Necessary conditions to enable SSMB in storage rings

...with nonlinear phase slippage

Arnold Kruschinski on behalf of the SSMB collaboration

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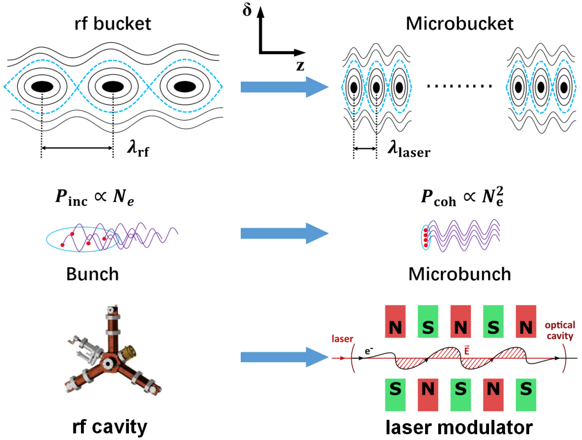
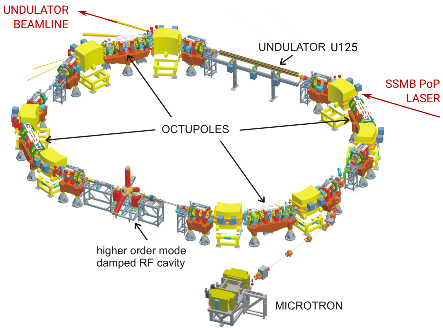
Table of Contents

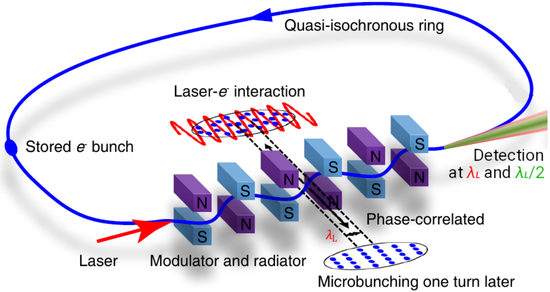
The idea of steady-state microbunching

- Goal: Coherent radiation from microbunched electron beams inside a storage ring
- Ingredients:
  - Longitudinal focusing at optical wavelengths
  - Ultra-low phase slippage
  - Suitable scheme to achieve bunch lengths $< 10\,\mathrm{nm}$
- Vision: kilowatt level EUV power for spectroscopy and lithography



The Metrology Light Source and the SSMB proof-of-principle experiment





Parameter	Symbol	User Op.	SSMB PoP
Circumference	C_0	48 m	48 m
Beam energy	E_0	629 MeV	250 MeV
Bunch charge	q_b	400 pC	< 10 pC
Phase slippage	η_0	0.03	$< 2 \cdot 10^{-5}$

Parameter	Symbol	Value
Undulator period	λ_u	125 mm
Number of periods	N	30
Laser wavelength	λ_L	1064 nm
Laser repetition rate		1.25 Hz
Laser pulse energy	E_L	400 mJ

A simple model for the microbunching process

One-turn maps for momentum deviation $\delta = \frac{\Delta p}{p_0}$ and longitudinal particle postion z:

$$\delta_{m+1} = \delta_m + A_m \sin\left(k_\mathrm{L} z_m\right),$$

$$z_{m+1} = z_m - C_0\, \eta\, \delta_{m+1}.$$

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Single shot modulation: energy modulation amplitude $A_0 = A$ for revolution number $m = 0$, $A_m = 0$ otherwise

Momentum compaction vs. phase slippage

Momentum compaction α:

$$\frac{\Delta C}{C_0} = \alpha \frac{\Delta p}{p_0}$$

Phase slippage η:

$$\frac{\Delta T}{T_0} = \eta \frac{\Delta p}{p_0}$$

$\searrow$ relation: $\eta = \alpha - \frac{1}{\gamma^2}$ $\swarrow$

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relation: $\eta = \alpha - \frac{1}{\gamma^2}$

$$\eta \approx \alpha$$

- MLS standard low-alpha: e^- @ 630 MeV: $\gamma^{-2} = 6.6 \cdot 10^{-7}$, $\alpha \approx 10^{-4}$

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- SSMB PoP:
  e^- @ 250 MeV:
  $\gamma^{-2} = 4.2 \cdot 10^{-6}$,
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- SSMB PoP: e^- @ 250 MeV: $\gamma^{-2} = 4.2 \cdot 10^{-6}$, $\alpha \approx 10^{-5}$

$\Rightarrow$ for SSMB, always use phase slippage!

