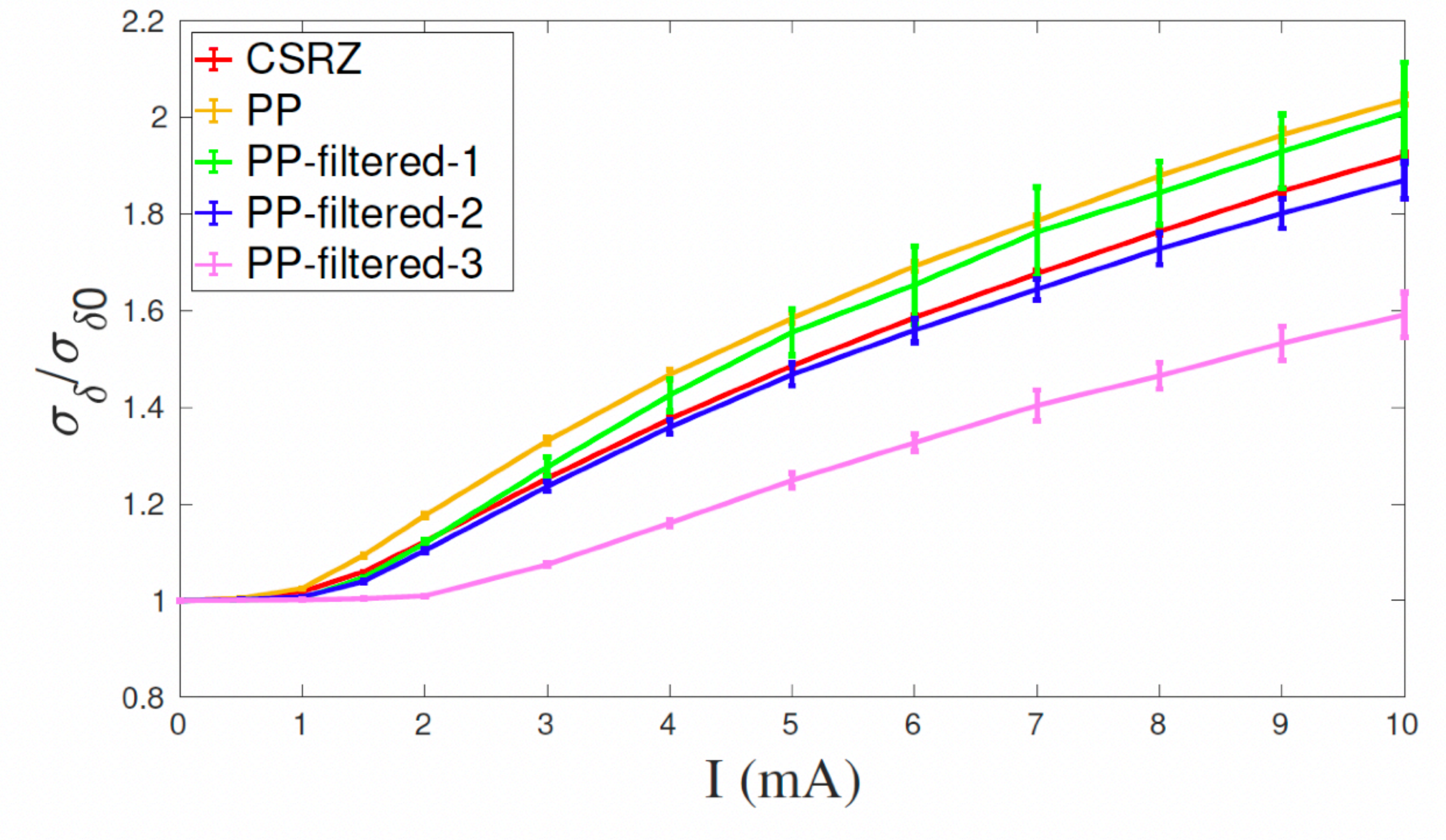
[1] S. Dastan, D. Zhou, et al., to be published.

CSR instability modelling

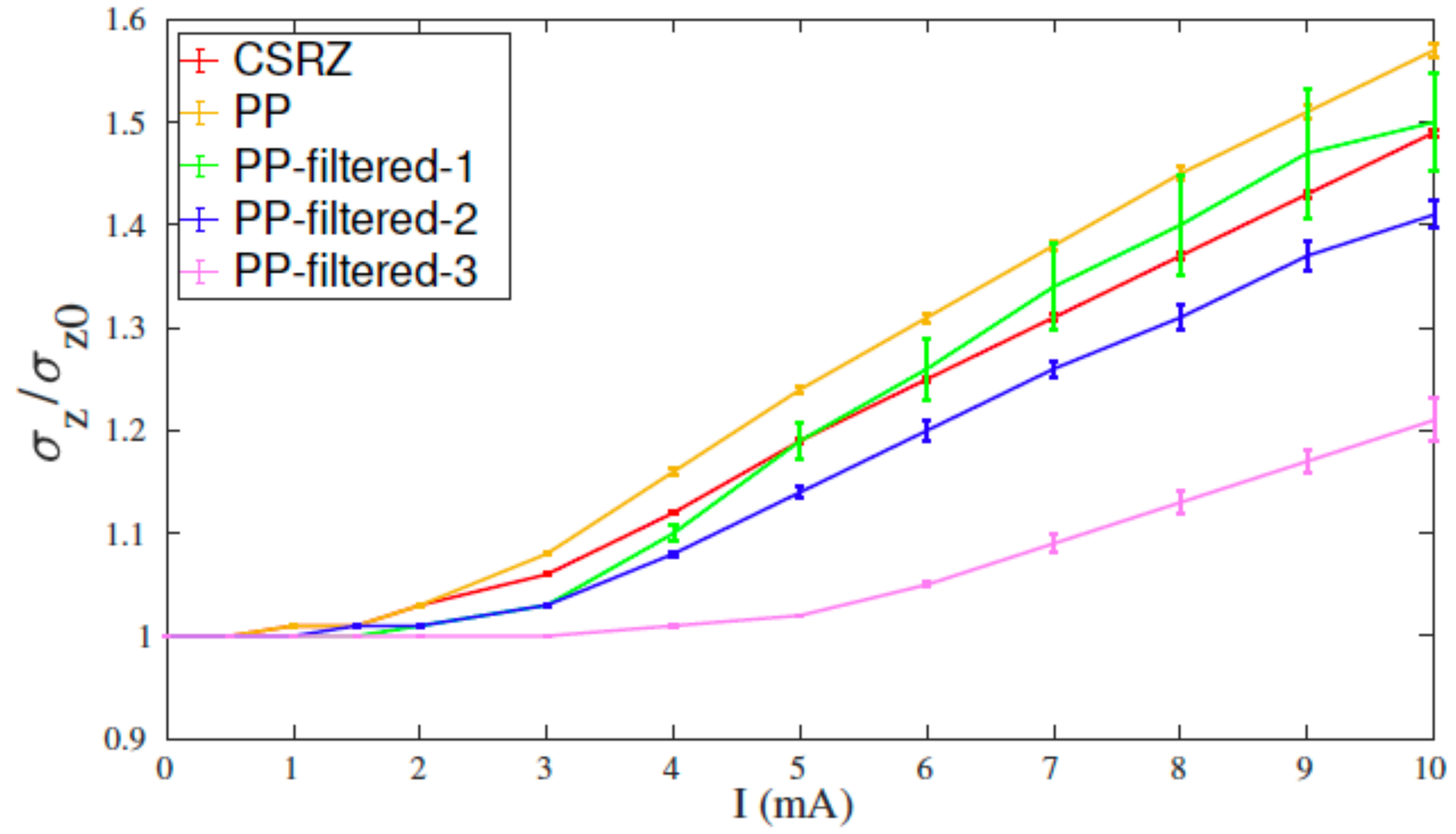
- Example-2: Elettra 2.0
  - Tracking simulations (Elegant) simulations (Only consider CSR)
    - General consistency between theory and Elegant simulations.
    - 3HC increases MWI threshold through reducing charge density as expected.

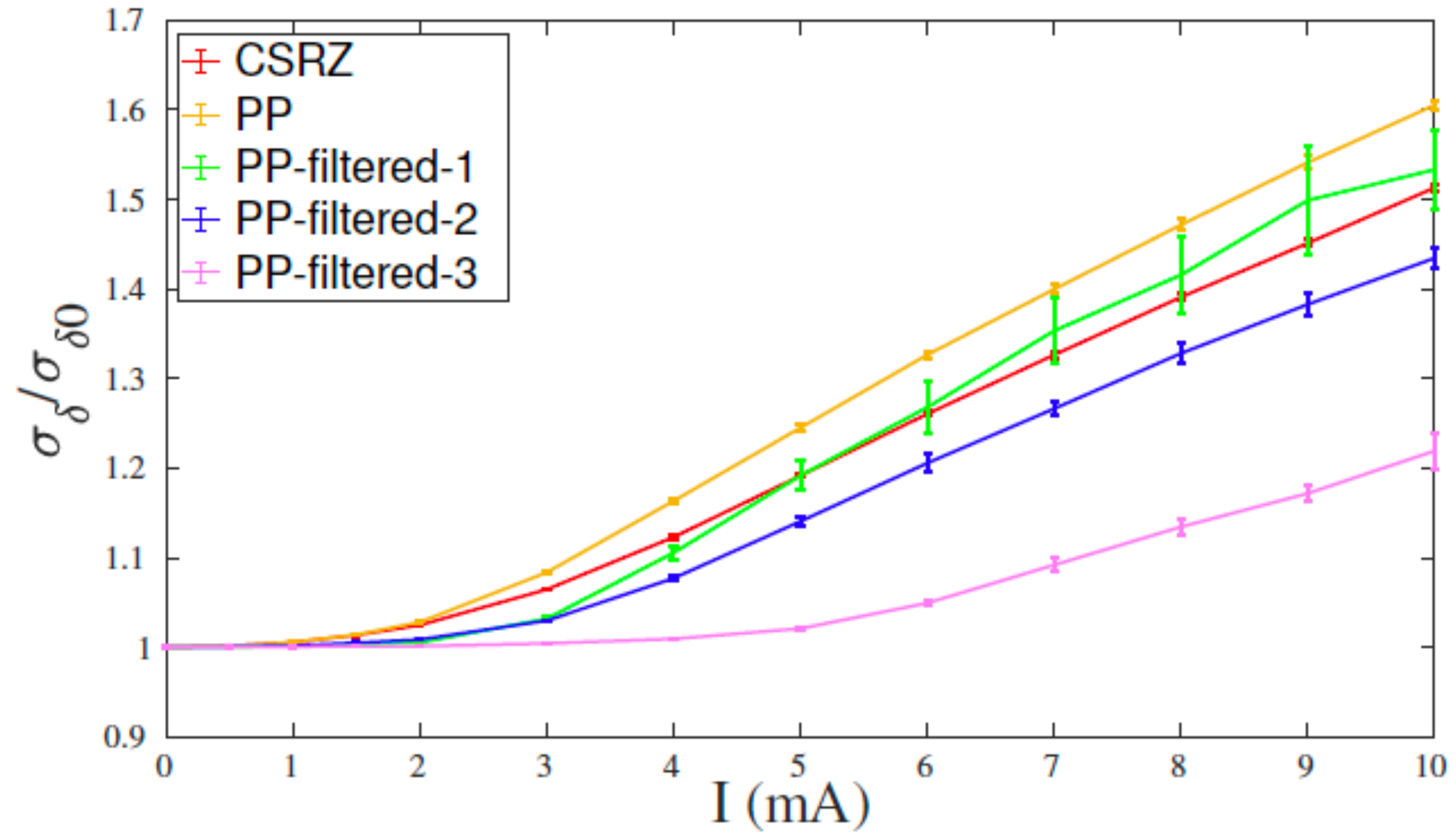
Elegant (10^6 particles, Δt=2x10^{-13} s, without 3HC)





Elegant (10^6 particles, Δt=2x10^{-13} s, with 3HC)

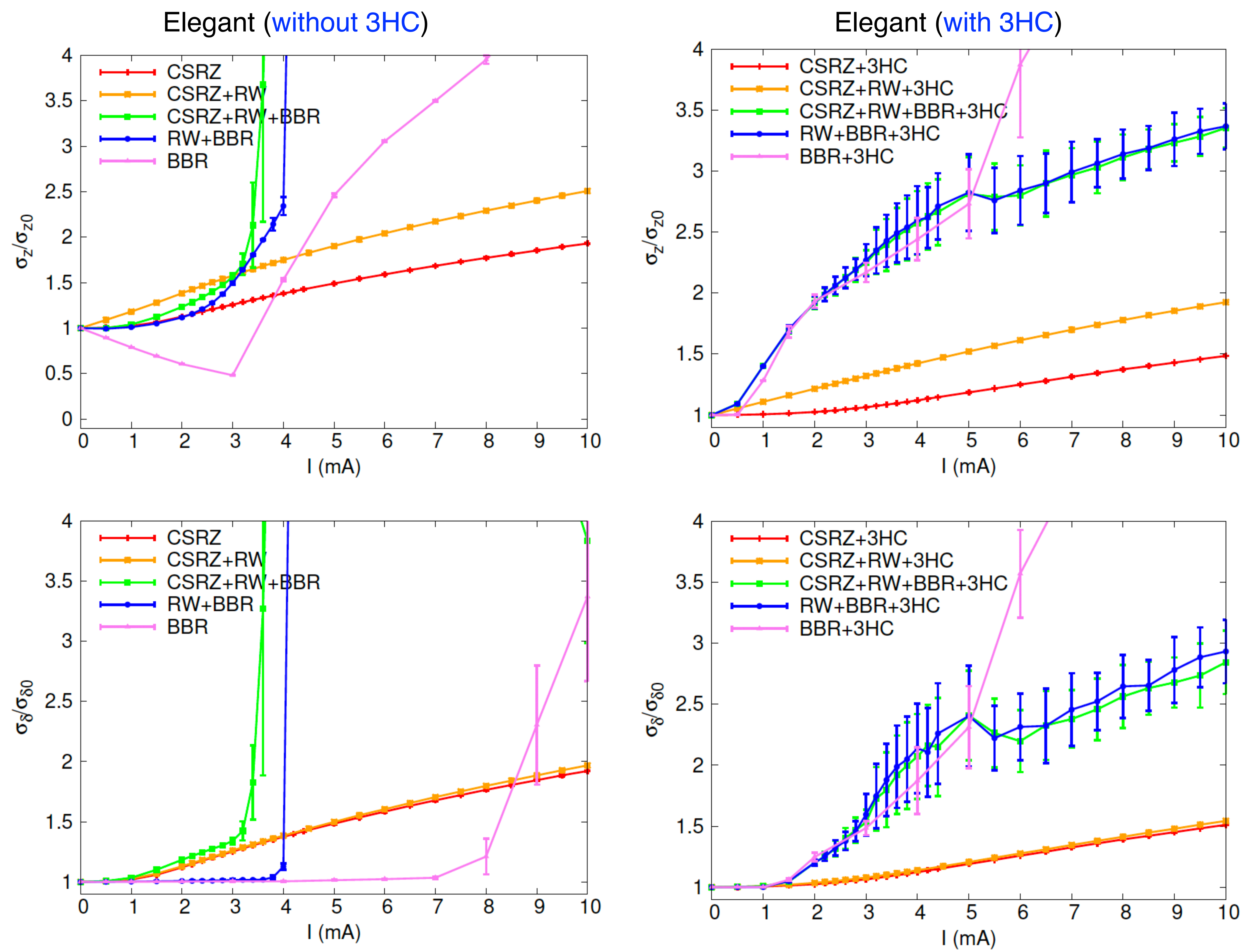




S. Dastan

CSR instability modelling

- Example-2: Elettra 2.0 [1]
  - Tracking simulations including CSR, RW and BBR:
    - With 3HC, MWI threshold is higher than design bunch current
  - All simulations can be well understood by theories
    - Without 3HC, CSR determines the MWI threshold
    - With 3HC, BBR impedance determines the MWI threshold
  - Future plan
    - Bottom-up impedance modelling to replace BBR

