

# Absolute radii of chlorine and potassium:

A heavyweight solution to a small problem

Michael Heines

On behalf of the muX collaboration

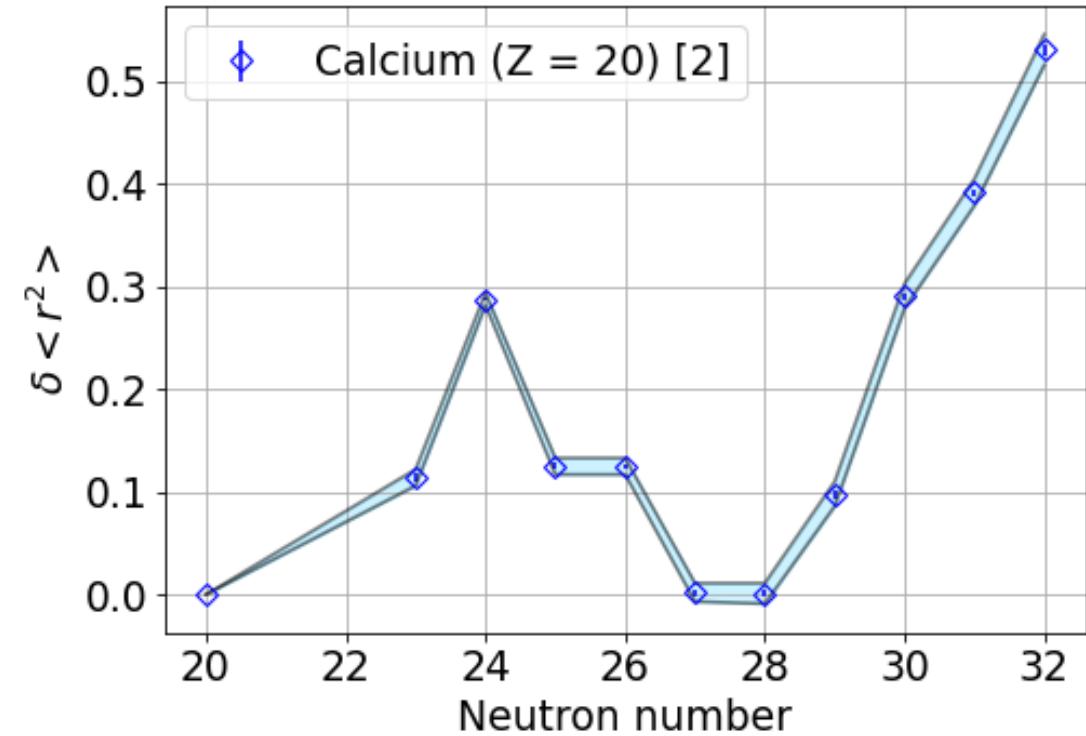
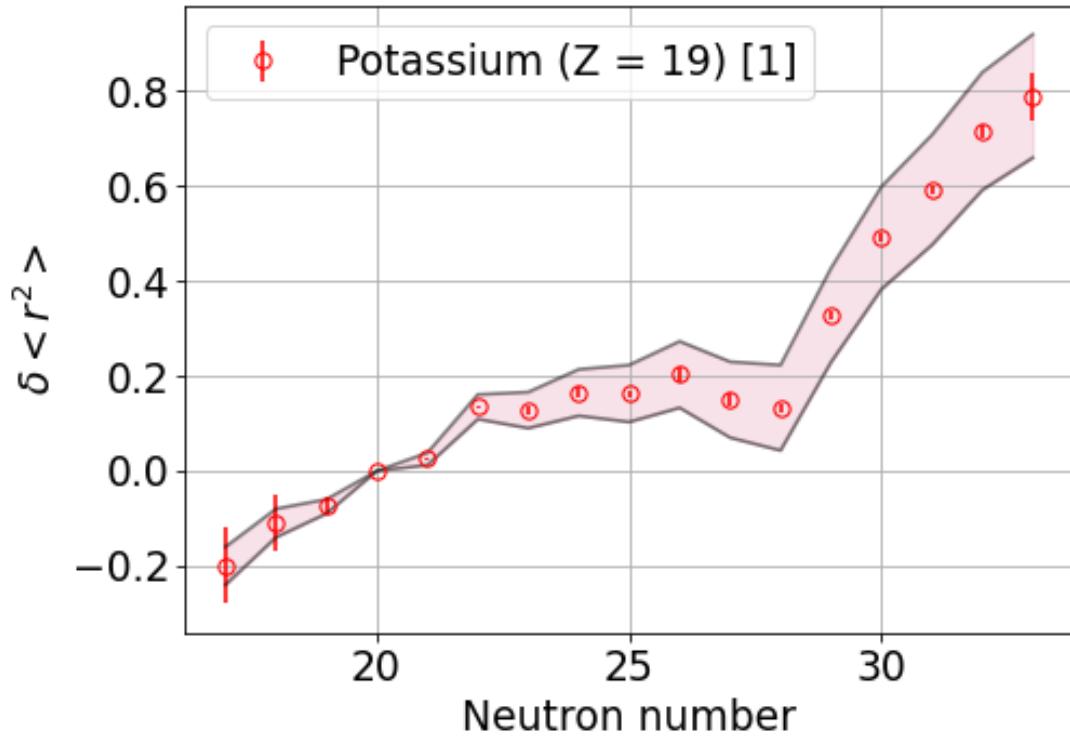
# Contents

- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Conclusion and outlook

# Contents

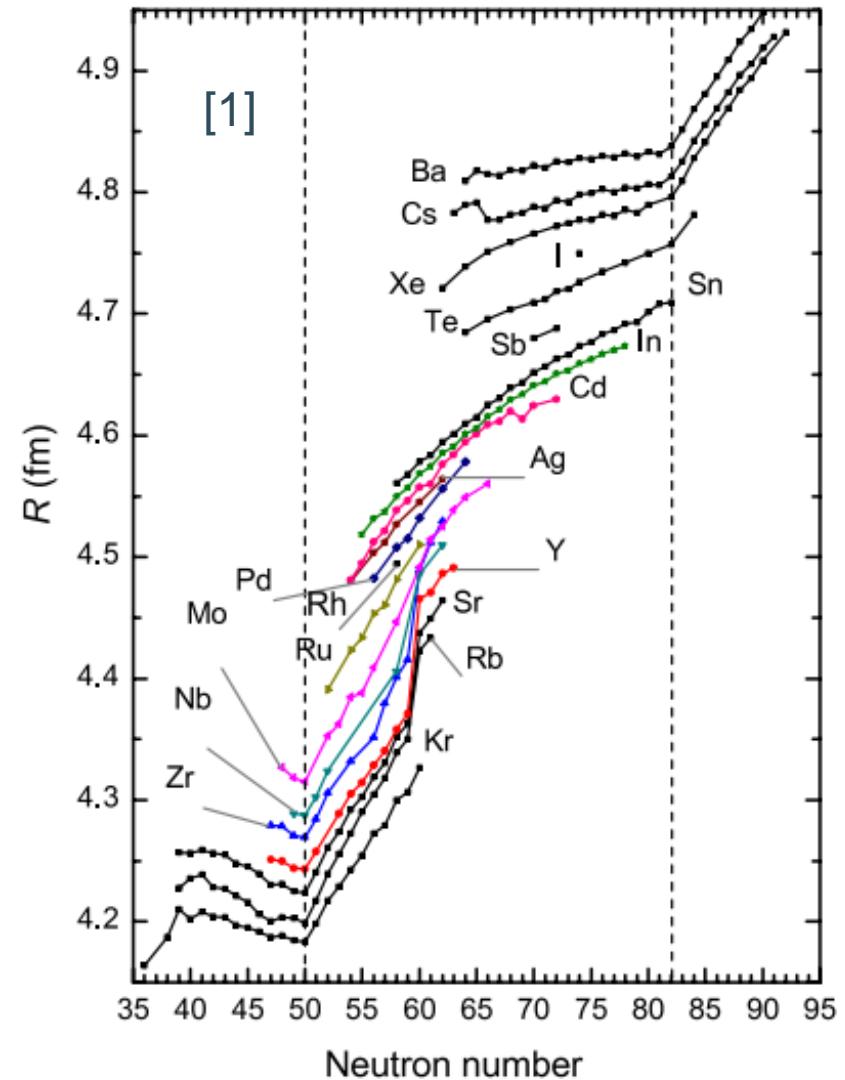
- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Conclusion and outlook

# Measurements of $\delta \langle r^2 \rangle$



# Benefit of absolute radii

- Visualizing global trends
- Input for other experiments
- Isotone shifts
- Mirror nuclei

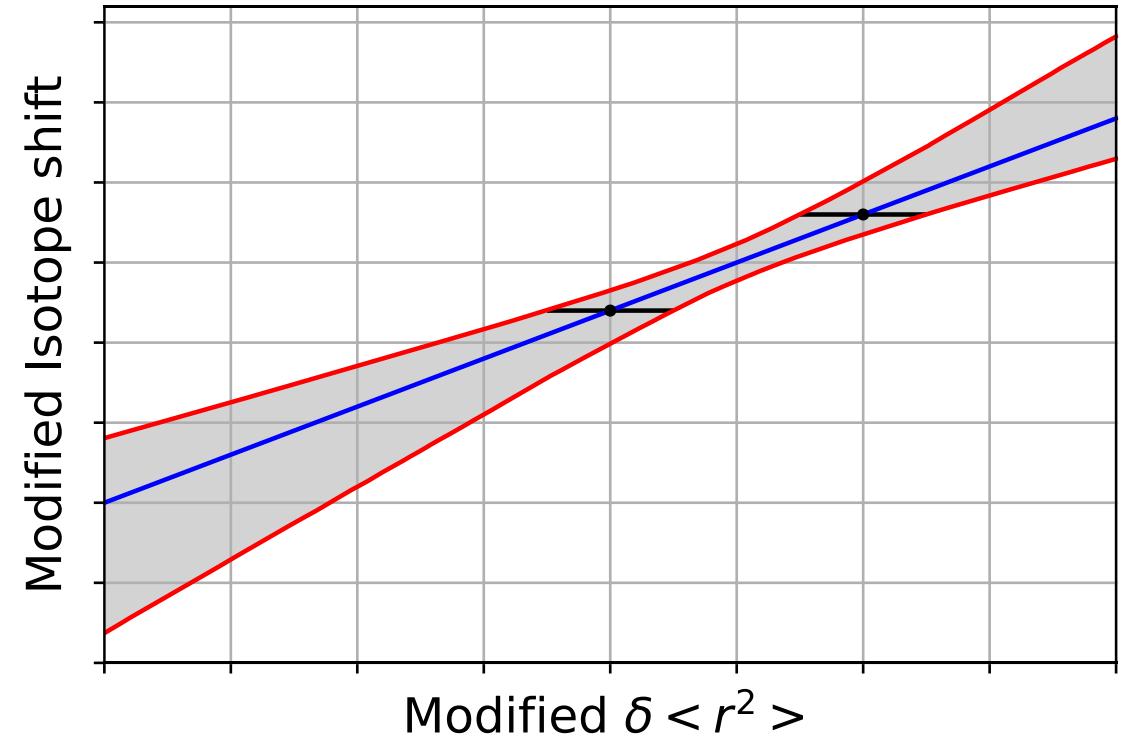


# King plot

$$\delta\langle r^2 \rangle^{A,A'} = \frac{1}{F_i} \left( \delta\nu_i^{A,A'} - \frac{A - A'}{AA'} M_i \right)$$

- $M_i$  : Mass shift factor
- $F_i$  : Field shift factor

$$\frac{AA'}{A - A'} \delta\nu_i^{A,A'} = M_i + F_i \frac{AA'}{A - A'} \delta\langle r^2 \rangle^{A,A'}$$

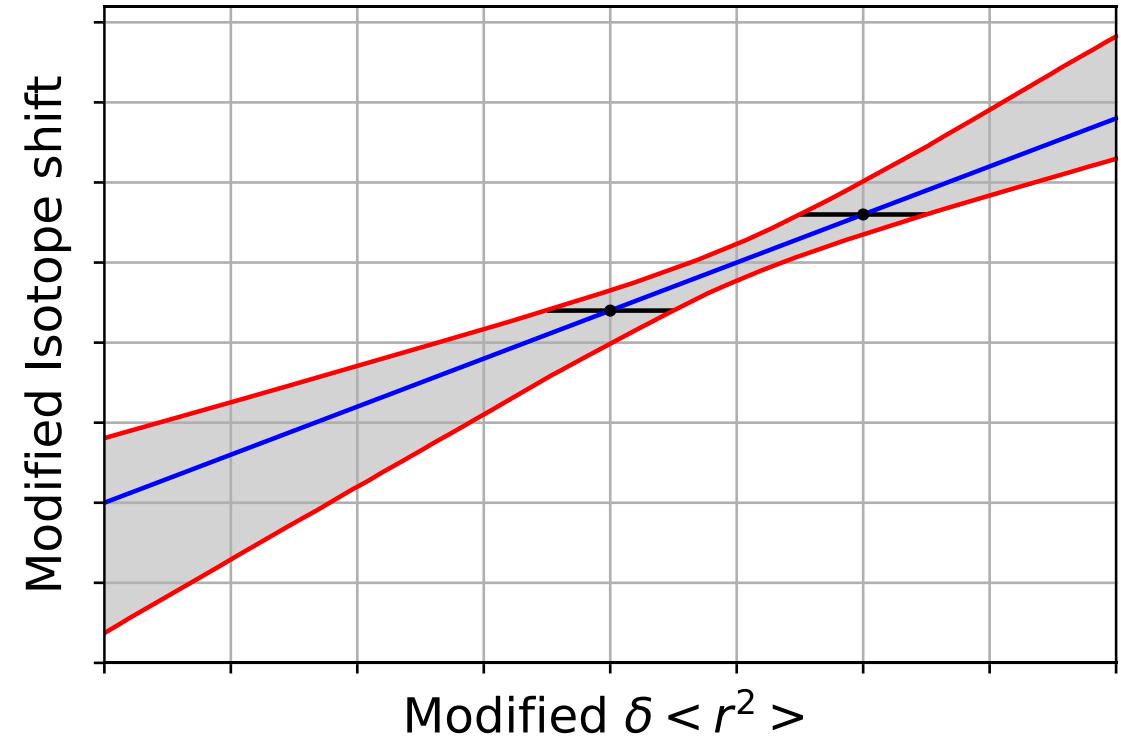


# King plot

$$\delta\langle r^2 \rangle^{A,A'} = \frac{1}{F_i} \left( \delta\nu_i^{A,A'} - \frac{A - A'}{AA'} M_i \right)$$

