Acceptance studies

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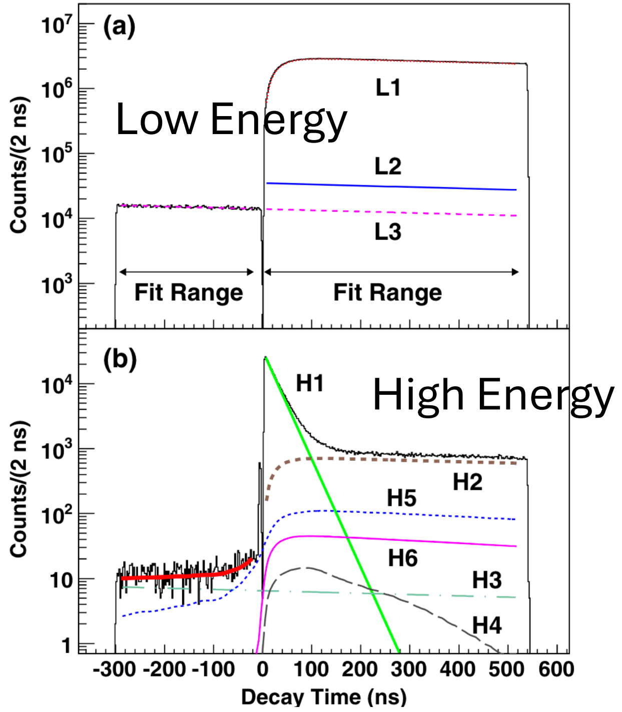
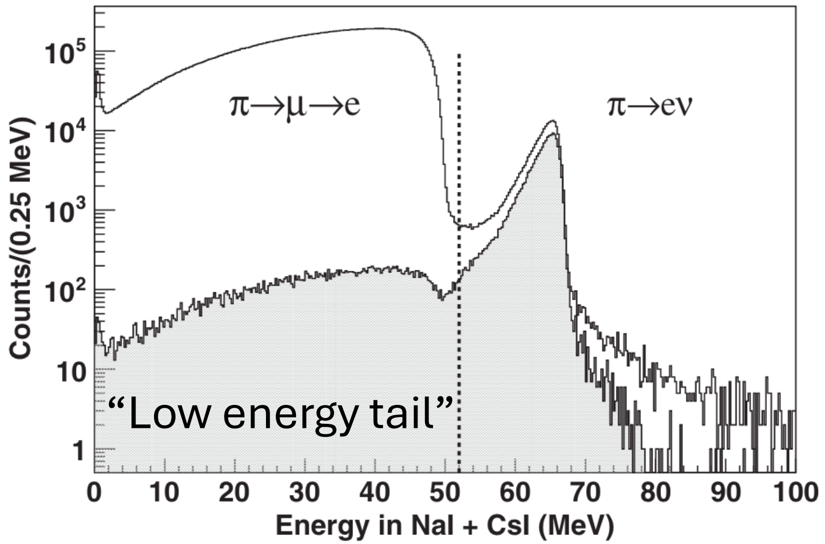
Motivation

- $R_{e/\mu} = \frac{N_{\pi e}(E>E_{th}) \cdot (1+C_{tail})}{N_{\pi\mu e}} \cdot R^{\epsilon}_{time} \cdot R^{\epsilon}_{energy} \cdot R^{\epsilon}_{angle} \cdot R_{topology}$
  - $\boldsymbol{N_{\pi e}(E > E_{th})}$ and $\boldsymbol{N_{\pi\mu e}}$ are obtained through fitting the timing spectrum
  - $\boldsymbol{C_{tail}}$ comes from a dedicated tail fraction measurement

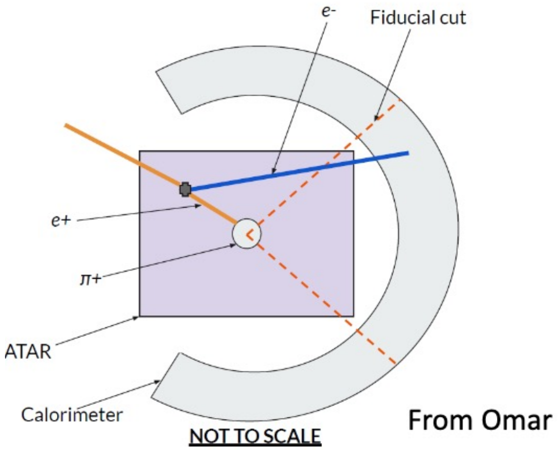


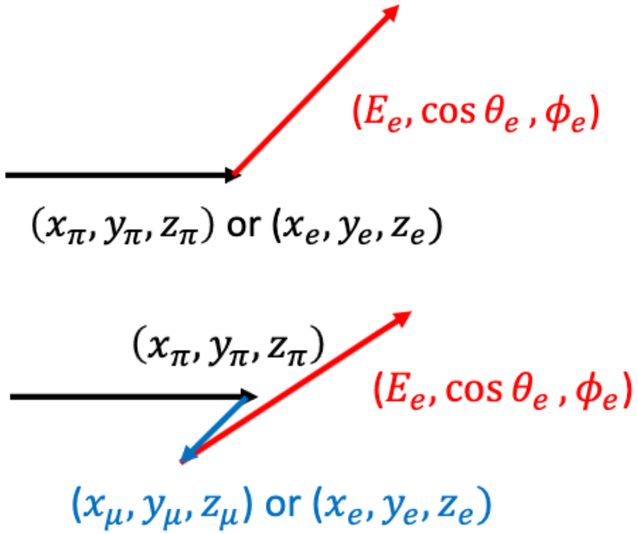


From PiENu

Motivation

- $R_{e/\mu} = \frac{N_{\pi e}(E>E_{th})\cdot(1+C_{tail})}{N_{\pi\mu e}} \cdot R^{\epsilon}_{time} \cdot R^{\epsilon}_{energy} \cdot R^{\epsilon}_{angle} \cdot R_{topology}$
  - $R^{\epsilon}_{time} \cdot R^{\epsilon}_{energy} \cdot R^{\epsilon}_{angle} \cdot R_{topology}$ are acceptance ratios of signal events ~ 1+/- O(10^{-4})
  - Defines fiducial phase space using (x_π, y_π, z_π) and $(\cos\theta_e, \phi_e)$ (DocDB-210-v2)
    - Beam momentum spread
    - Difference in detector response

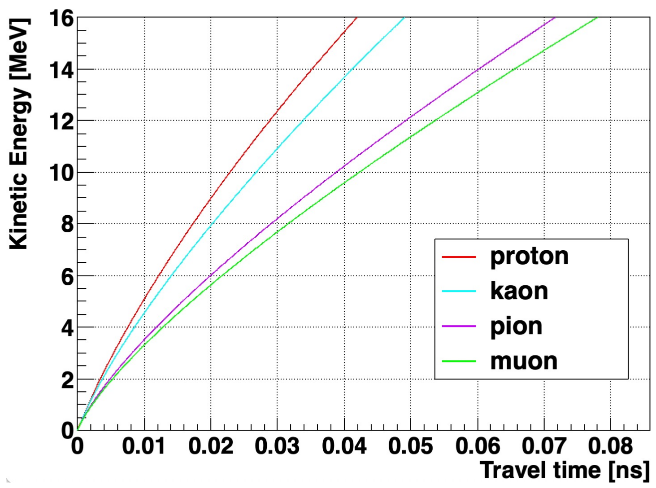


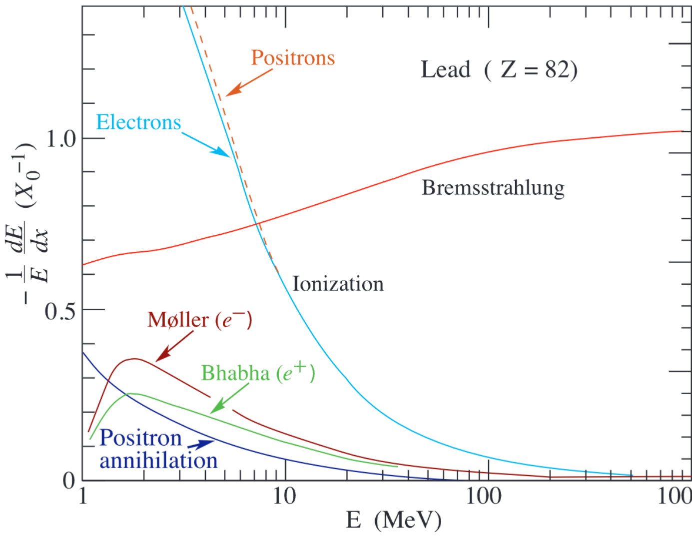


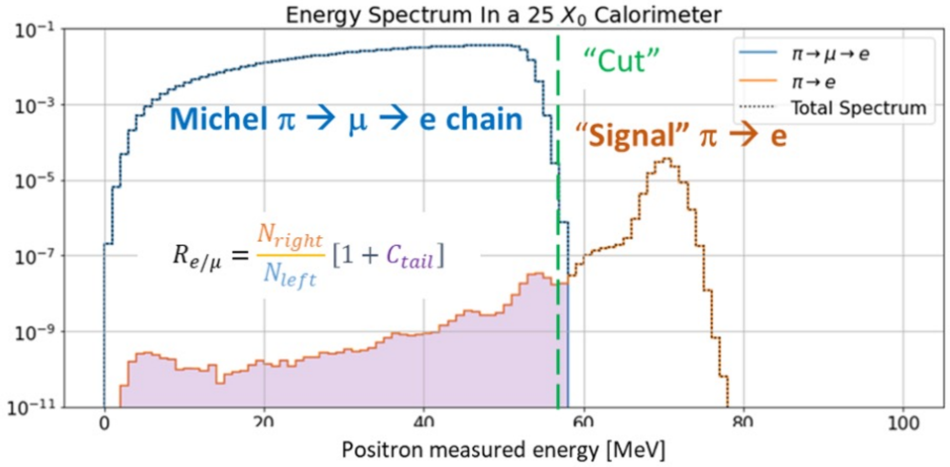
Acceptance definition

- Pion stop position (x_π, y_π, z_π)
  - Last layer of pion hits (by timing coincidence)
- Positron solid angle $(\cos\theta_e, \phi_e)$
  - Defined by standalone ATAR
    - First five hits of positron track
      - Low travel distance
        - small chance of e-Si scattering
        - Smaller energy deposition (R^{ϵ}_{energy})
      - Require high granularity and spatial resolution
  - Defined by ATAR & Tracker/CALO (PiENu)
    - Loose requirement on production point of e^+?
    - Suffers larger chance of Bhabha
    - Photons from annihilation
    - Missing, partially detected, detected by tracker/CALO

Focus on this talk

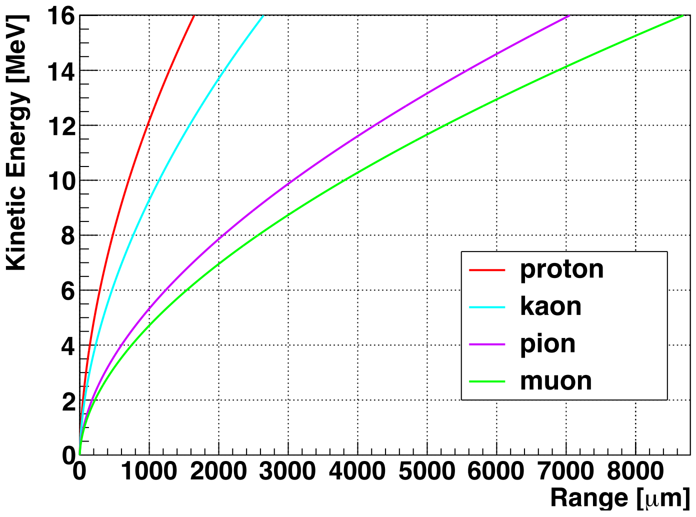


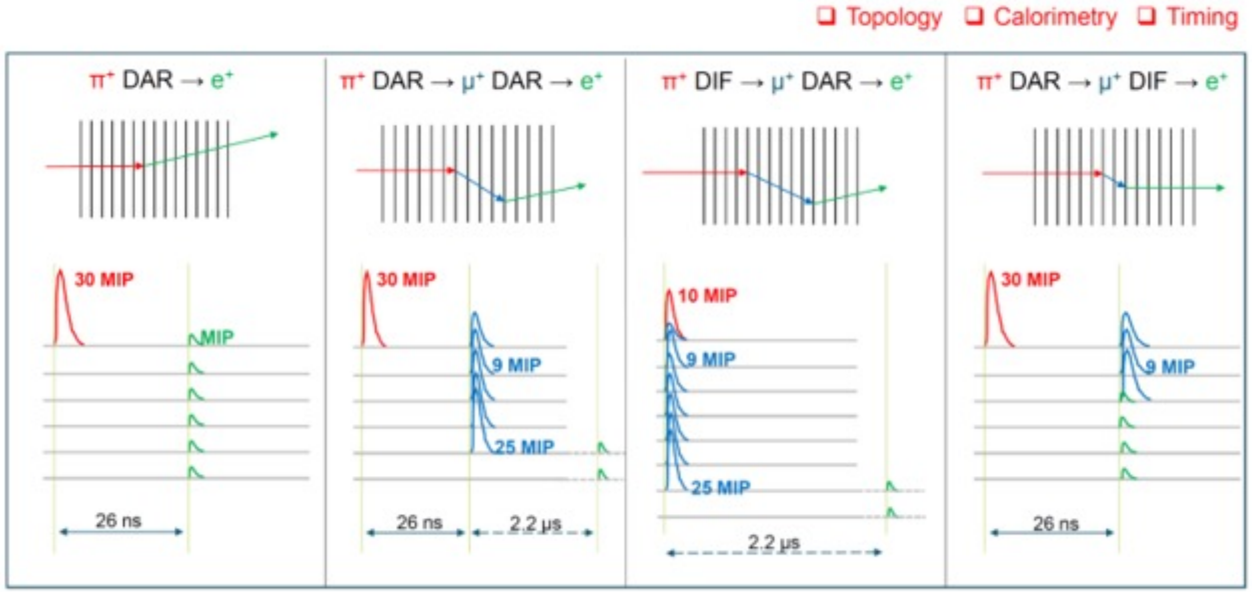




Selections on timing

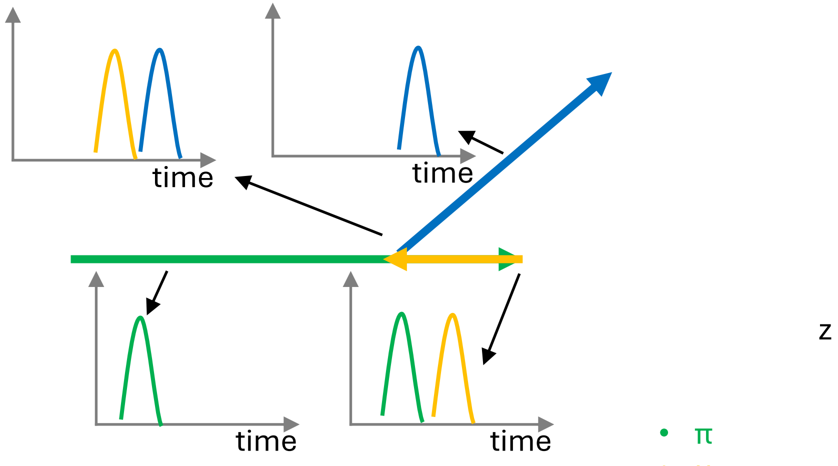
- In principle, signals
  - $N_{\pi e}$ defined by $\pi(DAR) \rightarrow e$
  - $N_{\pi\mu e}$ defined by $\pi(DAR) \rightarrow \mu(DAR) \rightarrow e$
- Sizable timing between pion, muon, positron
  - τ_π 26.033 ns
  - τ_μ 2197.03 ns
- Sizable travel distance for μDAR
  - Very likely to detect μDAR?
- Define signals using timing



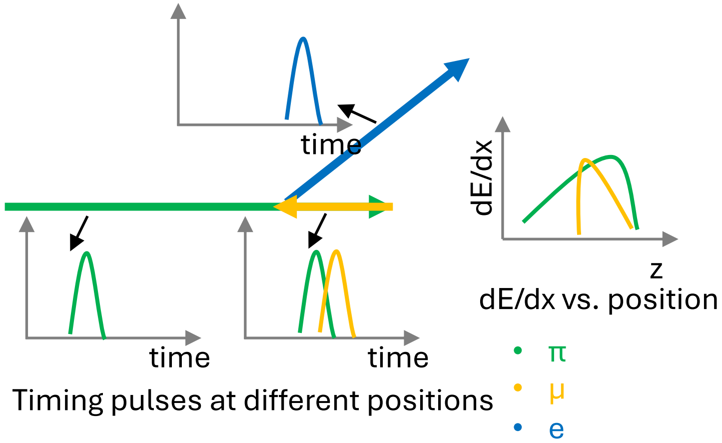


Selections on timing

- R^{ϵ}_{time} is defined by prompt (t_π) and delayed ($t' = t_\mu - t_\pi, t = t_e - t_\pi$) signals
  - Select $\pi(DAR) \rightarrow e$
    - by $\{t_e > t_\pi; t > \Delta t\}$
    - subject to $\int dt\, e^{-\frac{t}{\tau_\pi}}/\tau_\pi$
  - Select $\pi(DAR) \rightarrow \mu(DAR) \rightarrow e$
    - by $\{t_e > t_\mu > t_\pi;\ t' > \Delta t_\pi, t - t' > \Delta t_\mu\}$
    - subject to $\int dt \int_{\Delta t_\pi}^{t-\Delta t_\mu} dt' e^{-\frac{t-t'}{\tau_\mu}}/\tau_\mu e^{-\frac{t'}{\tau_\pi}}/\tau_\pi$
  - $\Delta t, \Delta t_\pi, \Delta t_\mu$ rely on timing resolution and pion lifetime
    - Single channel, $\sigma_t \sim$1ns/20ns (LGAD/PiN)
    - Inter-channel, $\sigma_t \sim$ 50ps/1ns (LGAD/PiN)
    - τ_π ~ 26ns



