JOHN TEMPLETON FOUNDATION
Inspiring Awe & Wonder

Spectral statistics of (complex) eigenvalues

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AURELIA'S QUANTUM LAB

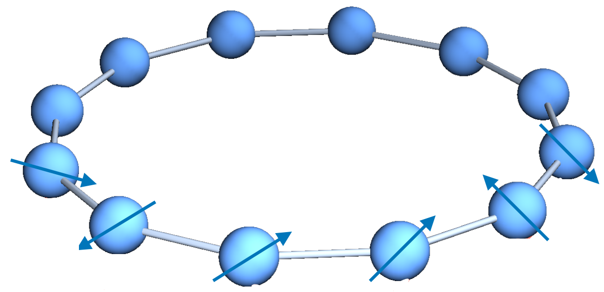
Quantum Chaos workshop
Bernoulli center, EPFL

uni.lu

Plot

- *Main characters*
- *Clues*
  - Quantum Chaos & its spectral signatures
  - Non-Hermitian Physics
  - Many-body Localization (MBL)
- *Results*
  - Two transitions: integrability and Hermiticity breaking
  - Dissipative quantum chaos (from SVD)

Physical model: Hermitian



XXZ model (hopping, interaction) + disorder:

$$\hat{H} = J \sum_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z \right) + \sum_i h_i \hat{S}_i^z$$

hopping: $\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y$

interaction: $\Delta \hat{S}_i^z \hat{S}_{i+1}^z$

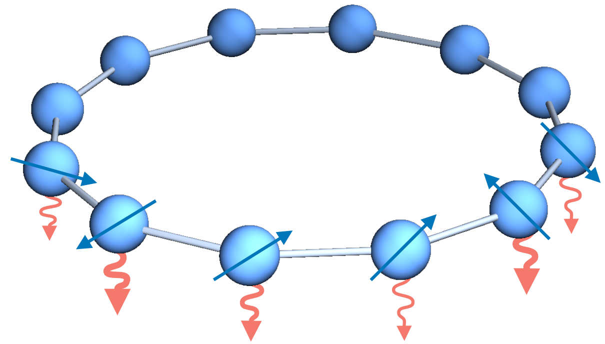
disorder: h_i sampled in $[-W, W]$

H Hermitian	weak disorder	strong disorder
eigenstates	**delocalized**	**(many-body) localization**
spectral statistics	**chaotic**	**integrable**

do these features survive for non-unitary dynamics?

What's broken first? Hermiticity or integrability?

Physical model: non-Hermitian



XXZ model — non-Hermiticity = disorder

$$\hat{H} = J\sum_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z \right) - \frac{i}{2}\sum_i \gamma_i \left(\hat{S}_i^z + \frac{1}{2} \right)$$

hopping — interaction

γ_i sampled in $[0, \gamma]$

$$\gamma_i \to 2ih_i$$

	Hermitian	non-Hermitian
Integrable		
Chaotic (RMT)		

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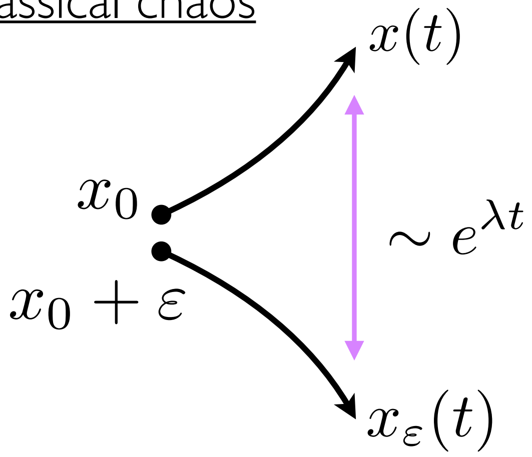
Chaos: Lyapunov exponent

<u>Classical chaos</u>



extreme sensitivity to initial conditions

…but Schrödinger Equation is linear

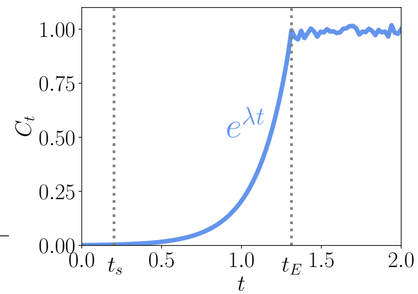
<u>Quantum chaos</u>

Out-of-time ordered correlator

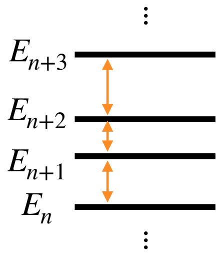
$$C_{\beta,t} = -\left\langle [\hat{A}_t, \hat{B}_0]^2 \right\rangle_\beta \sim e^{\lambda t}$$

Classical limit: Consider $\hat{A}_t = \hat{x}_t$, $\hat{B}_0 = \hat{p}_0$

$$C_t = \{x_t, p_0\}^2 = \left(\frac{\partial x_t}{\partial x_0}\right)^2 \sim e^{2\lambda_{\mathrm{cl}} t} \text{ then } \lambda \sim 2\lambda_{\mathrm{cl}}$$



Quantum Chaos



$$s_n = E_{n+1} - E_n$$

<u>Single number signature:</u>

r parameter = average of

$$r_n = \min(s_{n+1}, s_n)/\max(s_{n+1}, s_n)$$

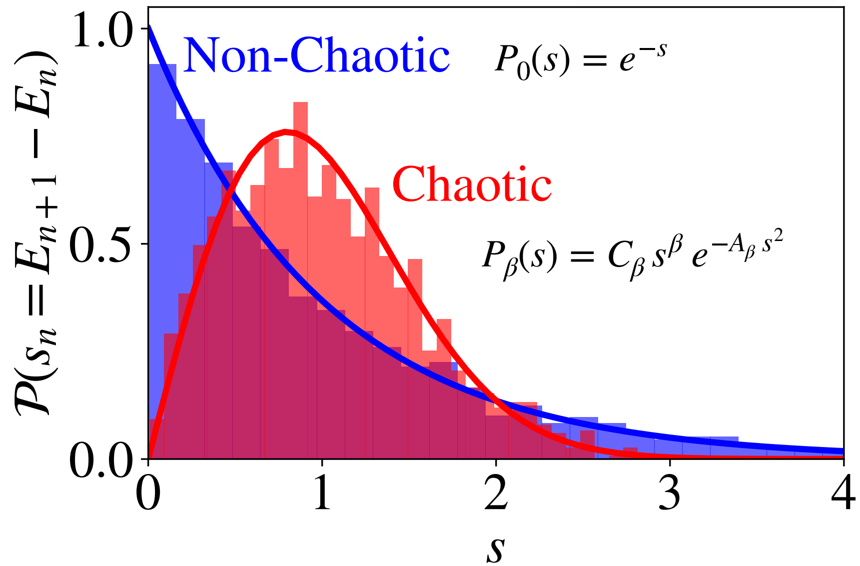
values and distributions known for Poisson & Gaussian ensembles

probe of chaotic/integrable crossover

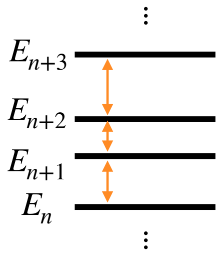
Oganesyan and Huse, PRB 2007
Atas et al., PRL 2013

Conjectures: level spacing distribution is
Poissionian → Integrable — Berry and Tabor '77
Wigner-Dyson → Chaotic — Bohigas-Giannoni-Schmit '84
Same in open Markovian dynamics — Grobe, Haake, Sommers '88

©P. Martinez-Azcona



Quantum Chaos



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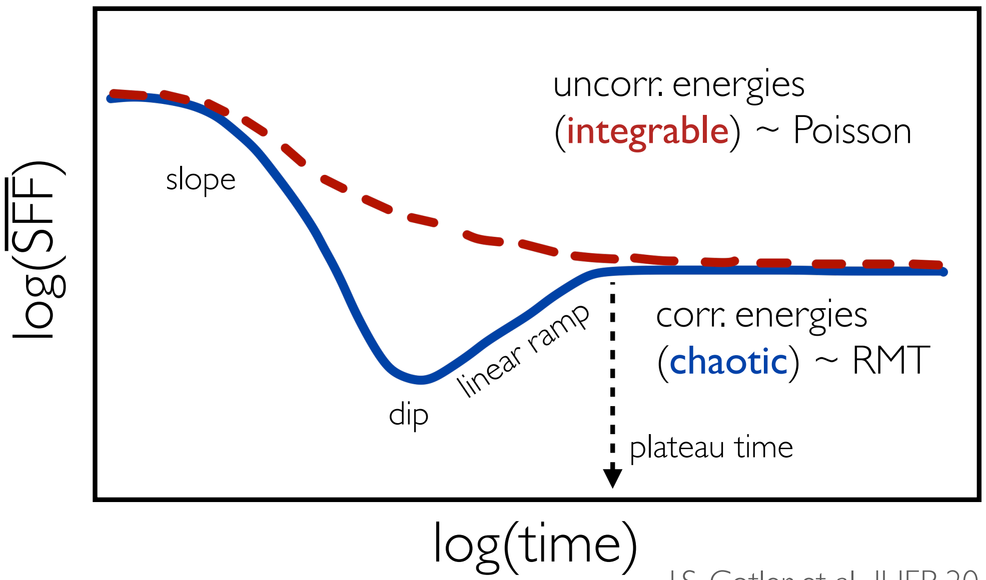
<u>Dynamical signature</u>: spectral form factor (SFF)

$$S_t = \frac{1}{N^2} \sum_{n,m}^{N} e^{-i(E_n - E_n)t}$$



J.S. Cotler et al., JHEP 2017

Beyond nearest neighbours

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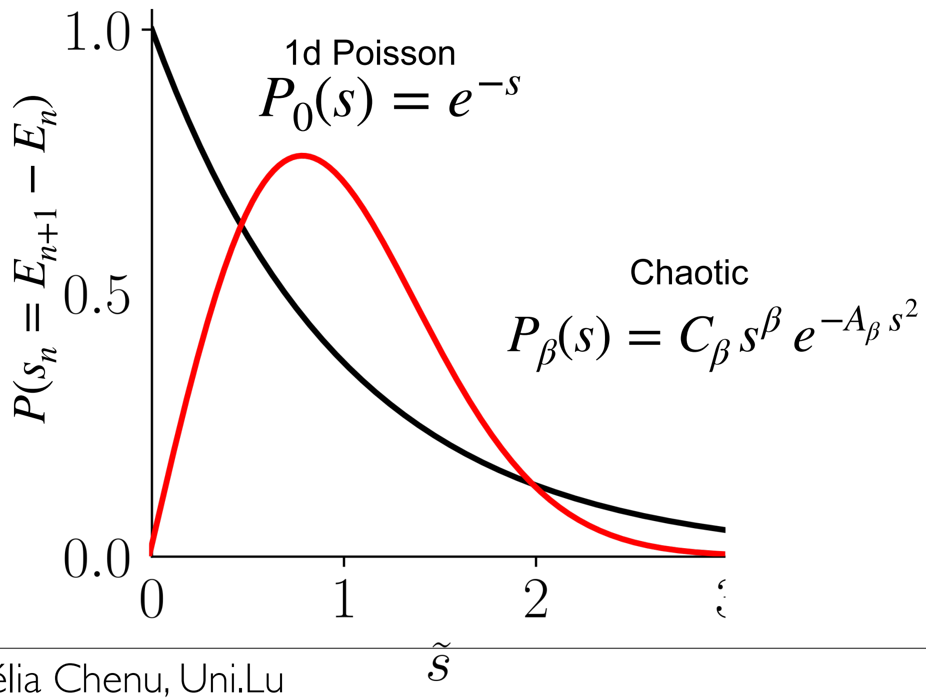
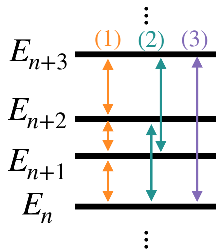


