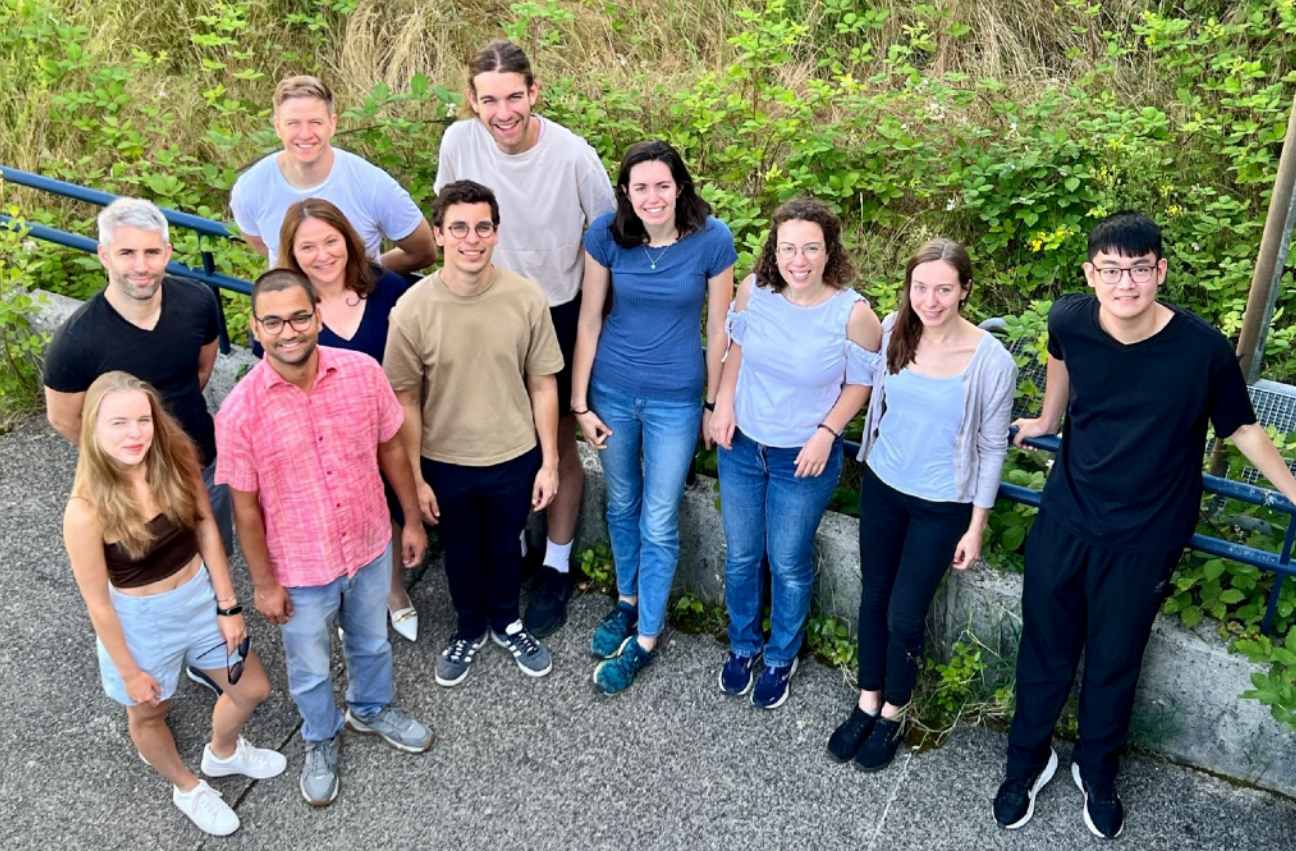


# All-to-all interacting quantum gases for quantum simulation

Jean-Philippe Brantut, EPFL



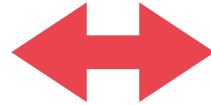
T. Zwettler	N. Sauerwein
G. Del Pace	F. Orsi
T. Bühler	E. Fedotova
A. Fabre	R. Bhatt
G. Bolognini	M. Eichenberger
Z. Xue	

***Theory collaborations:***

*S. Uchino (Waseda), H. Ritsch (Innsbrück), P. Hauke (Trento), G. Pupillo (Strasbourg), J. Sonner (Geneva), T. Giamarchi (Geneva), E. Demler (ETHZ)*

# Motivation: holography

Strongly correlated  
quantum matter

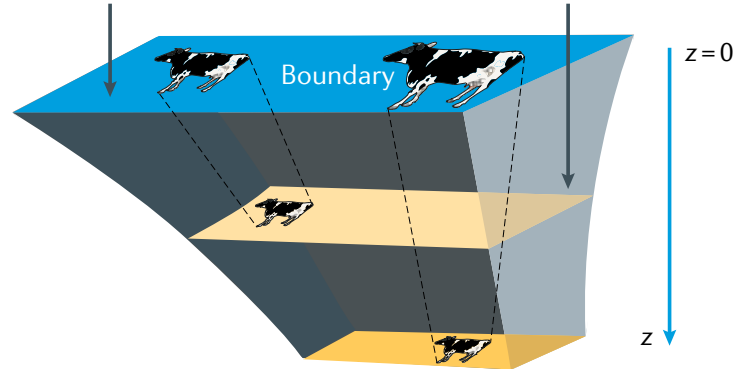


Quantum gravity

Many-body system  
without gravity in  
 $d$  spacetime dimensions

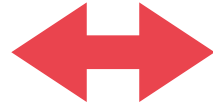
=

String theory,  
quantum gravity in  
 $d+1$  dimensions



# Motivation: holography

Strongly correlated  
quantum matter

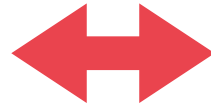


Quantum gravity

*Recipe for quantum simulation*

# Motivation: holography

Strongly correlated  
quantum matter



Quantum gravity

- No quasi-particles
- Good large-N limit
- Maximally chaotic

*Does such a material exist ?*

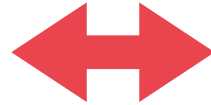
See for example:

J. Zhang *et al*, PNAS **116**, 19869 (2019)

A. Legros *et al*, Nature Physics **15**, 142 (2019)

# Motivation: holography

Strongly correlated  
quantum matter



Quantum gravity

Sachdev-Ye-Kitaev model:

$$\hat{H}_{SYK} = \sum_{ijkl} J_{ijkl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

*Degenerate Fermi gas*

*All-to-all, fully random interactions*

JT gravity in 1+1 dimensions

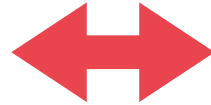
See for example:

M. Franz and M. Rozali, Nat Rev Mater **3**, 491–501 (2018)

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, Rev. Mod. Phys. **94** 035004 (2022)

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D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, Rev. Mod. Phys. **94** 035004 (2022)

# Proposals in various contexts

## Condensed-matter devices (topological systems, graphene dots)

A. Chew, A. Essin, and J. Alicea, Phys. Rev. B **96** 121119 (2017)

A. Chen, R. Ilan, F. De Juan, D. Pikulin, and M. Franz, Phys. Rev. X **7**, 031006 (2017)

A. Chen, R. Ilan, F. De Juan, D. Pikulin, and M. Franz, Phys. Rev. Lett. **121**, 036403 (2018)

M. Brzezińska, Y. Guan, O. V. Yazyev, S. Sachdev, and A. Kruchkov, arXiv:2208.01032v1

## Ultra-cold atoms

I. Danshita, M. Hanada, and M. Tezuka, Prog. Theor. Exp. Phys. **2017** 083I01 (2017)

C. Wei and T. A. Sedrakyan, Phys. Rev. A **103**, 013323 (2021)

## Direct digital simulation

L. Garcia-Alvarez, *et al*, Phys. Rev. Lett. **119**, 040501 (2017)

R. Babbush, D. W. Berry, and H. Neven, Phys. Rev. A **99**, 040301 (2019)



# Experiments

*Explicit programming of all the couplings*

NMR implementation (4 spins)

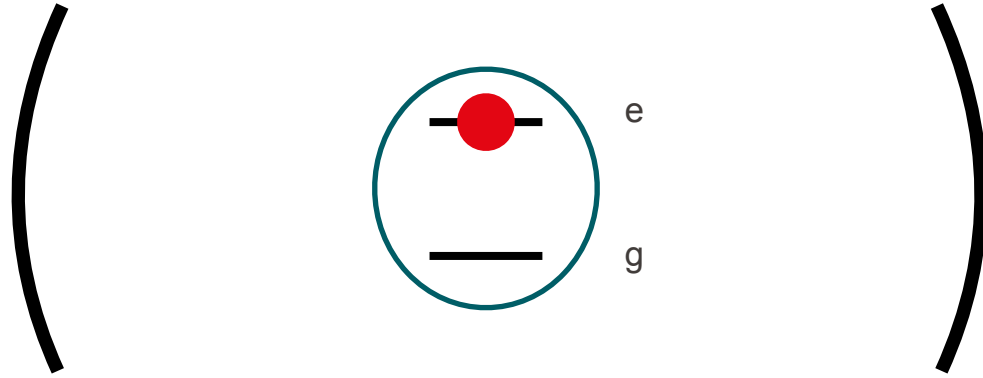
Z. Luo, Y.-Z. You, J. Li, C.-M. Jian, D. Lu, C. Xu, B. Zeng, and R. Laflamme,  
Npj Quantum Inf. **5**, 53 (2019)

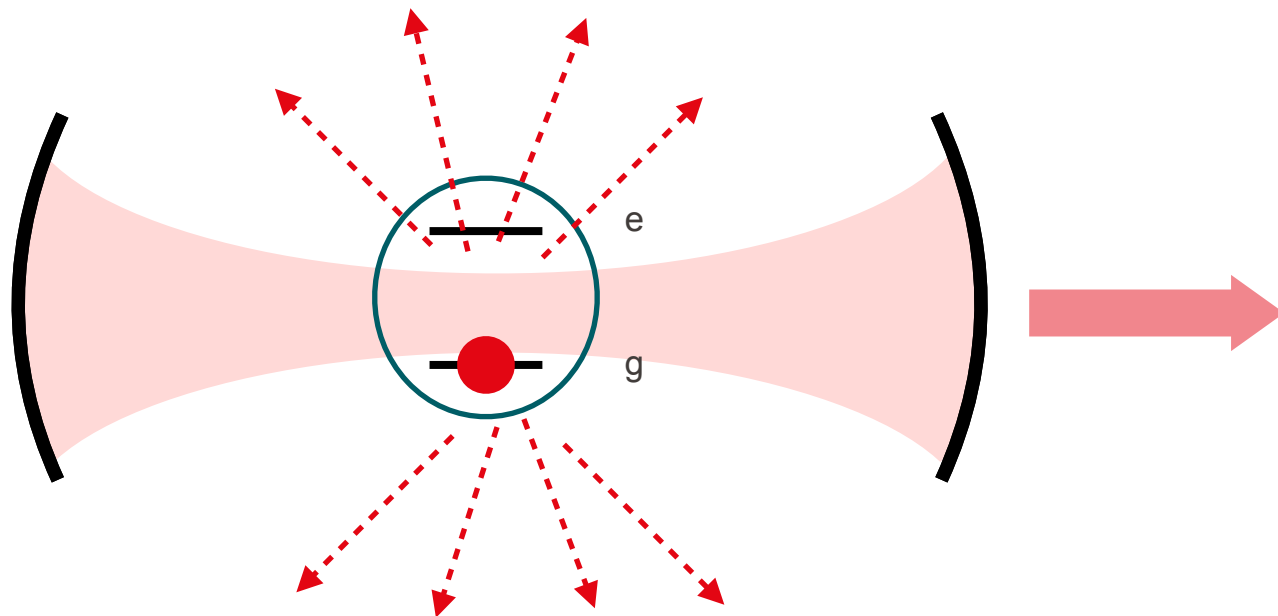
Superconducting circuit implementation (7 qubits)

D. L. Jafferis, *et al*,  
Nature **612**, 51 (2022)

B. Kobrin, T. Schuster, and N. Y. Yao,  
arXiv:2302.07897

# Cavity QED in a nutshell

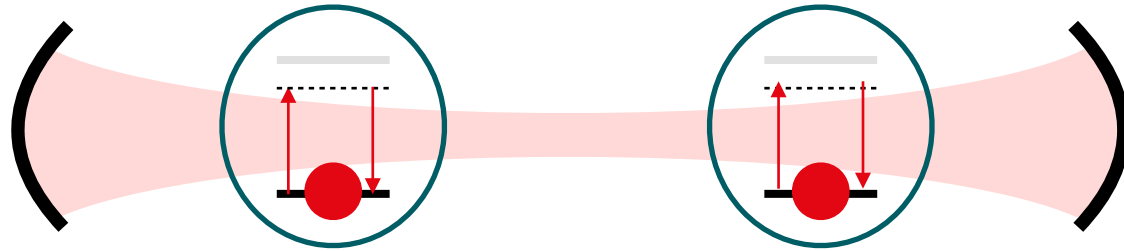




$$\text{Cooperativity } \eta = \frac{\Omega^2}{\kappa\Gamma} = \frac{24\mathcal{F}}{\pi k^2 w^2}$$

# Photon-mediated interactions





Rayleigh scattering channelled into the cavity mode



Interaction mediated by  
cavity photons :

$$V_{\text{cav}} = \frac{g_0^2}{\Delta_a} \frac{\alpha^2}{\Delta_a} \frac{1}{\Delta_c} \int d\mathbf{r} d\mathbf{r}' \hat{n}(\mathbf{r}) \hat{n}(\mathbf{r}') g_p(\mathbf{r}) g_c(\mathbf{r}) g_p(\mathbf{r}') g_c(\mathbf{r}')$$



Interaction mediated by  
cavity photons :

$$V_{\text{cav}} = D_0 \int d\mathbf{r} d\mathbf{r}' \hat{n}(\mathbf{r}) \hat{n}(\mathbf{r}') g_p(\mathbf{r}) g_c(\mathbf{r}) g_p(\mathbf{r}') g_c(\mathbf{r}')$$

Infinite-range, all-to-all  
'Local in  $k$  space'

$$g_{p,c}(\mathbf{r}) = \cos(\mathbf{k}_{p,c} \cdot \mathbf{r})$$

Pump and cavity modes



ETHZ, MIT, Tübingen, Berkeley, Hamburg, Stanford, JILA, Singapour,  
Shanghai, Vienna, Beijing...