Timing pulses at different positions

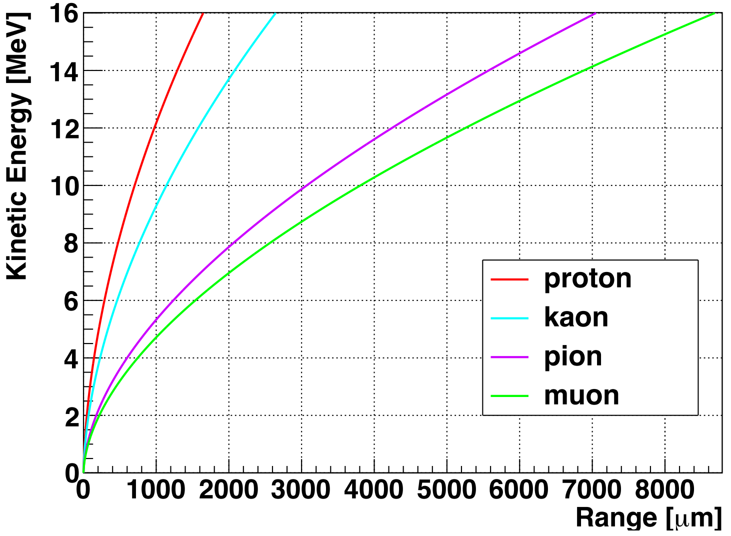


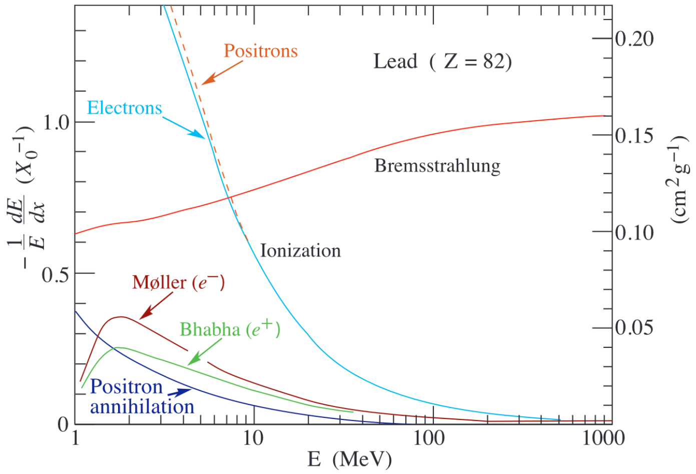
A quick look at $\Delta t = \Delta t_\pi = \Delta t_\mu$ =5ns

- Assume precision of pion lifetime is O(10^{-4}).
- Relative difference in acceptance caused by extrapolation is of order O(10^{-6})

