Radiative Decays

$$\pi^+ \to \mu^+ \nu \gamma$$
$$\pi^+ \to e^+ \nu \gamma$$
$$\mu^+ \to e^+ \nu \bar{\nu} \gamma$$

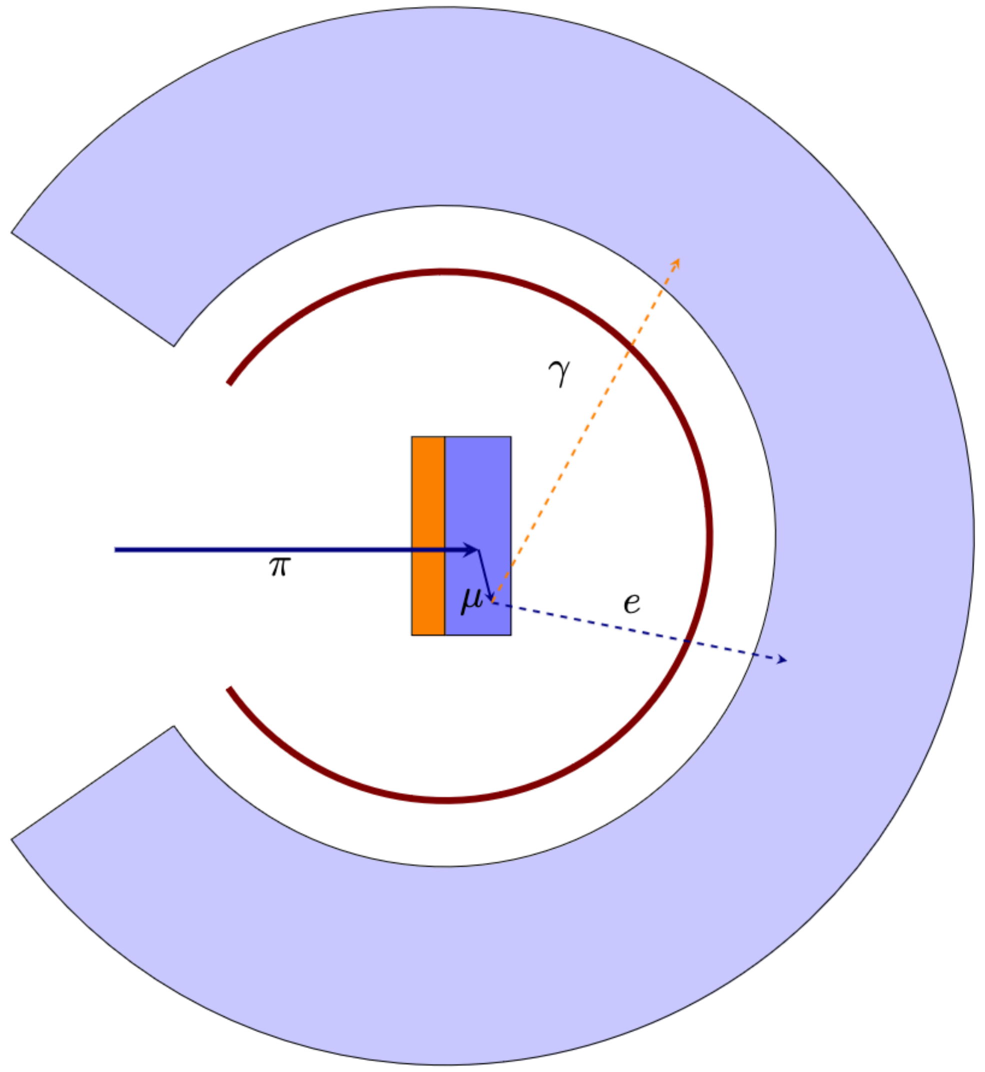
Patrick Schwendimann 19. June 2024

Why do we care about Radiative Decays?

Pion Decays:
Measure inclusive Branching Ratio

$$R_{e\mu} = \frac{\Gamma\left(\pi \to e\nu(\gamma)\right)}{\Gamma\left(\pi \to \mu\nu(\gamma)\right)}$$

Muon Decays:
Positron and Photon deposits can total up to $m_\mu \approx 106$ MeV if both observed.
→High Bin Background



Radiative Michel Decay

- $\pi \to \mu\nu\gamma$
  BR 2e-4 with $E_\gamma > 1$ MeV,
  G. Bressi et al.
- $\pi \to e\nu\gamma$
  BR 7.4e-7, $E_\gamma > 10$ MeV, $\alpha_{e\gamma} > 40°$
  PiBETA
- $\mu \to e\nu\nu\gamma$
  BR 6.03e-8,
  $p_e > 45$ MeV/c, $E_\gamma > 40$ MeV
  MEG
  BR: 4.365e-3, $E_\gamma > 10$ MeV, $\alpha_{e\gamma} > 30°$
  PiBETA
  https://arxiv.org/abs/1705.03782

Radiative Pion Decays

Replacing the insufficient Geant4 Generator

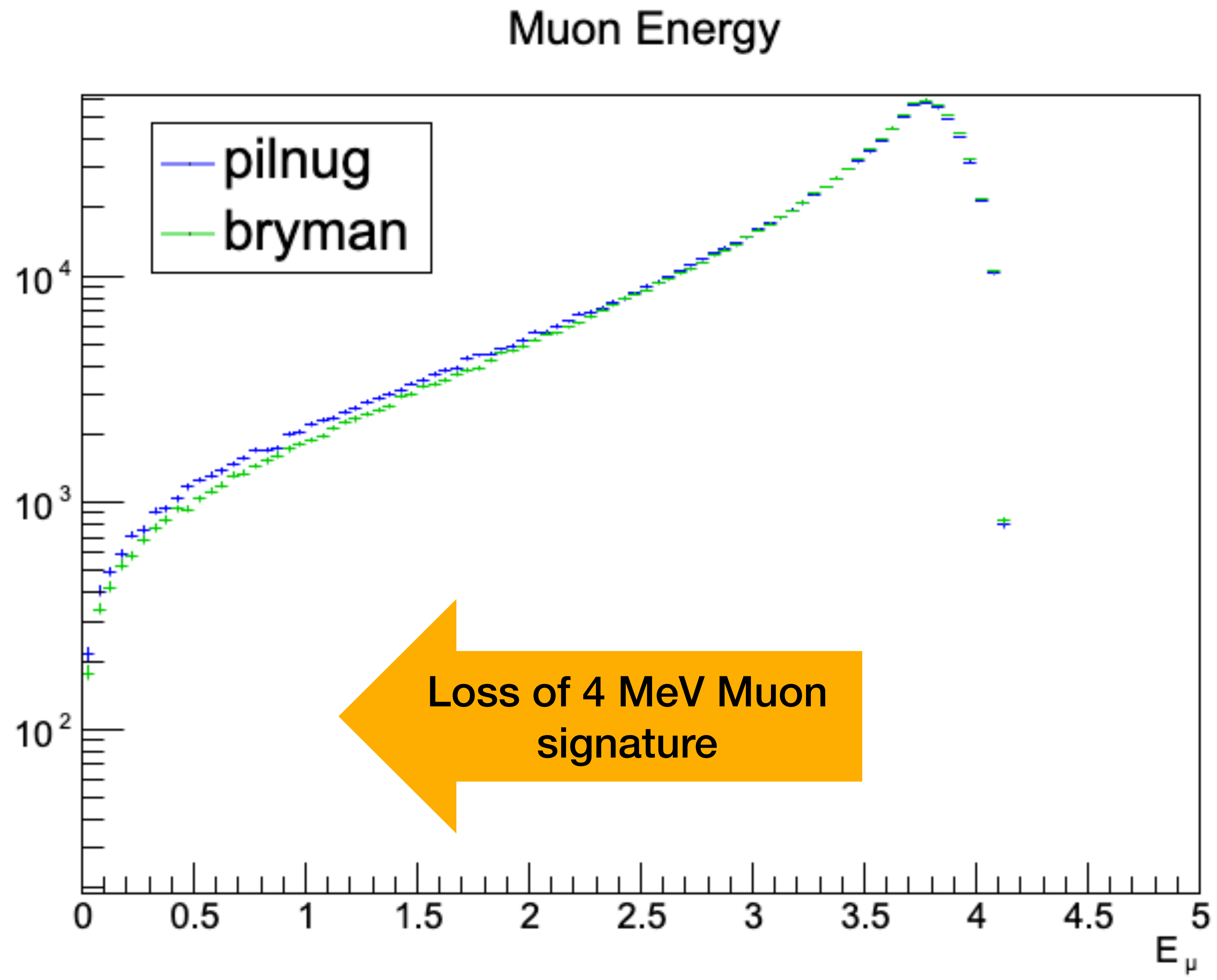
Luckily, other people already encountered radiative pion decays and shared the fruits of their labour:

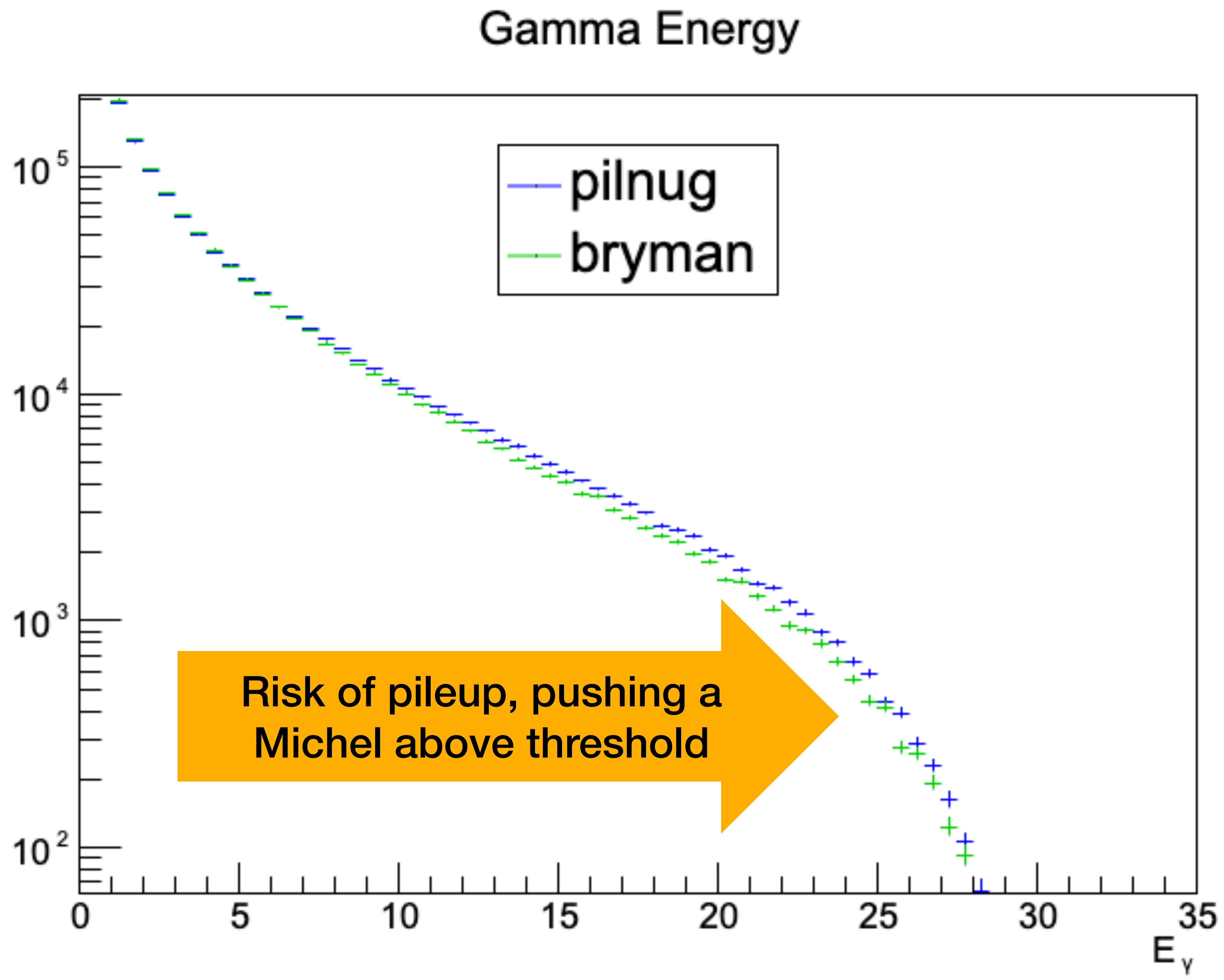
- `bryman`: based on https://doi.org/10.1016/0370-1573(82)90162-4
  It is internal bremsstrahlung (IB) only for $\pi \to \mu\nu\gamma$ and contains structure dependent (SD) and interference terms for $\pi \to e\nu\gamma$
  The case of $\pi \to \mu\nu\gamma$ was already reported here:
  https://pioneer.npl.washington.edu/cgi-bin/private/ShowDocument?docid=257

- `pilnug`: Based on (one of the) Fortran codes Richard Mischke sent me from their experiences with NA62 and Pienu. It contains IB, SD and interference terms as well as radiative corrections for $\pi \to e\nu\gamma$ and $\pi \to \mu\nu\gamma$.