### *Multimode cavity*

Y. Guo *et al.* *Nature* **599**, 211 (2021)  
V. D. Vaidya *et al.*, *Phys. Rev. X* **8** 011002 (2018)

### *Quantum dynamics*

Z. Wu *et al.*, *Phys. Rev. Lett.* **131** 243401 (2024)  
T. Zwickler *et al.*, arXiv:2405.18204 (2024)

### *Dissipation-induced phases*

D. Dreon *et al.* *Nature* **608**, 494 (2022)  
P. Kongkhambut *et al.*, *Science* **377** 670 (2022)

### *Long-range spin systems*

A. Periwai *et al.* *Nature* **600**, 630–635 (2021).  
D.J. Young *et al.* *Nature* **625**, 679 (2024)  
N. Sauerwein, *et al.* *Nature Physics* **19**, 1128 (2023)

### *Superradiant Mott-insulators*

R. Landig *et al.*, *Nature* **532** 476 (2016)  
J. Klinder *et al.*, *Phys. Rev. Lett.* **115** 230403 (2015)

Review: F. Mivehvar, F. Piazza, T. Donner and H. Ritsch, *Advances in Physics* **70** 1-153 (2021)



Density-wave ordering induced by light in a unitary Fermi gas

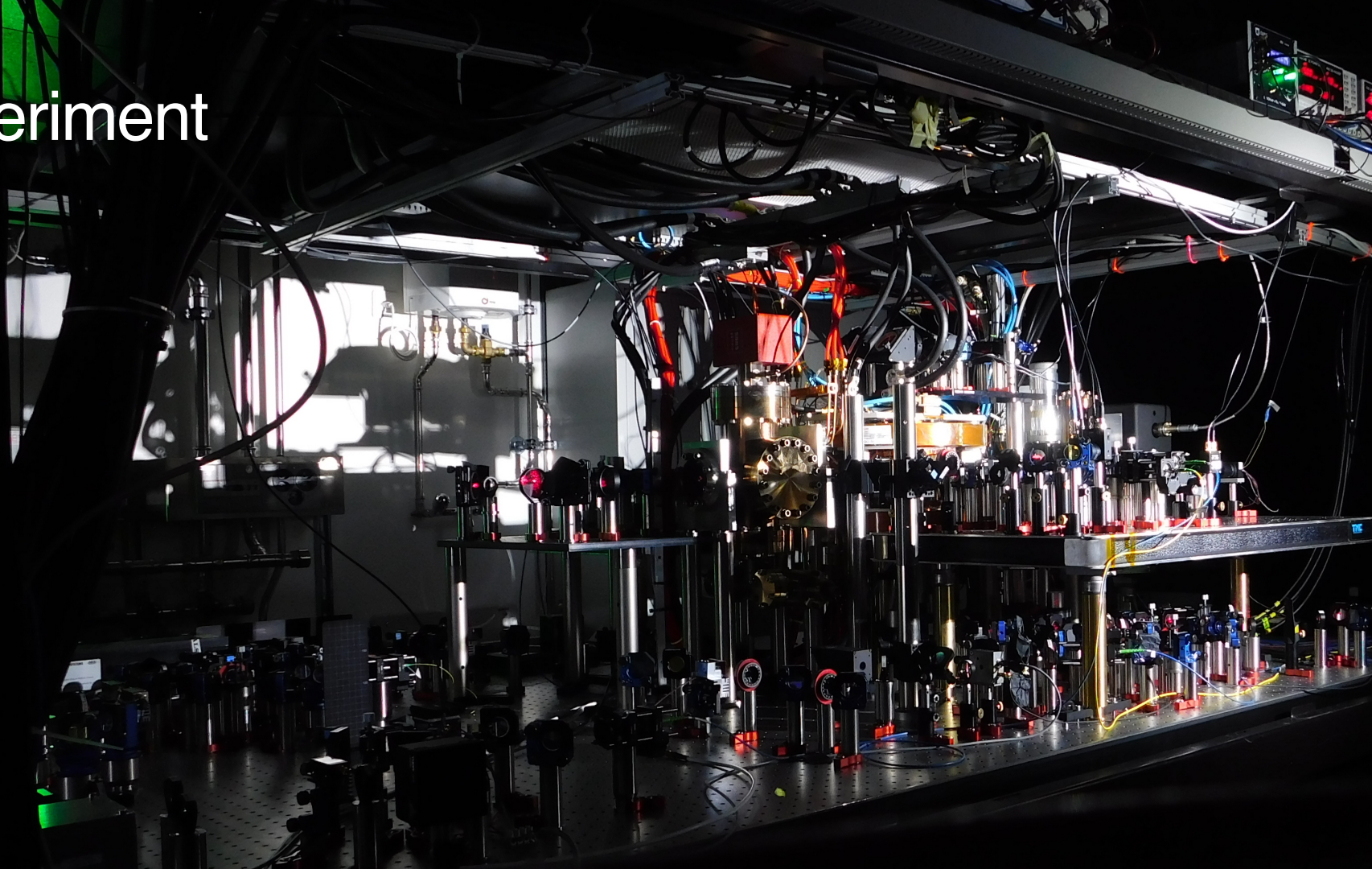
Competition between photon-mediated interaction and disorder

Prospects for realizing the SYK model

# Density-wave ordering induced by light in a unitary Fermi gas

*Theory:* H. Ritsch, E. Colella, F. Mivehvar (Innsbruck)

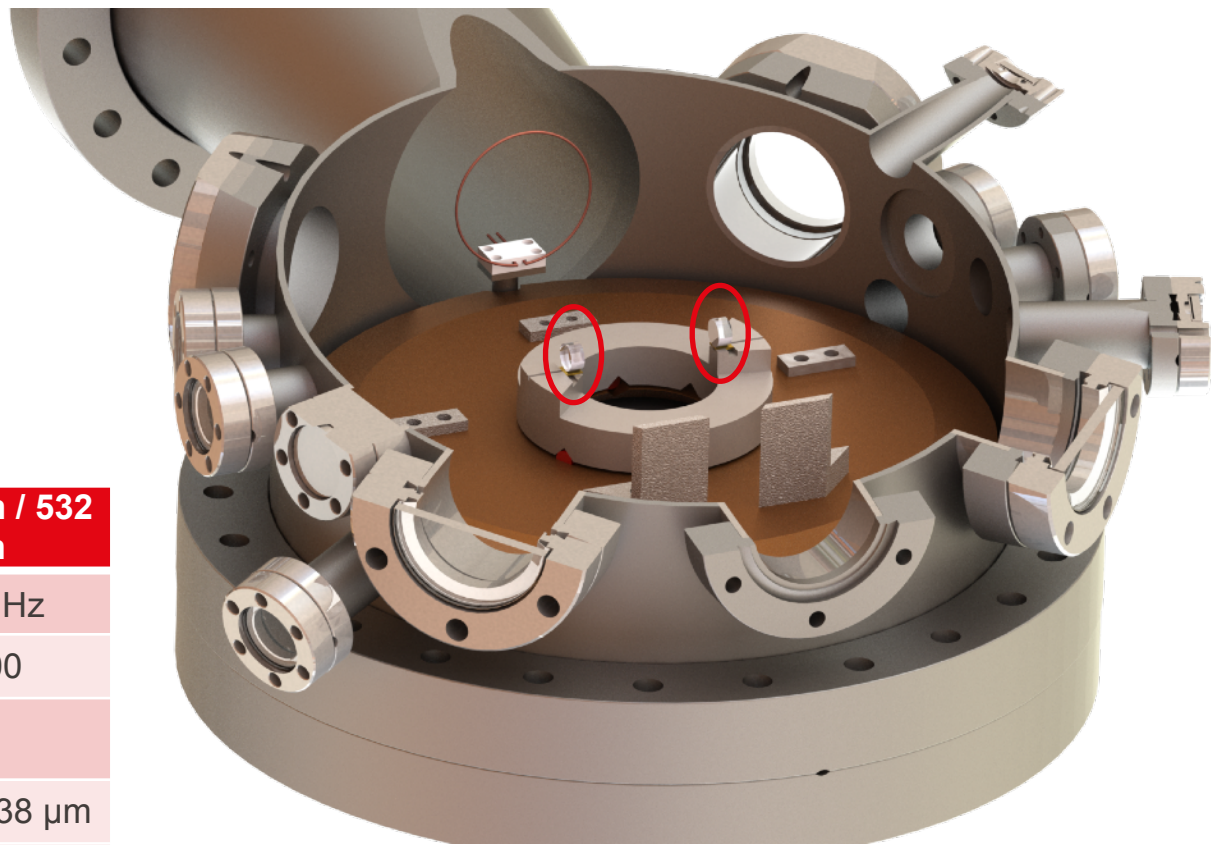
# Experiment



# Experiment

## High-finesse cavity

	671 nm	1064 nm / 532 nm
Linewidth	77 kHz	1.4 MHz
Finesse	47'000	2'800
Cooperativity	2.02	
Waist	45 $\mu\text{m}$	50 $\mu\text{m}$ / 38 $\mu\text{m}$
g	0.479 MHz	