- $M_i$  : Mass shift factor
- $F_i$  : Field shift factor

$$\frac{AA'}{A - A'} \delta\nu_i^{A,A'} = M_i + F_i \frac{AA'}{A - A'} \delta\langle r^2 \rangle^{A,A'}$$



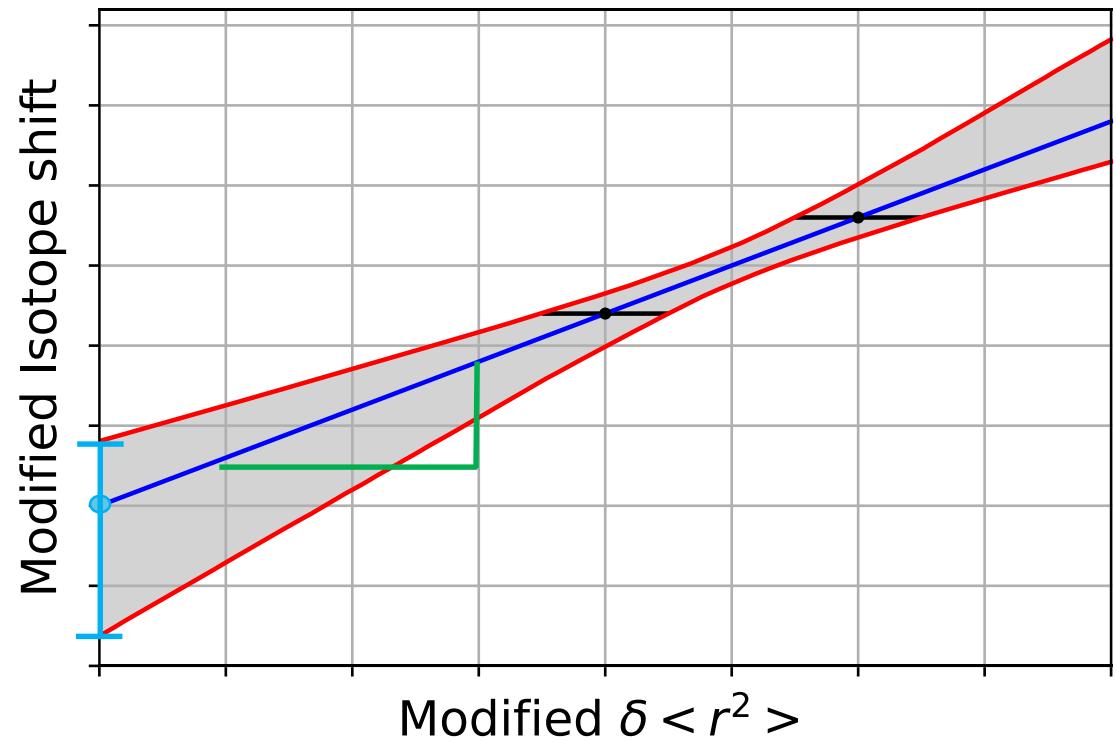
# King plot

$$\delta\langle r^2 \rangle^{A,A'} = \frac{1}{F_i} \left( \delta\nu_i^{A,A'} - \frac{A - A'}{AA'} M_i \right)$$

- $M_i$  : Mass shift factor
- $F_i$  : Field shift factor

$$\frac{AA'}{A - A'} \delta\nu_i^{A,A'} = M_i + F_i \frac{AA'}{A - A'} \delta\langle r^2 \rangle^{A,A'}$$

No odd-Z element with 3 stable isotopes!

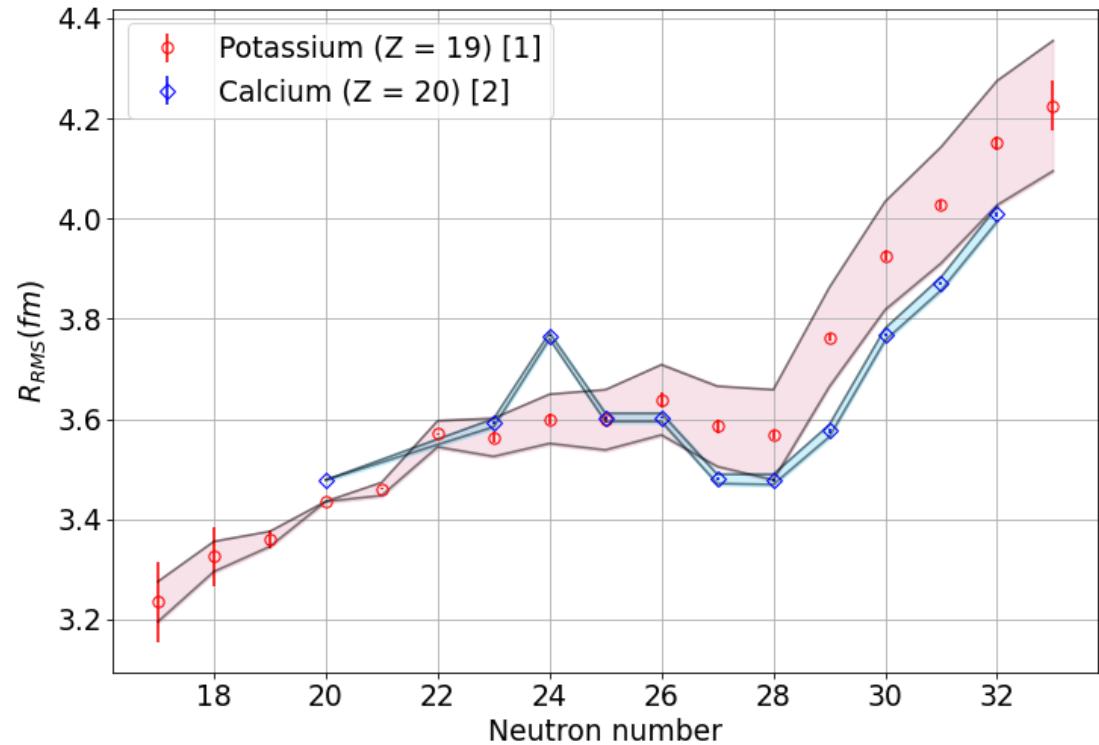


# King plot

$$\delta\langle r^2 \rangle^{A,A'} = \frac{1}{F_i} \left( \delta\nu_i^{A,A'} - \frac{A - A'}{AA'} M_i \right)$$

- $M_i$  : Mass shift factor
- $F_i$  : Field shift factor

$$\frac{AA'}{A - A'} \delta\nu_i^{A,A'} = M_i + F_i \frac{AA'}{A - A'} \delta\langle r^2 \rangle^{A,A'}$$

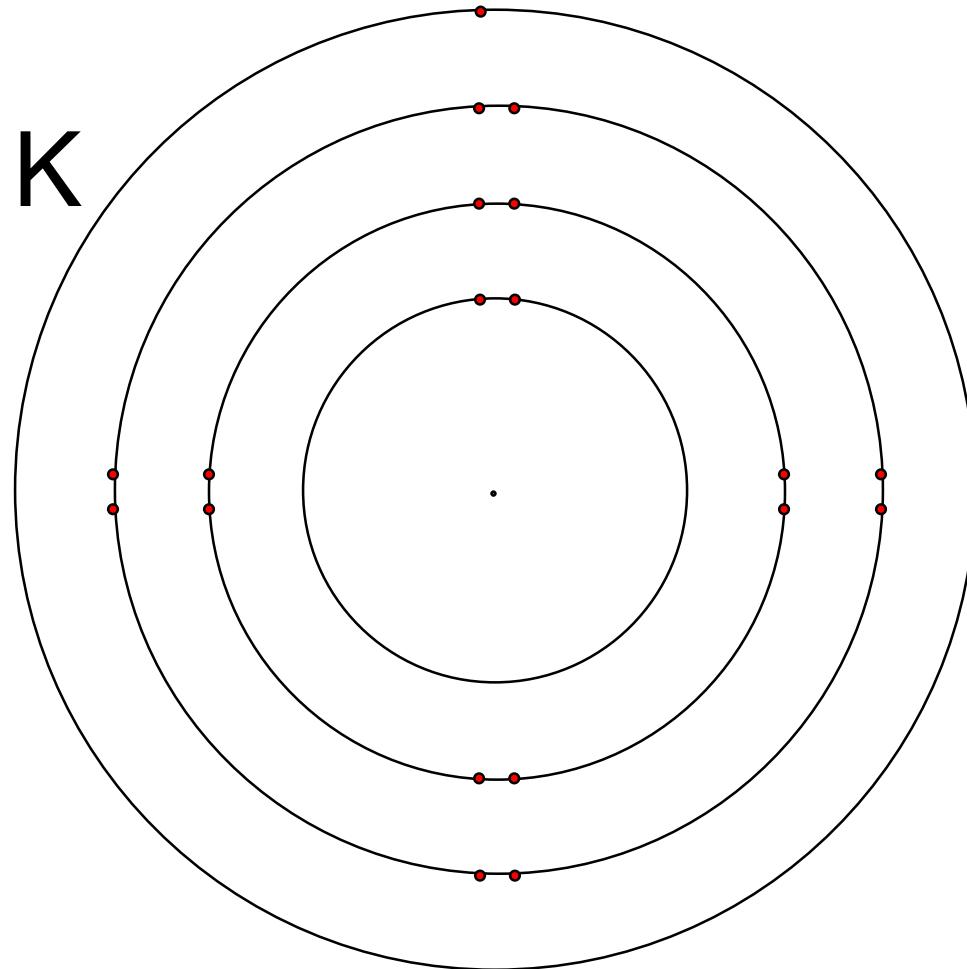
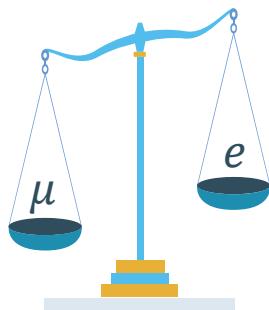


# Contents

- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Conclusion and outlook

# Muonic atoms

- Bohr model
  - $E_n \propto \frac{mZ^2}{n^2}$
  - $r_n \propto \frac{n^2}{mZ}$
- Muons:
  - $m_\mu \approx 207 m_e$
  - $\tau_\mu \approx 2.2 \mu\text{s}$



# Muonic atoms

- Bohr model

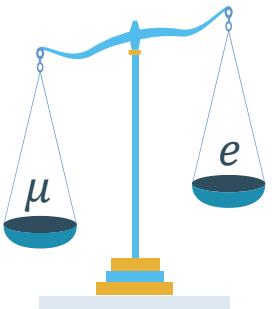
- $E_n \propto \frac{mZ^2}{n^2}$

- $r_n \propto \frac{n^2}{mZ}$

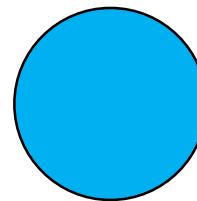
- Muons:

- $m_\mu \approx 207 m_e$

- $\tau_\mu \approx 2.2 \mu s$

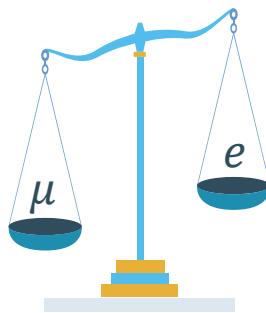


K

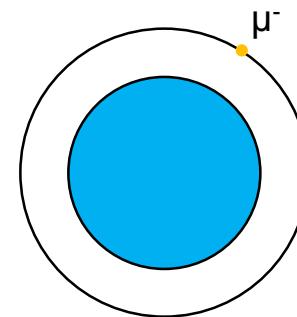


# Muonic atoms

- Bohr model
  - $E_n \propto \frac{mZ^2}{n^2}$
  - $r_n \propto \frac{n^2}{mZ}$
- Muons:
  - $m_\mu \approx 207 m_e$
  - $\tau_\mu \approx 2.2 \mu\text{s}$
- Effect:
  - Enhanced binding energy
  - Closer to the nucleus → More sensitive to nuclear effects

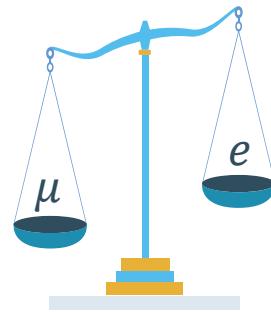


μK

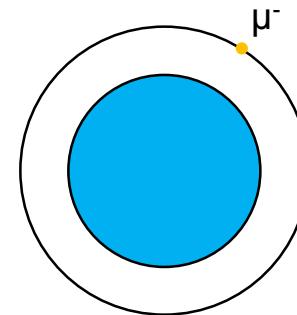


# Muonic atoms

- Bohr model
  - $E_n \propto \frac{mZ^2}{n^2}$
  - $r_n \propto \frac{n^2}{mZ}$
- Muons:
  - $m_\mu \approx 207 m_e$
  - $\tau_\mu \approx 2.2 \mu\text{s}$
- Effect:
  - Enhanced binding energy
  - Closer to the nucleus → More sensitive to nuclear effects