Selections on energy

- A MIP in silicon deposit ~3.87MeV/cm
  - 5 hits ~ 1000um < 0.5 MeV → difference of $O(10^{-6})$
- Chance for Bhabha scattering within 1000 um (DocDB 172-v3, X. Qian)
  - 69.3 MeV: 4.4×10^{-5}
  - Michel spectrum: 2.4×10^{-4}
- Annihilation
  - 11.0±2.5 mb per electron in a beryllium absorber for 50MeV incident positron (doi.org/10.1103/PhysRev.89.790)
  - Assume 14 free electrons in silicon






```
positron energy =  4 MeV.        xsec per electron = 8.11478e-26 cm^2.   m.f.p. = 17.6046 cm.    P under 1mm = 56.8034 E-4
positron energy = 10 MeV.        xsec per electron = 3.98278e-26 cm^2.   m.f.p. = 35.8687 cm.    P under 1mm = 27.8795 E-4
positron energy = 20 MeV.        xsec per electron = 2.32235e-26 cm^2.   m.f.p. = 61.514 cm.     P under 1mm = 16.2565 E-4
positron energy = 30 MeV.        xsec per electron = 1.68965e-26 cm^2.   m.f.p. = 84.5484 cm.    P under 1mm = 11.8275 E-4
positron energy = 50 MeV.        xsec per electron = 1.12685e-26 cm^2.   m.f.p. = 126.775 cm.    P under 1mm = 7.88796 E-4
positron energy = 65 MeV.        xsec per electron = 9.13133e-27 cm^2.   m.f.p. = 156.447 cm.    P under 1mm = 6.39193 E-4
positron energy = 70 MeV.        xsec per electron = 8.60218e-27 cm^2.   m.f.p. = 166.071 cm.    P under 1mm = 6.02152 E-4
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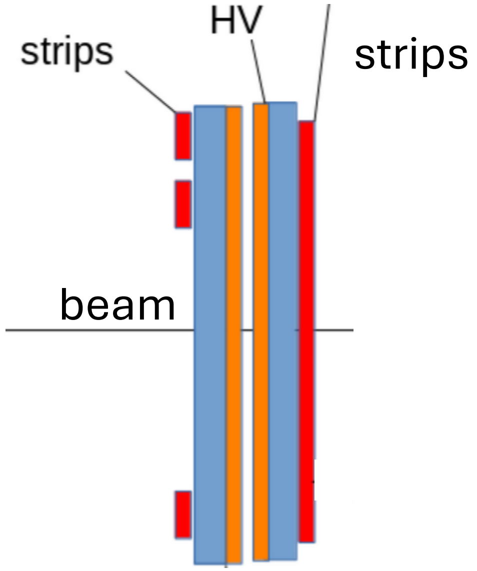
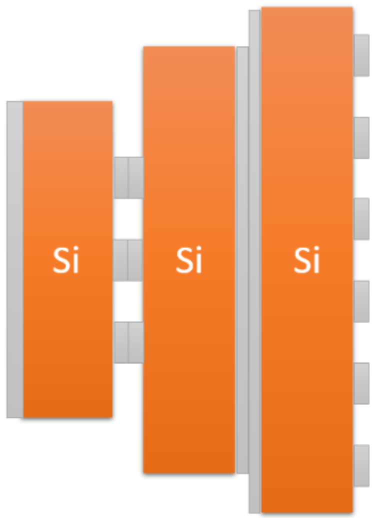
Selection on topology

- ATAR geometry
  - 1- vs. 2-side strips
- Pixel ← (x, y, z)
  - 1-side readout: 2 layers for 1 pixel
  - 2-side readout: 1 layer for 1 pixel
- Hit ← pixel + timing + energy deposition, (x, y, z, t, dE)
- Clustered hits ← hits
- Track ← clustered hits
- dE/dx ← track + energy deposition



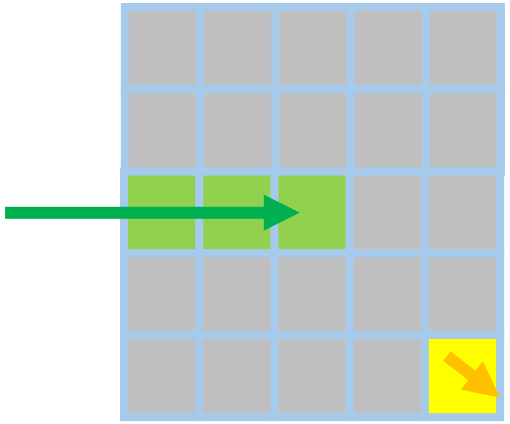


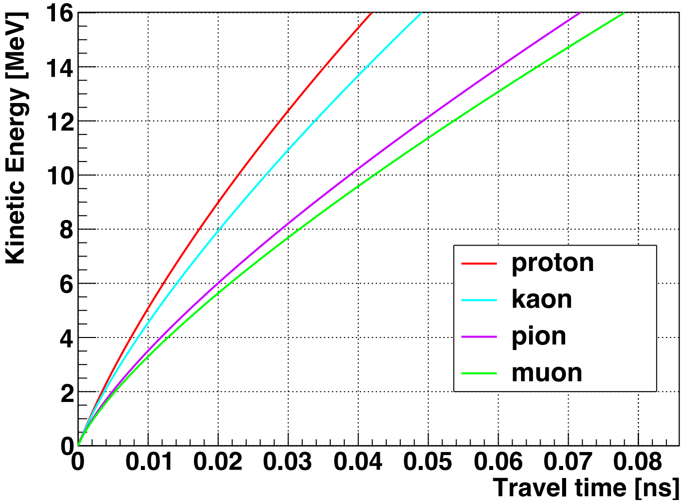
Parameters	Value
ATAR readout	Single/Double sided
ATAR layer gap	25um/4um
ATAR layer thickness	120um
ATAR strip pitch size	200um
σ_t for overlapping signal	0/1ns/20ns (ideal/LGAD/PIN)
σ_t for inter-channels	0/50ps/1ns (ideal/LGAD/PIN)
Energy resolution	0/10% / 0.1MIP (ideal/LGAD/PIN)

Used in simulation

Reconstruction and selections in simulation

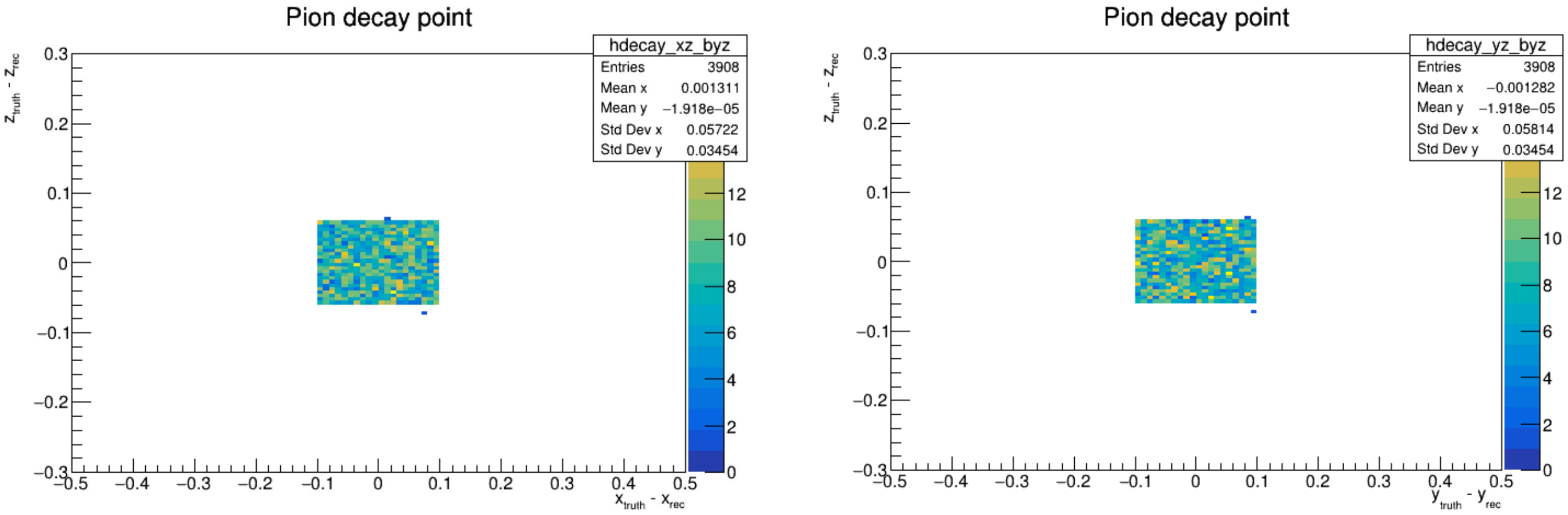
- Reconstruction of hits
  - Pixel is determined by geometry and <u>true energy deposition</u> 1/3 * 120 um * 3.875 MeV/cm = <u>0.0155 MeV</u>
  - Merge two hits at the same pixel if $|t_1 - t_2| < 1ns$
- Hit clustering in timing and space
