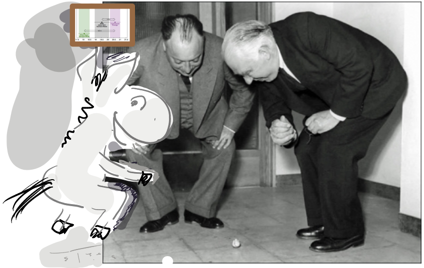
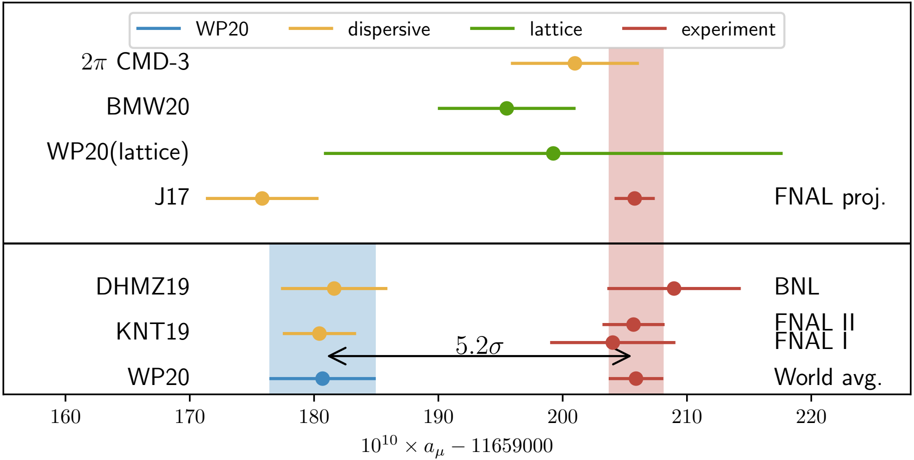
MUonE & Muon $g-2$ :: How theorists can help

