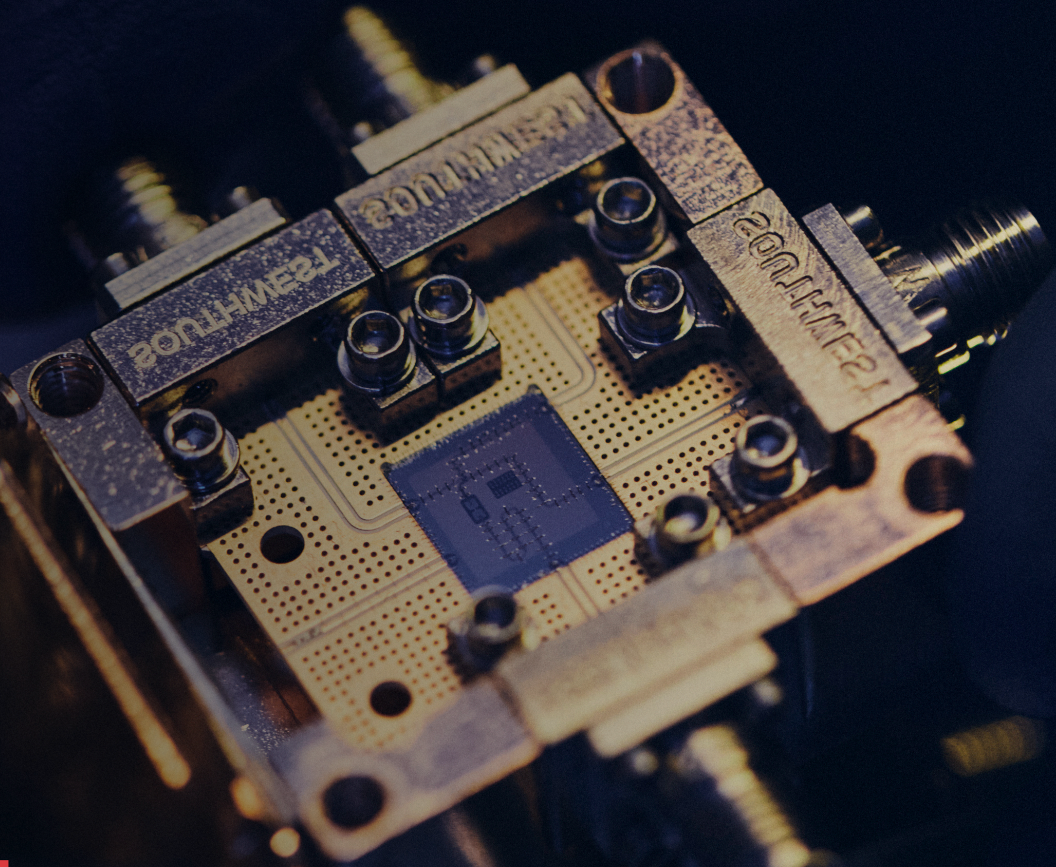
Chaos in open quantum systems

Fabrizio Minganti | 01/10/2024, Lausanne

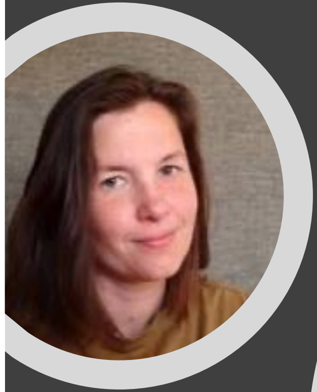


With a little help from my friends



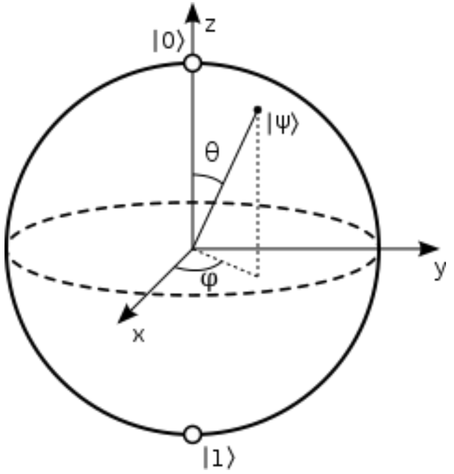







Setting the stage

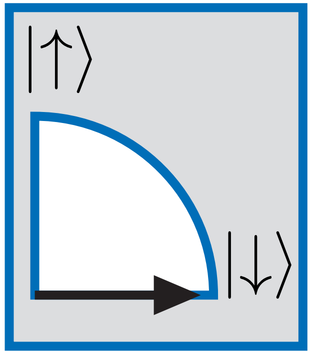
Quantum mechanics



First ingredient: a quantum system

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

Second ingredient: measurement