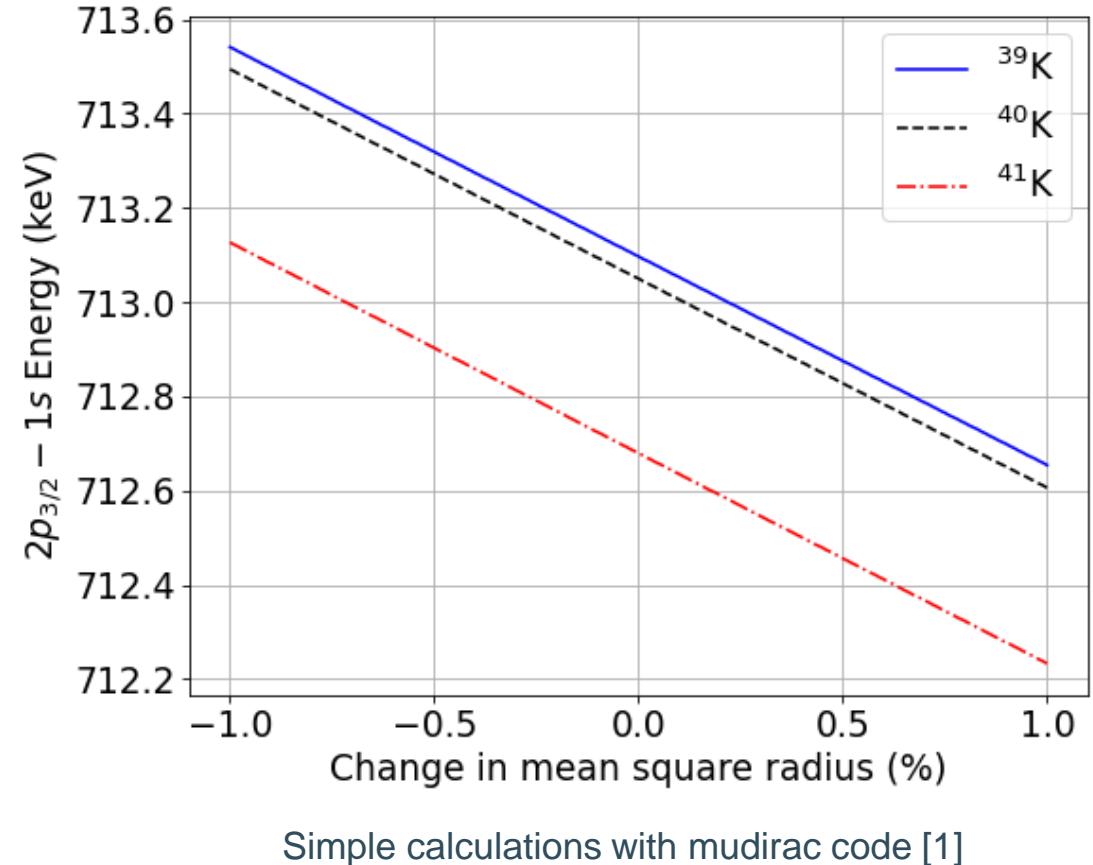
μK



Muon  $n = 14$  orbital is inside electron  $n = 1$  orbital  
→ electron correlation is negligible

# Extracting radii

- Finite size correction scales with  $\frac{1}{r^3} \approx 10^7$
- Calculate transition energy for many radii  
→ Compare with experiment
- Typical limitations:
  - Nuclear polarization (theory)
  - Nuclear shape (electron scattering)
  - Energy calibration



<sup>16</sup> [1] Sturniolo, Simone, and Adrian Hillier. "Mudirac: A Dirac equation solver for elemental analysis with muonic X-rays." *X-Ray Spectrometry* 50.3 (2021): 180-196.

# Extracting radii

- Finite size correction scales with  $\frac{1}{r^3} \approx 10^7$
- Calculate transition energy for many radii  
→ Compare with experiment
- Typical limitations:
  - Nuclear polarization (theory)
  - Nuclear shape (electron scattering)
  - Energy calibration

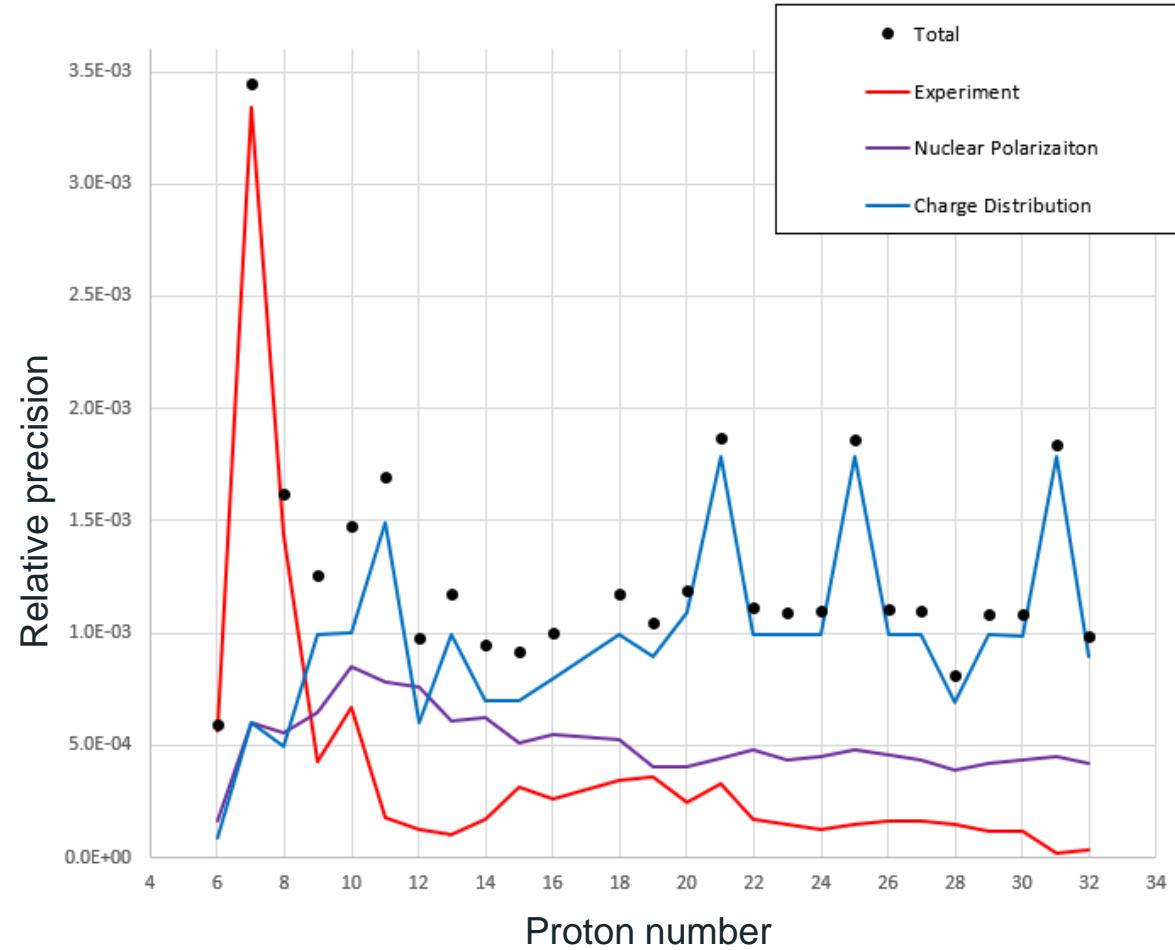
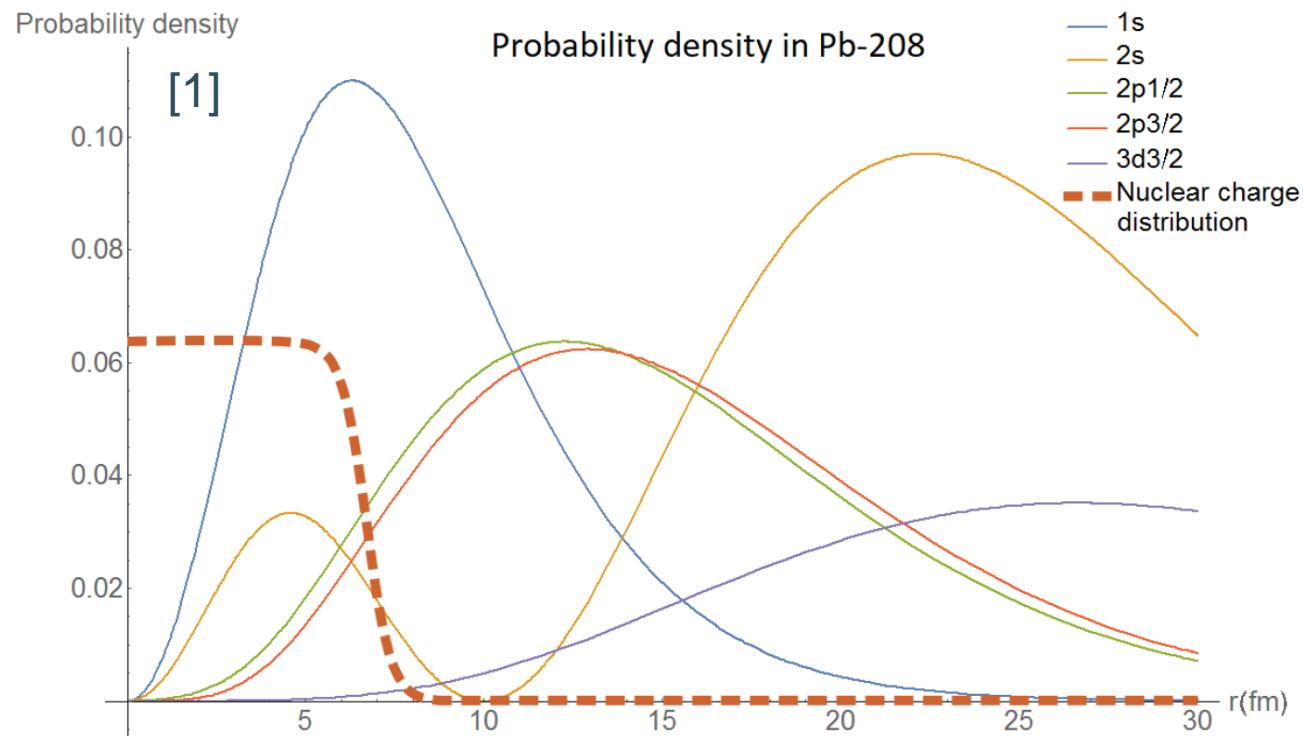


Image courtesy: Ben Ohayon

# How sensitive are we?

- Groundstate wavefunction has sizeable overlap with the nucleus
- Sensitivity increase:
  - Nuclear size:  $\left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$
  - Quadrupole:  $\left(\frac{m_\mu}{m_e}\right)^2 \approx 5 \times 10^4$
  - Octupole:  $\left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$



<sup>18</sup> [1] S. M. Vogiatzi. The fitting of the hyperfine splitting of the 5 → 4 transitions in muonic Re-185. Master's thesis, ETH Zurich, 2018.

# How sensitive are we?

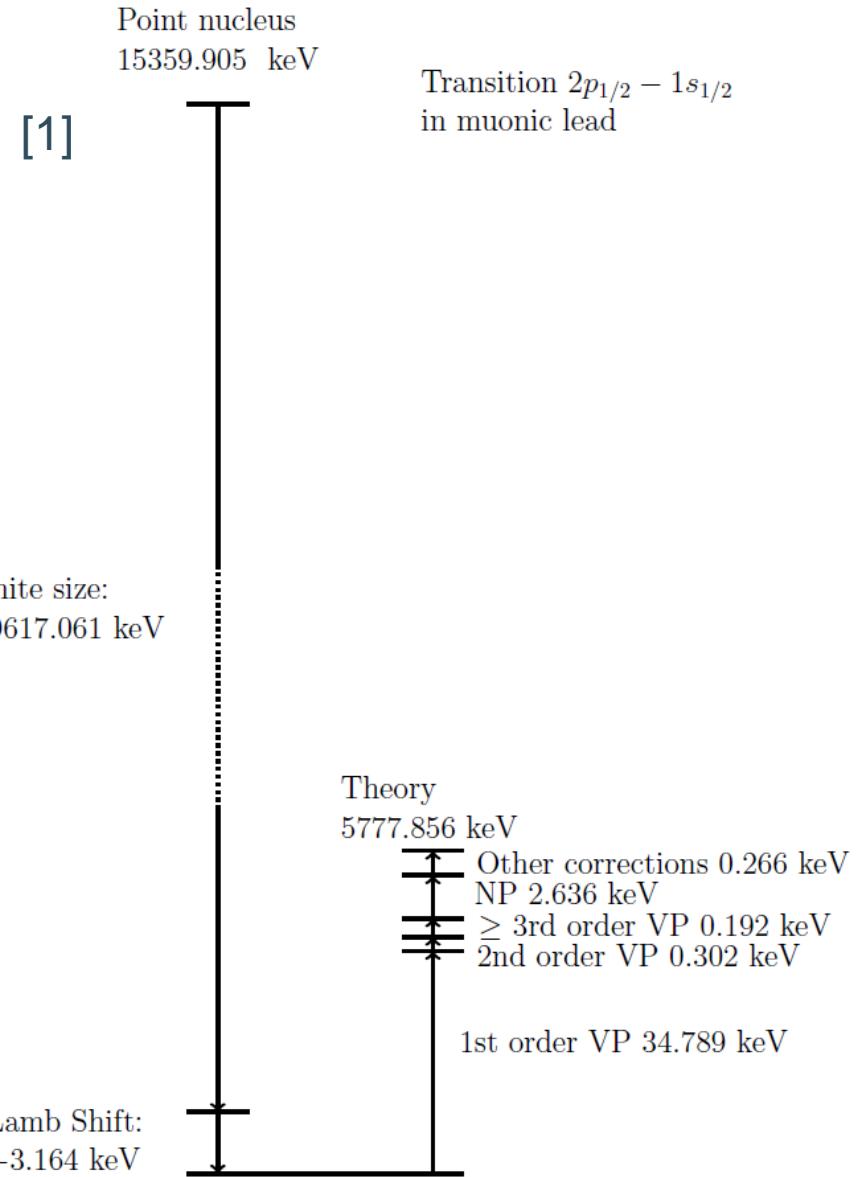
- Groundstate wavefunction has sizeable overlap with the nucleus

- Sensitivity increase:

- Nuclear size:  $\left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$

- Quadrupole:  $\left(\frac{m_\mu}{m_e}\right)^2 \approx 5 \times 10^4$

- Octupole:  $\left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$



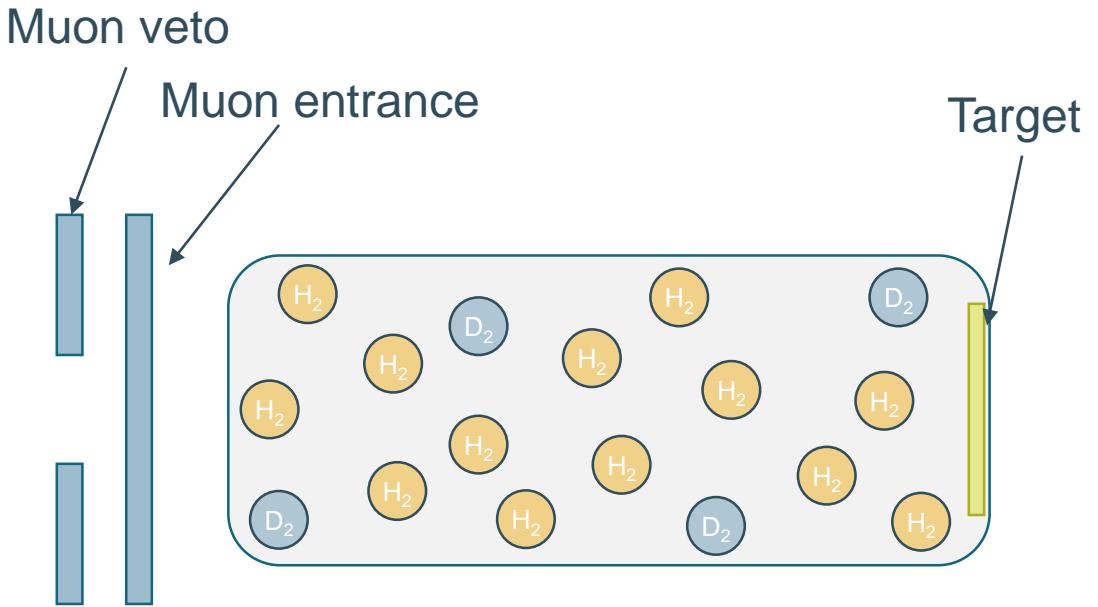
<sup>19</sup> [1] S. M. Vogiatzi. The fitting of the hyperfine splitting of the 5 → 4 transitions in muonic Re-185. Master's thesis, ETH Zurich, 2018.