$\rightarrow$ it is what we need to get the longitudinal slip Δz after one revolution:

$$\Delta z = -v\Delta T = -\beta c T_0 \eta \frac{\Delta p}{p_0}.$$

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$\rightarrow$ Calculate bunching factor from final distribution $\rho_m(z)$:

$$b_m(k) = \int_{-\infty}^{\infty} e^{-ikz}\, \rho_m(z)\, \mathrm{d}z = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-ikz_m(z_0,\delta_0)}\, \rho_0(z_0, \delta_0)\, \mathrm{d}z_0 \mathrm{d}\delta_0.$$

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→ Analytical formula can be derived under certain assumptions:

$$b_{n,m} \equiv b_m(nk_\mathrm{L}) = J_n(nmk_\mathrm{L}\eta_0 C_0 A) \cdot \exp\left[-\frac{(nk_\mathrm{L})^2}{2}\left\{(m\eta_0 C_0\sigma_\delta)^2 + 4\epsilon_{x,y}\mathcal{H}_{x,y}\sin^2(m\pi\nu_{x,y})\right\}\right]$$

Verifying the analytical formula in experiment

$$b_{n,m} \equiv b_m(nk_\mathrm{L}) = J_n(nmk_\mathrm{L}\eta_0 C_0 A) \cdot \exp\left[-\frac{(nk_\mathrm{L})^2}{2}\left\{(m\eta_0 C_0\sigma_\delta)^2 + 4\epsilon_{x,y}\mathcal{H}_{x,y}\sin^2(m\pi\nu_{x,y})\right\}\right]$$

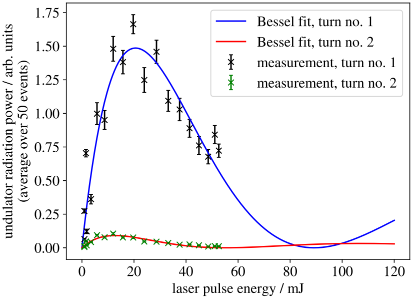
→ this formula has been shown experimentally to accurately predict the dependence of microbunching formation on a number of parameters.

→ Paper: A. Kruschinski, X. Deng et al., "Confirming the theoretical foudation of steady-state microbunching", *Communications Physics* **7**, 160 (2024)

Dependence on modulation amplitude

$$b_{n,m} \equiv b_m(nk_\mathrm{L}) = J_n(nmk_\mathrm{L}\eta_0 C_0 A) \cdot \exp\left[-\frac{(nk_\mathrm{L})^2}{2}\left\{(m\eta_0 C_0 \sigma_\delta)^2 + 4\epsilon_{x,y}\mathcal{H}_{x,y}\sin^2(m\pi\nu_{x,y})\right\}\right]$$

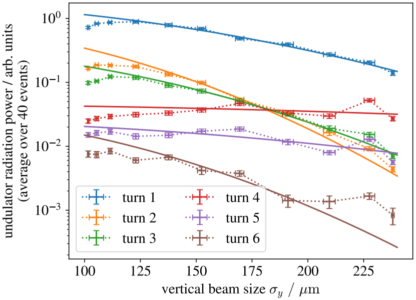
- Modulation amplitude proportional to laser electric field: $A \propto \mathcal{E}_\mathrm{L} \propto \sqrt{P_\mathrm{L}} \propto \sqrt{E_\mathrm{L}}$
- (Total laser pulse energy proportional to instantaneous power for constant pulse shape)
- Coherent radiation power $P_\mathrm{coh} \propto |b|^2$
- $\Rightarrow P_\mathrm{coh} = a_1 \left|J_1\left(a_2\sqrt{E_\mathrm{L}}\right)\right|^2$



Dependence on transverse-longitudinal coupling

$$b_{n,m} \equiv b_m(nk_\mathrm{L}) = J_n(nmk_\mathrm{L}\eta_0 C_0 A) \cdot \exp\left[-\frac{(nk_\mathrm{L})^2}{2}\left\{(m\eta_0 C_0\sigma_\delta)^2 + 4\epsilon_{x,y}\mathcal{H}_{x,y}\sin^2(m\pi\nu_{x,y})\right\}\right]$$

- Transverse-longitudinal coupling (TLC) can disrupt microbunching formation
- Impact proportional to transverse emittance
- Example on the right for the vertical plane; MLS tune: $\nu_y = 2.23$
- Impact of TLC is reduced if $m \cdot \nu_y$ is close to an integer



Verifying the analytical formula in experiment

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⇒ **But:** it assumes momentum-independent phase slippage $\eta = \eta_0 =$ const., and cannot describe impact of nonlinear phase slippage $\eta(\delta)$.