- The measurement returns a real number
- After the measurement, the system is in some state
- Measuring twice-in-a-row gives the same result

$$\hat{M} = \sum_m m\hat{P}_m$$

Measurement Operator

Outcome

Projection $\hat{P}_m^2 = \hat{P}_m$

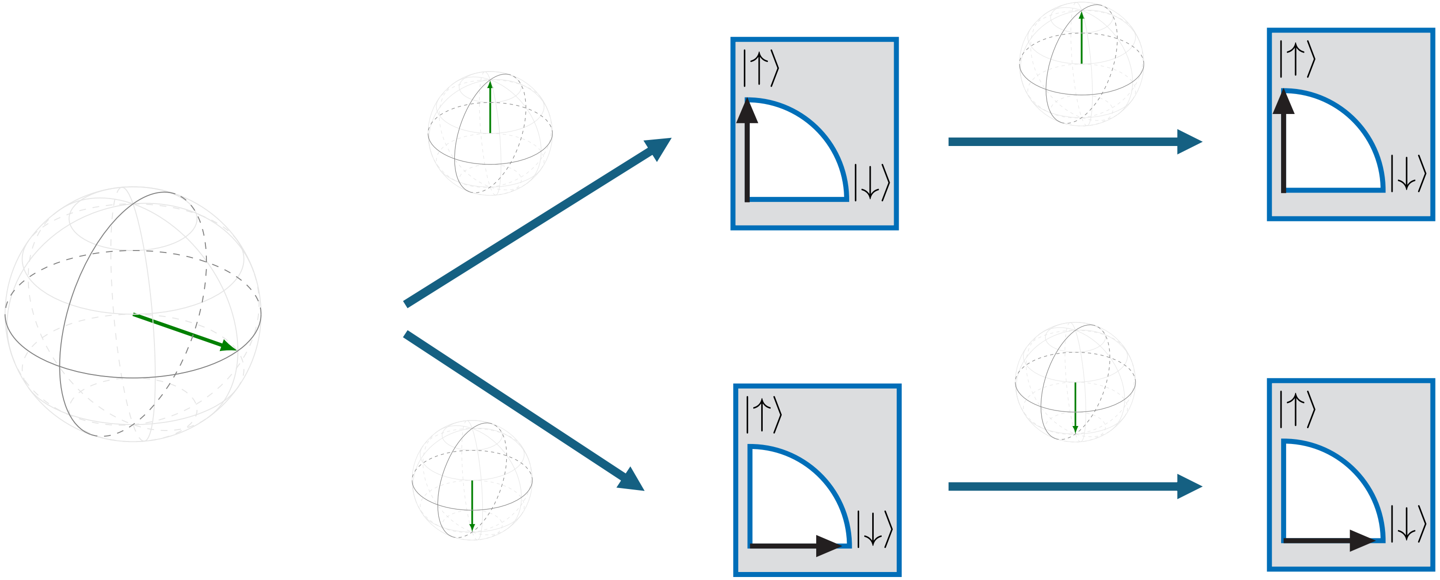
Properties:

- hermitian
- complete: $\sum_i \hat{P}_m = \hat{\mathbb{I}}$
- orthogonal: $\hat{P}_m \hat{P}_n = \delta_{mn} \hat{P}_m$