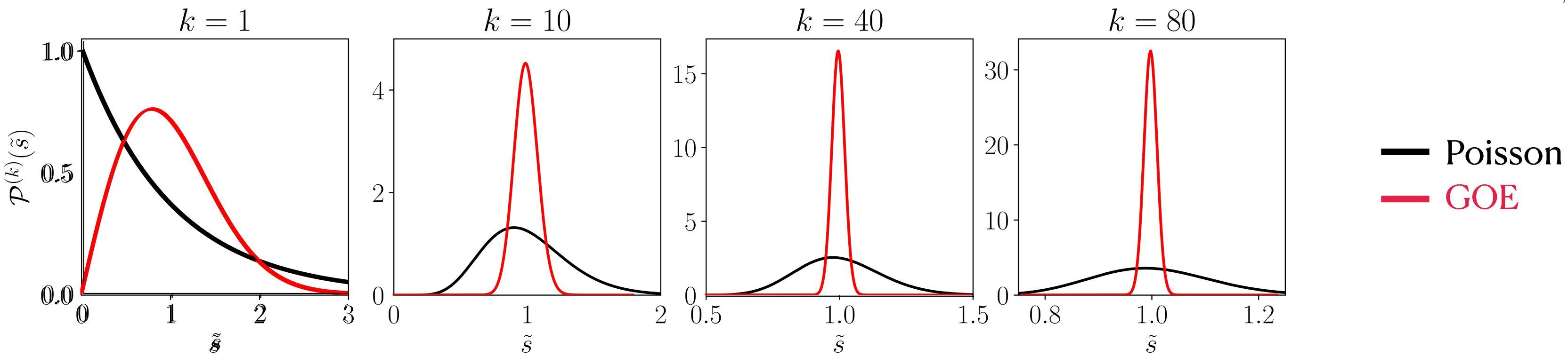
k-th neighbor Level spacings (knLS)



$$s_n = E_{n+1} - E_n$$
$$s_n^{(k)} = E_{n+k} - E_n$$
$$\tilde{s}_n^{(k)} = s_n^{(k)}/k$$

Engel, Main, Hunner, JPA (1998)
Abdul-Magd, Simbel PRE (1999)
Sakr, Nieminen PRE (2006)
Forrester, Comm MP (2009)
Tekur, Bhosale, Santhanam PRB (2018)
Rao, PRB (2020)

Beyond Wigner Surmise



NN (k=1): $P_\beta(s) = C_\beta\, s^\beta\, e^{-A_\beta\, s^2}$

$$P_\beta^{(k)}(s) \approx C_\alpha\, s^\alpha\, e^{-A_\alpha\, s^2}$$

with $\alpha = \frac{k(k+1)}{2}\beta + k - 1$ $\quad \omega_k = k\sqrt{\frac{\alpha}{\alpha+1}\langle \tilde{s}^2\rangle}$ $\quad \kappa^2 = \frac{\alpha+1}{2}$

$$P_\beta^{(k)}(\tilde{s}) \approx \tilde{s}^{-1}\frac{2}{\Gamma(\kappa^2)}\left(\kappa^2\frac{\tilde{s}^2}{\langle \tilde{s}^2\rangle}\right)^{\kappa^2} e^{-\kappa^2\frac{\tilde{s}^2}{\langle \tilde{s}^2\rangle}}$$

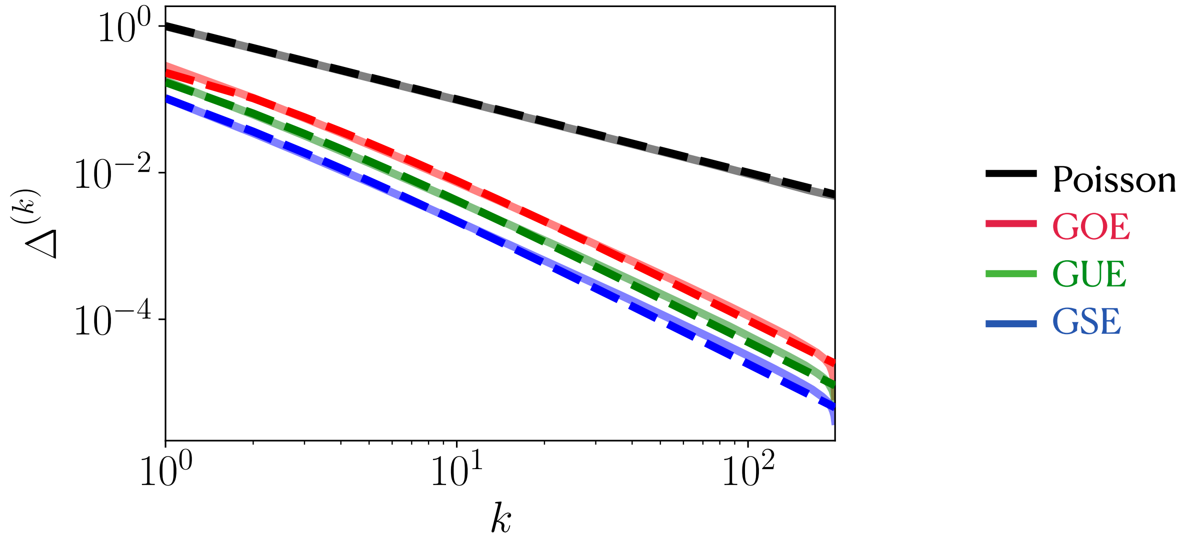
Variance of the kLS

Integrable
Poisson ensemble

$$\Delta_0^{(k)} = \frac{1}{k}$$

Chaotic
RMT

$$\Delta_\beta^{(k)} \equiv \langle \tilde{s}^2 \rangle - \langle \tilde{s} \rangle^2 = \frac{\alpha+1}{\alpha}\left(\frac{\omega_k}{k}\right)^2 - 1 \xrightarrow[k\to\infty]{} \frac{1}{\beta k^2}$$



R. Shir, P. Martinez-Azcona, AC, *arXiv*:2311.09292

k-th neighbor level spacing

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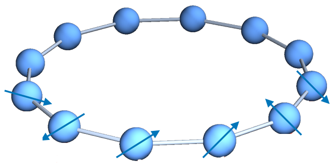
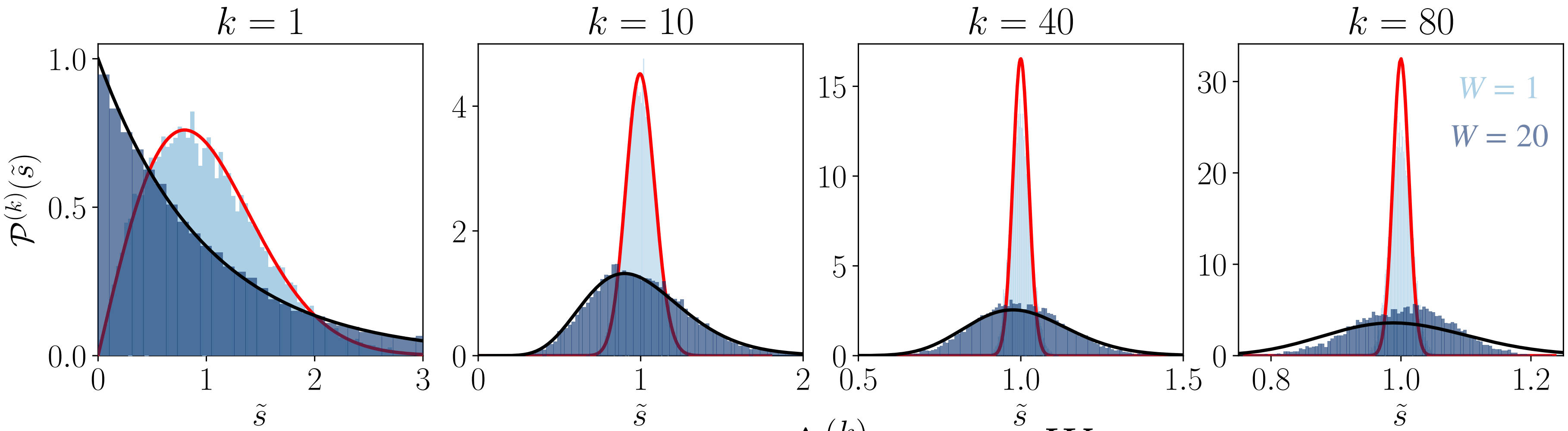
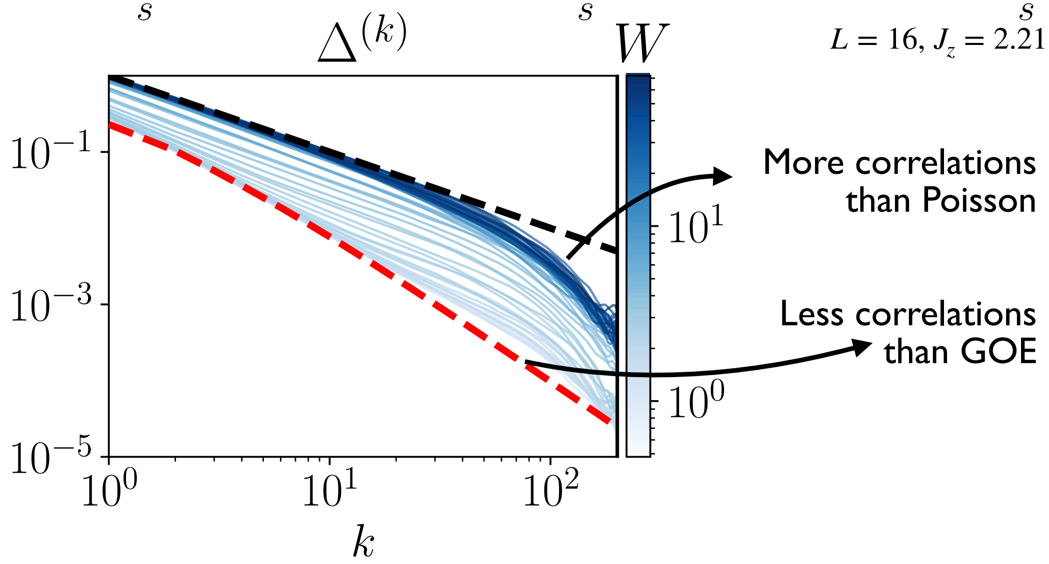




Variance of the kLS distribution: a refined measure of quantum chaos



Aurélia Chenu, Uni.Lu

arXiv:2311.09292

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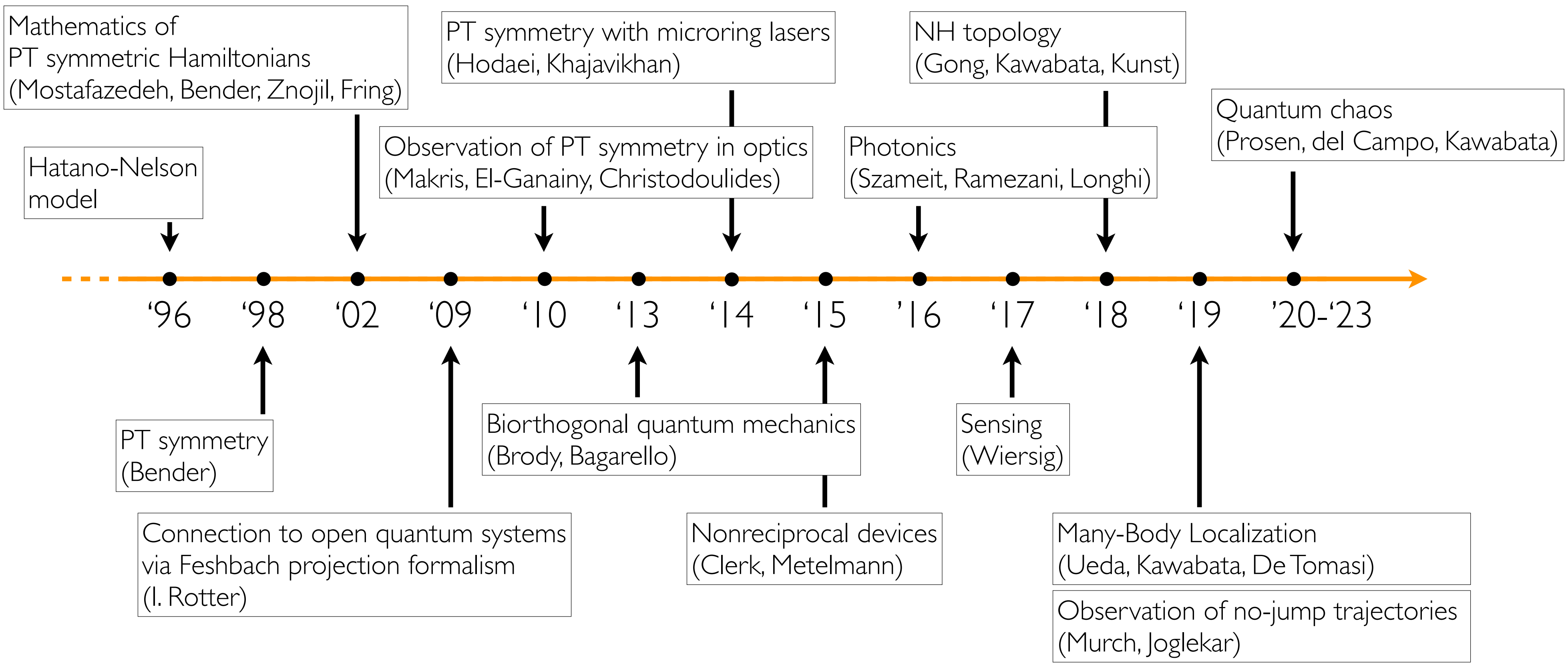
Non-Hermitian Physics