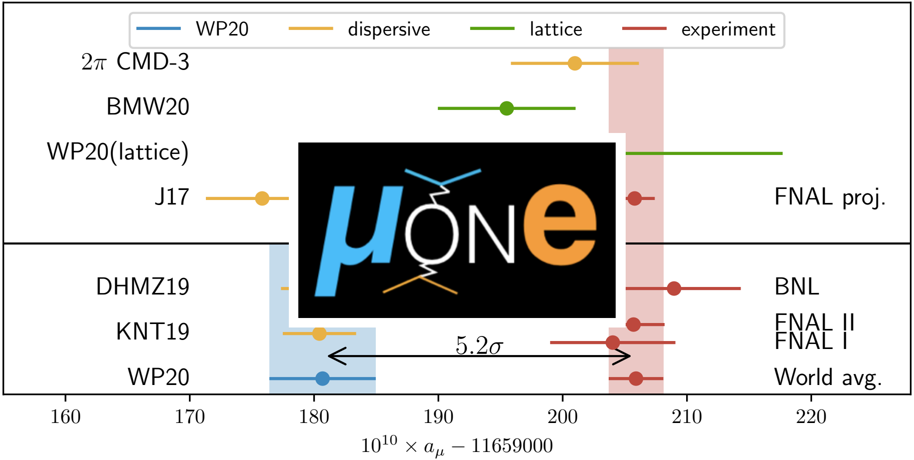
Marco Rocco

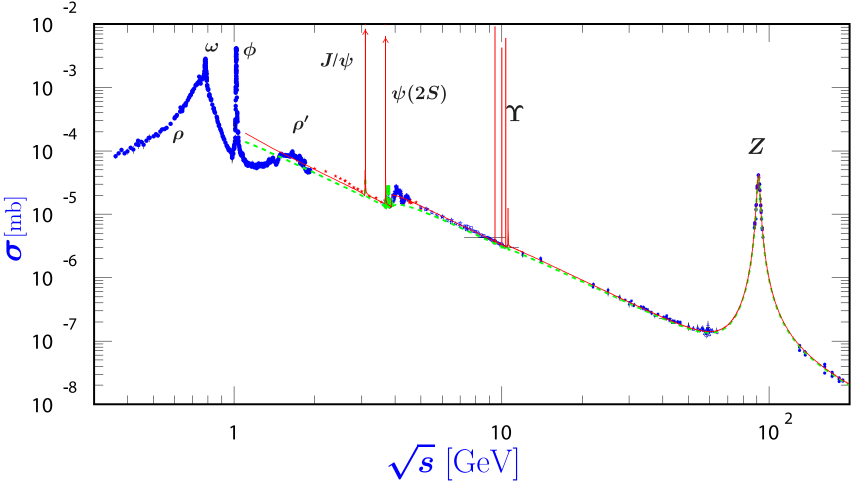


it's all about the Hadronic Vacuum Polarization

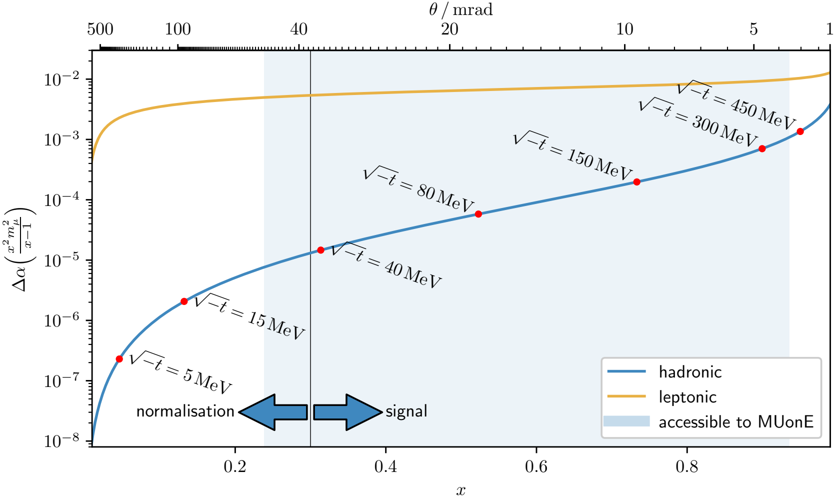


it's all about the Hadronic Vacuum Polarization

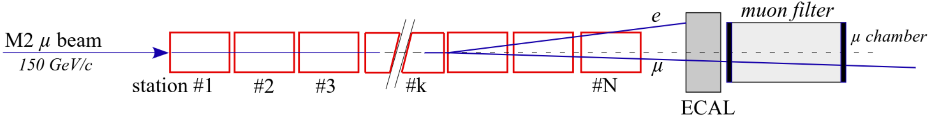




ω
ϕ
J/ψ
$\psi(2S)$
Υ
ρ
ρ'
Z
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
1
10
10^{2}
σ [mb]
$\sqrt{s}$ [GeV]

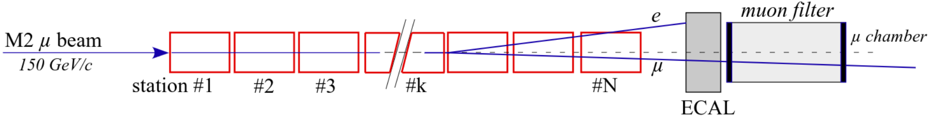


θ / mrad
500
100
40
20
10
5
1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta\alpha\left(\frac{x^2 m_\mu^2}{x-1}\right)$
$\sqrt{-t} = 450\,\mathrm{MeV}$
$\sqrt{-t} = 300\,\mathrm{MeV}$
$\sqrt{-t} = 150\,\mathrm{MeV}$
$\sqrt{-t} = 80\,\mathrm{MeV}$
$\sqrt{-t} = 40\,\mathrm{MeV}$
$\sqrt{-t} = 15\,\mathrm{MeV}$
$\sqrt{-t} = 5\,\mathrm{MeV}$
normalisation
signal
hadronic
leptonic
accessible to MUonE
0.2
0.4
0.6
0.8
x

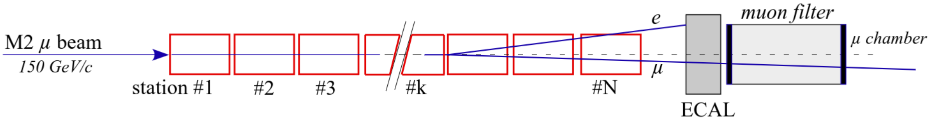


M2 μ beam
150 GeV/c
station #1
#2
#3
#k
#N
e
μ
ECAL
muon filter
μ chamber

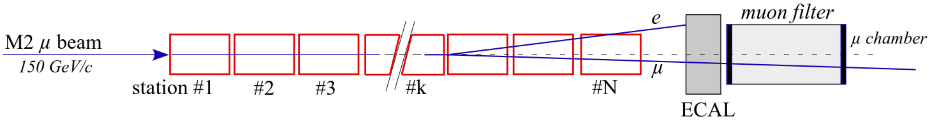
-
-
-
-



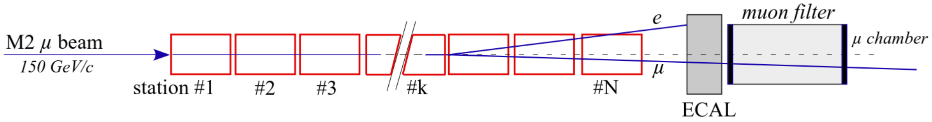
- measure θ_e and θ_μ very precisely
-
-
-