Radiative Pion Decay to Muon

$\pi \to \mu\nu\gamma$, BR 2e-4

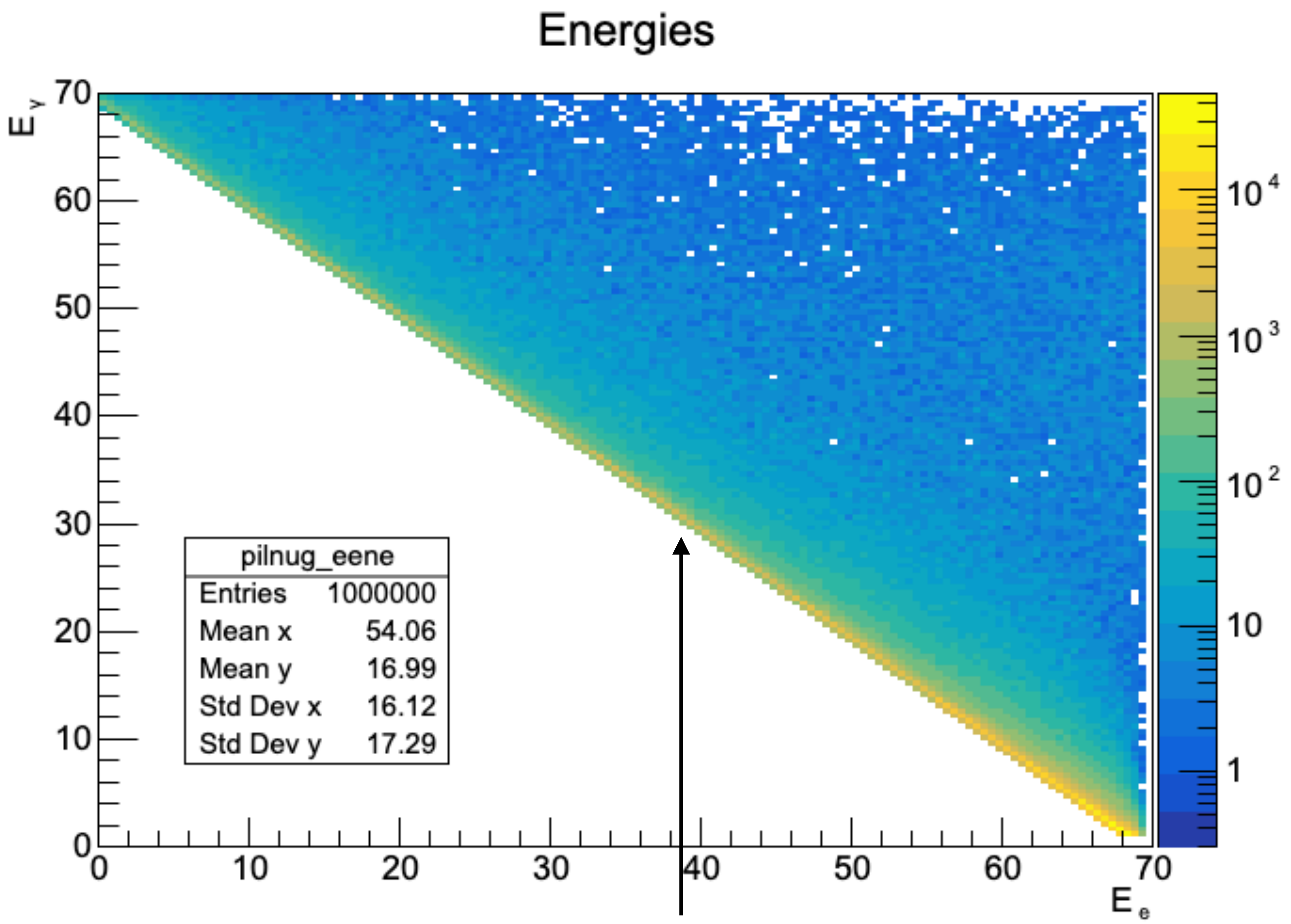




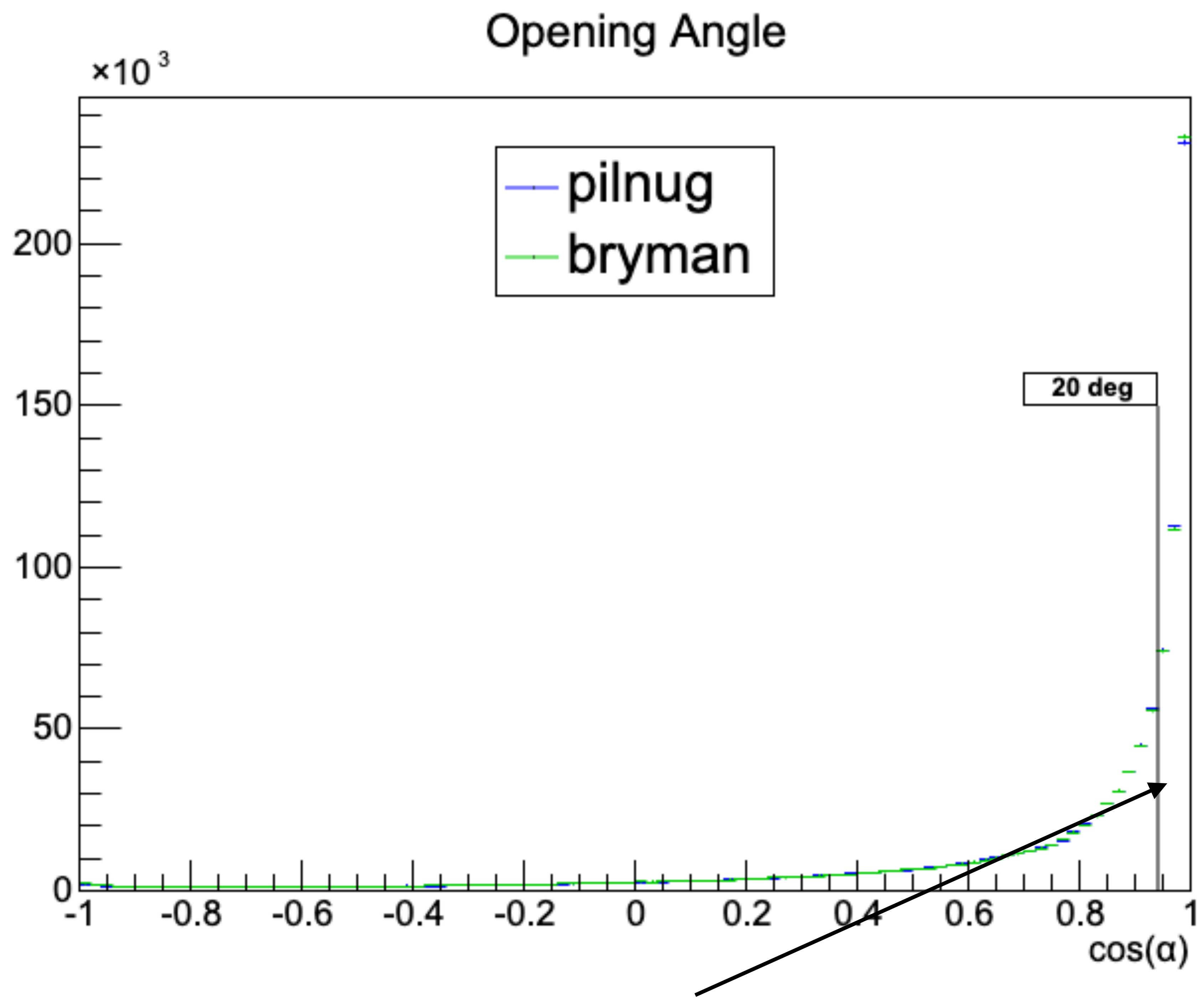
Both generators provide similar distributions.
Events that could foil our analysis strategy are few.

Radiative Pion Decay to Positron

$\pi \rightarrow e\nu\gamma$, BR: 7.4e-7



$E_\gamma + E_e \geq 70$ MeV



$\vec{p}_\gamma, \vec{p}_e$ perferrably collinear

Good chance to observe both particles and thus have a combined energy that ends up in the high bin.