K. Roux, V. Helson, H. Konishi and JPB, *New J. Phys.* **23** 043029 (2021)

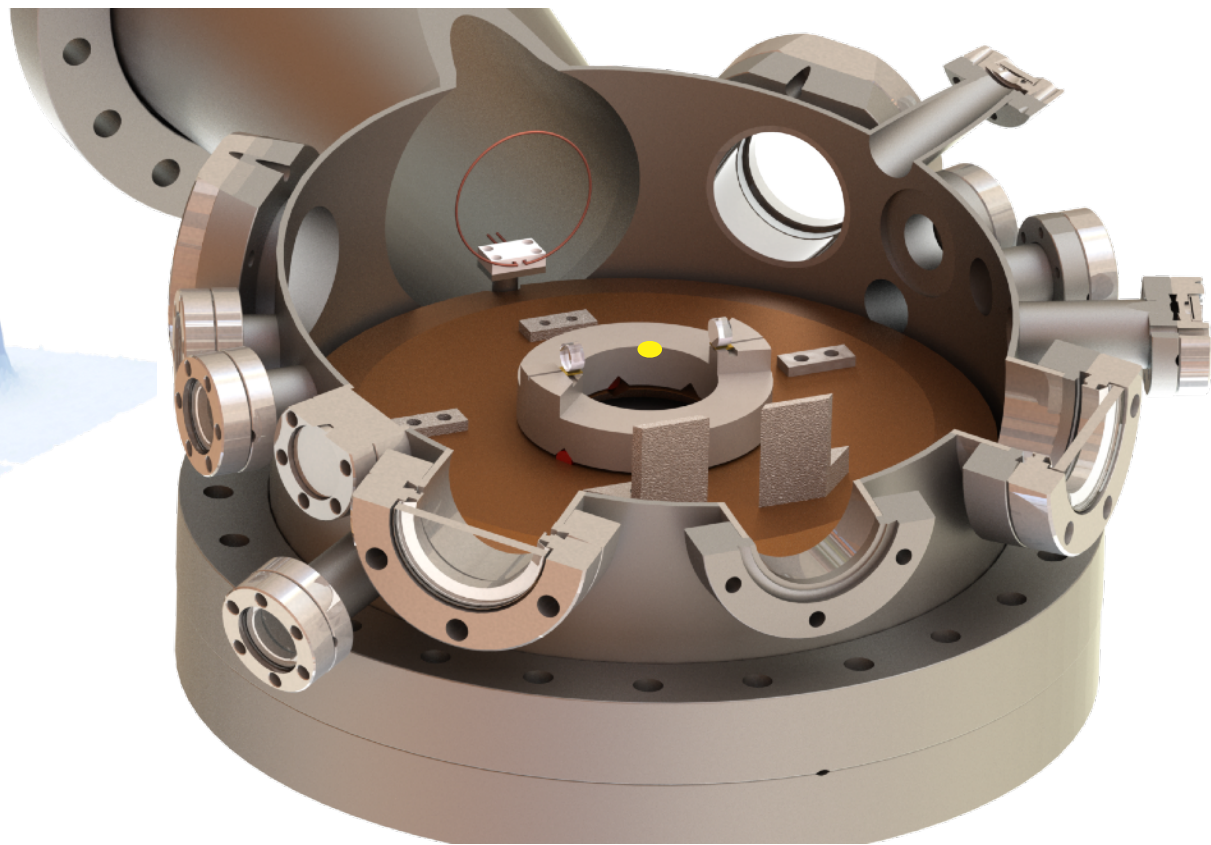
K. Roux, H. Konishi, V. Helson and JPB, *Nature Communications* **11** 2974 (2020)

# Experiment



Unitary Fermi gas

- 300'000  ${}^6\text{Li}$  atoms
- $T = 0.1 T_F$

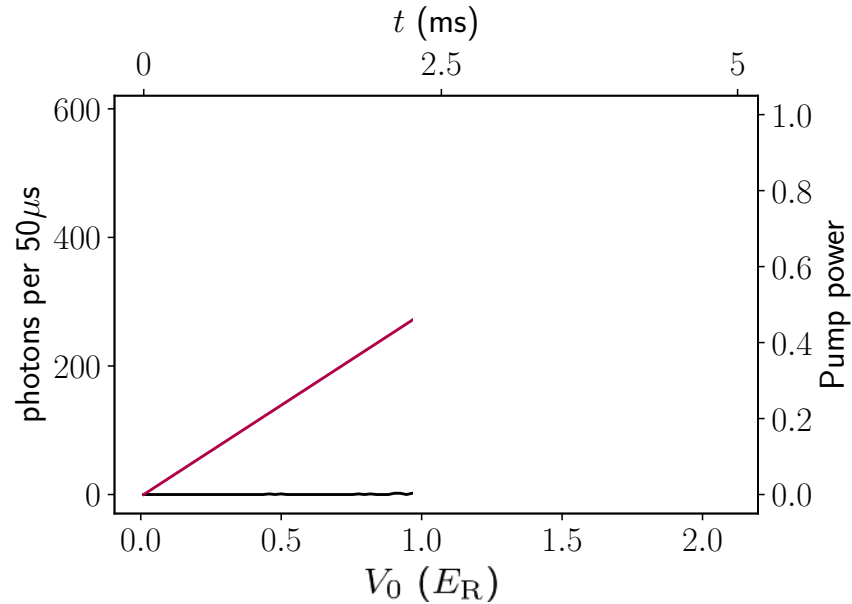
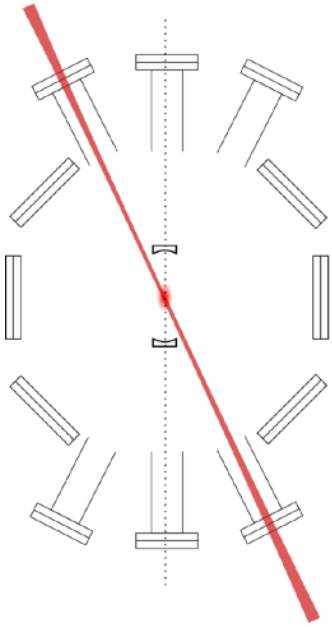


K. Roux, V. Helsen, H. Konishi and JPB, *New J. Phys.* **23** 043029 (2021)

K. Roux, H. Konishi, V. Helsen and JPB, *Nature Communications* **11** 2974 (2020)

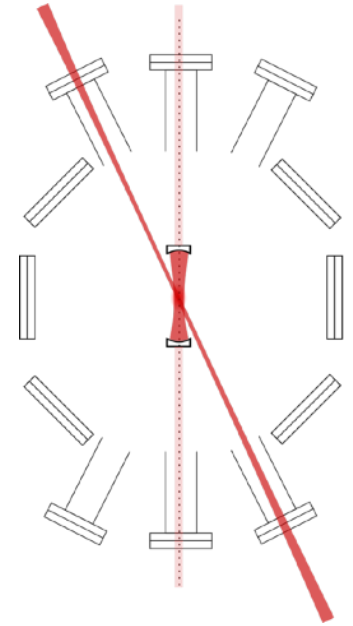
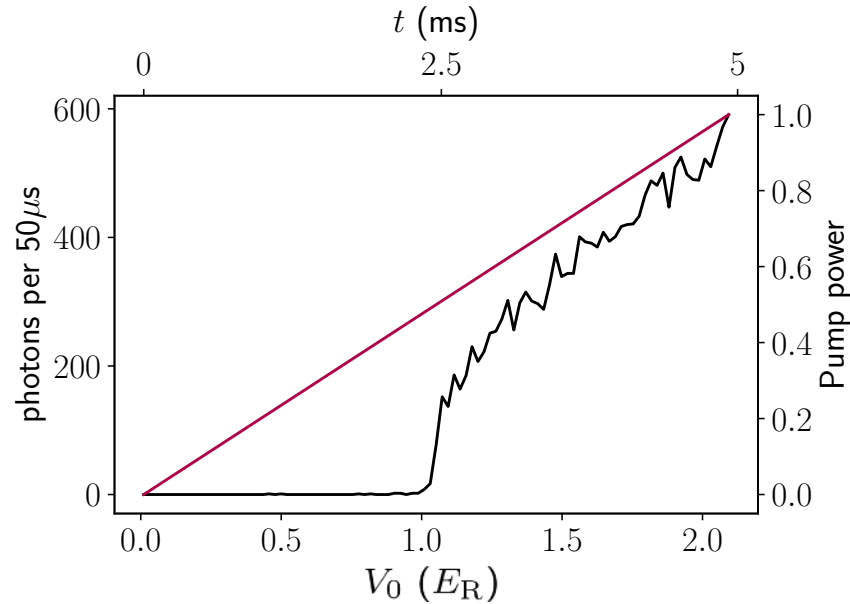
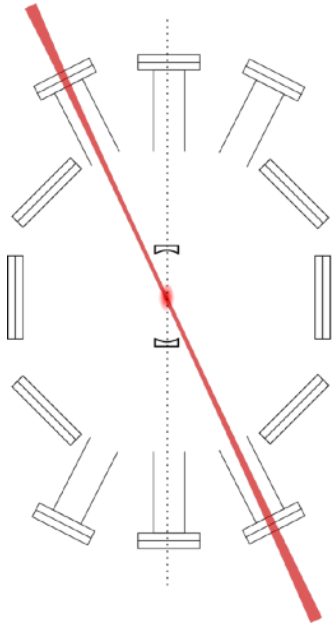
# Self-organization transition

Uniform unitary gas  $\longrightarrow$  'uniaxial' charge Density Wave Order

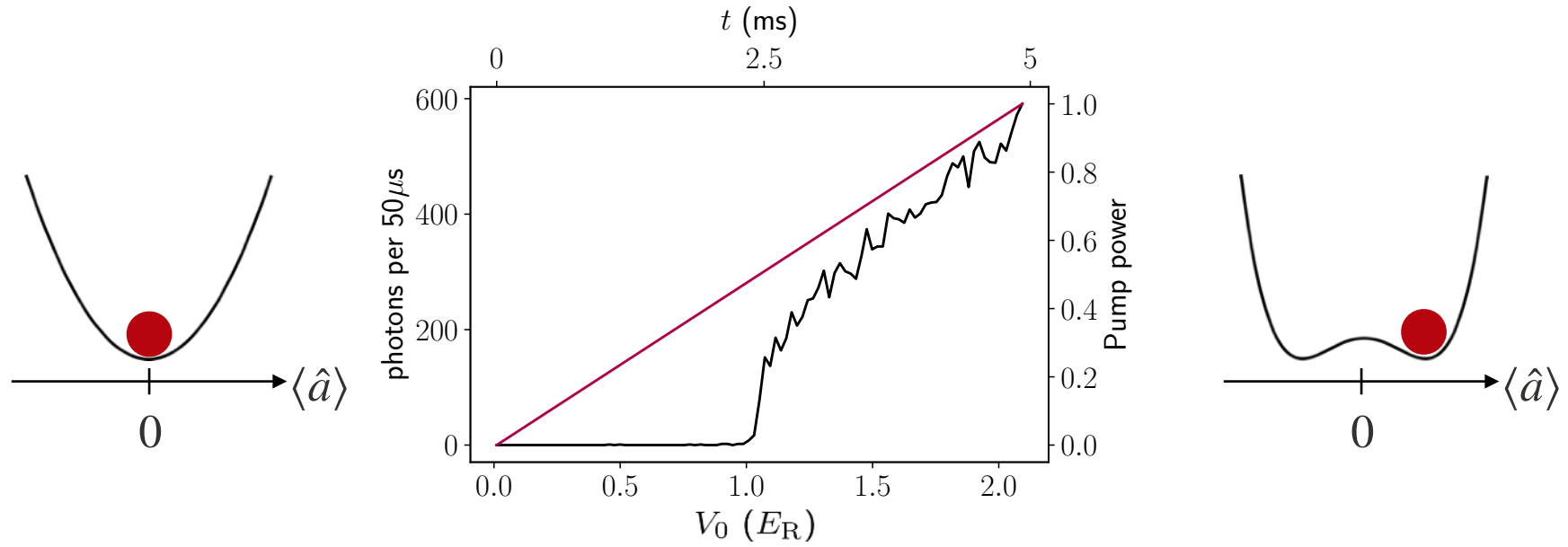


# Self-organization transition

Uniform unitary gas  $\longrightarrow$  'uniaxial' charge Density Wave Order



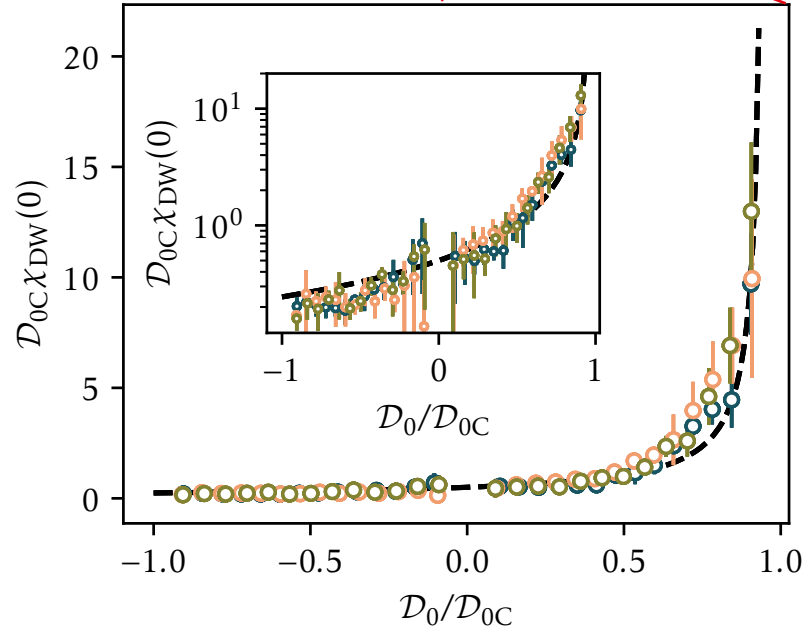
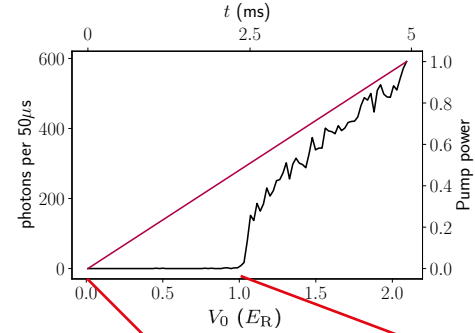
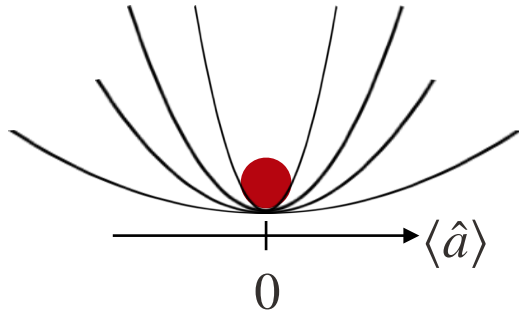
# Self-organization transition