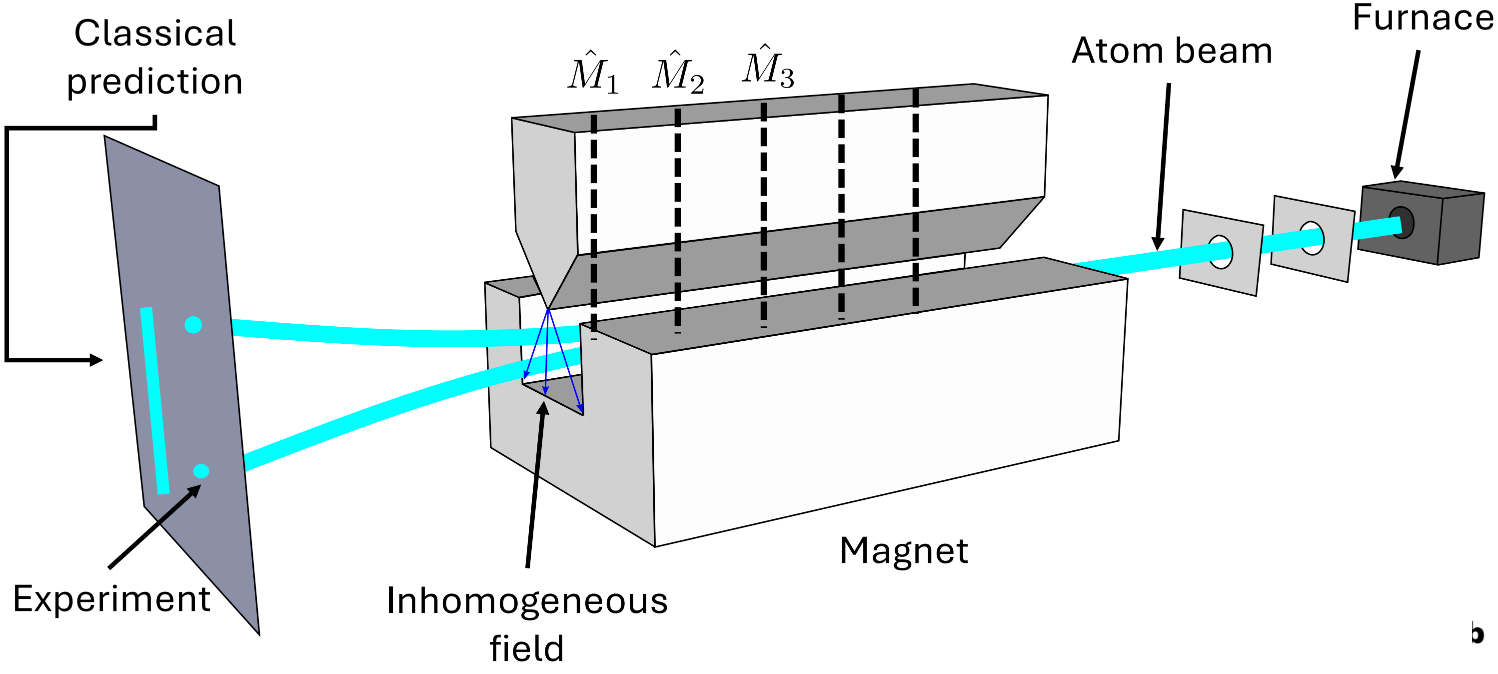
Measurements

After the measurement the state is $\frac{\hat{P}_m|\psi\rangle}{\sqrt{p(m)}}$

with probability $p(m) = \left\langle \psi \middle| \hat{P}_m \middle| \psi \right\rangle$