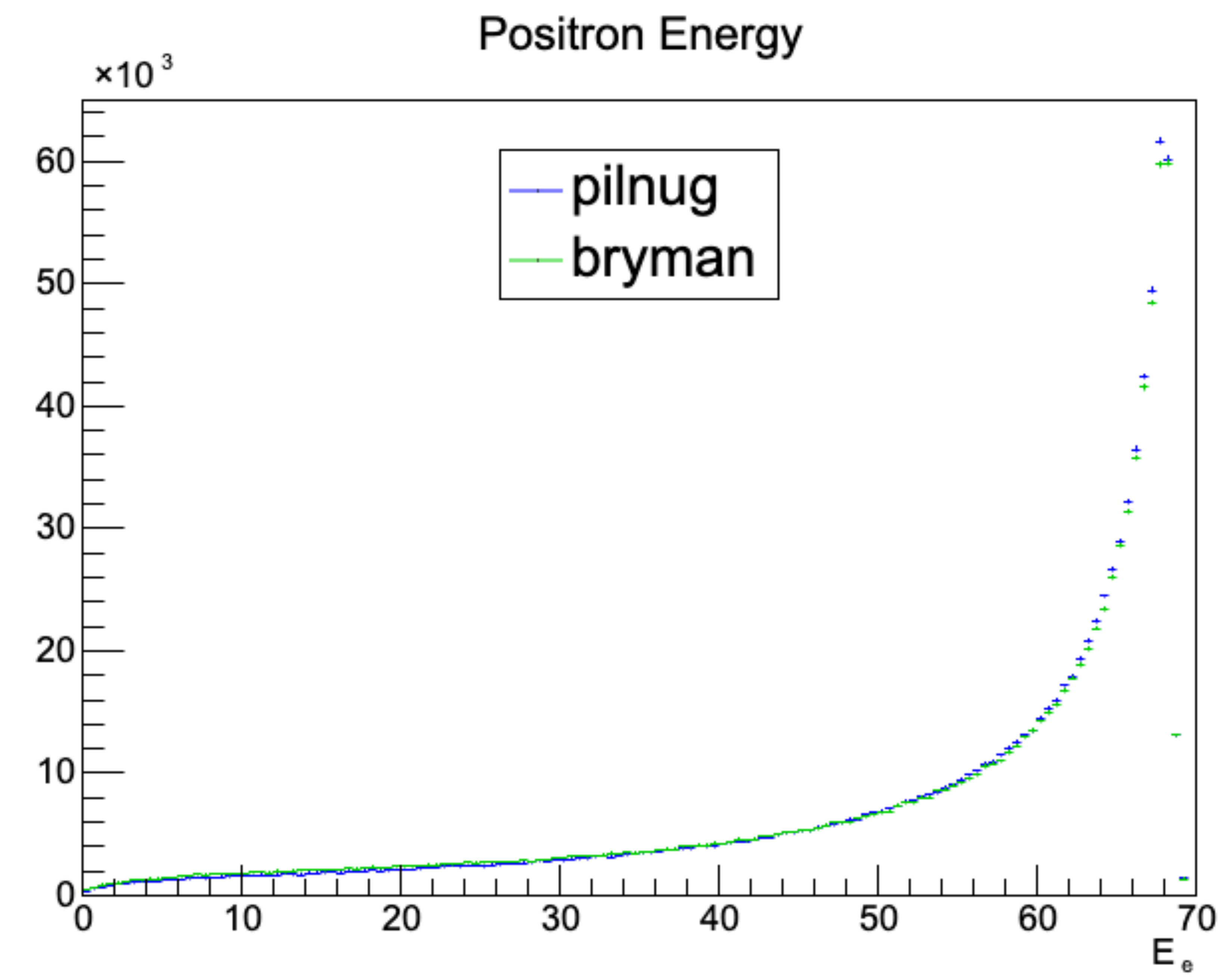
Two potential dangers

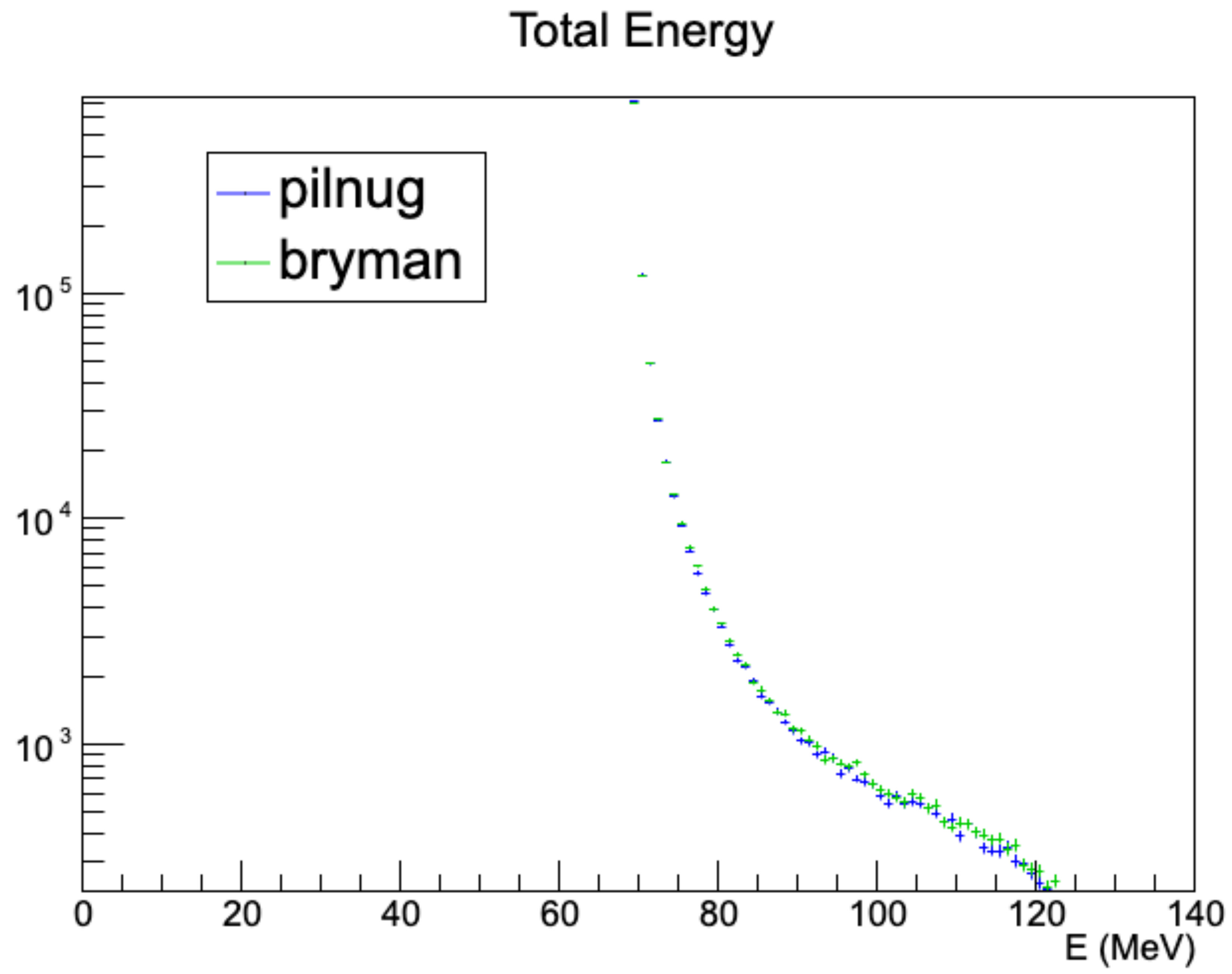
that come with the $\pi \rightarrow e\nu\gamma$ channel

Issue A: Non-detection of photon
The positron gets emitted in the fiducial region of the calorimeter. The photon is emitted towards entrance cone and does not hit calorimeter. This will be indistinguishable from other tail topologies and be a subdominant effect.

Issue B: Excessive Energy
Both particles are high energy and get detected. Total observed energy can be up to the pion mass. It exceeds the upper threshold of e.g. 85 MeV where we expect pile-up only.

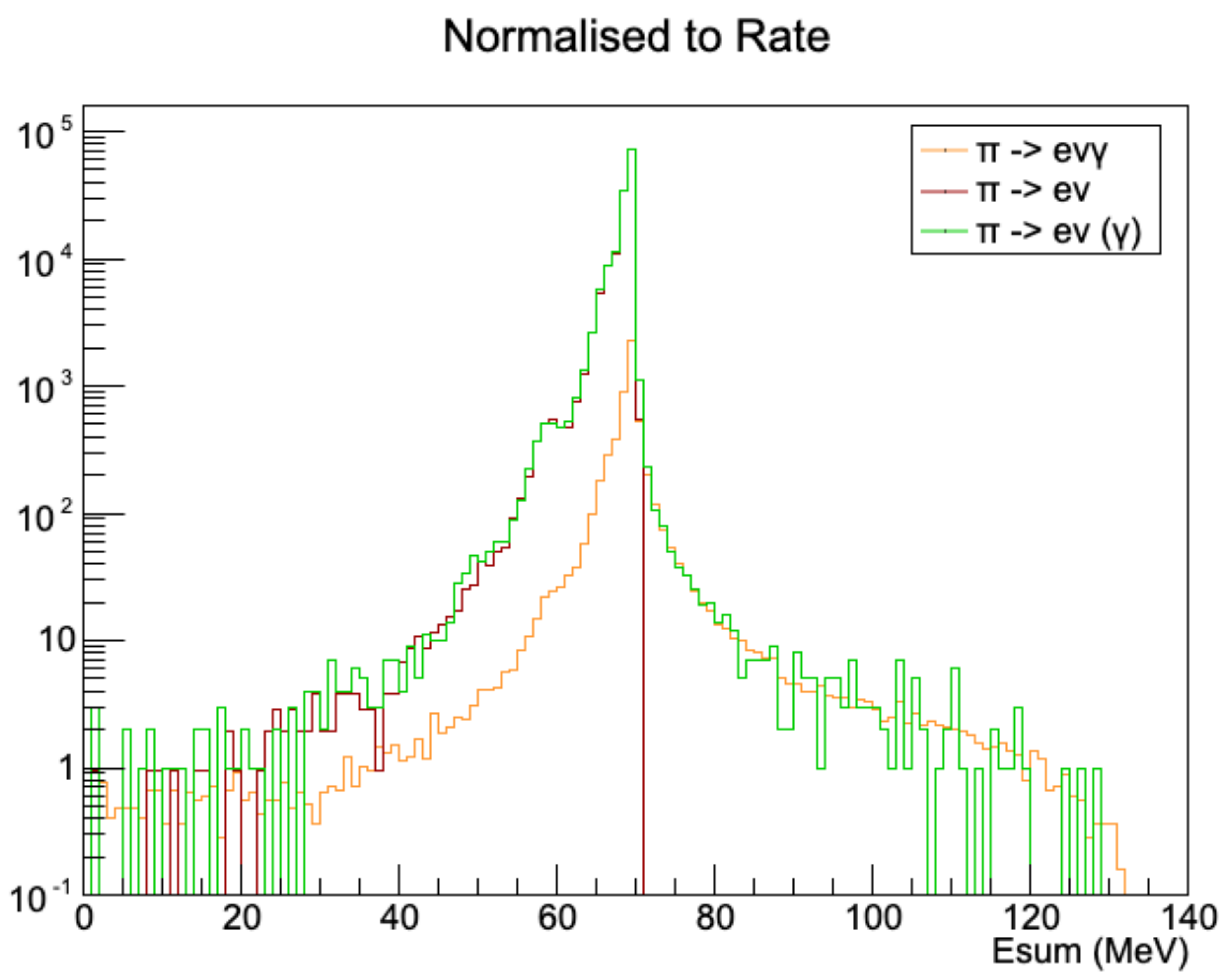
Require careful simulation in context of full geometry and reconstruction





Energy Spectrum Observed by PIONEER

Energy Range (MeV)	$\pi \rightarrow e\nu$	$\pi \rightarrow e\nu(\gamma)$
1 - 25	$1.0 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$
1 - 55	$3.4 \cdot 10^{-3}$	$3.8 \cdot 10^{-3}$
55 - 80	0.997	0.995
80 - 115	0.00	$1.1 \cdot 10^{-3}$
115 - 140	0.00	$7.6 \cdot 10^{-5}$



Radiative Decays with escaped photons make a 10% contribution to the tail and potentially force an expansion of our high bin region well above 100 MeV

Radiative Muon Decays

Radiative Michel Decay

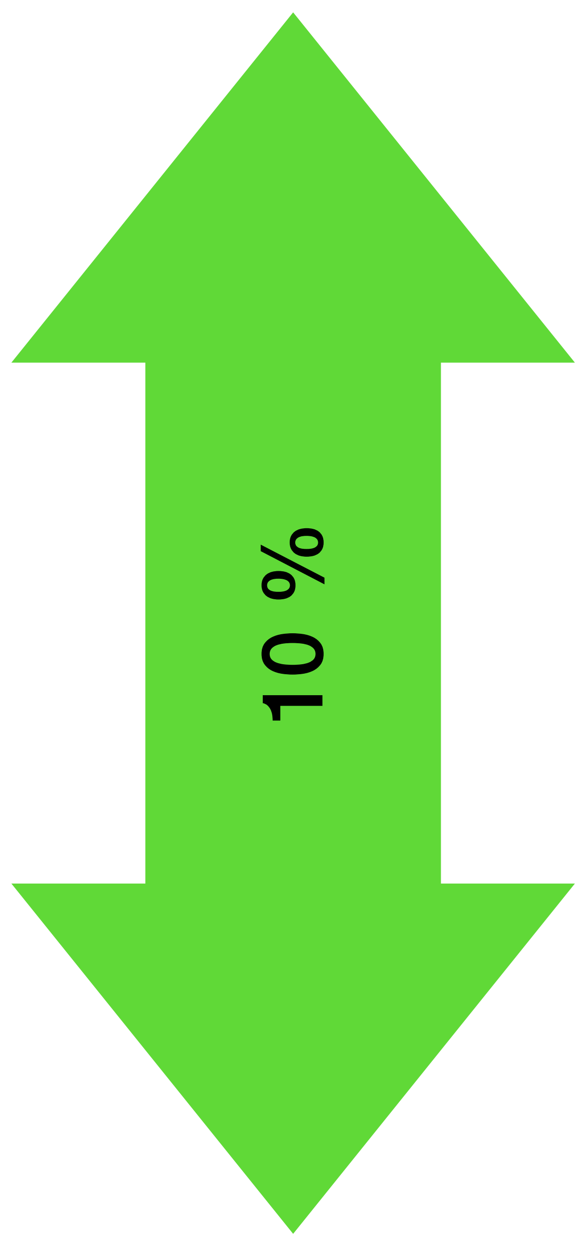
Validation of custom generator

- Expected number of events given BR and number of observed events: $N_0 = N_{gen}/BR$

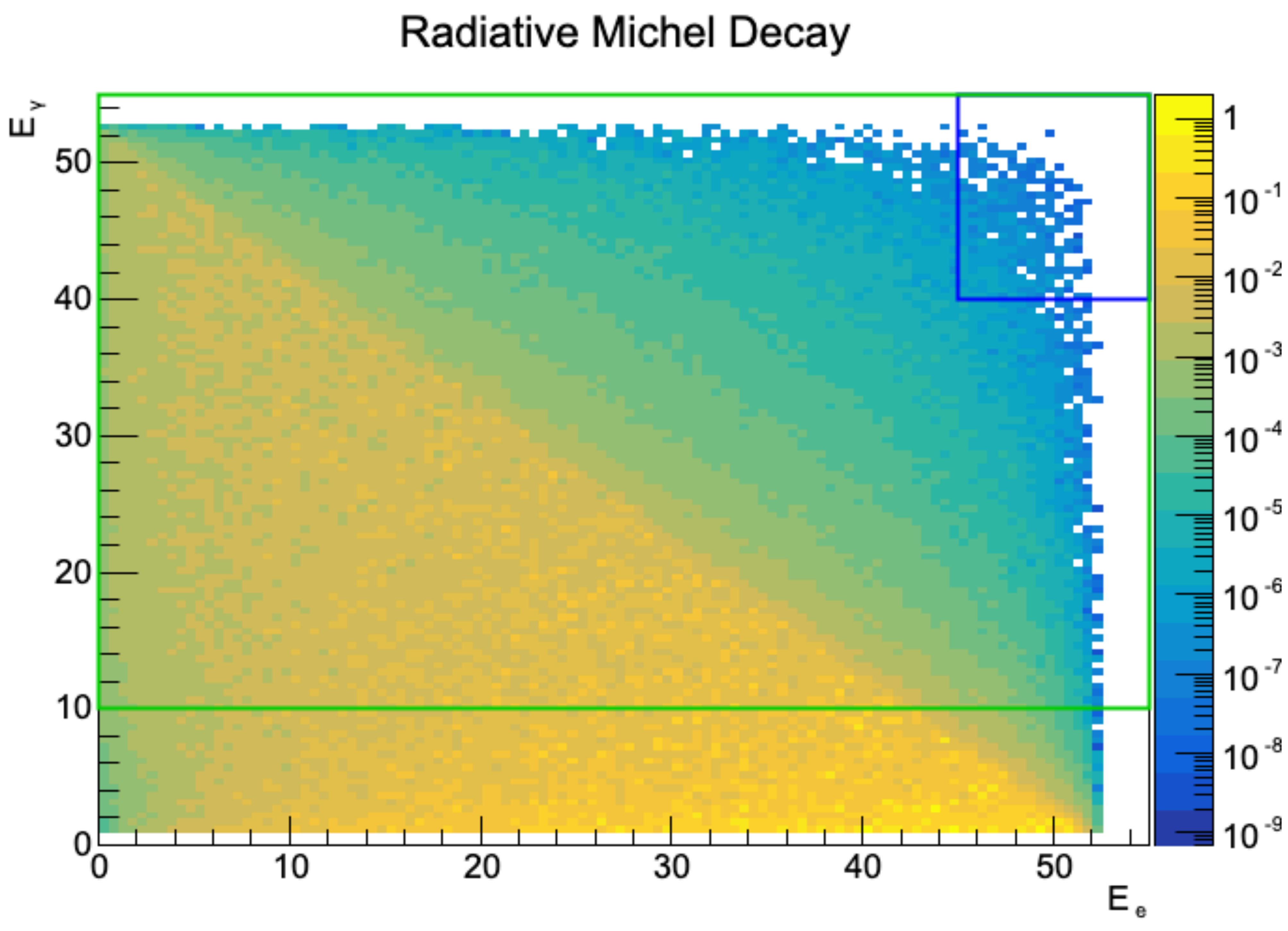
- **MEG II**: $N_0 = 1.8 \cdot 10^7$
  BR: 6.03e-8
  Cut: $p_e >$ 45MeV/c, $E_\gamma >$ 40MeV
  N_{gen}: 1