# Self-organization transition

Divergence of the susceptibility



# Self-organization transition

Divergence of the susceptibility

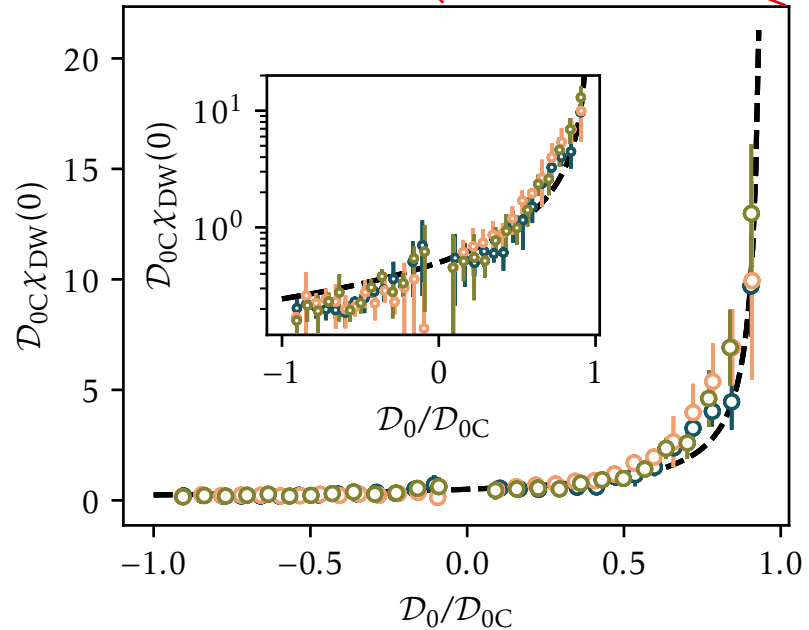
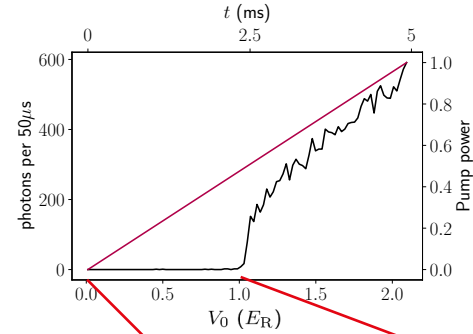
RPA result for the threshold:

$$D_{0c} = \frac{1}{2\chi_0}$$

$$\chi_0 \simeq \chi_{nn}^R(k_-, 0)$$

Divergence of the susceptibility

$$\chi_{DW} \propto \frac{1}{D_0 - D_{0c}}$$



# Ordering dynamics

*Theory:* L. Skolc, S. Chattopadhyay, F. Marijanovic, E. Demler (ETHZ)

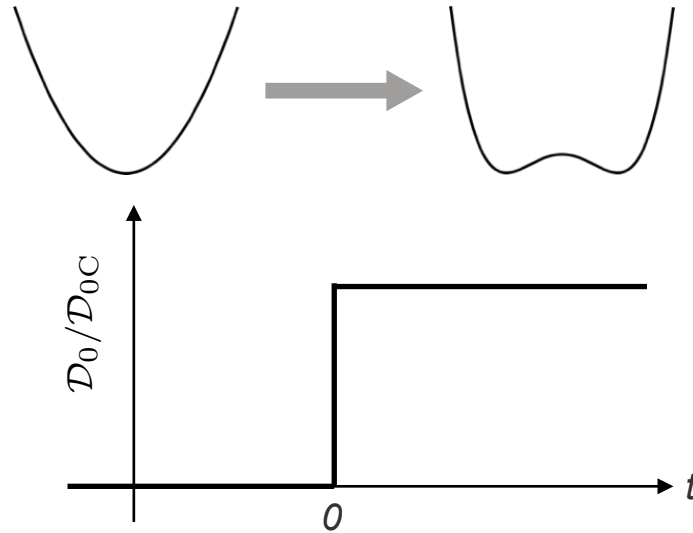
C.M. Halati, T. Giamarchi (Geneva)

S. Uchino (Waseda)

T. Zwettler *et al*, arXiv:2405.18204 (2024)

# Ordering dynamics

Instantaneous quench across the transition



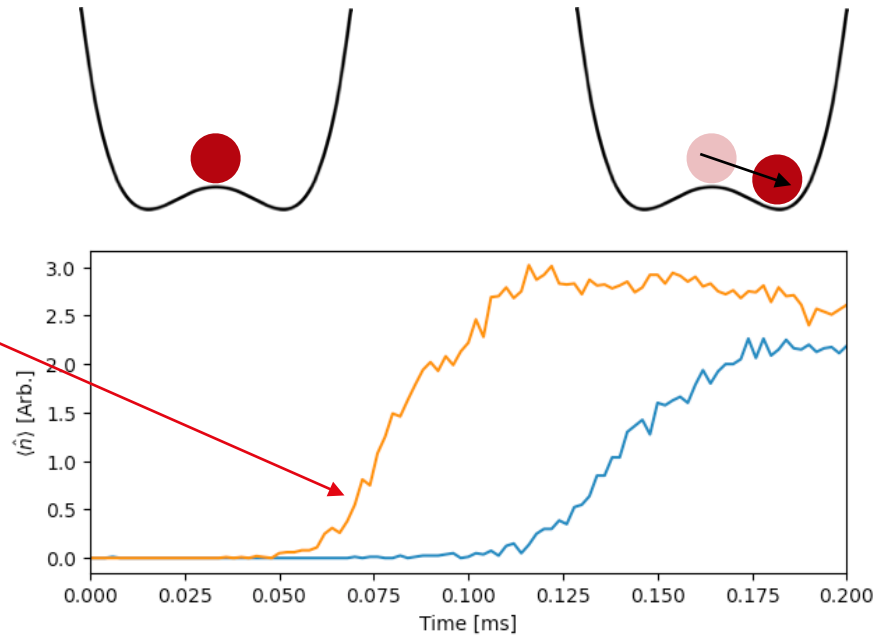
See also :

Z. Wu *et al*, Phys. Rev. Lett. **131** 243401 (2024)

T. Zwettler *et al*, arXiv:2405.18204 (2024)

# Ordering dynamics

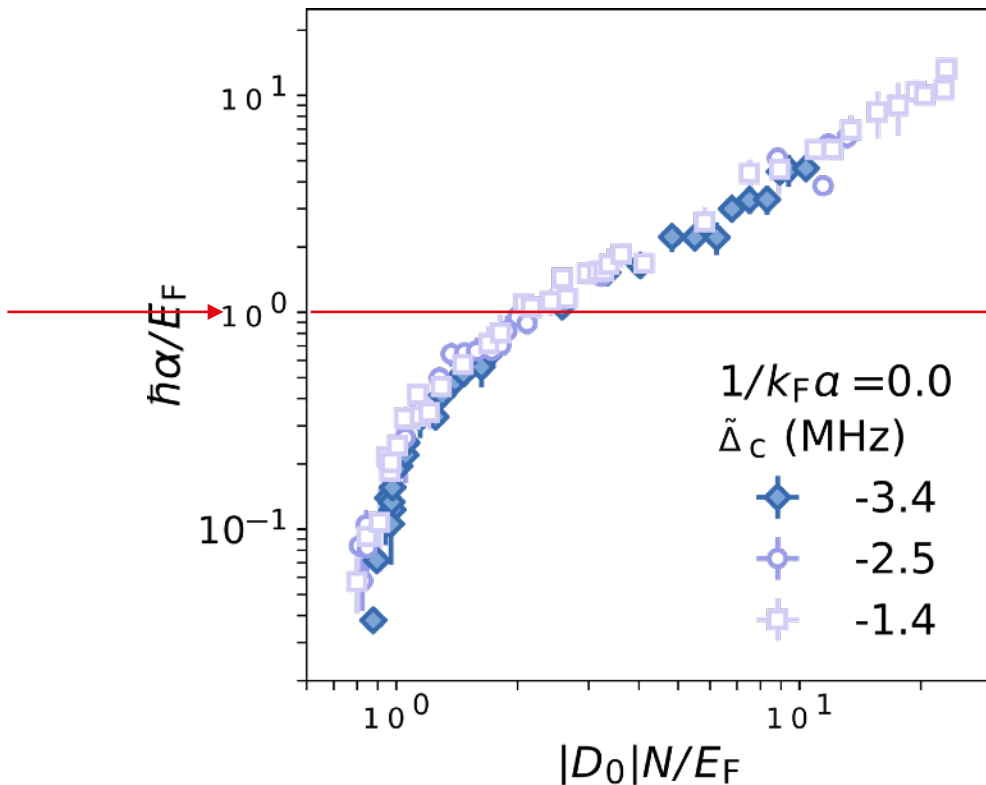
exponential rise  
rate  $\alpha$



# Ordering dynamics

Quench in the unitary Fermi gas

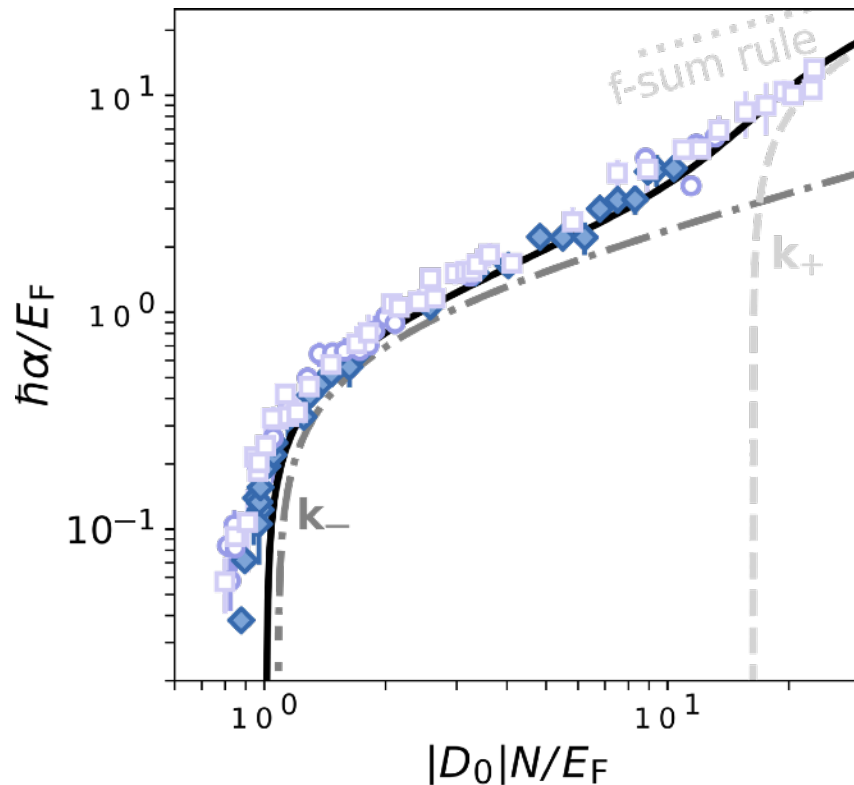
No speed limit at the Fermi time



# Ordering dynamics

Quench in the unitary Fermi gas  
Early-time linear instability analysis  
RPA theory for the unitary gas

