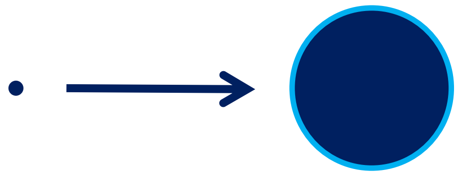
Nuclear polarization correction: what, why and how

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

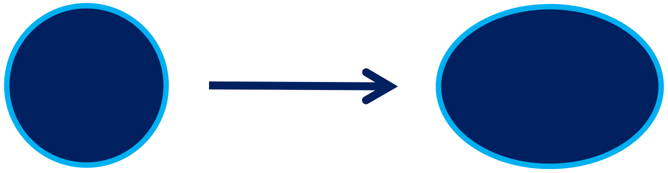
Bird’s-eye view

Nuclear effects

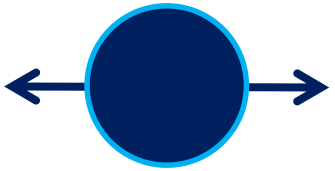
finite nuclear size



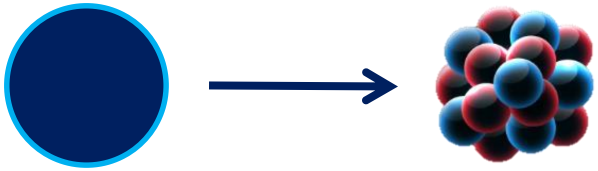
nuclear deformation



nuclear recoil

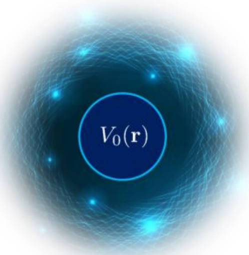


nuclear polarization

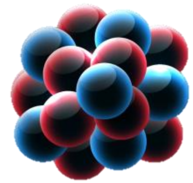


Nuclear polarization: a simple view

$$|\mathcal{I}\rangle = |i\rangle\,|I\rangle$$

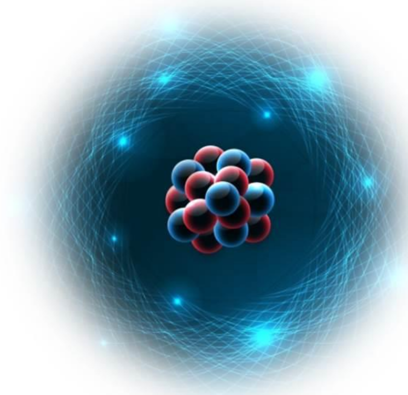


$\otimes$



$$\widehat{H}^0_\mu\,|i\rangle = E^0_i\,|i\rangle \qquad \widehat{H}_N\,|I\rangle = E^0_I\,|I\rangle$$

$\rightarrow$



$$\widehat{H}^0_\mu = \boldsymbol{\alpha}\cdot\mathbf{p} + \beta m_\mu + V_0(\mathbf{r})$$

$$\rho(\mathbf{r}') \rightarrow \rho(\mathbf{r}', \mathbf{r}_{N_i}) = \sum_i^Z \delta(\mathbf{r}' - \mathbf{r}_{N_i})$$

$$V_0(\mathbf{r}) = -\alpha \int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,d^3\mathbf{r}' \xrightarrow{\Delta V = V - V_0} V(\mathbf{r}, \mathbf{r}_{N_i}) = -\alpha \sum_i^Z \frac{1}{|\mathbf{r}-\mathbf{r}_{N_i}|}$$

$$\Delta E_{\mathcal{I}} = \sum_{\mathcal{N}}{}' \frac{\langle\mathcal{I}|\Delta V|\mathcal{N}\rangle\,\langle\mathcal{N}|\Delta V|\mathcal{I}\rangle}{E^0_{\mathcal{I}} - E^0_{\mathcal{N}}}$$