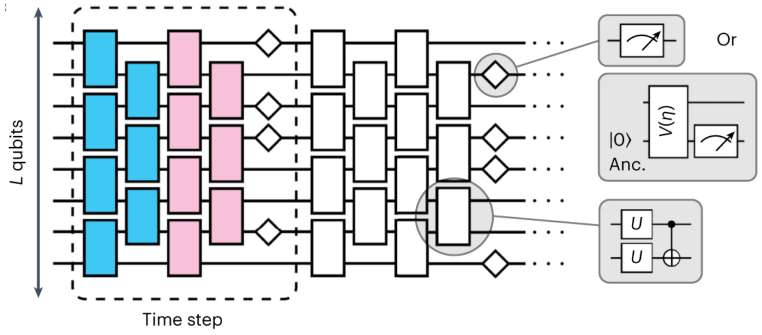


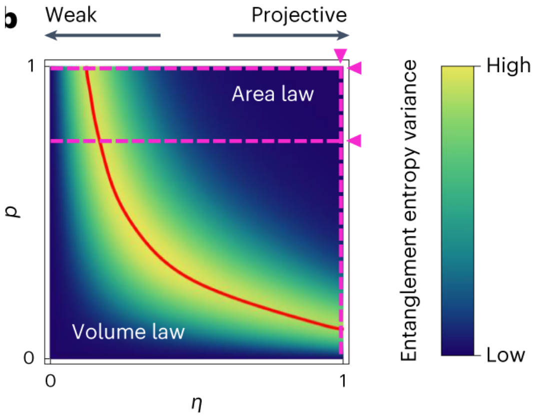
Stern-Gerlach



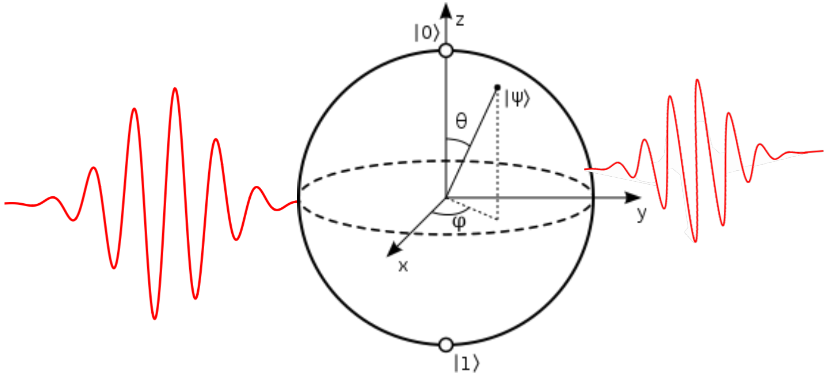
Measurements: nonclassical dynamics

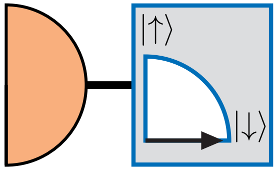
Measurement-induced transition





Photodetection





Q: How can we describe this apparatus

t

Initial state

Entangling

Measuring

$$|\Psi(t)\rangle = |\psi(t)\rangle|\theta(t)\rangle = \hat{U}(T_1)\left(|\psi(t)\rangle|\theta(t)\rangle\right)$$

Qubit **Photon**

$$|\Psi_r(t+T)\rangle = \frac{|r\rangle\langle r|\hat{U}(T_1)|\psi(t)\rangle|\theta(t)\rangle}{\sqrt{p_r}}$$

Q: What if we do not care about the photon?