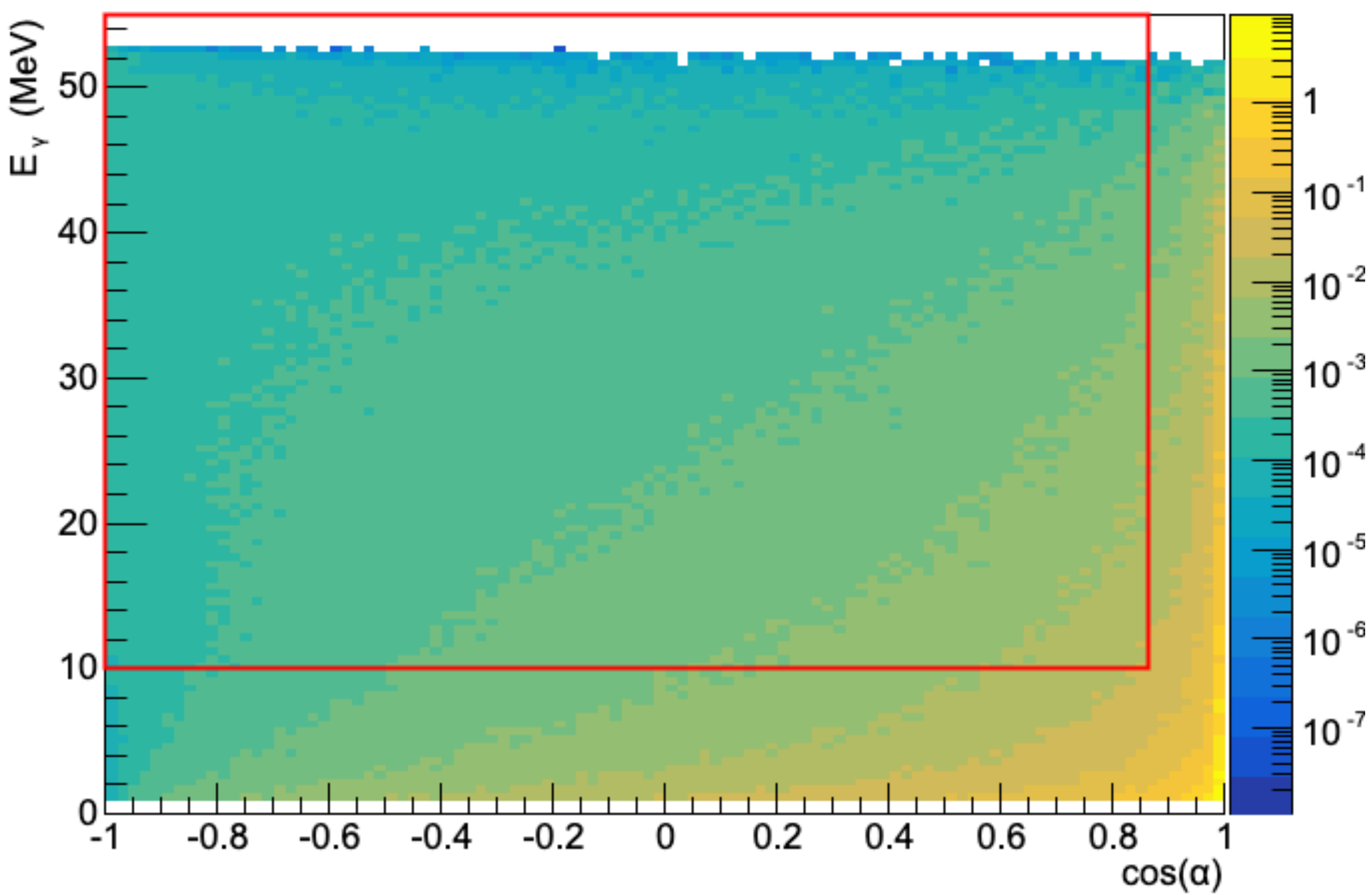
- **PiBeta**: $N_0 = 1.9 \cdot 10^7$
  BR: 4.365e-3
  Cut: $\omega_0 >$ 10MeV, $\Theta_{e\gamma} > 30°$
  N_{gen}: 83k

- **Theory**: $N_0 = 2 \cdot 10^7$
  BR: 1.289e-2
  Cut: $\omega_0 >$ 10MeV
  N_{gen}: 256k



https://arxiv.org/abs/1705.03782

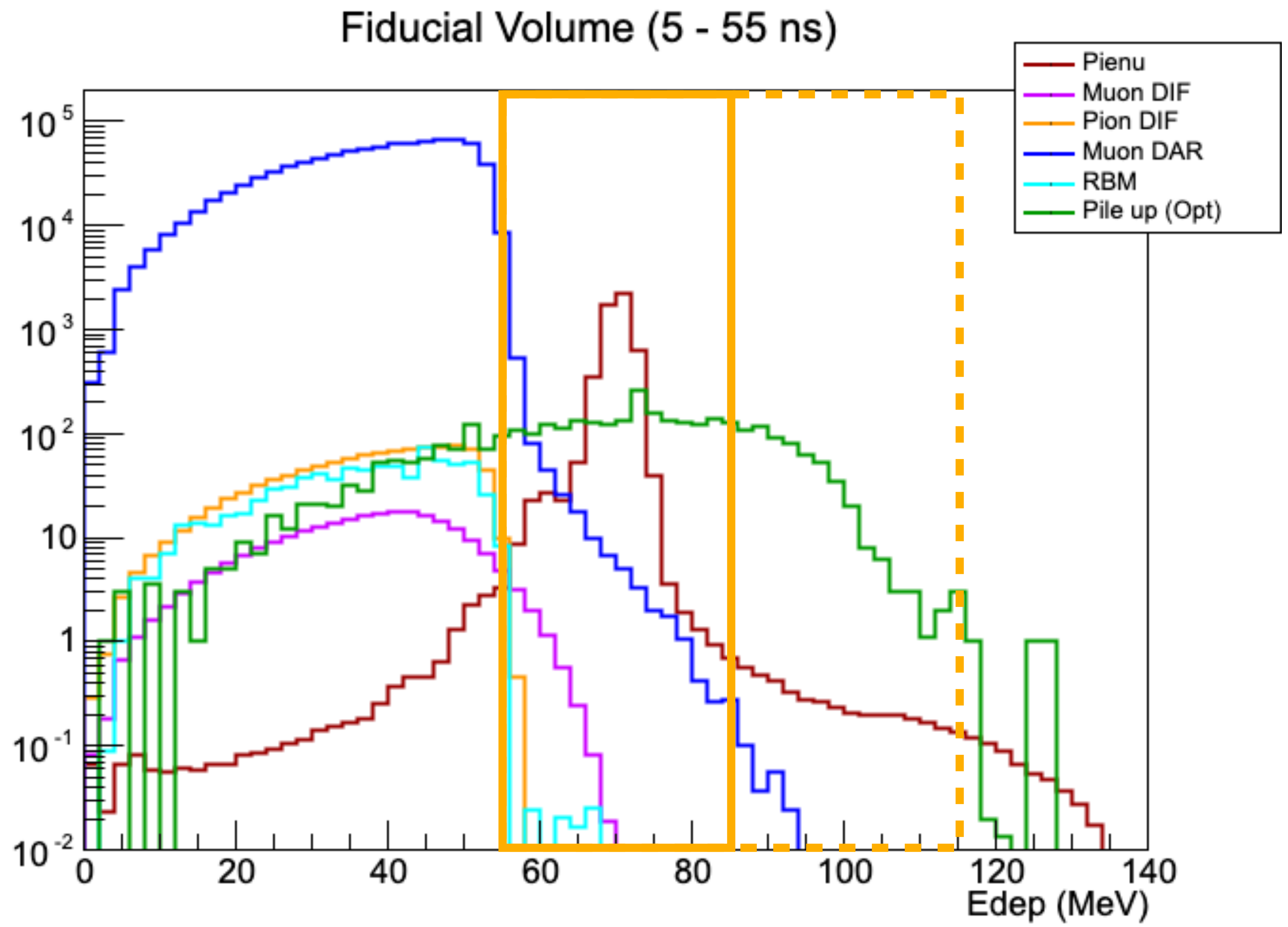




Impact on Energy Spectra

- Asume LXe detector
  - Polar angle up to $\theta_{calo} = 145°$
  - No separation power between positron and photon (if both hit)

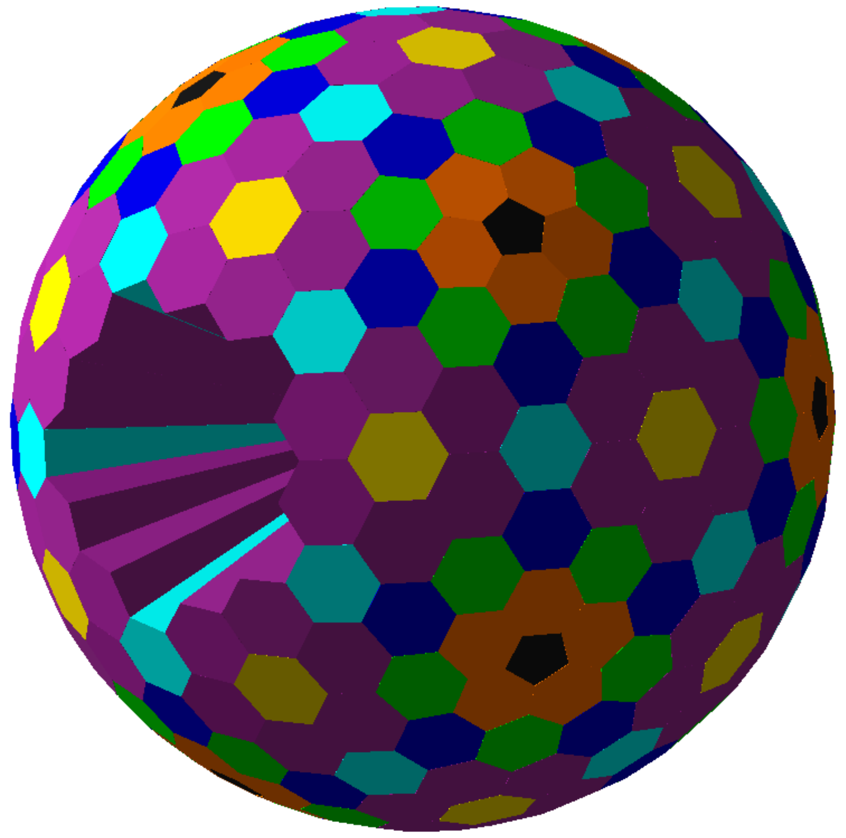
- Energy spectra get a high-energy tail. Major leakage to high bin range for muon decay at rest.
  - 0.01% of pienu events above 115 MeV.