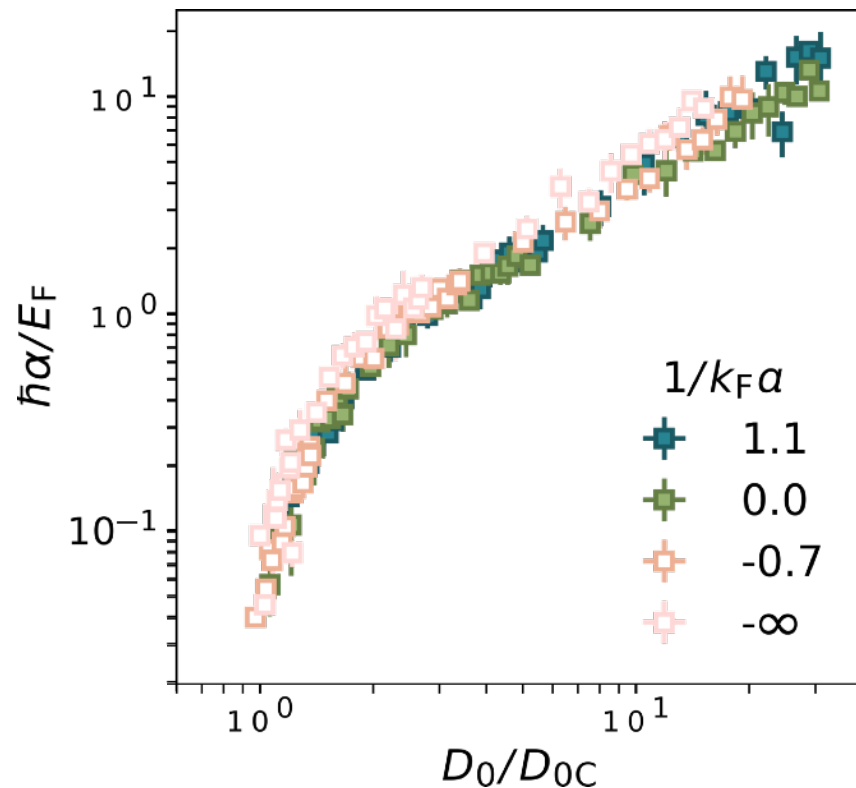
$$1 = \frac{D_0}{4} \sum_{q=k_{\pm}} \int_0^{\infty} \frac{d\omega}{\pi} \frac{\omega \text{Im}[\chi^R(q, \omega)]}{\alpha^2 + \omega^2},$$



# Ordering dynamics

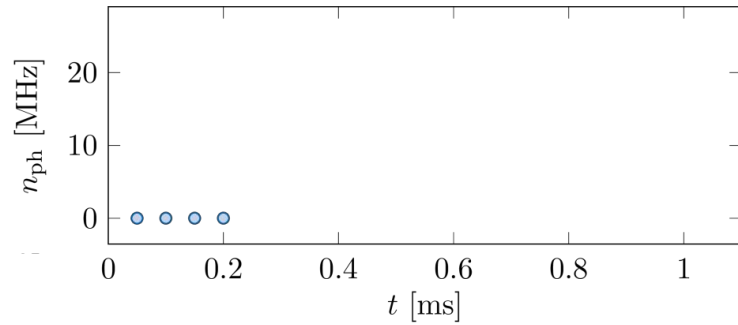
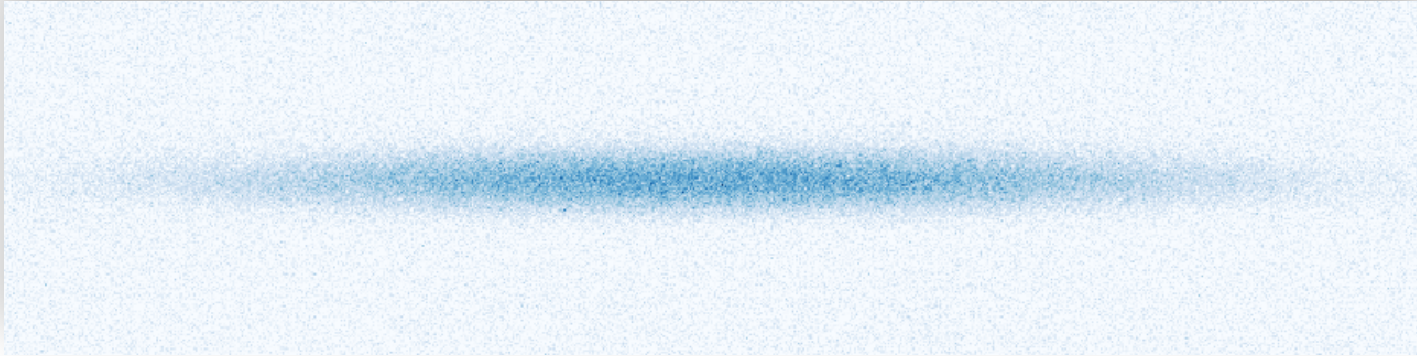
Quench in the BEC-BCS crossover

- Constraints by sum-rules at the largest pump-power
- Weak dependence on interactions close to critical point

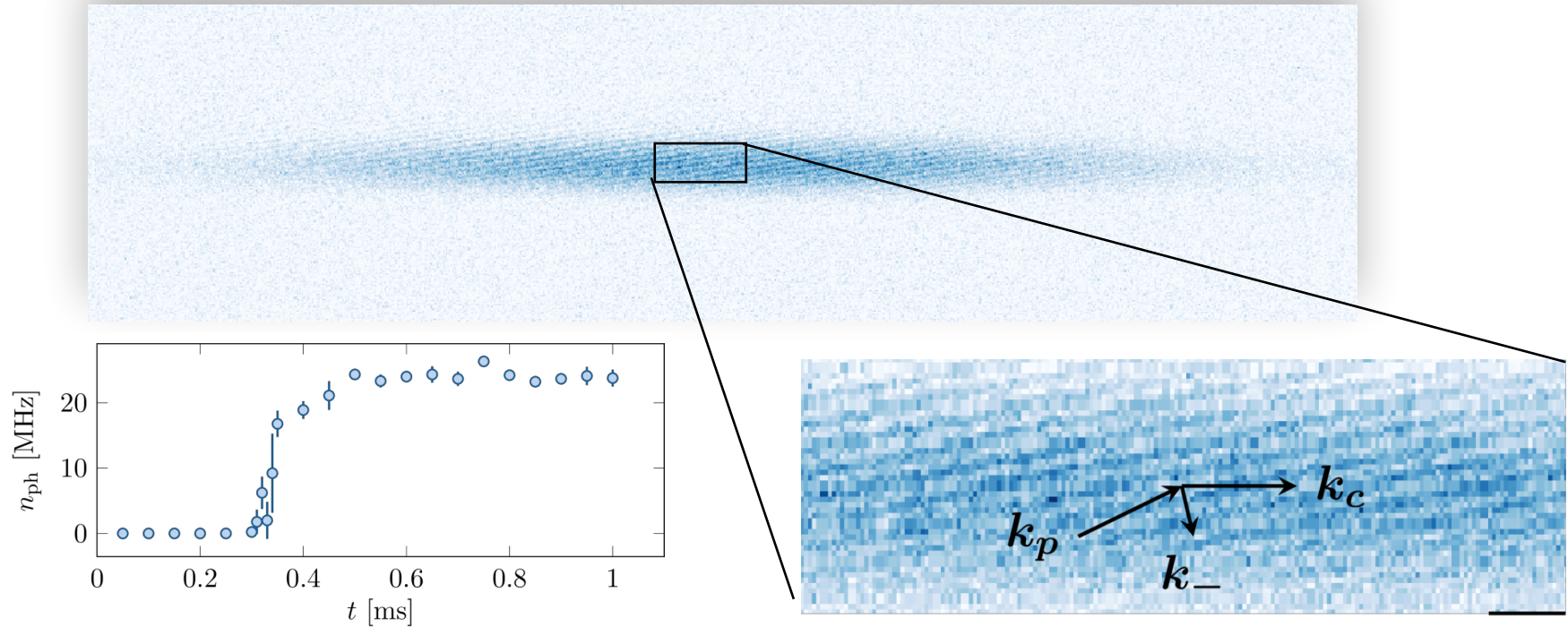




# In-situ observation



# In-situ observation

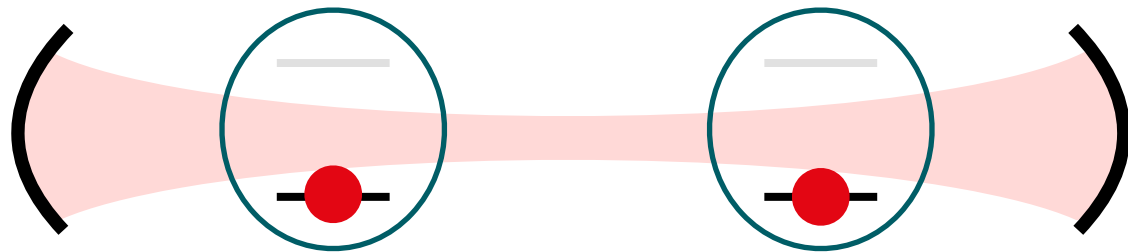


# Competition of photon-mediated interaction with disorder

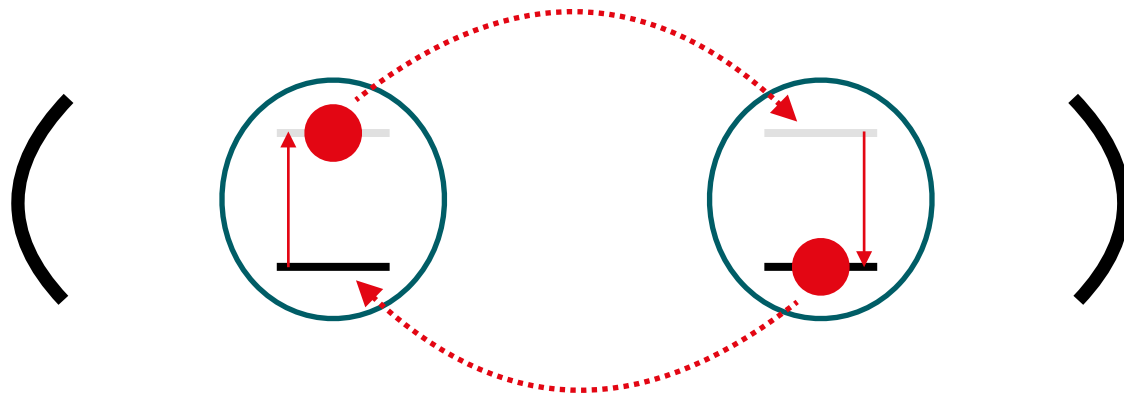
*Theory:* P. Uhrich, S. Bandyopadhyay and P. Hauke (Trento)  
F. Mattiotti and G. Pupillo (Strasbourg)







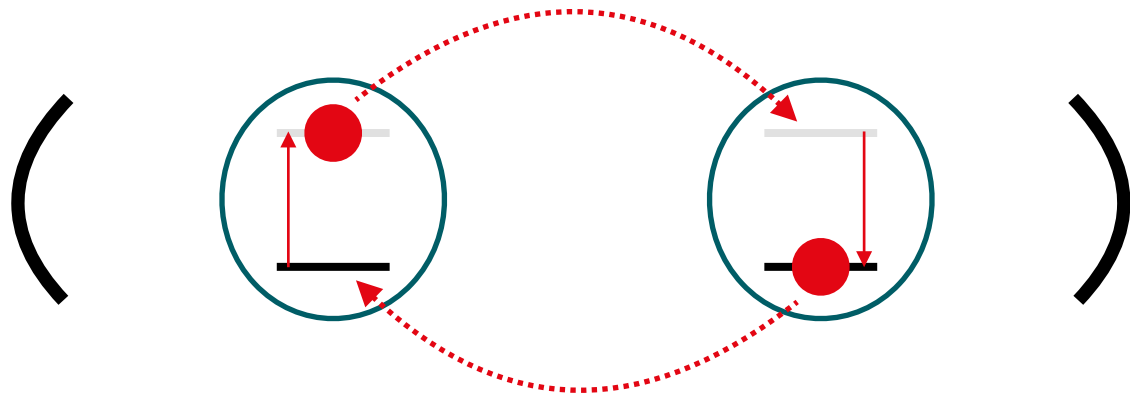




Spin exchange mediated by  
cavity photon

$$\hat{H}_{\text{int}} = \frac{g_0^2}{\Delta} \sum_{i,j} f(r_i) f(r_j) \hat{\sigma}_i^+ \hat{\sigma}_j^- + hc$$