Generalized measures

$$|\psi_r(t+T)\rangle = \frac{\hat{M}_r|\psi(t)\rangle}{\sqrt{p_r}}$$

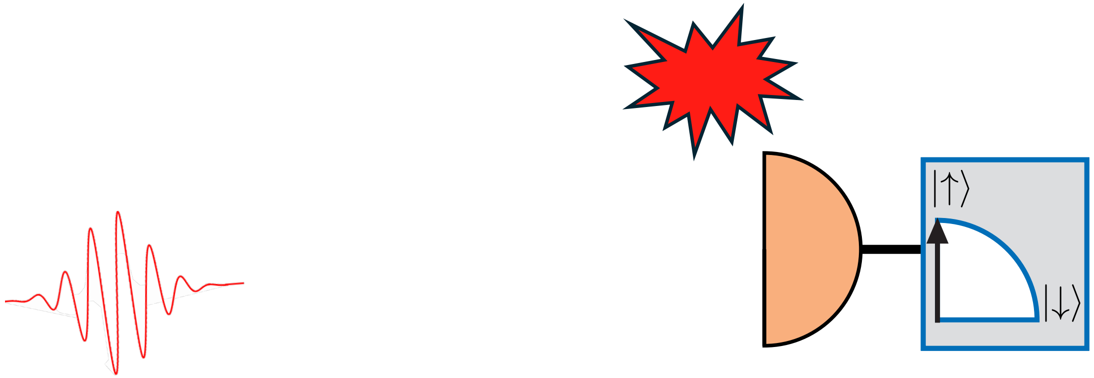
- **The measurement returns a real number**
- **After the measurement, the system is in some state**
- ~~**Measuring twice-in-a-row gives the same result**~~

Q: What are the properties of $\widehat{M}_r$

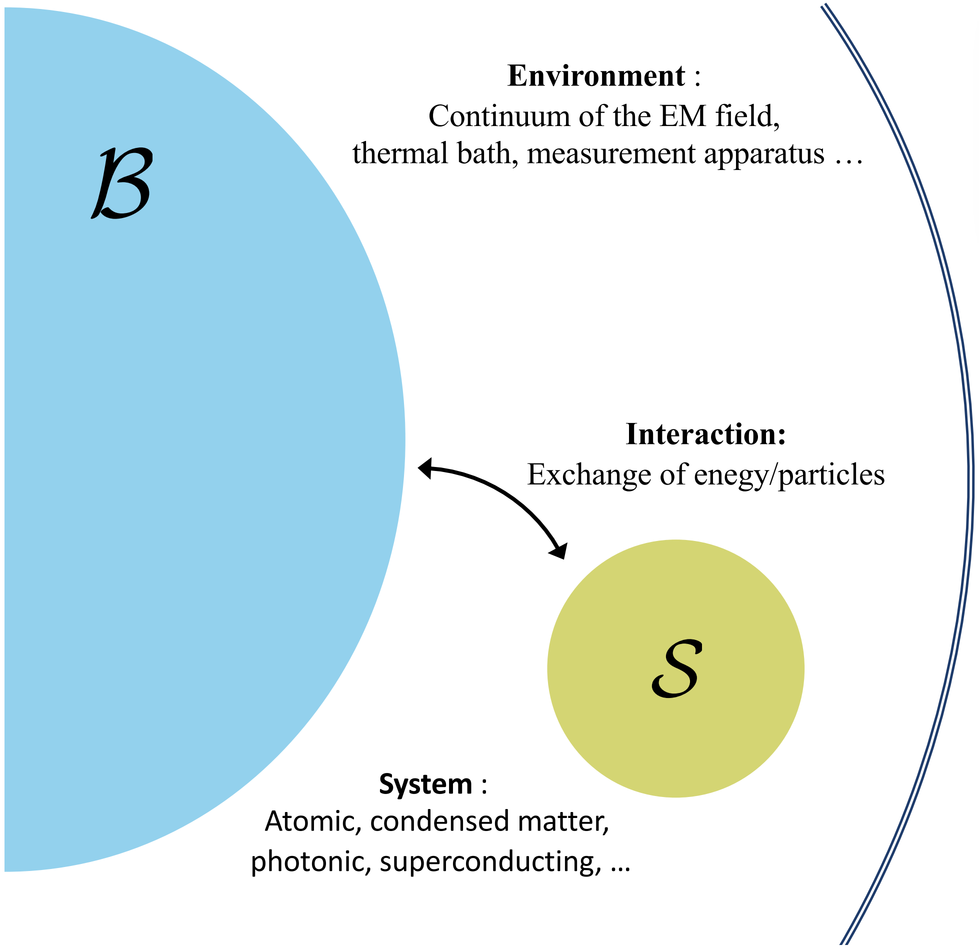
$$p_r = \left\langle \psi(t) \middle| \hat{M}_r^\dagger \hat{M}_r \middle| \psi(t) \right\rangle$$