56 - 85 MeV: 66% Pienu, 25% Pileup, 9% Muon DAR
56 - 115 MeV: 60% Pienu, 31% Pileup, 9% Muon DAR

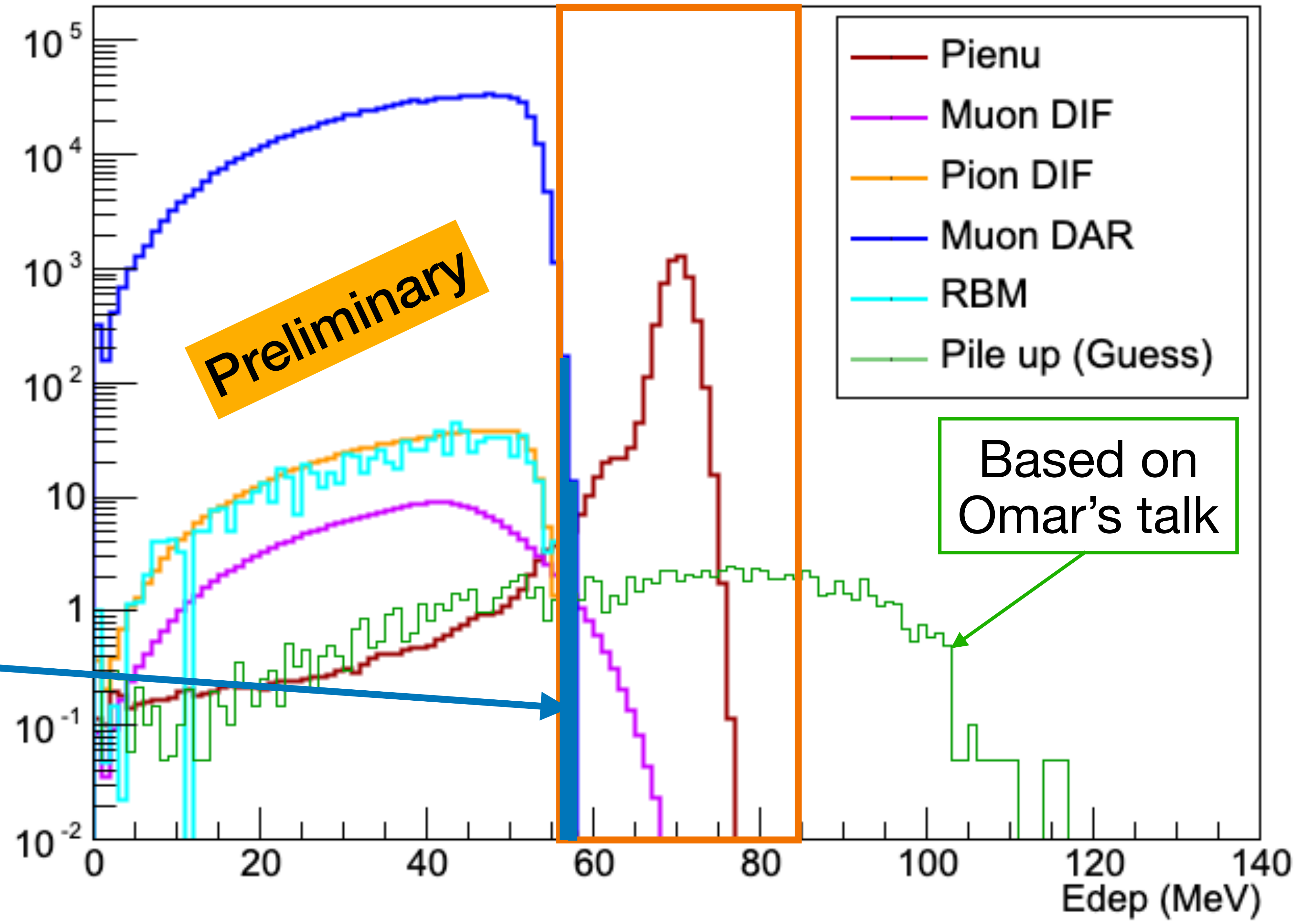
Identify radiative decays or add Pileup!

Benefits of a Segmented Calorimeter



LYSO Crystal Calo Concept

- Assume we can distinguish photon and positron if they are further than 45° apart.
  - $\approx 3.7 R_M$ @ 10 cm
  - $\approx 5.3 R_M$ @ 15 cm
- Tail Fractions for $\pi \rightarrow e\nu(\gamma)$:
  - 0.6% below 56 MeV, 0 above 80 MeV
- High Bin Composition (56 - 85 MeV):
  - 95 % $\pi \rightarrow e\nu(\gamma)$
  - 4 % Muon decay at rest
  - 1 % Pileup



Clean high bin at cost of larger tail

Challenges and Opportunities

Radiative Pion Decay to Muon

$\pi \to \mu\nu\gamma$

Characteristics
- Low energy muon + photon
- IB dominated
- BR: 2e-4 with $E_\gamma > 1$ MeV

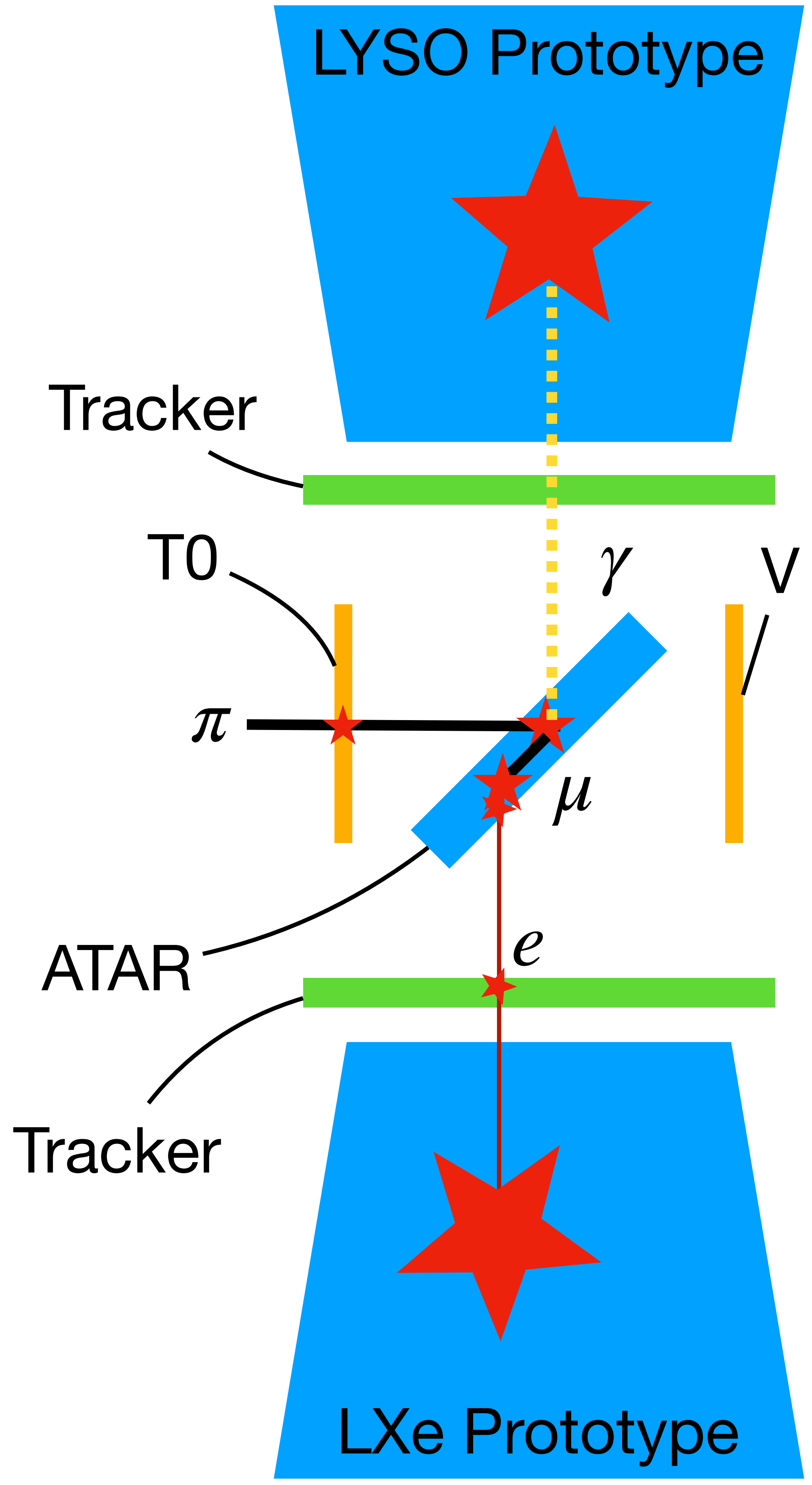
Challenges
- Wash out 4 MeV μ signature.
- Photon can pile up

Opportunity
- Measurement before PSI shutdown
- Proof of detectors and analysis strategy

Chance

Can we use it as prototype test?

Would require:
- Pion beam
- Enough ATAR to stop pions.
- Extra detectors?
- Simulations!



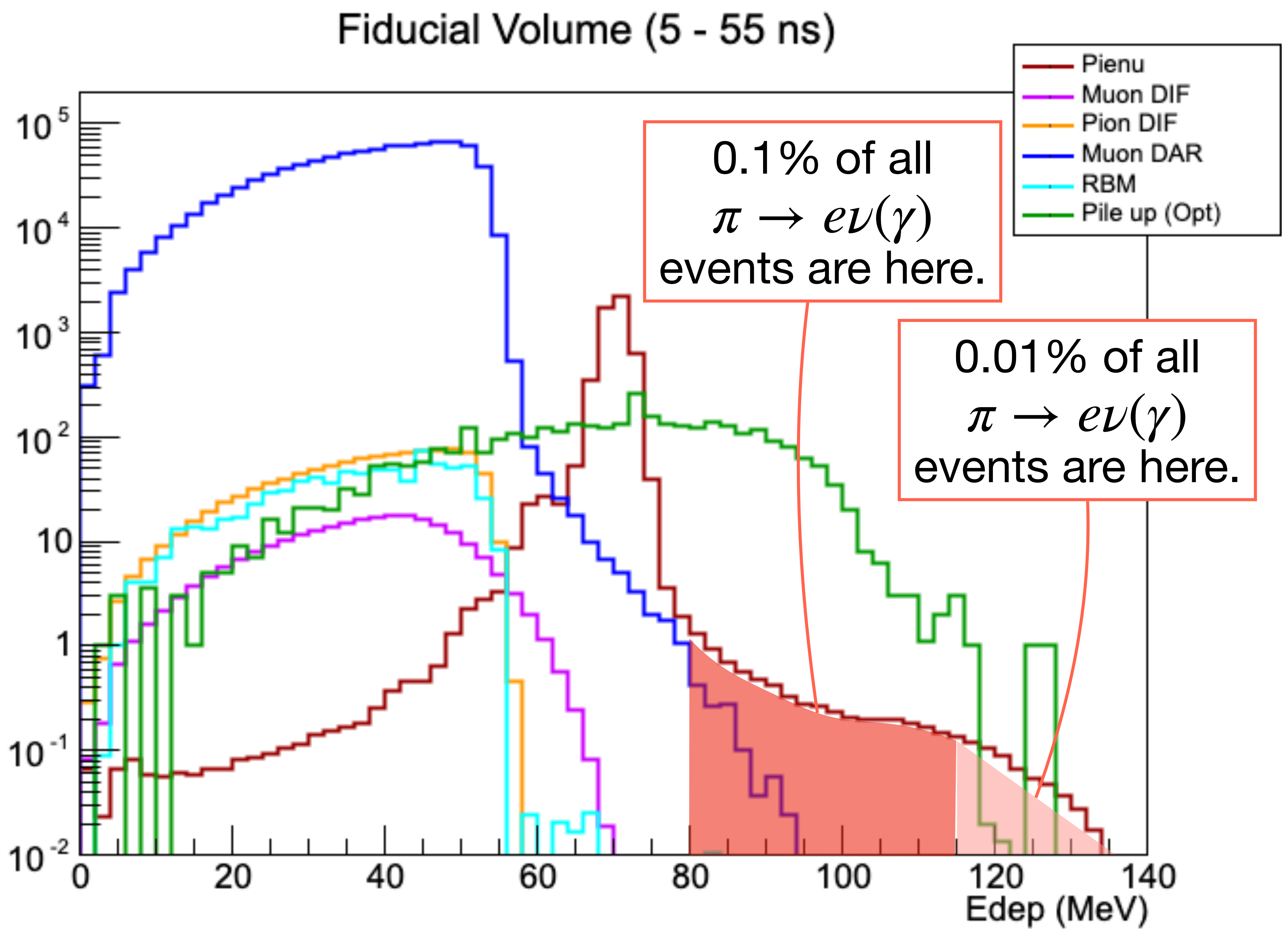
Radiative Pion Decay to Positron

$\pi \to e\nu\gamma$

Characteristics
- Energetic positron + photon
- Structure dependent coupling
- BR 7.4e-7, $E_\gamma > 10$ MeV, $\Theta_{e\gamma} > 40°$

Challenges
- 10% contribution to tail
- Sum of energy pushes upper threshold

Opportunity
Potential measurement of SD terms?



Need to get $\pi \to e\nu\gamma$ to at least 10 % precision.