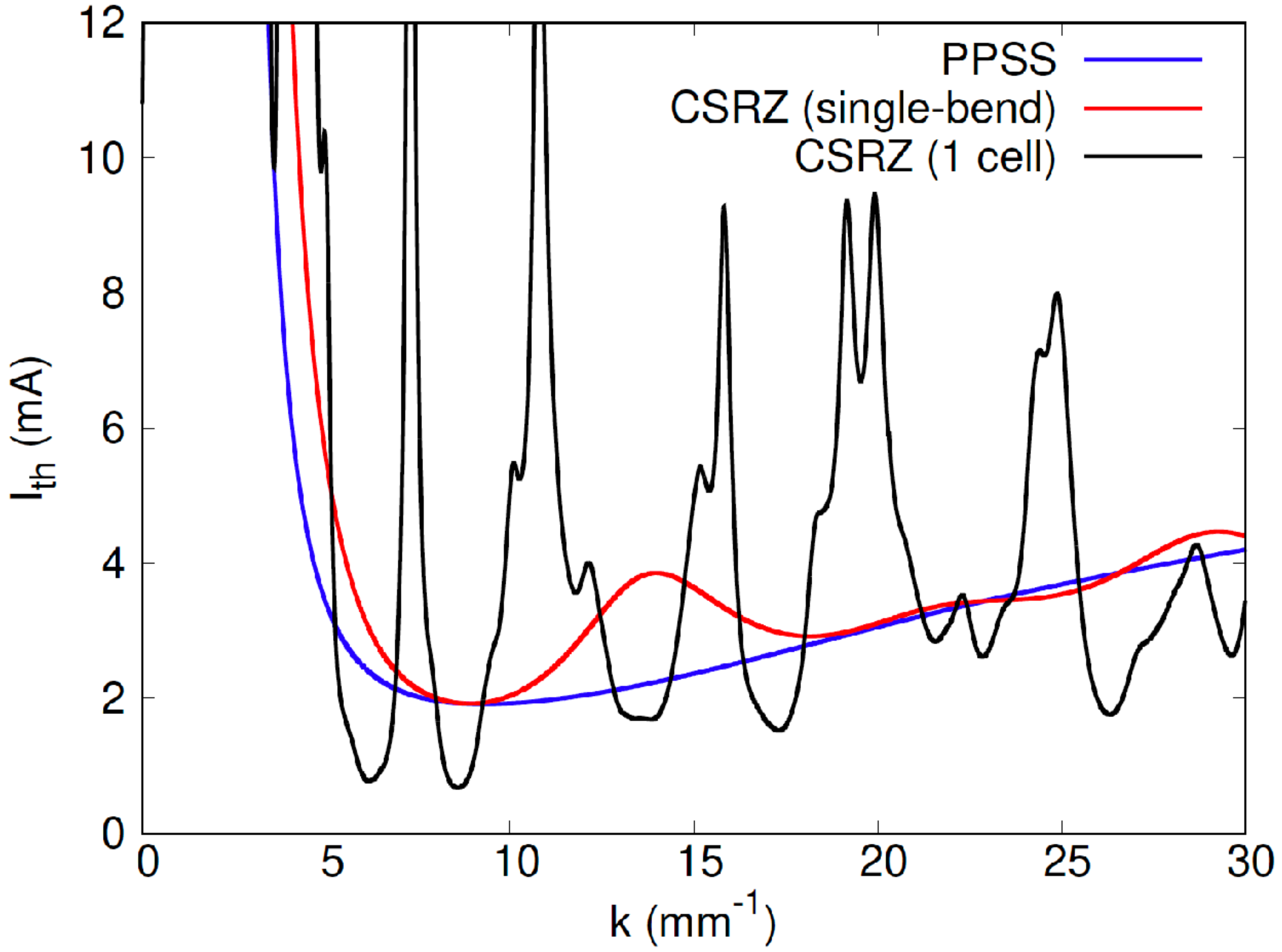


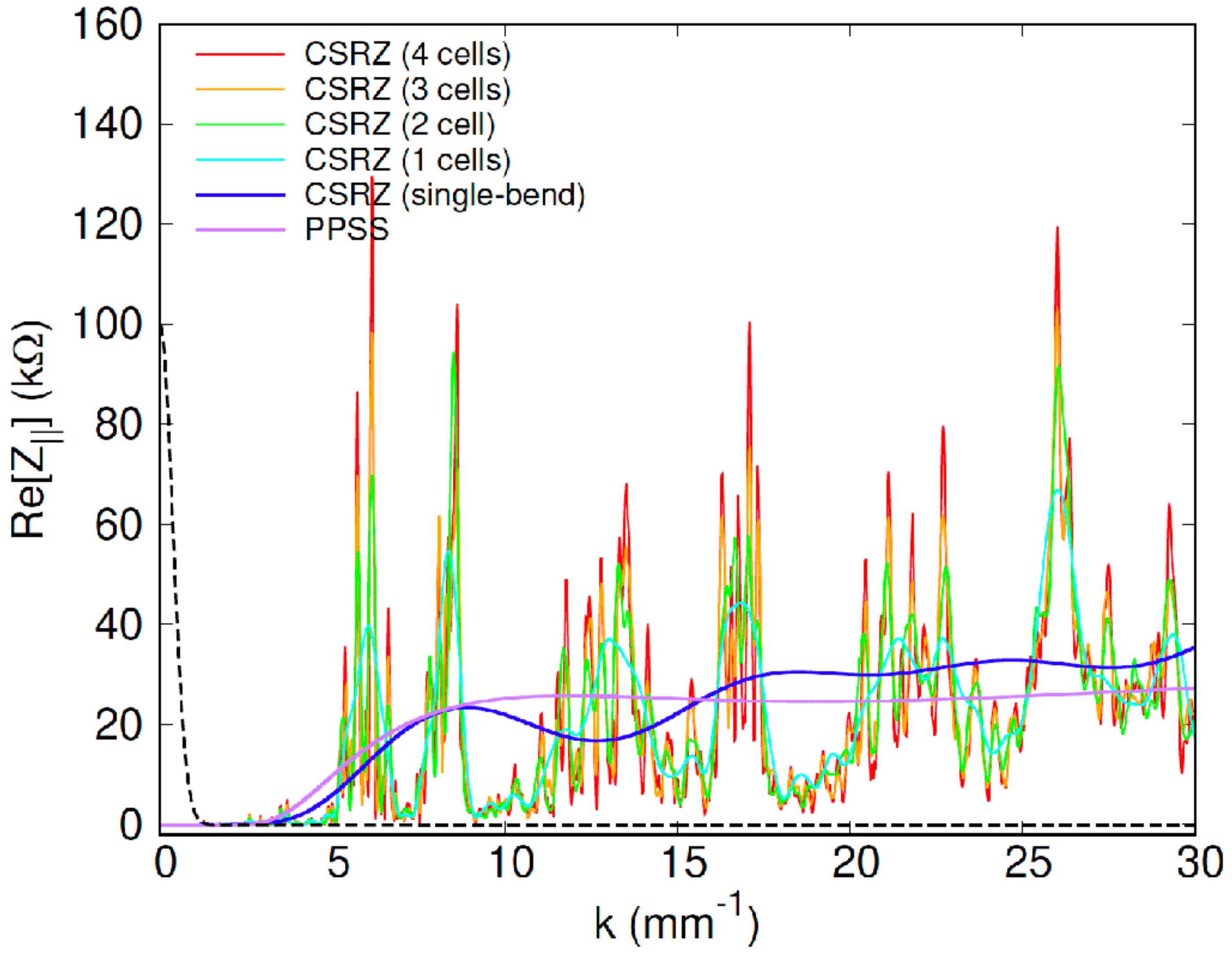
S. Dastan

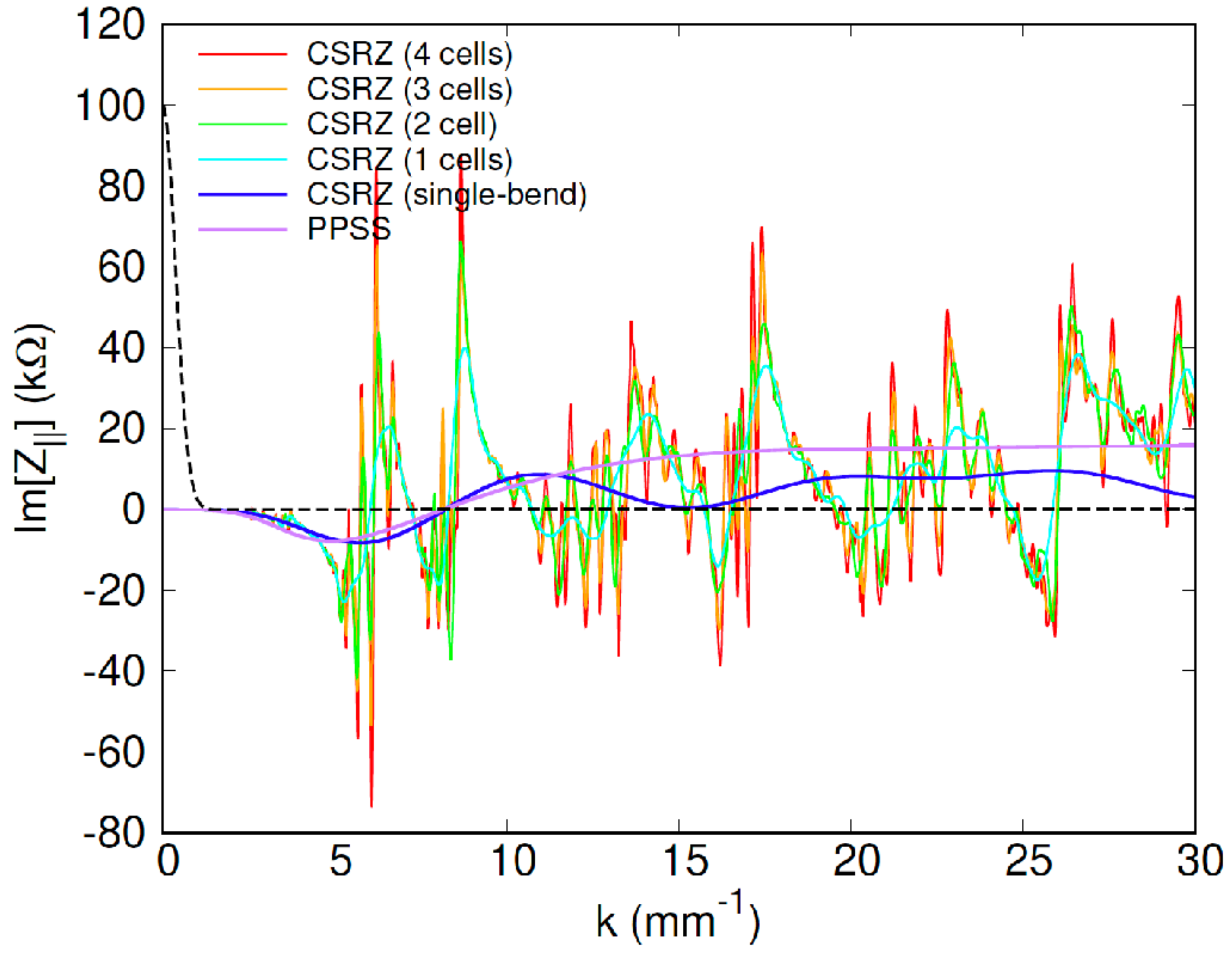
[1] S. Dastan, D. Zhou, et al., to be published.