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$$z_{m+1} = z_m - C_0\, \eta\, \delta_{m+1}.$$

$\rightarrow$ Instead of analytical formula, use simulation with nonlinear phase slippage

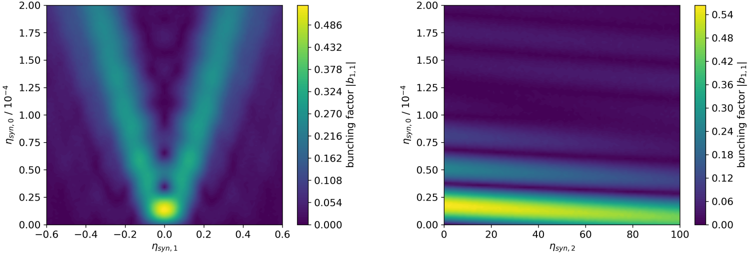
$$\eta(\delta) = \eta_0 + \eta_1\, \delta + \eta_2\, \delta^2 + \dots$$

and calculate bunching factor from the final distribution using DFT.

Simulation results: Microbunching with nonlinear dynamic phase slippage

Simulation set up with a uniform particle distribution around $\delta = 0$.

Results: bunching factor at first laser harmonic, one turn after modulation vs. first three orders of dynamic phase slippage:



$\Rightarrow$ Optimal bunching factor at $\eta_{\text{syn},1} = 0$ and suitable value of $\eta_{\text{syn},0} \approx 2 \cdot 10^{-5}$, depending on modulation amplitude. Value of $\eta_{\text{syn},2}$ is uncritical as long as it is not too large.

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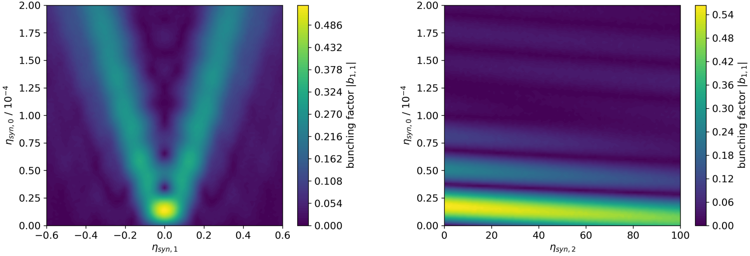

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⇒ What is “dynamic phase slippage” η_{syn}?

How to define momentum-dependent phase slippage

$$\frac{\Delta T}{T_0} = \eta(\delta)\,\delta, \quad \text{with } \delta = \frac{\Delta p}{p_0}$$

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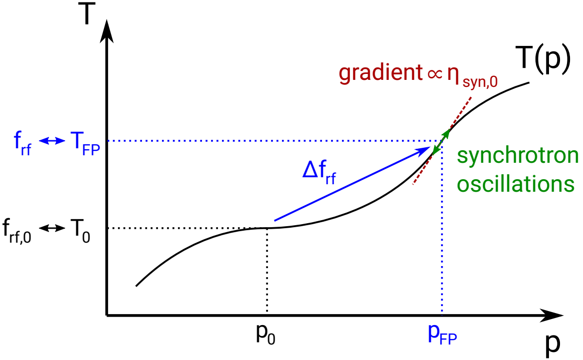
⇒ Momentum of reference particle with orbit through centers of all magnets. But this orbit is only stationary if the rf frequency matches!

A step back: look at absolute function $T(p)$:



⇒ To deal with changes of fixed point momentum, define a dynamic phase slippage relative to the fixed point momentum p_{FP}.

Defining static and dynamic phase slippage

Static reference momentum p_0 from static reference rf frequency $f_{\mathrm{rf},0}$:

$$T_0 = T(p_0) = \frac{h}{f_{\mathrm{rf},0}}.$$

$\Rightarrow$ Static phase slippage η:

$$\frac{T - T_0}{T_0} = \eta \cdot \frac{p - p_0}{p_0}.$$

Reference momentum p_{FP} from rf frequency f_{rf} (which may be varied):

$$T_{\mathrm{FP}} = T(p_{\mathrm{FP}}) = \frac{h}{f_{\mathrm{rf}}}.$$

$\Rightarrow$ Dynamic phase slippage η_{syn}:

$$\frac{T - T_{\mathrm{FP}}}{T_{\mathrm{FP}}} = \eta_{\mathrm{syn}} \cdot \frac{p - p_{\mathrm{FP}}}{p_{\mathrm{FP}}}$$

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$$\frac{T - T_{\text{FP}}}{T_{\text{FP}}} = \eta_{\text{syn}} \cdot \frac{p - p_{\text{FP}}}{p_{\text{FP}}}$$

$\Rightarrow$ Calculate fixed point momentum shift δ_{FP} from static phase slippage and rf frequency:

$$\eta \cdot \delta_{\text{FP}} \equiv \eta \cdot \frac{p_{\text{FP}} - p_0}{p_0} = \frac{T_{\text{FP}} - T_0}{T_0} = \frac{f_{\text{rf}}^{-1} - f_{\text{rf},0}^{-1}}{f_{\text{rf},0}^{-1}} \approx -\frac{f_{\text{rf}} - f_{\text{rf},0}}{f_{\text{rf},0}}.$$