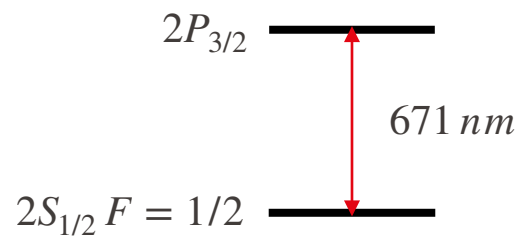


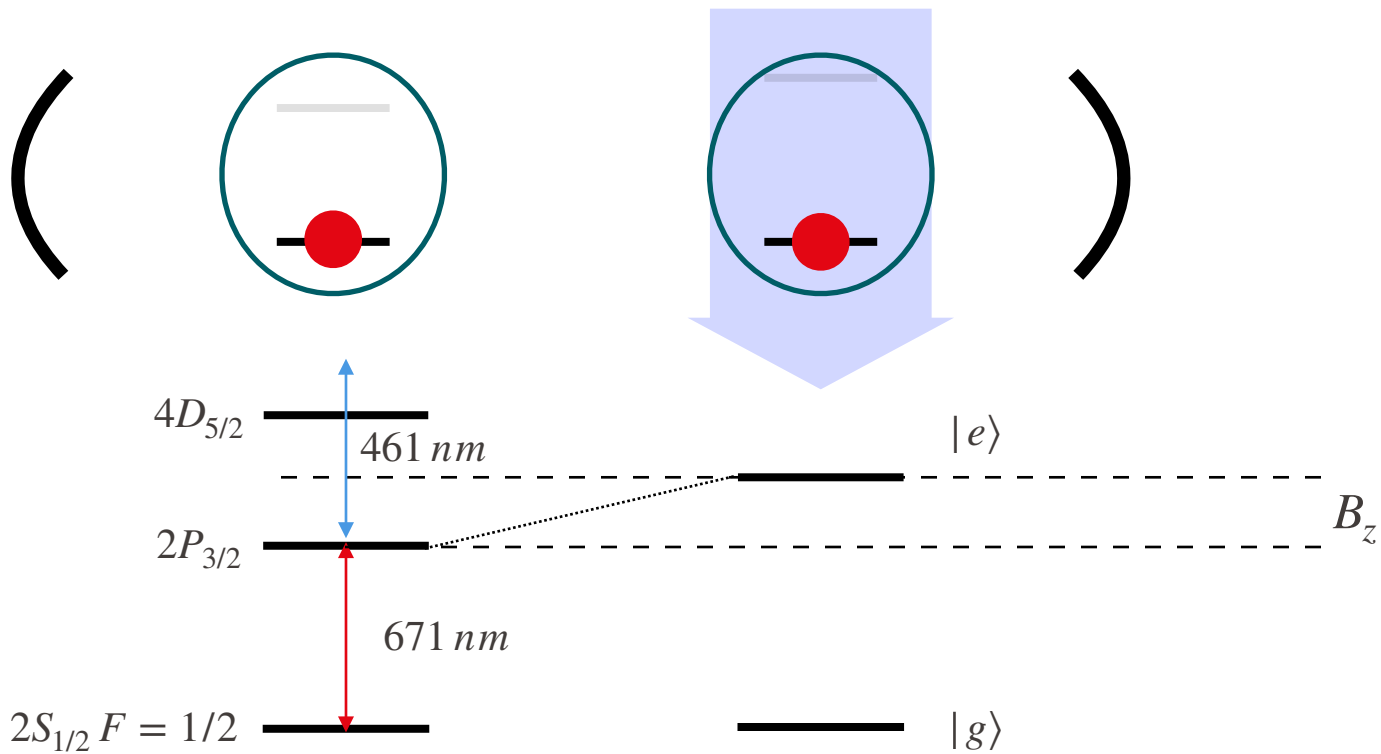


Spin exchange mediated by  
cavity photon

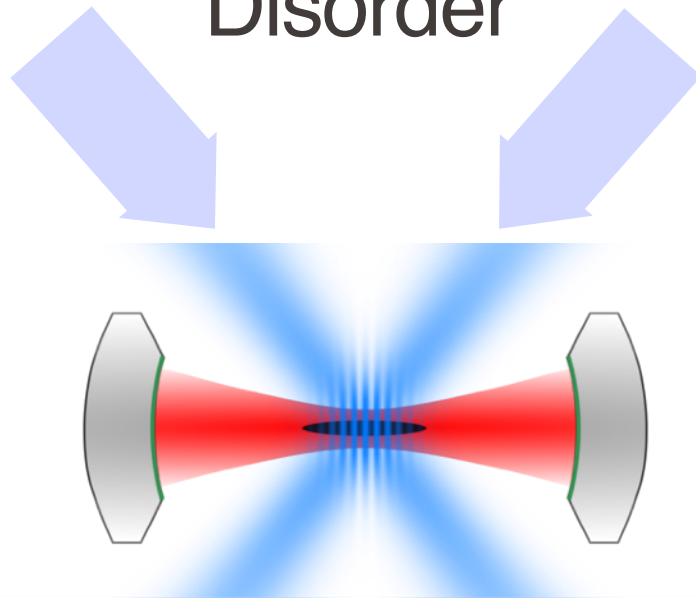
Collective spin description  $\hat{J}^{\pm} = \sum_i f(r_i) \hat{\sigma}_i^{\pm}$

$$\begin{aligned} \hat{H}_{\text{int}} &= \frac{g_0^2}{\Delta} \sum_{i,j} f(r_i) f(r_j) \hat{\sigma}_i^+ \hat{\sigma}_j^- + hc \\ &= \frac{g_0^2}{\Delta} \hat{J}^+ \hat{J}^- \end{aligned}$$

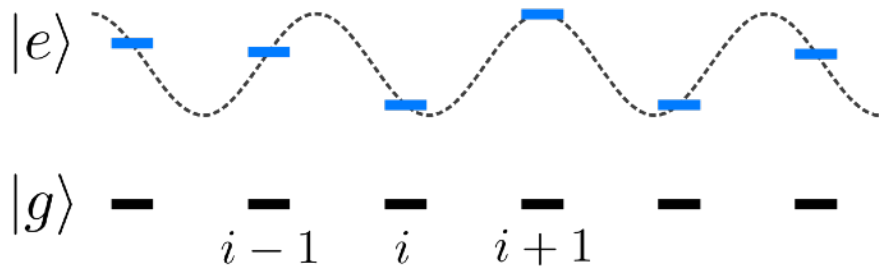
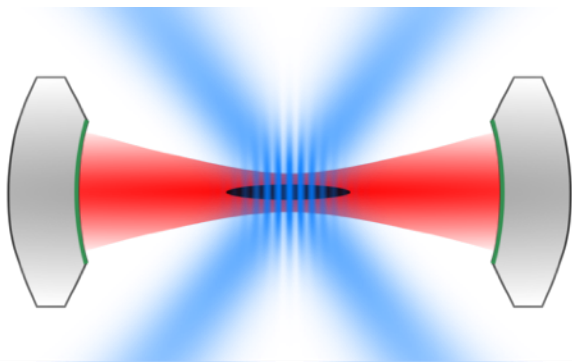




