# $R_{e/\mu}$ analysis strategy

**Quentin Buat (University of Washington)**

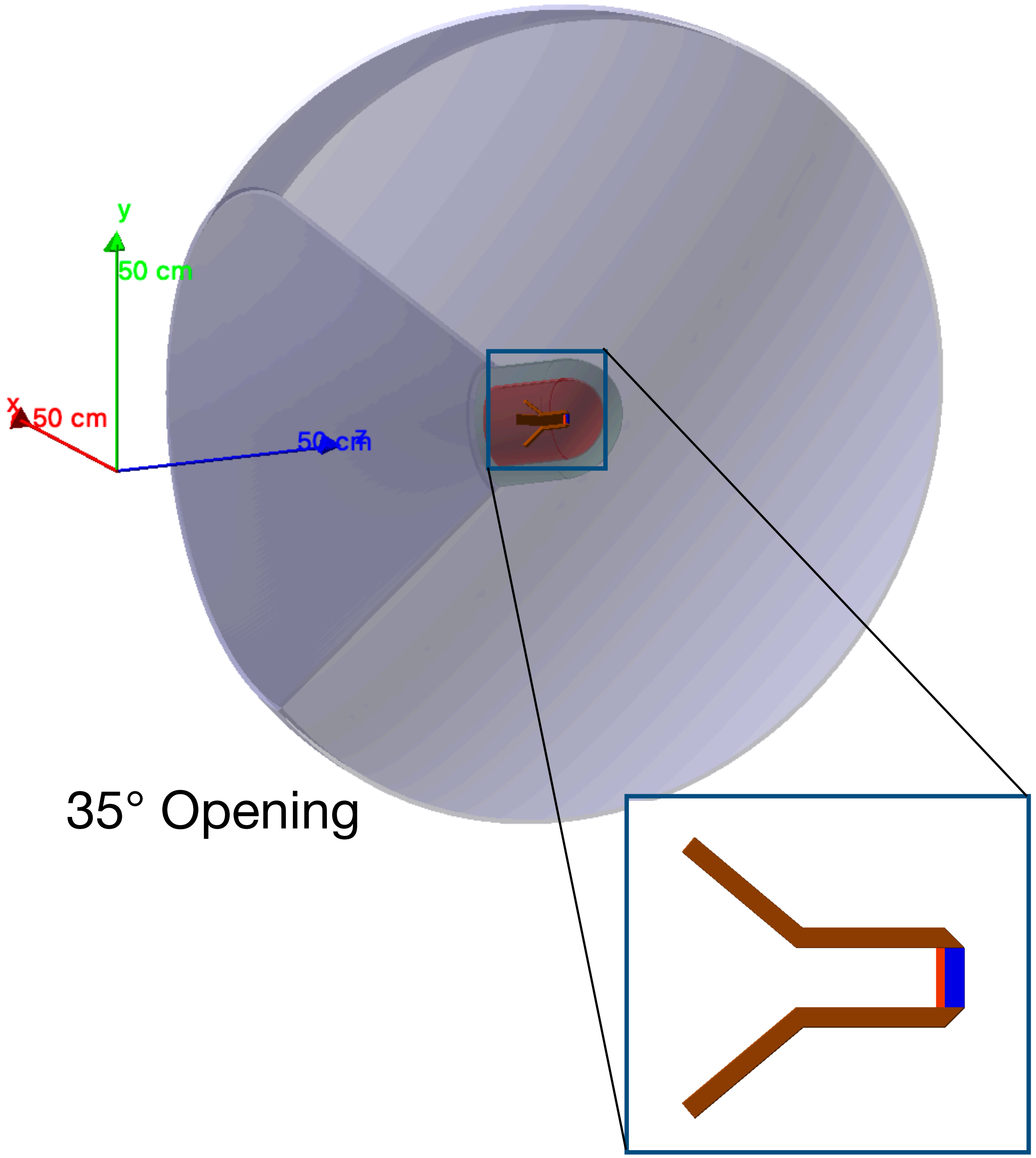
**PIONEER Collaboration Meeting — June 19, 2024**

# Introduction

- In previous talks:
  - Specific (critical!) aspects were tackled individually
  - Patrick presented an overview of the simulation
- This talk:
  - From output histograms of the simulation framework to $\Delta R_{e/\mu}/R_{e/\mu}$
- Results shown in this talk come from the latest simulated samples
  - Every sample considered was simulated with $10^8$ pions
    *(PIONEER Phase 1 goal is to record 2e8 $\pi^+ \to e^+ \nu$)*

# The Simulated Geometry

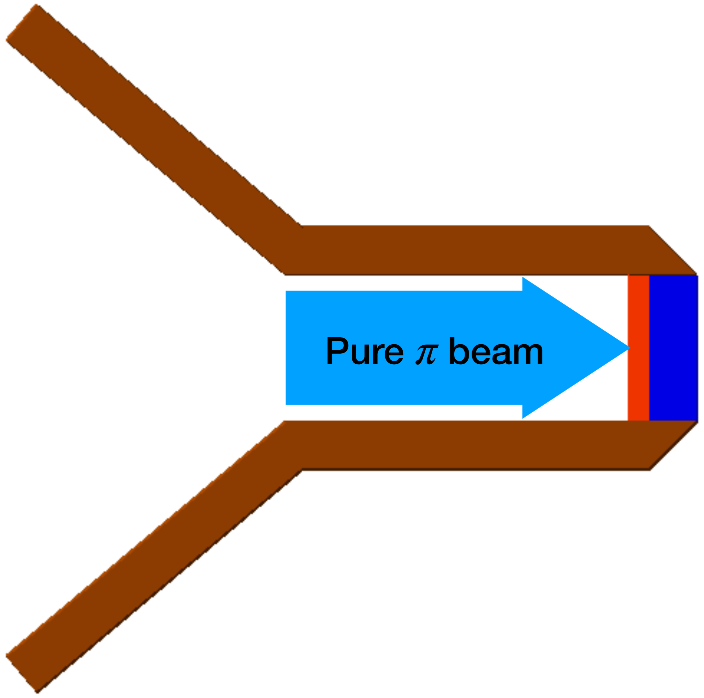
- 25 $X_0$ LXe
- Opening: 35°
- 2x2 mm Ti-6Al-4V alloy windows
- ATAR Cables (Al)
- Mock-up Tracker
- ATAR + DTAR
- Missing:
    - DTAR Cables
    - ATAR + DTAR Supports

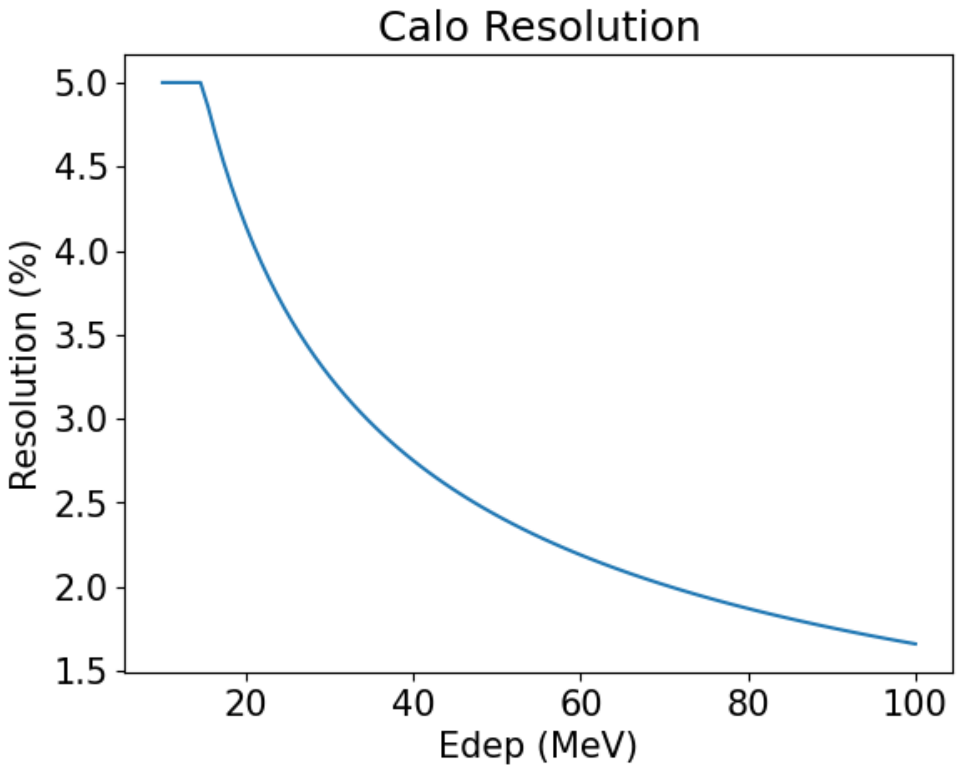



35° Opening

# Further Simulation Assumptions

- Pure, cylindrical $\pi$ beam 5 cm upstream of ATAR with 1 cm radius
- Run the full reconstruction
- Recover ATAR and DTAR energies with a 10% resolution
- Energy Dependent Calo resolution