Image source: https://new-science.ru/chto-nahoditsya-vnutri-atomnogo-yadra/

Transverse part of muon-nucleus interaction

$$\widehat{H} = \widehat{H}_N + \boldsymbol{\alpha} \cdot \mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

more complete

$$\widehat{H} = \widehat{H}_N + \boldsymbol{\alpha} \cdot (\mathbf{p} - e\mathbf{A}(\mathbf{r}, \mathbf{r}_{N_i})) + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

- not negligible for the 1^- (electric dipole) contribution
- important for the splitting between $np_{1/2}$ and $np_{3/2}$ states

[1] Y. Tanaka and Y. Horikawa, Nucl. Phys. A580, 291 (1994).

[2] A. Haga, Y. Horikawa, and Y. Tanaka, *Phys. Rev. A* **66**, 034501 (2002).

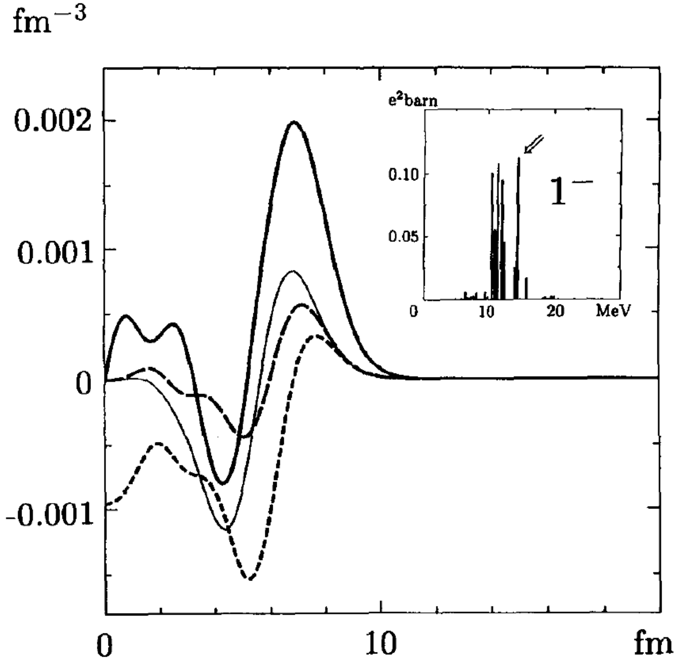
[3] I. A. Valuev, N. S. Oreshkina, arXiv:2401.05904 [physics.atom-ph] (2024).

What is needed from the nuclear side

$$\mathrm{NP} \to \sum_{|\lambda\rangle} [\text{the entire nuclear spectrum}]$$

- excitation energies $\omega_\lambda = E_\lambda - E_0$
- reduced matrix elements:
  - transition (charge) densities
    $\varrho_J^\lambda(\mathrm{x}) = \langle\lambda|| \int d\Omega_{\mathbf{x}}\, Y_J(\Omega_{\mathbf{x}})\hat{\rho}_{\mathrm{N}}(\mathbf{x})||0\rangle$
  - transition current densities
    $\mathcal{J}_{JL}^\lambda(\mathrm{x}) = \langle\lambda|| \int d\Omega_{\mathbf{x}}\, \mathbf{Y}_{JL}(\Omega_{\mathbf{x}}) \cdot \hat{\mathbf{J}}_{\mathrm{N}}(\mathbf{x})||0\rangle$

  for different excitation modes:
  $0^+, 1^-, 2^+, 3^-, (4^+, 5^-, 1^+)$
  in the laboratory frame