How to break Hermiticity

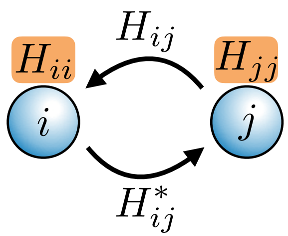
Hermite City (Dieuze, France)

- Let H be a general (many-body, interacting,…) Hamiltonian
- Let's write it in some basis $H_{ij} = \langle e_i | H | e_j \rangle$

$$H \sim \sum_{ij} H_{ij}\, \hat{c}_i^\dagger \hat{c}_j$$

single particle hopping
in a (possibly complicated) graph

Hermitian



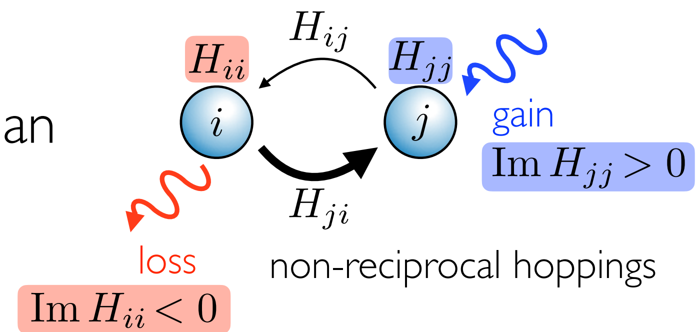
real on-site
energies

Two ways to break Hermiticity:

A. Complex diagonal terms (gain/loss)

B. Non-reciprocal hoppings

non-Hermitian



non-reciprocal hoppings

Non-Hermitian Physics

our approach:

Effective non-Hermitian Hamiltonian = GKSL equation conditioned to no jumps

$$\dot{\rho}_t = -i[\hat{H}, \rho_t] + \sum_j \mathcal{D}[\sqrt{\gamma_j}\hat{L}_j]\rho_t \qquad \mathcal{D}[\hat{\mathcal{O}}]\rho = \hat{\mathcal{O}}\rho\hat{\mathcal{O}}^\dagger - \frac{1}{2}\{\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}, \rho\}$$

Non-Hermitian Physics

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$$\dot{\rho}_t = -i[\hat{H}, \rho_t] + \sum_j \mathcal{D}[\sqrt{\gamma_j}\hat{L}_j]\rho_t \qquad \mathcal{D}[\hat{\mathcal{O}}]\rho = \hat{\mathcal{O}}\rho\hat{\mathcal{O}}^\dagger - \tfrac{1}{2}\{\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}, \rho\}$$

rearrange

$$\dot{\rho}_t = -i(\hat{H}_{\text{eff}}\rho_t - \rho_t\hat{H}_{\text{eff}}^\dagger) + \sum_j \gamma_j \hat{L}_j \rho_t \hat{L}_j^\dagger \qquad \hat{H}_{\text{eff}} = \hat{H} - \frac{i}{2}\sum_j \gamma_j \hat{L}_j^\dagger \hat{L}_j$$

Minganti et al., PRA 2019

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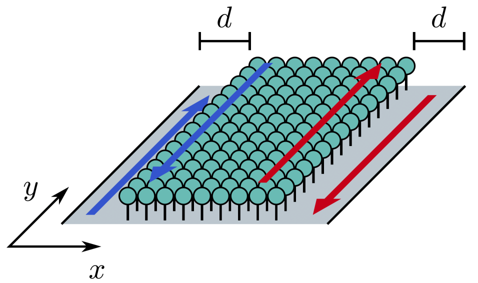
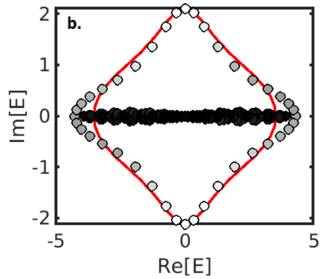
Current topics (~2019 - now)

NH topology in quantum optics



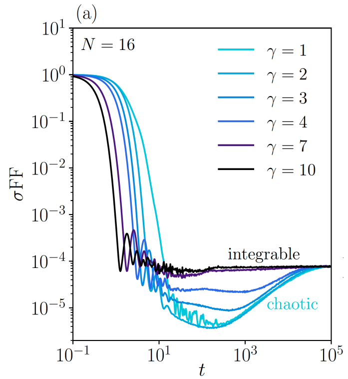
Roccati et al., Optica 2022
Gong et al., PRL 2022
Roccati et al., Nat Comm. 2023

Many-body NH topology



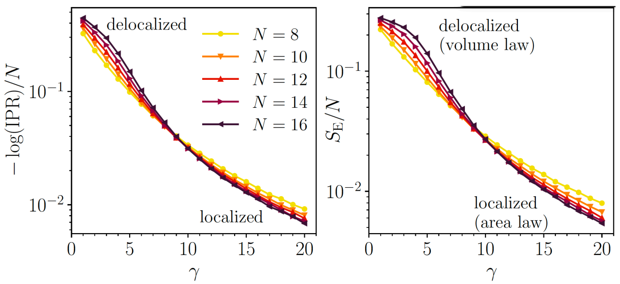
Kawabata et al., PRB 2022
Faugno and Ozawa, PRL 2022

Dissipative quantum chaos



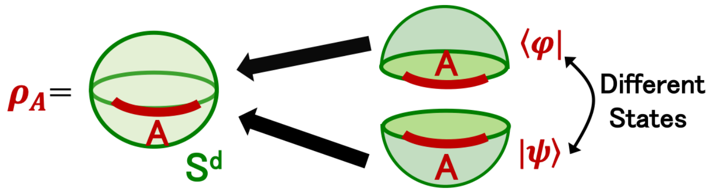
Sá et al., PRX 2020
Xu et al., PRB 2021
Cornelius et al., PRL 2022
Matsoukas-Roubeas et al., JHEP 2023
Kawabata et al., PRX Quantum 2023
Roccati et al., PRB letter 2024
Akemann et al., ArXiv 2024

NH many-body localization



Hamazaki et al., PRL 2019
De Tomasi and Khaymovich, PRB 2022
Roccati et al., PRB letter 2024

NH density matrices in dS/CFT



Doi et al., PRL 2023

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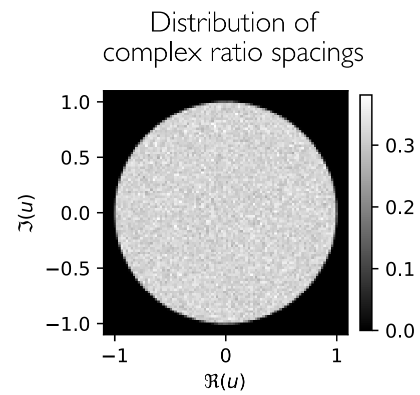
Models

d-Dimensional Poisson



	Hermitian ↔	non-Hermitian
Integrable	**1D Poisson**	
Chaotic (RMT)		

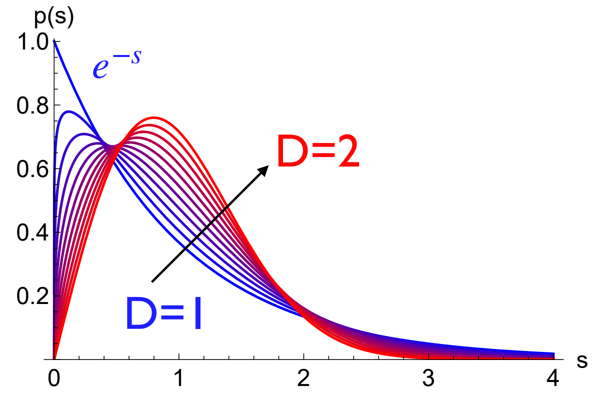
Models: d-Dimensional Poisson

NN spacing distribution

Haake

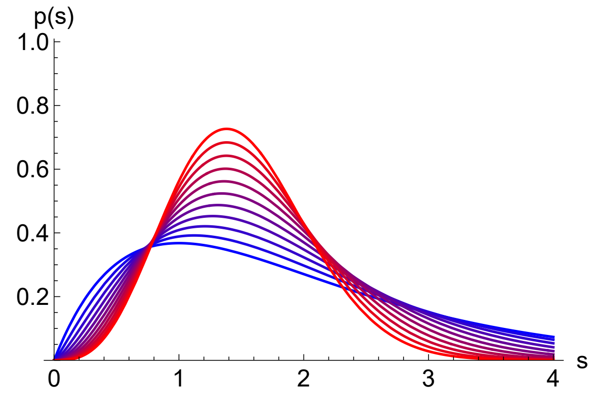
Sa et al., PRX 2020

$$P_D^{NN}(s) = D\,\Gamma\left(1+\frac{1}{D}\right)^D s^{D-1}\, e^{-\Gamma\left(1+\frac{1}{D}\right)^D s^D}$$



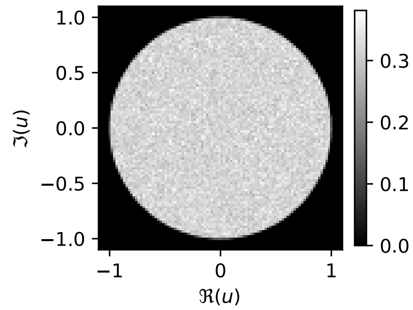
NNN spacing distribution

$$P_D^{NNN}(s) = D\,\Gamma\left(1+\frac{1}{D}\right)^{2D} s^{2D-1}\, e^{-\Gamma\left(1+\frac{1}{D}\right)^D s^D}$$



Models

Distribution of
complex ratio spacings



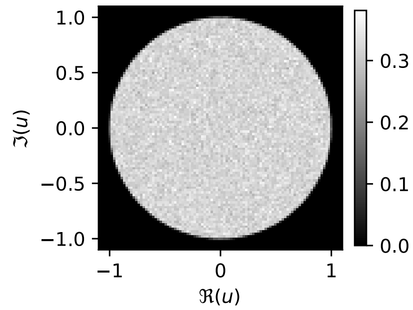
d-Dimensional Poisson

	Hermitian ⟷	non-Hermitian
Integrable	**1D Poisson**	**2D Poisson**
Chaotic (RMT)	**GOE** **GUE** **GSE**	

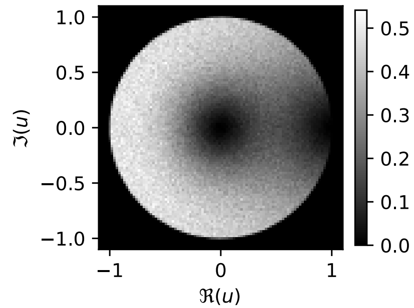
Models

d-Dimensional Poisson

Distribution of
complex ratio spacings



	Hermitian ⟷	non-Hermitian
Integrable	**1D Poisson**	**2D Poisson**
Chaotic (RMT)	**GOE** **GUE** **GSE**	**Class A (complex Ginibre ensemble)** **Class AI^{+} (complex symmetric)** **Class AII^{+} (complex self-dual)**



Among all 38 classes of NH random matrices, only 3 are relevant to capture local level spacing statistics in the bulk

Hamazaki et al., PRR 2020

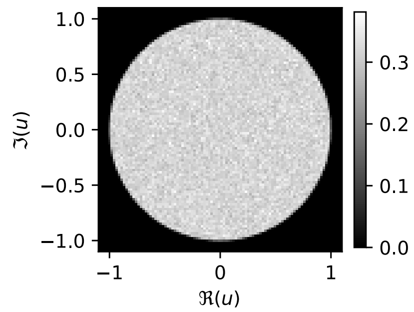
Models

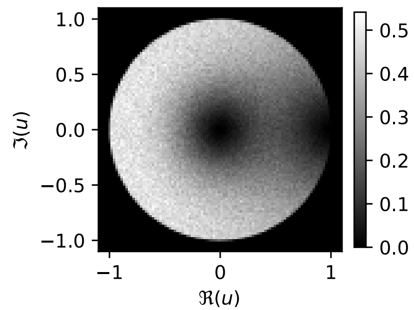
d-Dimensional Poisson

	Hermitian ↔	non-Hermitian
Integrable	**1D Poisson**	**2D Poisson**
Chaotic (RMT)	**GOE** **GUE** **GSE**	**Class A** **Class AI$^+$** **Class AII$^+$**