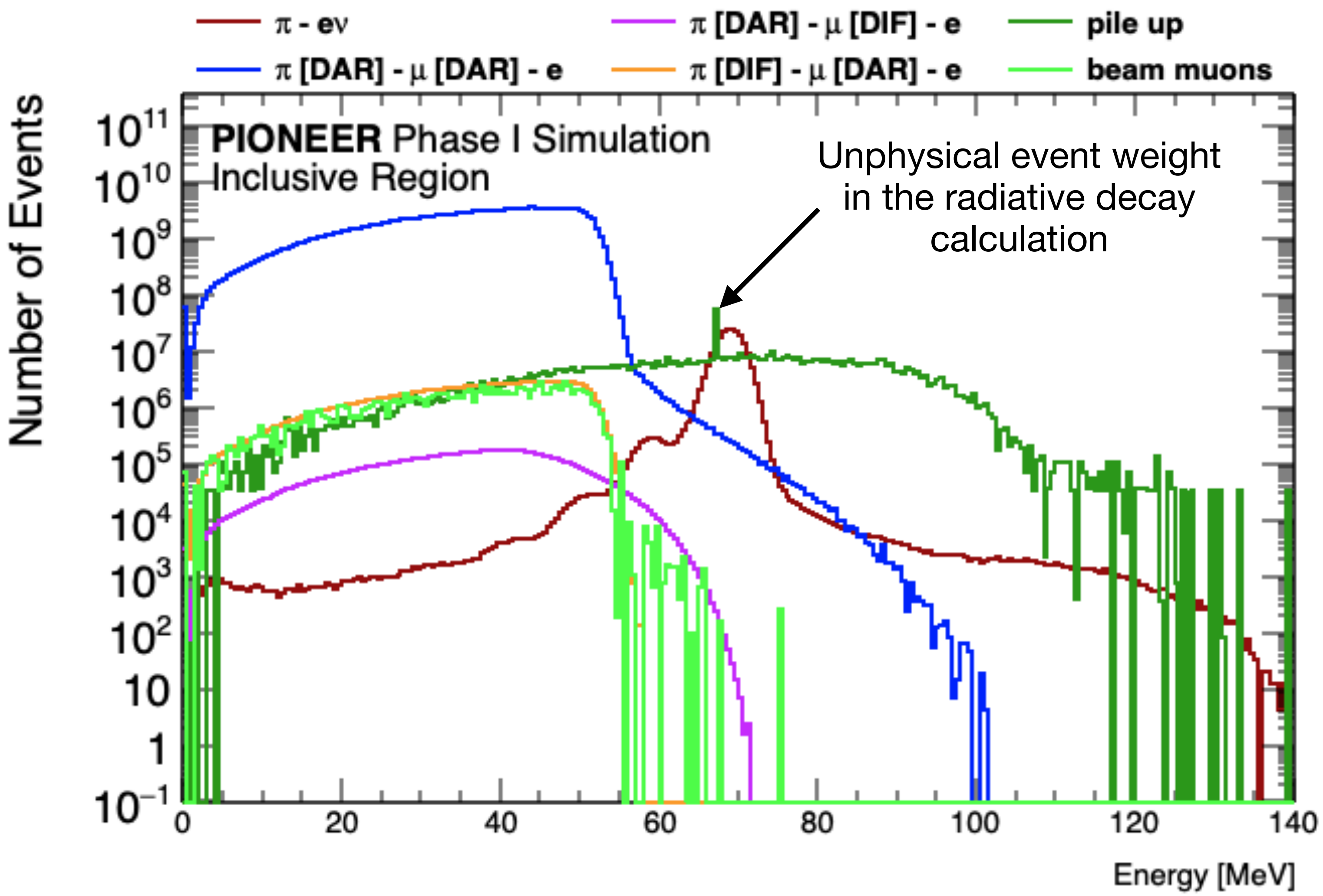



# What our data could look like

## Our latest best guess

Fiducial region defined with true positron momentum

Histograms scaled up to match expectation for 2e8 $\pi^+ \to e^+\nu$ collected events




| Sample | Yields | Uncert. (Rel. uncert. in %) | | Composition (%) |
|---|---|---|---|---|
| pienu | 199’999’999.66 | 28958.04 | (0.014) | 0.05 |
| michel | 379'191’543’116.89 | 569562545.83 | (0.150) | 99.43 |
| pileup | 1’459’044’812.34 | 49957046.76 | (3.424) | 0.38 |
| mudif | 9’652’972.38 | 2733.28 | (0.028) | 0.00 |
| pidif | 293’190’817.08 | 195515.18 | (0.067) | 0.08 |
| beam muons | 221’834’948.78 | 3116556.18 | (1.405) | 0.06 |

# Expressing $R_{e/\mu}$

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1+c_{tail}) \times R^{\epsilon}$$

$\frac{\delta R^{\epsilon}}{R^{\epsilon}} = 10^{-4}$ Do we control the π-e/π-μ-e event selection efficiency ratio at the $10^{-4}$ level?

$c_{tail} \approx 1\,\%\,, \frac{\delta c}{c} \approx 1\,\%$ Can we reveal the tail while maintaining a sufficient signal efficiency?

$\frac{\delta N}{N} = 10^{-4}$ High Energy bin: can we extract $N_{\pi-e}$ with a time fit at the desired precision?

$\frac{\delta N}{N} = 10^{-4}$ Low Energy bin: can we extract $N_{\pi-\mu-e}$ with a time fit at the desired precision?

# Acceptance

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times \boxed{R^{\epsilon}}$$

$$R^{\epsilon}_{time} \times R^{\epsilon}_{energy} \times R^{\epsilon}_{angle} \times R^{\epsilon}_{topology}$$

Evaluated from simulations

Goal

$$\frac{\delta R^{\epsilon}}{R^{\epsilon}} = 10^{-4}$$

| Quantity | Value |
|---|---|
| pienu time corr [5, 500] ns | 0.8253 |
| pimue time corr [5, 500] ns | 0.1938 |
| R_time | 0.2348 |

# Acceptance

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

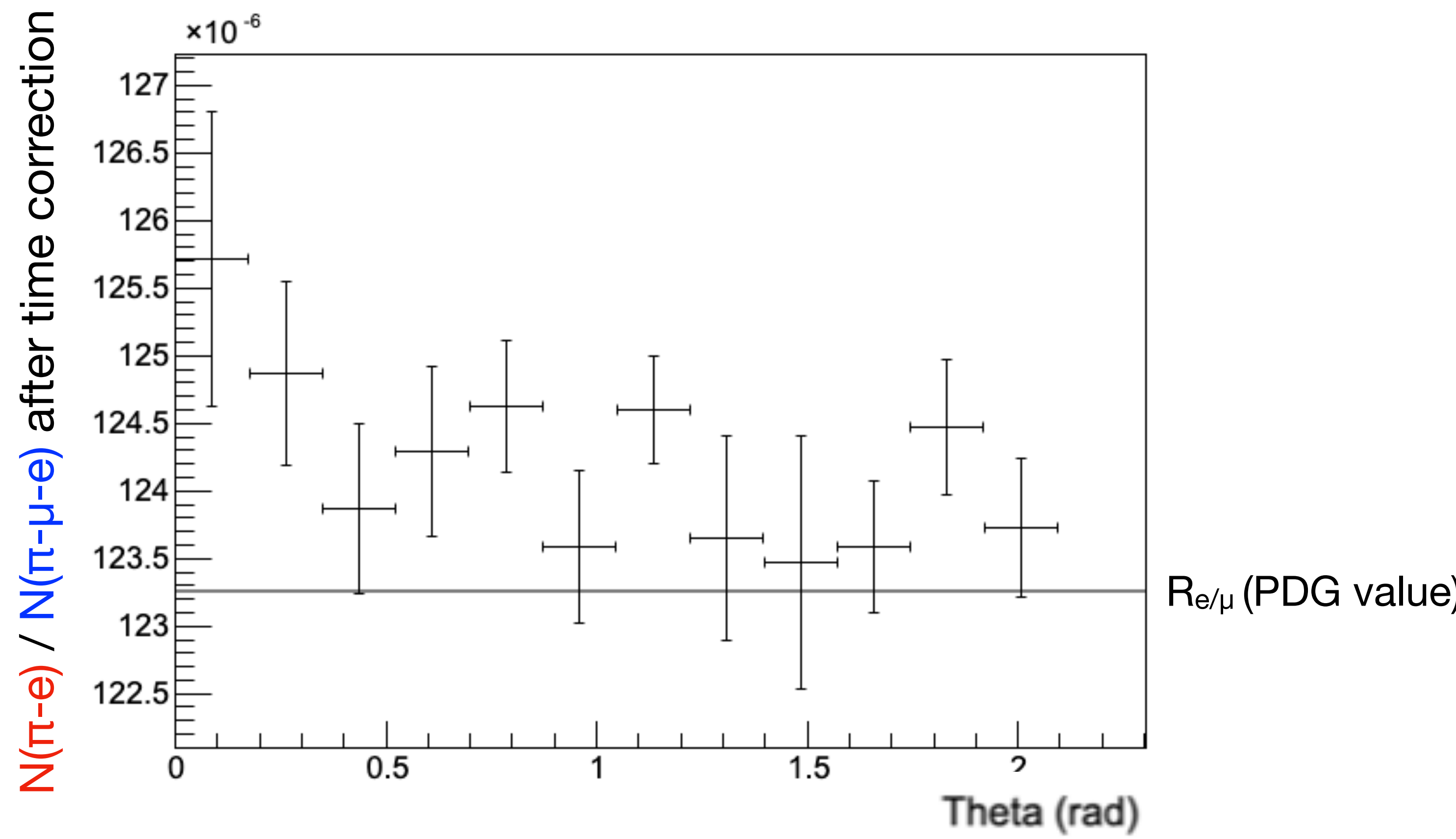
$$R^{\epsilon}_{time} \times R^{\epsilon}_{energy} \times R^{\epsilon}_{angle} \times R^{\epsilon}_{topology}$$

Evaluated from simulations

Goal

$$\frac{\delta R^{\epsilon}}{R^{\epsilon}} = 10^{-4}$$

See Youzhen Zhang's talk




Good agreement with the expectations, within $10^{-3}$ precision level

Some theta dependencies? Need more stat to decisively conclude.

# Acceptance

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times \boxed{R^{\epsilon}}$$

$$R^{\epsilon}_{time} \times \underline{R^{\epsilon}_{energy} \times R^{\epsilon}_{angle} \times R^{\epsilon}_{topology}}$$

Evaluated from simulations

Goal

$$\frac{\delta R^{\epsilon}}{R^{\epsilon}} = 10^{-4}$$

Numbers from a simulations of $10^8$ pions 'sent to' PIONEER

| Quantity | Value | Uncert |
|---|---|---|
| N(pienu events) (scaled down by R_pi) | 5879.2354 | 0.85126 |
| N(pimue events) | 11146781.7563 | 16742.96147 |
| **R(e/mu) ('Perfect' PIONEER) (X 1e4)** | **1.2386** | **0.00796** |
| **R(e/mu) (SM) (X 1e4)** | **1.2352** | **0.00015** |
| **R(e/mu) ('Perfect' PIONEER) / R(e/mu) (SM)** | **1.0027** | **0.00644** |

True $R_{e/\mu}$ obtained from the simulation samples is consistent with the SM expectations within 0.1%

Very promising but need samples at least 10 times larger to monitor the acceptance at the desired precision

# Tail Fraction

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + \boxed{c_{tail}}) \times R^{\epsilon}$$

Goal

$c_{tail} \approx 1\,\%,\ \frac{\delta c}{c} \approx 1\,\%$

# Tail Fraction

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + \boxed{c_{tail}}) \times R^{\epsilon}$$