# Contents

- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Conclusion and outlook

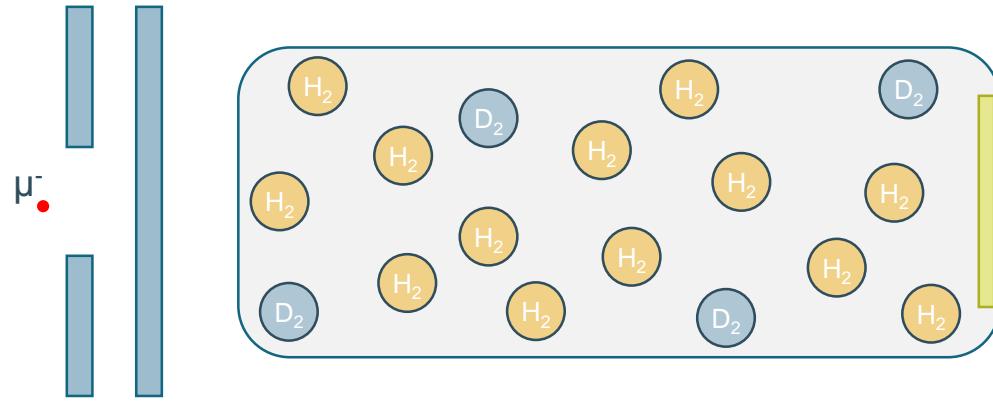
# Measuring microgram materials

- Traditionally: Limited to target mass  $O(10\text{-}100 \text{ mg})$
- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - Limited to  $O(5 \mu\text{g})$
  - Down to 20 year half-life (radioprotection)



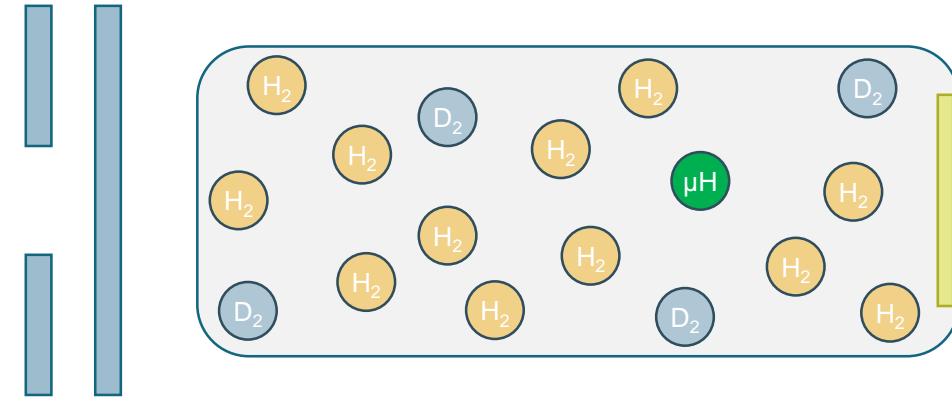
# Measuring microgram materials

- Traditionally: Limited to target mass  
 $O(10\text{-}100 \text{ mg})$
- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - Limited to  $O(5 \mu\text{g})$
  - Down to 20 year half-life (radioprotection)



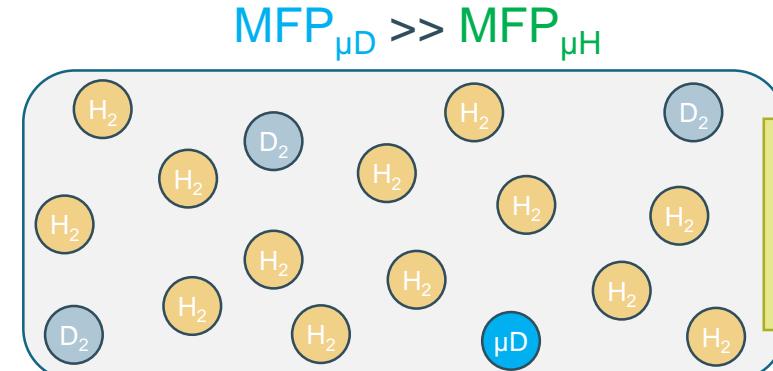
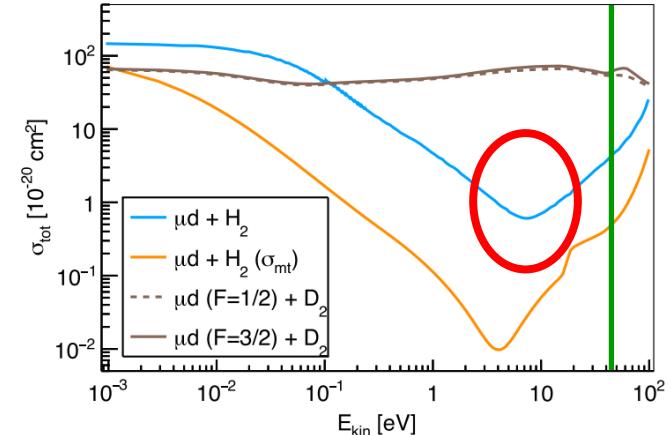
# Measuring microgram materials

- Traditionally: Limited to target mass  
 $O(10\text{-}100 \text{ mg})$
- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - Limited to  $O(5 \mu\text{g})$
  - Down to 20 year half-life (radioprotection)



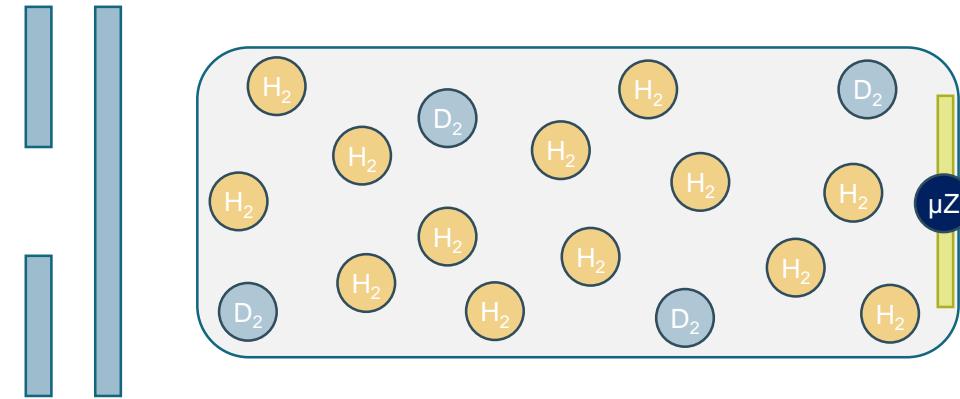
# Measuring microgram materials

- Traditionally: Limited to target mass  $O(10\text{-}100 \text{ mg})$
- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - Limited to  $O(5 \mu\text{g})$
  - Down to 20 year half-life (radioprotection)



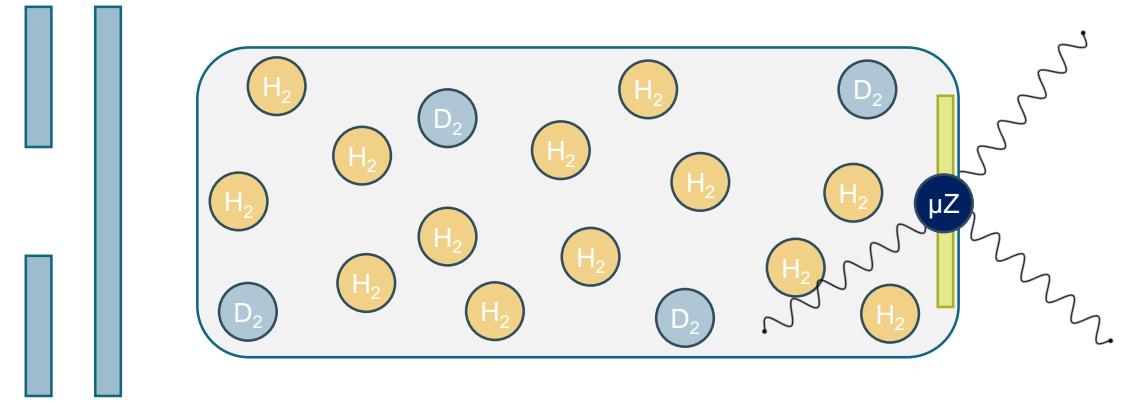
# Measuring microgram materials

- Traditionally: Limited to target mass  
 $O(10\text{-}100 \text{ mg})$
- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - Limited to  $O(5 \mu\text{g})$
  - Down to 20 year half-life (radioprotection)



# Measuring microgram materials

- Traditionally: Limited to target mass  
 $O(10\text{-}100 \text{ mg})$
- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - Limited to  $O(5 \mu\text{g})$
  - Down to 20 year half-life (radioprotection)



# Contents

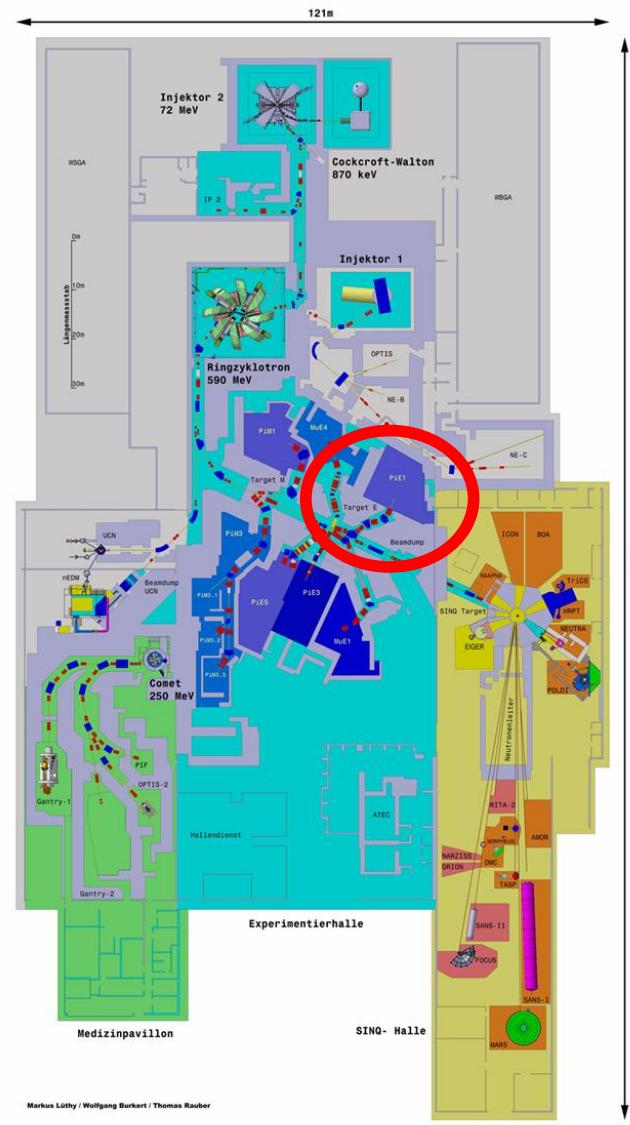
- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Conclusion and outlook

# Primary goals

- First measurement of isotopically pure  $^{35, 37}\text{Cl}$  (macroscopic target)
- Remeasurement of  $^{39, 41}\text{K}$  (macroscopic target)
- First measurement of  $^{40}\text{K}$  (microscopic implanted target)