CSR instability modelling

- Example-3: ALBA-II
  - Chamber full height/width=16/16 mm (approximation of CSRZ code).
  - Beam line configuration: 5BA lattice described in [1] (4 types of bend taken into account: QD/QDS/QF/QFS).
  - Maximum frequency for impedance modeling: k_{max}=30 mm^{-1}.
  - Beam parameters: σ_z=2.7 mm (w/o HC), σ_δ=1.2E-3, α_c=1.3E-4.
  - Instability analysis predicts MWI threshold (the same for PP-SS and CSRZ single-bend model): $I_{th} \approx$ 1.9 mA
  - Instability analysis not well applicable for narrowband impedances



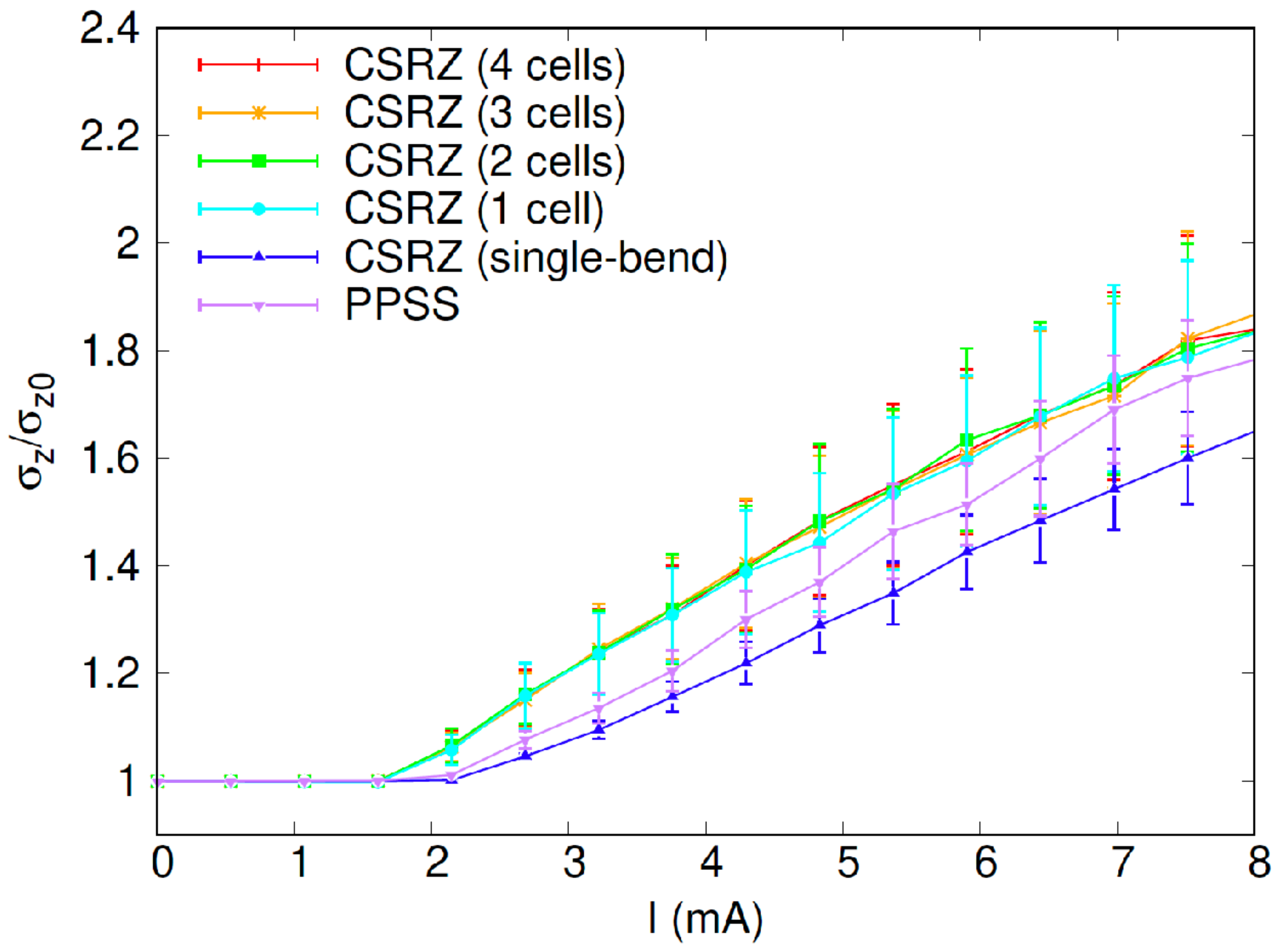




[1] M. Garla et al., THPC01, IPAC'24.

CSR instability modelling

- Example-3: ALBA-II
  - Simulated MWI threshold using PP-SS and CSRZ single-bend models: $I_{th} \approx$ 2.2 mA, close to instability analysis.
  - Simulated MWI threshold using CSRZ multi-bend models (arc cells connected with long straight sections, same as lattice configuration): $I_{th} \approx$ 1.7 mA. Narrowband CSR impedance seems to play a role, but not as much as predicted in [1].
  - From wakefield viewpoint, the CSR fields trailing behind the beam are not felt by the beam if the bunch is not too long.



χ=4.1 for ALBA-II

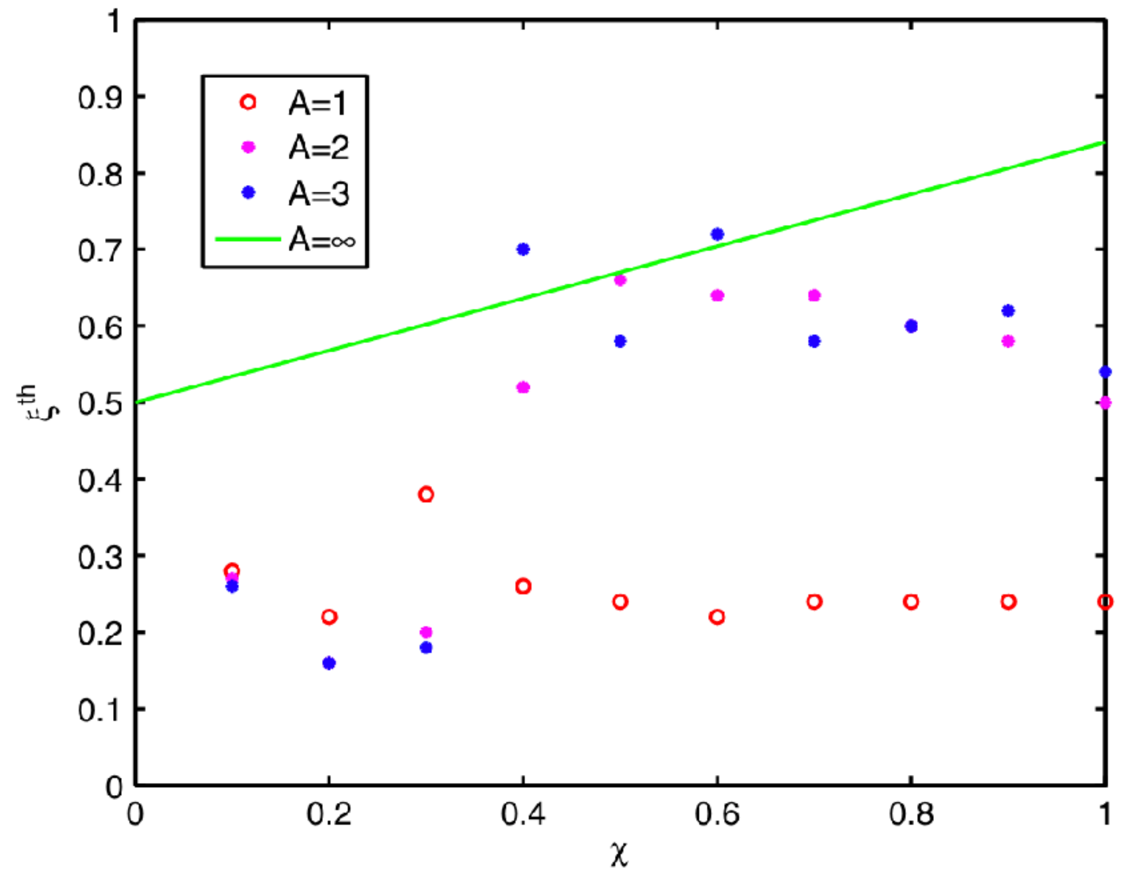
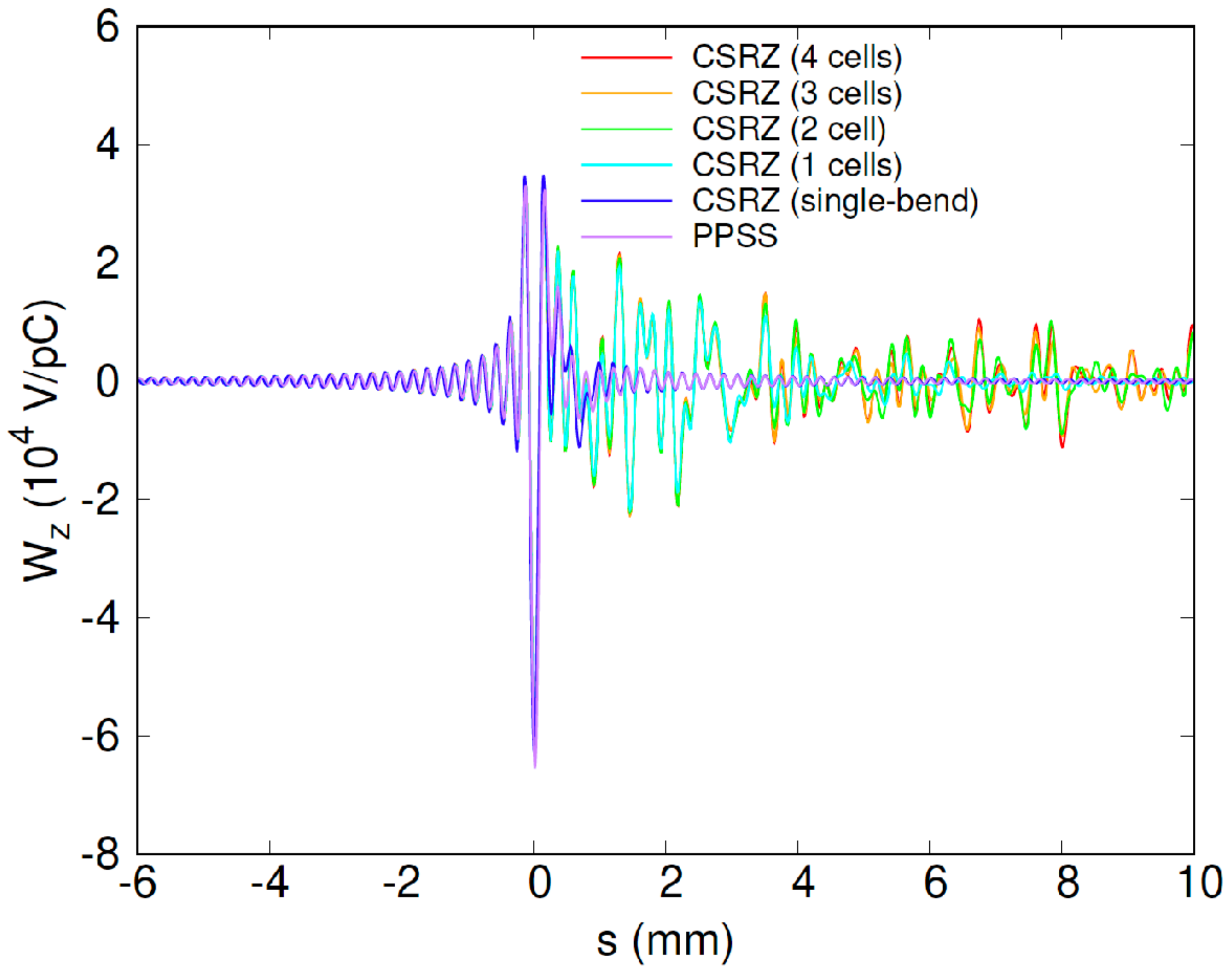
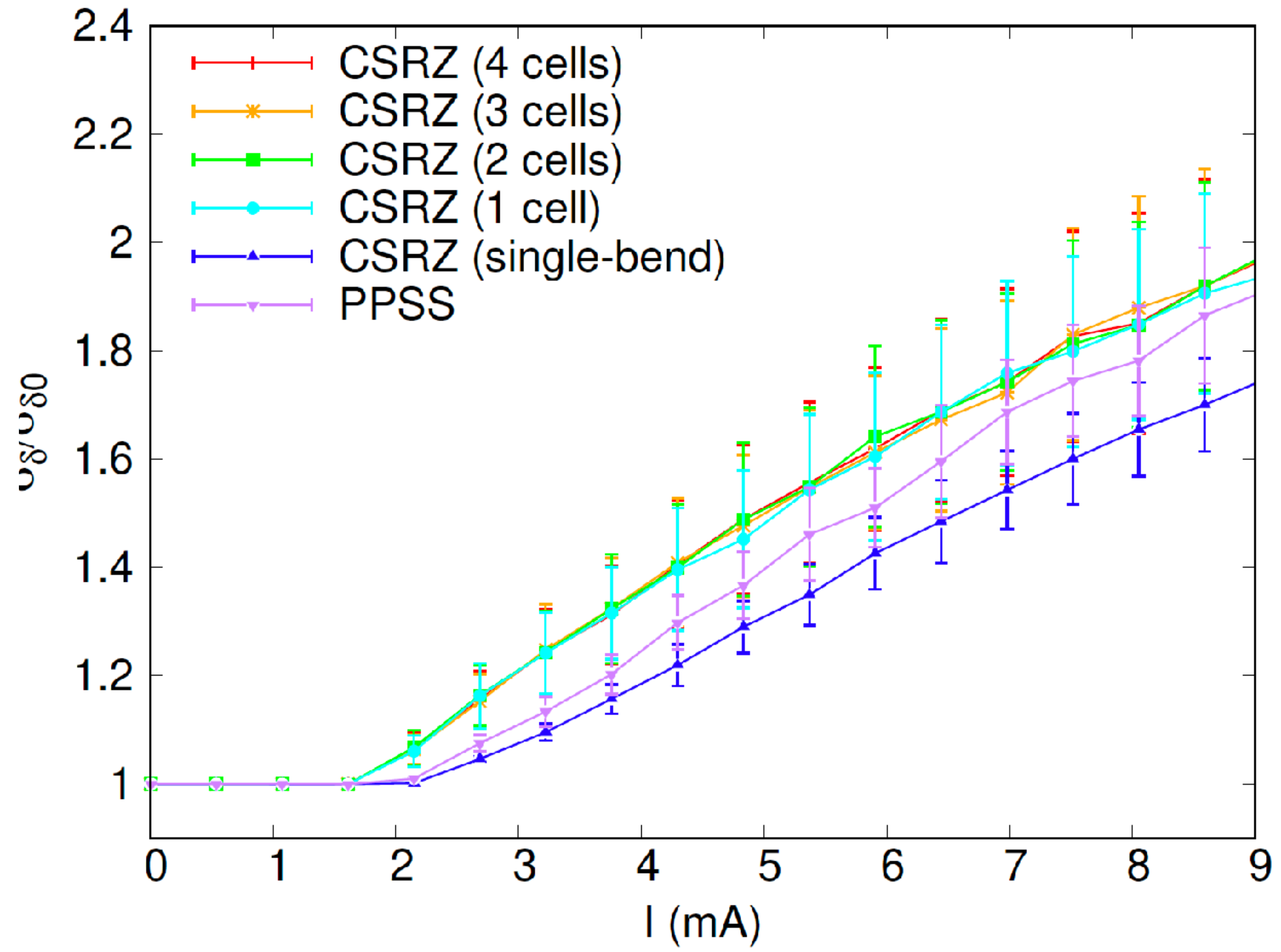


FIG. 6 (color online). Simulation results of the threshold as a function of the shielding parameter for rectangular chambers with various aspect ratios. The solid line represents $\xi^{th} = 0.5 + 0.34\chi$ for the parallel plate model.