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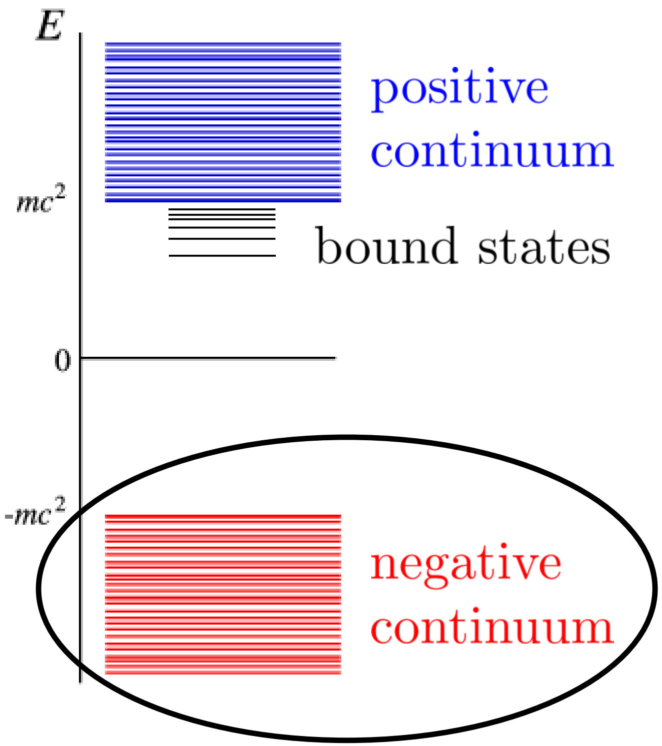
*simplifications are possible in terms of transition probabilities $B(EL)$

Some more details

Summation over the muonic spectrum



Dirac equation:

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m_\mu + V_0(\mathbf{r})]\, \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$$\rho_{\text{Fermi}}(r) = \frac{\rho_0}{1 + e^{(r-c)/a}}$$

$$\Delta E_{\mathcal{I}} = {\sum_{\mathcal{N}}}' \frac{\langle \mathcal{I} | \Delta V | \mathcal{N} \rangle \, \langle \mathcal{N} | \Delta V | \mathcal{I} \rangle}{E^0_{\mathcal{I}} - E^0_{\mathcal{N}}}$$

Naive extension of the summation leads to a wrong expression in the energy denominator

$\rightarrow$ can only be resolved in a *field-theoretical* description

Field-theoretical approach

$$\hat{J}^{\mu}_{\mathrm{N,\,total}}(x) = J^{\mu}_{\mathrm{N,\,stat}}(\mathbf{x}) + \hat{J}^{\mu}_{\mathrm{N,\,fluc}}(x)$$

$$\hat{A}^{\mu}_{\mathrm{total}}(x) = \mathcal{A}^{\mu}_{\mathrm{stat}}(\mathbf{x}) + \underbrace{\hat{A}^{\mu}_{\mathrm{fluc}}(x) + \hat{A}^{\mu}_{\mathrm{free}}(x)}_{:= \hat{A}^{\mu}_{\mathrm{rad}}(x)}$$

$$iD_{\mu\nu}(x - x')$$

$$= \langle 0 | T[\hat{A}^{\mathrm{free}}_{\mu}(x)\hat{A}^{\mathrm{free}}_{\nu}(x')] | 0 \rangle$$

dressed muon propagator

modified photon propagator:

$$i\mathcal{D}_{\mu\nu}(x, x') = \langle 0 | T[\hat{A}^{\mathrm{rad}}_{\mu}(x)\hat{A}^{\mathrm{rad}}_{\nu}(x')] | 0 \rangle$$

$$= iD_{\mu\nu}(x - x') + \langle 0 | T[\hat{A}^{\mathrm{fluc}}_{\mu}(x)\hat{A}^{\mathrm{fluc}}_{\nu}(x')] | 0 \rangle$$