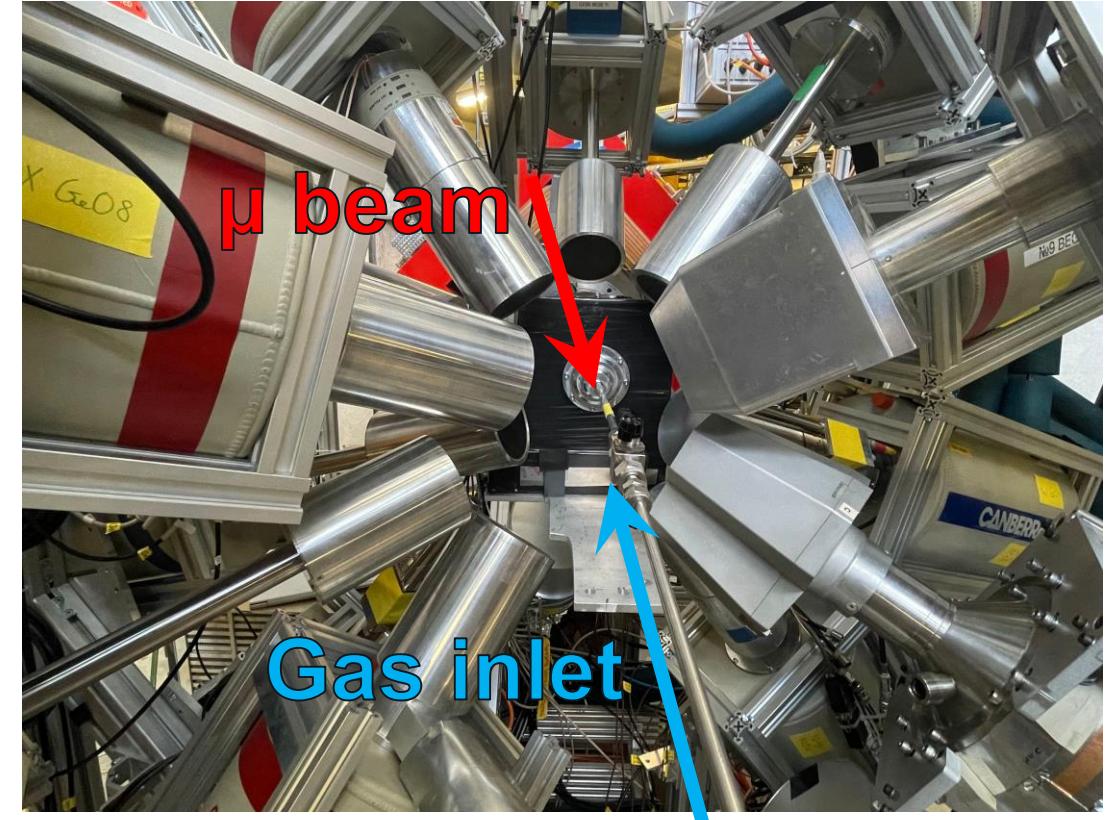
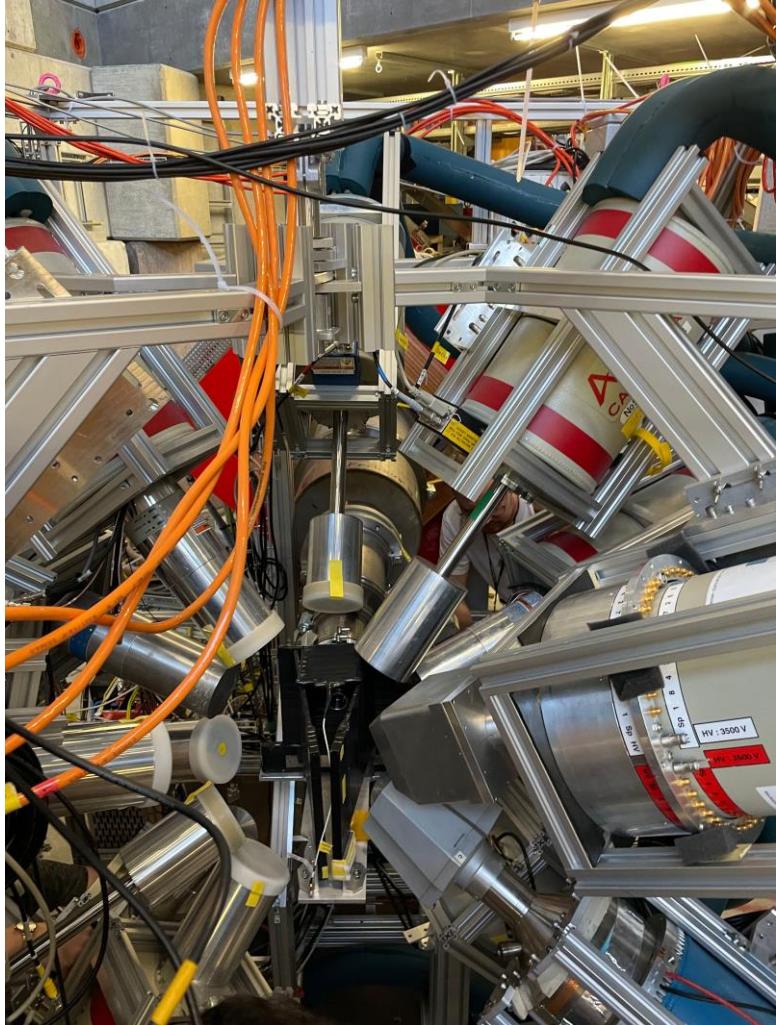
# PSI – High-intensity proton



# Setup

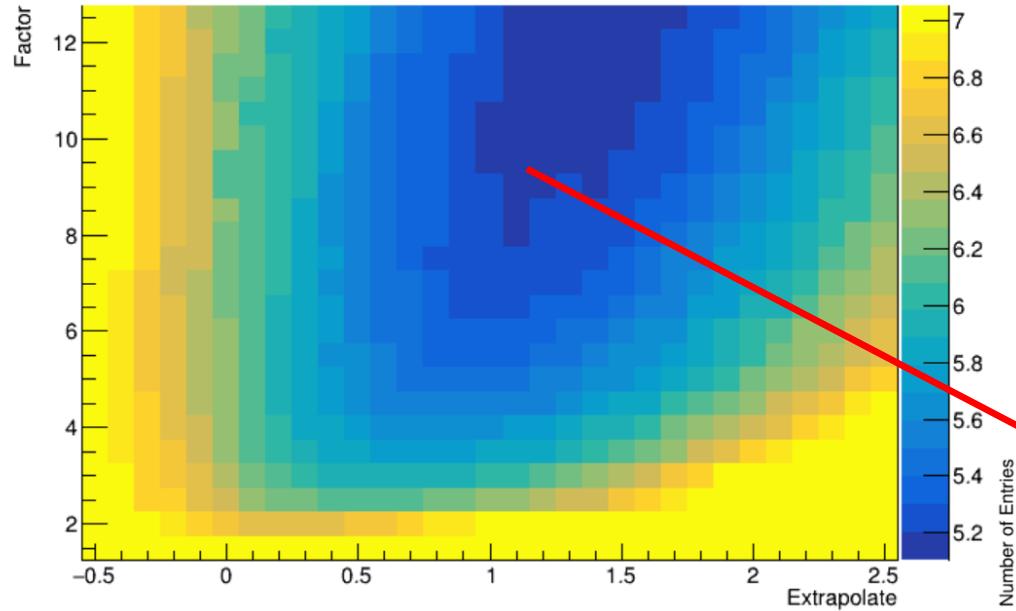


# Setup

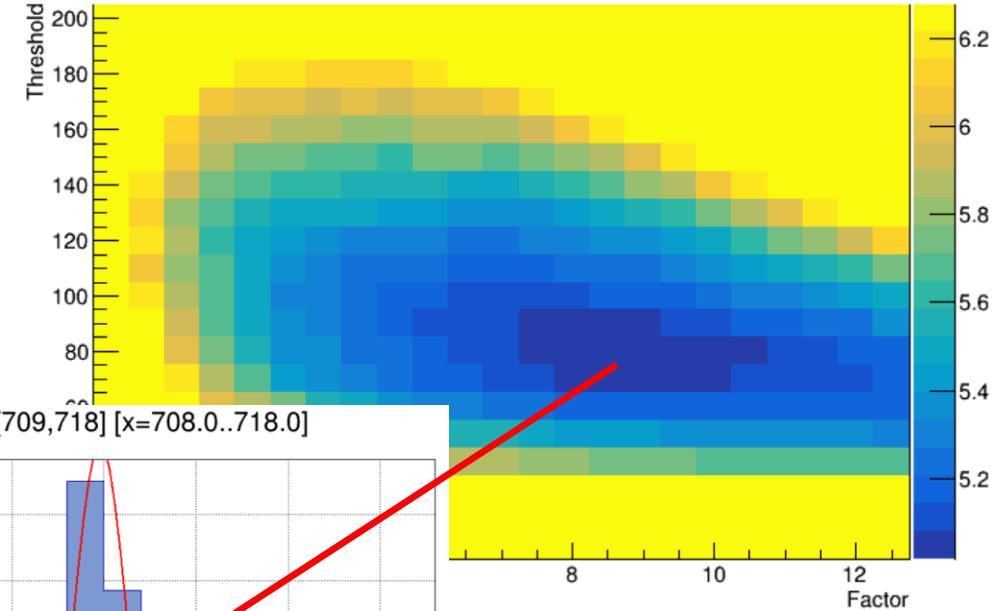


# Data filtering: Timing optimization

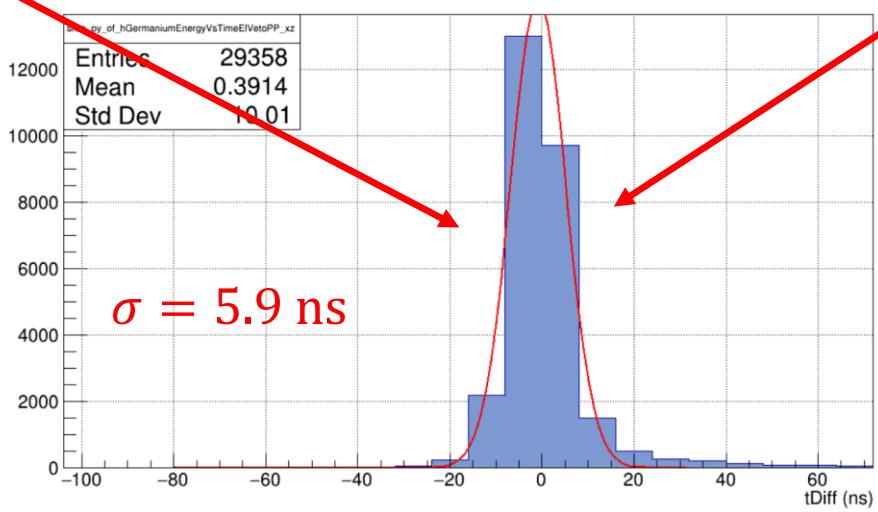
Time resolution yz projection



Time resolution xy projection

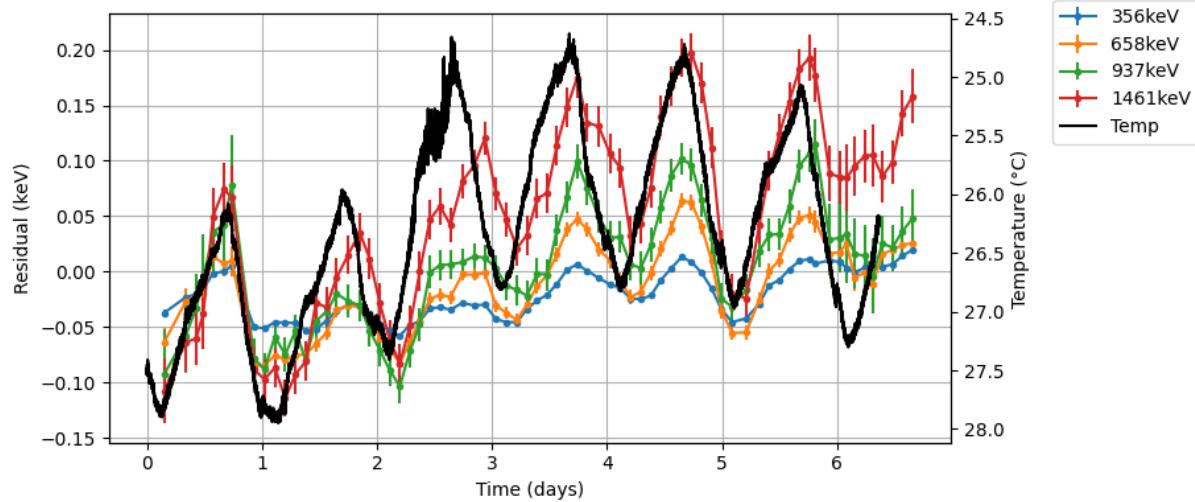


ProjectionY of binx=[709,718] [x=708.0..718.0]