[1] Y. Cai, PRST-AB 17, 020702 (2014).

Summary

- A practical approach for investigation of CSR instability in rings [1]
  - Examine alternative CSR impedance models
  - Apply theories of instability analysis for predictions of CSR instability
  - Include resistive wall (RW) and other conventional impedances
  - Apply instability theories to guide and interpret the numerical simulations
  - Run detailed numerical simulations with specific machine configurations

[1] S. Dastan, D. Zhou, et al., to be published.

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- Takeaway messages on CSR instability in low-emittance e- rings [1]
  - In most cases, the scaling law with PP-SS model is a good estimate
  - Instability analysis is useful to incorporate other high-frequency impedances
  - If a low MWI threshold is seen, check/increase $\gamma\eta\sigma_\delta^2\sigma_z$
  - Instability analysis and convergence studies are useful to guide MWI simulations

$$I_{th} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z}{Z_0 h} \cdot 0.384$$

[1] S. Dastan, D. Zhou, et al., to be published.

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- Not covered in this talk:
  - Narrow-band CSR impedance and its impact on MWI [2]
  - Accurate prediction of beam dynamics in the region well above MWI threshold

[1] S. Dastan, D. Zhou, et al., to be published. [2] Y. Cai, Phys. Rev. ST Accel. Beams **17**, 020702 (2014).

Backup

Introduction

- Standard theories of wakefields [1]
  - Panofsky-Wenzel theorem
  - Causality and resulting Hilbert-transform relation of impedance

Definition of wake function:

$$\tau = d/v = (z_0 - z)/v \qquad \overline{\vec{F}}(\vec{r}, \vec{r}_0; \tau) = \int_{-\infty}^{\infty} dt \ v \ \vec{F}(\vec{R}, \vec{R}_0; t)\Big|_{z_0 = vt, z = vt - d}.$$

$$w_z(\vec{r}, \vec{r}_0; d) = -\frac{1}{q_0 q_1} \overline{F}_z(\vec{r}, \vec{r}_0; \tau), \qquad w_\perp(\vec{r}, \vec{r}_0; d) = \frac{1}{q_0 q_1} \overline{F}_\perp(\vec{r}, \vec{r}_0; \tau),$$

Fourier-transform pair of wake function and impedance:

$$w_z(\vec{r}, \vec{r}_0; d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \ Z_\parallel(\vec{r}, \vec{r}_0; \omega) e^{-i\omega\tau}, \qquad w_\perp(\vec{r}, \vec{r}_0; d) = \frac{1}{2\pi\kappa} \int_{-\infty}^{\infty} d\omega \ Z_\perp(\vec{r}, \vec{r}_0; \omega) e^{-i\omega\tau}.$$

Hilbert-transform relation of impedance:

$$\mathrm{Re}\{Z(\omega)\} = \frac{2}{\pi} \mathrm{P.V.} \int_0^{\infty} \frac{\omega' \mathrm{Im}\{Z(\omega')\}}{\omega'^2 - \omega^2} d\omega', \qquad \mathrm{Im}\{Z(\omega)\} = -\frac{2\omega}{\pi} \mathrm{P.V.} \int_0^{\infty} \frac{\mathrm{Re}\{Z(\omega')\}}{\omega'^2 - \omega^2} d\omega'.$$

[1] A. Chao, “Physics of collective beam instabilities in high energy accelerators”, 1993.

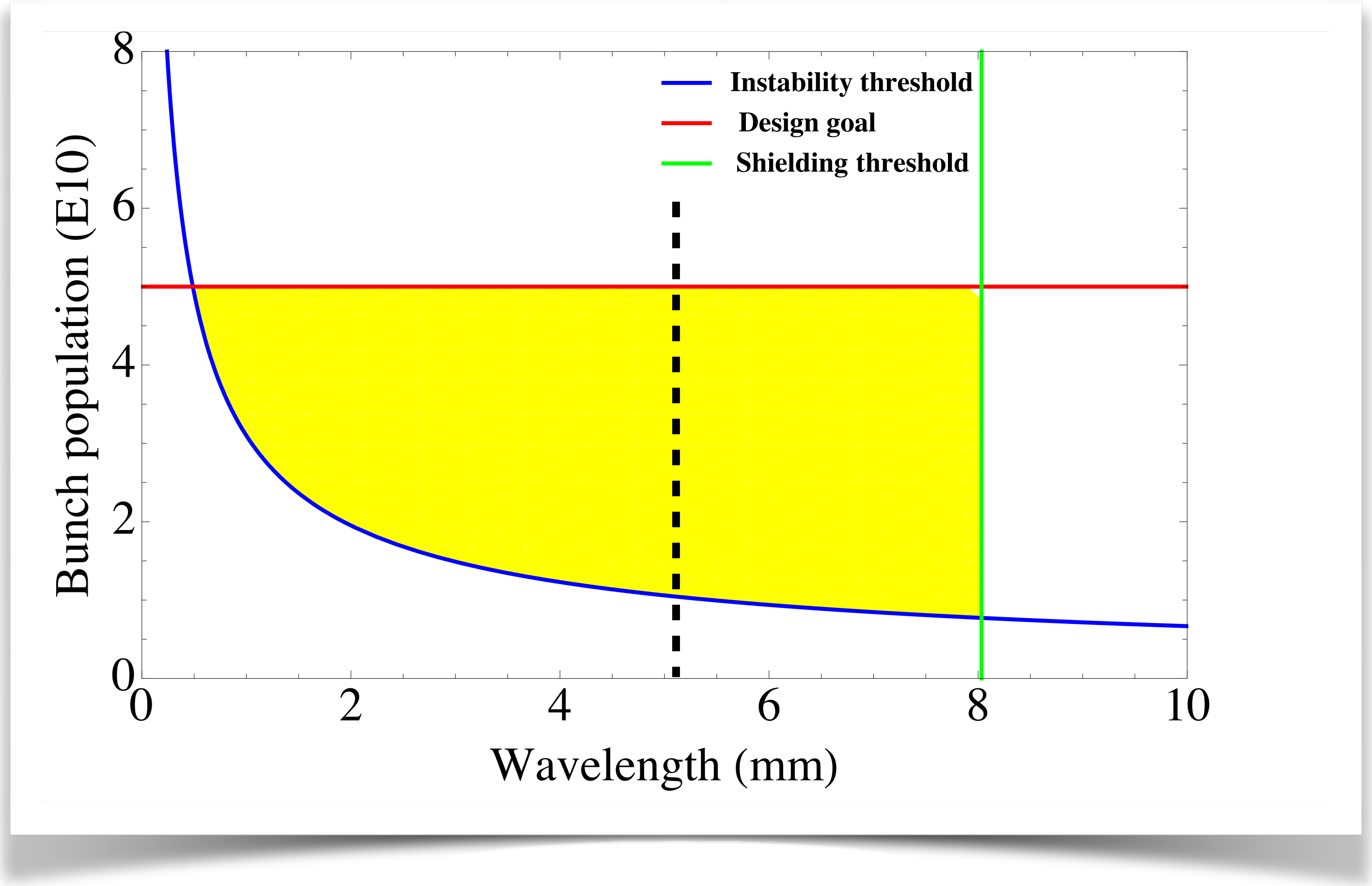
CSR instability theories

- Apply S-H theory to electron storage rings
  - Quick estimate of CSR instability.
  - Very useful in the design stage of a storage ring.
  - “Yellow region” indicates “severity of instability”.
  - For rings where CSR is marginally of concern, MWI simulations are required.

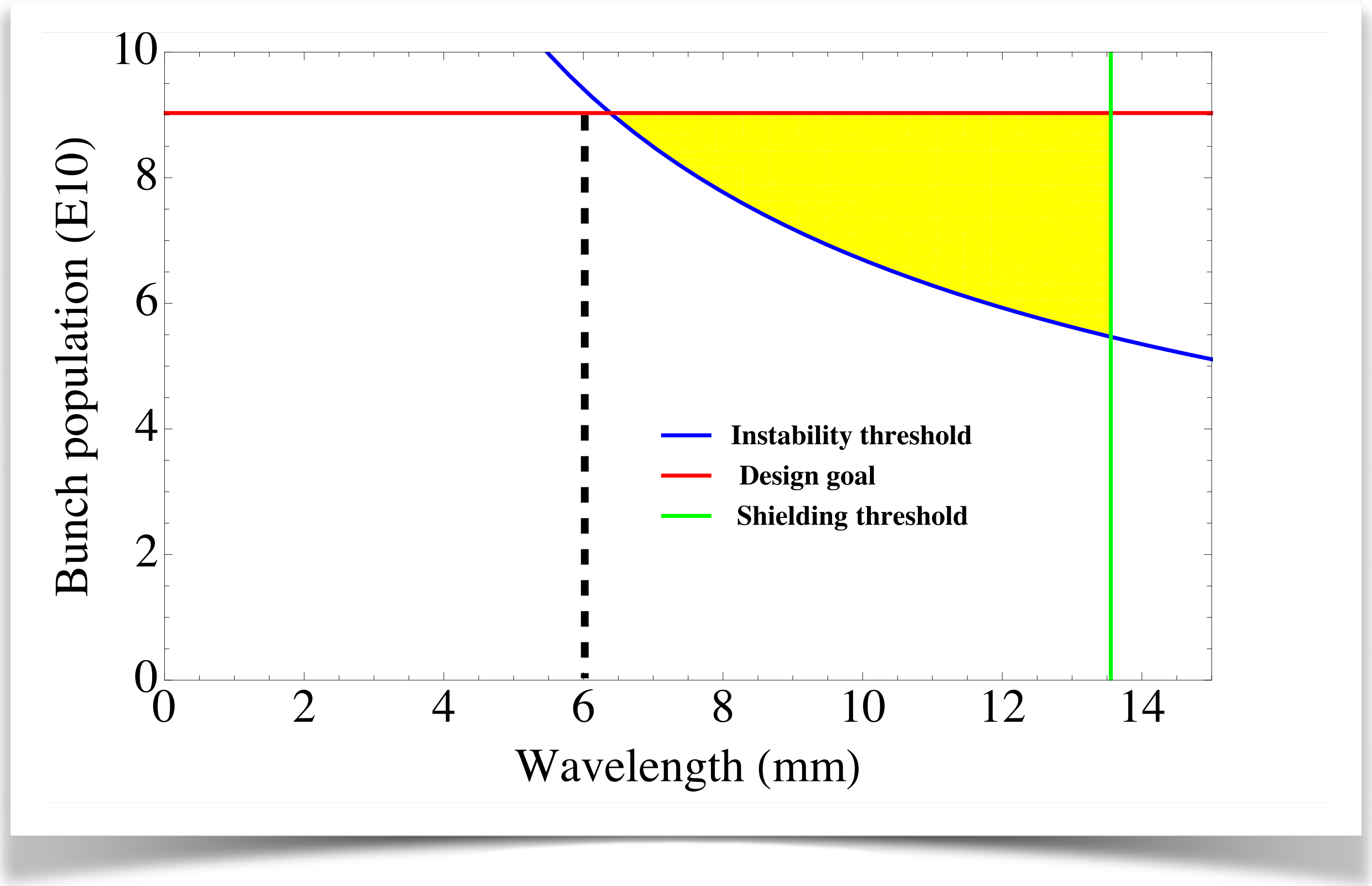
Parameters	SuperKEKB DR[1)]	SLC DR[2)]	ATF[3)]	SuperKEKB LER[4)]	SuperKEKB LER[5)]	PEP-II LER[6)]	ALS[6)]	KEKB LER[7)]
Circumference (m)	135.5	35.27	138.6	3016	3016	2200	196	3016
Energy (GeV)	1	1.21	1.54	4	3.5	3.1	1.5	3.5
Bending radius	2.43623	2.0372	5.73	15.87	15.87	13.7	4	15.87
Mom. compaction	3.43E-03	0.01814	2.17E-03	2.74E-04	2.74E-04	1.31E-03	1.41E-03	3.31E-04
Energy spread(10^{-4})	5.44	7.3	5.56	8.14	7.13	8.1	7.1	7.27
Bunch length (mm)	5.1	5.9	5	6	3	10	7	4.58
Bunch population (10^{10})	5	5	2	9.03	11.7	9.16	12.3	6.47
Pipe height@bends (mm)	34	15.6	24	90	90	50	40	94
Total bend. radius(2π)[8)]	1	1	1	1	1	1	1	1

1) Design Version 1.140, Apr. 2010
2) SLC design handbook, Dec. 1984
3) ATF design and study report, KEK Internal 95-4
4) Nano-beam option design, Feb. 2008
5) High-current option design
6) G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002)
7) Machine operating parameters, Jun.17, 2009
8) Assumed

SuperKEKB DR (Design Ver. 1.140)