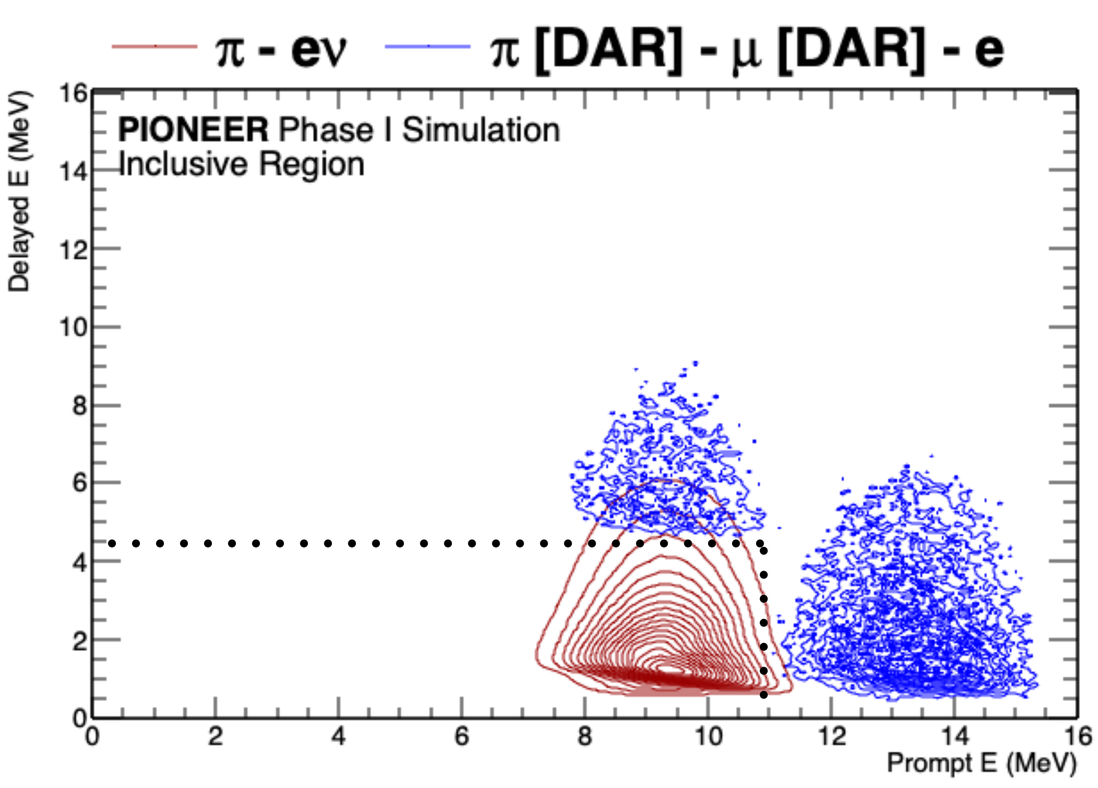
Goal

$c_{tail} \approx 1\,\%\,, \frac{\delta c}{c} \approx 1\,\%$

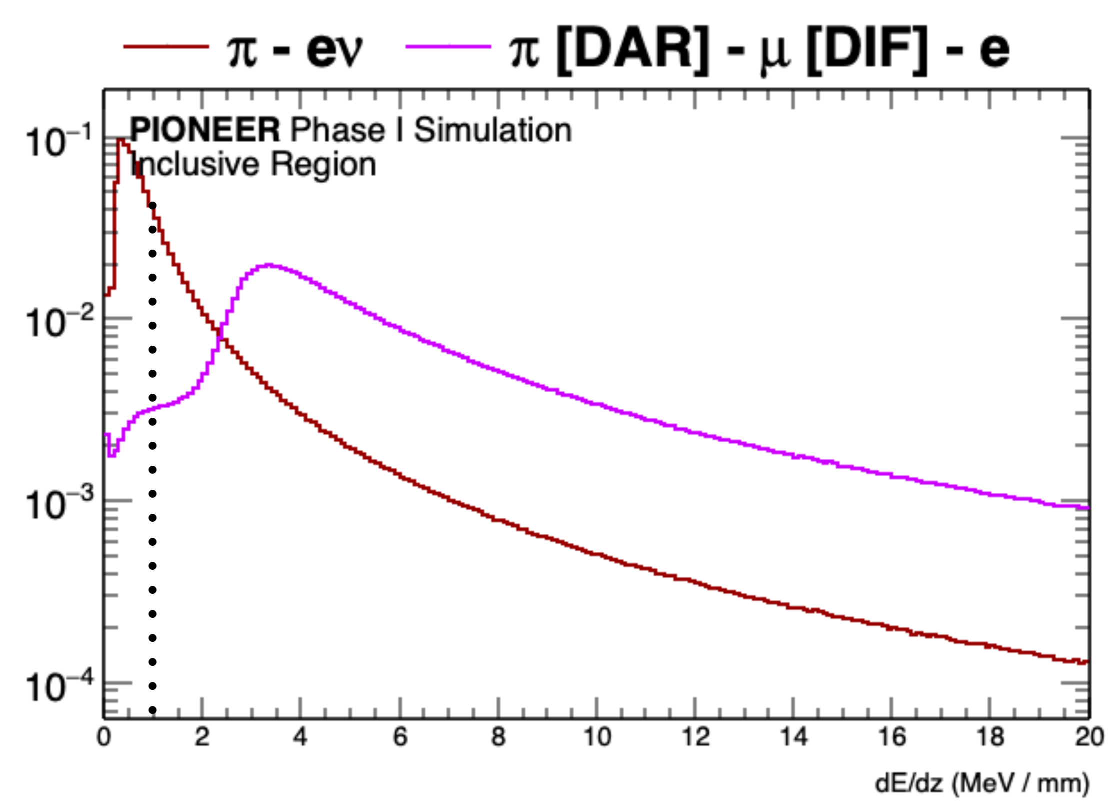
Series of cut implemented in the event reconstruction
to suppress the different backgrounds

Cuts are placed on quantities that can be defined at detector level

BUT for now we rely on pattern and tracklet formations
which are still (almost completely) truth-based