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$\Rightarrow$ Dynamic phase slippage η_{syn}:

$$\frac{T - T_{\mathrm{FP}}}{T_{\mathrm{FP}}} = \eta_{\mathrm{syn}} \cdot \frac{p - p_{\mathrm{FP}}}{p_{\mathrm{FP}}}$$

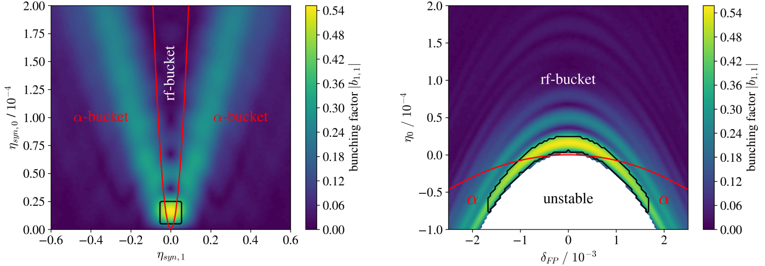
$\Rightarrow$ Dynamic phase slippage can be calculated from static phase slippage for any value of δ_{FP}:

$$\eta_{\mathrm{syn}}(\delta_{\mathrm{syn}}, \delta_{\mathrm{FP}}) = \frac{1}{1 + \eta(\delta_{\mathrm{FP}})\,\delta_{\mathrm{FP}}} \sum_{i=1}^{\infty} \frac{1}{i!} (1 + \delta_{\mathrm{FP}})^i \left[\frac{\mathrm{d}^i}{\mathrm{d}\delta^i}\, \eta(\delta)\,\delta\right]_{\delta=\delta_{\mathrm{FP}}} \delta_{\mathrm{syn}}^{i-1}$$

Transforming the simulation results to static phase slippage

Directly accessible in the experiment is not the dynamic phase slippage, but only the static phase slippage (which can be manipulated with multipole magnets) and the fixed point momentum (via changes to the rf frequency).

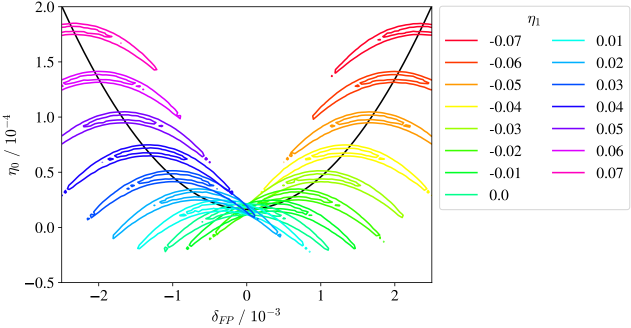
Transform the local but general simulation results from dynamic to static phase slippage:



→ For the plot on the right, $\eta_1 = 0$ and $\eta_2 = 10$ are assumed. Different values of η_2 would change the curvature of the parabolic shapes.

Transforming the simulation results to static phase slippage

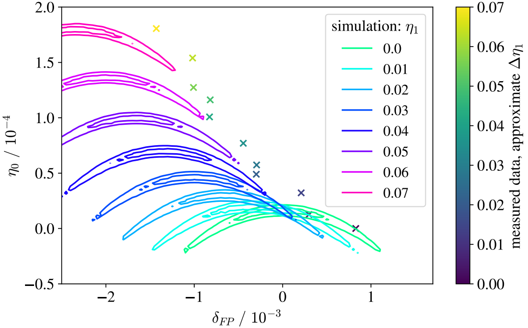
For different values of η_1, the area of maximum bunching factor (depicted is $|b| > 0.5$) does not change shape but only moves along a parabola in the η_0-δ_{FP} plane:



$\Rightarrow$ Changes of η_1 away from the optimum can be compensated by adjusting η_0 and δ_{FP}!

Experimental confirmation

In the experiment at the MLS, sextupoles are used to manipulate η_1. η_0 (via quadrupoles) and δ_{FP} (via rf frequency) are adjusted to recover microbunching: This is possible over a wide range of η_1, and the necessary changes also quantitatively align with the simulation results:



(the horizontal displacement of experimental points can be explained by a systematic offset.)

Conclusion

- Concept of static and dynamic phase slippage is useful not only for SSMB, also for any other applications of extreme low-alpha

- Microbunching process shows complex dynamics with a nonlinear phase slippage function
- Results obtained from simulation have been validated in experiment
- Insights into the general behavior of the SSMB PoP experiment were gained, important for preparation of next phase

- Important for any future SSMB storage ring:
  - Accurate control over higher orders of phase slippage (sextupoles, octupoles)
  - Stable conditions (minimize fluctuation of η_0, η_1, best with permanent magnets)

Thank you for you attention!

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