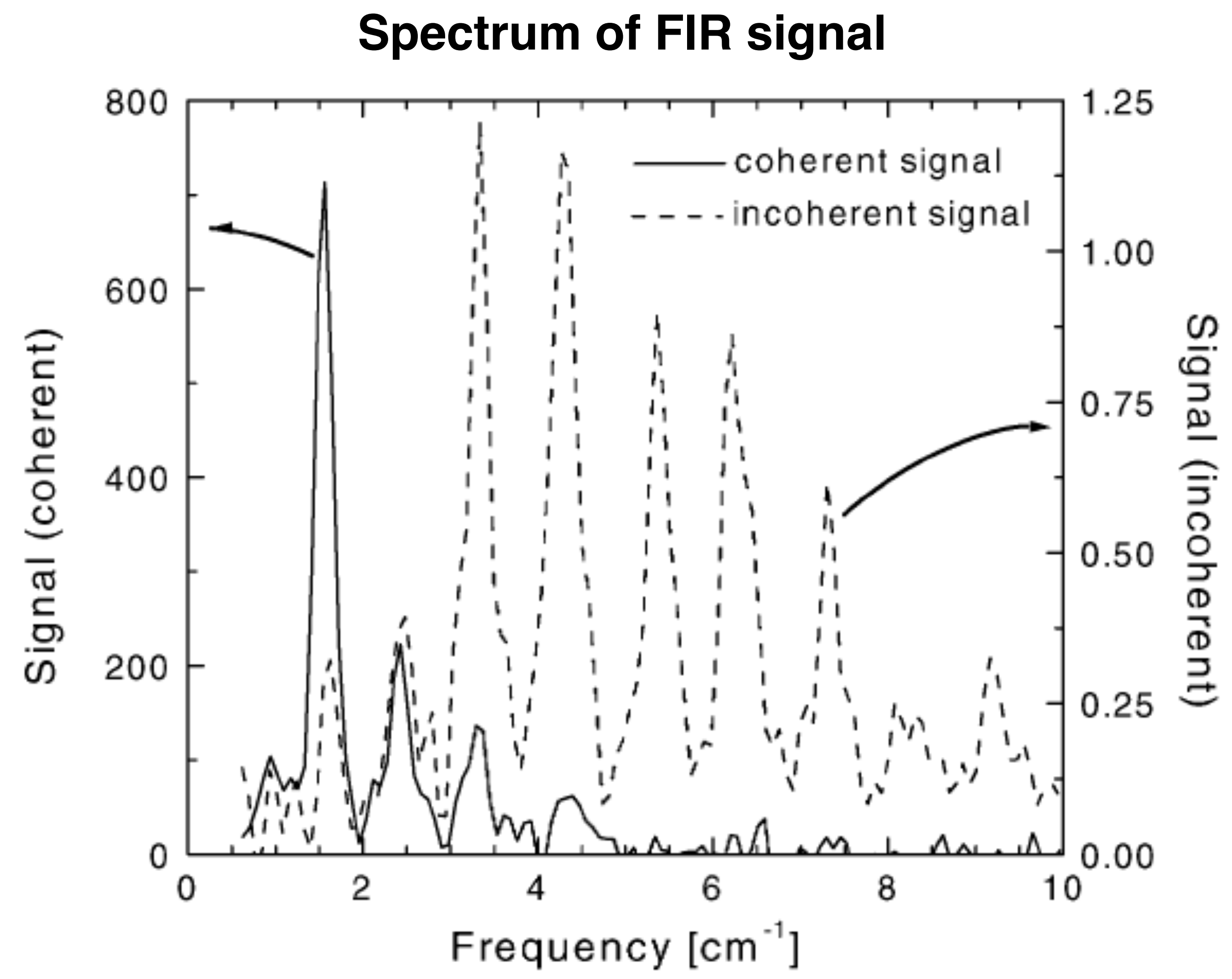


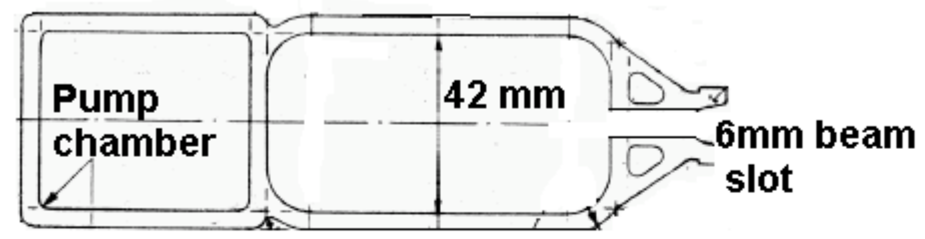
SuperKEKB LER (Design nano-beam option)

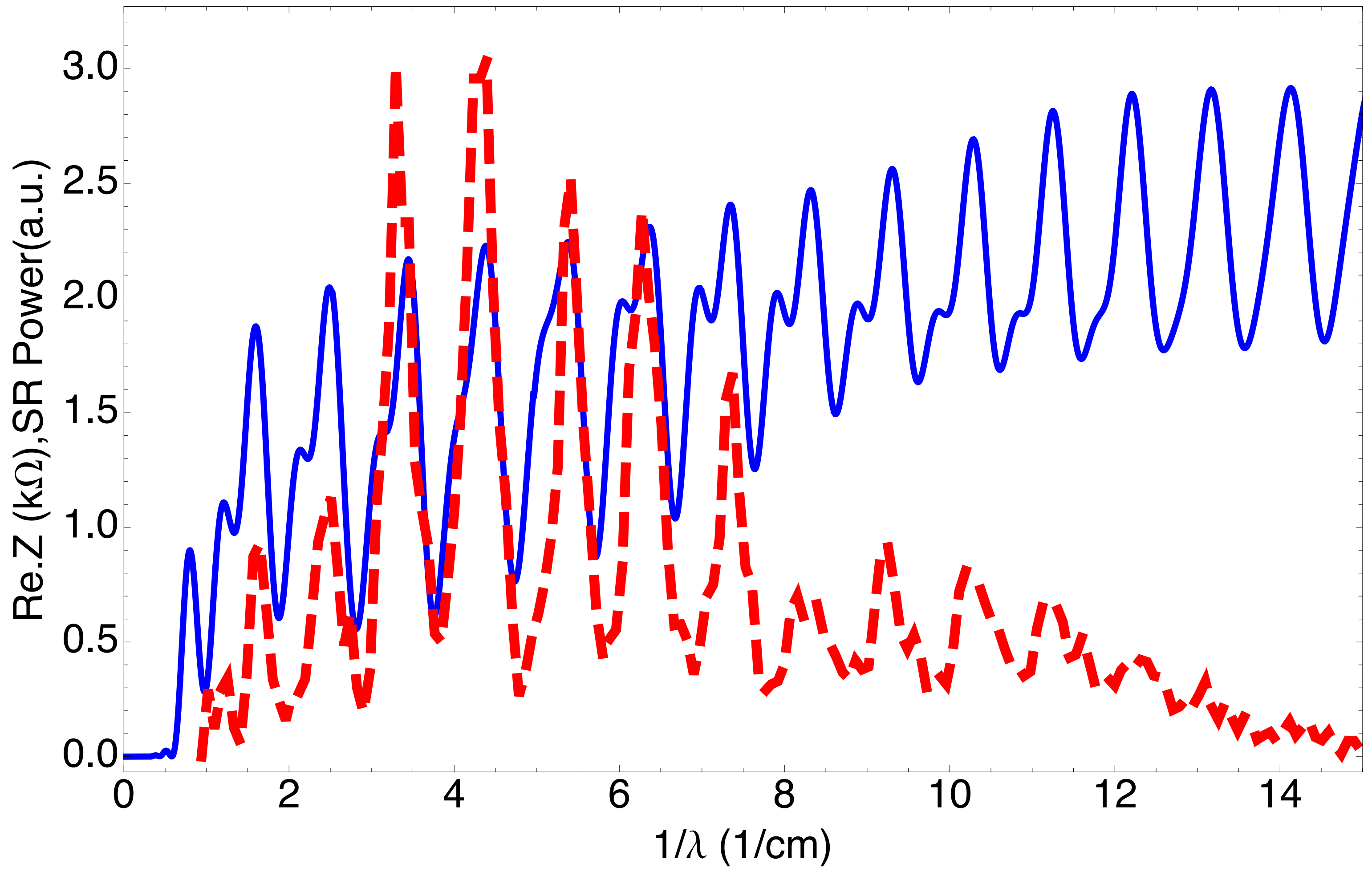


CSR impedance modelling

- Examples of CSR impedance by CSRZ
  - NSLS VUV as an example: a/b=80/42 mm, Lbend=1.5 m, R=1.91 m (Collaboration with S. Kramer)
  - Measured SR spectrum showed similar pattern of CSR impedance [6,8,10]. This is an evidence of multi-bend interference of CSR, or CSR in “whispering gallery modes”.







[1] D. Zhou, IPAC’12, MOOBB03. [2] R. Warnock, arXiv:1708.05500. [3] G. Carr et al., NIM-A 463, 387 (2001).

CSR impedance modelling

- Analytic theory compared with CSRZ calculations [1]
  - Single bend

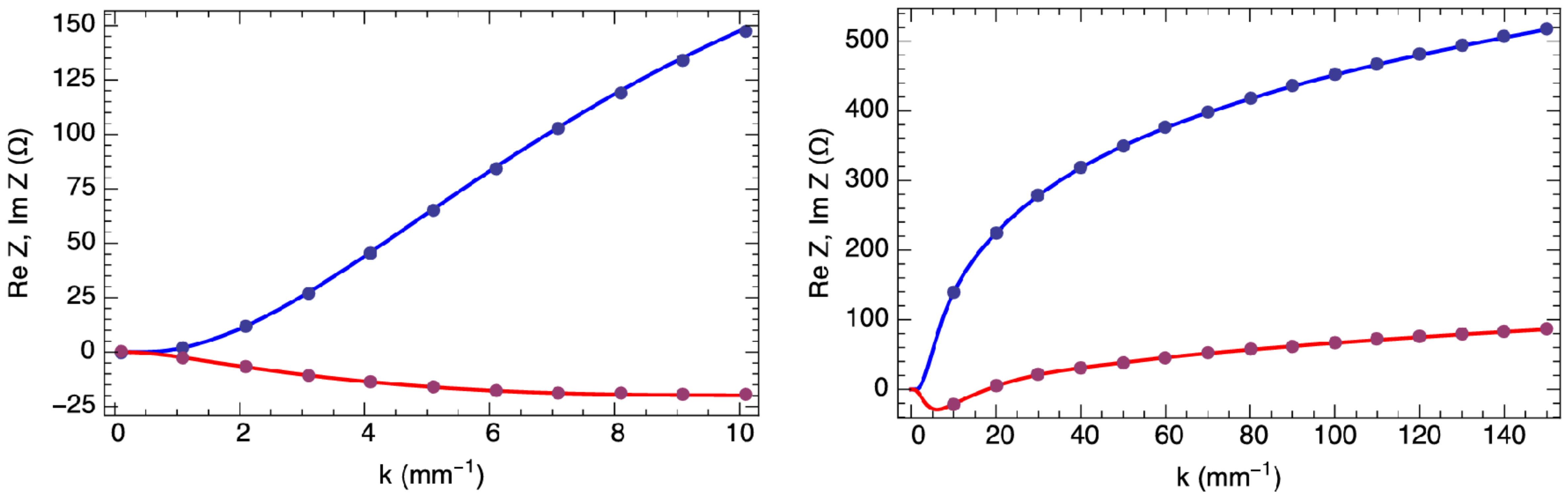
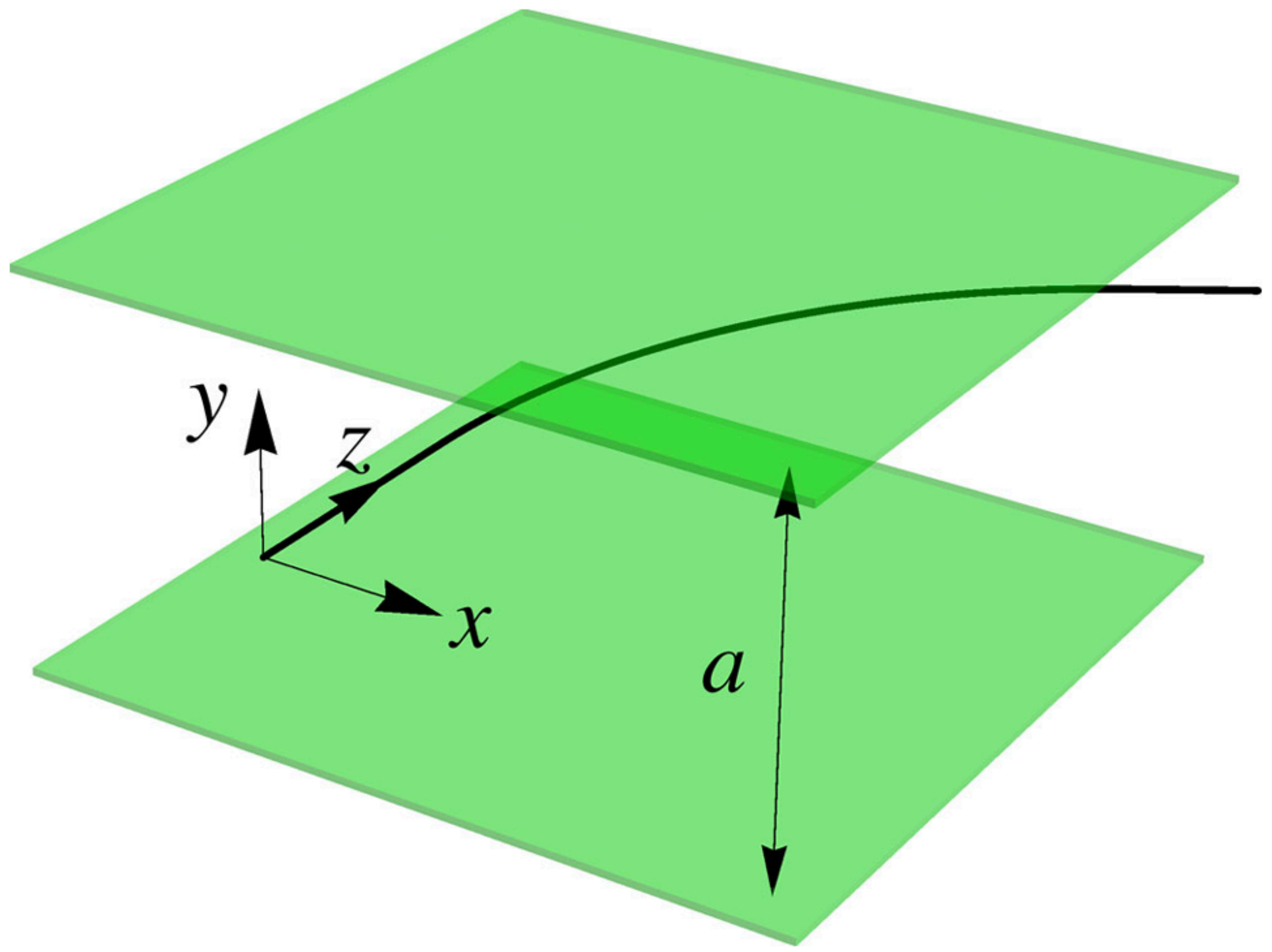


FIG. 7. Comparison of analytical calculations (shown by dots) with computer simulations (shown by solid lines): ReZ (blue) and ImZ (red).