2d Coulomb gas f(β)

Distribution of complex ratio spacings



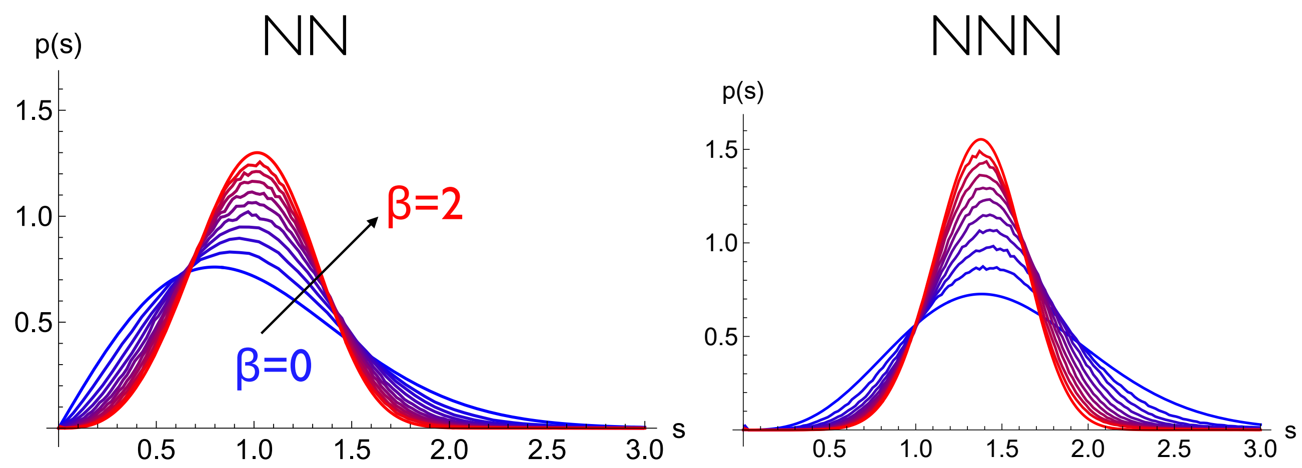


Models: 2d Coulomb gas f(β)

$z \in \mathbb{R}$ β=1; 2; 4 GOE, GUE, GSE

Joint distribution in the complex plane $z \in \mathbb{C}$

$$\mathscr{P}_{N,\beta}(z_1, z_2, \ldots, z_N) \propto \exp\left[-\sum_{i=1}^{N} |z_i|^2 + \frac{\beta}{2} \sum_{i \neq j}^{N} \ln |z_i - z_j| \right]$$



β=0 ↔ 2d Poisson process

Surmise at small β

β=2 ↔ Complex Ginibre ensemble (Class A)

β=1.4 ~ Class AI$^+$ for NN (and NNN)

Akemann et al., PRE 2022

β=2.6 ~ Class AII$^+$ for NN (and NNN)

Akemann et al., PRE 2022

Models: class AI$^+$ ~ 2dCG with β=1.4

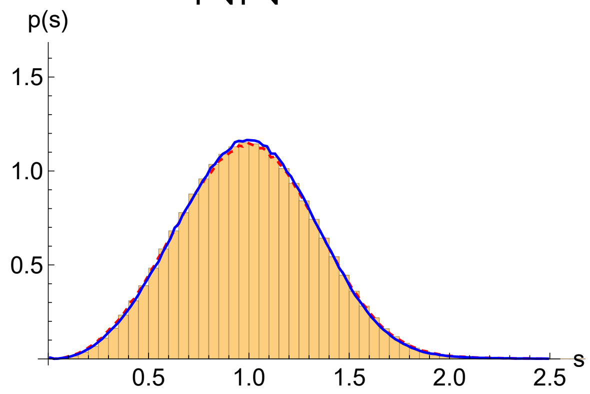
Gaussian distribution

$$P_N(J) \propto e^{-\mathrm{Tr}(JJ^\dagger)}$$

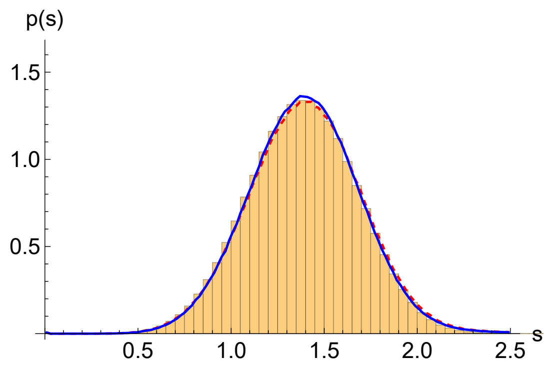
with a NxN symmetric matrix $J = J^T$

fitted	β	1.1	1.2	1.3	1.4	1.5	1.6
NN	σ	3.3	2.2	1.1	**0.9**	1.7	2.7
	d	1.7	1.2	0.6	**0.3**	0.7	1.2
NNN	σ	3.4	2.0	**0.8**	1.4	2.7	4.0
	d	3.0	1.8	0.8	**0.6**	1.7	2.7

NN



NNN



Methods: Comparison of two distributions

Standard deviation

$$\sigma = \left[\frac{1}{n}\sum_{j=1}^{n}[p_1(s_j) - p_2(s_j)]^2\right]^{\frac{1}{2}}$$

- ○ Cut the distribution into *n* bins
- ✓ Fast

Kolmogorov-Smirnov distance

$$d = \max_{x \geq 0} \; F_1(x) - F_2(x) \quad \in [0,1]$$

Cumulative distribution

- ✓ No binning
- ○ Slow

Methods: unfolding the spectra

Remove system-specific data

$$\rho(E) = \bar{\rho}(E) + \rho_{\text{fl}}(E)$$

1d: unique method Guhr et al., Phys. Rep. 1998

2d: different possible choices Akemann et al., PRL 2019

$$\rho(x, y) = \frac{1}{N} \sum_{i=1}^{N} \delta^{(2)}(z - z_i)$$

$$\downarrow$$

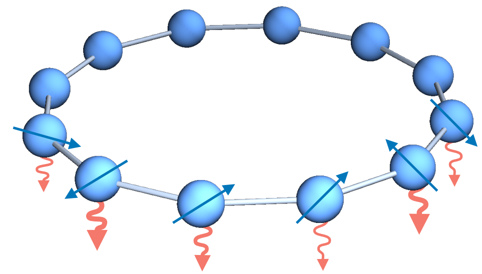
$$\bar{\rho}(x, y) = \frac{1}{2\pi\Sigma^2 N} \sum_{i=1}^{N} e^{-\frac{z - z_i \;\, 2}{2\Sigma^2}}$$

Unfolded spacings

$$s_i^{\prime NN} = \; z_i - z_i^{NN} \; \sqrt{\bar{\rho}(x_i, y_i)}$$

$$s_i^{\prime NNN} = \; z_i - z_i^{NNN} \; \sqrt{\bar{\rho}(x_i, y_i)}$$

Physical model:



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$$\hat{H} = J \sum_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z \right) - \frac{i}{2} \sum_i \gamma_i \left(\hat{S}_i^z + \frac{1}{2} \right)$$

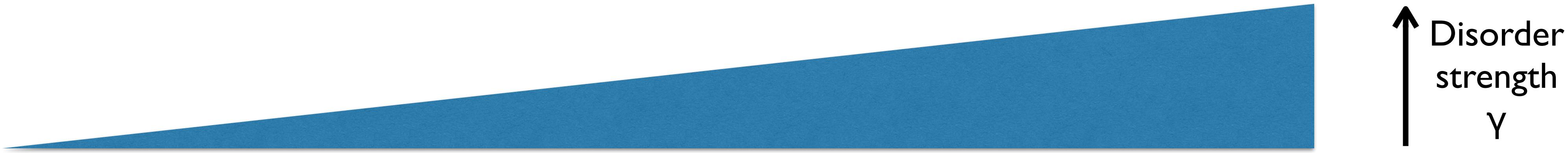
hopping — interaction

γ_i sampled in $[0, \gamma]$

- its Hermitian version $\gamma_i \to 2ih_i$ displays chaotic/integrable & localization crossovers
- effective non-Hermitian Hamiltonian from the GKSL equation

$$\dot{\rho} = -i[\hat{H}_{\mathrm{XXZ}}, \rho] + \mathcal{D}[\sqrt{\gamma_i} \hat{S}_i^-]\rho$$

- Complex symmetric (pseudo-Hermitian matrix $H = S\, H^\dagger S^{-1}$) ⟶ **Class AI$^+$**



Hermiticity breaking
From 1d to 2d poisson

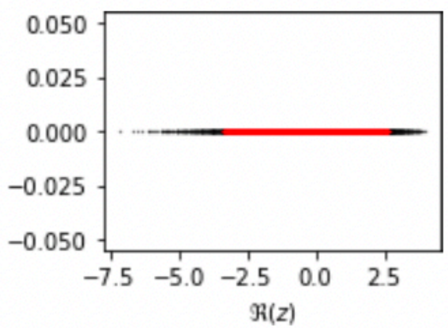
Integrability breaking
From 2dCG (β>0) to AI$^+$