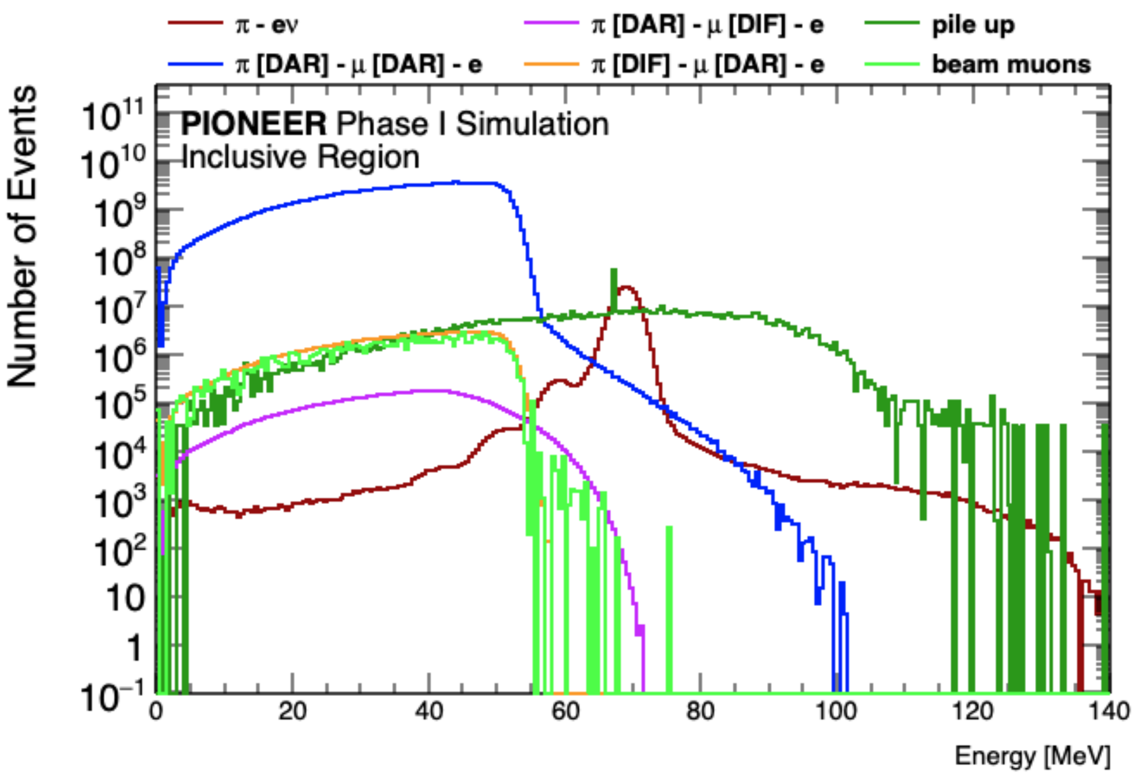

Energy cuts to suppress Michel events

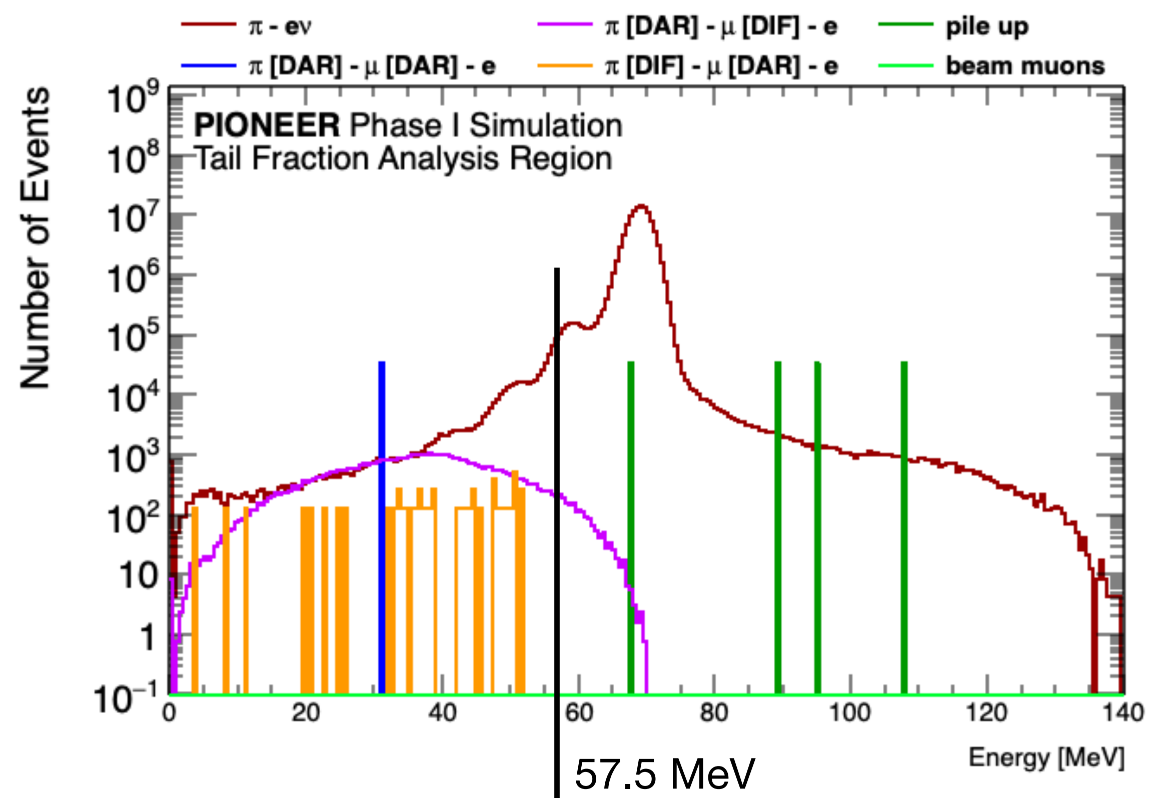



dE/dx cut along the z-direction to suppress muon decay in flight

# Tail Fraction Analysis

## Energy spectrum

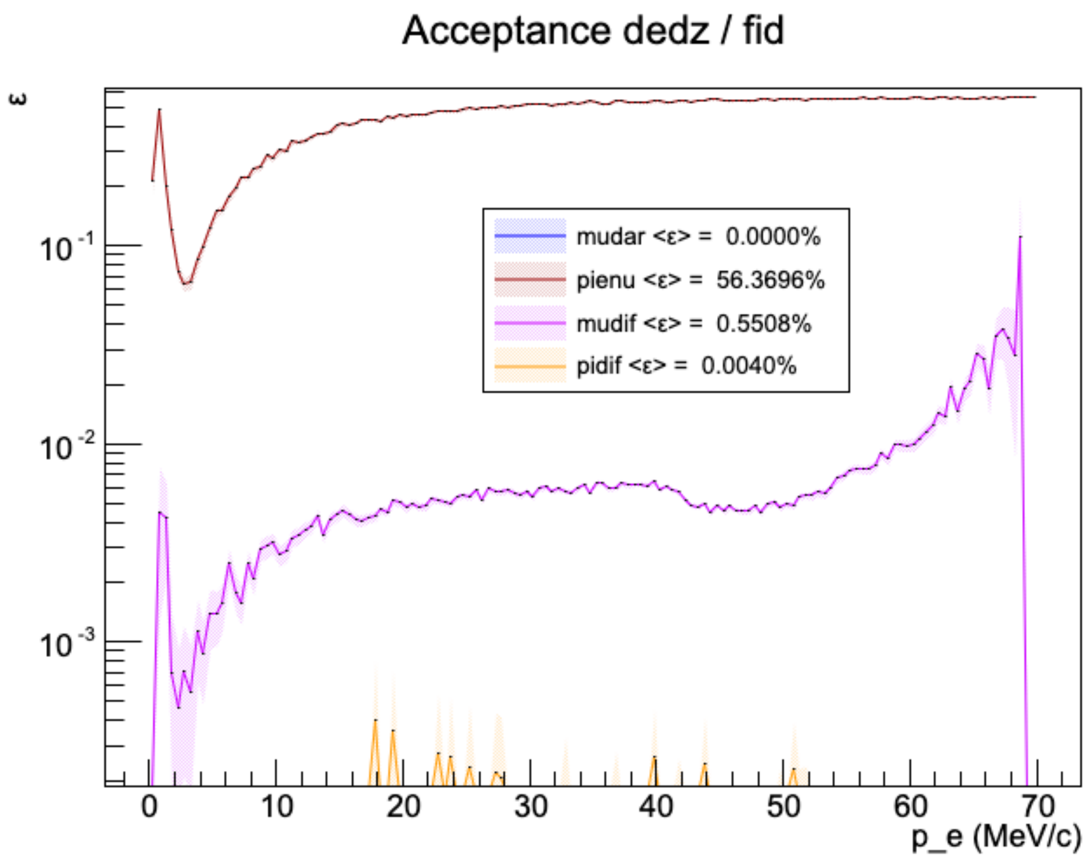


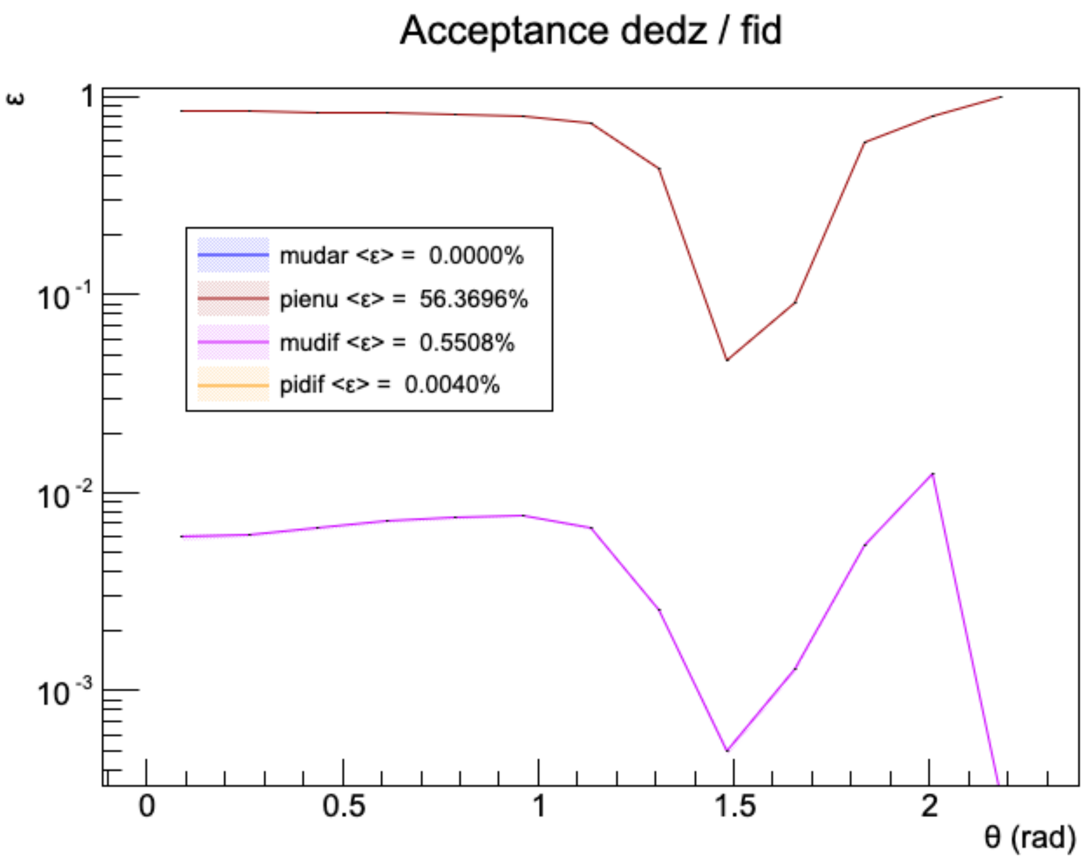




| Sample | Composition (%) | Composition (%) |
|---|---|---|
| pienu | 89.37 | 99.96 |
| michel | 4.45 | 0.00 |
| pileup | 0.00 | 0.03 |
| mudif | 6.01 | 0.00 |
| pidif | 0.17 | 0.00 |
| beam_muons | 0.00 | 0.00 |

# Tail Fraction Analysis

## Signal Efficiency and background rejection




While maintaining a signal efficiency of 56%
We suppress all the backgrounds at the targeted level

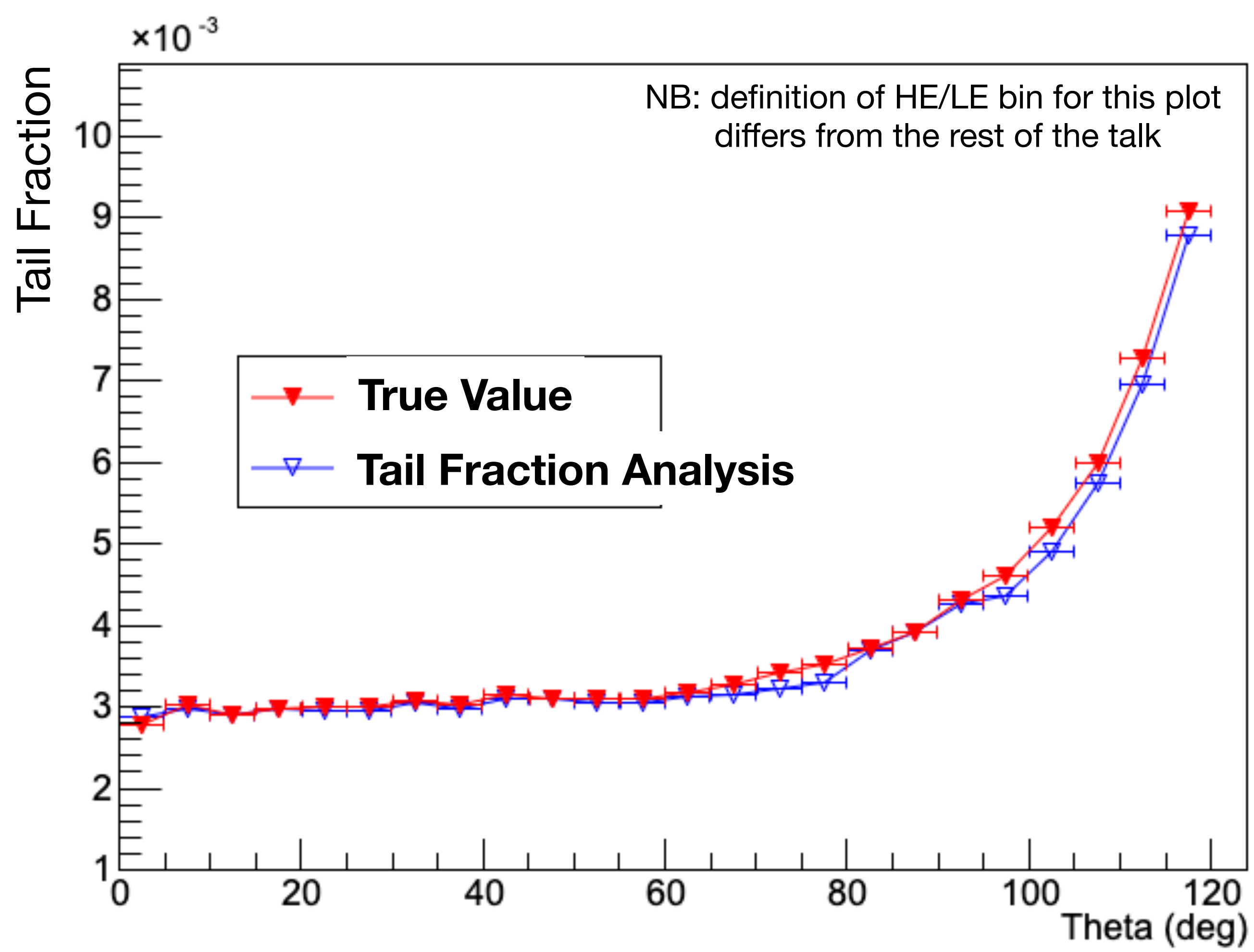
Cuts need to be tuned to maintain
a flat signal efficiency