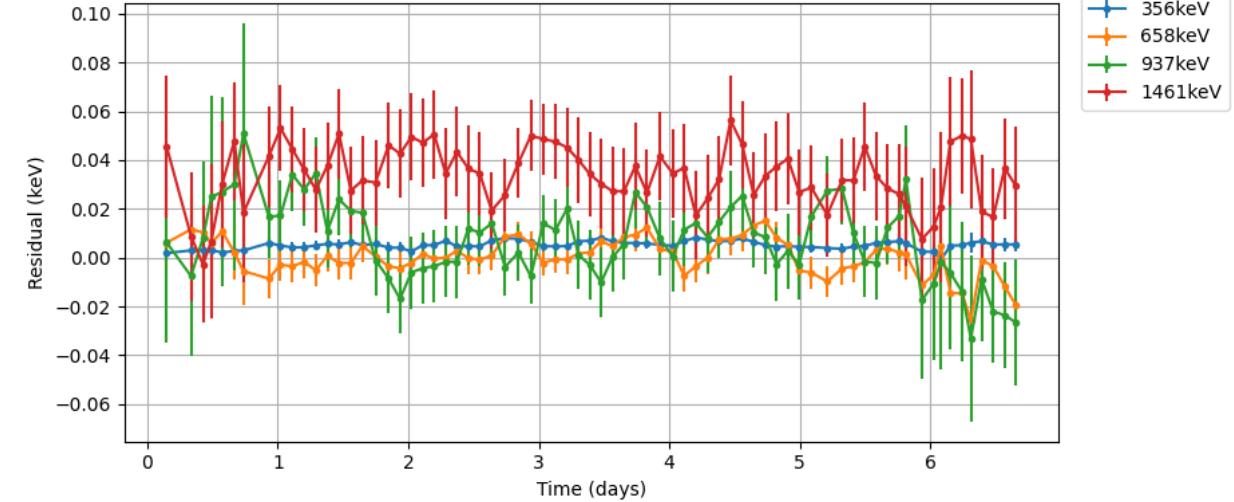


# Data filtering: Gain drift correction

- Before correcting

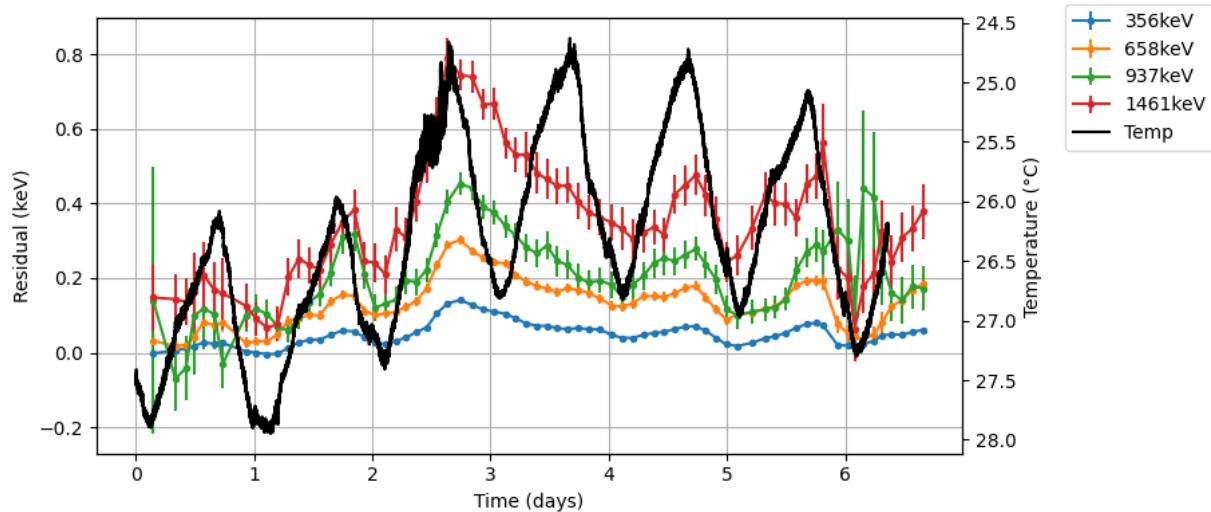


- After correcting

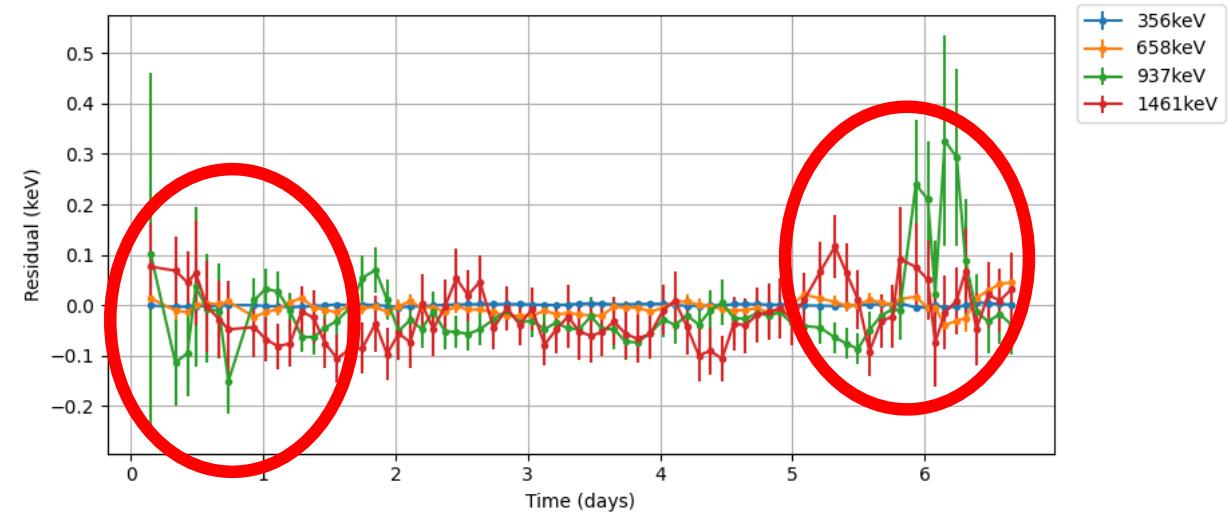


# Data filtering: Gain drift correction

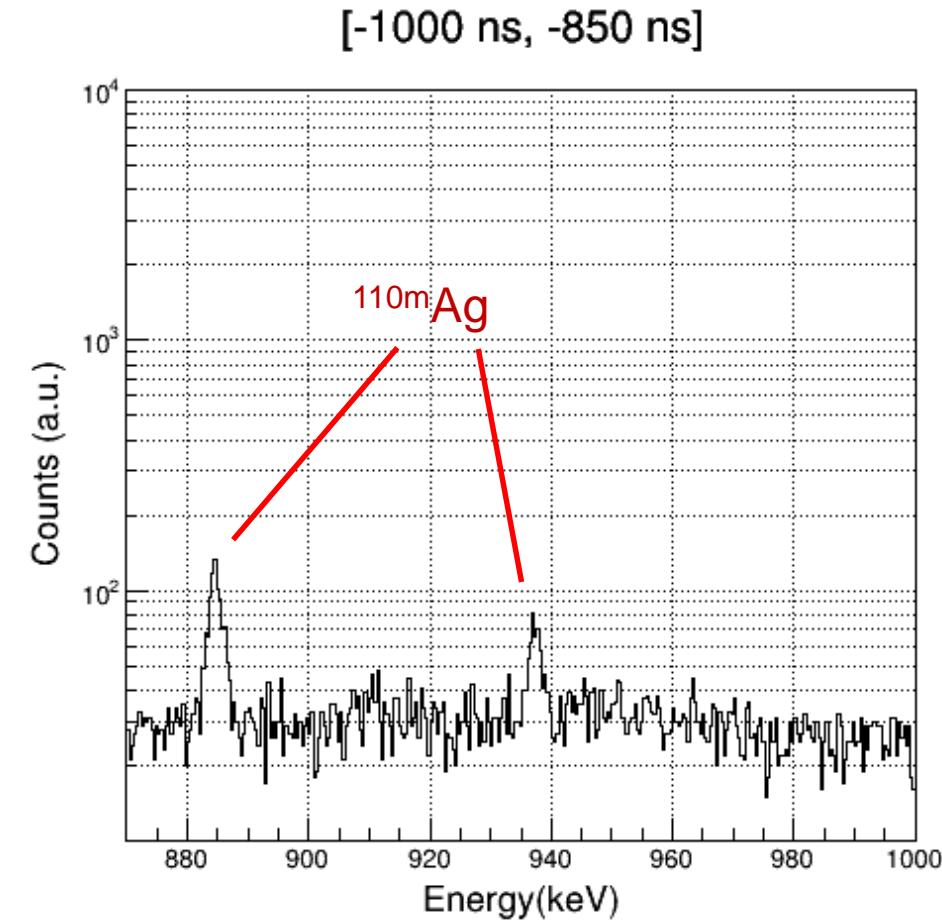
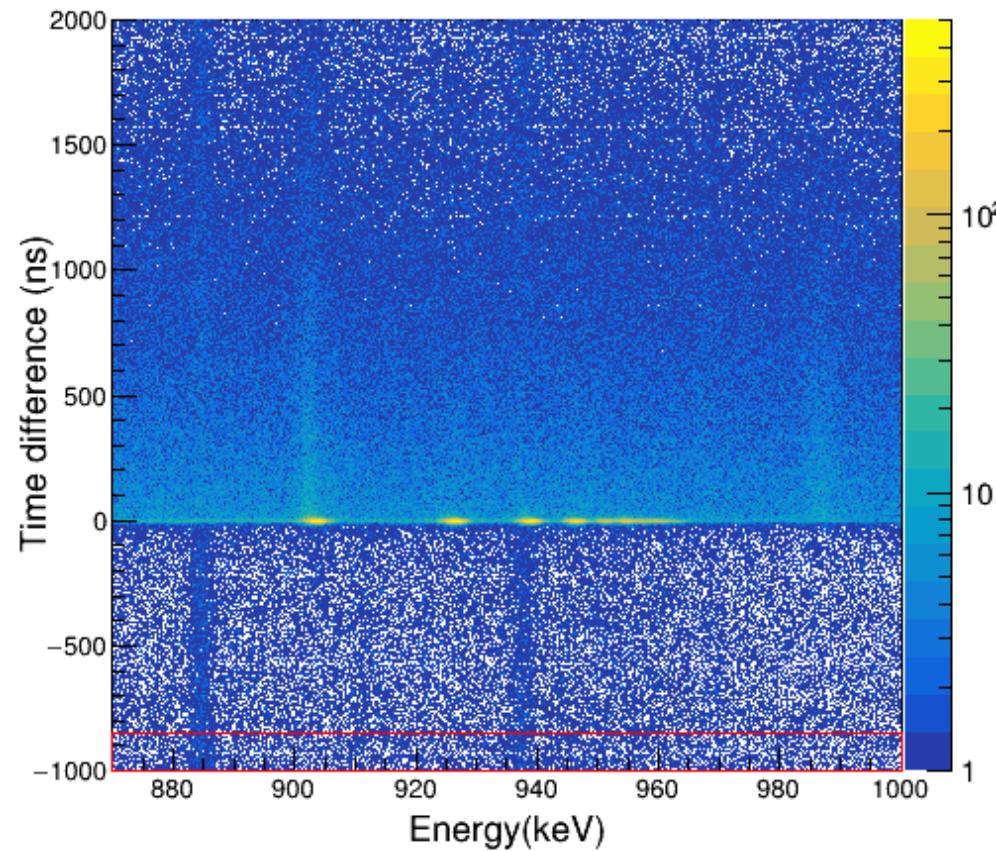
- Before correcting



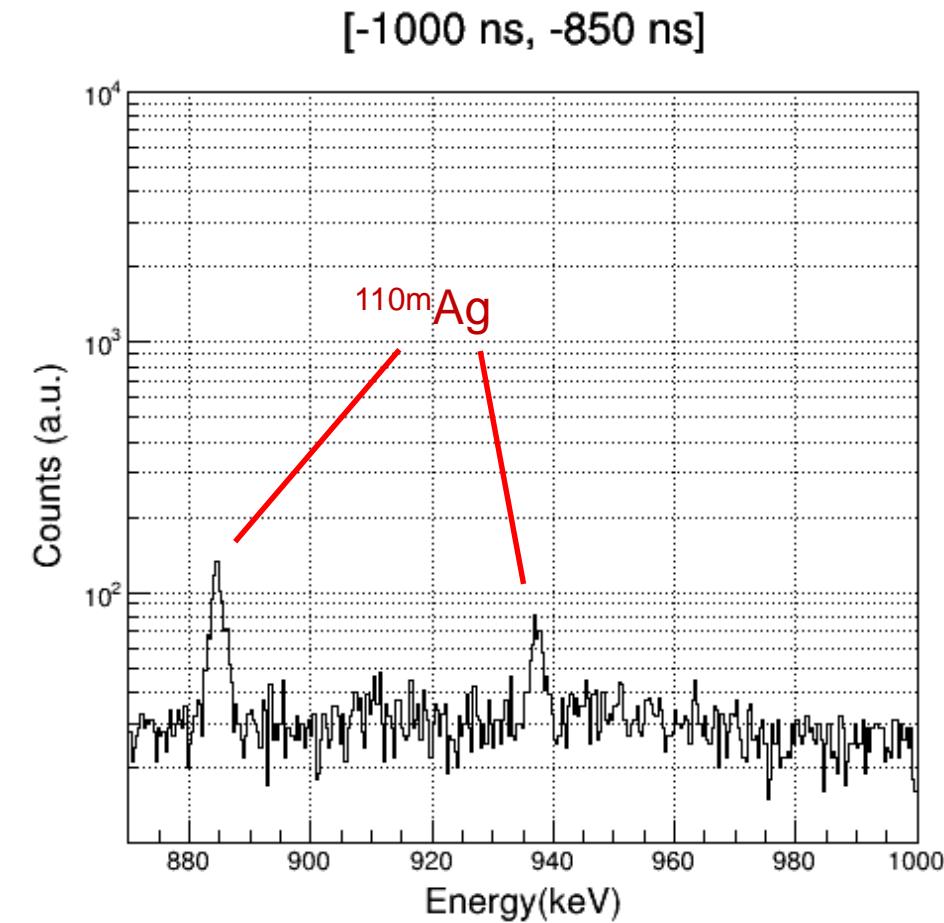
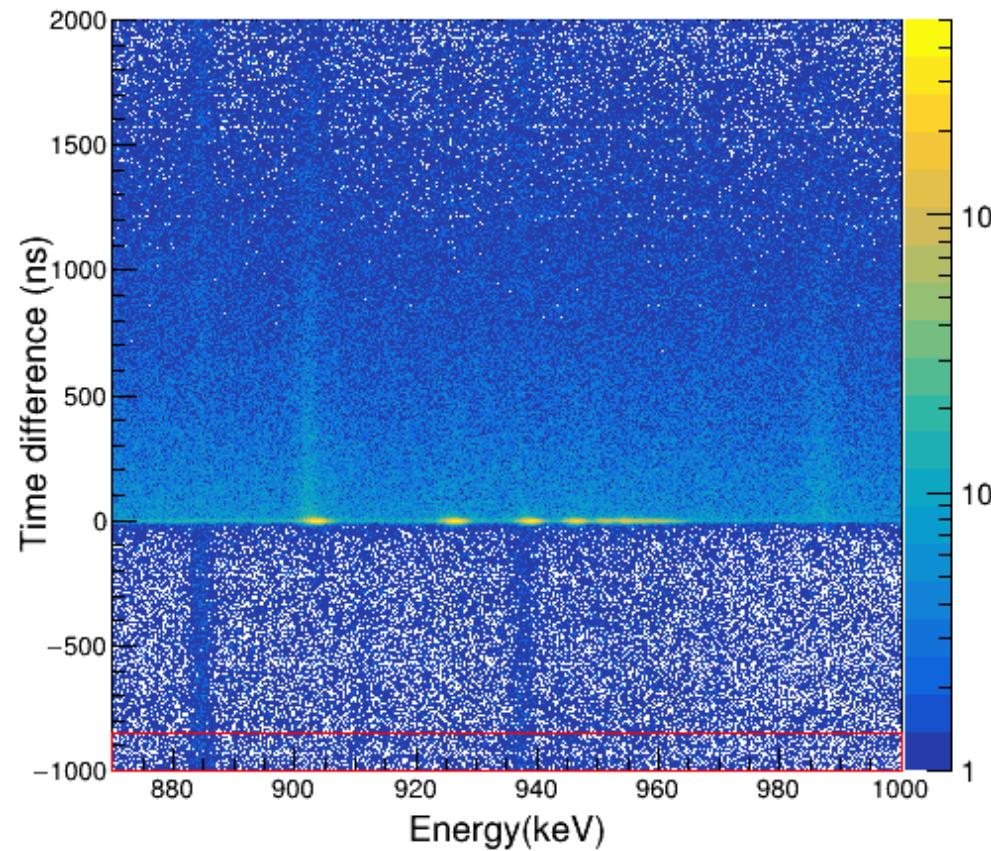
- After correcting