  - Cluster pixels by timing if $|t_1 - t_2| < 1ns$ (alternative way to study σ_t for inter-channels → 1ns)
    - Collections of hits $\{t_\pi^i; t_\pi^i < t_\pi^{i+1}\}, \{t_\mu^i; t_\mu^i < t_\mu^{i+1}\}, \{t_e^i; t_e^i < t_e^{i+1}\}$
    - Identify the timing of each particle by t_π^0, t_μ^0, t_e^0
  - Cluster pixels by position by requiring adjacent pixels within $<\sqrt{200^2 + 200^2 + 120^2} \times 3$ um
- Event selection
  - $|t_\pi - t_\mu| > 5ns$ and $|t_\mu - t_e| > 5ns$ for $\pi\mu e$
  - $|t_\pi - t_e| > 5ns$ for $\pi e\nu$





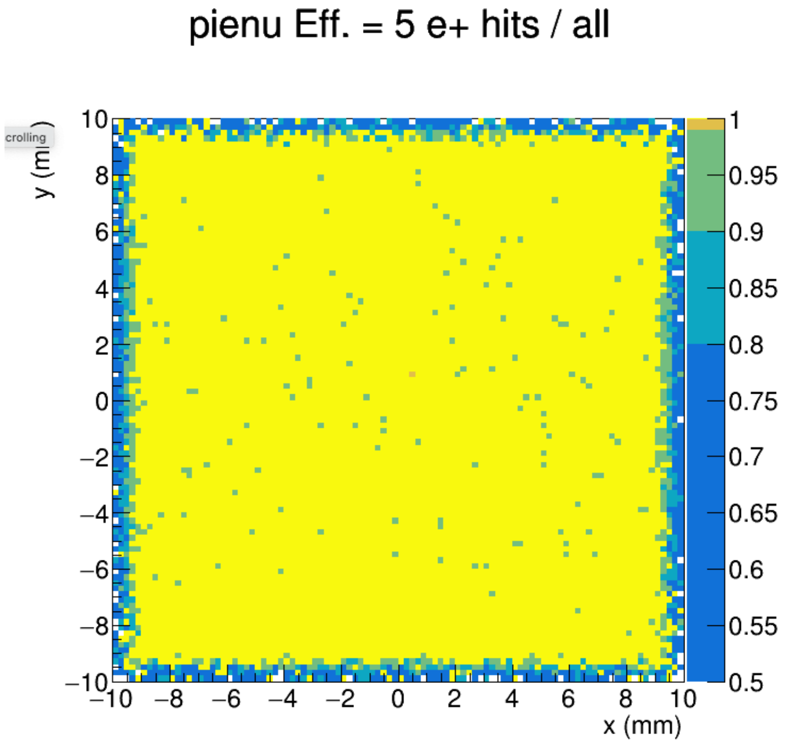
Pion decay vertex for $\pi \to \mu \to e$

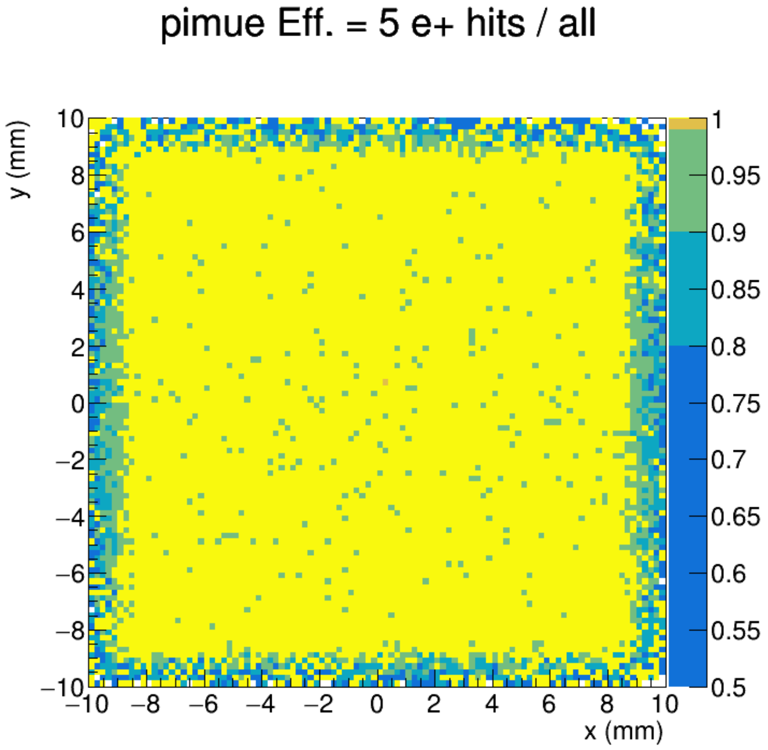
- Select last layer in z
- Pion beam is approximately perpendicular to ATAR and travels forward (positive-z)
- Not cover Q. Buat's studies for precise determination of pion vertex, (DocDB-242)

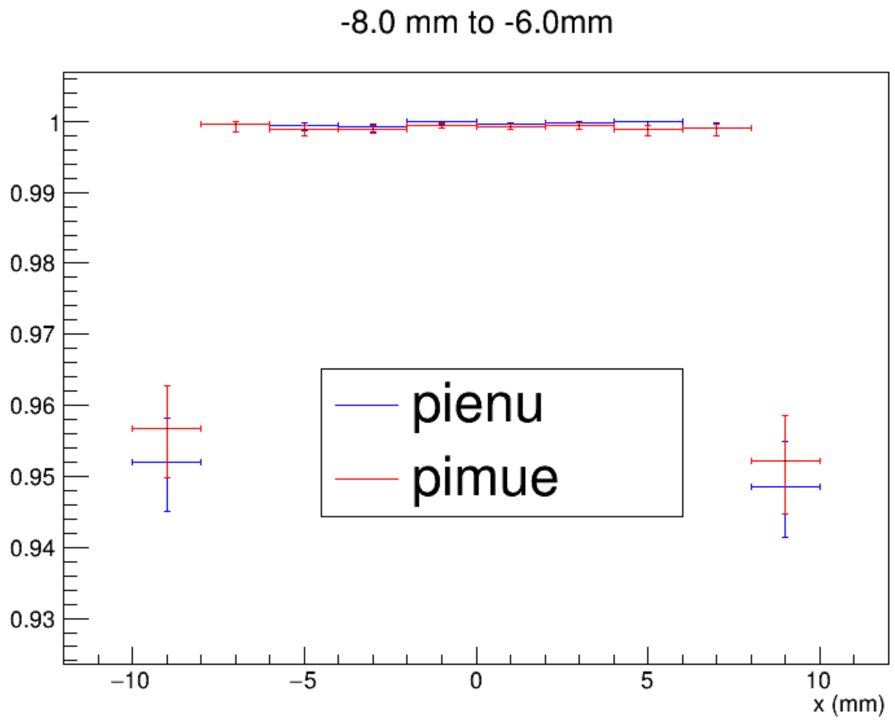


On requiring 5 hits

- Edges of $\pi \rightarrow \mu \rightarrow e$ span ~ 2mm
- Choose 2mm as binning size
  - Consistency within 0.2%, sample size O(400k)

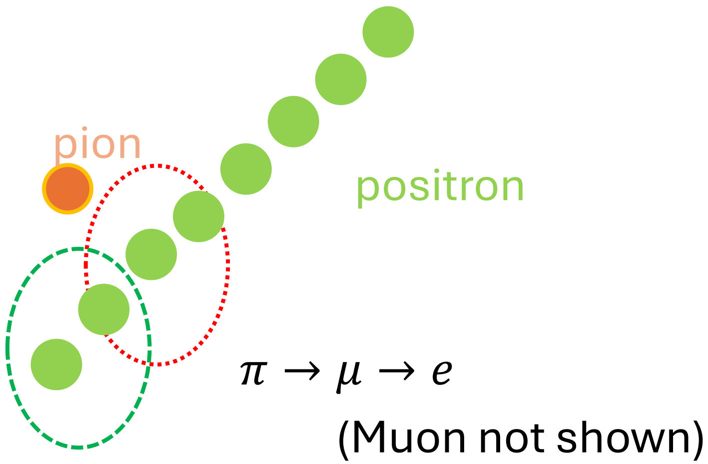


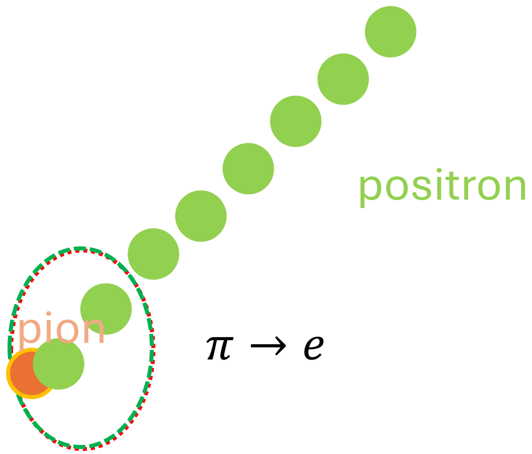




The algorithm for positron direction

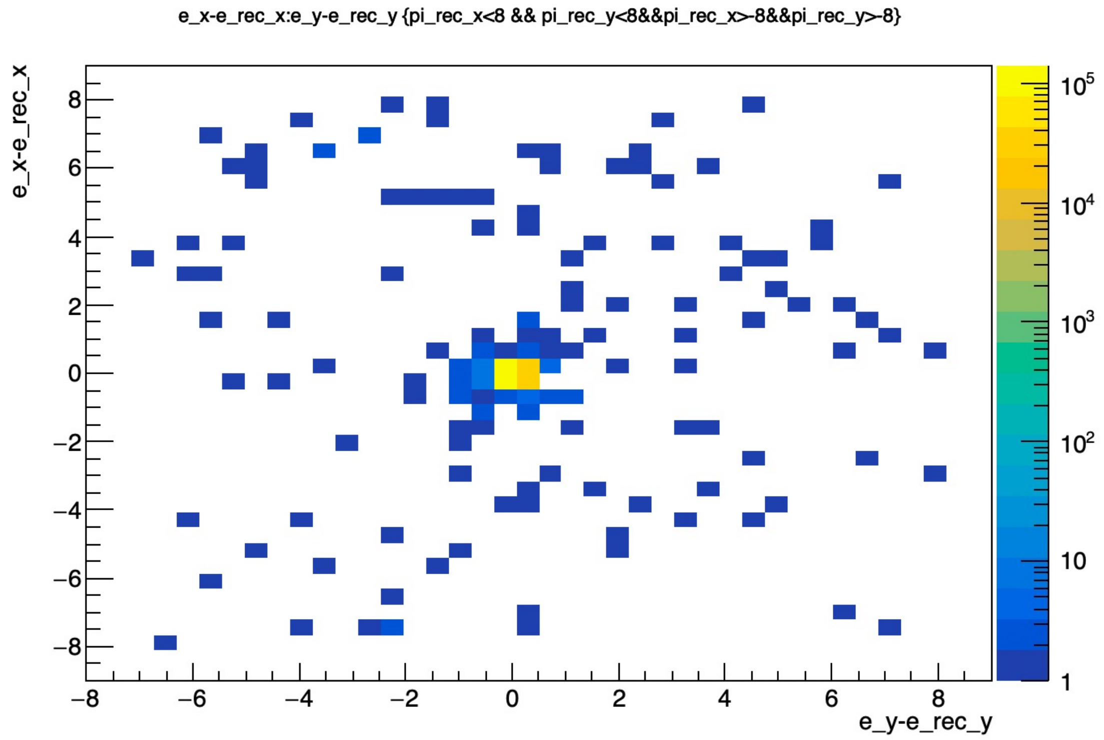
- Known pion decay point (x_π, y_π, z_π)
- Find nearest 5 hits to (x_π, y_π, z_π)
- Fit a preliminary direction $\hat{n}$
- Rank positron hits by $\vec{d}(\pi, e) \cdot \hat{n}$
- Find the extremes in ranked hits $\{e_{start}, e_{end}\}$
- Find the nearest point (x_e, y_e, z_e) to (x_π, y_π, z_π)
- Find nearest 5 hits to (x_e, y_e, z_e)
- Fit the direction $\widehat{n_e}$
- Flip the direction if necessary
  - Positive sign for $\vec{d}(e_{start}, e_{end}) \cdot \widehat{n_e}$
  - Validate (x_e, y_e, z_e)





Validation of positron starting point

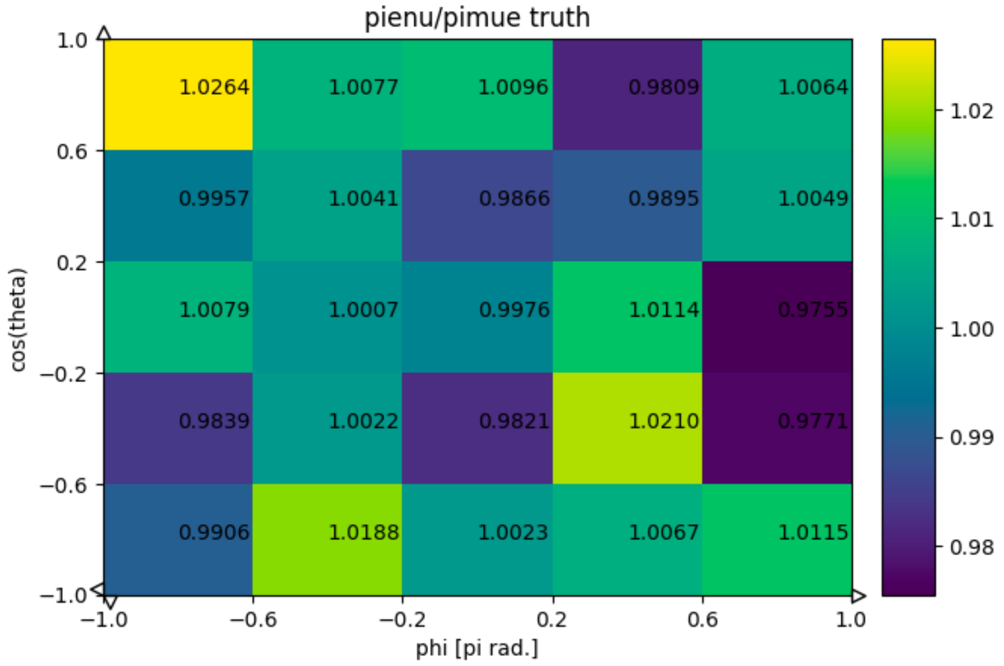
- Pion decay in the center → |x|<8mm && |y|<8mm
- Studied $\pi \to e$ (sample size O(300k))

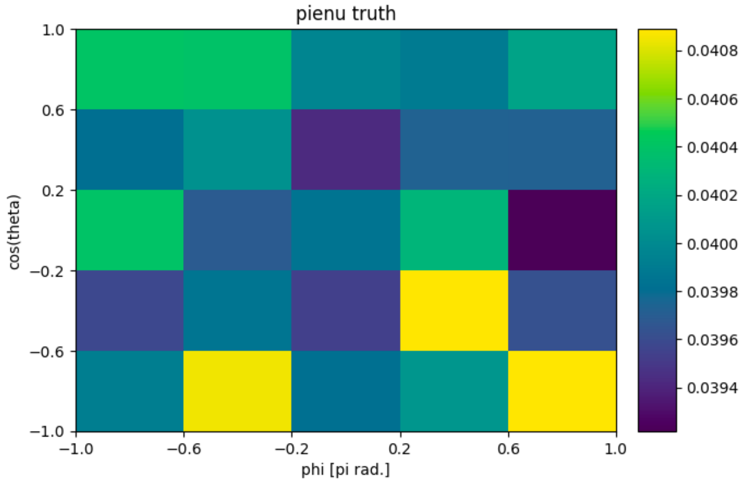


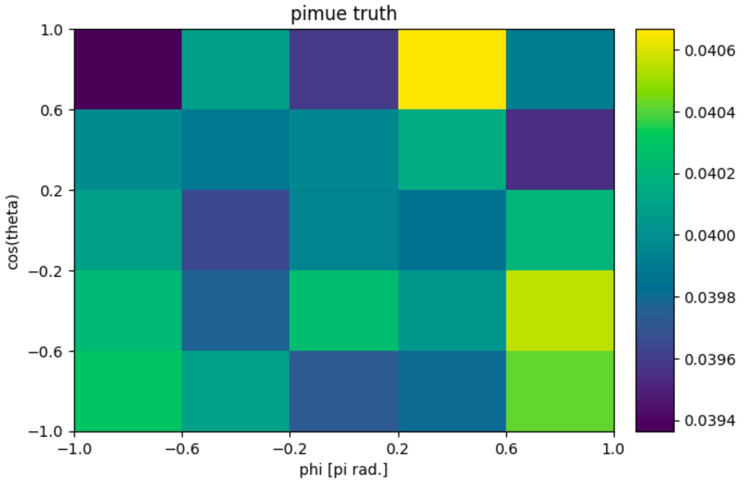
Require pion in the center

Study the positron outgoing direction

- Select events with
  - pion decay in the center of ATAR, $|x| < 8mm, |y| < 8mm$
  - 5 hits of positrons are found
- True momenta of positrons
  - Plot normalized to number of events
  - Uniform distribution implies 1/(5*5) ~ 0.04

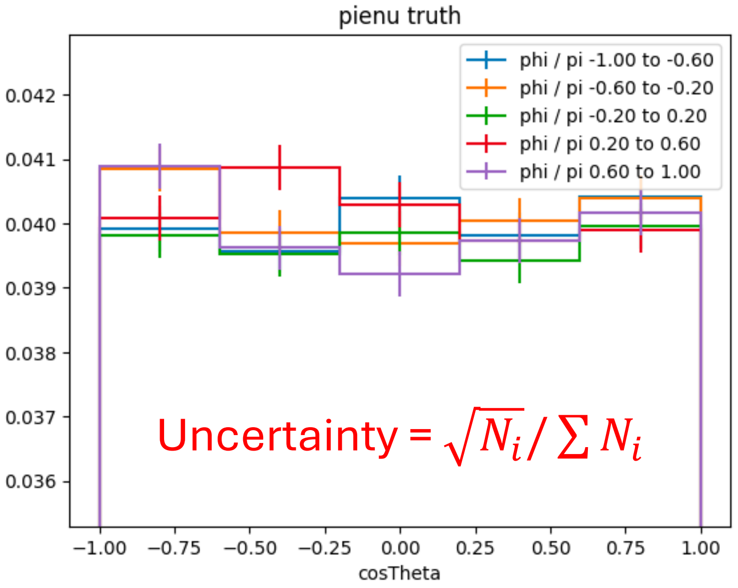


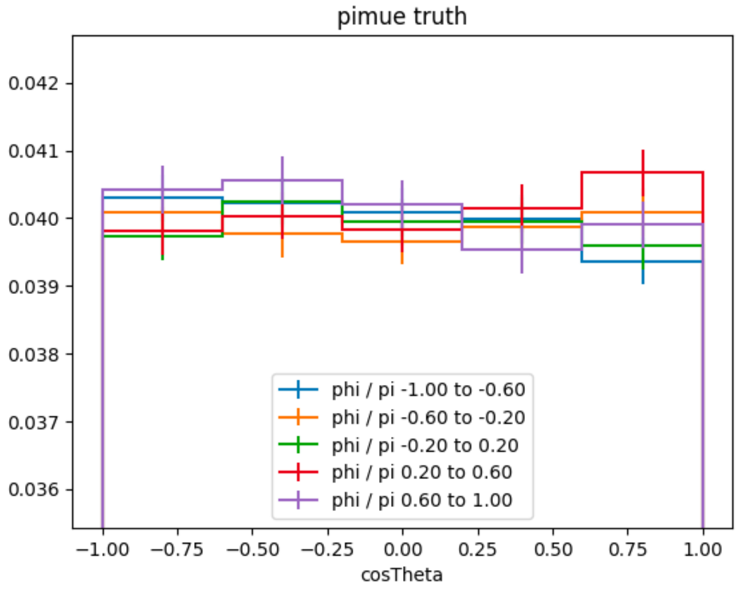


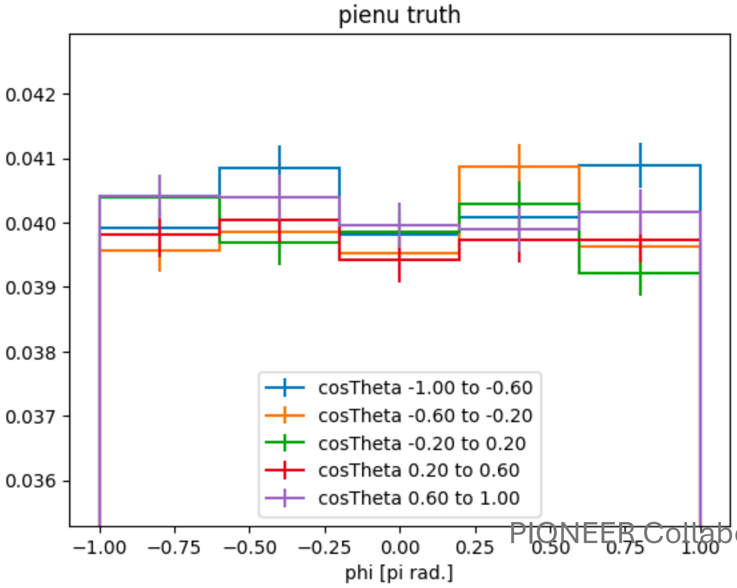


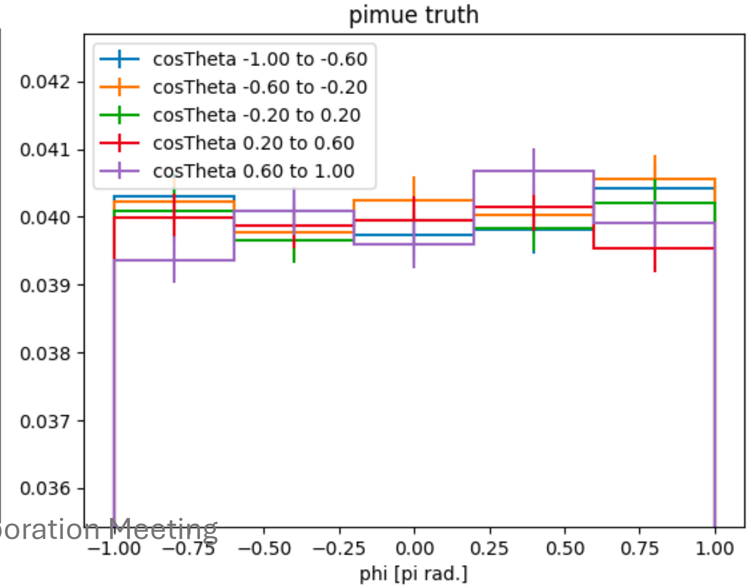
Study the positron outgoing direction