$$\sum_r \hat{M}_r^\dagger \hat{M}_r = \hat{1}$$

Kraus operators



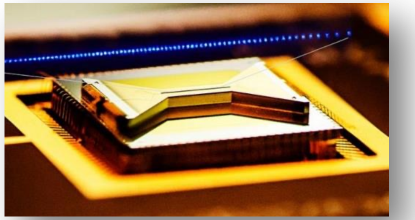
The environment



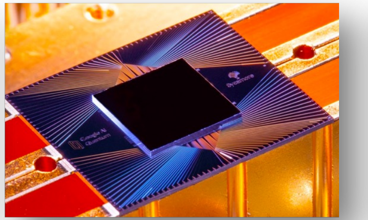
Environment :
Continuum of the EM field,
thermal bath, measurement apparatus …

Interaction:
Exchange of enegy/particles

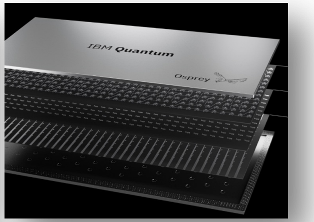
System :
Atomic, condensed matter,
photonic, superconducting, …



[Trapped ion chip @IonQ]



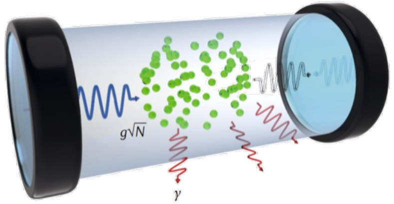
[Sycamore chip@Google]

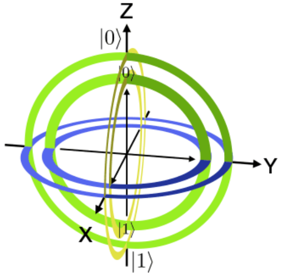


[Osprey chip@IBM]

The environment induces:

- Loss and gain of particles
- Loss of information

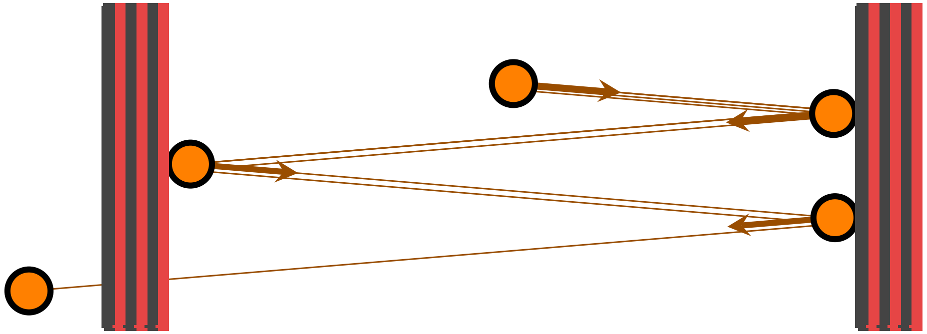




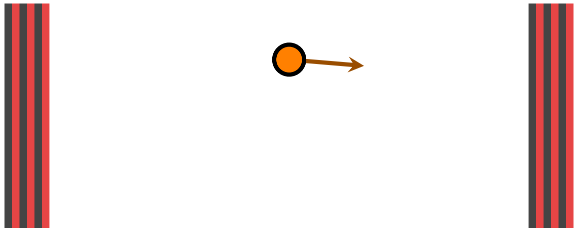
GOAL: Control, manipulate, an preserve many-body quantum states

Environment effect (1)

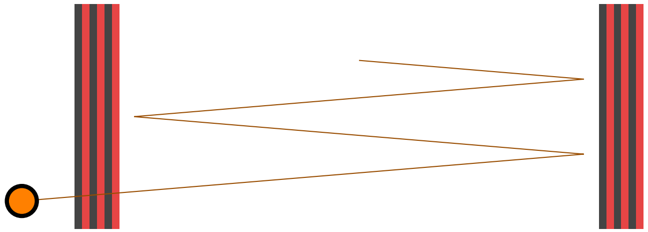
Let us consider a simple system



How can we describe the field in the cavity?