Back to integrable
AI$^+$ to 2d poisson

d-Dimensional Poisson

$\gamma = 0.0$

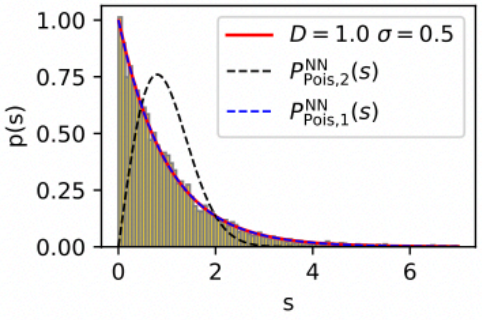
Eigenvalues



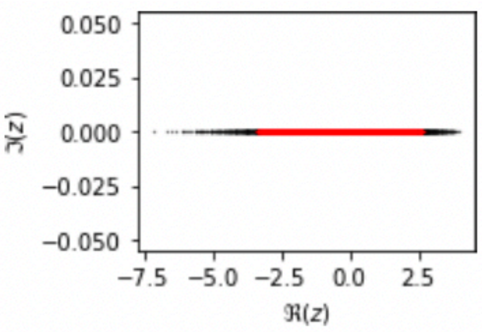
$\gamma = 0.0$

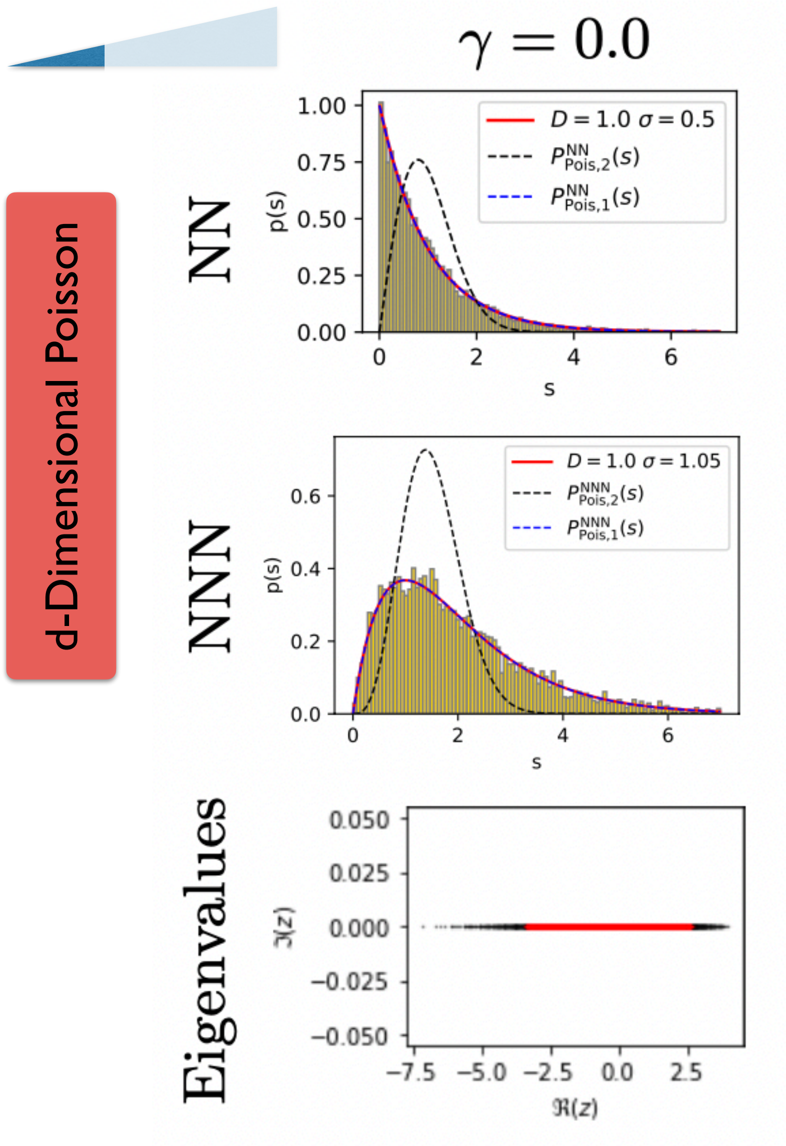
d-Dimensional Poisson

NN

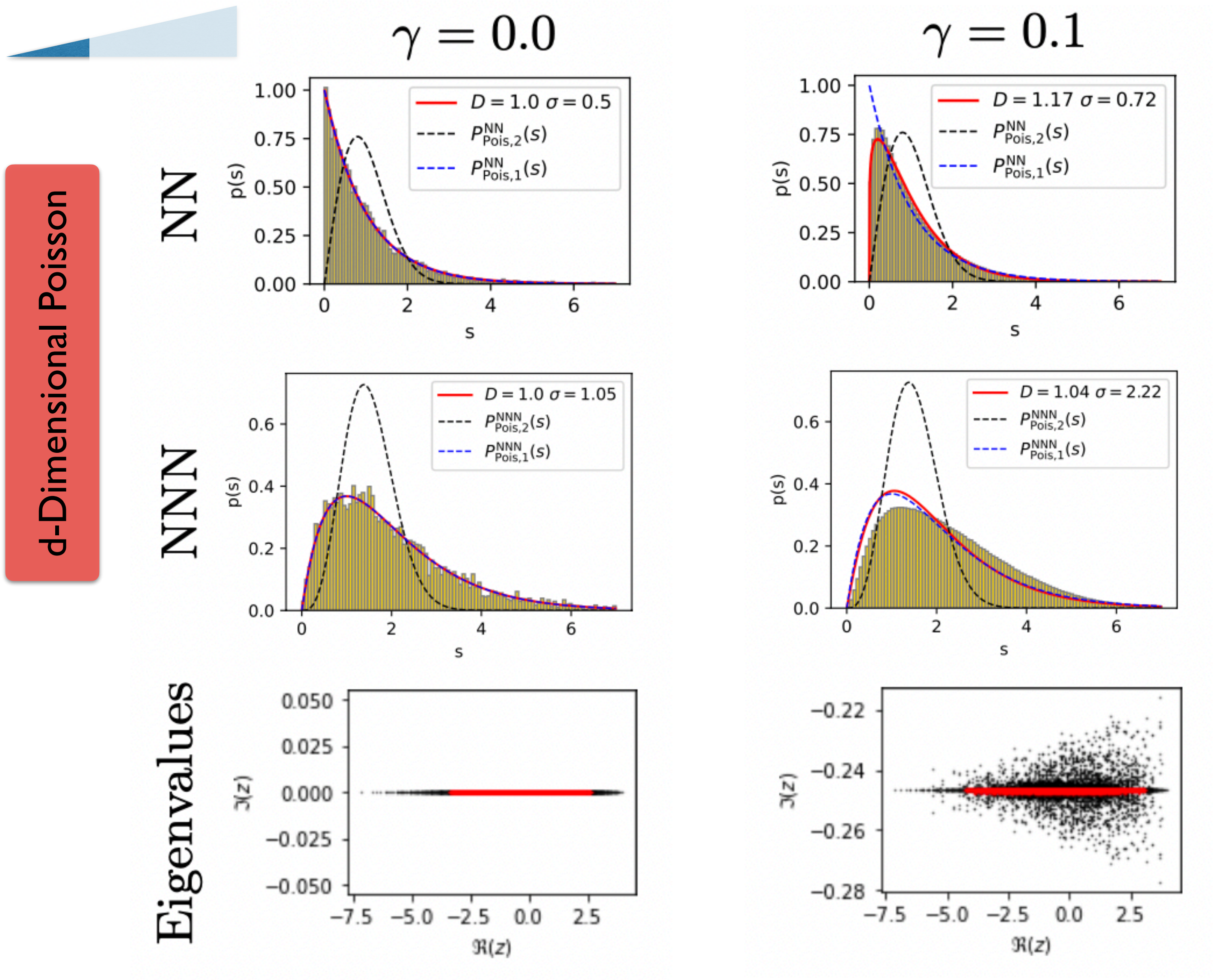


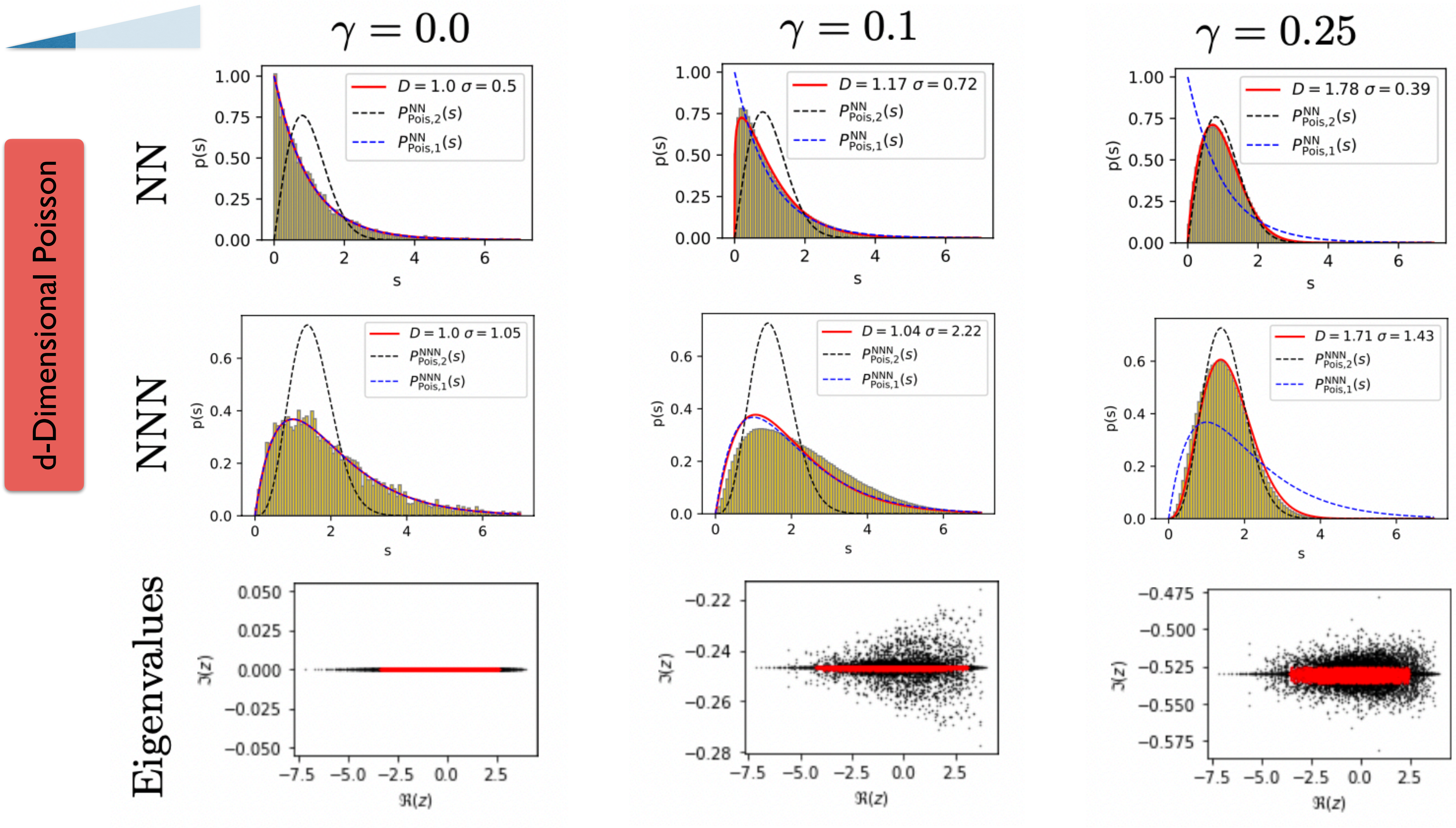
Eigenvalues





γ = 0.0
d-Dimensional Poisson
NN
NNN
Eigenvalues
D = 1.0 σ = 0.5
P^NN_Pois,2(s)
P^NN_Pois,1(s)
p(s)
s
D = 1.0 σ = 1.05
P^NNN_Pois,2(s)
P^NNN_Pois,1(s)
ℑ(z)
ℜ(z)

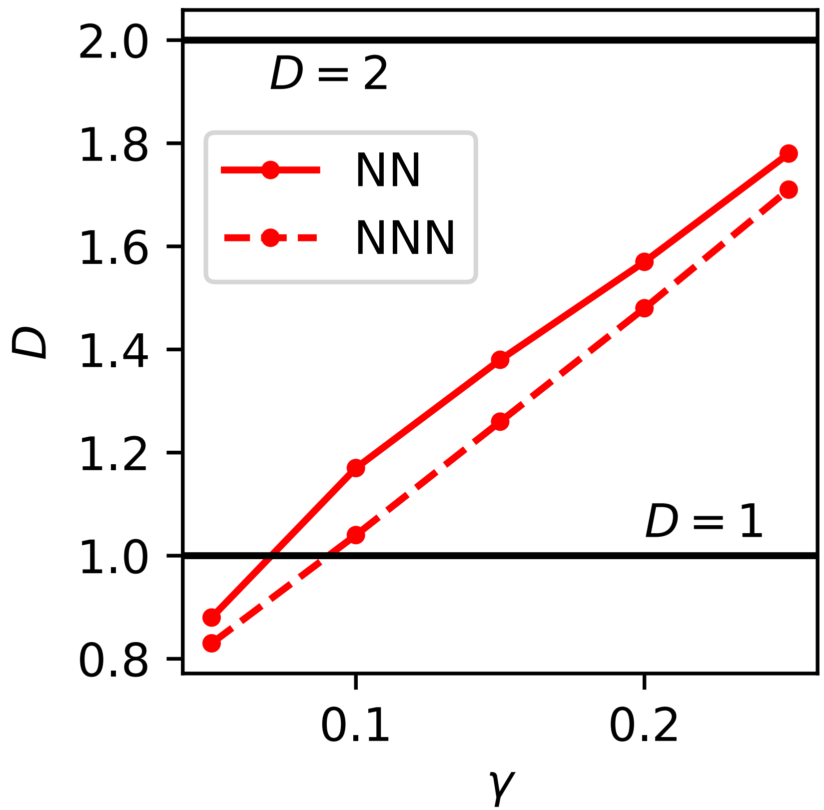

d-Dimensional Poisson
γ = 0.0
γ = 0.1
NN
NNN
Eigenvalues
D = 1.0 σ = 0.5
D = 1.17 σ = 0.72
D = 1.0 σ = 1.05
D = 1.04 σ = 2.22
p(s)
s
ℜ(z)
ℑ(z)