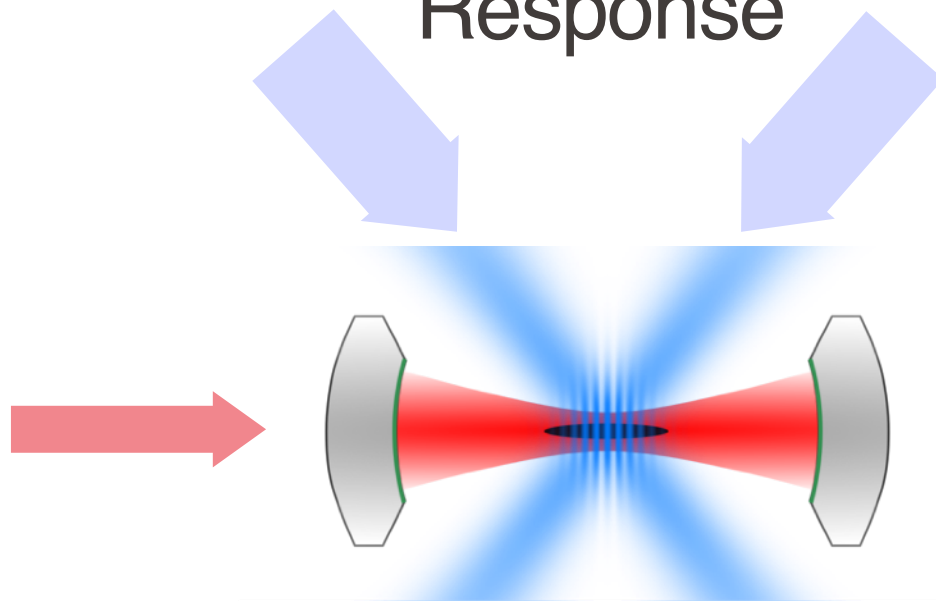
Disorder



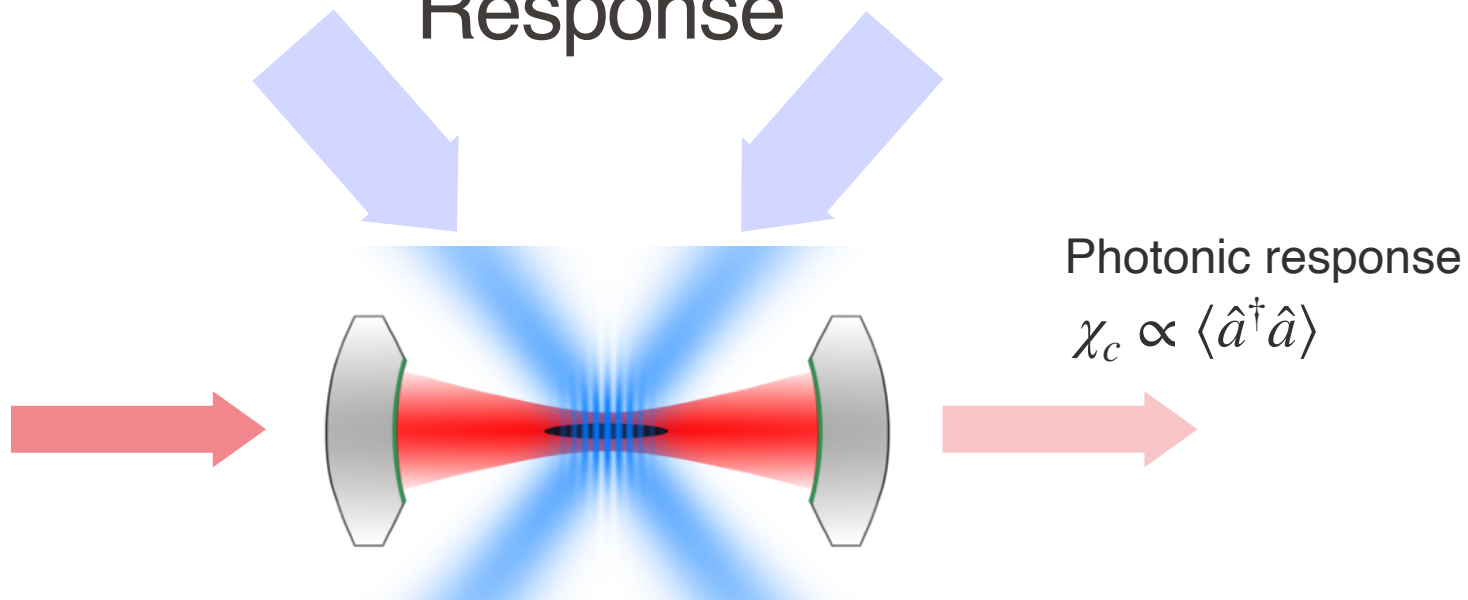
# Disorder



Response



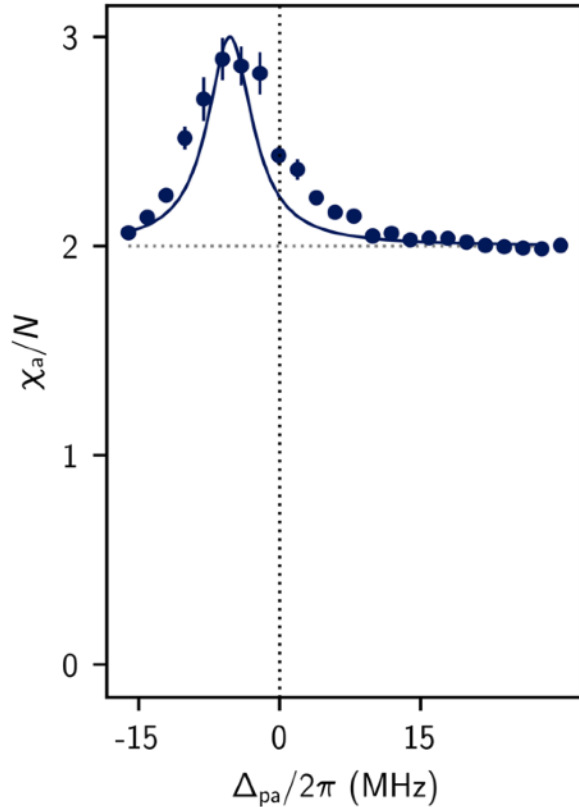
Response



Photonic response

$$\chi_c \propto \langle \hat{a}^\dagger \hat{a} \rangle$$

# Lipkin-Meshkov-Glick model



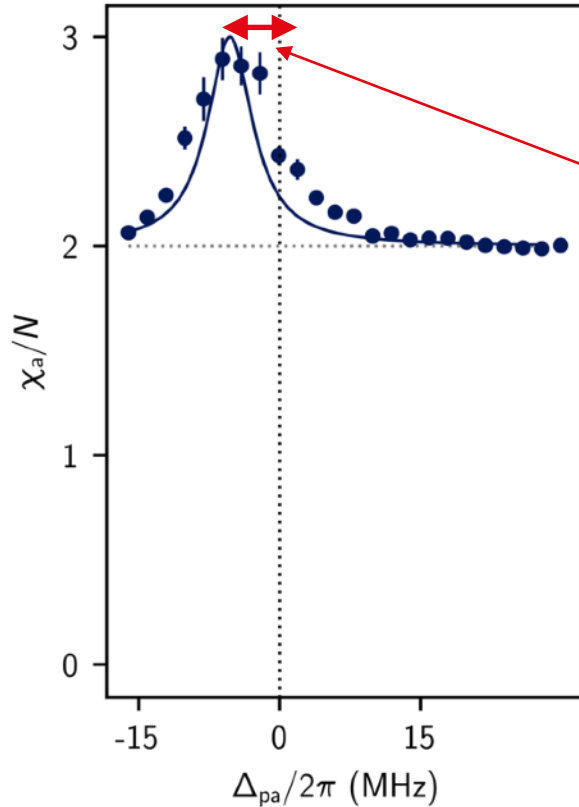
$$\hat{H}_{\text{int}} = \frac{g_0^2}{\Delta_{ac}} \hat{J}_+ \hat{J}_- + \Delta_{pa} \hat{J}_z$$

Muniz, Juan A., et al. *Nature* **580** 602-607 (2020)

Lewis-Swan, Robert J., et al. *Physical Review Letters* **126** 173601 (2021)



# Lipkin-Meshkov-Glick model



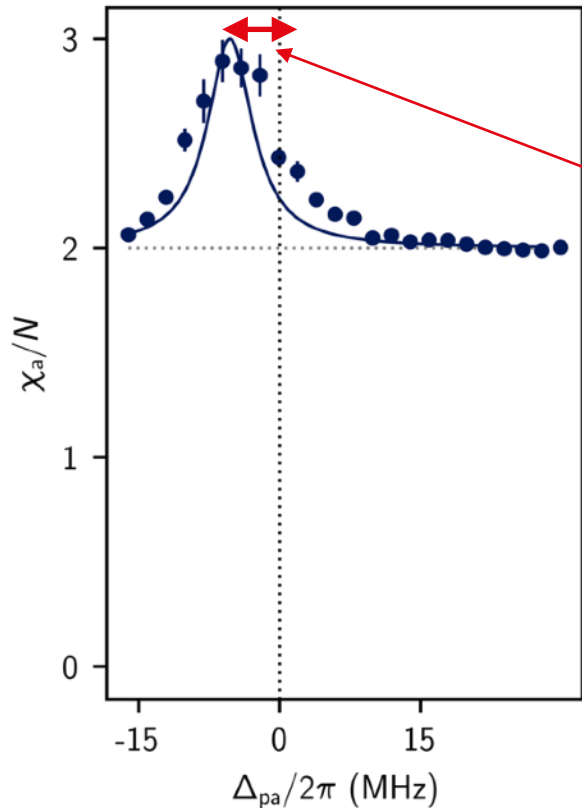
$$\hat{H}_{\text{int}} = \frac{g_0^2}{\Delta_{ac}} \hat{J}_+ \hat{J}_- + \Delta_{pa} \hat{J}_z$$

Ferromagnetic gap

Muniz, Juan A., et al. *Nature* **580** 602-607 (2020)

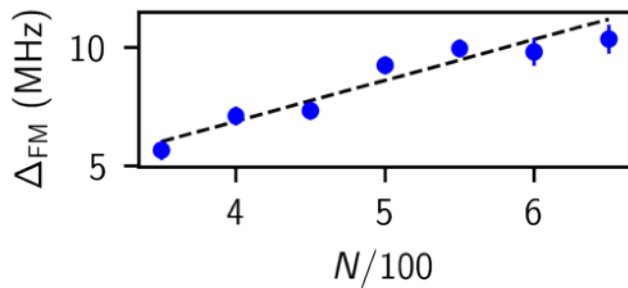
Lewis-Swan, Robert J., et al. *Physical Review Letters* **126** 173601 (2021)

# Lipkin-Meshkov-Glick model

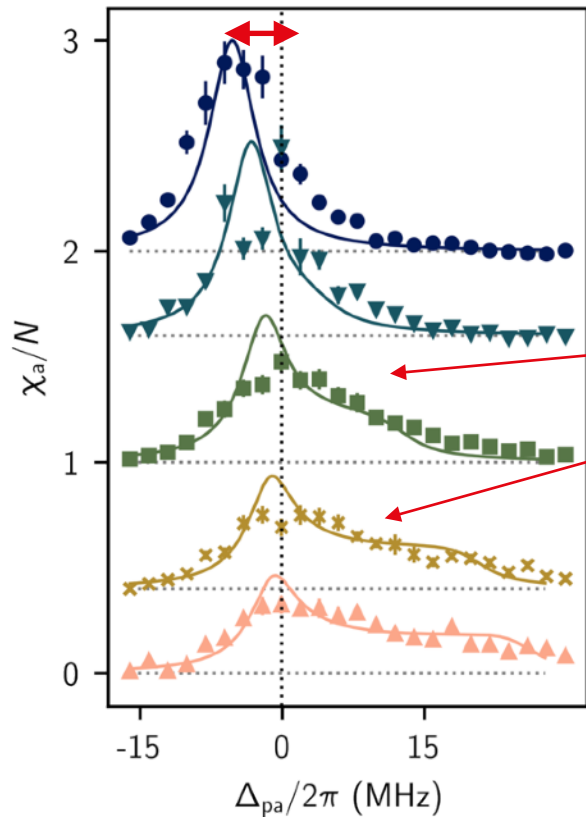


$$\hat{H}_{\text{int}} = \frac{g_0^2}{\Delta_{ac}} \hat{J}_+ \hat{J}_- + \Delta_{pa} \hat{J}_z$$

Ferromagnetic gap



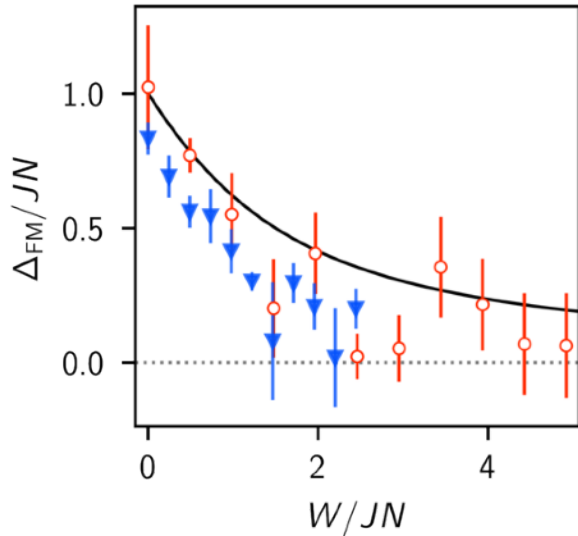
# Lipkin-Meshkov-Glick model



$$\hat{H}_{\text{int}} = \frac{g_0^2}{\Delta_{ac}} \hat{J}_+ \hat{J}_- + \Delta_{pa} \hat{J}_z + \sum_i B_{i,z} \hat{\sigma}_{i,z}$$

Broad response

# Break-down of collective coupling

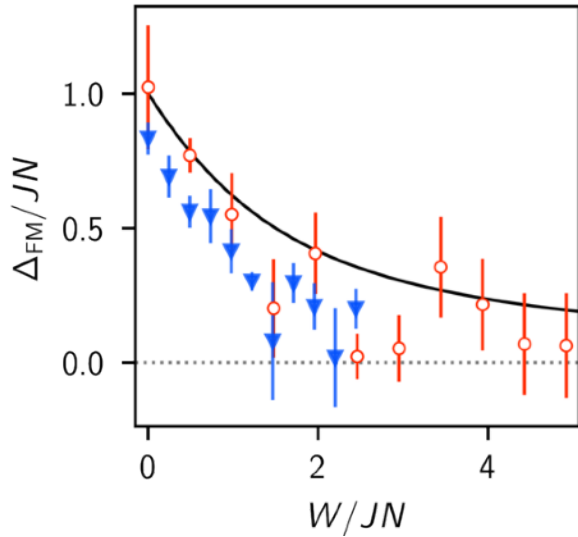


Ferromagnetic gap  $\longrightarrow 0$  for  $W \longrightarrow \infty$

- For fixed atom number: disappearance of energy resonances

*Ferromagnetic*  $\longrightarrow$  *paramagnetic crossover*

# Break-down of collective coupling



Ferromagnetic gap  $\longrightarrow 0$  for  $W \longrightarrow \infty$

- For fixed atom number: disappearance of energy resonances

*Ferromagnetic*  $\longrightarrow$  *paramagnetic crossover*

- For fixed  $W$  : infinite number of resonances at the thermodynamic limit

*No disorder induced phase transition*

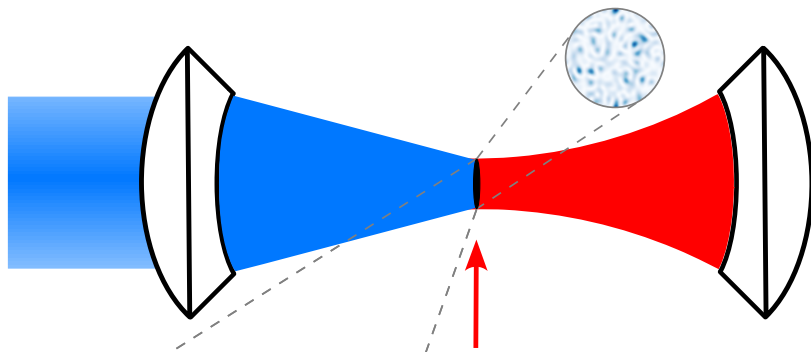
P. Hauke (Trento)  
J. Sonner (Geneva)



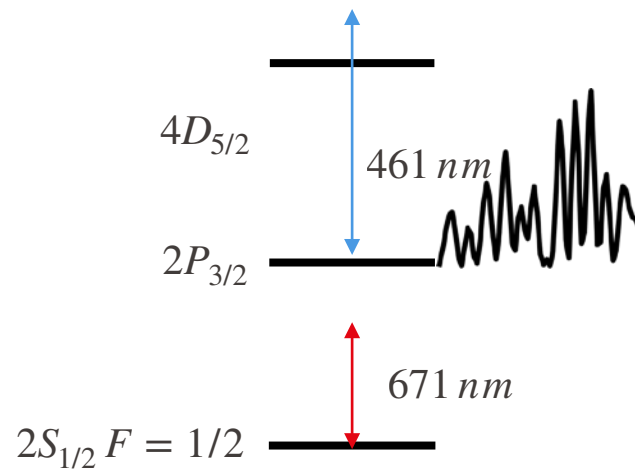
# Prospects for realizing the SYK model

# Prospects for realizing the SYK model

Fermionic motional  
degrees-of-freedom



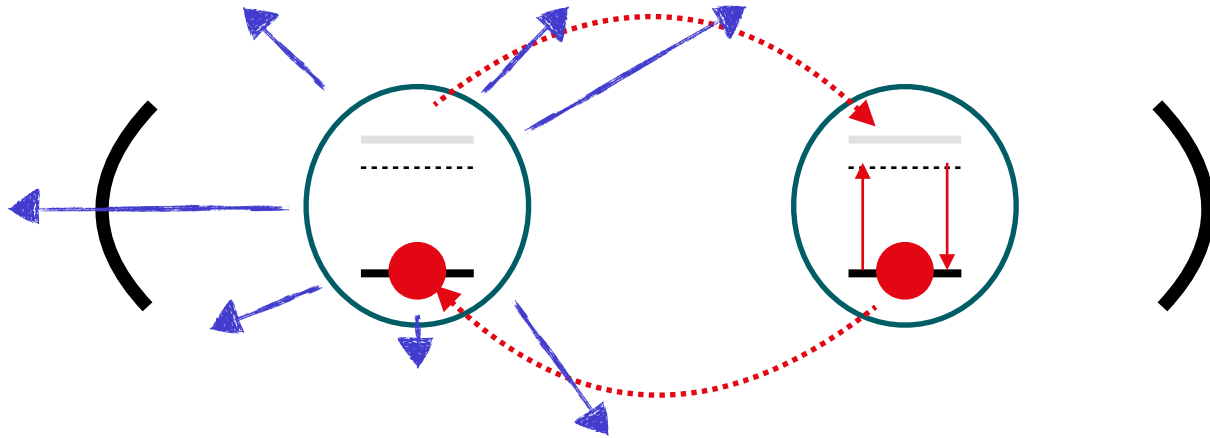
Disorder in the excited-  
state manifold