Radiative Michel Decay

$\mu \rightarrow e\nu\nu\gamma$

Characteristics

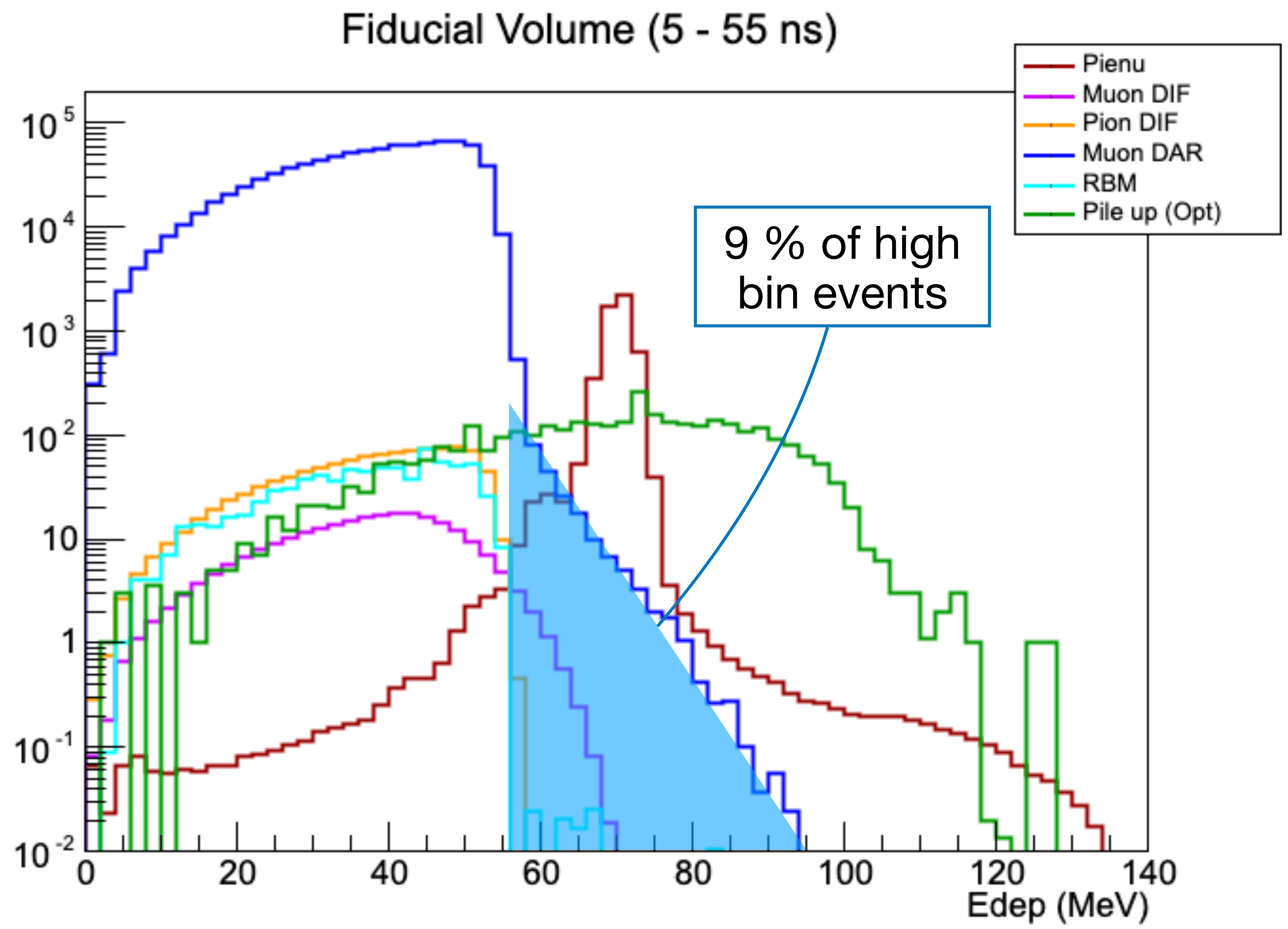
- Energetic positron + photon
- BR 6.03e-8 $p_e > 45$ MeV, $E_\gamma > 40$ MeV
- BR 4.365e-3 $E_\gamma > 10$ MeV, $\alpha_{e\gamma} > 30°$

Challenge

- Main single event leakage to high bin.

Opportunity

Measurement of Michel parameters from radiative decay?



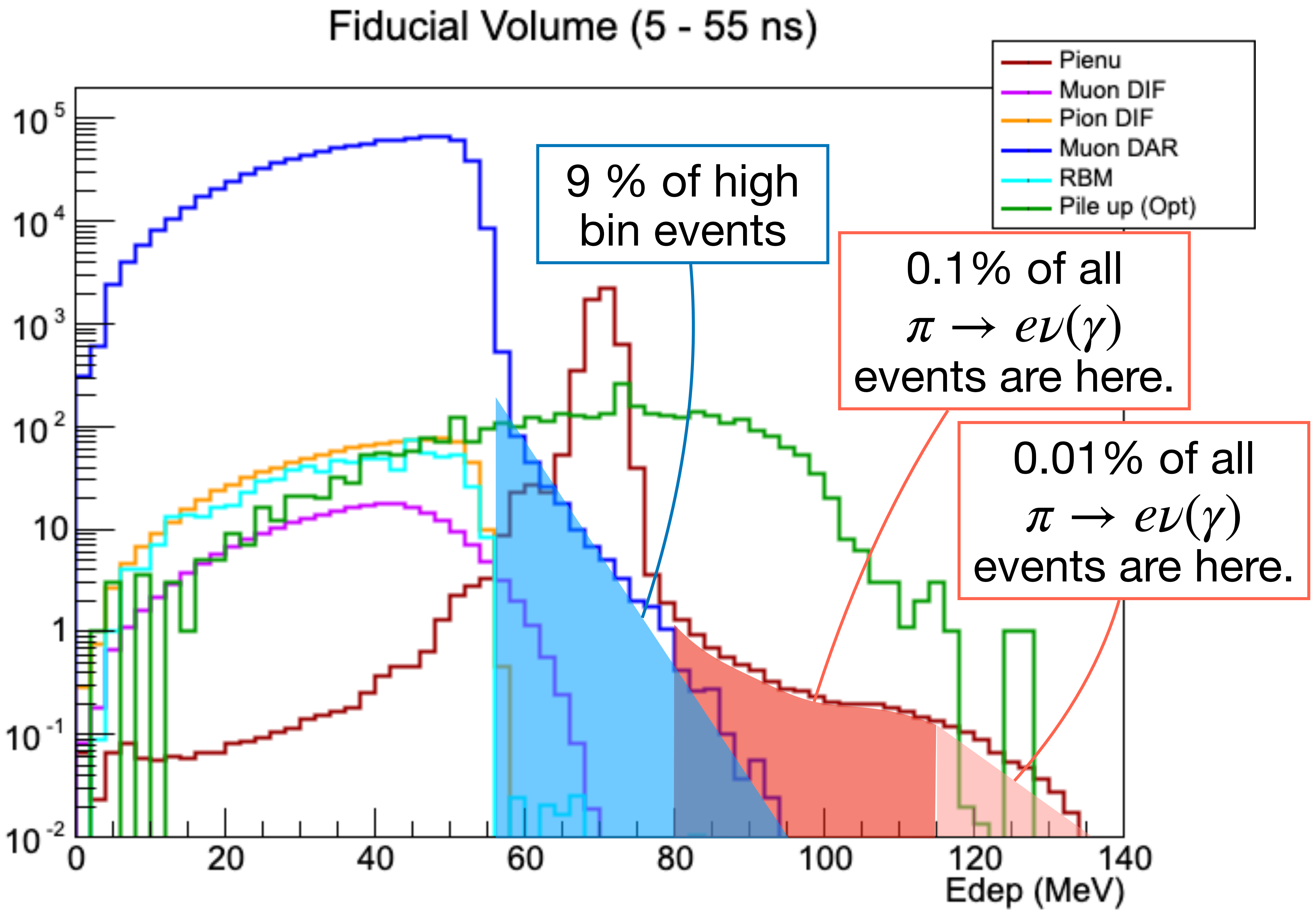
Need to know radiative Michel decay to 0.1 %.

We need to get radiative decays right

- Measure inclusive Branching Ratio

$$R_{e\mu} = \frac{\Gamma\left(\pi \rightarrow e\nu(\gamma)\right)}{\Gamma\left(\pi \rightarrow \mu\nu(\gamma)\right)}$$

- Radiative decays can shift events out of their native bin.
- Require a calorimeter that can resolve two simultaneous but spatially separated energy deposits.

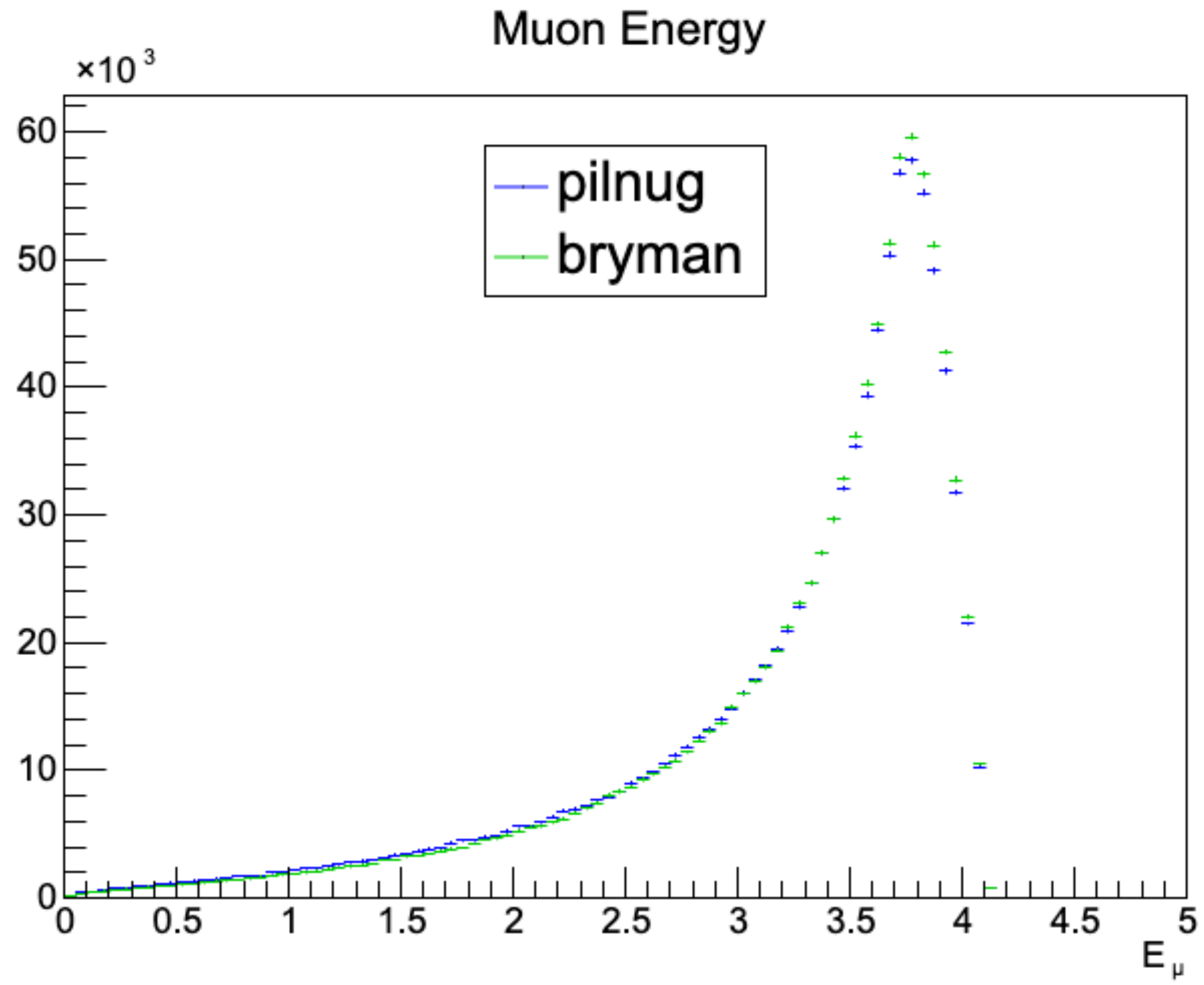


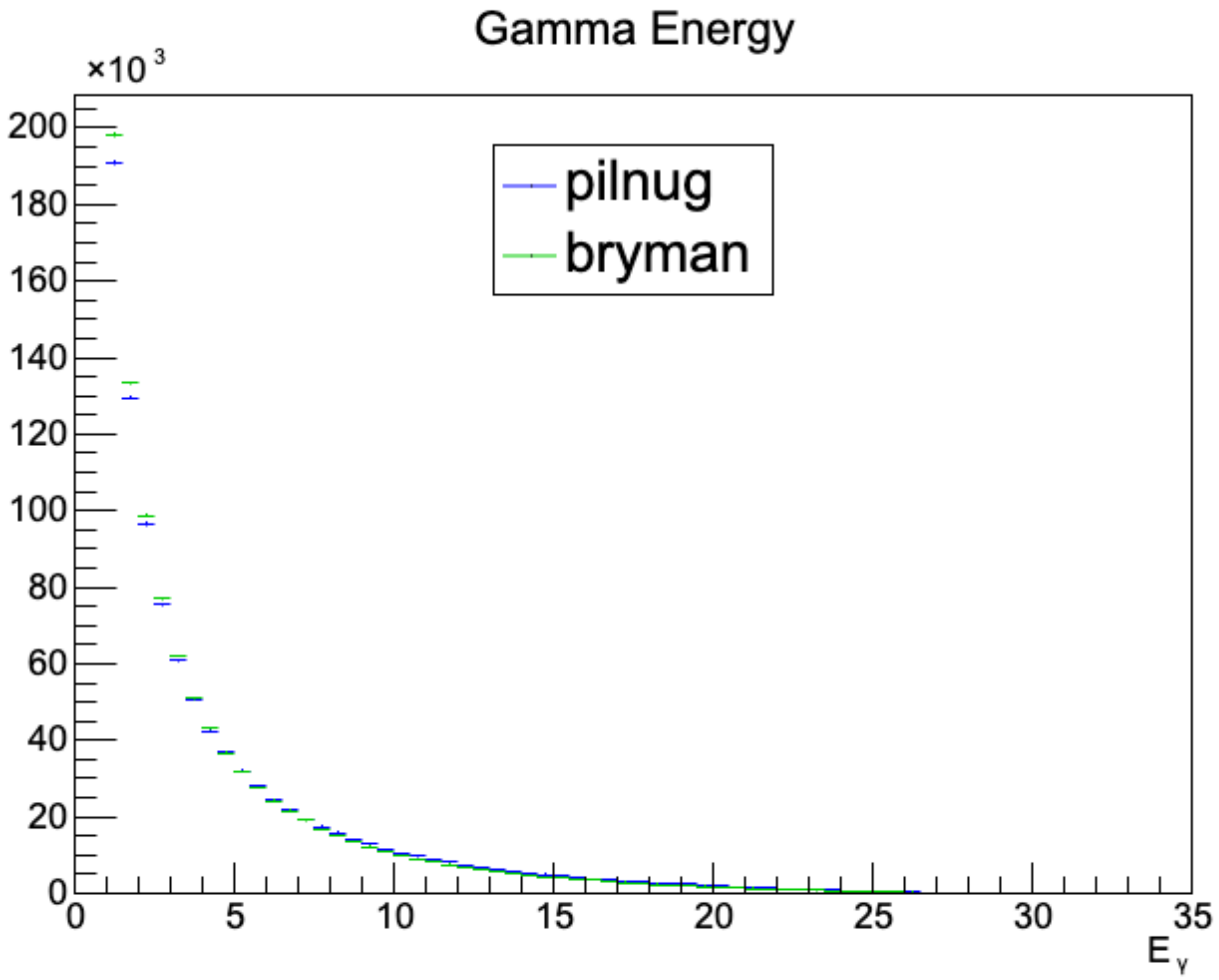
If we can handle them properly, they potentially offer additional physics measurements and detector benchmarks

Additional Material

Radiative Pion Decay to Muon - linear scale

$\pi \to \mu\nu\gamma$, BR 2e-4





Both generators provide similar distributions.
Events that could foil our analysis strategy are few.