  - Wiggler

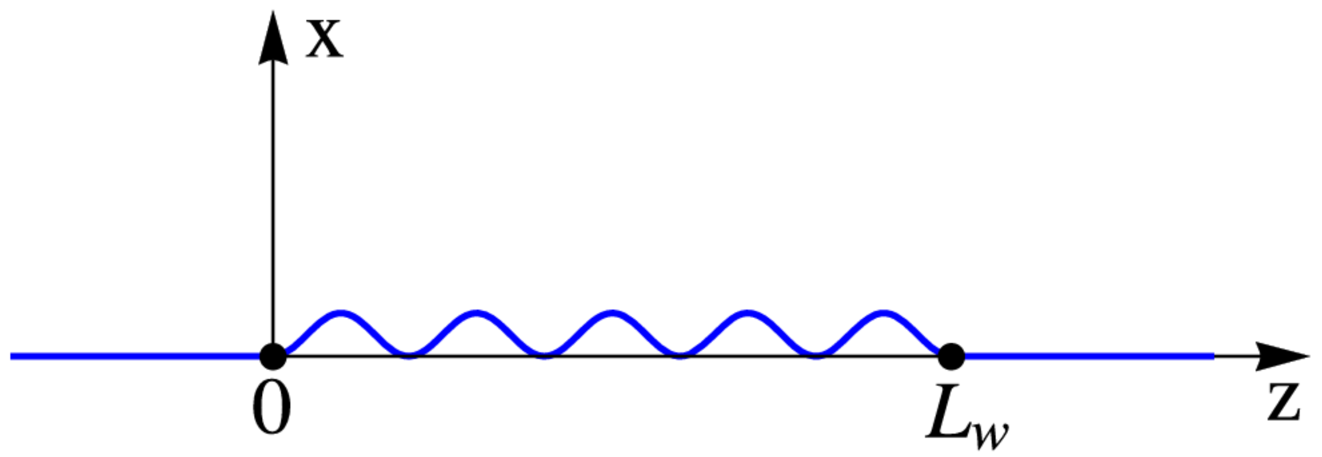
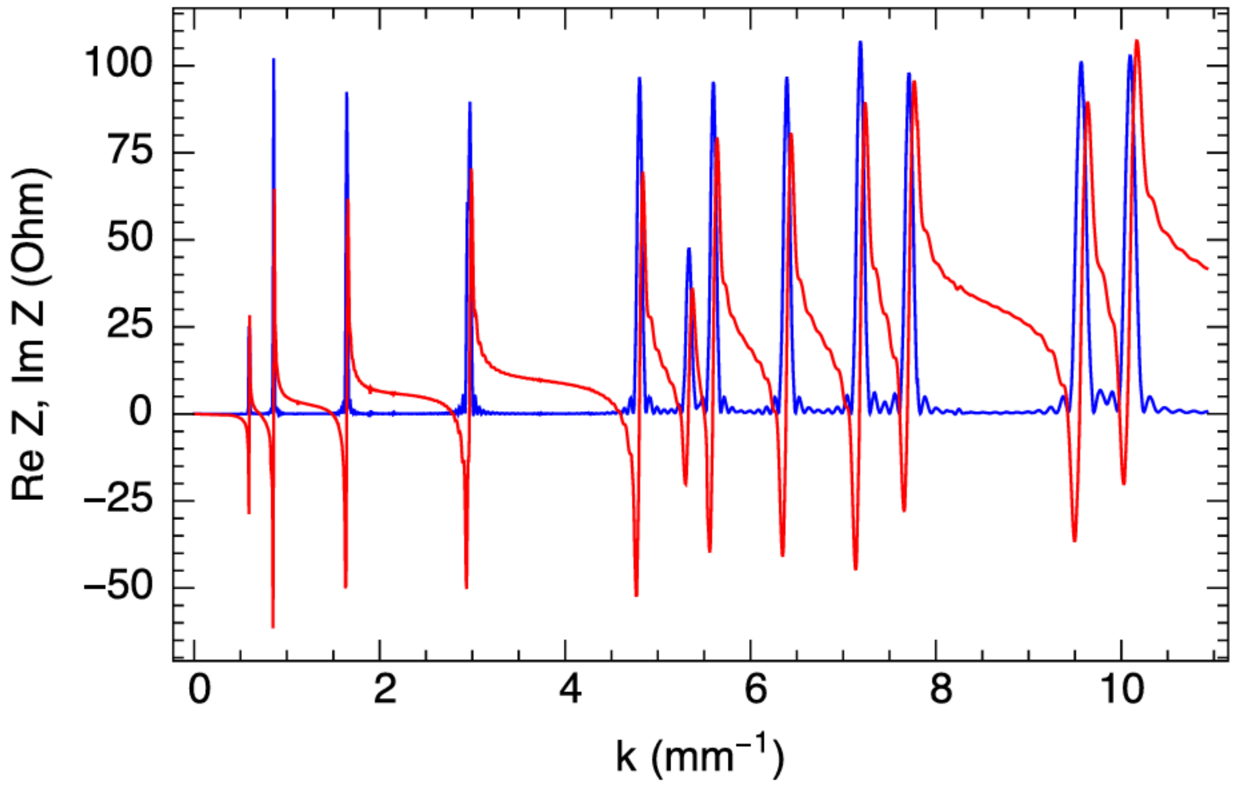
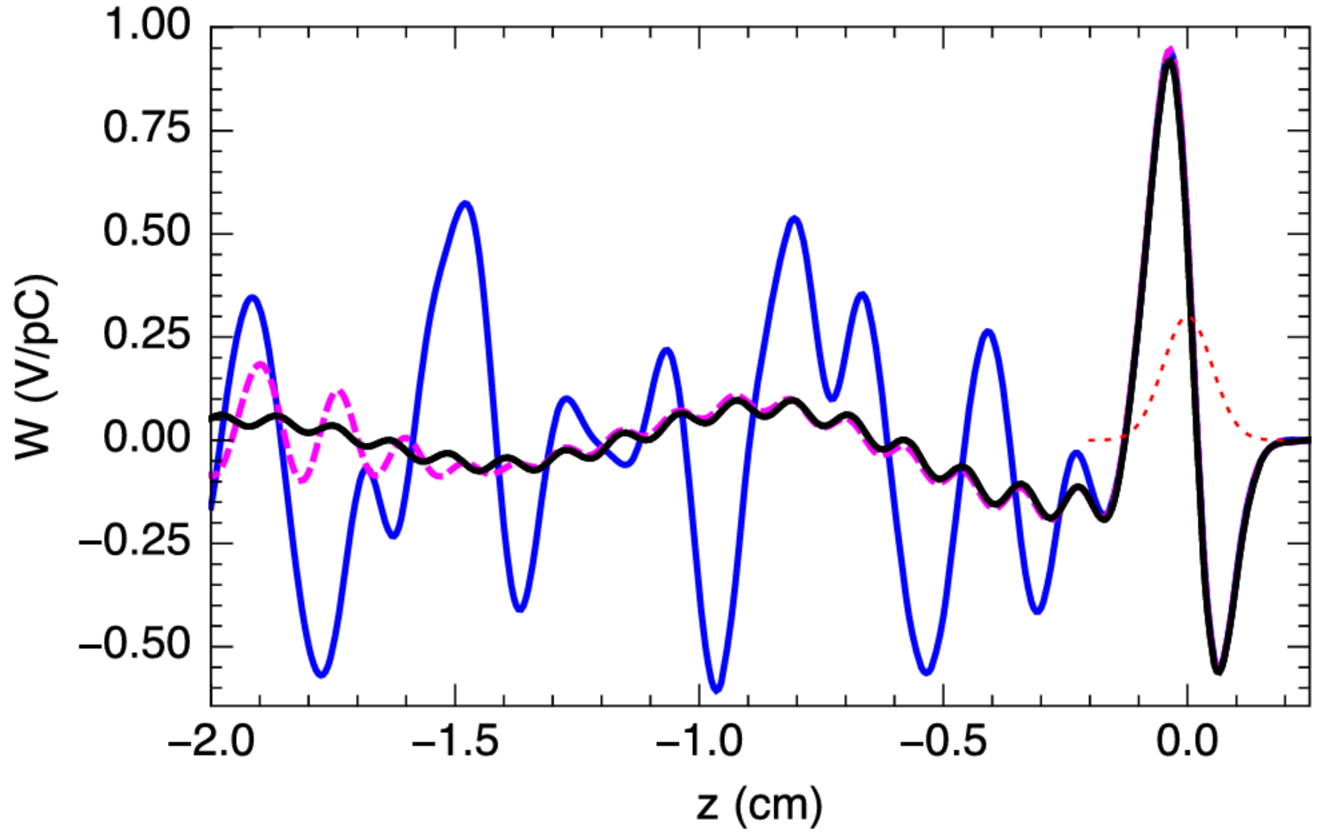


FIG. 8. Wiggler of length $L_w = N_w\lambda_w$ with the orbit shown by blue line.





[1] G. Stupakov and D. Zhou, PRAB 19, 044402 (2016).

CSR impedance modelling

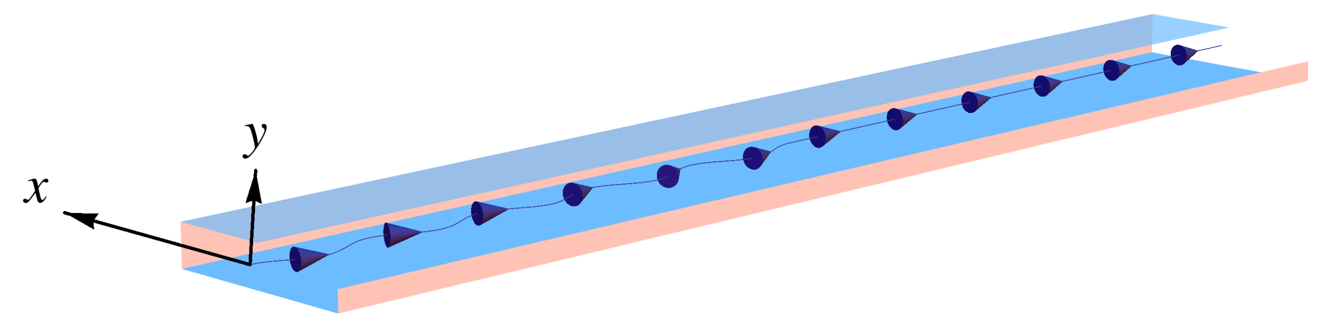
- Examples of CSR impedance by CSRZ
  - CSR in a wiggler/undulator: Coherent wiggler/undulator radiation (CWR or CUR).
  - The CWR spectrum can be calculated analytically (for example, see Refs.[1,2]):

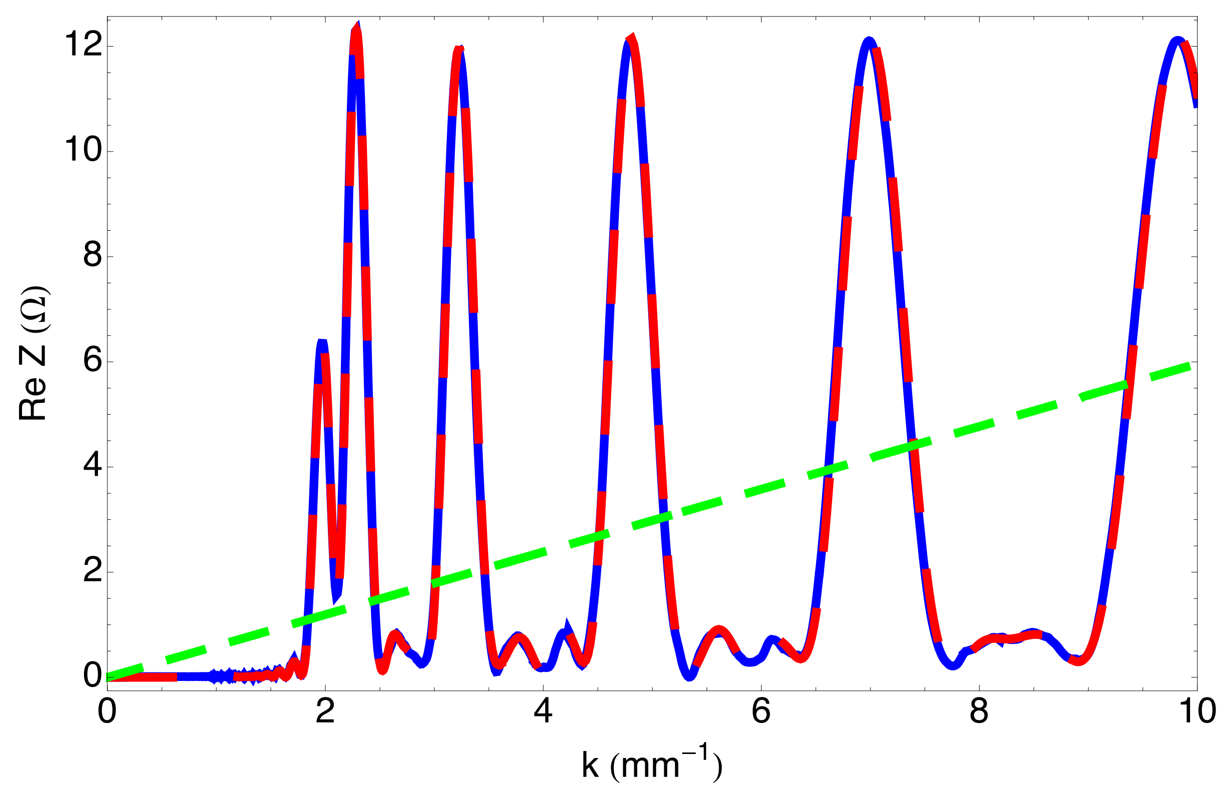
$$\mathrm{Re}\ Z(k) = \frac{4Z_0}{abR_0^2}\sum_{m=0}^{\infty}\sum_{p=1}^{\infty}\frac{k}{(1+\delta_{m0})k_z}\frac{\sin^2\left((k-k_z-k_w)L_w/2\right)}{(k-k_z)^2-k_w^2}$$