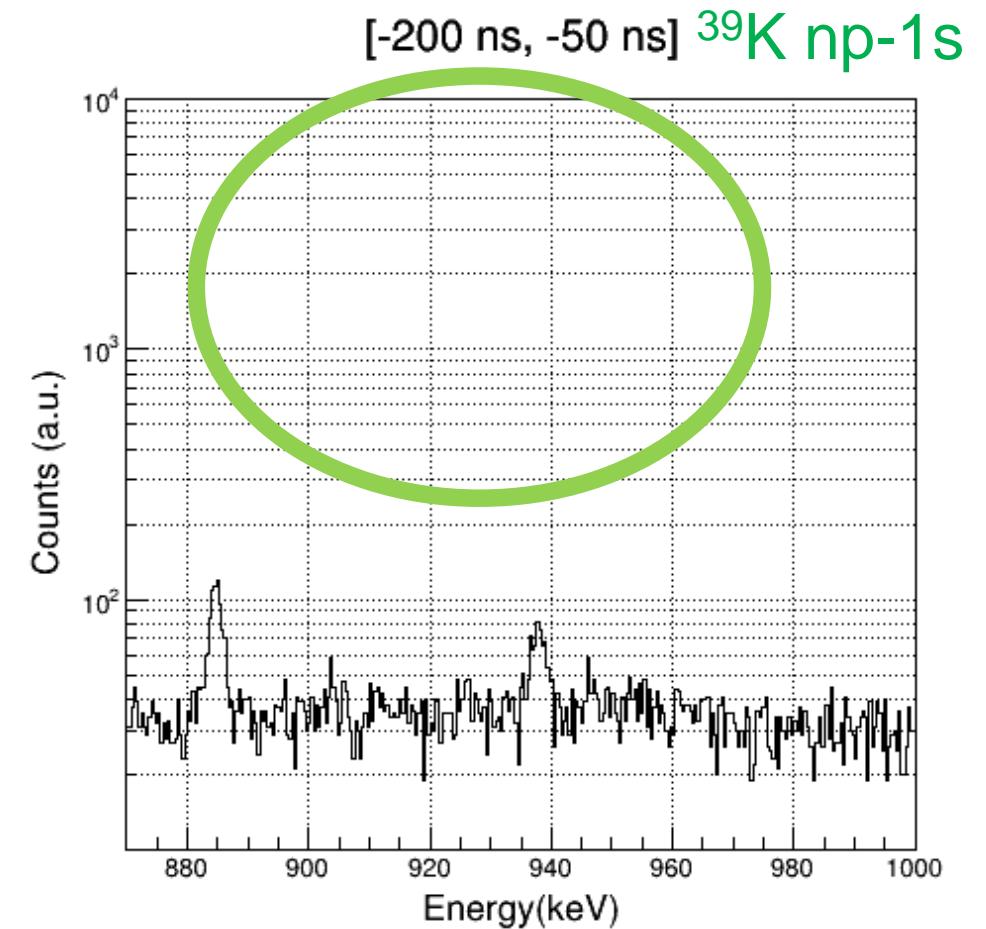
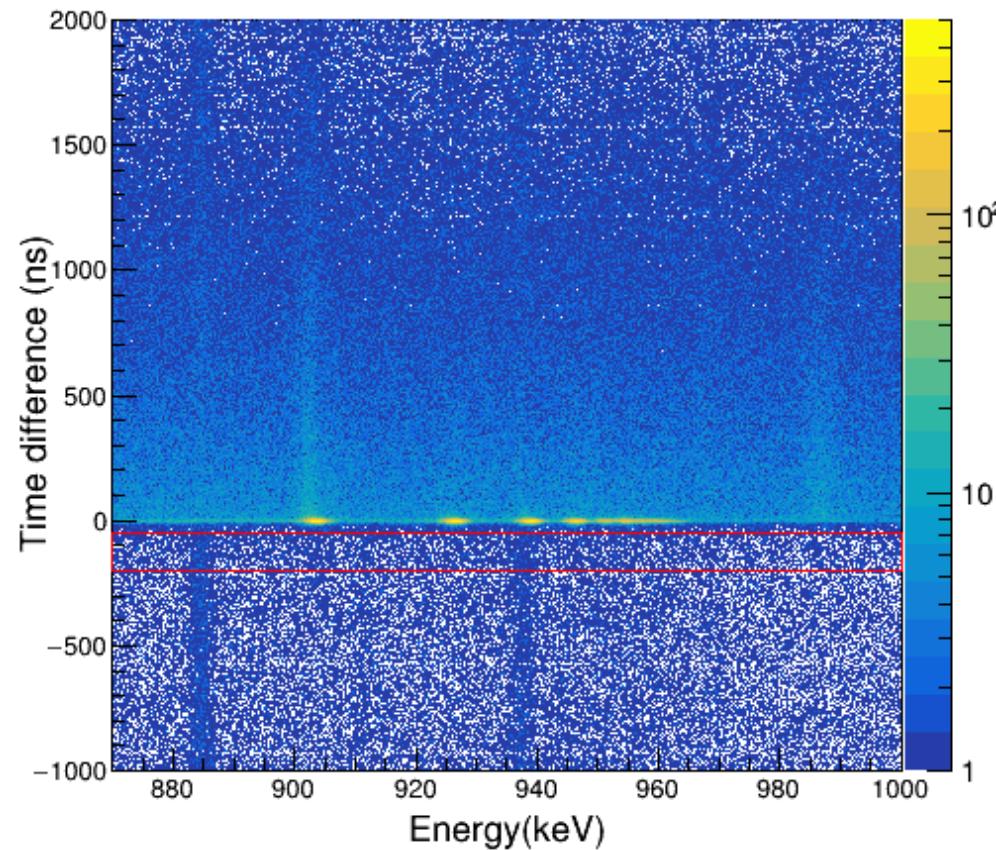
# Interpreting energy Vs time plots



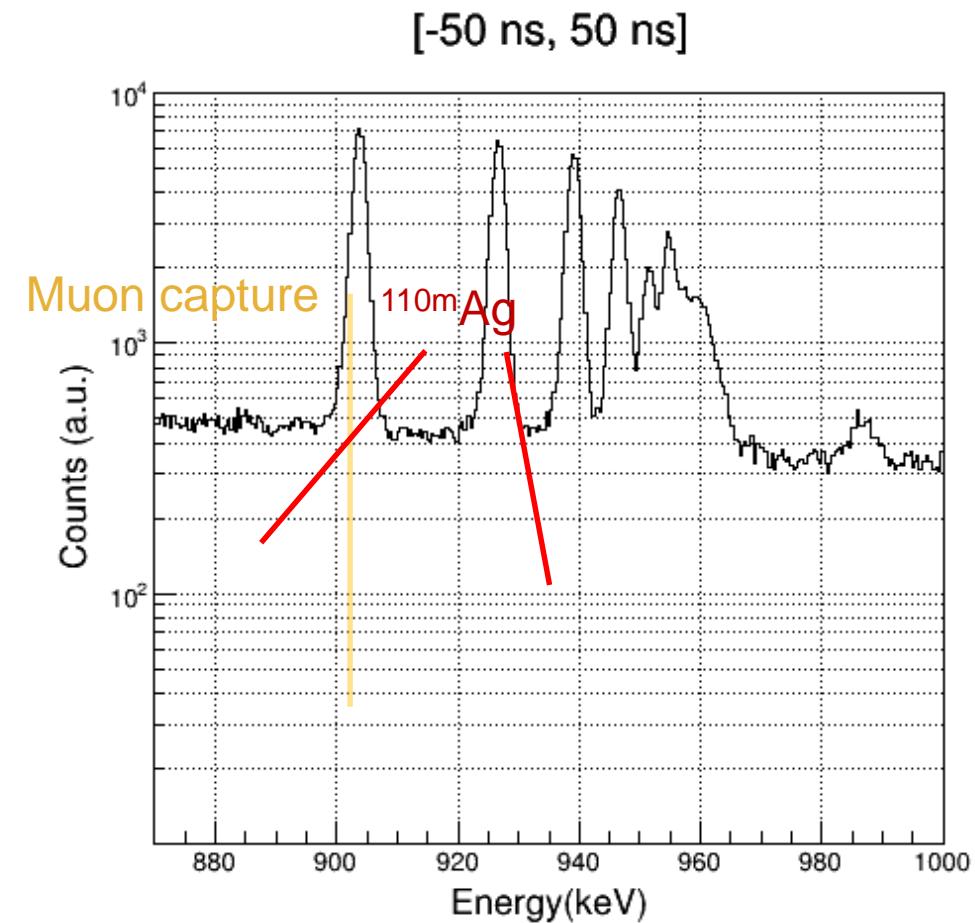
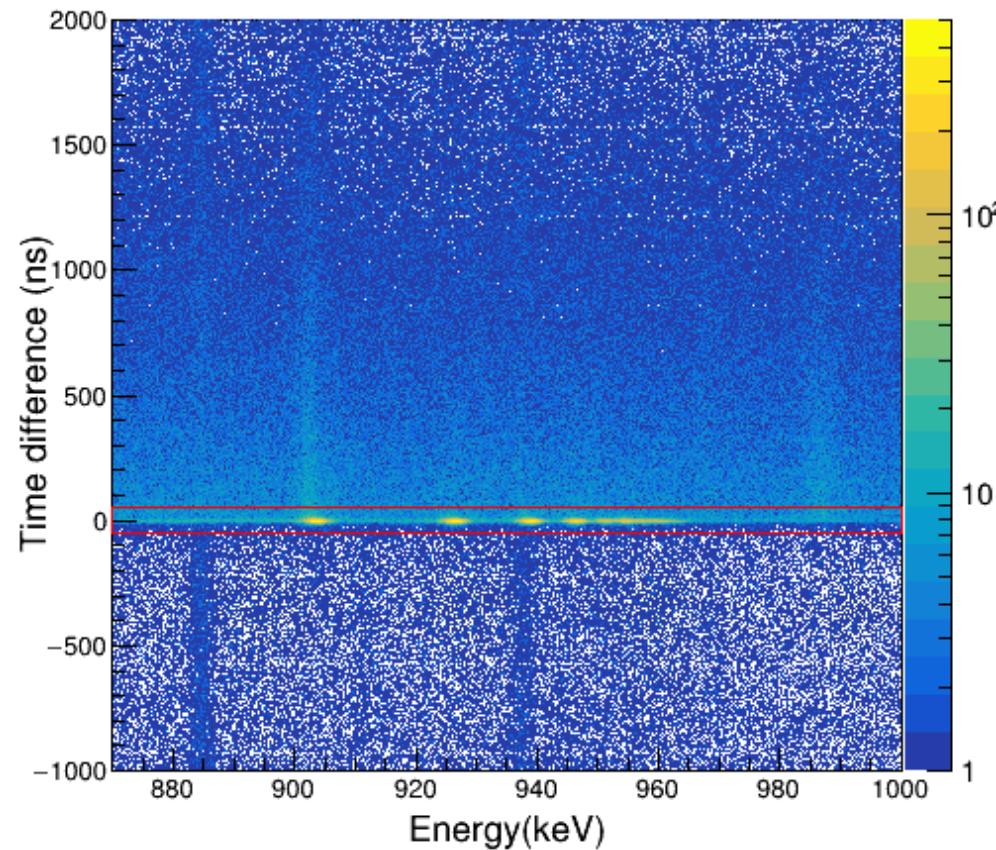
# Interpreting energy Vs time plots