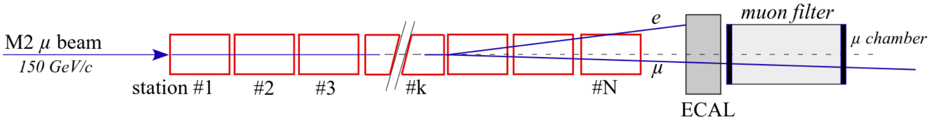
- measure θ_e and θ_μ very precisely
- extract HVP from differential xsec shape (with a fit)
-
-



- measure θ_e and θ_μ very precisely
- extract HVP from differential xsec shape (with a fit)
- BUT HVP is $10^{-3}\times$ background, and competitive if @10^{-2} precision
-

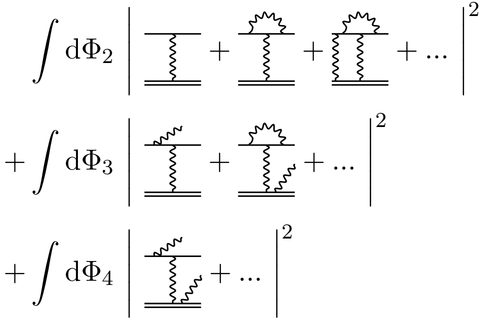


- measure θ_e and θ_μ very precisely
- extract HVP from differential xsec shape (with a fit)
- BUT HVP is $10^{-3}\times$ background, and competitive if @10^{-2} precision
- what is the background?



- measure θ_e and θ_μ very precisely
- extract HVP from differential xsec shape (with a fit)
- BUT HVP is $10^{-3}\times$ background, and competitive if @10^{-2} precision
- what is the background? lots of γ's, π^0 and pair production, nuclear scattering

(part of) the theory



1. fully-differential PS integration
   $\rightarrow$ FKS$^\ell$
2. virtual amplitudes with massive particles
   $\rightarrow$ one-loop: OpenLoops
   $\rightarrow$ two-loop: massification
3. numerical instabilities due to pseudo-singularities
   $\rightarrow$ next-to-soft stabilisation

$$\mu\, e \to \mu\, e \quad @\ \text{NNLO}$$

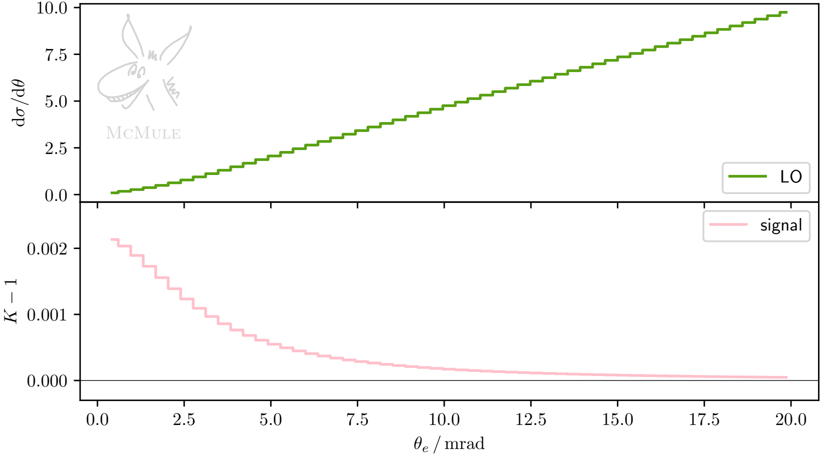
- kinematical setup mimics MUonE:

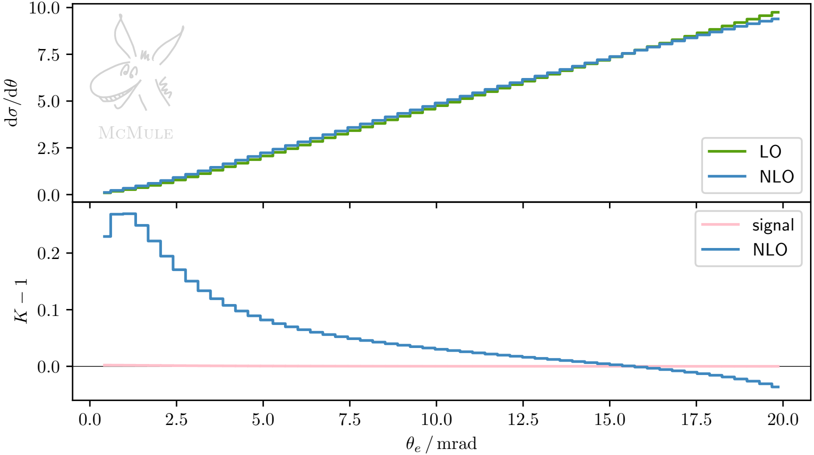
$$E_{\mu,i} = 160\,\text{GeV} \qquad E_{e,f} > 1\,\text{GeV} \qquad \theta_{\mu,f} > 0.3\,\text{mrad}$$

- results for different kinematical scenarios and any IR safe observable
- no mass is neglected

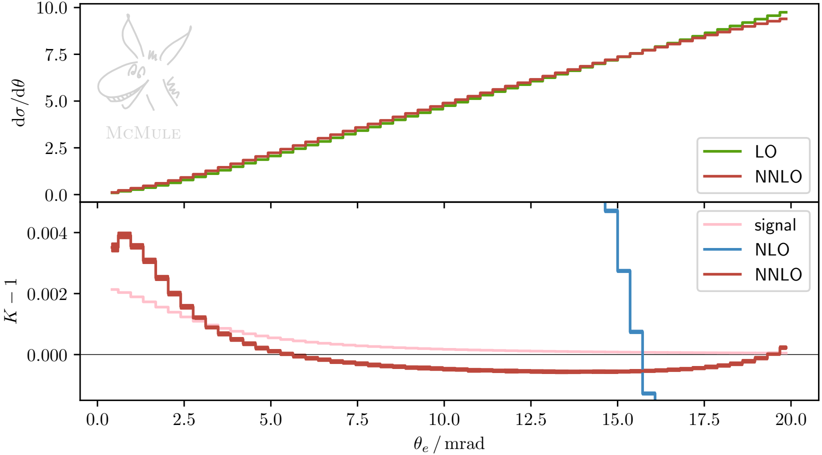
* *a restless yet smiling mule after 2.5 CPU years of work*

S1 θ_e spectrum

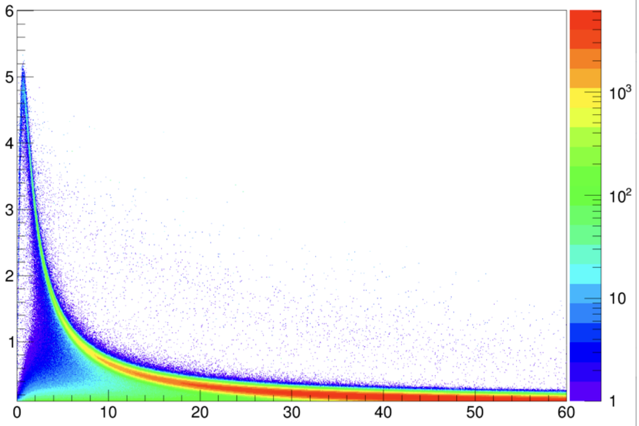



10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NLO
0.2
0.1
0.0
K − 1
signal
NLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