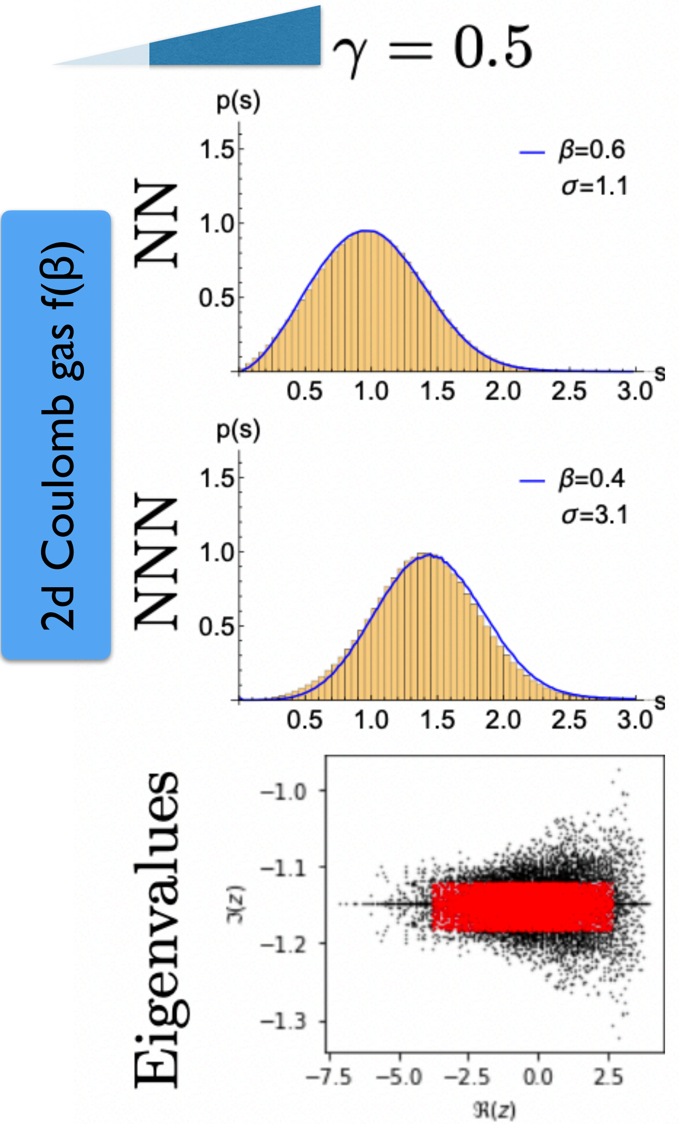


d-Dimensional Poisson
$\gamma = 0.0$
$\gamma = 0.1$
$\gamma = 0.25$
NN
NNN
Eigenvalues
$D = 1.0\ \sigma = 0.5$
$D = 1.17\ \sigma = 0.72$
$D = 1.78\ \sigma = 0.39$
$D = 1.0\ \sigma = 1.05$
$D = 1.04\ \sigma = 2.22$
$D = 1.71\ \sigma = 1.43$
$P^{NN}_{Pois,2}(s)$
$P^{NN}_{Pois,1}(s)$
$P^{NNN}_{Pois,2}(s)$
$P^{NNN}_{Pois,1}(s)$
p(s)
s
$\Im(z)$
$\Re(z)$

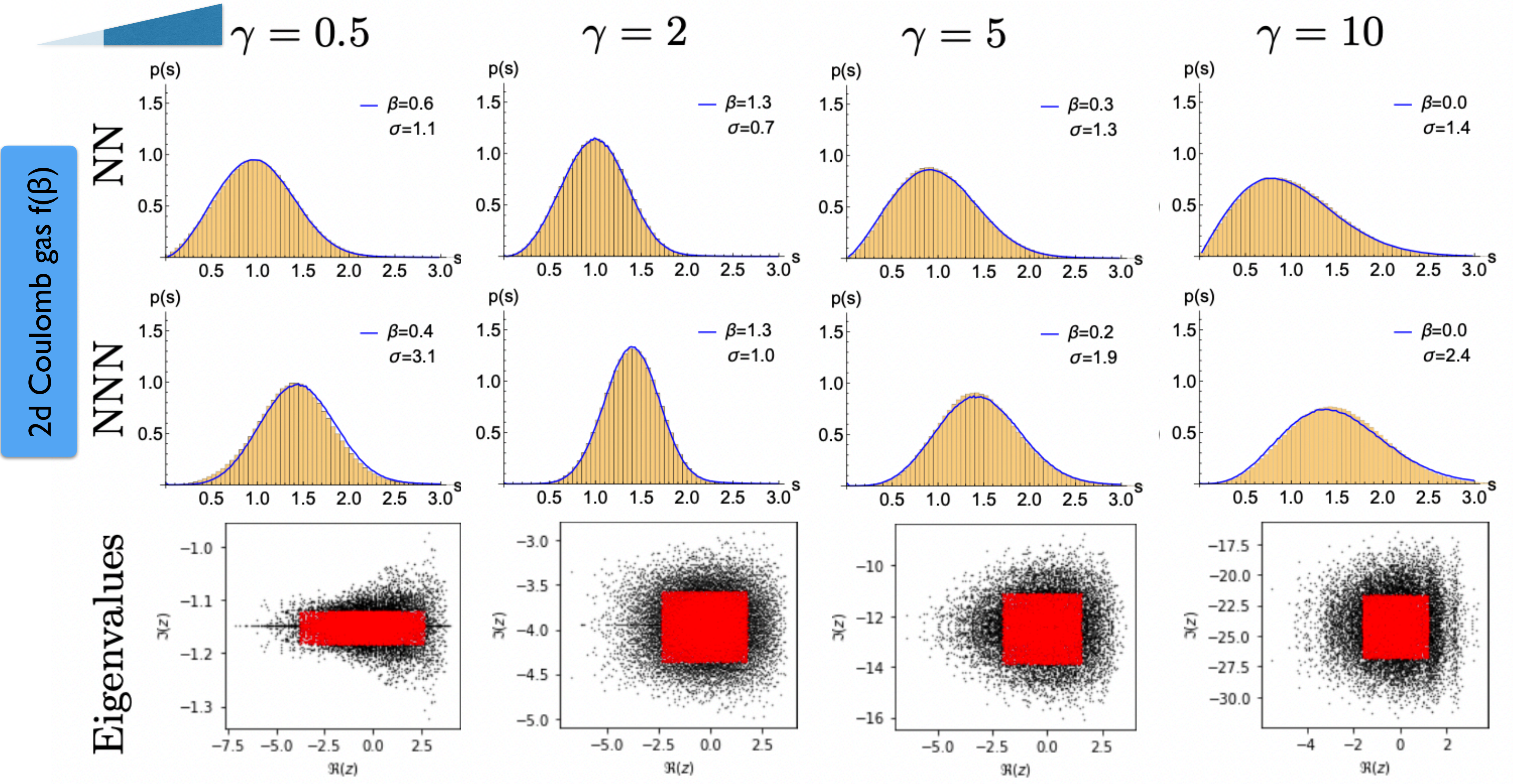
Hermiticity breaking

From 1d to 2d poisson





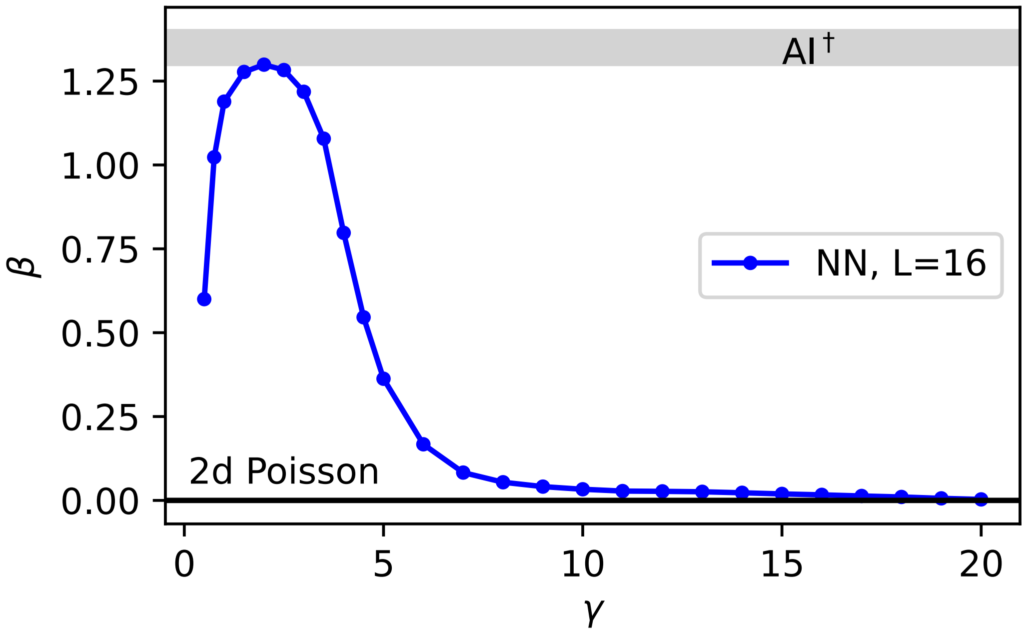
$\gamma = 0.5$
2d Coulomb gas f(β)
NN
NNN
Eigenvalues
p(s)
β=0.6
σ=1.1
β=0.4
σ=3.1
s
ℑ(z)
ℜ(z)



2d Coulomb gas f(β)
γ = 0.5
γ = 2
γ = 5
γ = 10
NN
NNN
Eigenvalues
p(s)
s
β=0.6
σ=1.1
β=1.3
σ=0.7
β=0.3
σ=1.3
β=0.0
σ=1.4
β=0.4
σ=3.1
β=1.3
σ=1.0
β=0.2
σ=1.9
β=0.0
σ=2.4
ℜ(z)
ℑ(z)