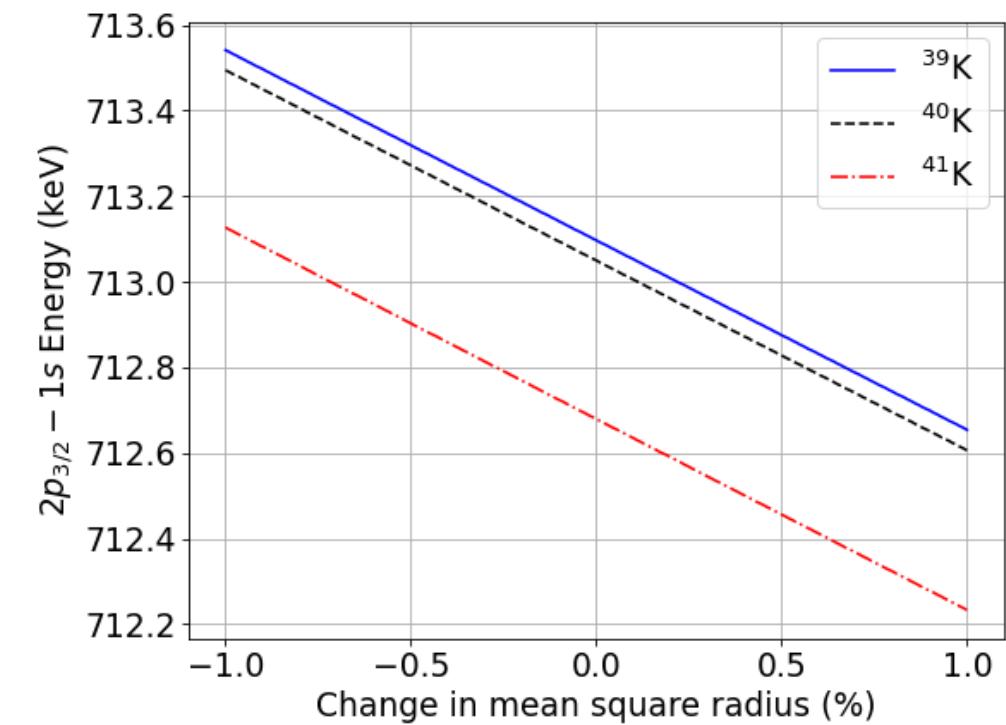
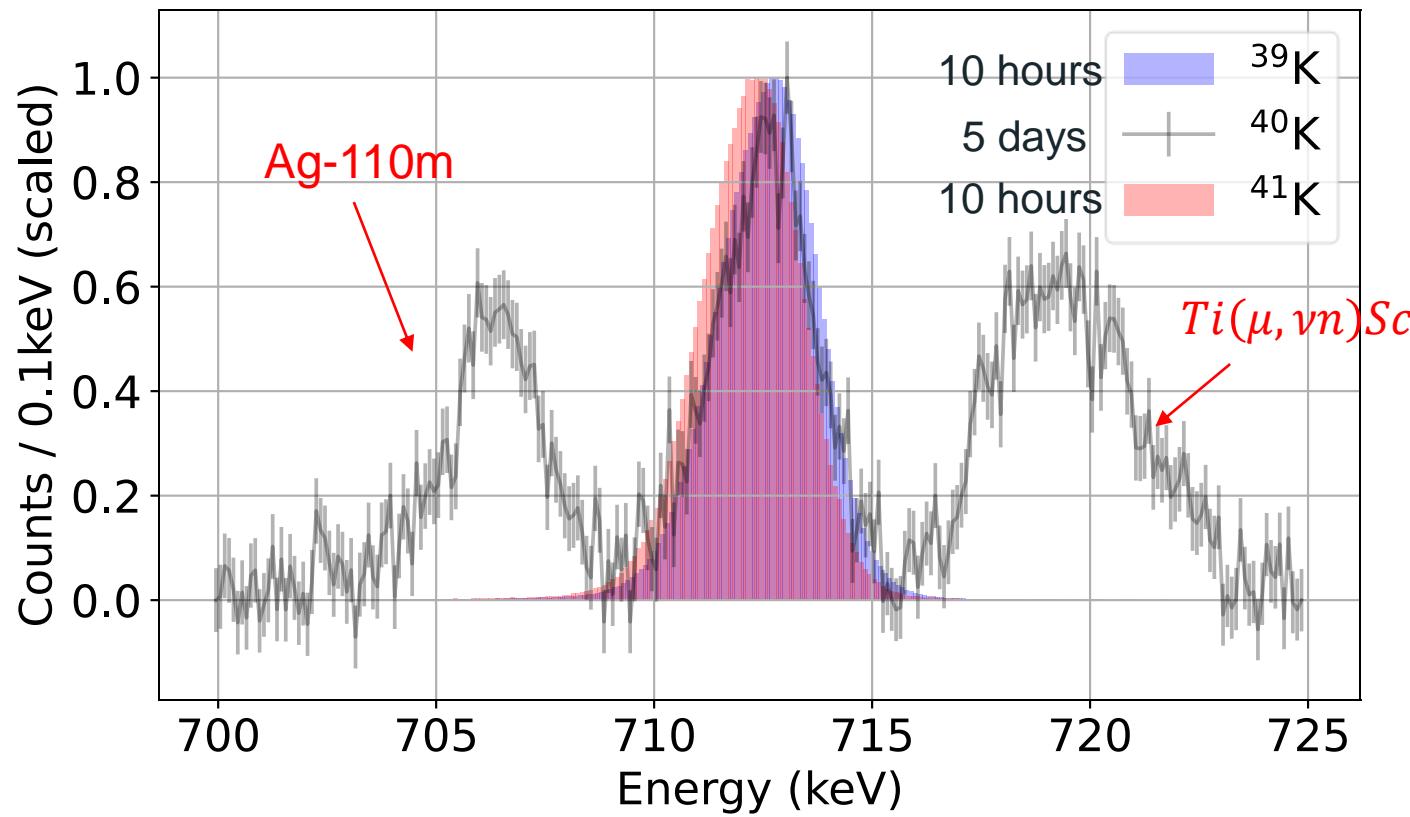
# Interpreting energy Vs time plots



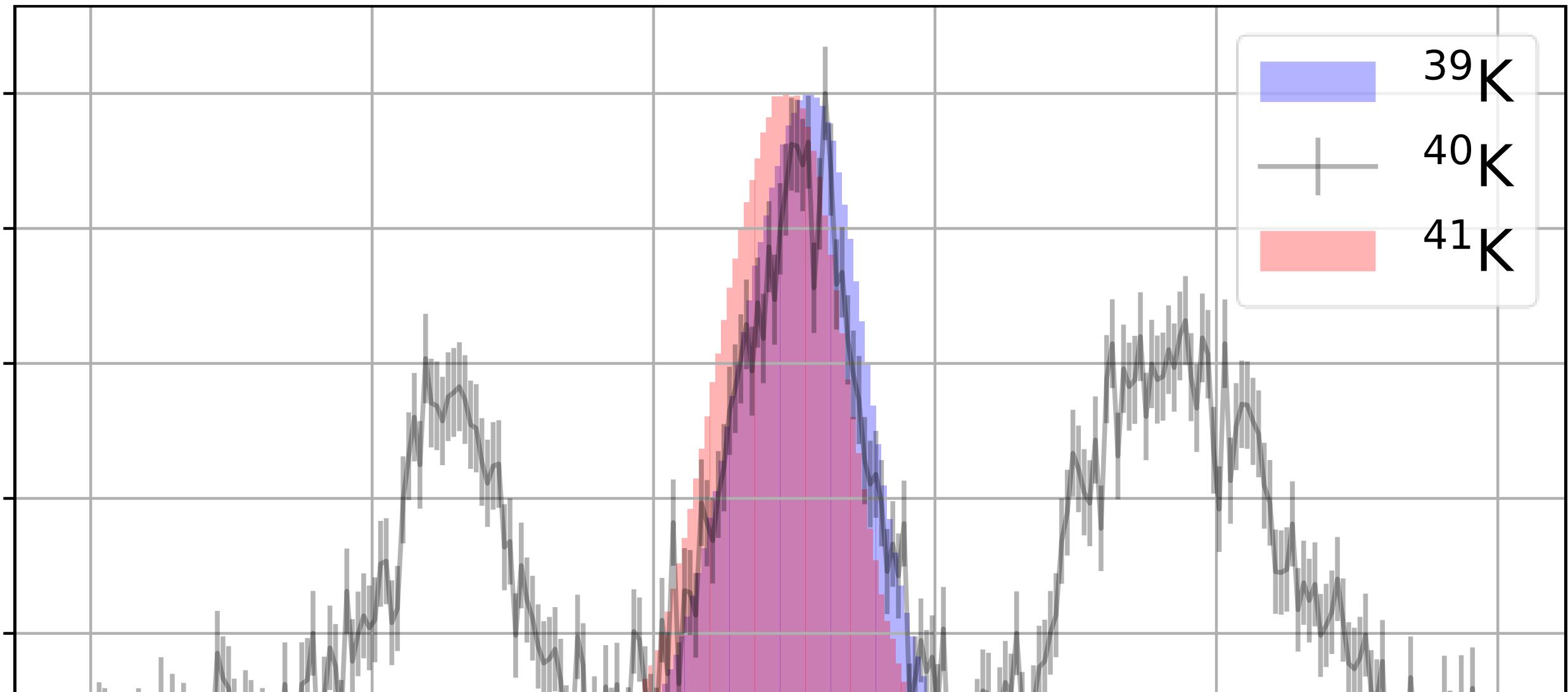
# Interpreting energy Vs time plots



# Potassium muonic isotope shift

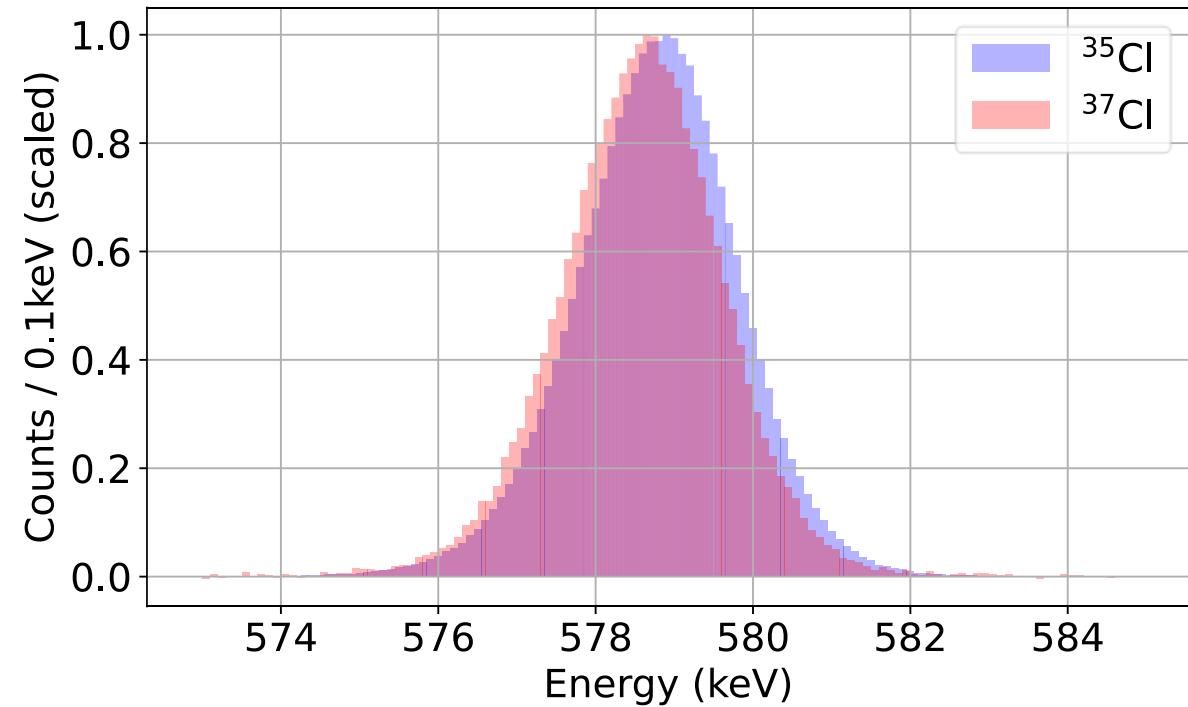


# Potassium muonic isotope shift

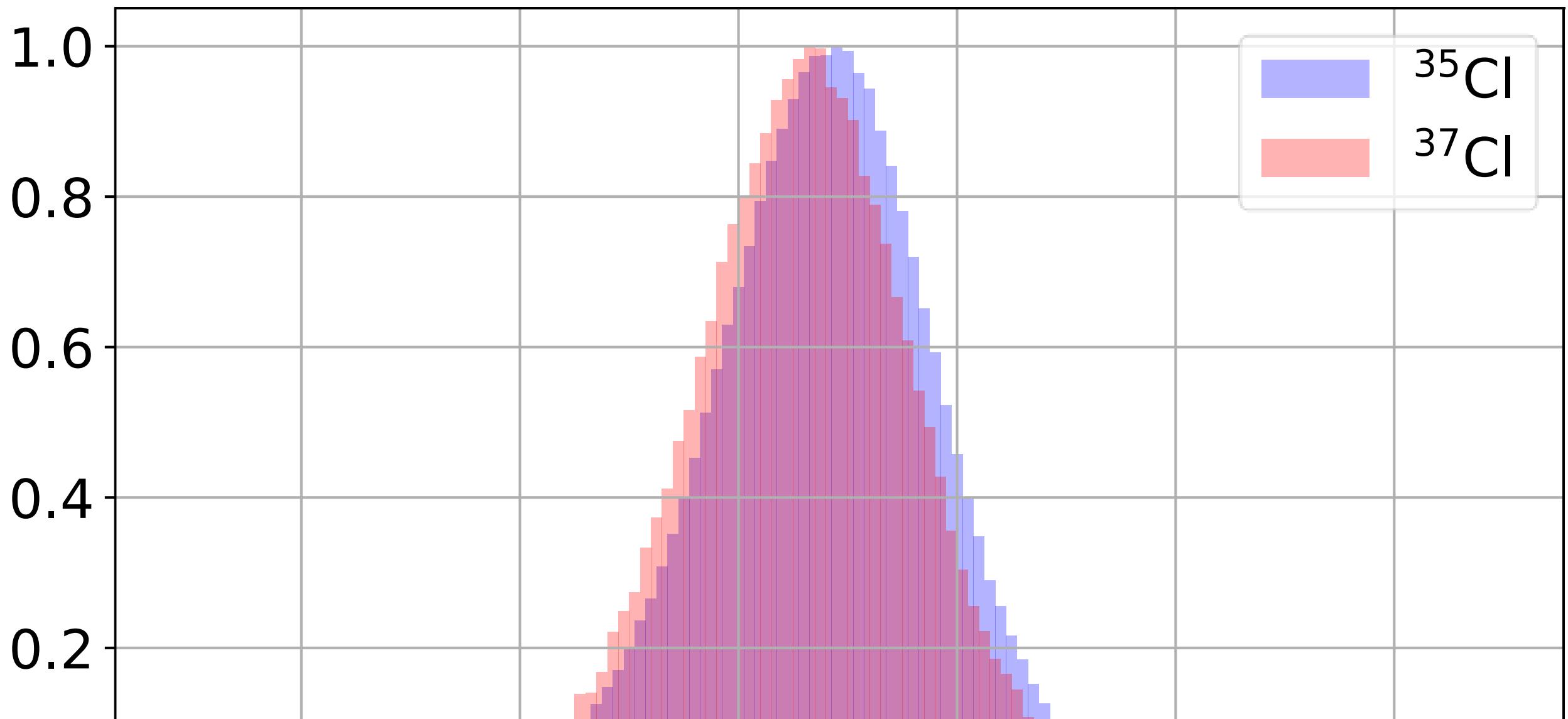


# Chlorine measurement

- Muonic 2p-1s energy:  
 ${}^{nat}\text{Cl}$ : 578.56(30) keV
- Expected improvement on 2p-1s transition energy:  
300 eV → Most likely < 30 eV
- Expected improvement on radii:  
0.45% → ~0.10-0.15 % (including systematics)



# Chlorine measurement

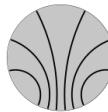


# Conclusion and outlook

- Muonic atoms can be used as precise probes for the nucleus
  - Giving input for laser spectroscopy
  - Radii comparison across elements
  - Inputs for other experiments
- Measured transition energies of Cl and K
  - Theory calculations have been initiated
  - In-depth analysis ongoing (ideal precision < 20 eV)
- Some of our other work:
  - Low-Z: Li, Be, B, C
  - Medium-Z: Cl, K, Ag
  - High(er)-Z: Re, Cm

# Thank you for your attention!

Thanks to the muX collaboration and the QUARTET collaboration:



KIRCHHOFF-  
INSTITUT  
FÜR PHYSIK



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ



# Backup slides



# Muonic x rays

- Captured in high-n state → Cascade down
- X rays emitted in atomic transition
  - Electronic atoms: < 100 keV
  - Muonic atoms: Up to 10 MeV
- Information about energy levels → Extract nuclear properties