- Project to 1D and show uncertainty – uniform distribution in truth



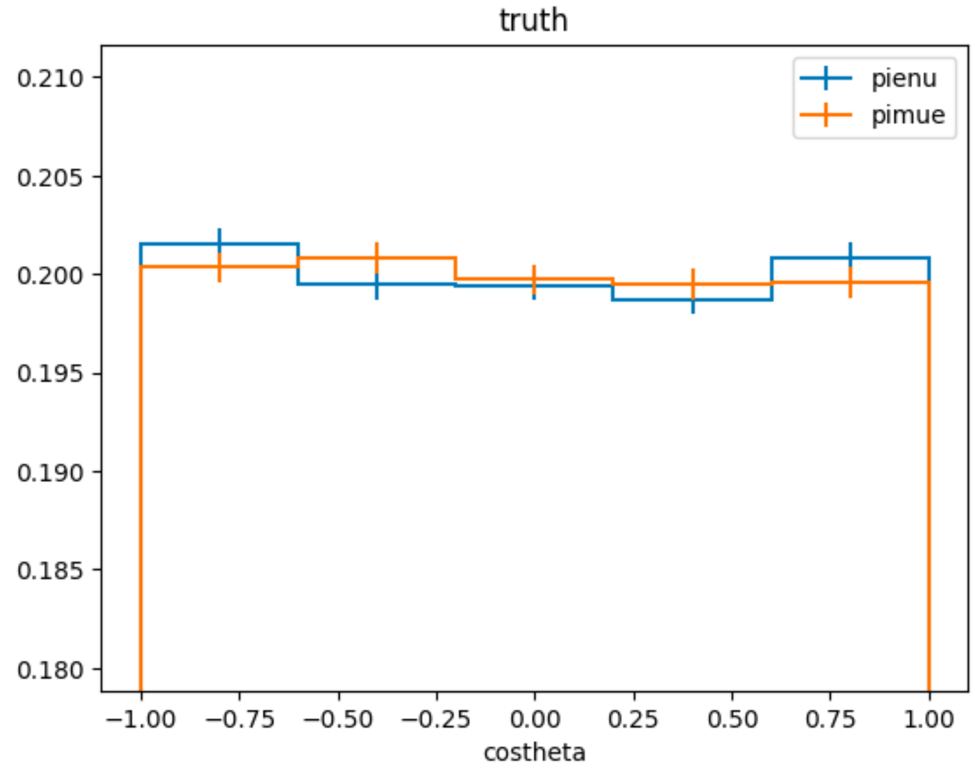


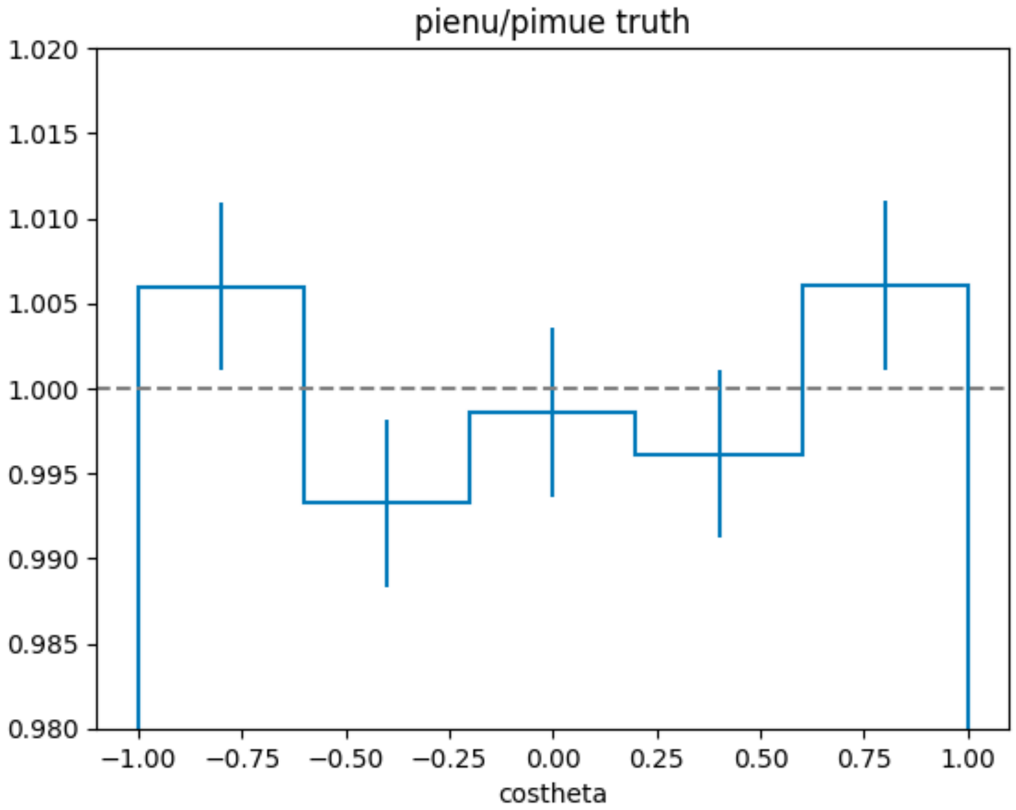




Study the positron outgoing direction

- Reduce $(\cos\theta, \varphi)$ to $(\cos\theta)$ – difference at O(0.5%) for true positron momenta



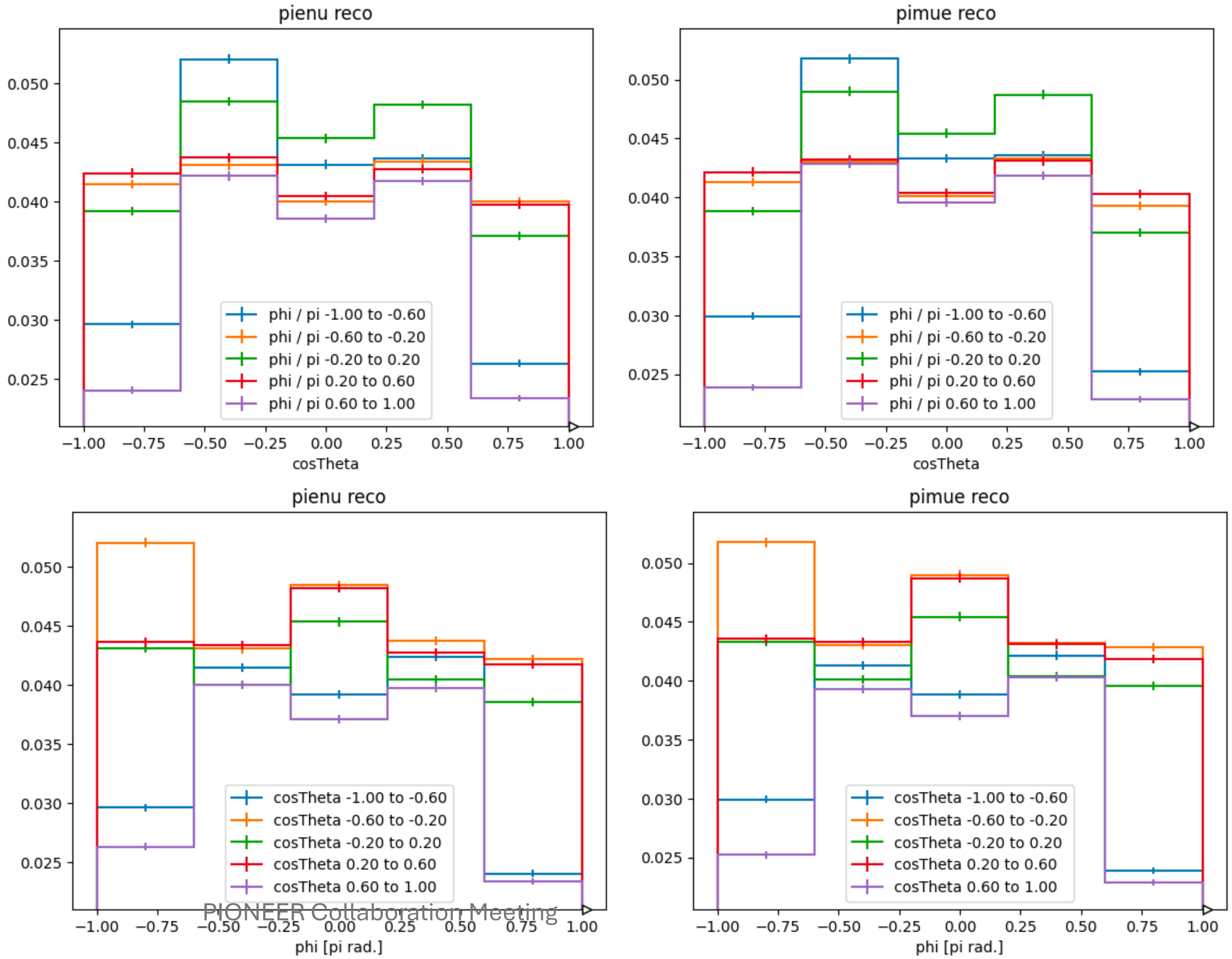


Uncertainty = $\sqrt{N_i} / \sum N_i$

Uncertainty = $\frac{\sqrt{N_1}N_2}{(N_1+N_2)^2} \oplus \frac{\sqrt{N_2}N_1}{(N_1+N_2)^2}$

Study the positron outgoing direction

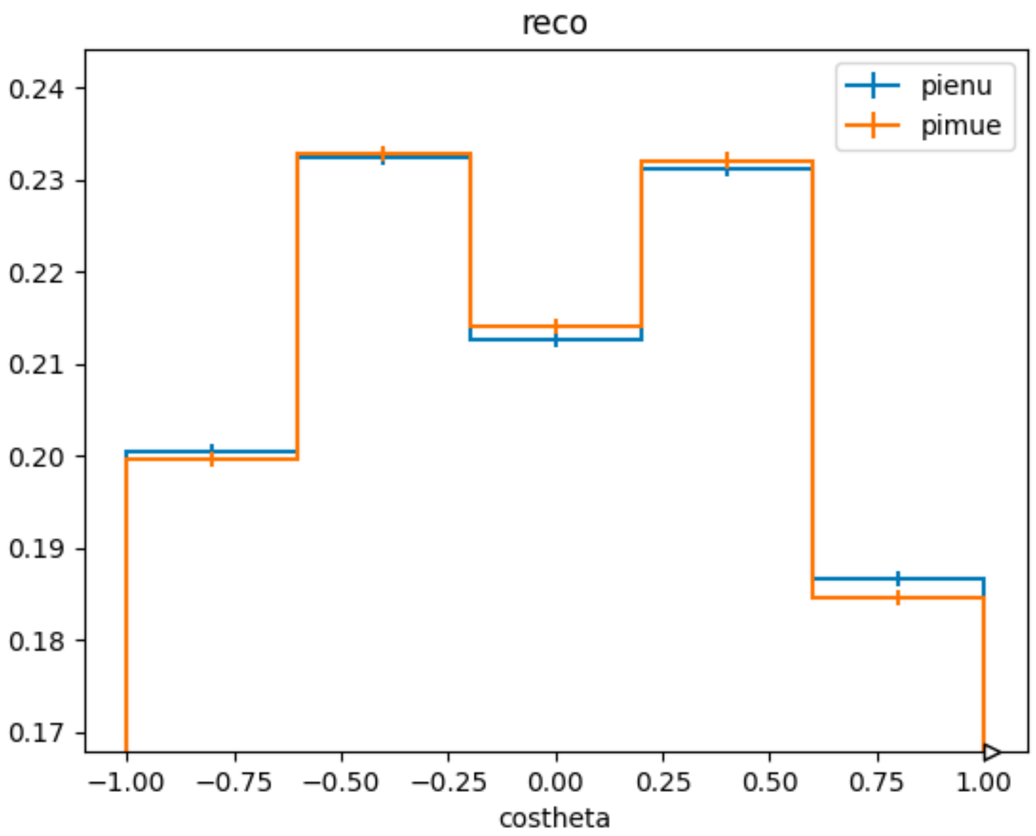
- Project to 1D and show uncertainty – reco
- Non-uniform distribution in both decay channels
  - Expected as direction from discrete pixels
  - Similar behavior

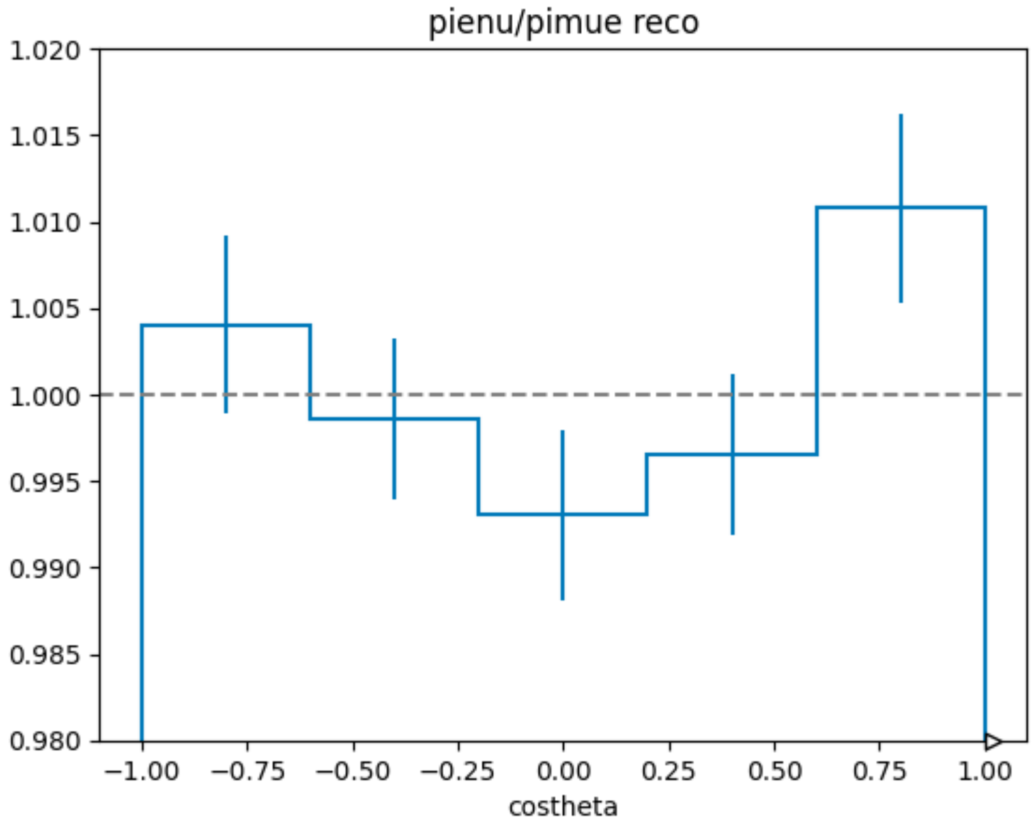


Uncertainty = $\sqrt{N_i} / \sum N_i$

Study the positron outgoing direction

- Reduce $(\cos\theta, \varphi)$ to $(\cos\theta)$ -- difference at O(0.5%) for reconstructed positron direction





Uncertainty = $\sqrt{N_i}/\sum N_i$

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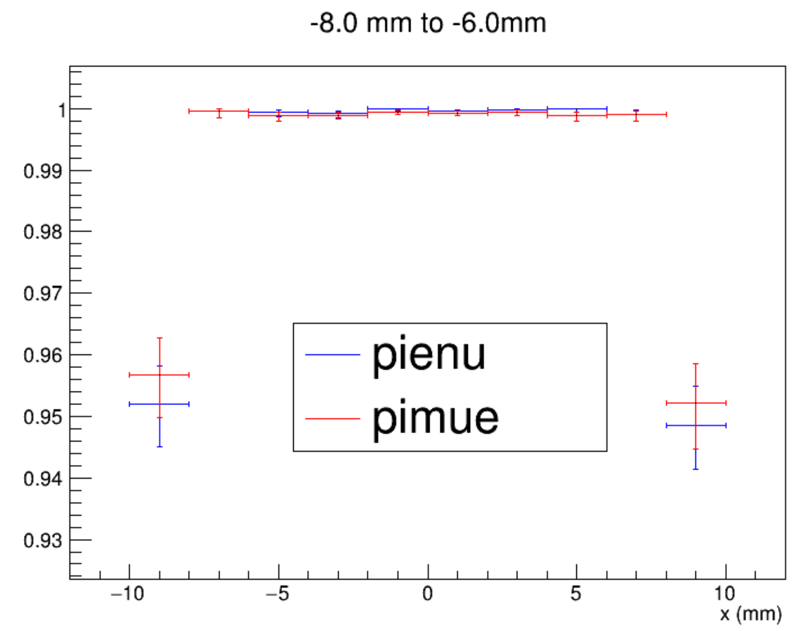
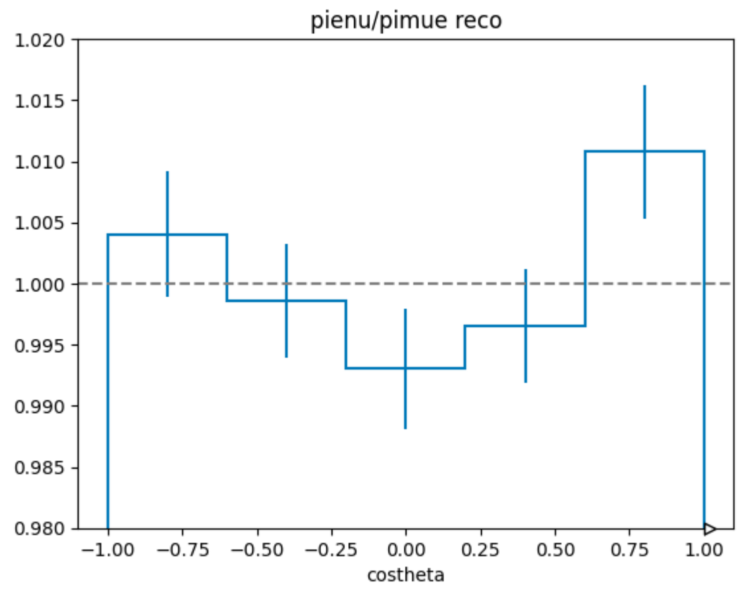
Summary and discussions

- Consistent acceptance between $\pi e\nu$ and $\pi\mu e$ given uncertainty at $O(0.5\%)$
  - Timing and topology cuts imposed
- Uniformity of angle distribution breaks in RECO but still give a relatively equal behavior between $\pi e\nu$ and $\pi\mu e$
- Future efforts
  - Improving the uncertainty estimate to O(0.01%)
    - Larger size of samples
    - Incorporating selections from other studies
    - …
  - Changing detector geometry and setup
    - timing/resolution resolution
    - …
  - Integrating studies to central software
  - …

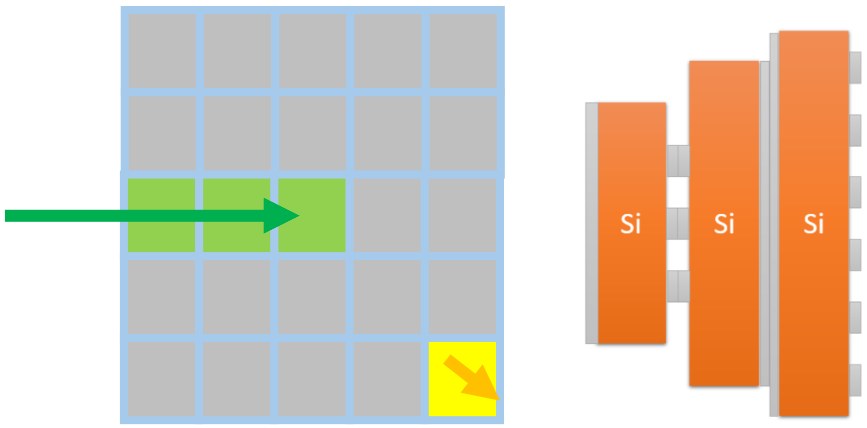


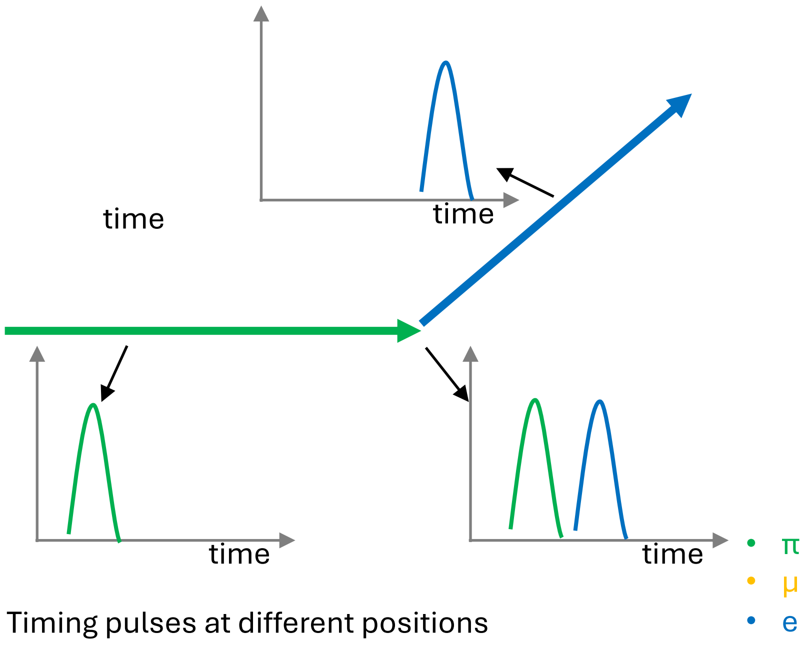


Backup

Studies of in-out definition

- Pure pion beam at 1200 mm upstream
  - Pion beam momentum 55 +/- 1.1 MeV
  - Gaussian-shaped with waist sigma ~ 10mm
- Truth level
  - Pion DAR (K.E. < 1keV, to be tuned)
- RECO level
  - ATAR-only Geometry
    - Double-sided shared readout with little gap (=4um)
    - Thickness = 120 um
    - Strip pitch size = 200 um
  - Reconstruction
    - Hits are combinations of two strips and layer, abstracted by pixel location + timing
    - Pixel is determined by geometry and true energy deposition 1/3 * 120 um * 3.875 MeV/cm = 0.0155 MeV
    - Mean ionization energy for pure silicon I_0 = 3.62 eV, 4k electrons for 0.0155MeV
    - Merge two hits at the same pixel if $|t_1 - t_2| < 1ns$ (to be tuned)