# Tail Fraction Analysis

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$c_{tail} \approx 1\,\%\,, \frac{\delta c}{c} \approx 1\,\%$




Very good agreement at low theta, deviation observed at higher values

# Tail Fraction Analysis

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + \boxed{c_{tail}}) \times R^{\epsilon}$$

Goal

$c_{tail} \approx 1\,\%\,, \frac{\delta c}{c} \approx 1\,\%$

Numbers from a simulations of $10^8$ pions 'sent to' PIONEER

```
Energy threshold = 57.5 MeV
Tail Fraction Calculation          Value (%)     MC stat uncertainty
-----------------------------     -----------   ---------------------
True Value                           0.70425                  0.00122
Tail Fraction Analysis Region        0.68880                  0.00161
Analysis Region / True Value        0.978074                  0.28406
```

Tail Fraction measured from the Tail Fraction analysis is off by **about 2%** from expectations

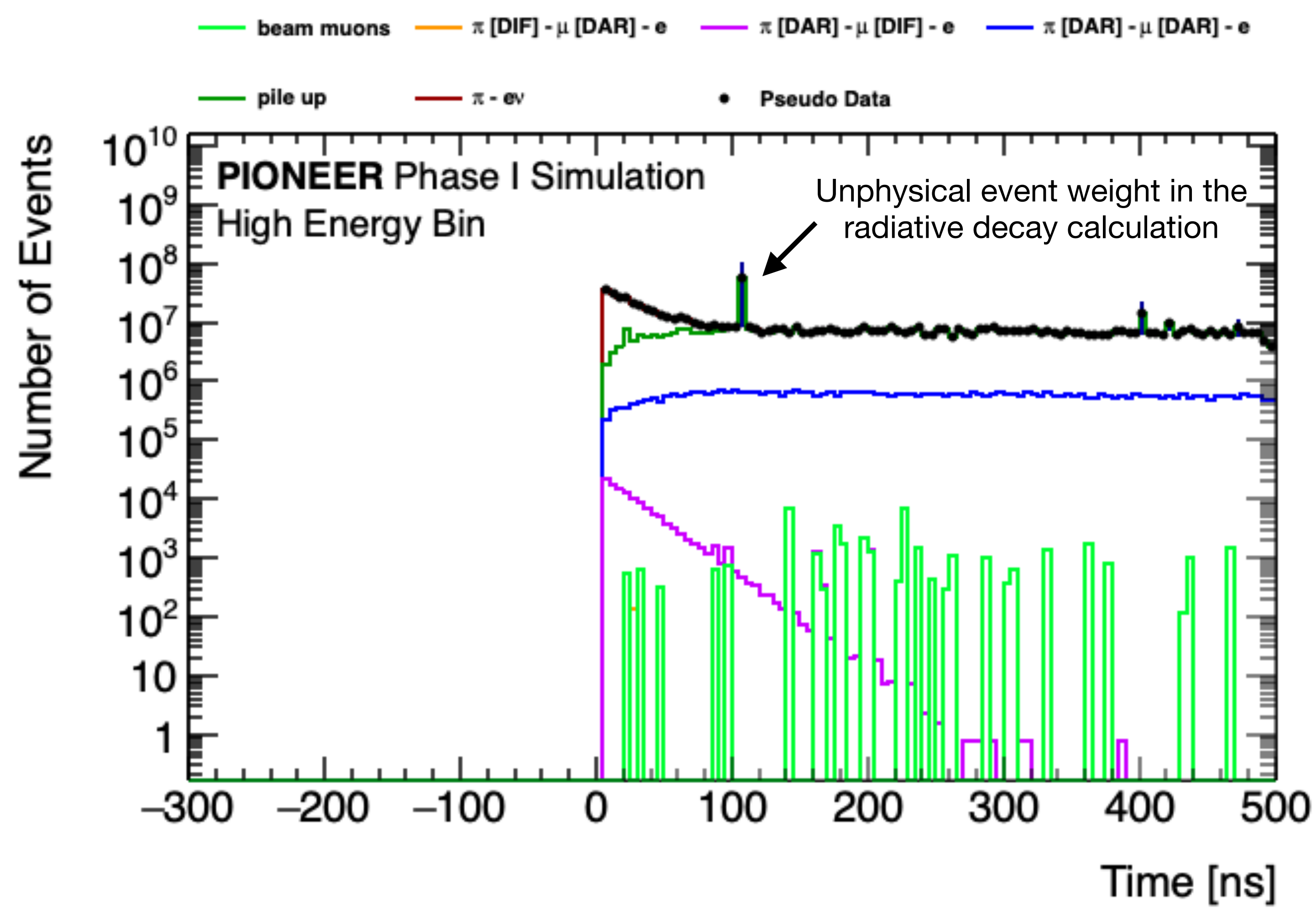
Cut optimisation is most likely the culprit

# High Energy Bin

$$R_{e/\mu} = \frac{\boxed{N_{\pi-e}(E > E_{th})}}{N_{\pi-\mu-e}} \times (1+c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$




Can we extract $N_{\pi-e}(E > E_{th})$?

# A side note on fitting

## An LHC perspective

- A lot of LHC analysis are done by extracting the parameter of interest with a **template fit**
  - **Histograms** of the expectations are fitted to the **binned data**

$$f(\boldsymbol{n}, \boldsymbol{a} \,|\, \boldsymbol{\eta}, \boldsymbol{\chi}) = \underbrace{\prod_{c \in \text{channels}} \prod_{b \in \text{bins}_c} \text{Pois}\left(n_{cb} \,|\, \nu_{cb}\left(\boldsymbol{\eta}, \boldsymbol{\chi}\right)\right)}_{\substack{\text{Simultaneous measurement} \\ \text{of multiple channels}}} \underbrace{\prod_{\chi \in \boldsymbol{\chi}} c_{\chi}(a_{\chi} | \, \chi)}_{\substack{\text{constraint terms} \\ \text{for "auxiliary measurements"}}} ,$$

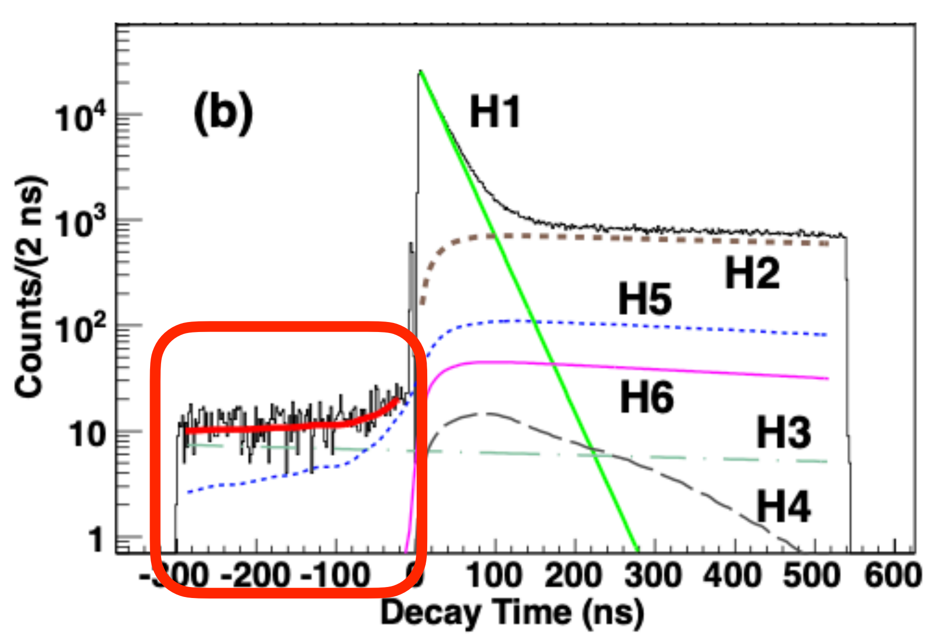
- These tools are also very powerful to study sensitivity
  - We can encode our (assumed) knowledge using (gaussian) constraints of some parameters (for example normalisation of bkg components)
- Some slides with more details on the tool in the elog
  - https://maxwell.npl.washington.edu/elog/pienuxe/Simulation+and+software/25
  - Maybe a topic for an upcoming general meeting?

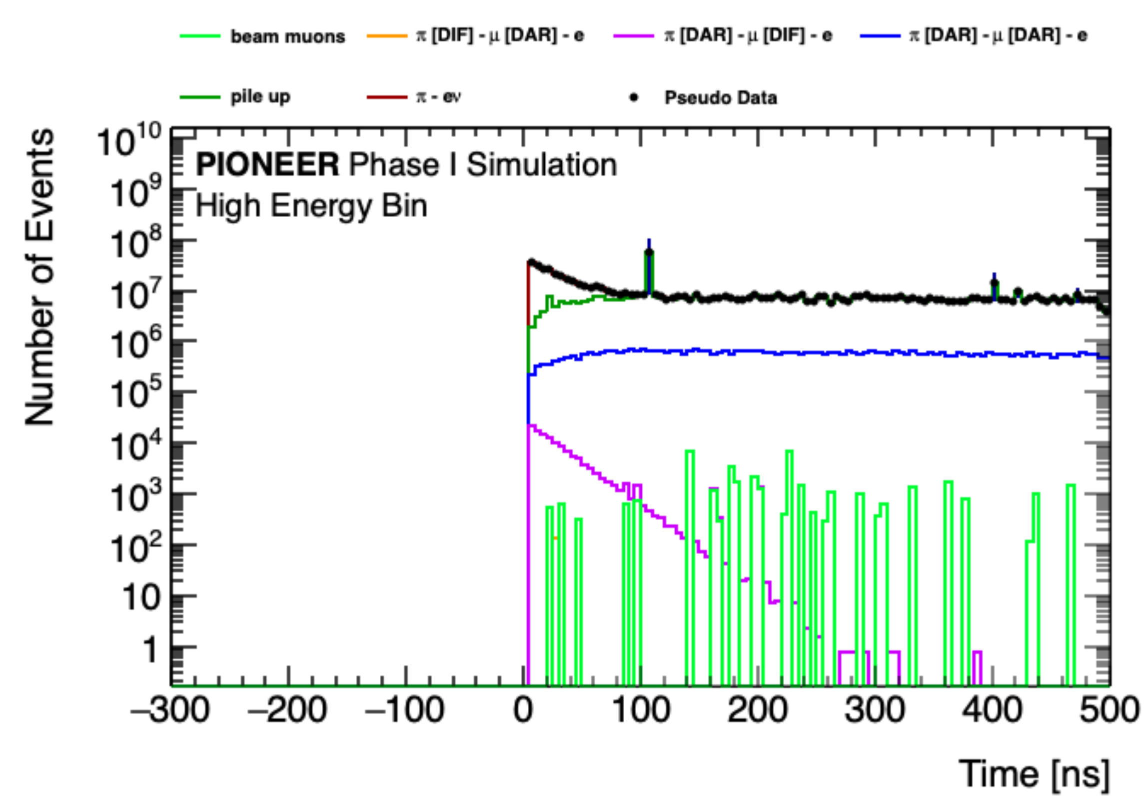
# High Energy Bin

## Fit Setup

| Name | Type | Range | Desc. |
|---|---|---|---|
| pie_HE | NormFactor | Floating (starting at 1) | Floating normalisation of pi-e |
| pidar_mudar_HE | NormFactor | Floating (starting at 1) | Floating norm of pi[dar]-mu[dar] bkg |
| alpha_mudif_HE | OverallSys | 1±0.1 | Constrained uncert on mudif in HE |
| alpha_pileup_HE | OverallSys | 1±0.001 | Constrained uncert on PU in HE |
| alpha_beam_muon_HE | OverallSys | 1±0.001 | Constrained uncert on beam muon in HE |