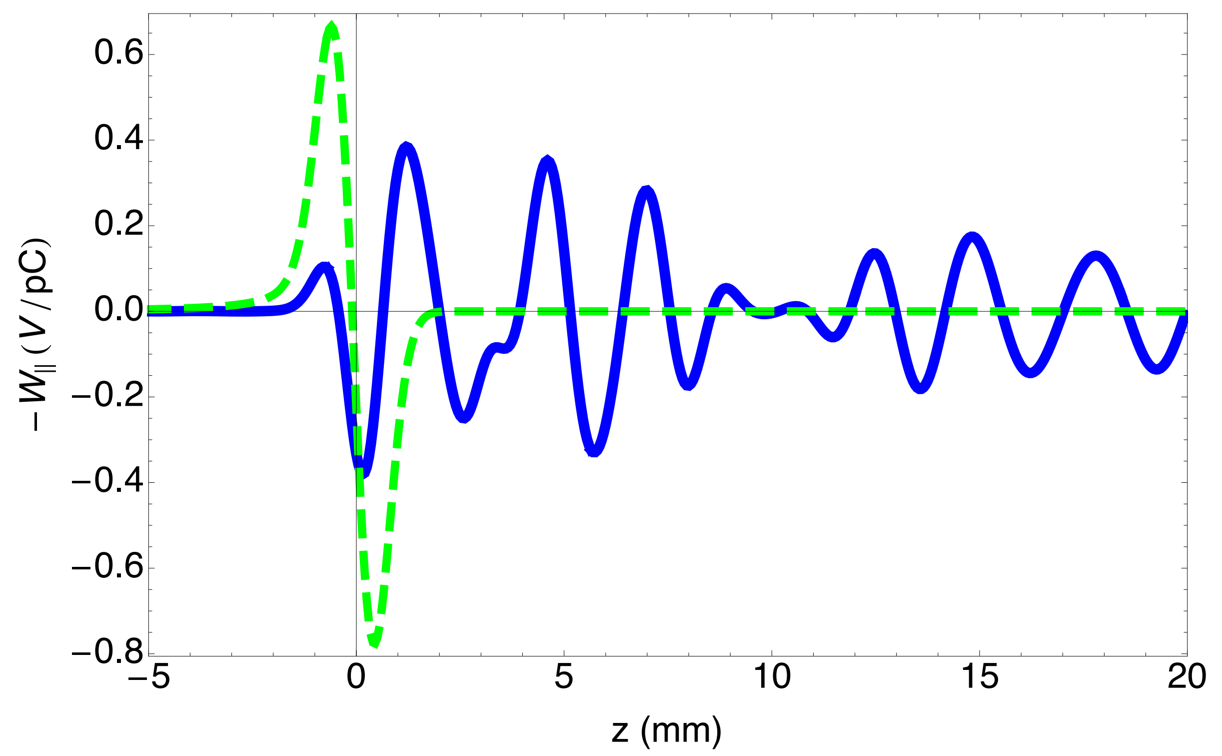
  - A weak wiggler: a/b=100/20 mm, λw=1 m, R0=100 m, N_{period}=10
  - Blue line by CSRZ; Red line by analytic theory with rectangular chamber; Green line by analytic theory in free space [3].

$$Z(k) = \frac{1}{4}Z_0 L_w k\frac{k_w}{k_0}\left(1-\frac{2i}{\pi}\left(\log\frac{4k}{k_0}+\gamma_E\right)\right)$$

  - For storage-ring light sources or THz FELs, it might be interesting to look at the interference of CUR + SC + RW.



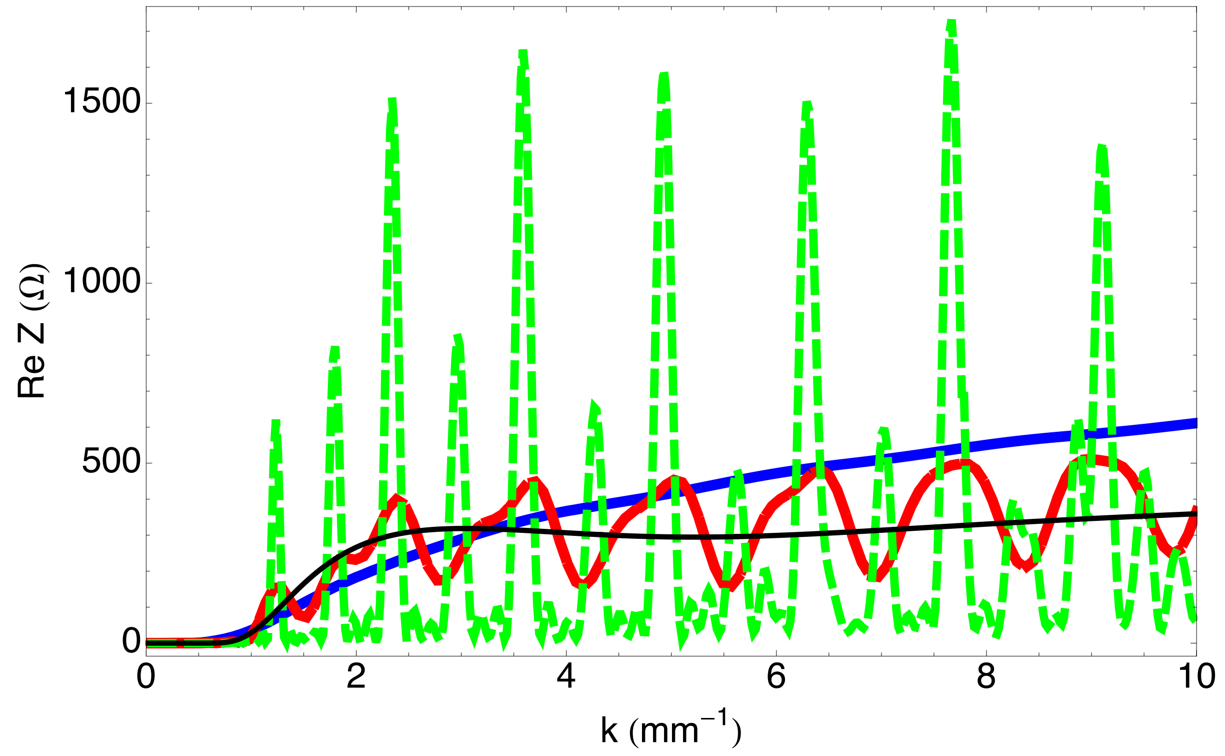


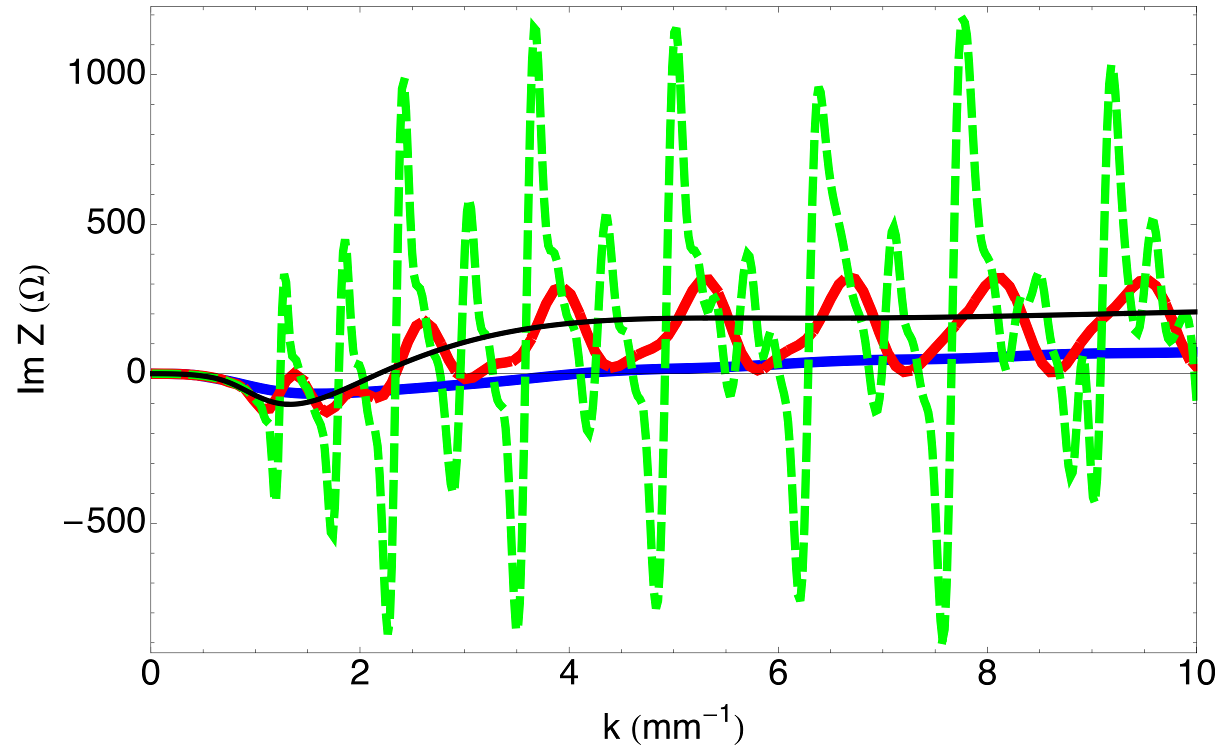


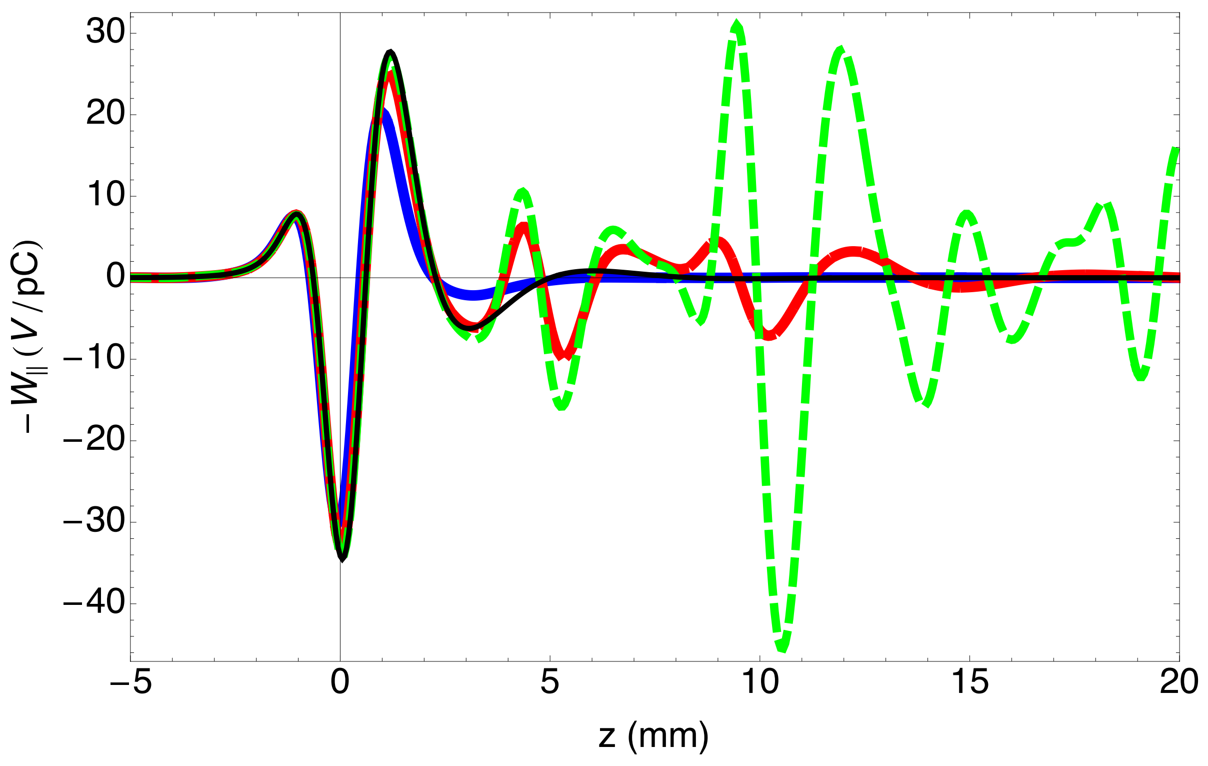
[1] Y. Chin, LBL-29981, 1990.
[2] G. Stupakov and D. Zhou, KEK Preprint 2010-43.
[3] J. Wu et al., Phys. Rev. ST Accel. Beams **6**, 040701 (2003).

CSR impedance modelling

- Examples of CSR impedance by CSRZ
  - A single bend with varied length: w/h=30/15 mm, R=5 m, L_{bend}=0.5/2/8 m.
  - Black/Blue/Red/Green lines: Steady-state parallel-plates/L=0.5/L=2/L=8 m. For convenience of comparison, the impedance amplitude is scaled to L=1 m.
  - “Short bend”: Transient effect at the entrance and exit is important.
  - “Long bend”: Excited eigenmodes of a toroidal chamber (or “whispering gallery modes” by R. Warnock [1]).
  - “Overtaking field”: Short-range wake fields, space charge like.
  - “Trailing field”: Long-range wake fields, relevant to excited eigenmodes.