    - Cluster pixels by timing if $|t_1 - t_2| < 1ns$
    - Cluster pixels by position by requiring adjacent pixels within two units, sqrt(0.2*0.2*2 + 0.12*0.12)*3 um
  - Selections (event-level)
    - $|t_\pi - t_e| > 5ns$ (to be tuned)
    - $|t_\pi - t_\mu| > 5ns$ and $|t_\mu - t_e| > 5ns$
    - Subject to the exponential distribution as discussed previously

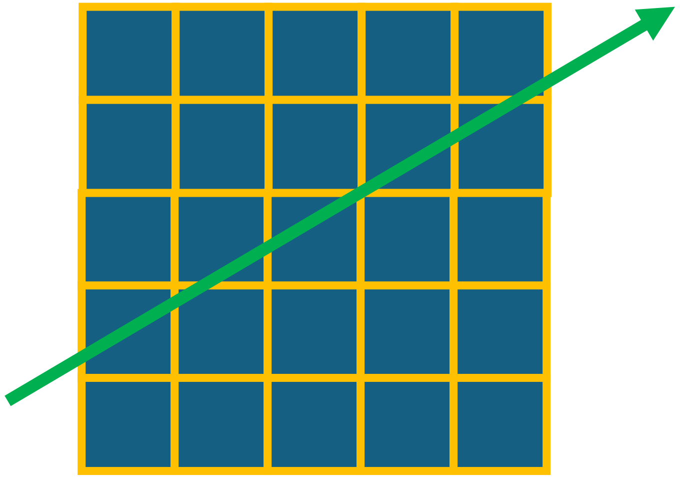




Timing pulses at different positions

Strategy

- Divide G4Step into smaller pieces
- Each piece is assigned to a pixel in grid defined by double-sided strips
- Each piece is a “rec hit”
- Rec hit position is defined as the center of pixel
- Rec hits in the same cell merged together as long as $\Delta t < 1$ns



Separation between μ and μ→e – continued

- A quick look at $(t_\mu - t_{\mu\to e}) > 5\text{ns}$
  - Assume precision of pion lifetime is $O(10^{-4})$.
  - Relative difference in acceptance caused by extrapolation is of order $O(10^{-6})$

In[1]:= `t1 = 2197.03`

Out[1]= 2197.03

In[2]:= $$\partial_{t2}\left(\left(\int_0^{700} e^{-x/t2}/t2\,dx\right)\Big/\left(\int_0^{700}\int_{x+5}^{700} e^{-(y-x)/t1}/t1\, e^{-x/t2}/t2\,dy\,dx\right)\right)$$

Out[2]= $$-\frac{700\,e^{-700/t2}}{t2^2\left(0.997727+\frac{1597.59}{-2197.03+1.\,t2}+e^{-700./t2}\left(-0.997727-\frac{2197.03}{-2197.03+1.\,t2}\right)\right)}-\frac{\left(1-e^{-700/t2}\right)\left(-\frac{1597.59}{(-2197.03+1.\,t2)^2}+\frac{2197.03\,e^{-700./t2}}{(-2197.03+1.\,t2)^2}+\frac{700.\,e^{-700./t2}\left(-0.997727-\frac{2197.03}{-2197.03+1.\,t2}\right)}{t2^2}\right)}{\left(0.997727+\frac{1597.59}{-2197.03+1.\,t2}+e^{-700./t2}\left(-0.997727-\frac{2197.03}{-2197.03+1.\,t2}\right)\right)^2}$$

In[3]:= `t2 = 26.033`

Out[3]= 26.033

In[4]:= `N[%2] * t2 * 0.0001`

Out[4]= 0.0000128696

In[5]:= $$N\left[\left(\int_0^{700} e^{-x/t2}/t2\,dx\right)\Big/\left(\int_0^{700}\int_{x+5}^{700} e^{-(y-x)/t1}/t1\, e^{-x/t2}/t2\,dy\,dx\right)\right]$$

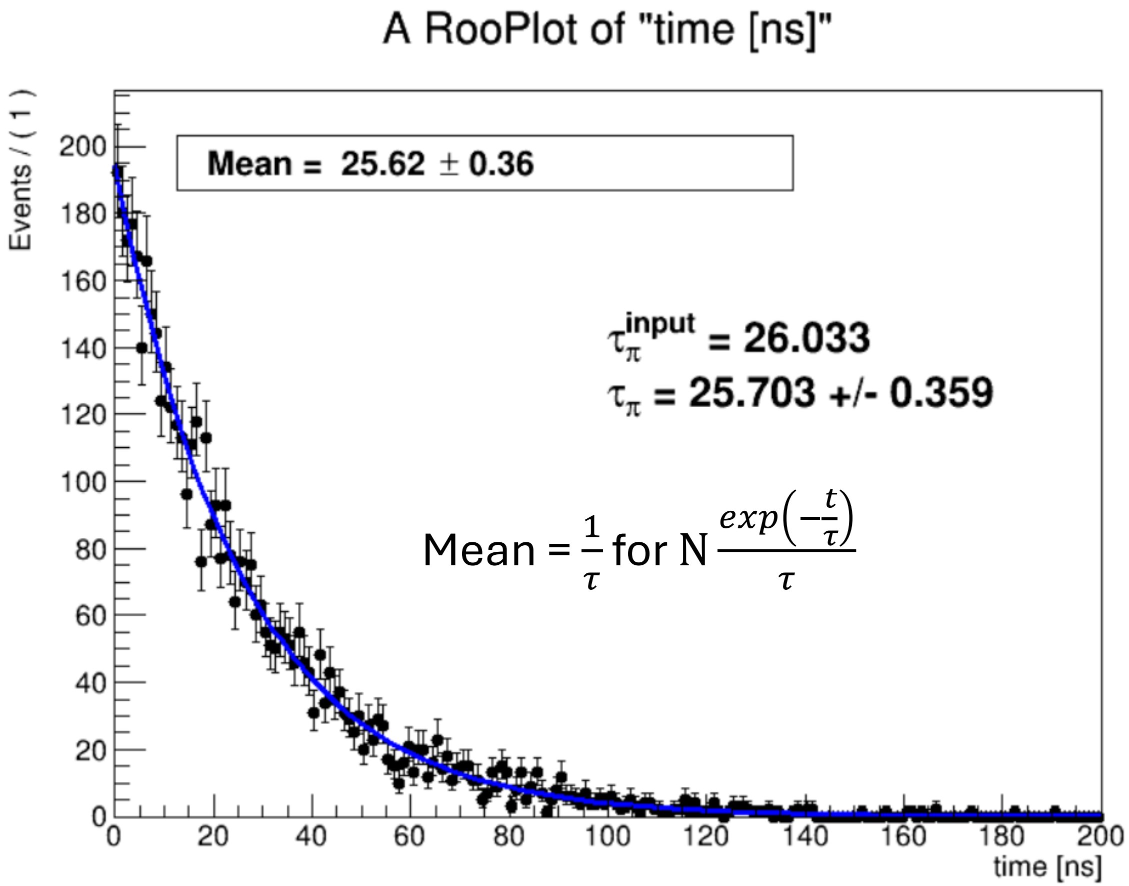
Out[5]= 3.81899

In[7]:= `%4 / %5`

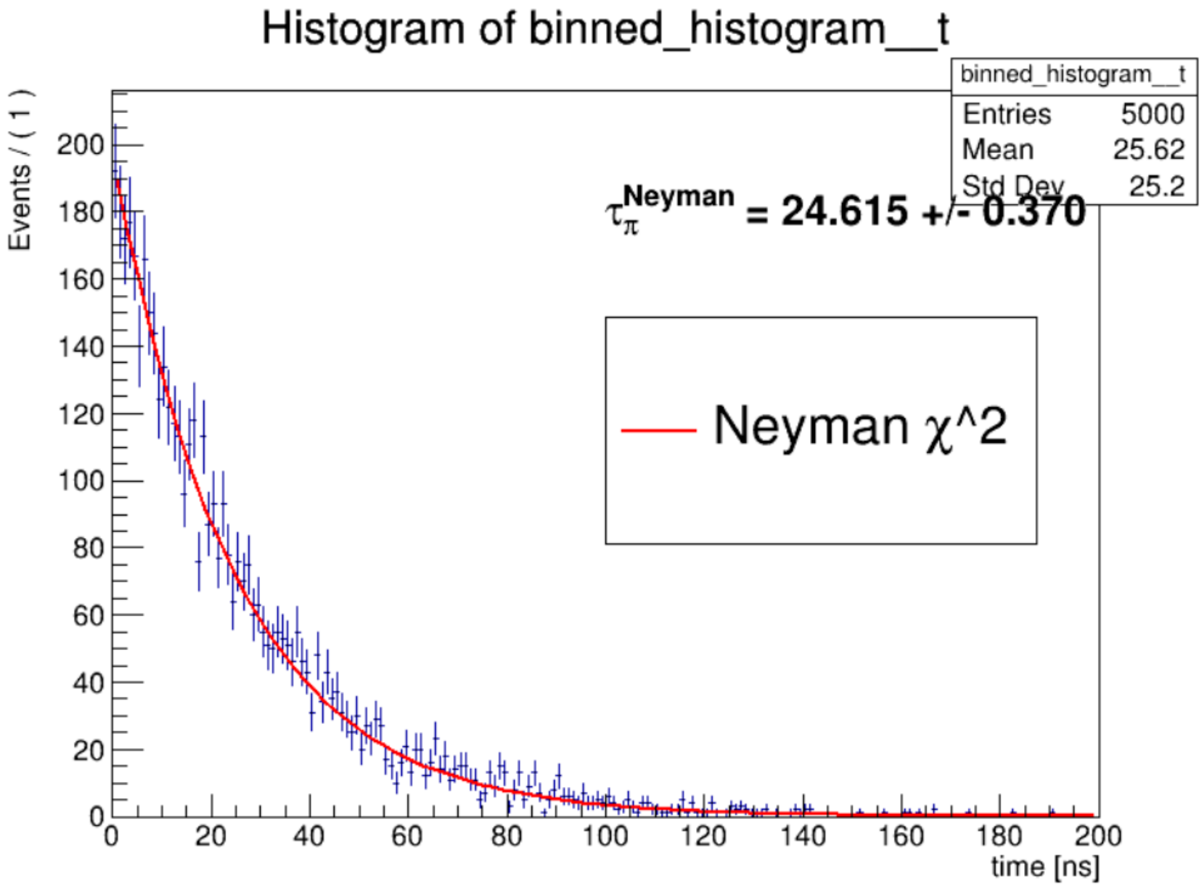
Out[7]= 3.36991×10^{-6}

Bias from binned chi2 fit

- Observe differences between unbinned maximum likelihood fit and binned chi2 fit

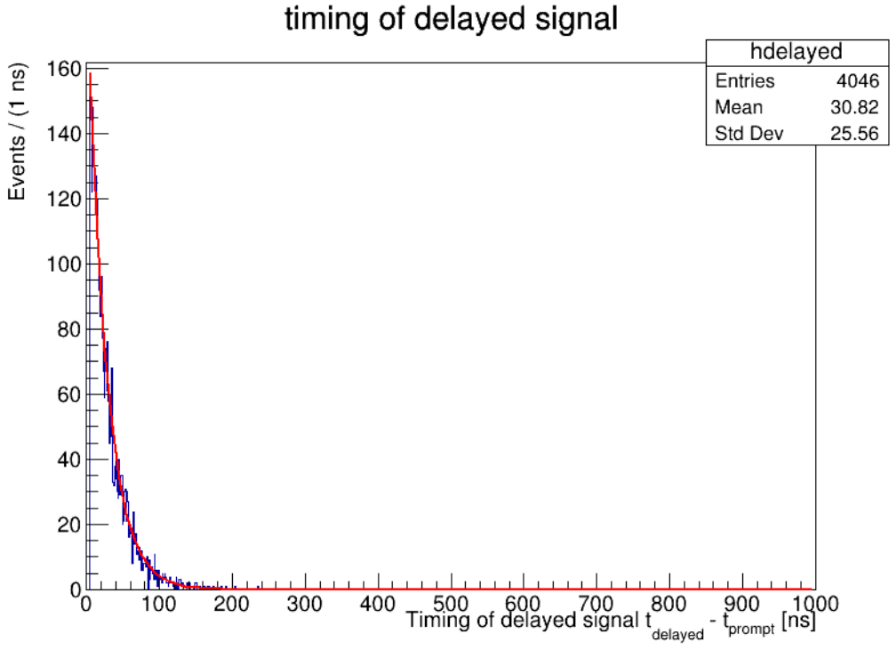


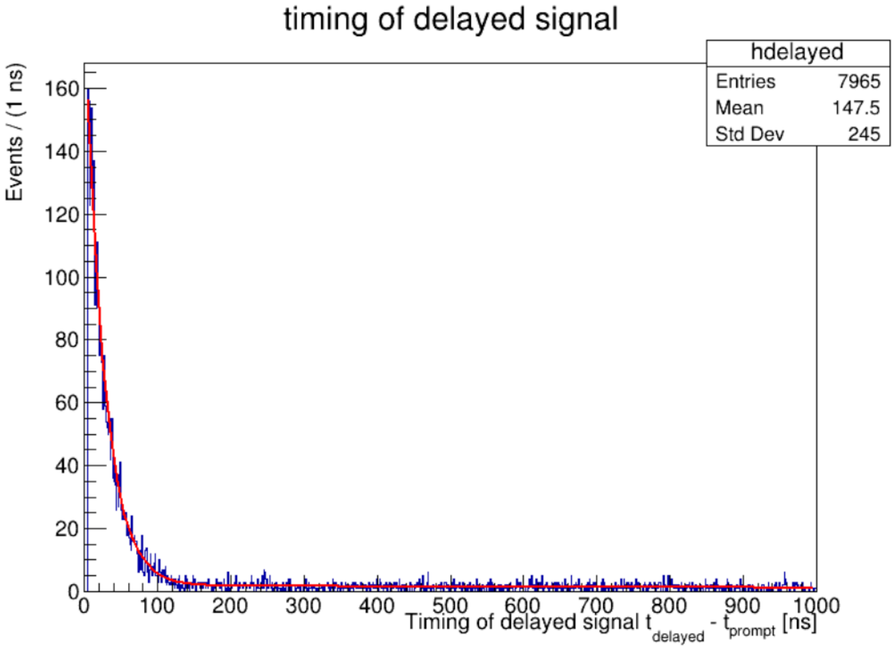
$\text{Mean} = \frac{1}{\tau}$ for $N\frac{exp\left(-\frac{t}{\tau}\right)}{\tau}$



Compare with expectation

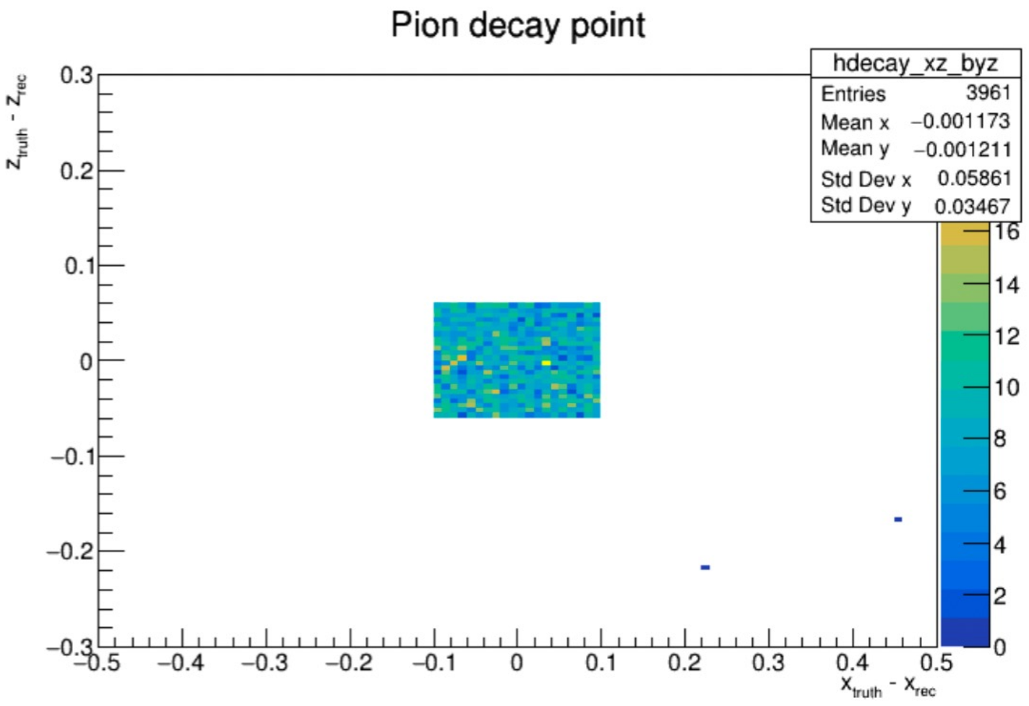
- Left: Red curve: function: $\mathrm{N}e^{-\frac{t}{\tau_\pi}}/\tau_\pi$, parameters are plugged in, no fit