**Phys. Rev. Lett. 115, 071601 (2015)**



# High Energy Bin

$$R_{e/\mu} = \frac{\boxed{N_{\pi-e}(E > E_{th})}}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$

Fit Result

| Floating Parameter | FinalValue +/- | Error |
|---|---|---|
| alpha_beam_muons_HE | -5.2387e-05 +/- | 9.93e-01 |
| alpha_mudif_HE | 2.1547e-03 +/- | 9.92e-01 |
| alpha_pileup_HE | 3.1258e-05 +/- | 3.24e-01 |
| pidar_mudar_HE | 1.0000e+00 +/- | 3.81e-03 |
| **pie_HE** | **1.0000e+00 +/-** | **1.05e-04** |

Can we extract $N_{\pi-e}(E > E_{th})$?

Yes

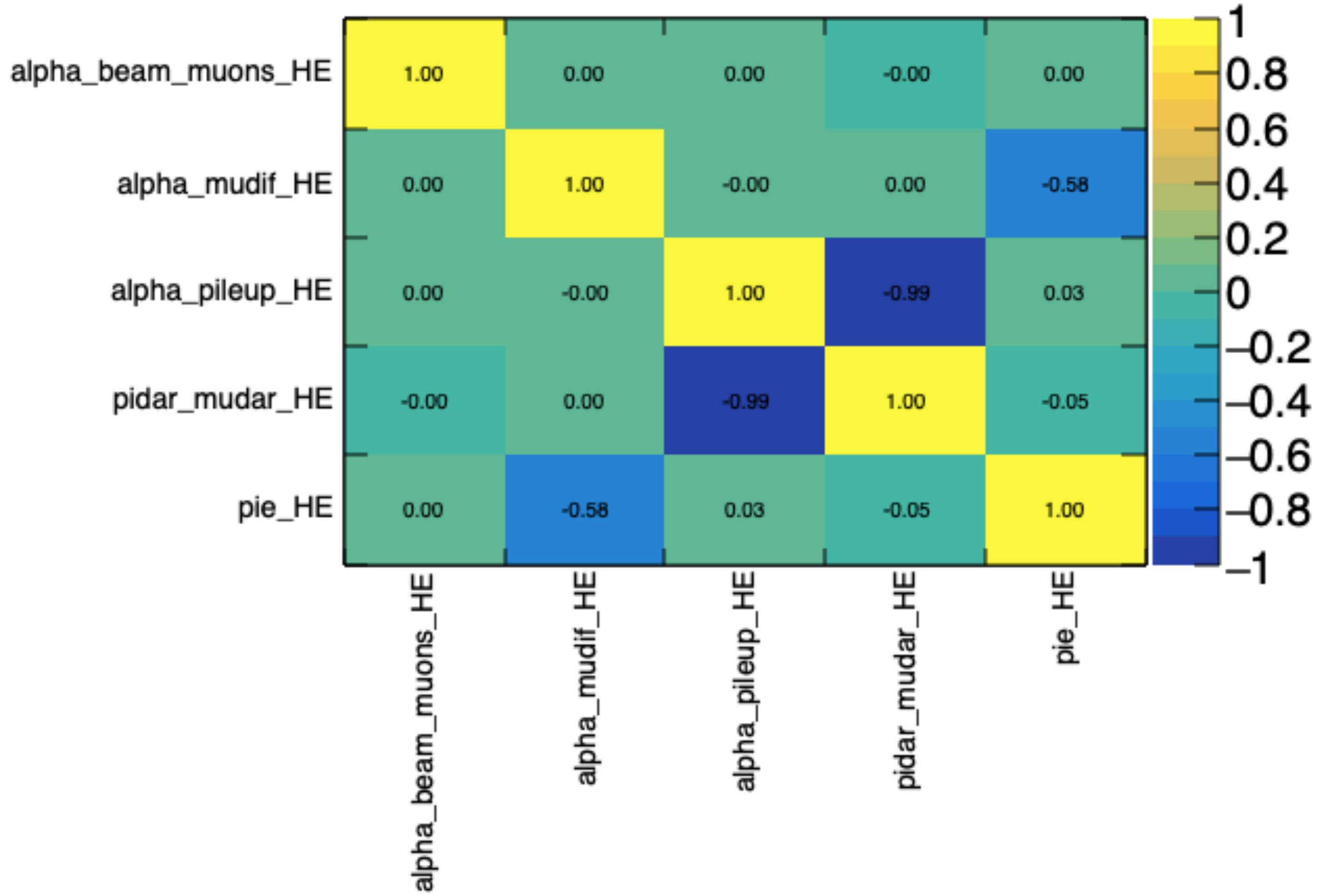
# High Energy Bin

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$

What else did we learn?



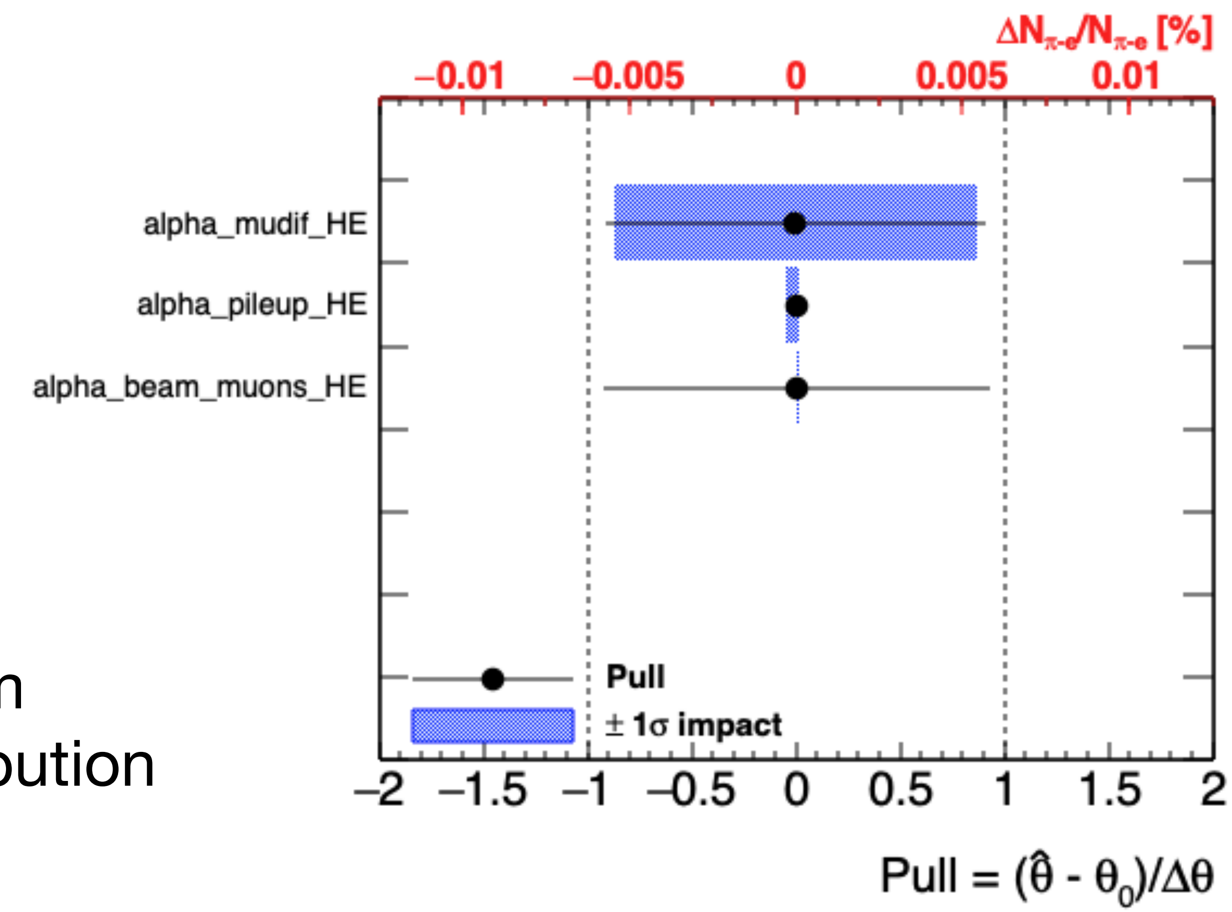
# High Energy Bin

$$R_{e/\mu} = \frac{\boxed{N_{\pi-e}(E > E_{th})}}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

What else did we learn?