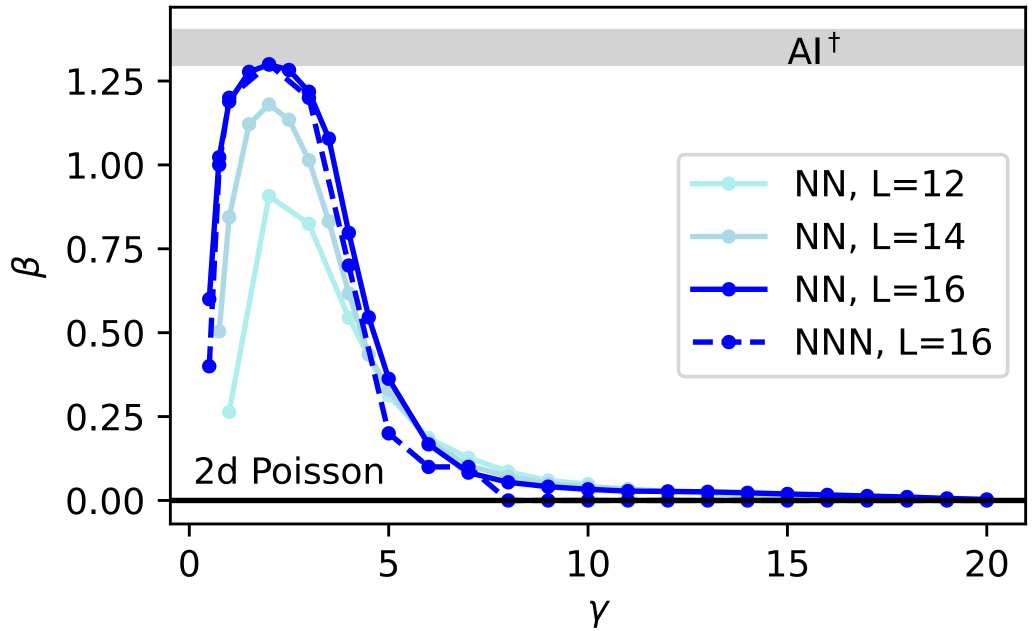
Integrability breaking

From 2dCG to AI^{+} to 2d poisson



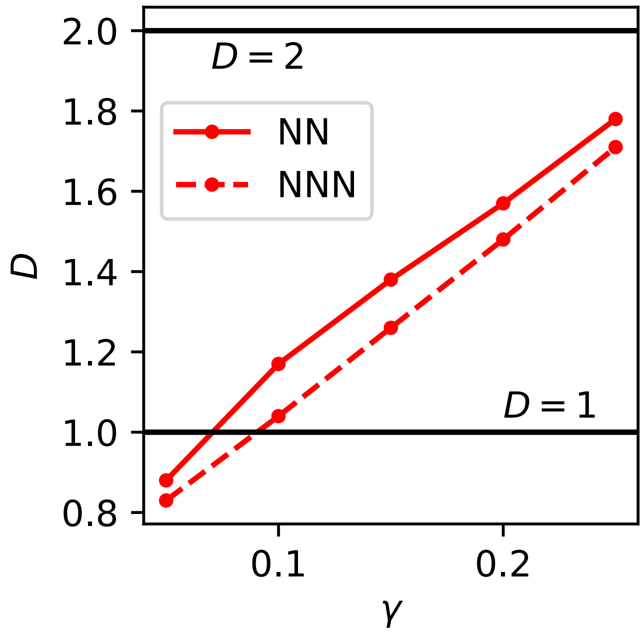
Integrability breaking

From 2dCG to AI⁺ to 2d poisson

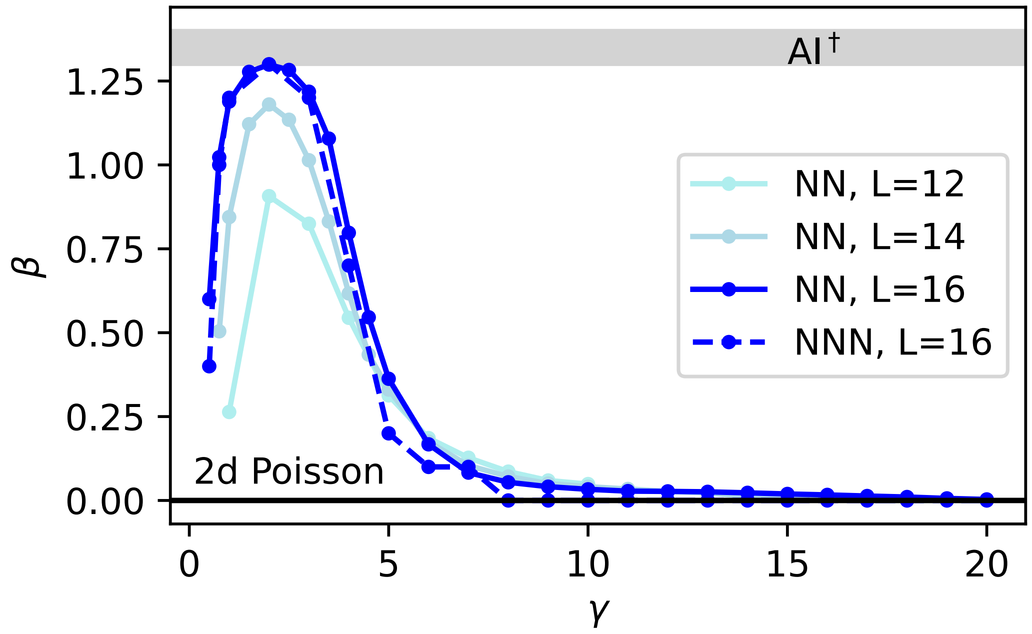


Two transitions

Hermiticity breaking
From 1d to 2d poisson



Integrability breaking
From 2dCG to AI† to 2d poisson



Summary

what we did:

- introduce tools for signature of quantum chaos (Hermitian setup)
- diagnostic tool for dissipative quantum chaos (NH setup - singular form factor)
- study the MBL of singular vectors
- toy model: XXZ + random losses (disorder = non-Hermiticity)
  - Rich complex spectral statistics
  - characterised the NN and NNN spacing distributions
  - Witness Hermiticity and integrability breaking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Letter

Diagnosing non-Hermitian many-body localization and quantum chaos via singular value decomposition

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Many-Body Localization

Single-body case: Anderson localization

hopping   disorder

$$\hat{H} = J \sum_{\langle ij \rangle} (|i\rangle\langle j| + \text{ H.c.}) + \sum_i h_i |i\rangle\langle i|$$

h_i sampled in $[-h, h]$

Scardicchio and Thiery arXiv:1710.01234

Many-Body Localization

Single-body case: Anderson localization

$$\hat{H} = \underbrace{J \sum_{\langle ij \rangle} (|i\rangle\langle j| + \text{ H.c.})}_{\text{hopping}} + \underbrace{\sum_i h_i \, |i\rangle\langle i|}_{\text{disorder}}$$

hopping

disorder

h_i sampled in $[-h, h]$

Two (extreme) regimes:

Scardicchio and Thiery arXiv:1710.01234

Many-Body Localization

Single-body case: Anderson localization

hopping disorder

$$\hat{H} = J \sum_{\langle ij \rangle} (|i\rangle\langle j| + \text{ H.c.}) + \sum_i h_i \, |i\rangle\langle i|$$

sampled in $[-h, h]$

Two (extreme) regimes:

<u>no disorder</u>

- delocalized eigenmodes
- non-random spectrum

<u>no hoppings</u>

- localized eigenmodes $|i\rangle$
- Poisson level spacing

Scardicchio and Thiery arXiv:1710.01234

Many-Body Localization

Single-body case: Anderson localization

hopping | disorder

$$\hat{H} = J \sum_{\langle ij \rangle} (|i\rangle\langle j| + \text{H.c.}) + \sum_i h_i \, |i\rangle\langle i|$$

sampled in $[-h, h]$

Two (extreme) regimes:

<u>no disorder</u>

- delocalized eigenmodes
- non-random spectrum

<u>no hoppings</u>

- localized eigenmodes $|i\rangle$
- Poisson level spacing

Scardicchio and Thiery arXiv:1710.01234

Many-Body Localization

Single-body case: Anderson localization

$$\hat{H} = \underbrace{J \sum_{\langle ij \rangle} (|i\rangle\langle j| + \text{ H.c.})}_{\text{hopping}} + \underbrace{\sum_i h_i \, |i\rangle\langle i|}_{\text{disorder}}$$

h_i sampled in $[-h, h]$

Two (extreme) regimes:

no disorder	in between	no hoppings
• delocalized eigenmodes	$\langle \mathbf{x} \| w_n \rangle \sim e^{-\|\mathbf{x}-\mathbf{x}_n\|/\xi}$	• localized eigenmodes $\|i\rangle$
• non-random spectrum	• Wigner-Dyson level spacing	• Poisson level spacing

Scardicchio and Thiery arXiv:1710.01234

Many-Body Localization

Many-body case: existence of a MBL phase is debated   De Luca and Scardicchio, EPL 2013

(vague) definition of MBL: non-thermalization under unitary dynamics

thermalization (expected for interacting many-body systems):

$$\lim_{t\to\infty}\lim_{L\to\infty}\langle\psi(t)|O_A|\psi(t)\rangle = \lim_{L\to\infty}\mathrm{Tr}\left(O_A\frac{e^{-\beta(H-\mu\hat{N})}}{Z}\right)$$

local observable (O_A) starting from an initial MB state ($|\psi(t)\rangle$)

Scardicchio and Thiery arXiv:1710.01234

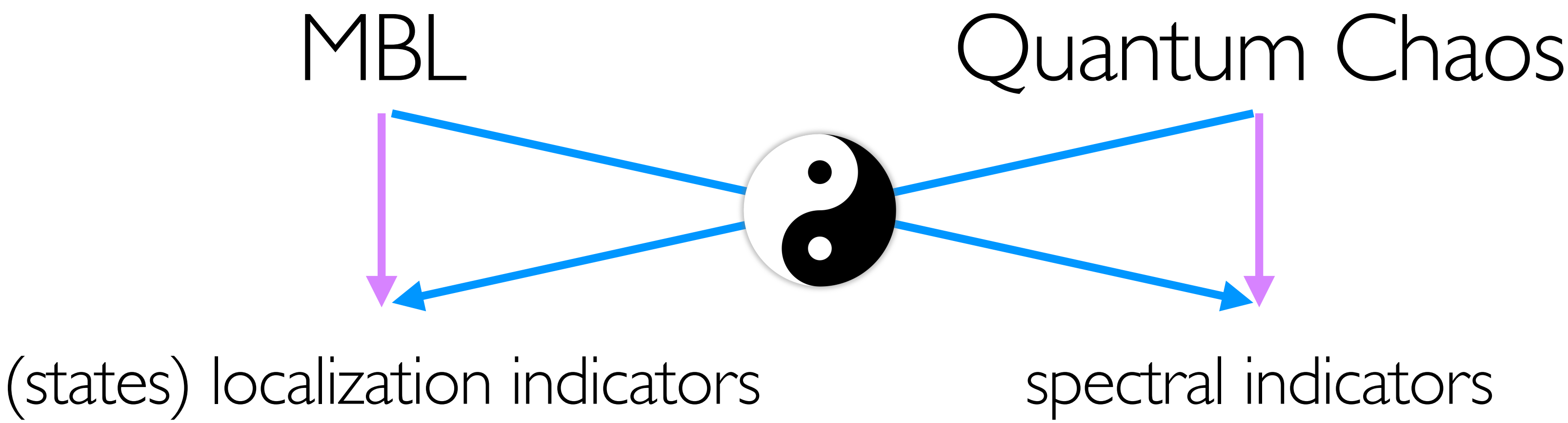
MBL

(states) localization indicators

Quantum Chaos

spectral indicators

actually


MBL
Quantum Chaos
(states) localization indicators
spectral indicators

State of the art

restoring/making sense of Hermitian results with non-Hermitian Hamiltonians

<u>MBL</u>

typically: resilience to dissipation

- Non-Hermitian MBL: Hamazaki et al., PRL 2019
  localization of right eigenstates

- Lindbladian MBL: Hamazaki et al., arXiv:2206.02984
  localization of Lindbladian eigenstates

notable exception:

De Tomasi and Khaymovich, arXiv:2311.00019

<u>Quantum Chaos</u>

- spectral form factor for open systems

$$\mathrm{SFF}(t) = \langle\psi_0|\underbrace{\Lambda_t(|\psi_0\rangle\langle\psi_0|)}_{\text{quantum map}}\underbrace{|\psi_0\rangle}_{\text{coherent Gibbs state}}$$

quantum map

coherent Gibbs state

Xu et al., PRB 2021
Cornelius et al., PRL 2022
Matsoukas-Roubeas et al., JHEP 2023

- complex r parameter: Sá et al., PRX 2020
  distribution of amplitude and phase

…recently: singular values statistics

Kawabata et al., PRX Quantum 2023

Biorthogonal quantum mechanics

- Right/Left eigenstates $$H\left|\psi_k^R\right\rangle = E_k\left|\psi_k^R\right\rangle \qquad \left\langle\psi_k^L\right| H = E_k\left\langle\psi_k^L\right|$$
- Left eigenstates are not in general H.c. of the right ones $$\left\langle\psi_k^L\right| \neq \left(\left|\psi_k^R\right\rangle\right)^\dagger$$
- Bi-orthogonality $$\left\langle\psi_k^{R/L}\middle|\psi_{k'}^{R/L}\right\rangle \neq \delta_{kk'} \qquad \left\langle\psi_k^L\middle|\psi_{k'}^R\right\rangle \propto \delta_{kk'}$$
- Completeness $$\sum_k \left|\psi_k^{R/L}\right\rangle\!\left\langle\psi_k^{R/L}\right| \neq \mathbb{1} \qquad \sum_k \frac{\left|\psi_k^R\right\rangle\!\left\langle\psi_k^L\right|}{\left\langle\psi_k^L\middle|\psi_k^R\right\rangle} = \mathbb{1}$$

Brody J. Phys. A 2013

Plot twist: the SVD

Eigen- vs Singular Value Decomposition of a non-Hermitian Hamiltonian:

	when?
Eigen-	diagonaliz. (no EPs)
Singular-	always

Plot twist: the SVD

Eigen- vs Singular Value Decomposition of a non-Hermitian Hamiltonian:

	when?	decomposition
Eigen-	diagonaliz. (no EPs)	$\hat{H} = \sum_n E_n \|R_n\rangle\langle L_n\|$ $\hat{H}\|R_n\rangle = E_n \|R_n\rangle$ $\langle L_n\| \hat{H} = E_n \langle L_n\|$
Singular-	always	$\hat{H} = \sum_n \sigma_n \|u_n\rangle\langle v_n\|$ $\hat{H}\|v_n\rangle = \sigma_n \|u_n\rangle$ $\langle u_n\| \hat{H} = \sigma_n \langle v_n\|$

Plot twist: the SVD

Eigen- vs Singular Value Decomposition of a non-Hermitian Hamiltonian:

	when?	decomposition	-values
Eigen-	diagonaliz. (no EPs)	$\hat{H} = \sum_n E_n \|R_n\rangle\langle L_n\|$ $\hat{H}\,\|R_n\rangle = E_n\,\|R_n\rangle$ $\langle L_n\|\,\hat{H} = E_n\,\langle L_n\|$	$E_n \in \mathbb{C}$
Singular-	always	$\hat{H} = \sum_n \sigma_n \|u_n\rangle\langle v_n\|$ $\hat{H}\,\|v_n\rangle = \sigma_n\,\|u_n\rangle$ $\langle u_n\|\,\hat{H} = \sigma_n\,\langle v_n\|$	$\sigma_n \geq 0$