- Right: Red curve: $\mathrm{N}(e^{-\frac{t}{\tau_\pi}} + \int_5^{t-5} dt' e^{-\frac{t-t'}{\tau_\mu}} e^{-\frac{t'}{\tau_\pi}}/(\tau_\mu \tau_\pi)\ )$, parameters are plugged in, no fit

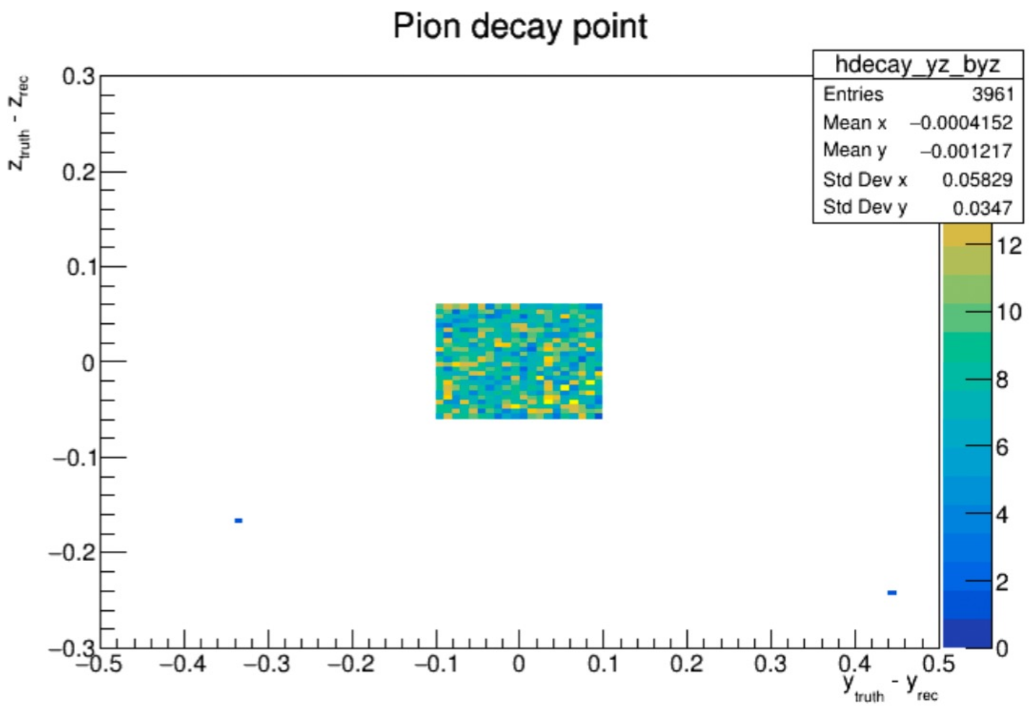




Pion decay vertex for $\pi \rightarrow e$

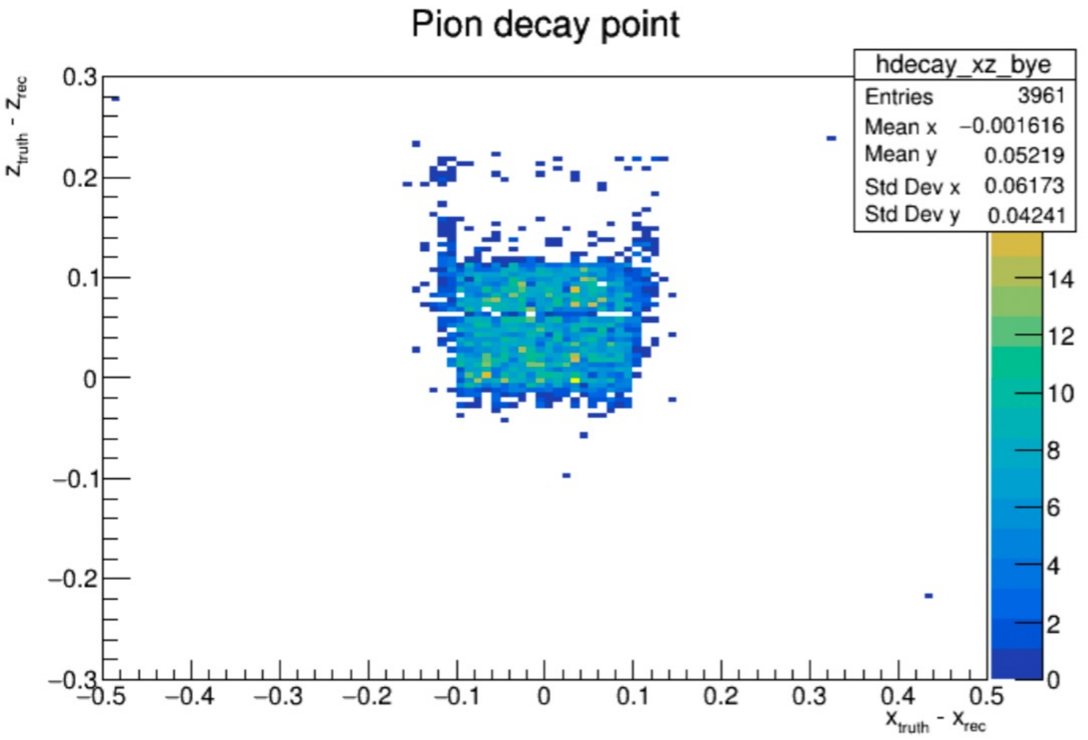
- Select last layer in z
- Pion beam is approximately perpendicular to ATAR and travels forward (positive-z)
- Not cover Q. Buat's studies for precise determination of pion vertex, (DocDB-242)

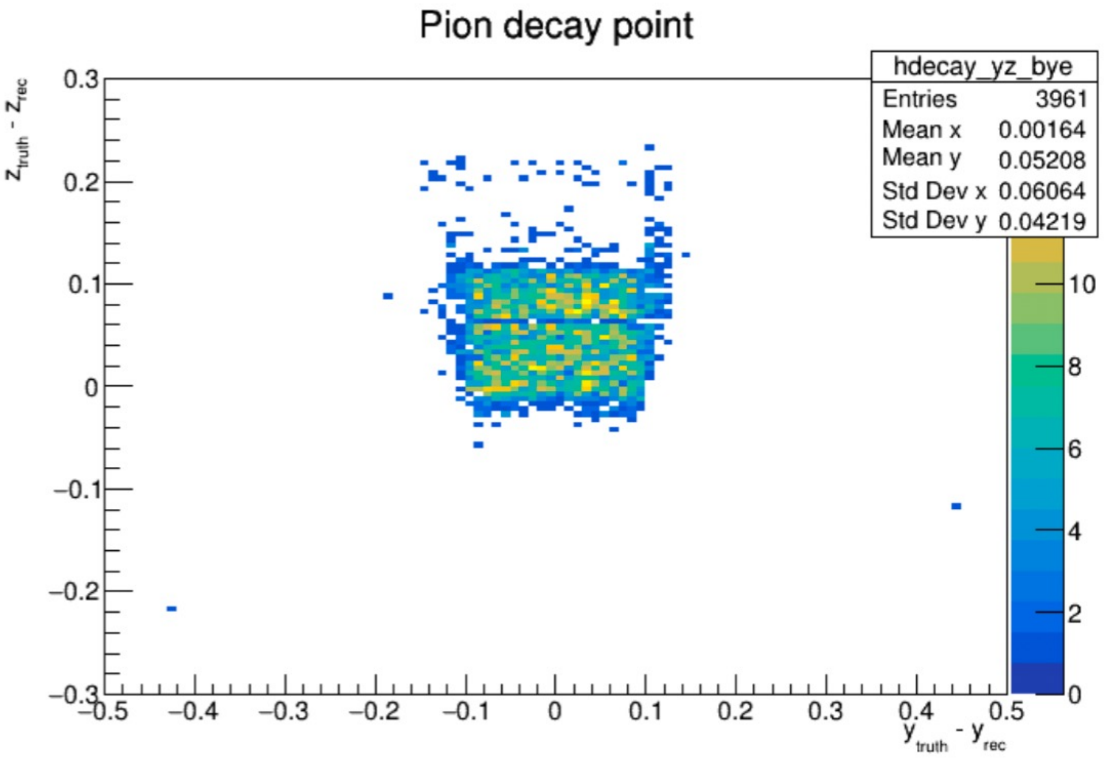




Pion decay vertex by largest dE

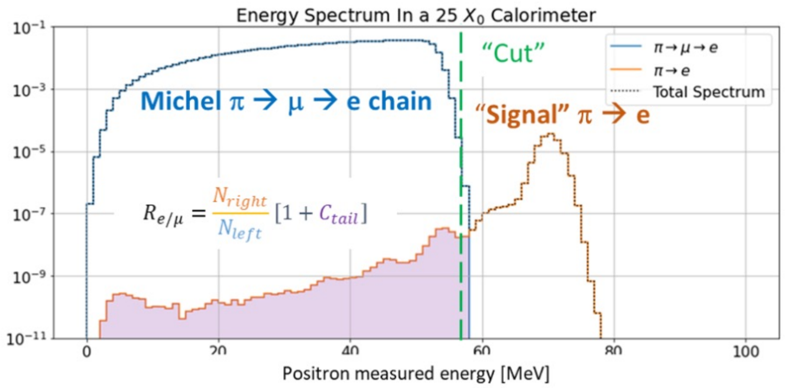
- dE/dx is increasing as momentum falls. Pion stops at Bragg Peak
  - dE/dx * dx for small dx may not be largest at decay vertex

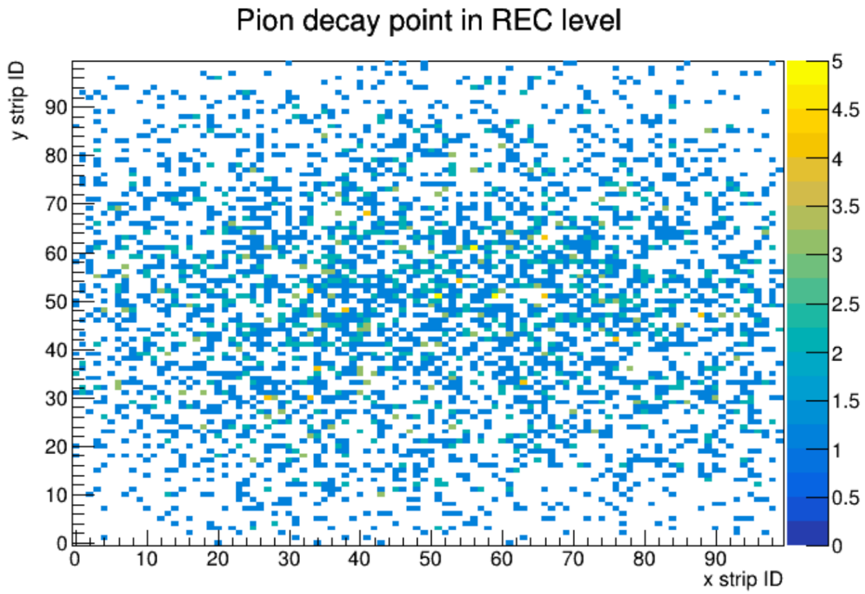




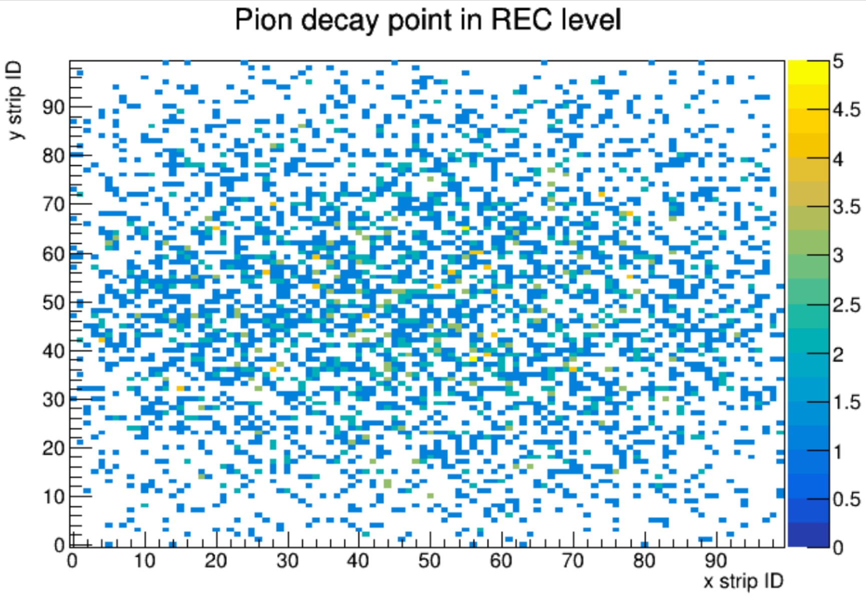
Pion decay position distribution

- A requirements of 5-hits for positrons introduce an energy threshold for positrons





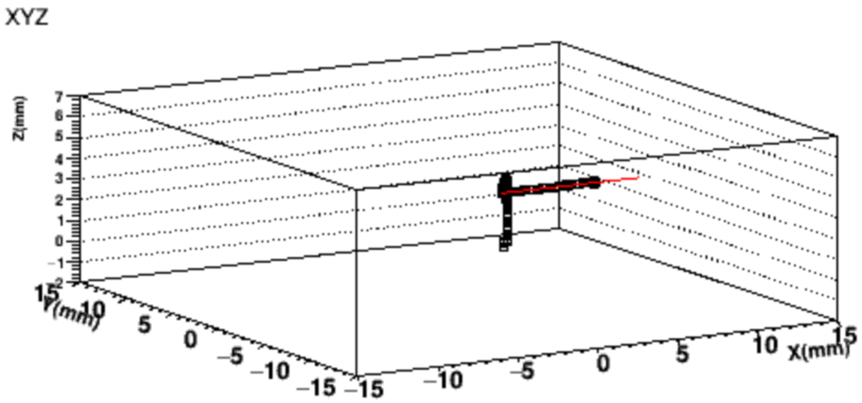
Counts by requiring 5 hits for $\pi \to e$



Counts by requiring 5 hits for $\pi \to \mu \to e$

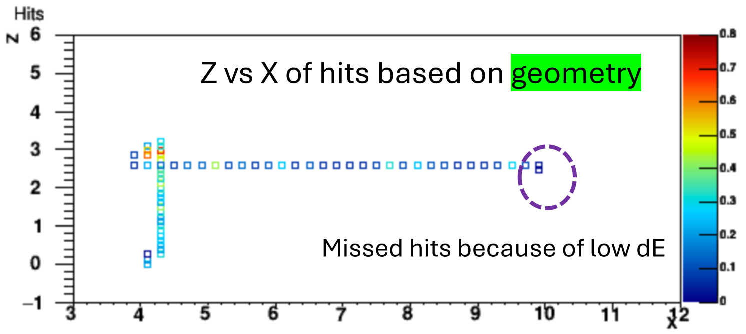
Event display of hit and track

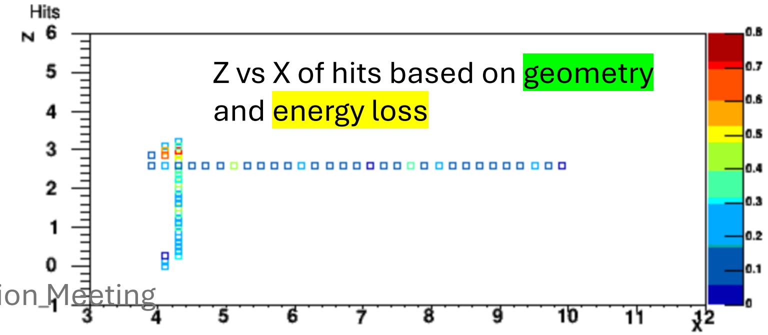
- Fitting positron direction using first 5 hits



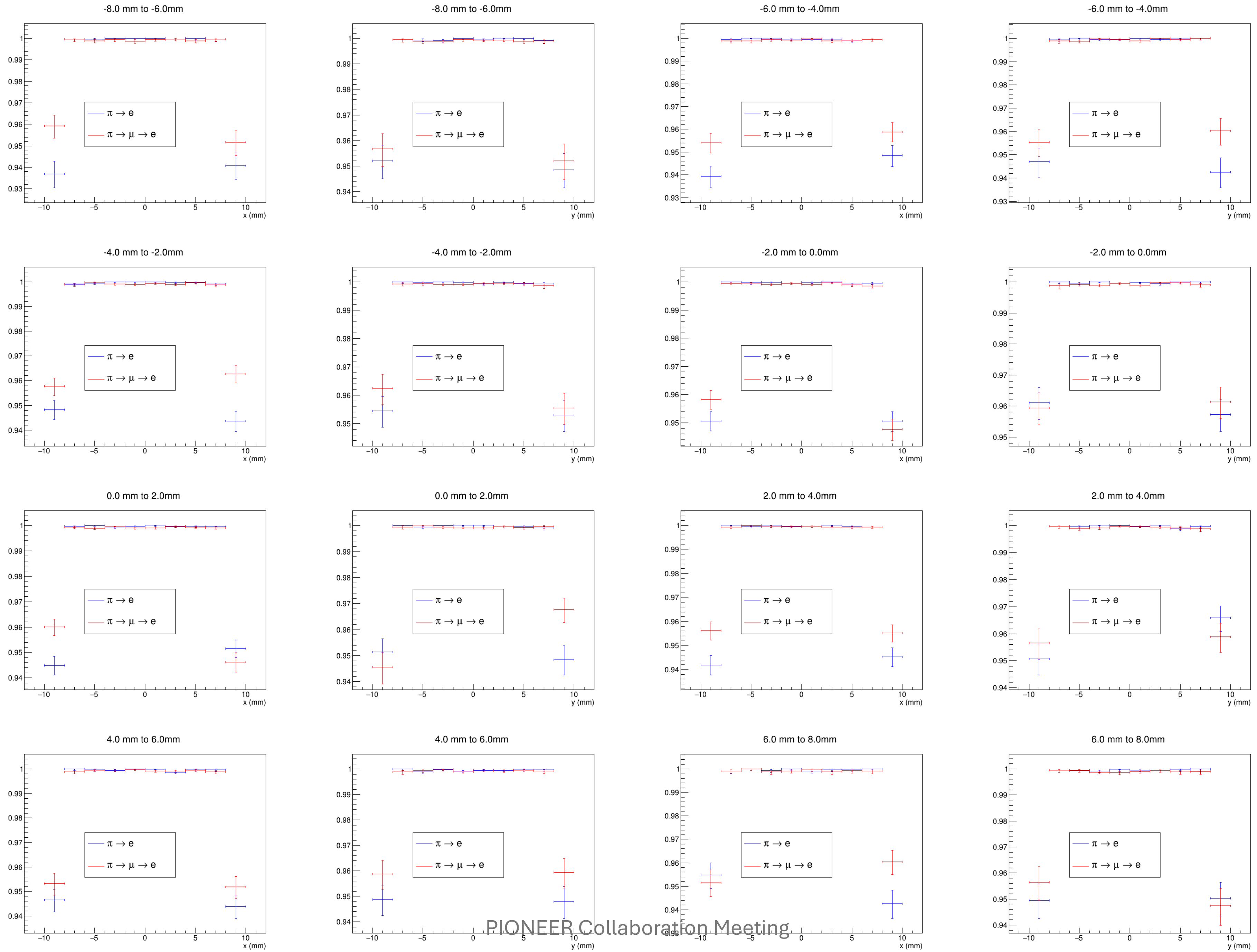
- An illustration of $\pi \to \mu \to e$ events.
- Red line shows fitted direction and extrapolates to very far side.

- Put a (tunable) energy threshold on "hits"
  - Energy depositions of merged ATAR responses must exceed threshold
  - A tentative cuts is 1/3 * 120 um * 3.875 MeV/cm = 0.0155 MeV



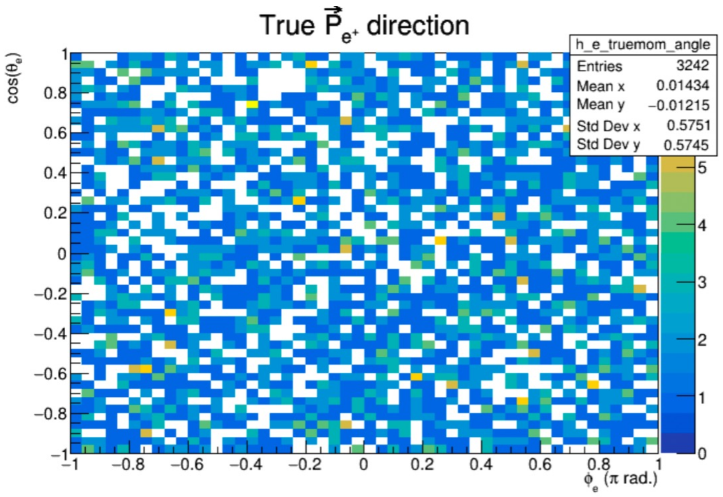


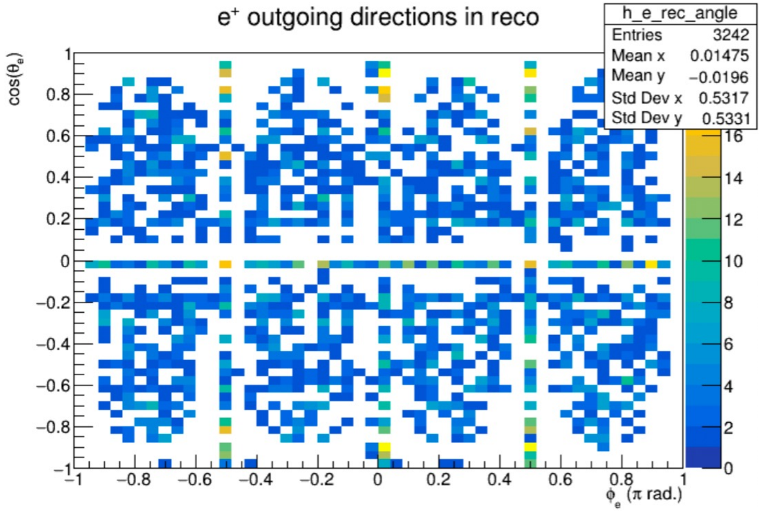
- $\pi \to \mu \to e$ event
- Same event as the left
- X and Z are in mm
- Color represents dE

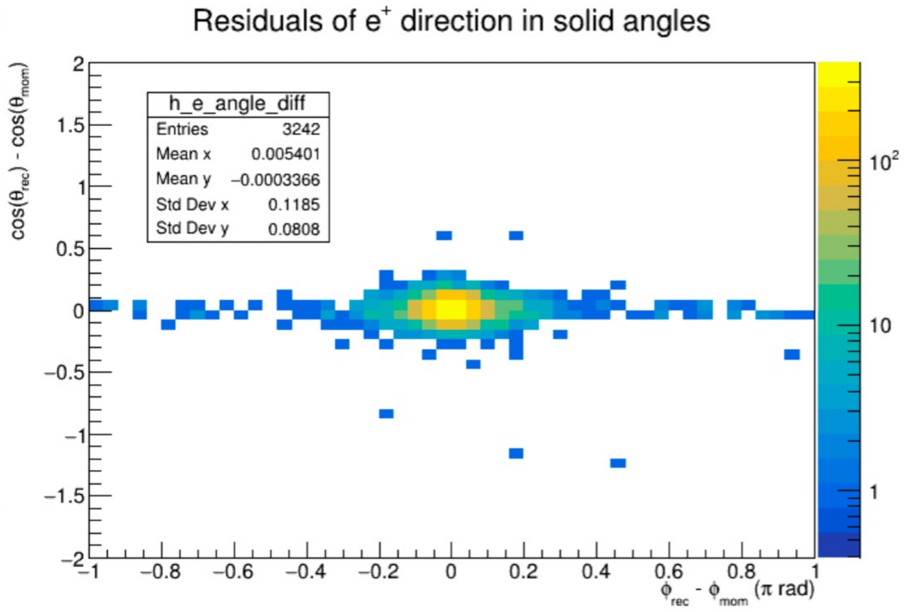

-8.0 mm to -6.0mm
x (mm)
-8.0 mm to -6.0mm
y (mm)
-6.0 mm to -4.0mm
x (mm)
-6.0 mm to -4.0mm
y (mm)
-4.0 mm to -2.0mm
x (mm)
-4.0 mm to -2.0mm
y (mm)
-2.0 mm to 0.0mm
x (mm)
-2.0 mm to 0.0mm
y (mm)
0.0 mm to 2.0mm
x (mm)
0.0 mm to 2.0mm
y (mm)
2.0 mm to 4.0mm
x (mm)
2.0 mm to 4.0mm
y (mm)
4.0 mm to 6.0mm
x (mm)
4.0 mm to 6.0mm
y (mm)
6.0 mm to 8.0mm
x (mm)
6.0 mm to 8.0mm
y (mm)
π → e
π → μ → e

Validation of the fitting algorithm

- Require pion decay in the center, |x|<8mm && |y|<8mm
- Studied $\pi \to e$ (sample size O(3k))
- Better determination of the positron outgoing direction



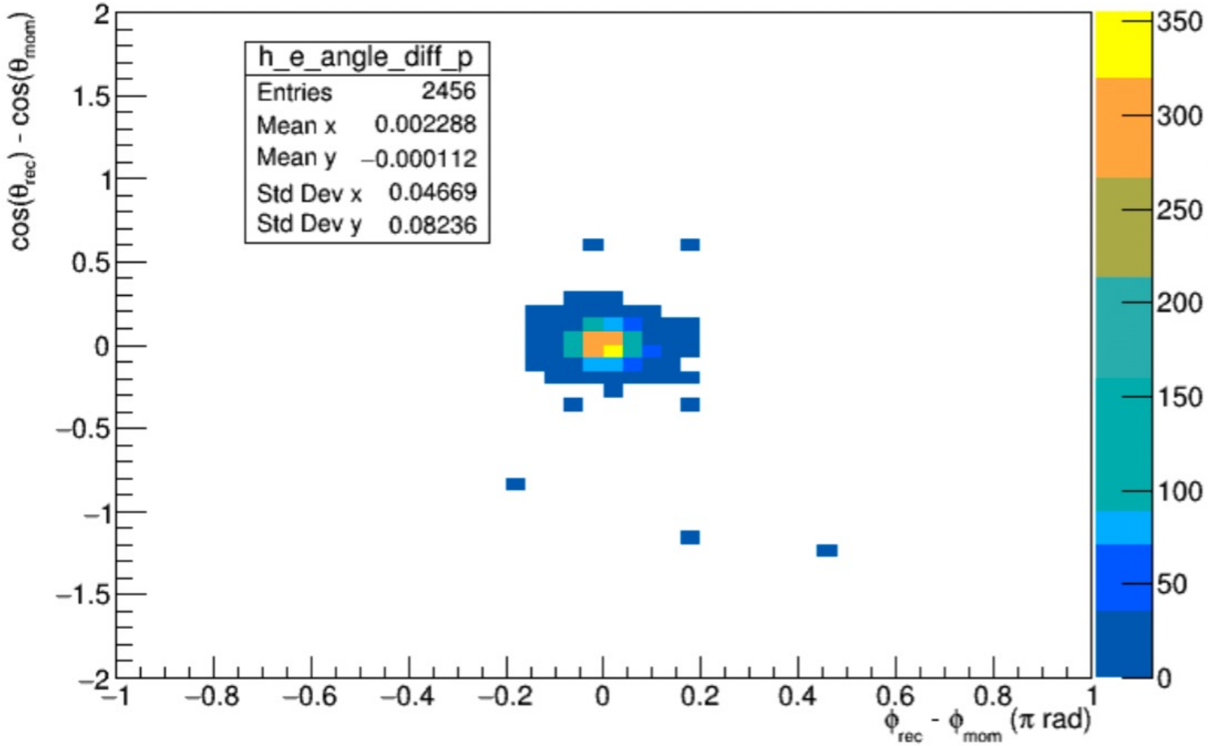