Goal

$$\frac{\delta N}{N} = 10^{-4}$$




High Energy bin (positive) time spectrum has constraining power on the pileup contribution

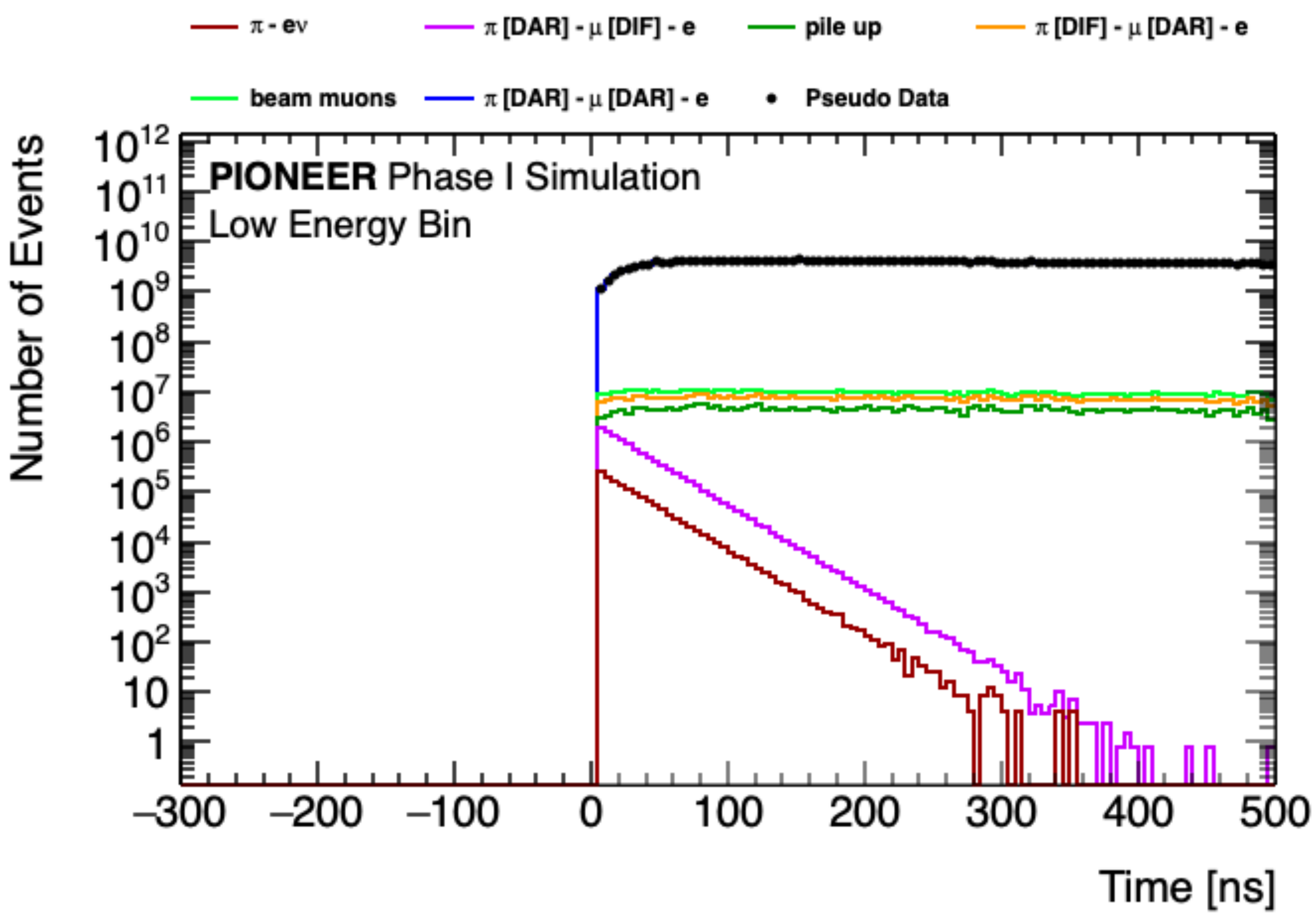
Muon Decay-in-Flight uncertainty has a large impact on $N_{\pi-e}(E > E_{th})$

# Low Energy Bin

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{\boxed{N_{\pi-\mu-e}}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$




Can we extract $N_{\pi-\mu-e}$?

# Low Energy Bin

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{\boxed{N_{\pi-\mu-e}}} \times (1 + c_{tail}) \times R^{\epsilon}$$

| Name | Type | Range | Desc. |
|---|---|---|---|
| pidar_mudar_LE | NormFactor | Floating (starting at 1) | Floating norm of pi[dar]-mu[dar] |
| pidif_LE | NormFactor | Floating (starting at 1) | Floating normalisation of pi(dif) bkg |
| beam_muon_LE | NormFactor | Floating (starting at 1) | Floating norm for beam muon |
| alpha_mudif_LE | OverallSys | 1±0.001 | Constrained uncert on mudif in HE |
| alpha_pileup_HE | OverallSys | 1±0.001 | Constrained uncert on pileup |
| pienu | Fixed | | |

Fit is a lot more complex in the Low Energy bin with more components with similar (positive) time spectra

# Low Energy Bin

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{\boxed{N_{\pi-\mu-e}}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$

| Floating Parameter | FinalValue +/- | Error |
|---|---|---|
| alpha_mudif_LE | -5.8064e-03 +/- | 1.24e-02 |
| alpha_pileup_LE | -5.5502e-03 +/- | 2.50e-07 |
| beam_muons_LE | 1.0001e+00 +/- | 2.14e-05 |
| **pidar_mudar_LE** | **1.0000e+00 +/-** | **2.03e-06** |
| pidif_LE | 9.9993e-01 +/- | 3.58e-05 |

Can we extract $N_{\pi-\mu-e}$?

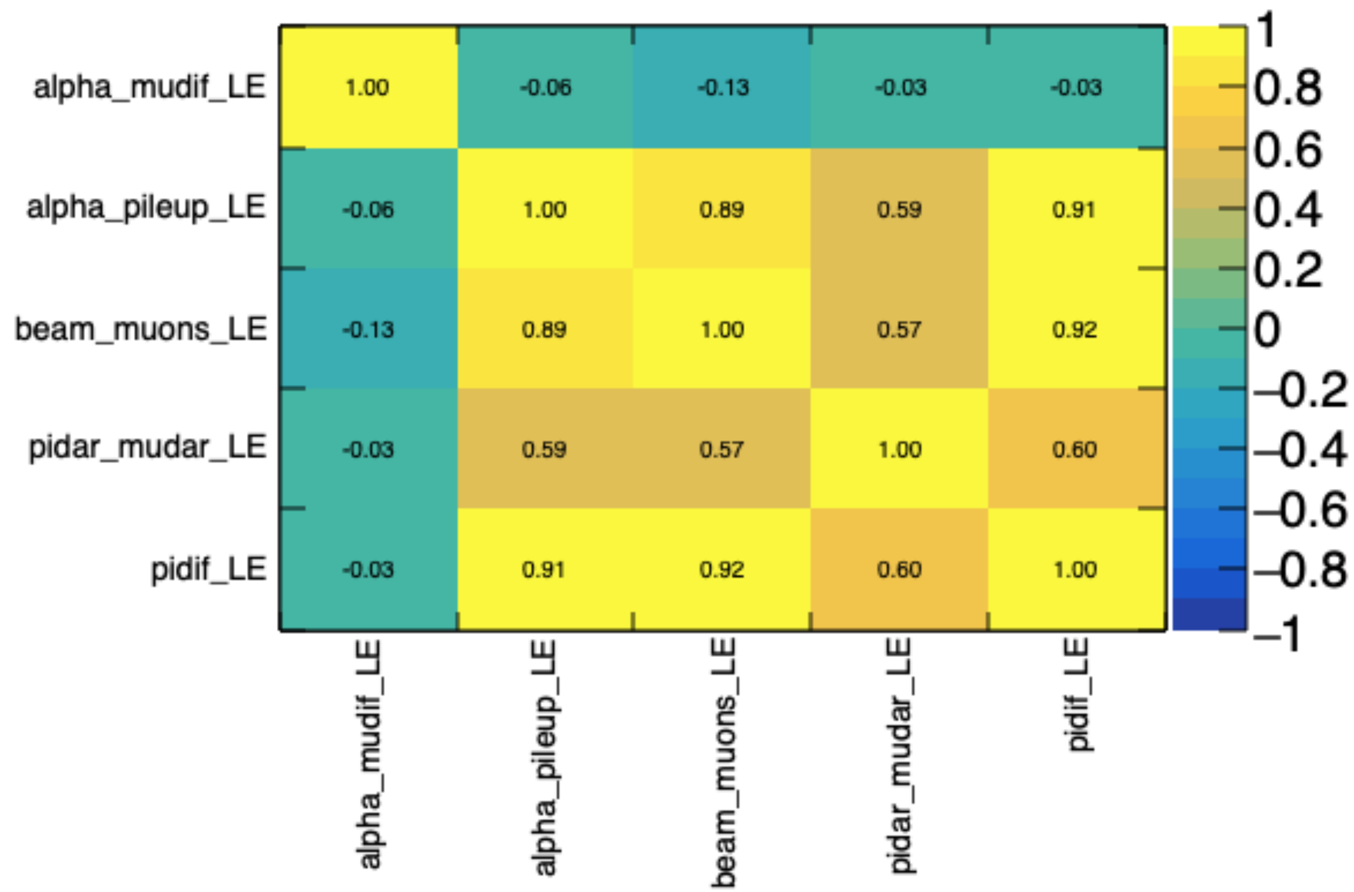
Yes

# Low Energy Bin

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{\boxed{N_{\pi-\mu-e}}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$




A lot of correlations between the parameter of interest (pidar_mudar_LE) and the other ones.

Potential dangerous source of bias!

# Fitting it all together

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1+\boxed{c_{tail}}) \times R^{\epsilon}$$

Fit model for the Tail Fraction Region

| **Name** | **Type** | **Range** | **Desc.** |
|---|---|---|---|
| pie_tf_HE | NormFactor | Floating (starting at 1) | Floating norm of pienu in the high energy bin |
| pie_tf_LE | NormFactor | Floating (starting at 1) | Floating norm of pienu in the low energy bin |
| alpha_mudif_tf_uncert | OverallSys | 1±0.5 | 50% uncertainty on mudif in the tail fraction analysis |
| alpha_pidif_tf_uncert | OverallSys | 1±0.5 | 50% uncertainty on pidif in the tail fraction analysis |