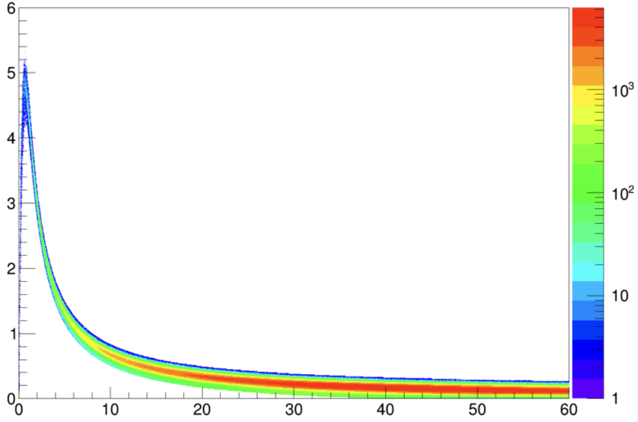
S1 θ_e spectrum



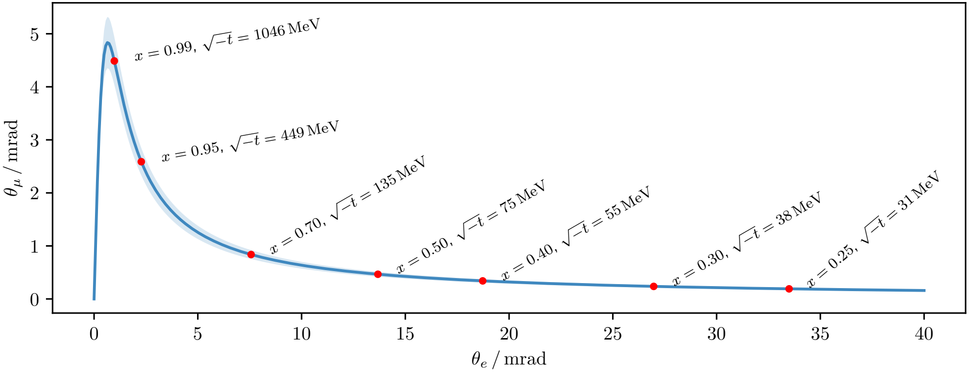
experimental *simulation*



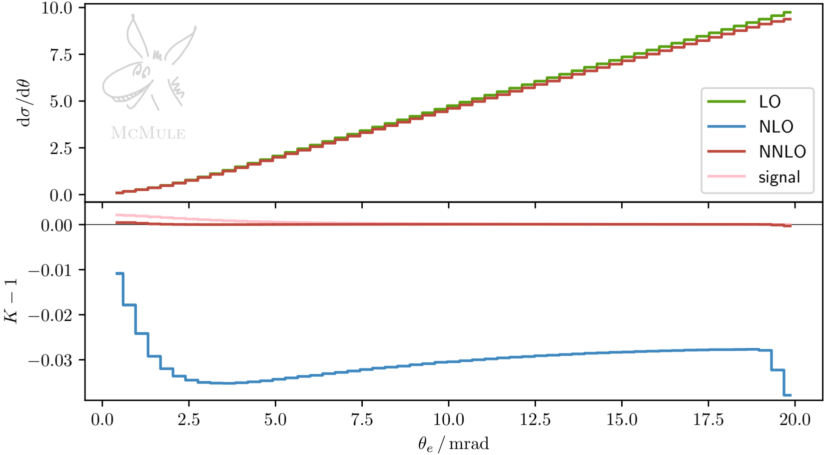
same, with an elasticity cut



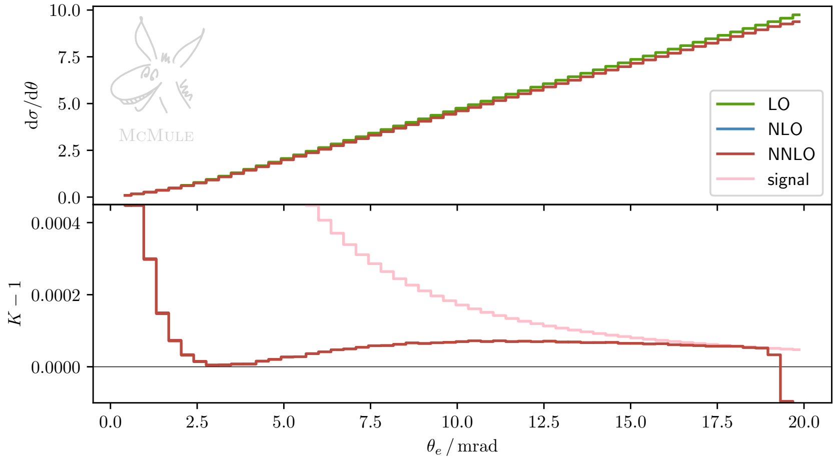
theoretically



S2 θ_e spectrum



S2 θ_e spectrum



- NNLO with different external masses [2212.06481]
- precision now $\mathcal{O}(10^{-3/-4})$, would like to reach $\mathcal{O}(10^{-5})$
- we have started thinking about $\mathrm{N}^3\mathrm{LO}$ dominant corrections
- resummation (analytic & parton shower)