Radiative Michel Decay

- Expected number of events given BR and number of observed events: $N_0 = N_{gen}/BR$

- **MEG II**:
  BR: 6.03e-8
  Cut: $p_e > 45$MeV/c, $E_\gamma > 40$MeV
  N_{gen}: 0.6k

  $N_0 = 1 \cdot 10^{10}$

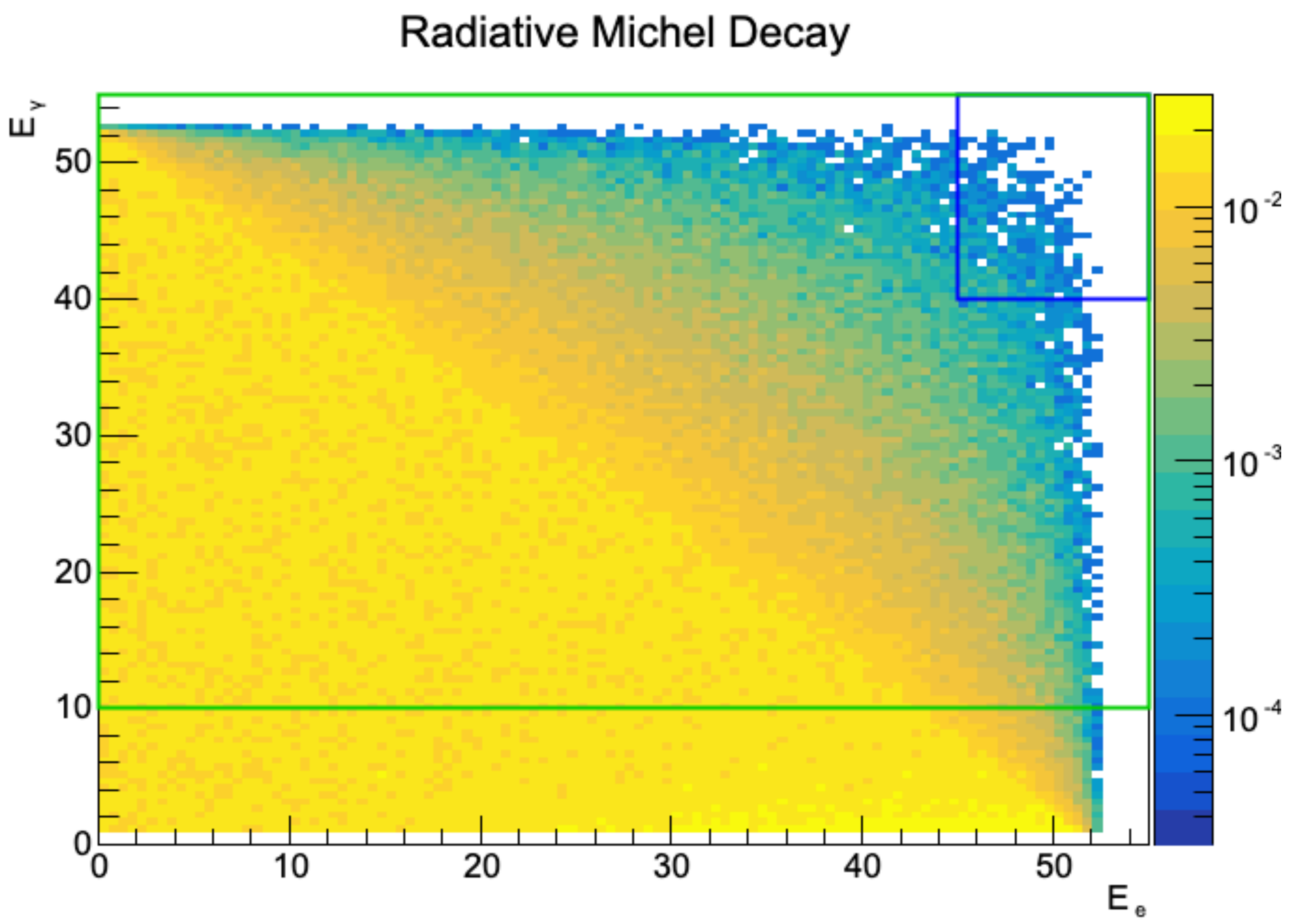
- **PiBeta**:
  BR: 4.365e-3
  Cut: $\omega_0 > 10$MeV, $\Theta_{e\gamma} > 30°$
  N_{gen}: 0.68M