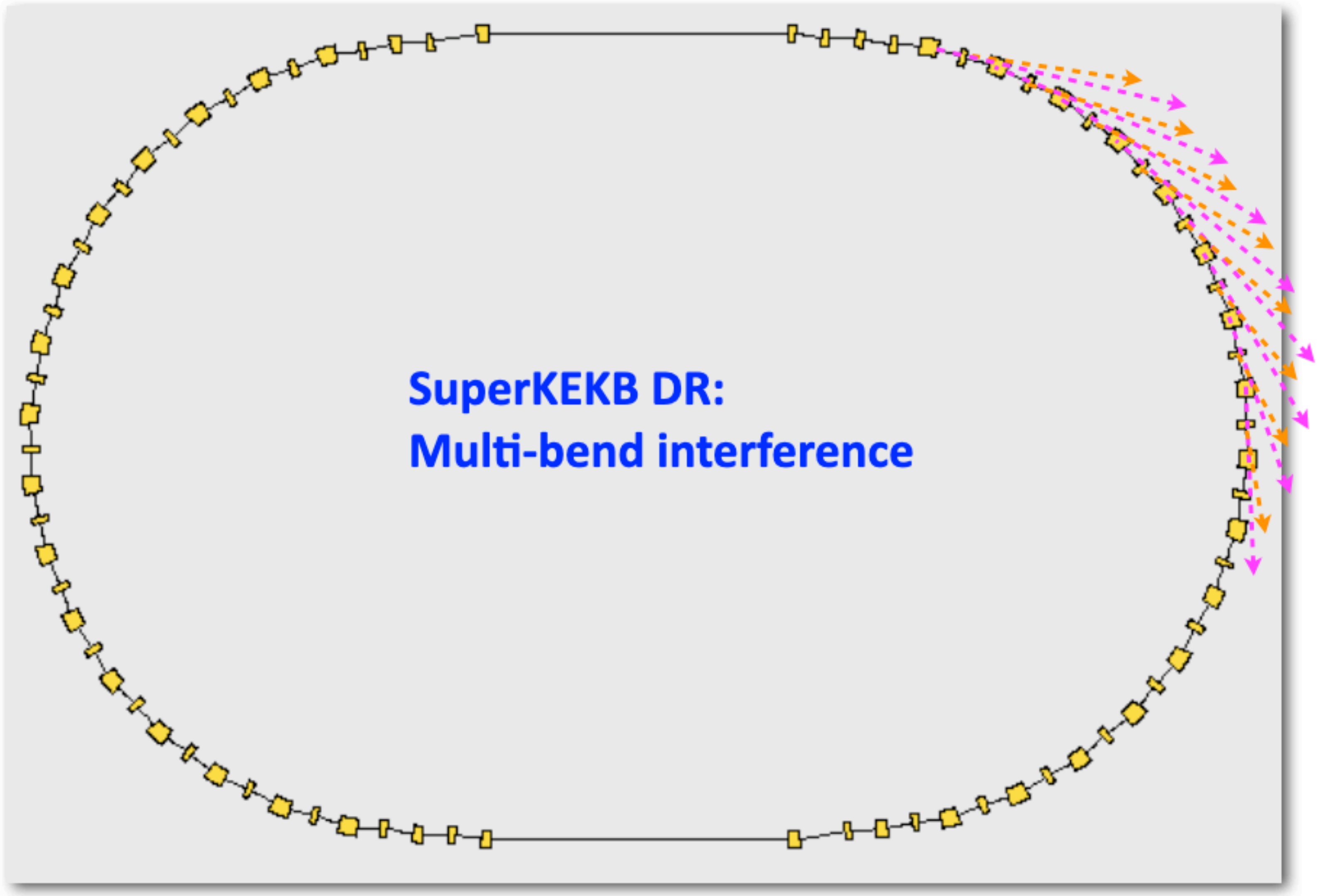


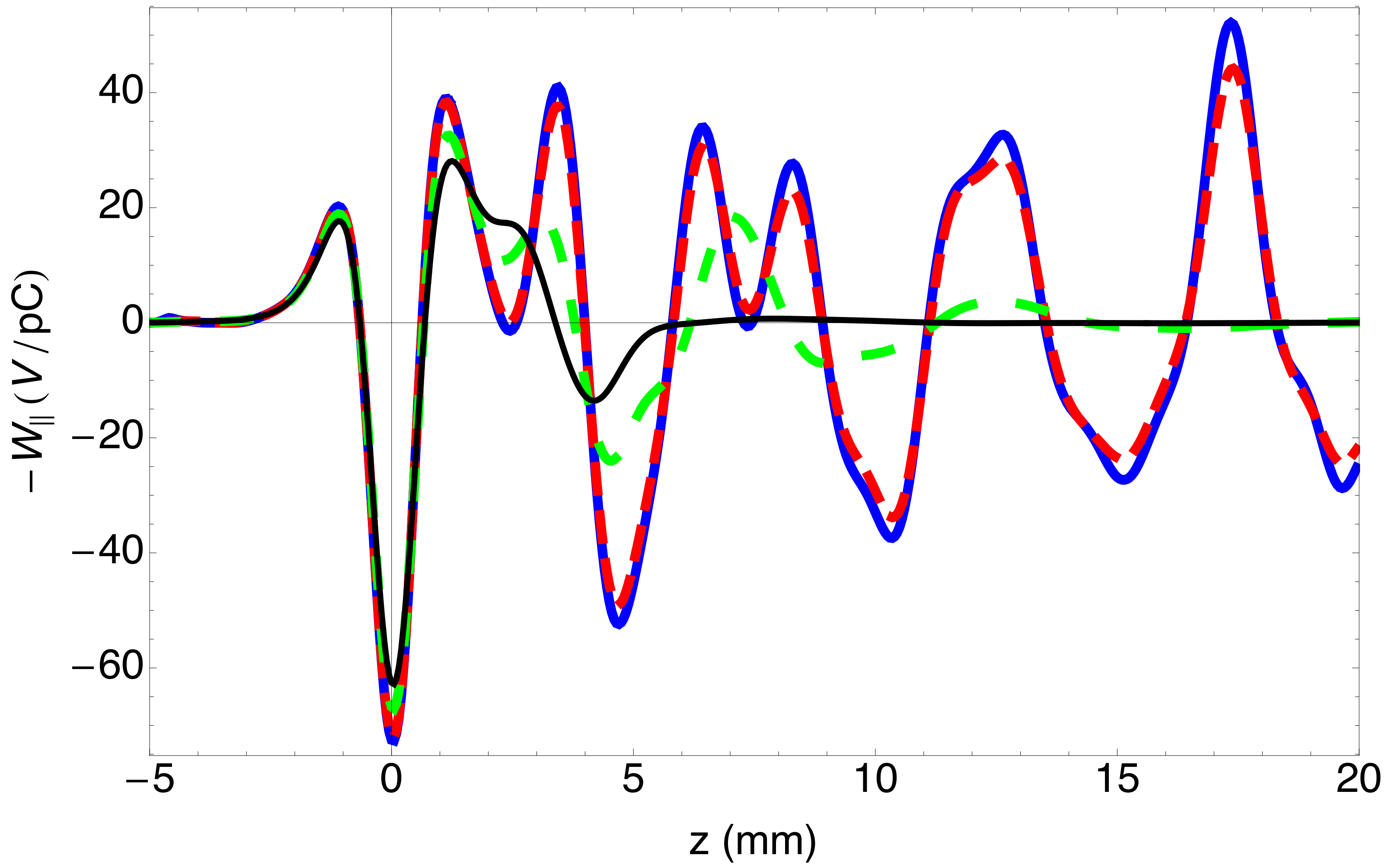


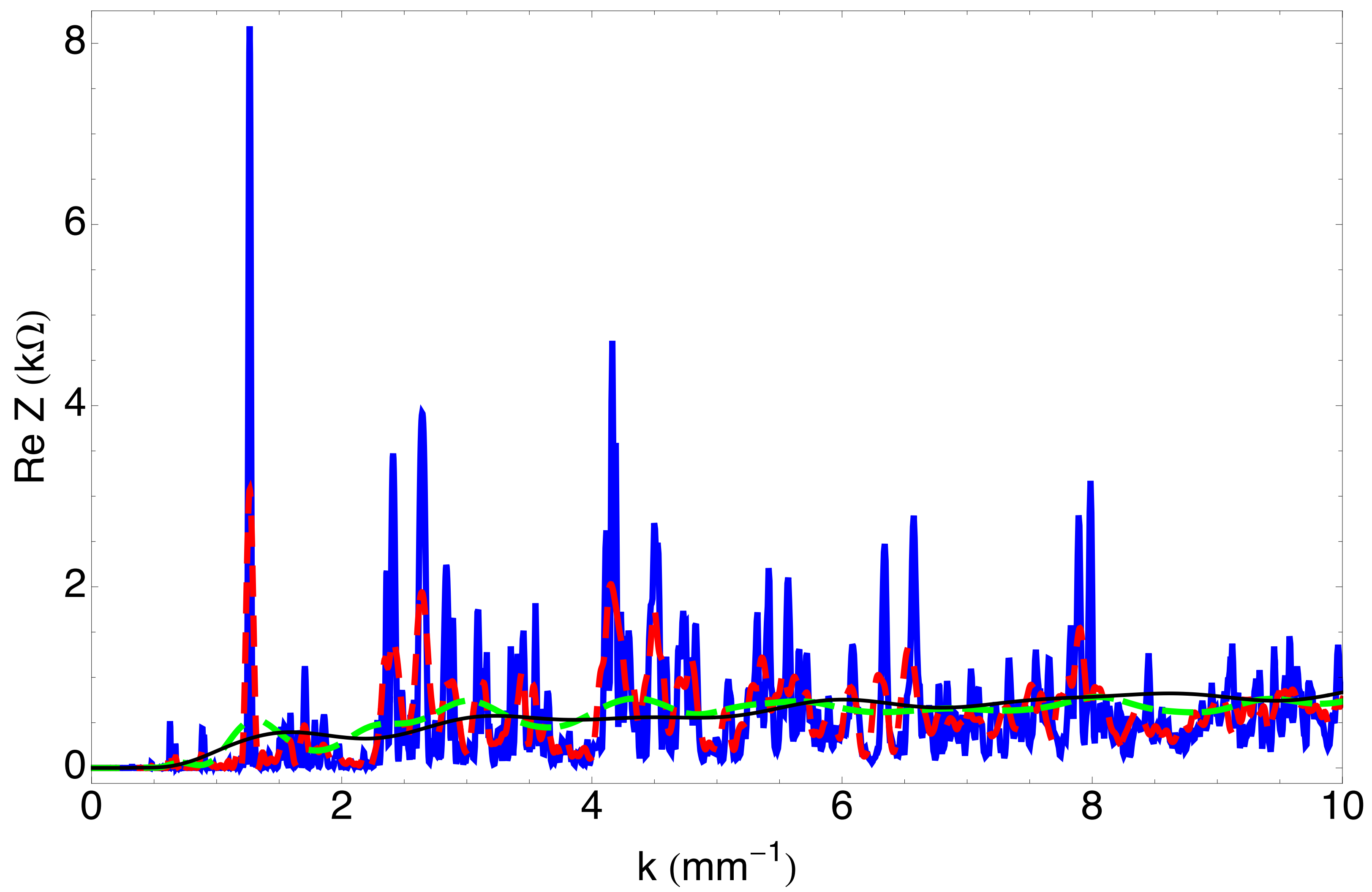
[1] R. Warnock, arXiv:1708.05500.

CSR impedance modelling

- Examples of CSR impedance by CSRZ
  - A realistic ring with multiple-bends: assuming smooth chamber.
  - SuperKEKB DR as an example [1]: a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends), L_{drift}=0.9 m, N_{cell}=1/6/16.
  - Multi-bend interference: CSR fields generated by multiple bends propagate along the chamber together with the beam. The fields interfere to produce a pattern of “narrow-band spikes”.
  - The real part of CSR impedance should correspond to SR spectrum in measurement.







[1] D. Zhou, et al., Jpn. J. Appl. Phys. 51 (2012) 016401.

CSR instability modelling

- Effective inductance for BBR [1,2]

$$L_{eff} = L_0 \Lambda_r(\Omega_r)$$

$$\Lambda_r(\Omega_r) = -\frac{2\sqrt{\pi}Q\Omega_r}{Q'}\left[\frac{\Omega_r Q'}{\sqrt{\pi}Q} + \mathrm{Im}\left[\Omega_1^2 w(\Omega_1)\right]\right]$$

$$L_0 = R_s/(Qk_r c)$$

$$\Omega_r = k_r \sigma_z$$

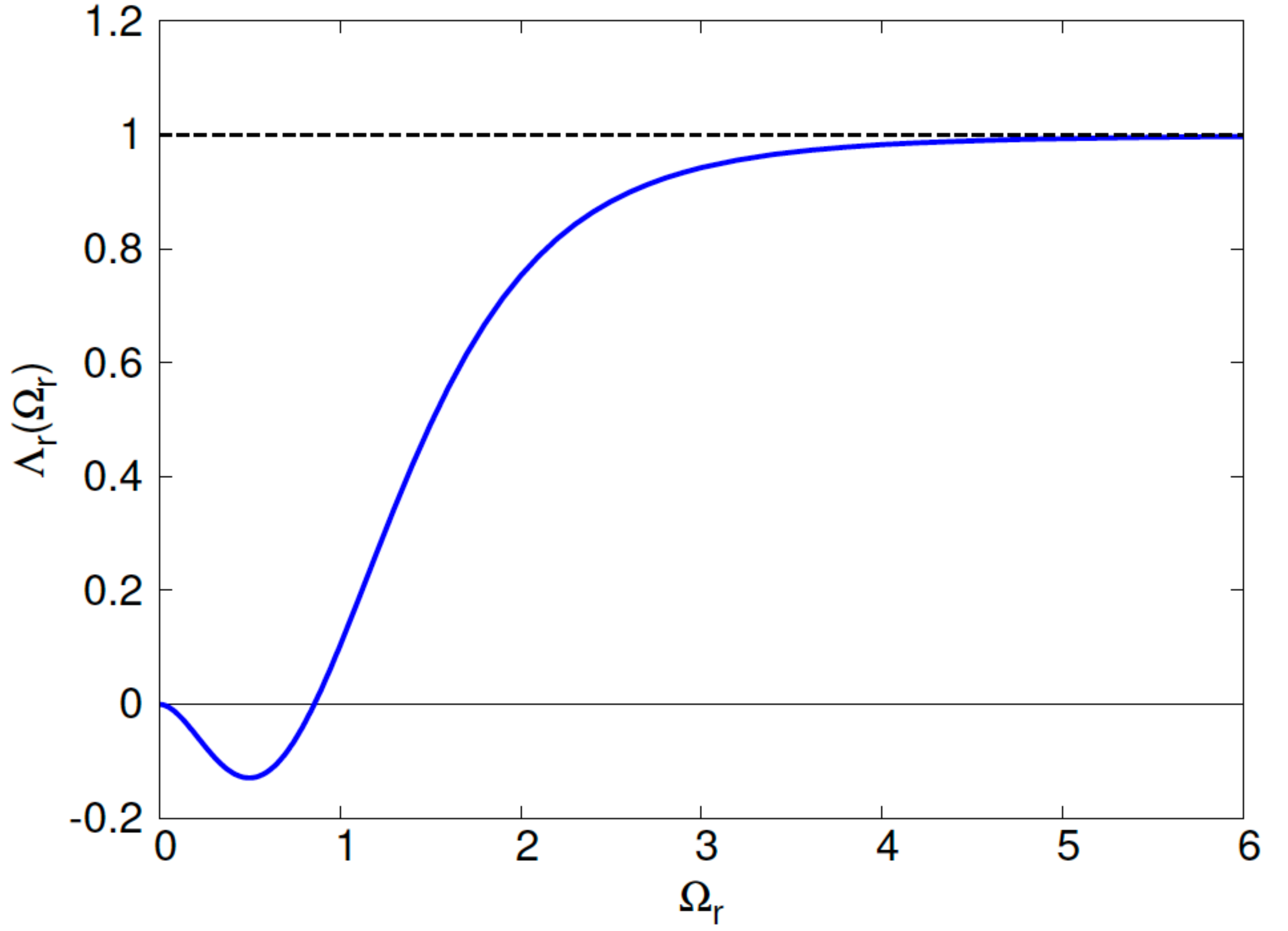


FIG. 16. $\Lambda_r(\Omega_r)$ for the effective inductance of a BBR impedance model. The dashed line indicates $\Lambda_r(\Omega_r)$=1 when $\Omega_r \to \infty$.

[1] S. Dastan, D. Zhou, et al., to be published. [2] D. Zhou, et al., NIM-A Volume 1063, 169243 (2024)