- hopefully MUonE does not stop as well

① problem: PS integration

FKS$^\ell$ + DIMREG

$$d\Phi_{n+r} \equiv d\Phi_n \prod_{i=1}^{r} d\Phi_{i,\gamma}$$

$$= d\Phi_n \prod_{i=1}^{r} d\Phi_{i,\gamma}^{d=4-2\epsilon}\, \xi^2\, \xi_i^{-1-2\epsilon}\, d\xi_i\, d\Upsilon_i^{d=4-2\epsilon}$$

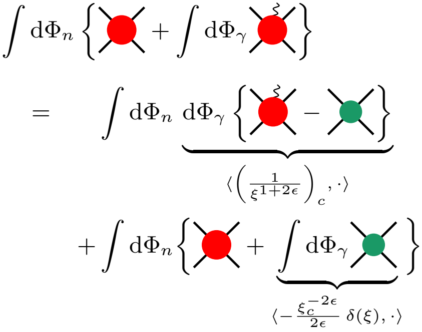
❶ reproduce and isolate IR behaviour from regions of the phase space where (one or more) real photons are soft:

$$\lim_{\xi\to 0} \xi^2\, \mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\, M_n^{(\ell)}$$

❷ isolate IR-divergent behaviour from virtual amplitudes:

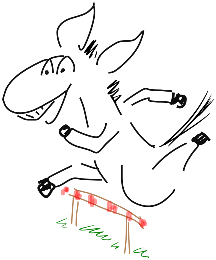
$$\sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = e^{-\alpha\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)\, f}$$

❸ cancel analytically IR divergences and then integrate numerically in $d = 4$ over the non-radiative phase space



full muone 2-loop amplitude with $M \neq 0$, $m = 0 \rightarrow$ [Bonciani et al. 21]

full muone 2-loop amplitude with $M \neq 0$, $m \neq 0 \rightarrow$ [??]

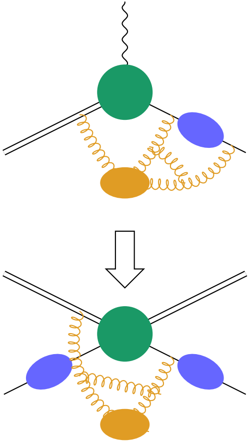


$\rightarrow$ exploit scale hierarchy $m^2 \ll M^2, Q^2$

simple process ($\mu \to e\nu\nu$ or $t \to b\ell\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

[Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18]

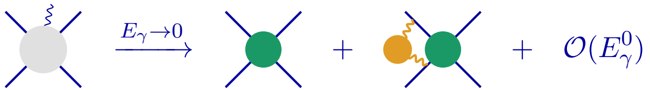


different process ($\mu e \to \mu e$)

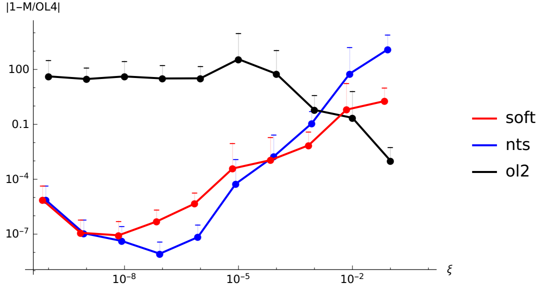
- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

based on SCET and
method of regions as calculational tool

$\to$ massify [Bonciani et al. 21] $\to$ enhanced + constant terms real-virtual corrections ‘trivial’ in principle, extremely delicate numerically

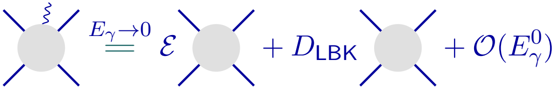


- soft limit (of collinear emission)
- OL4 $\equiv$ OpenLoops in quadruple-precision mode
- OL2 $\equiv$ OpenLoops in double/hybrid-precision mode
- OL2 vs next-to-soft limit
- stability problem solved



OpenLoops → [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

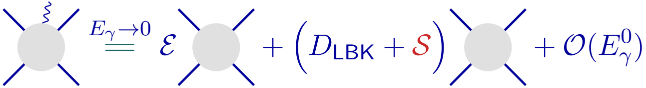
LBK theorem @ tree-level [Low 58, Burnett, Kroll 67]



LBK theorem @ one-loop [Engel, Signer, Ulrich 21]

→ D_{LBK} yields hard contribution in language of MoR (HQET)

→ generic soft contribution $\mathcal{S}$



→ introduce next-to-soft stabilisation [McMule 21, 22]