Modified photon propagator

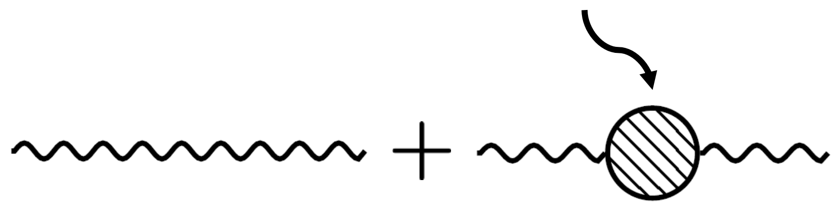
$$\mathcal{D}_{\mu\nu}(x,x') = D_{\mu\nu}(x-x') + D^{\mathrm{NP}}_{\mu\nu}(x,x')$$

NP tensor

"seagull" term

$$D^{\mathrm{NP}}_{\mu\nu}(x,x') = \int d^4x_1\, d^4x_2\, D_{\mu\xi}(x-x_1) \underbrace{\left[\Pi^{\xi\zeta}_{\mathrm{N}}(x_1,x_2) + S^{\xi\zeta}_{\mathrm{N}}(x_1,x_2)\right]} D_{\zeta\nu}(x_2-x')$$

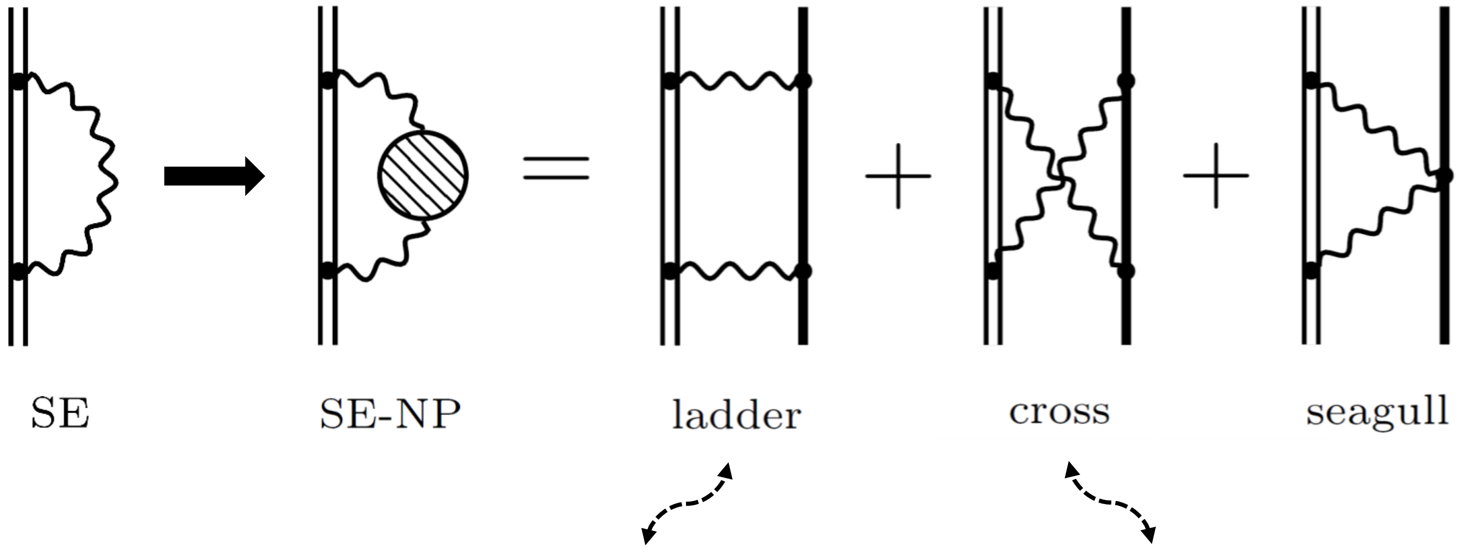
$\mathcal{D}_{\mu\nu}(x,x') =$



NP insertion

$$i\Pi^{\xi\zeta}_{\mathrm{N}}(x_1,x_2) = \langle 0|T[\hat{J}^{\xi}_{\mathrm{N,\,fluc}}(x_1)\hat{J}^{\zeta}_{\mathrm{N,\,fluc}}(x_2)]|0\rangle$$