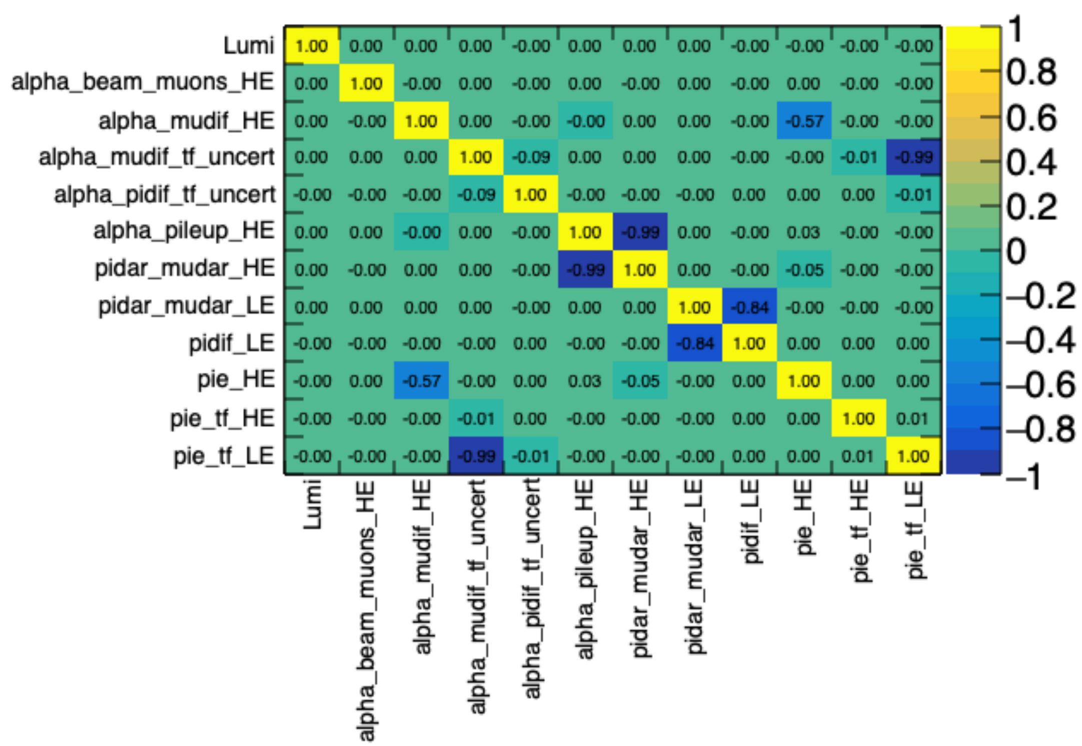
Full Analysis Likelihood Function: High Energy Bin X Low Energy Bin X Tail Fraction Region

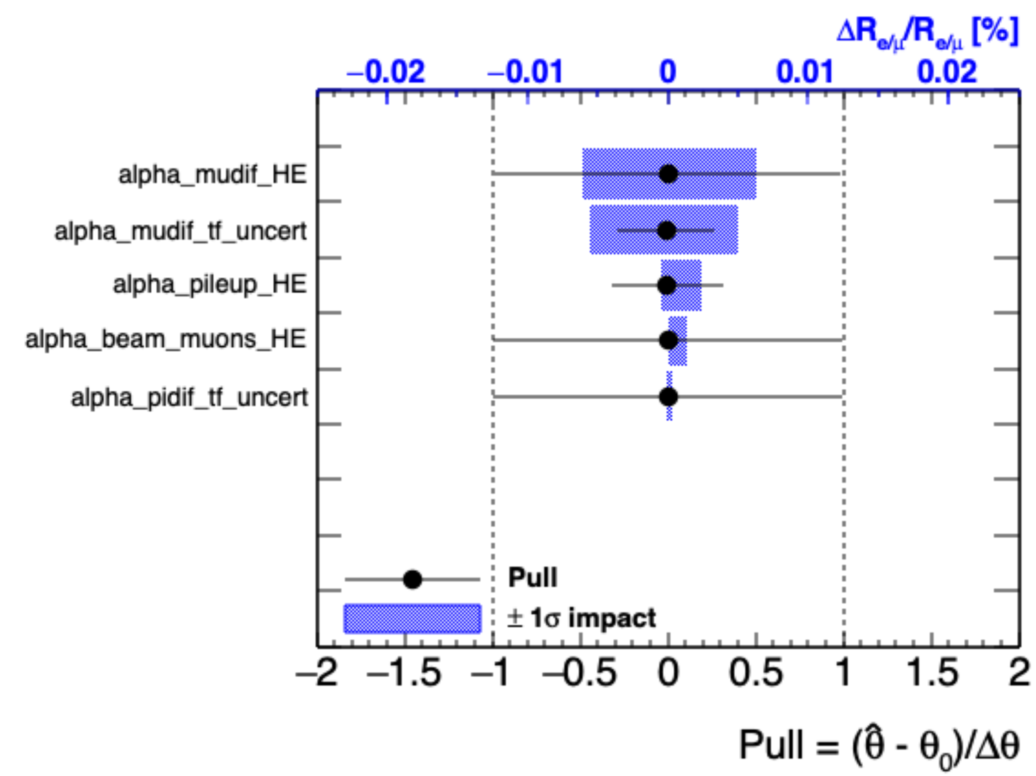
# Fitting it all together

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

| Source | $R_{e/\mu}$ x 1e4 | Delta($R_{e/\mu}$) x 1e4 | Delta($R_{e/\mu}$) / $R_{e/\mu}$ (%) |
|---|---|---|---|
| **PIONEER** | **1.23519** | **0.000150613** | **0.0121935** |
| PIONEER (w/o $c_{tail}$) | 1.22655 | 0.000125561 | 0.0101653 |
| PIENU | 1.2327 | 0.0023 | 0.186582 |
| sm | 1.23524 | 0.00015 | 0.0121434 |

| | Quantity x 1e2 | Uncertainty x 1e2 | Relative Uncertainty (%) |
|---|---|---|---|
| $c_{tail}$ | 0.68899 | 0.00652504 | 0.947043 |



# Conclusion

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

- Simulation samples can be used to conduct a full LFU analysis
  - Sample size are still a limiting factor for accurate background estimate in the Tail Fraction Analysis and acceptance correction studies at the $10^{-4}$ precision level
- Using templated fits, we can estimate PIONEER sensitivity from simulated samples
  - Many studies can be conducted to understand our measurement
- Event reconstruction still relies heavily on truth information and (very) naive detector response
  - A lot of work ahead of us, priorities need to be defined

# Additional material

# Tail Fraction Analysis

## Unscaled yields

```
==================================================
Yield reports in the LE bin of the TFA (unscaled)

Sample          Yields  Uncert. (Rel. uncert. in %)     Composition (%)
----------    --------  ------------------------------  -----------------
pienu            20.07  0.05 (0.25)%                              89.37
michel            1.00  1.00 (100.00)%                             4.45
pileup            0.00  <3.00 (95% CL)                             0.00
mudif             1.35  0.01 (0.41)%                               6.01
pidif             0.04  0.01 (31.62)%                              0.17
beam_muons        0.00  <3.00 (95% CL)                             0.00
```

```
==================================================
Yield reports in the HE bin of the TFA (unscaled)

Sample          Yields  Uncert. (Rel. uncert. in %)     Composition (%)
----------    --------  ------------------------------  -----------------
pienu          2893.71  0.60 (0.02)%                              99.96
michel            0.00  <3.00 (95% CL)                             0.00
pileup            1.00  1.00 (100.00)%                             0.03
mudif             0.04  0.00 (2.40)%                               0.00
pidif             0.00  <3.00 (95% CL)                             0.00
beam_muons        0.00  <3.00 (95% CL)                             0.00
```

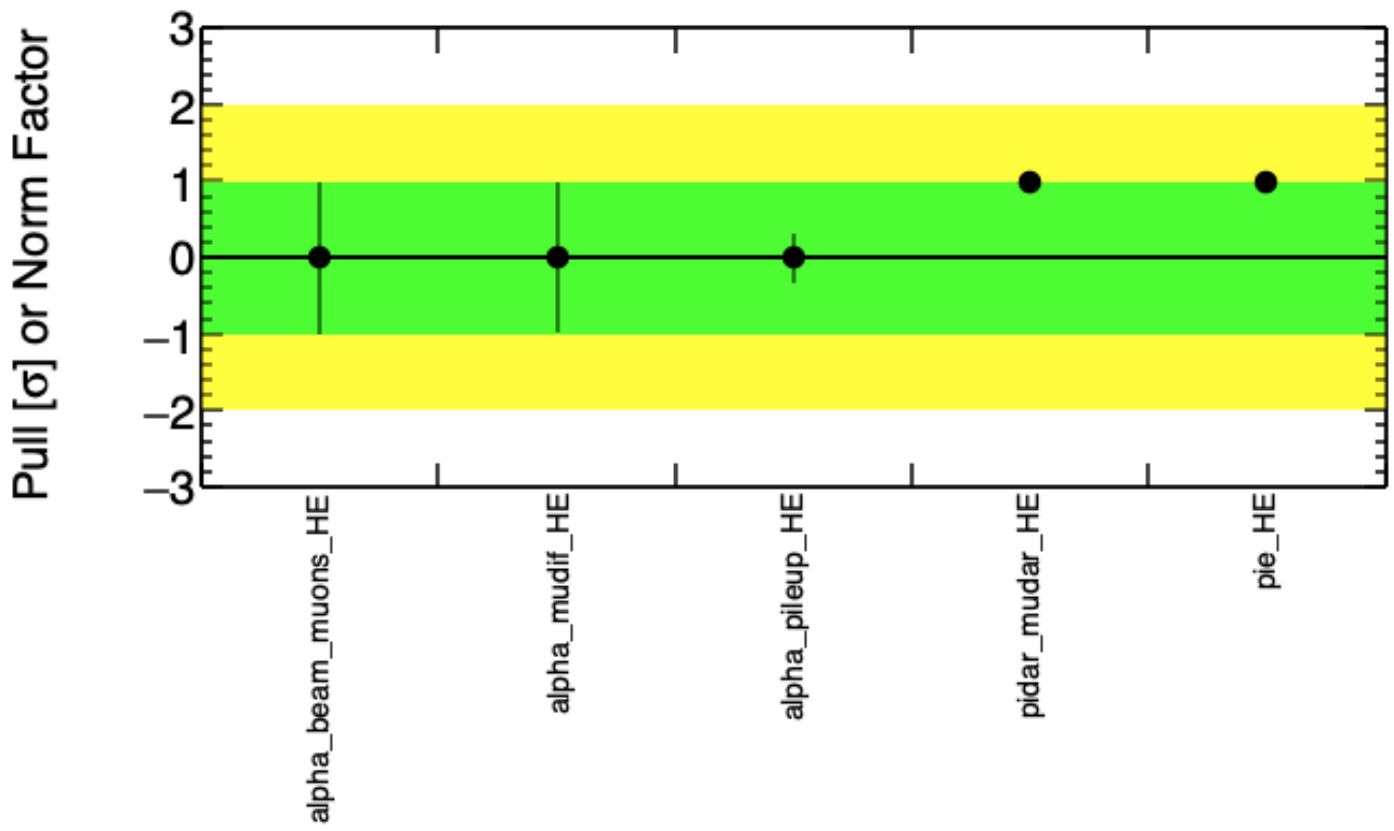
# High Energy Bin

$$R_{e/\mu} = \frac{N_{\pi-e}(E > E_{th})}{N_{\pi-\mu-e}} \times (1 + c_{tail}) \times R^{\epsilon}$$

Goal

$$\frac{\delta N}{N} = 10^{-4}$$

What else did we learn?




High Energy bin (positive) time spectrum has constraining power on the pileup contribution