- Pick events with delta phi ~pi to check if their theta ~ 0 or pi

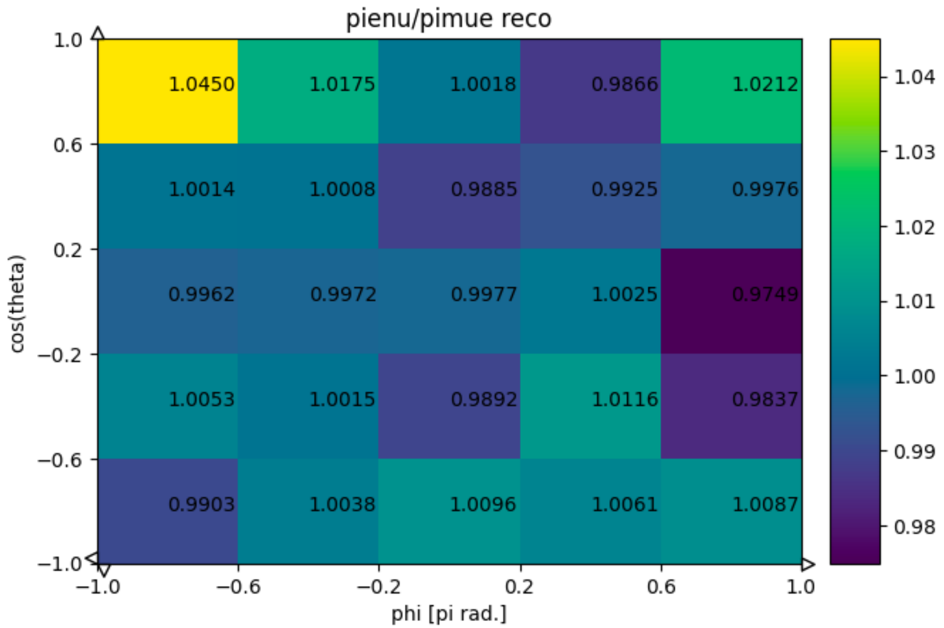
Impact from polar angle

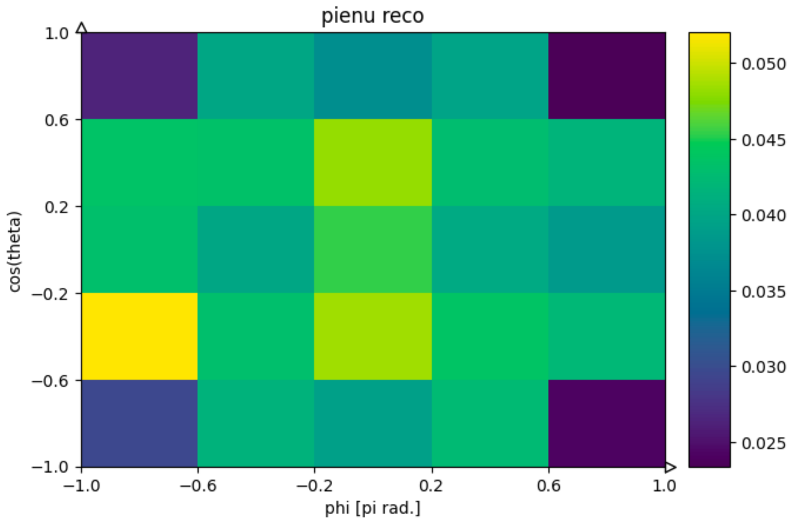
- Require pion decay in the center, |x|<8mm && |y|<8mm
- Studied $\pi \rightarrow e$
- Require at least one pixel to be 200um (one-strip) away in x- or y-direction from the pixel where positron starts
- Separate samples according to polar angle is necessary
  - Reduce $(\cos\theta_e, \phi_e)$ to $(\cos\theta_e)$ maybe useful for straightly forward/backward positrons

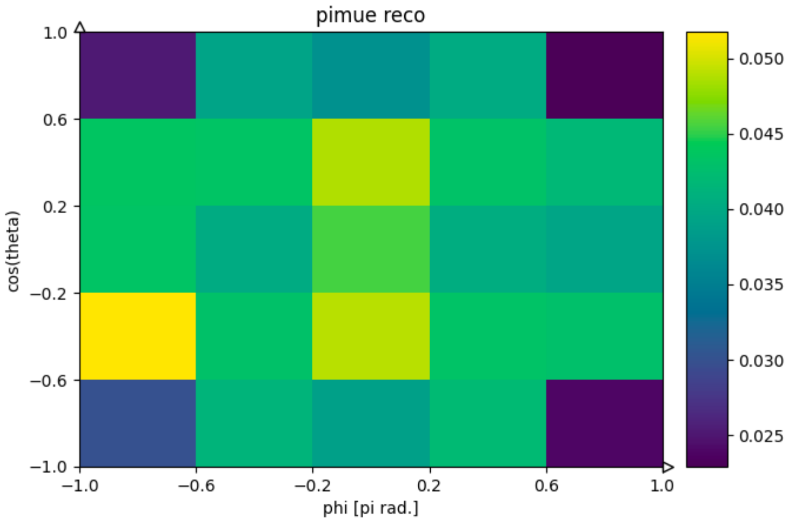


Study the positron outgoing direction

- Select events with
  - pion decay in the center of ATAR, $|x| < 8mm, |y| < 8mm$
  - 5 hits of positrons are required
- RECO information employed
  - Plot normalized to number of events
  - Uniform distribution implies 1/(5*5) ~ 0.04

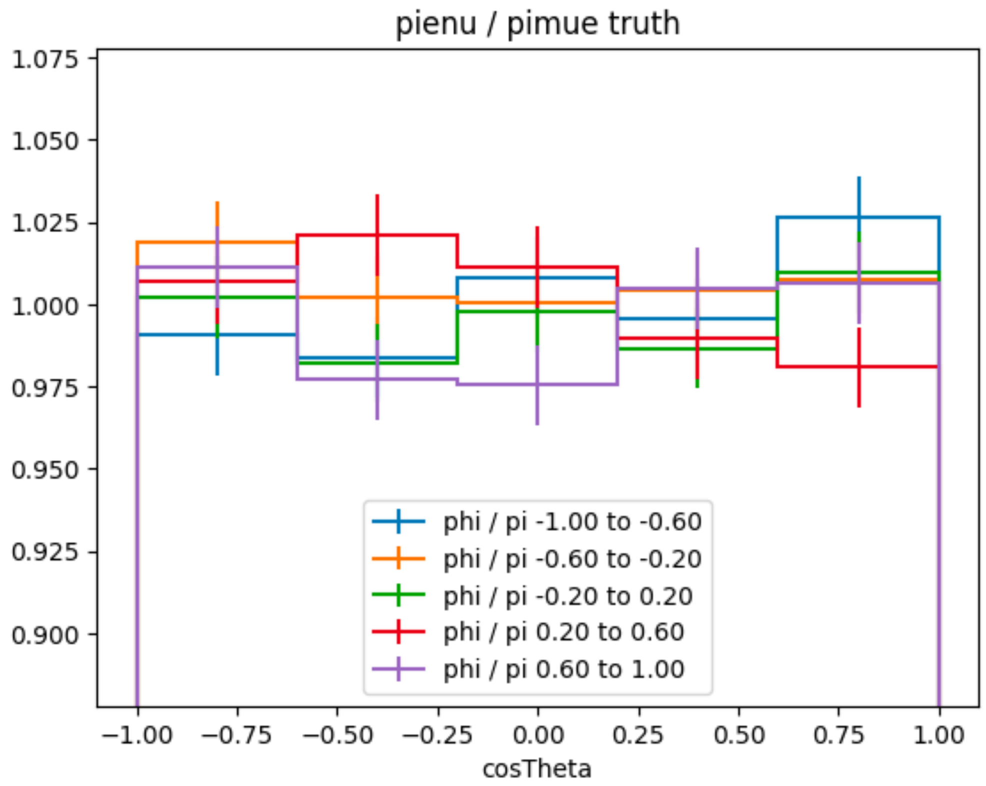


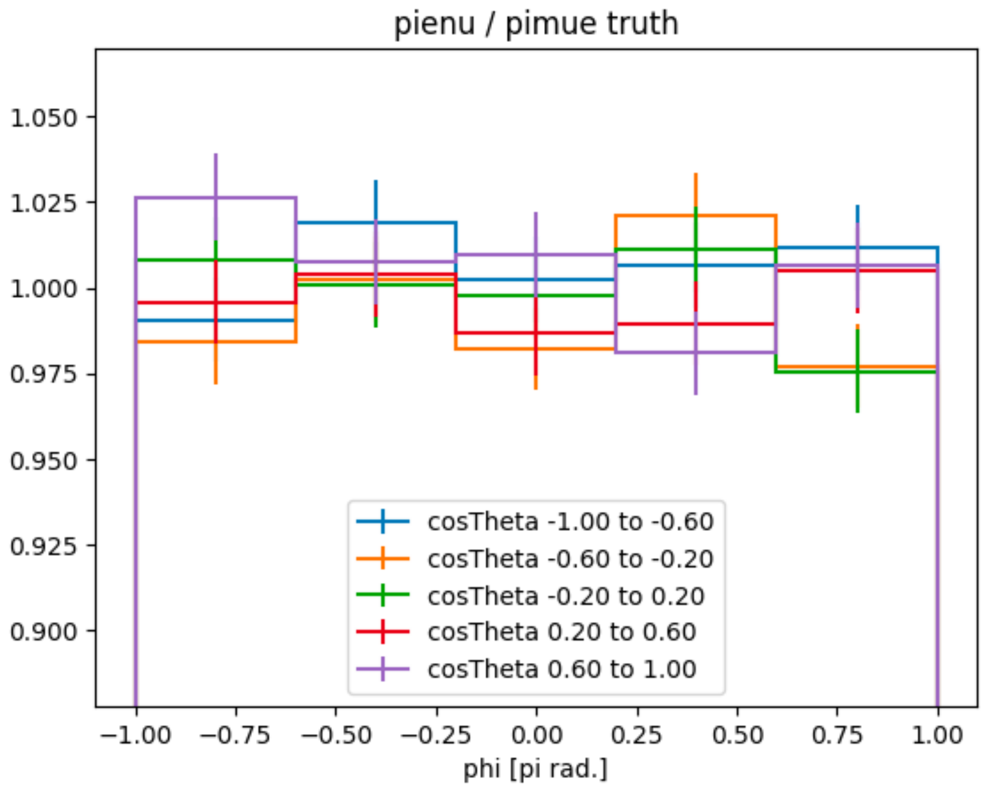




Study the positron outgoing direction

- Ratio between $\pi e \nu$ and $\pi \mu e$ – no difference found in truth

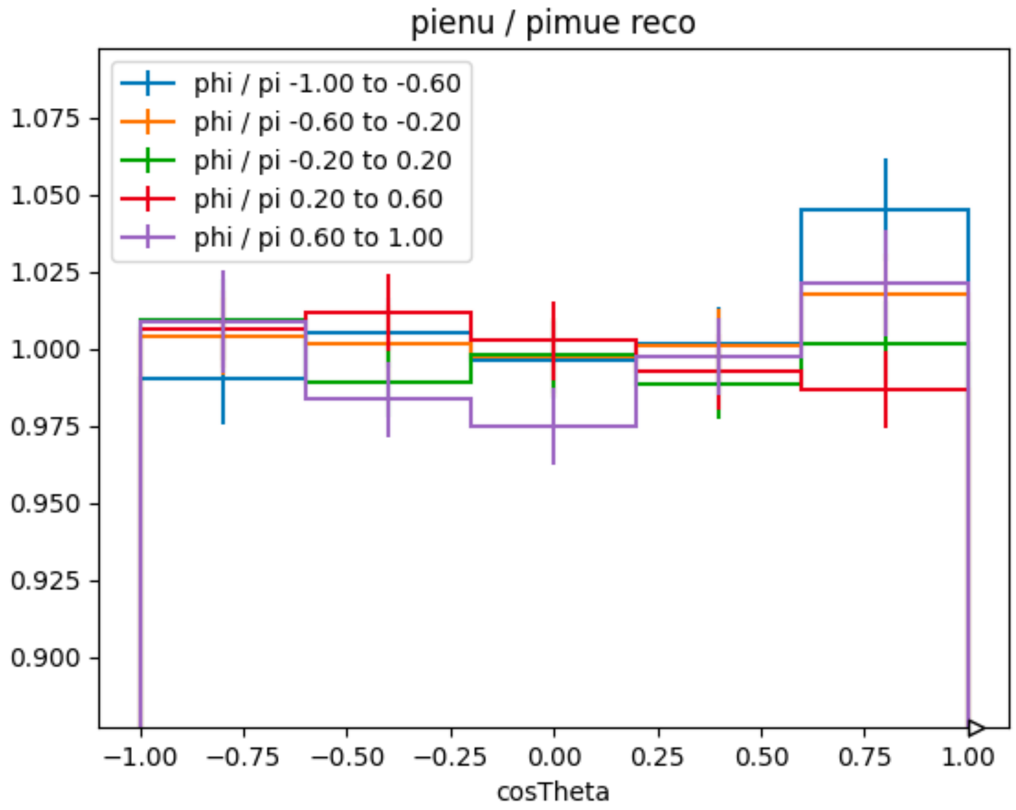


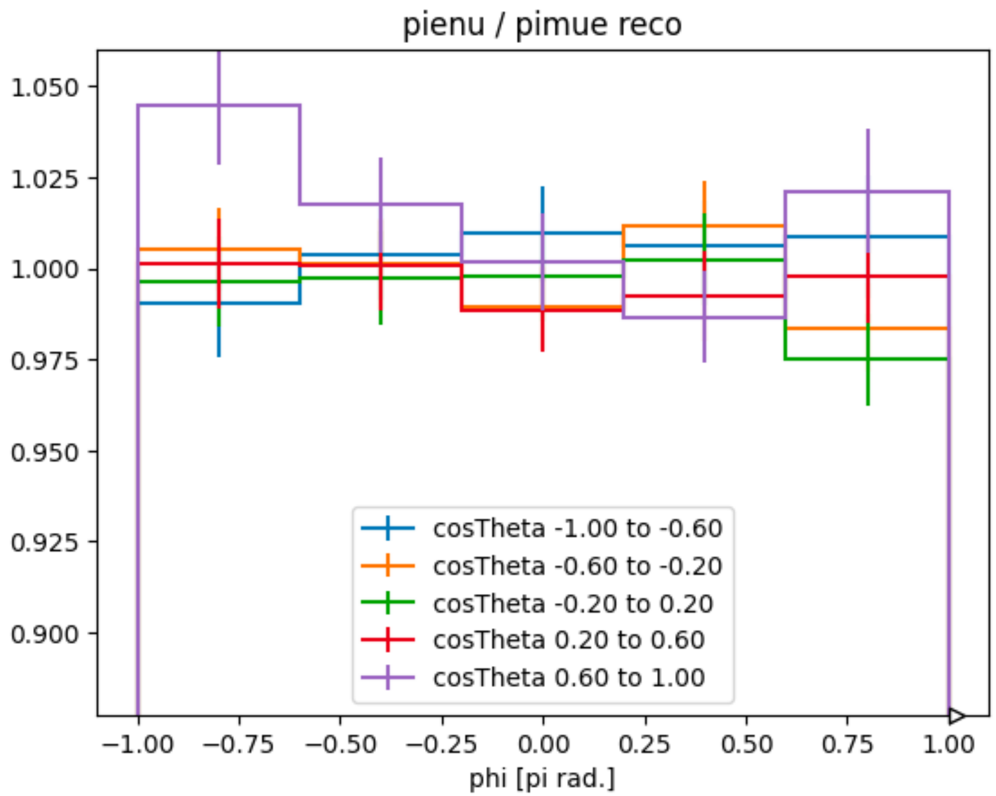


Uncertainty = $\sqrt{N_i} / \sum N_i$

Study the positron outgoing direction

- Ratio between $\pi e \nu$ and $\pi \mu e$ – no bias found in RECO





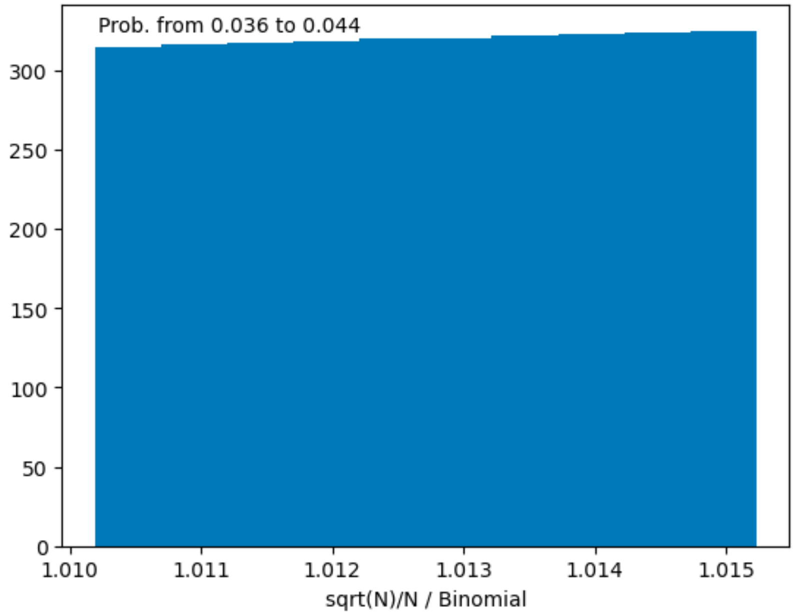
Uncertainty = $\sqrt{N_i} / \sum N_i$

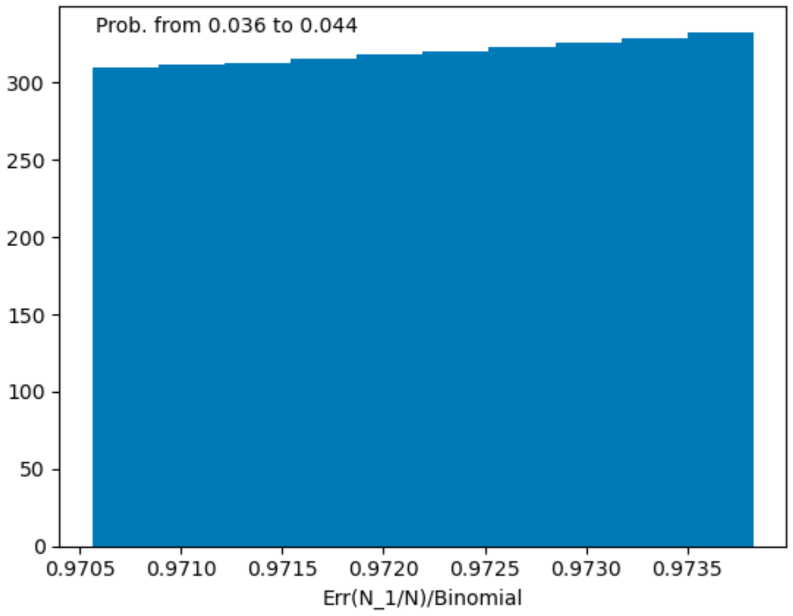
Estimates uncertainties

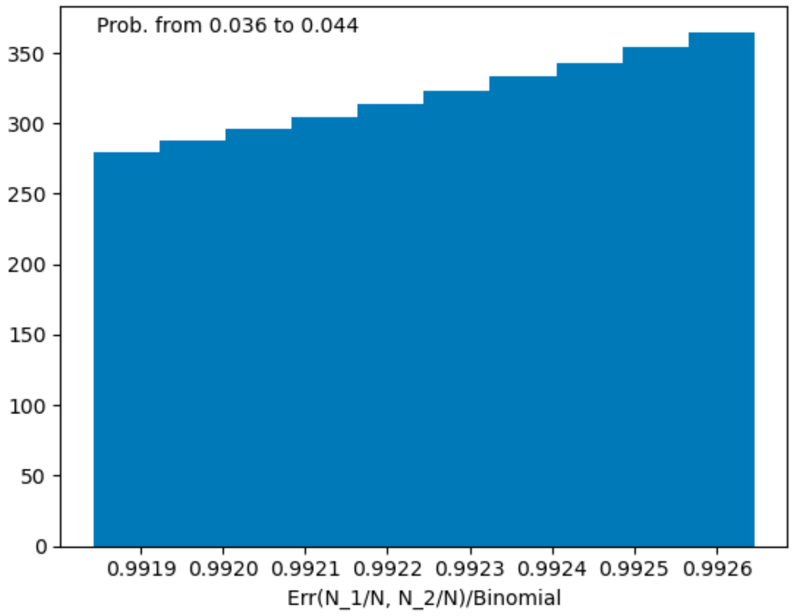
- Binominal proportion confidence interval
  - I choose the larger error among two upper and lower error bars
- Considering Poisson + multinomial uncertainties
  - Independent between bins
  - Normalized counts = ${}^{N_1}/_{(N_1+N_2)}$
    - $\delta = {}^{\delta N_1}/_{(N_1+N_2)}$
    - $\delta = {}^{N_2 \delta N_1}/_{(N_1+N_2)^2}$
    - $\delta = {}^{N_2 \delta N_1}/_{(N_1+N_2)^2} \oplus {}^{N_1 \delta N_2}/_{(N_1+N_2)^2}$

Probability from 0.036 to 0.044

- Close to binomial interval by a few percents

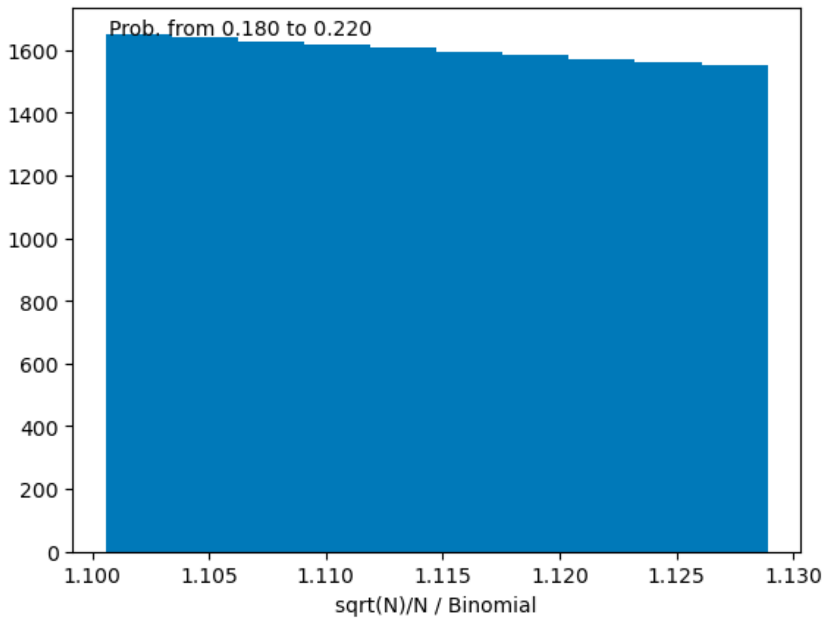


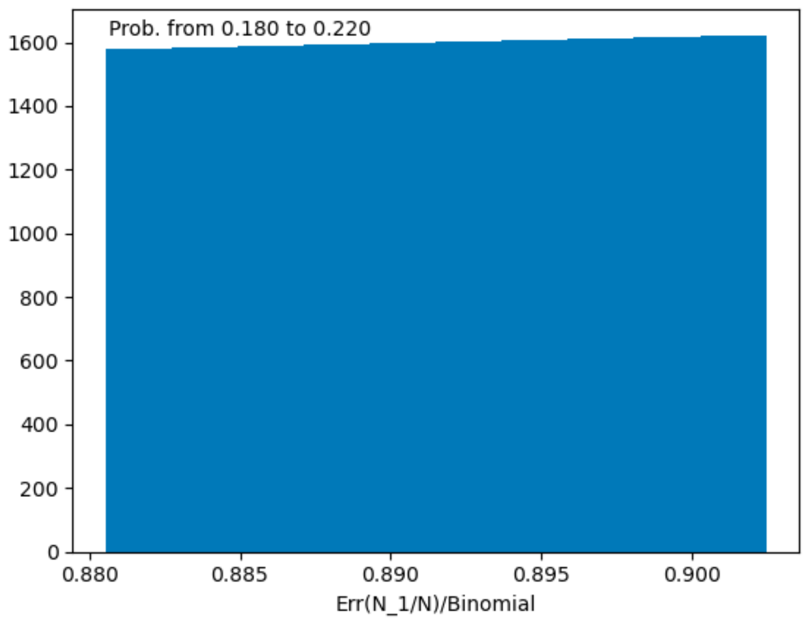


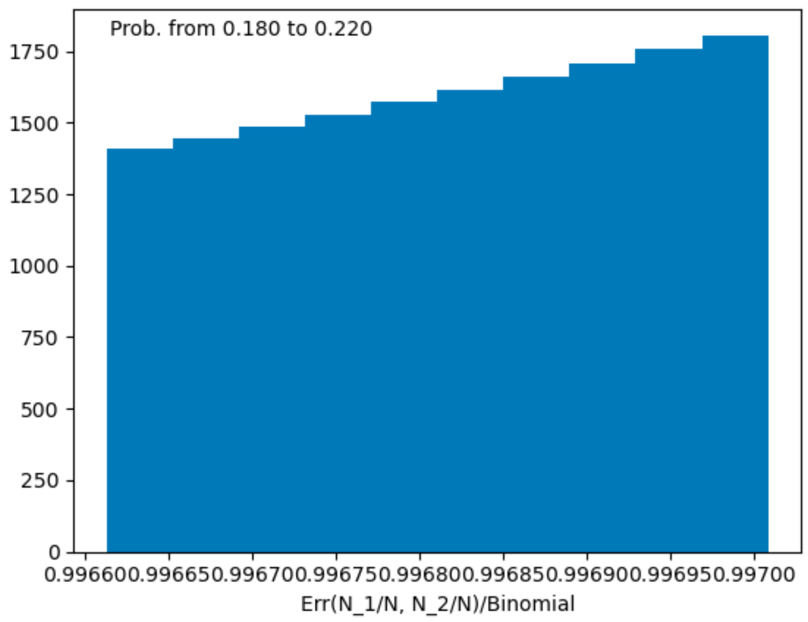


Probability from 0.18 to 0.22

- 10% deviation for the first two estimates

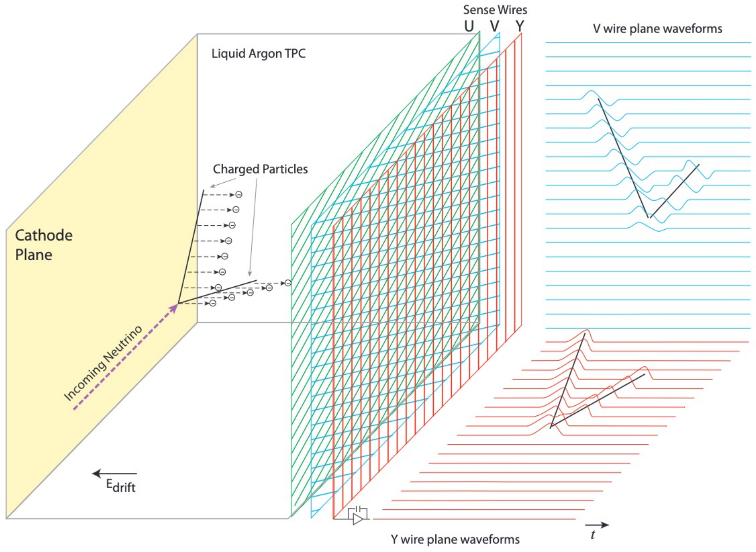


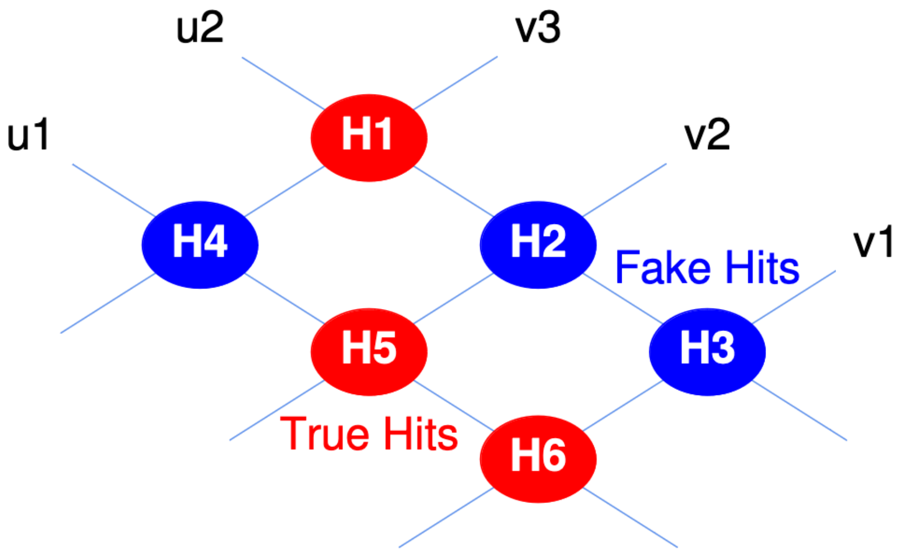




Resolve ambiguity of strips

- Idea from wire-cell at LArTPC





$$\begin{pmatrix} u1 \\ u2 \\ v1 \\ v2 \\ v3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} H1 \\ H2 \\ H3 \\ H4 \\ H5 \\ H6 \end{pmatrix}$$