Nuclear polarization as effective self-energy



$$\widetilde{\Pi}_{\mathrm{N}}^{\xi\zeta}(\omega, \mathbf{x}_1, \mathbf{x}_2) = \sum_{\lambda} \left(\frac{\langle 0| \hat{J}_{\mathrm{N}}^{\xi}(\mathbf{x}_1)|\lambda\rangle\langle\lambda|\hat{J}_{\mathrm{N}}^{\zeta}(\mathbf{x}_2)|0\rangle}{\omega - \omega_\lambda + i0} - \frac{\langle\lambda| \hat{J}_{\mathrm{N}}^{\xi}(\mathbf{x}_1)|0\rangle\langle 0|\hat{J}_{\mathrm{N}}^{\zeta}(\mathbf{x}_2)|\lambda\rangle}{\omega + \omega_\lambda - i0} \right)$$

for non-relativistic nuclear charge-current operators:

$$\widetilde{S}_{\mathrm{N}}^{\xi\zeta}(\omega, \mathbf{x}_1, \mathbf{x}_2) = \frac{|e|\langle 0|\hat{\rho}_{\mathrm{N}}(\mathbf{x}_1)|0\rangle}{M_{\mathrm{p}}} \delta^{\xi\zeta}\delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_2)$$

$$(\delta^{00} = 0)$$

Non-relativistic nuclear charge-current operators

Charge: $$\hat{\rho}_{\mathrm{N}}(\mathbf{x}) = \sum_{i}^{A} |e|\, \delta^{(3)}(\mathbf{x}-\mathbf{x}_i)\, \frac{1-\tau_{3,i}}{2},$$

Current: $$\hat{\mathbf{J}}_{\mathrm{N}}(\mathbf{x}) = \hat{\mathbf{J}}_{\mathrm{N,c}}(\mathbf{x}) + \hat{\mathbf{J}}_{\mathrm{N,m}}(\mathbf{x}),$$

Convection part:

$$\hat{\mathbf{J}}_{\mathrm{N,c}}(\mathbf{x}) = \sum_{i}^{A} |e|\, \delta^{(3)}(\mathbf{x}-\mathbf{x}_i)\, \frac{1-\tau_{3,i}}{2}\, \frac{\overrightarrow{\nabla}_{\mathbf{x}_i} - \overleftarrow{\nabla}_{\mathbf{x}_i}}{2Mi},$$

Magnetization part:

$$\hat{\mathbf{J}}_{\mathrm{N,m}}(\mathbf{x}) = \hat{\mathbf{J}}^{\mathrm{p}}_{\mathrm{N,m}}(\mathbf{x}) + \hat{\mathbf{J}}^{\mathrm{n}}_{\mathrm{N,m}}(\mathbf{x}) = \nabla \times \hat{\boldsymbol{\mu}}(\mathbf{x}),$$

$$\hat{\boldsymbol{\mu}}(\mathbf{x}) = \sum_{i}^{A} \delta^{(3)}(\mathbf{x}-\mathbf{x}_i)\, \frac{|e|}{2M} \left(\frac{1-\tau_{3,i}}{2}\mu_{\mathrm{p}} + \frac{1+\tau_{3,i}}{2}\mu_{\mathrm{n}} \right) \boldsymbol{\sigma}_i,$$

Relativistic vs. non-relativistic for the muonic part

very light nuclei:

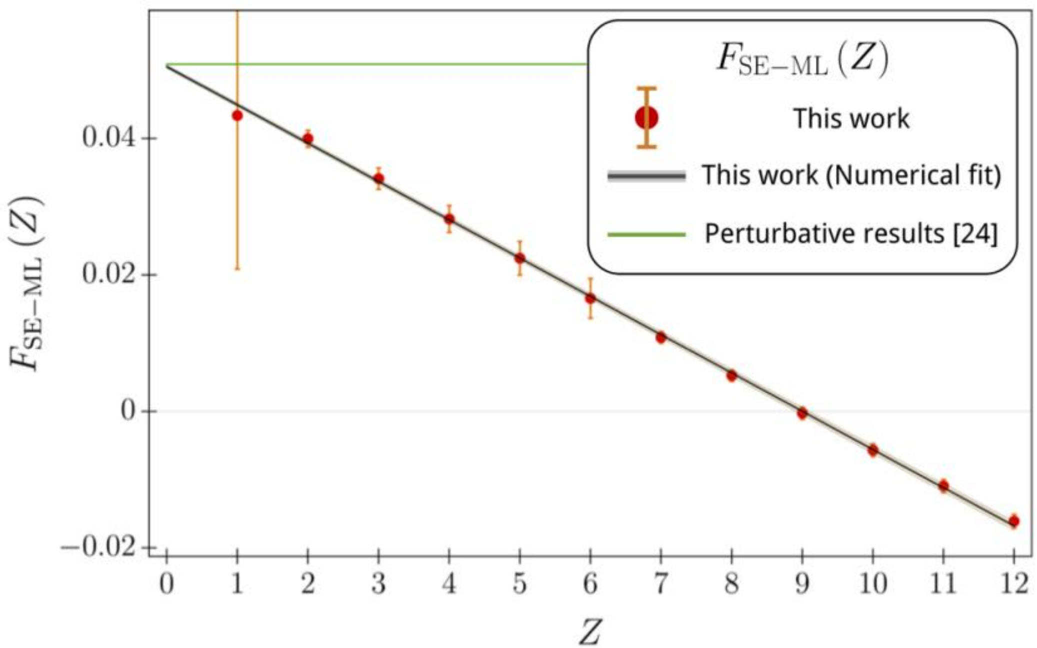
- *ab initio* nuclear
- non-relativistic muonic $\to (Z\alpha)$ expansion

$$\delta_{\rm NP} = -\frac{16\pi^2}{9}(Z\alpha)^2\phi^2(0)\int_0^\infty d\omega\sqrt{\frac{2m_r}{\omega}}S_{D_1}(\omega)+\ldots$$

muonic wave-function normalization constant

C. Ji, S. Bacca, N. Barnea, O. J. Hernandez, and N. N. Dinur, *Journal of Physics G: Nuclear and Particle Physics*, **45**(9), 093002 (2018).

However, even for electronic systems:



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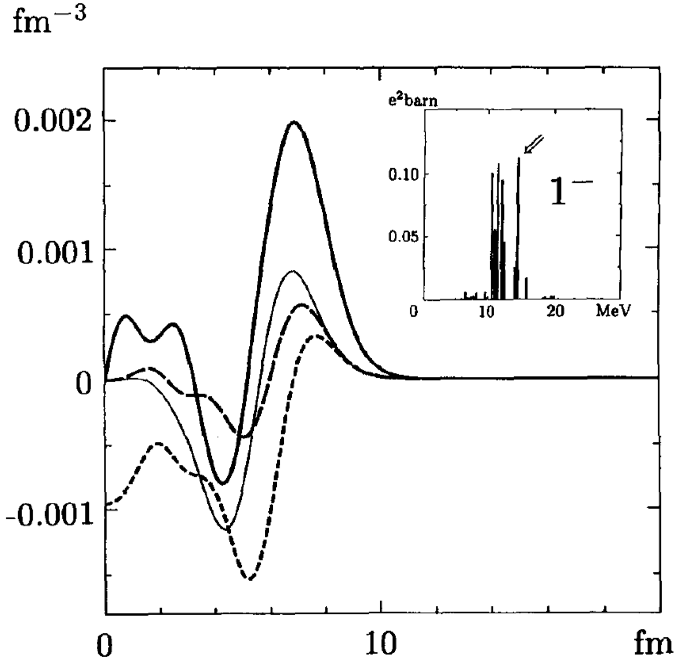

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