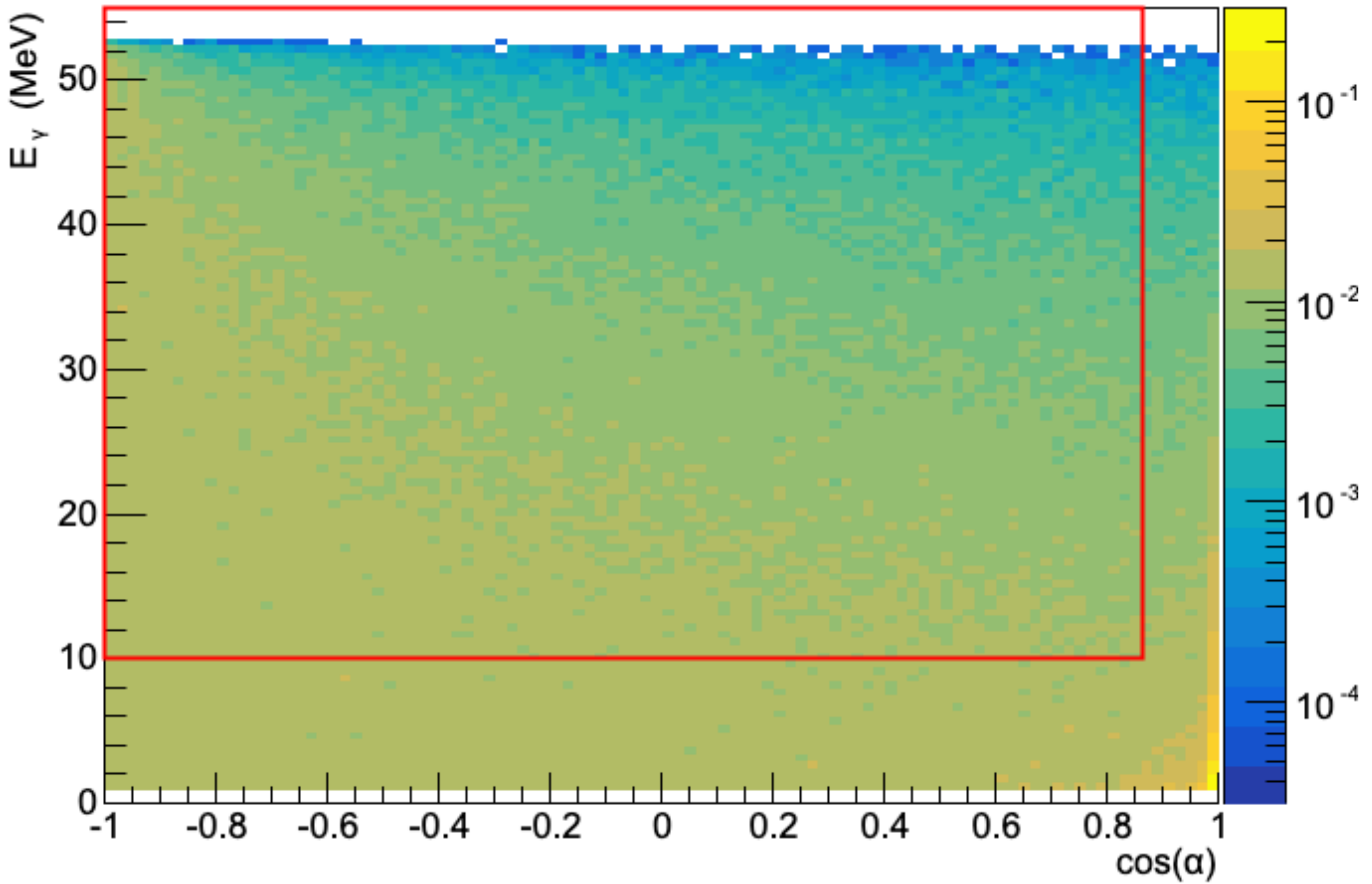
  $N_0 = 1.6 \cdot 10^{8}$

- **Theory**:
  BR: 1.289e-2
  Cut: $\omega_0 > 10$MeV
  N_{gen}: 0.73M

  $N_0 = 6 \cdot 10^{7}$

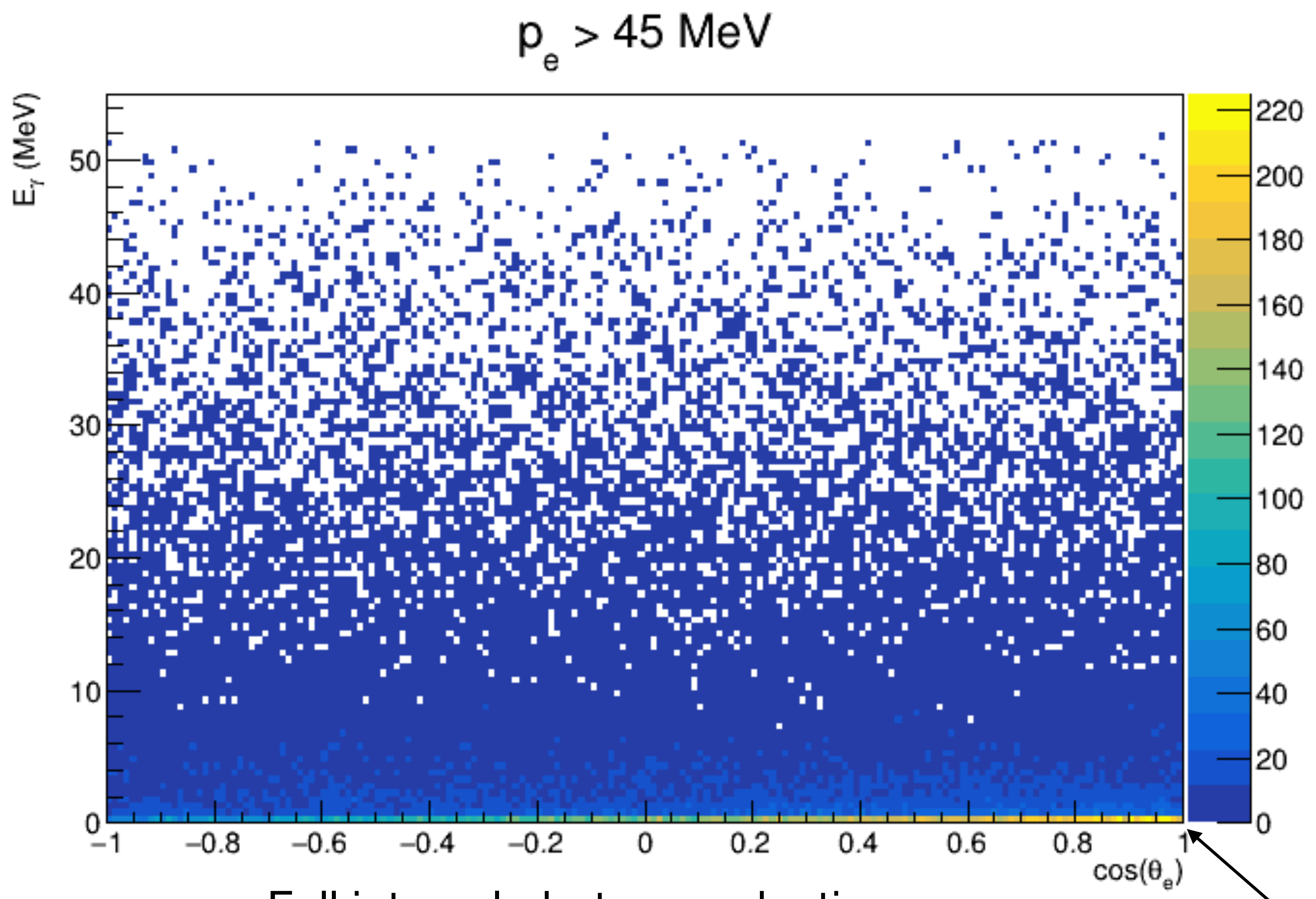
2 Orders of Magnitude



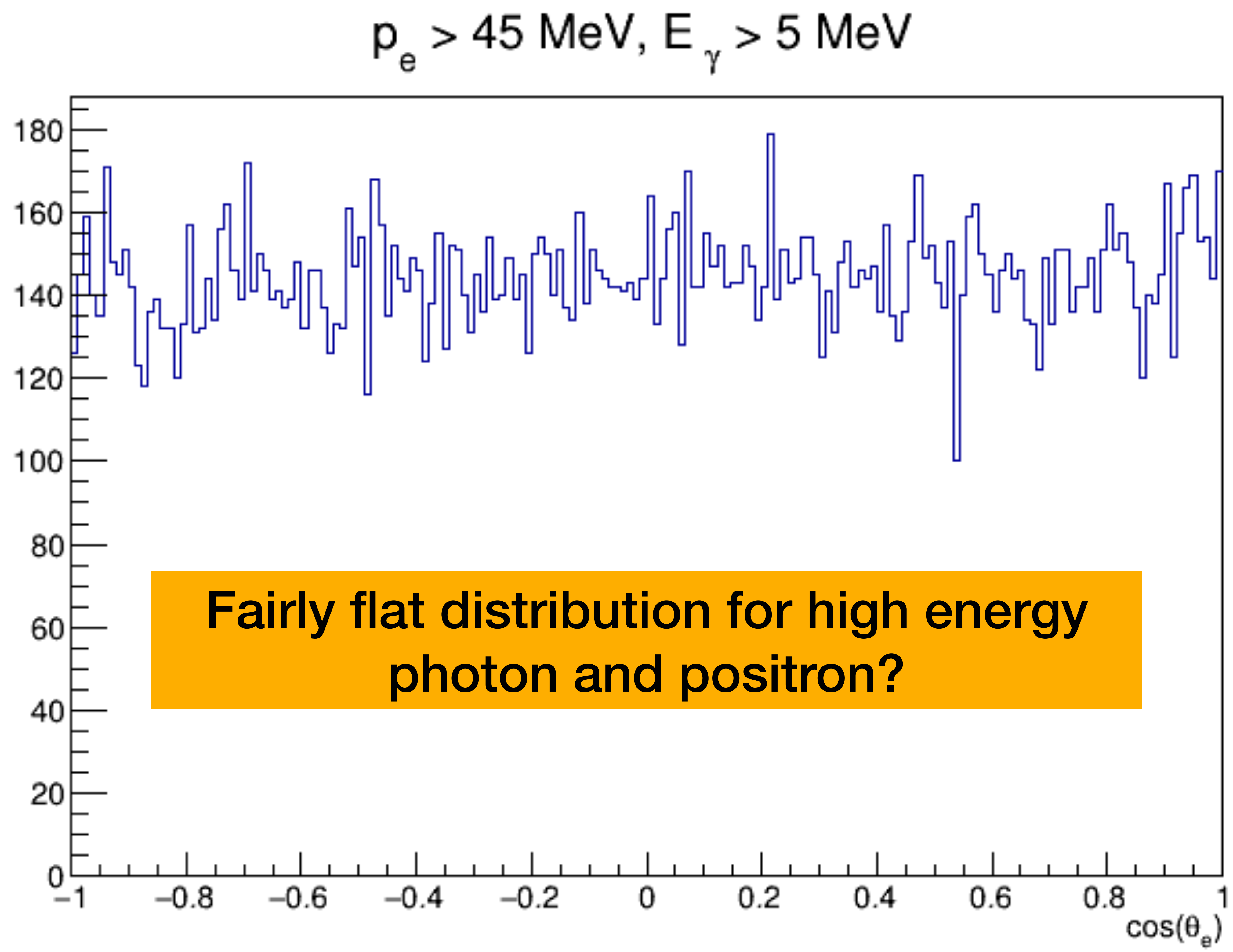


https://arxiv.org/abs/1705.03782

Positron Angle w.r.t. Muon Spin



Full internal photon production range



Fairly flat distribution for high energy photon and positron?

Clear Asymmetry for low energy photons

Positron Angle w.r.t. Muon Spin

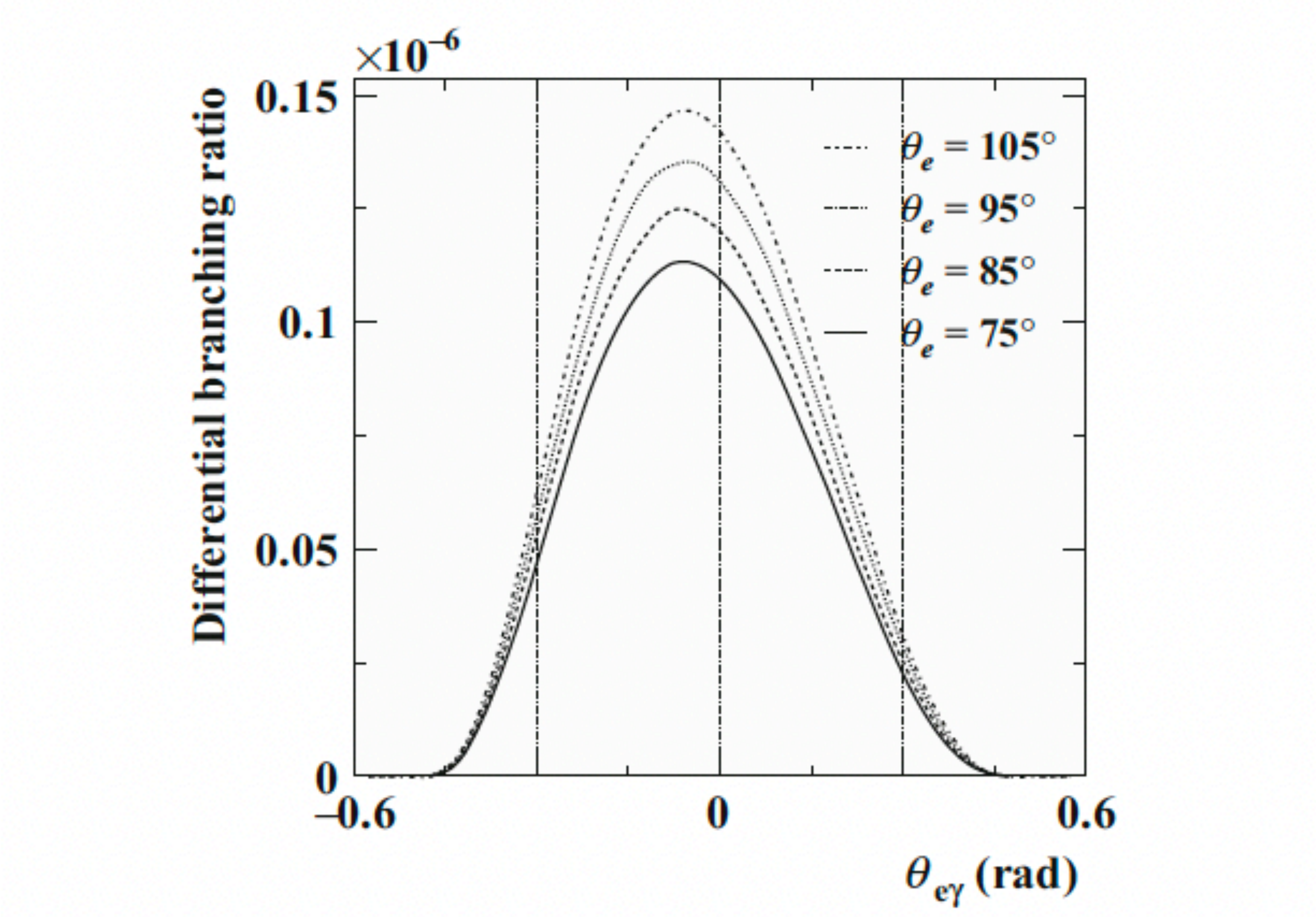
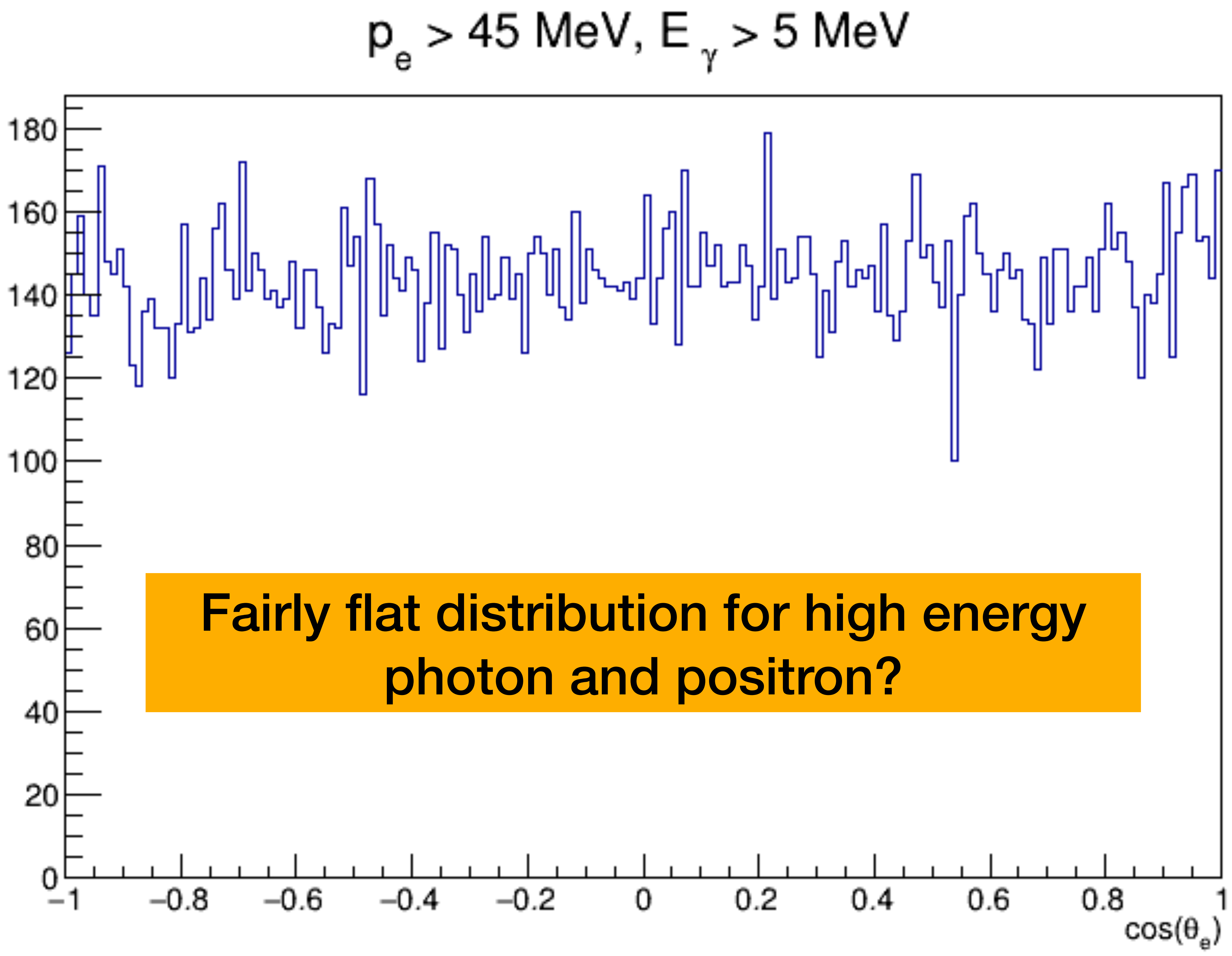


Fig. 3 Differential branching ratio of RMD for $P_{\mu^+} = -0.85$ as a function of $\theta_{e\gamma}$ for four different values of positron polar angle θ_e. These distributions are obtained by the numerical integration of Eq. (2) over $E_e > 45$, $E_\gamma > 40$ MeV, and $\phi_{e\gamma}$

MEG Paper suggest some dependency

https://doi.org/10.1140/epjc/s10052-016-3947-6



Fairly flat distribution for high energy photon and positron?

We need our own RMD generator to get the spectrum right