# Prospects for realizing the SYK model

Fermionic motional  
degrees-of-freedom

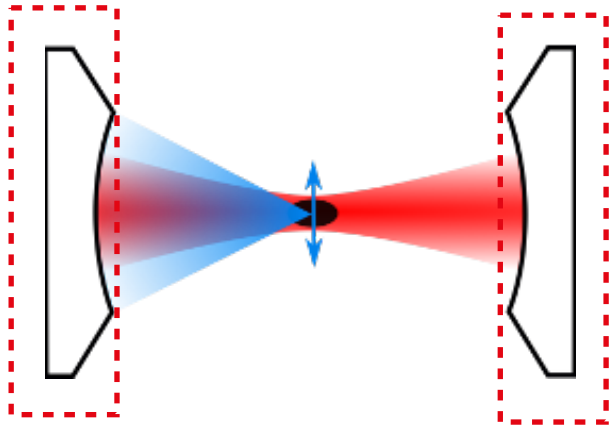
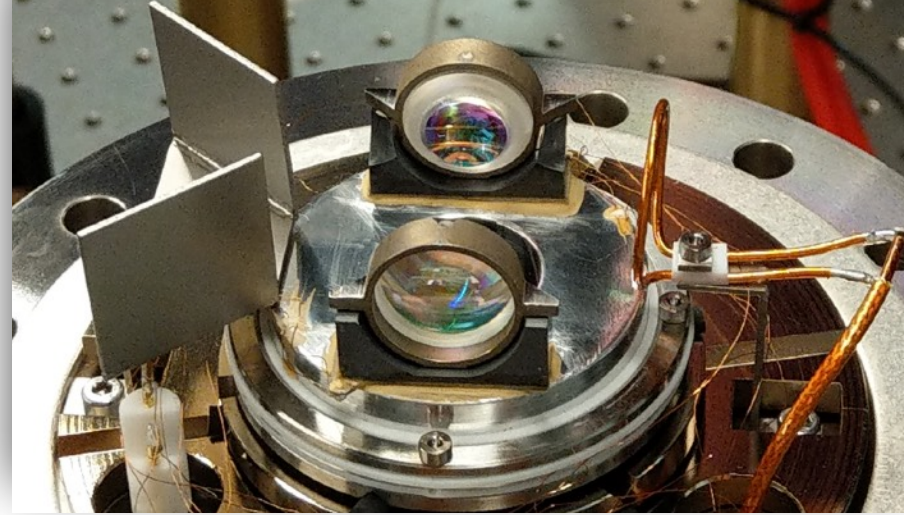
Disorder in the excited-  
state manifold



*Engineering of the density-density interaction*

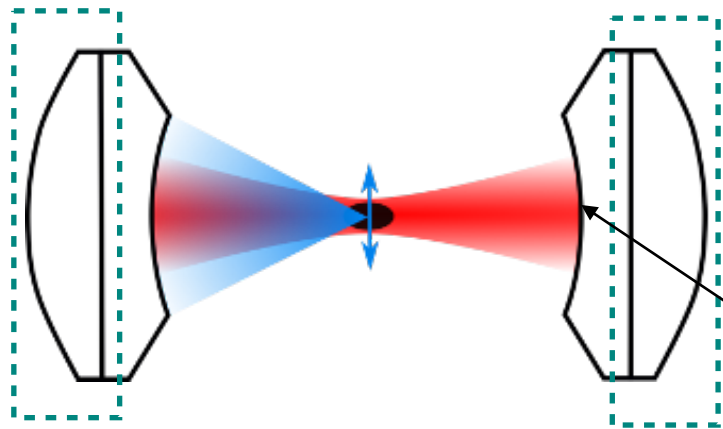


# Cavity-microscope



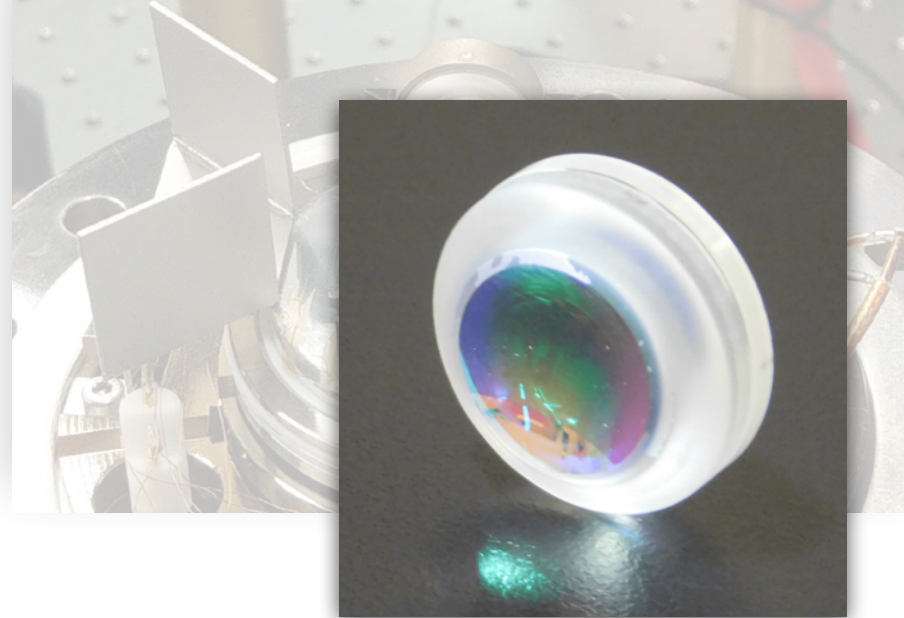
← High-finesse cavity mirrors

# Cavity-microscope

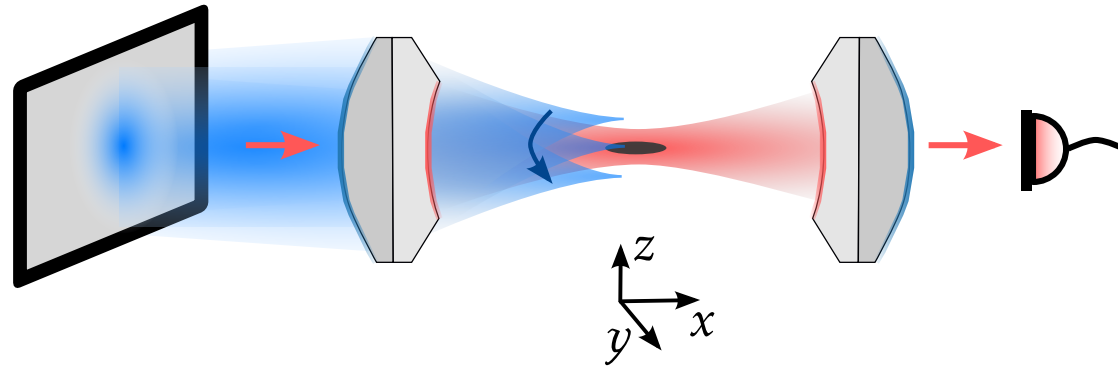


0.37 NA aspherical lens pair  
*Optically contacted on the mirrors*

HR 671 + 1342 nm, HT 780 + 461 nm

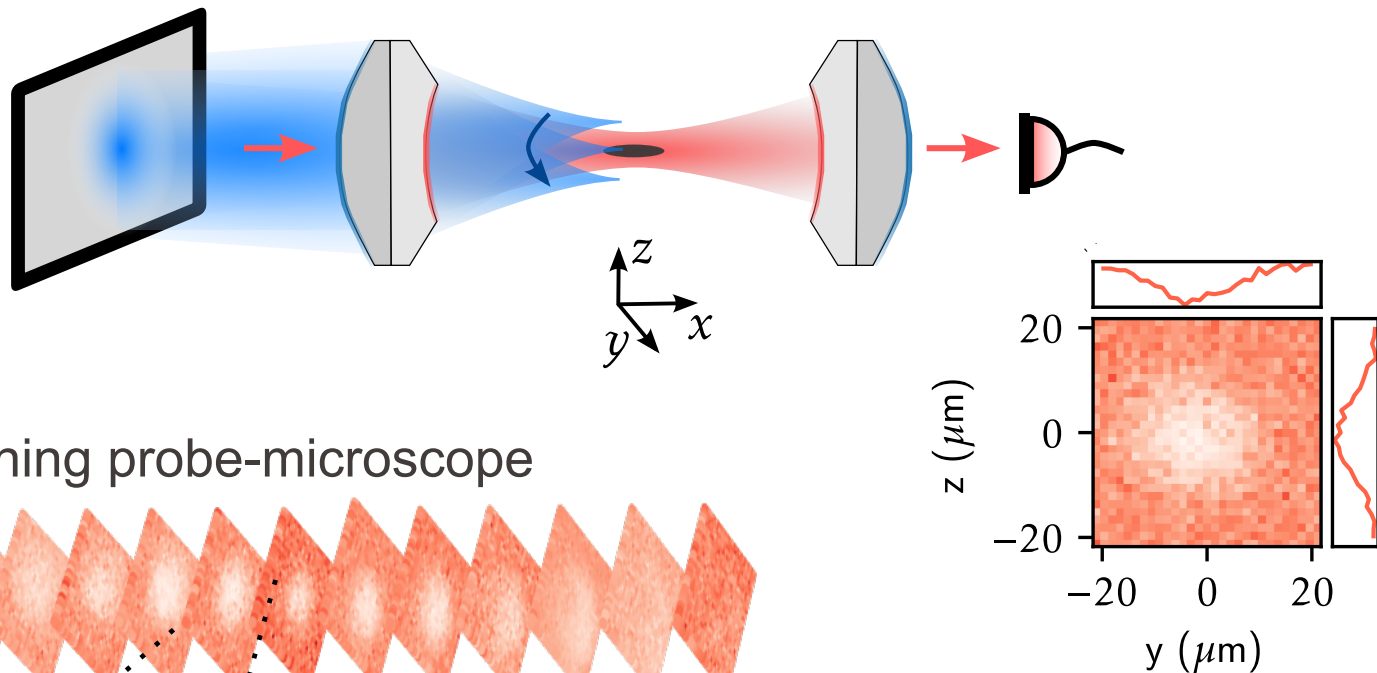


# Cavity-microscope



Spatial programming of photon-atom coupling

# Cavity-microscope



Example: scanning probe-microscope

# SYK simulation: challenge

$$\hat{H}_{SYK} = \sum_{ijkl} J_{ijkl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

$\sim N^4$  **independent** random couplings

*Photon-exchange interactions lead to correlations*

$$J_{ijkl} = f_{ij} f_{kl}$$

# SYK simulation: challenge

$$\hat{H}_{SYK} = \sum_{ijkl} J_{ijkl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

$\sim N^4$  independent random couplings

*Photon-exchange interactions lead to correlations*

$$J_{ijkl} = f_{ij} f_{kl} \longrightarrow \sum_{\alpha}^M \frac{f_{ij}^{(\alpha)} f_{kl}^{(\alpha)}}{1 + \delta_{\alpha}}$$

*Use multimode structure of the cavity*

*Time-dependent disorder*



- Fermions with strong, long-range, all-to-all interactions are available in the lab

V. Helson, T. Zwettler, E. Collela, F. Mivhevar, K. Roux, H. Konishi, H. Ritsch and JPB  
Nature **618**, 716 (2023)

- Controlled disorder with all-to-all interacting systems is available in the lab

N. Sauerwein, F. Orsi, P. Uhrich, S. Bandyopadhyay, F. Mattiotti, T. Cantat-Moltrecht, G. Pupillo, P. Hauke and JPB  
Nature Physics **19**, 1128 (2023)

- Blue-print for the implementation of the SYK model

P. Uhrich, S. Bandyopadhyay, N. Sauerwein, J. Sonner, JPB, P. Hauke  
arXiv:2303.11343 (2023)