Plot twist: the SVD

Eigen- vs Singular Value Decomposition of a non-Hermitian Hamiltonian:

	when?	decomposition	-values	-vectors
Eigen-	diagonaliz. (no EPs)	$\hat{H} = \sum_n E_n \|R_n\rangle\langle L_n\|$ $\hat{H}\|R_n\rangle = E_n \|R_n\rangle$ $\langle L_n\| \hat{H} = E_n \langle L_n\|$	$E_n \in \mathbb{C}$	$\|R_n\rangle$ $(\|L_n\rangle)$ non-orthonormal but biorthonormal
Singular-	always	$\hat{H} = \sum_n \sigma_n \|u_n\rangle\langle v_n\|$ $\hat{H}\|v_n\rangle = \sigma_n \|u_n\rangle$ $\langle u_n\| \hat{H} = \sigma_n \langle v_n\|$	$\sigma_n \geq 0$	$\|v_n\rangle$ $(\|u_n\rangle)$ orthonormal → physical states

Plot twist: the SVD

Eigen- vs Singular Value Decomposition of a non-Hermitian Hamiltonian:

	when?	decomposition	-values	-vectors	completeness
Eigen-	diagonaliz. (no EPs)	$\hat{H} = \sum_n E_n \|R_n\rangle\langle L_n\|$ $\hat{H}\|R_n\rangle = E_n \|R_n\rangle$ $\langle L_n\| \hat{H} = E_n \langle L_n\|$	$E_n \in \mathbb{C}$	$\|R_n\rangle$ $(\|L_n\rangle)$ non-orthonormal but biorthonormal	$\mathbb{1} = \sum_n \|R_n\rangle\langle L_n\|$
Singular-	always	$\hat{H} = \sum_n \sigma_n \|u_n\rangle\langle v_n\|$ $\hat{H}\|v_n\rangle = \sigma_n \|u_n\rangle$ $\langle u_n\| \hat{H} = \sigma_n \langle v_n\|$	$\sigma_n \geq 0$	$\|v_n\rangle$ $(\|u_n\rangle)$ orthonormal → physical states	$\mathbb{1} = \sum_n \|v_n\rangle\langle v_n\|$

Plot twist: the SVD

Eigen- vs Singular Value Decomposition of a non-Hermitian Hamiltonian:

	when?	decomposition	-values	-vectors	completeness	spectral theorem
Eigen-	diagonaliz. (no EPs)	$\hat{H} = \sum_n E_n \|R_n\rangle\langle L_n\|$ $\hat{H}\|R_n\rangle = E_n \|R_n\rangle$ $\langle L_n\| \hat{H} = E_n \langle L_n\|$	$E_n \in \mathbb{C}$	$\|R_n\rangle$ $(\|L_n\rangle)$ non-orthonormal but biorthonormal	$\mathbb{1} = \sum_n \|R_n\rangle\langle L_n\|$	$f(\hat{H}) = \sum_n f(E_n) \|R_n\rangle\langle L_n\|$
Singular-	always	$\hat{H} = \sum_n \sigma_n \|u_n\rangle\langle v_n\|$ $\hat{H}\|v_n\rangle = \sigma_n \|u_n\rangle$ $\langle u_n\| \hat{H} = \sigma_n \langle v_n\|$	$\sigma_n \geq 0$	$\|v_n\rangle$ $(\|u_n\rangle)$ orthonormal → physical states	$\mathbb{1} = \sum_n \|v_n\rangle\langle v_n\|$	$f(\hat{H}) \neq \sum_n f(\sigma_n) \|u_n\rangle\langle v_n\|$

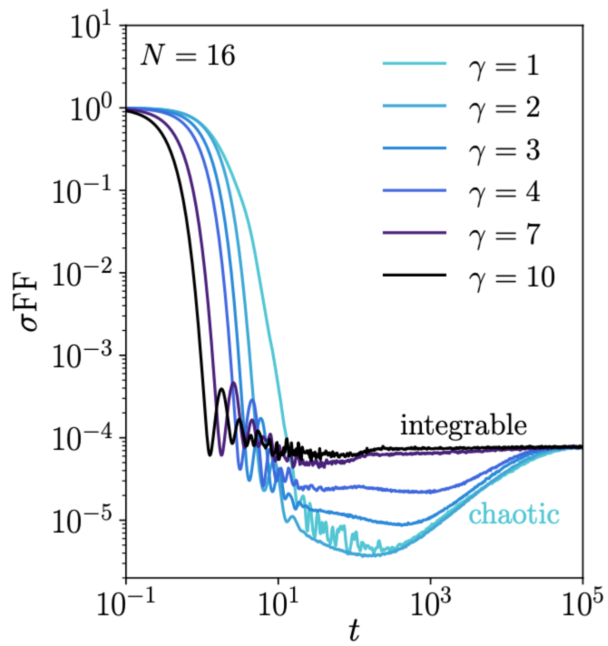
Dissipative Quantum chaos

Dynamical signature: singular form factor (σFF)

$$\mathrm{SFF}(t) = \left| \frac{1}{D} \sum_n e^{-iE_n t} \right|^2$$

$$\downarrow$$

$$\sigma\mathrm{FF}(t) = \left| \frac{1}{D} \sum_n e^{-i\sigma_n t} \right|^2$$



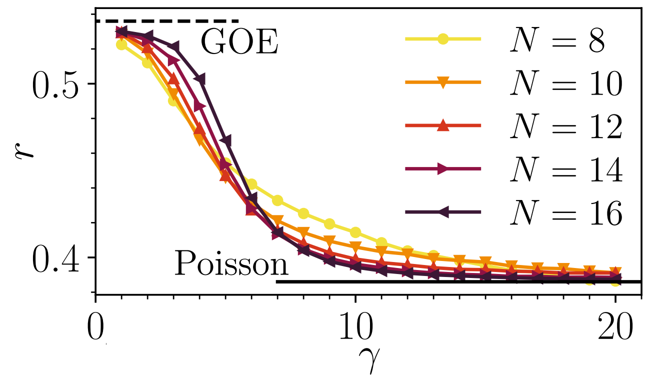
Singular values statistics

$$s_n = \sigma_{n+1} - \sigma_n$$

r parameter = average of

$$r_n = \min(s_{n+1}, s_n)/\max(s_{n+1}, s_n)$$

introduced in Kawabata et al., PRX Quantum 2023



cleaner results wrt complex r statistics

Dissipative MBL of singular vectors

inverse participation ratio (IPR)

= ensemble average of

$$\sum_{k=1}^{D} |\langle e_k | v_n \rangle|^4 / D$$

singular vectors

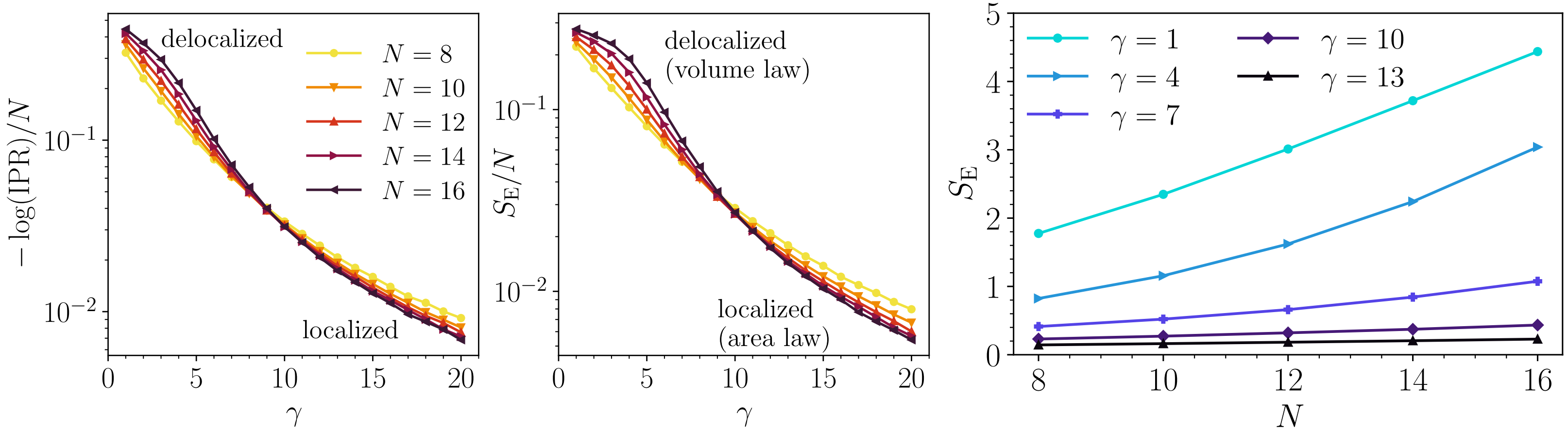
computational basis

entanglement entropy (S_E)

= ensemble average of

$$S_E = -\operatorname{tr} \rho_A \log \rho_A$$

$\rho_A = \mathrm{Tr}_B(|\phi\rangle\langle\phi|)$ $\quad A \cup B$ bipartition

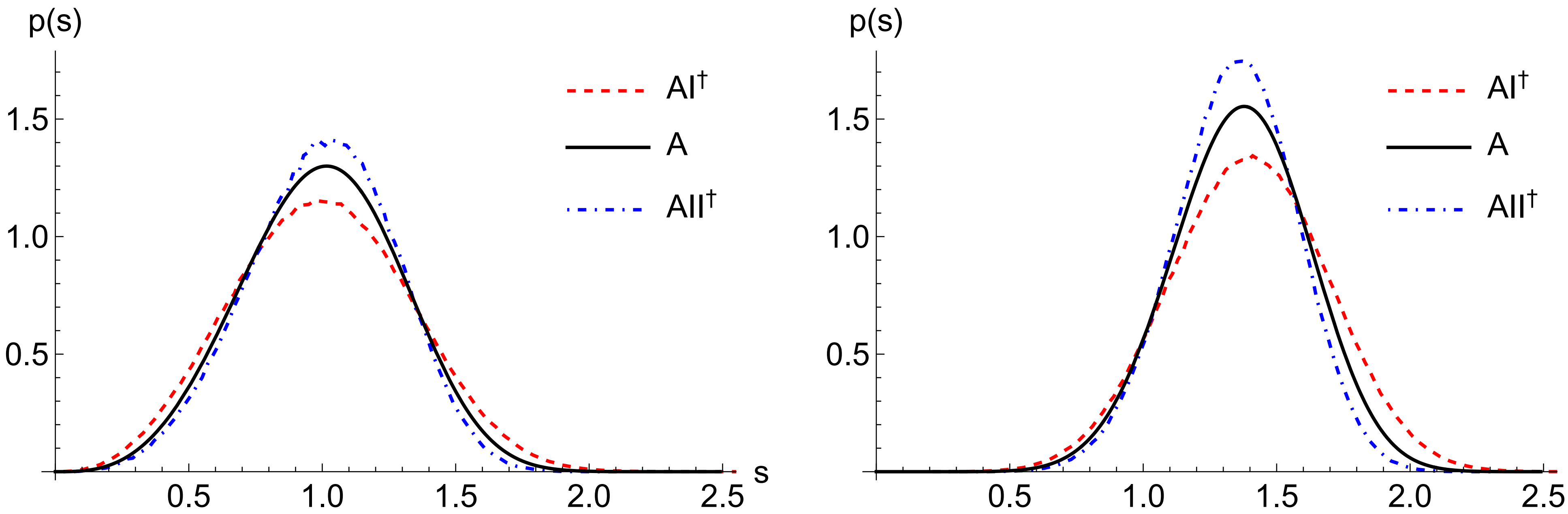


volume law

area law

FR et al., *arXiv*:2311.16229

- *Main ingredients:*
  - Hermitian / non-Hermitian systems
  - Quantum Chaos / Integrability
  - Many-Body Localization (MBL)
- *Main tools:*
  - SVD
  - Unfolding 2D spectra
  - Comparing spectral distributions
- *Results:*
  - Tools to characterize deviation from ideal models
  - dissipative chaos & MBL via SVD
  - Breaking of integrability and Hermiticity


p(s)
1.5
1.0
0.5
0.5
1.0
1.5
2.0
2.5
s
AI†
A
AII†
p(s)
1.5
1.0
0.5
0.5
1.0
1.5
2.0
2.5
s
AI†
A
AII†

Methods: complex ratio spacing

Remove system-specific data

$$u_i = \frac{z_i^{NN} - z_i}{z_i^{NNN} - z_i}$$

1d Oganesyan and Huse, PRB 2007

2d Sa et al., PRX 2020

↓