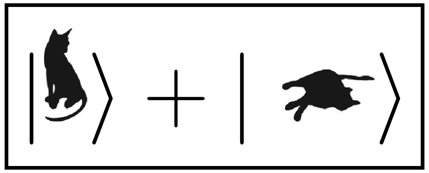
$$\partial_t \hat{\rho}(t) = -i[\hat{H}, \hat{\rho}(t)]$$



$$\hat{\rho}(0) = |1\rangle\langle 1| \longrightarrow \hat{\rho}(t \gg 1) = \hat{a}\hat{\rho}(0)\hat{a}^\dagger$$

Environment effect (2)

What about quantum superposition ?



$$\hat{\rho}(0) = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{\langle 0| + \langle 1|}{\sqrt{2}}$$

$$\Big\downarrow$$

$$\hat{\rho}(t \gg 1) = \hat{a}^\dagger \hat{a} \hat{\rho}(t) + \hat{\rho}(t) \hat{a}^\dagger \hat{a}$$

$$\partial_t \hat{\rho}(t) = \underset{\uparrow}{-i[\hat{H}, \hat{\rho}(t)]} + \gamma \left(\underset{\uparrow}{\hat{a}\hat{\rho}(t)\hat{a}^\dagger} - \underset{\uparrow}{\frac{\hat{a}^\dagger \hat{a} \hat{\rho}(t) + \hat{\rho}(t) \hat{a}^\dagger \hat{a}}{2}} \right)$$

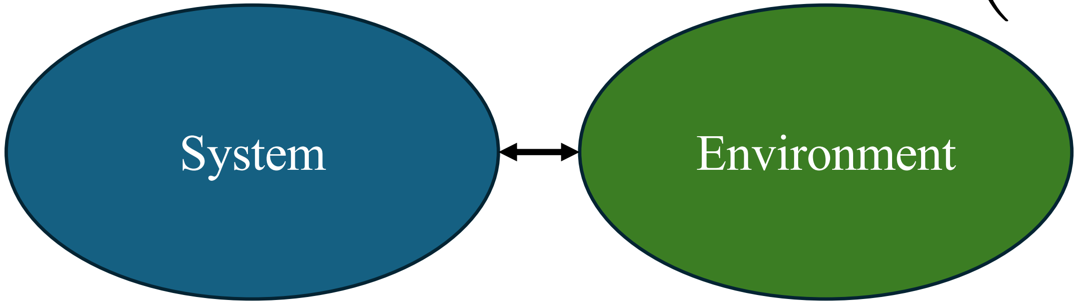
Coherent Evolution

Spontaneous emission

Loss of coherence

Open quantum systems

$$\partial_t \hat{\rho}(t) = -i[\hat{H}, \hat{\rho}(t)] + \gamma \left(\hat{a}\hat{\rho}(t)\hat{a}^\dagger - \frac{\hat{a}^\dagger\hat{a}\hat{\rho}(t) + \hat{\rho}(t)\hat{a}^\dagger\hat{a}}{2} \right)$$



System

Environment

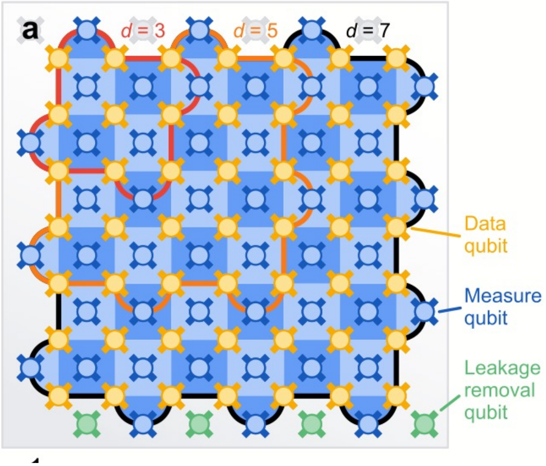
Hamiltonian

Hamiltonian + Measure

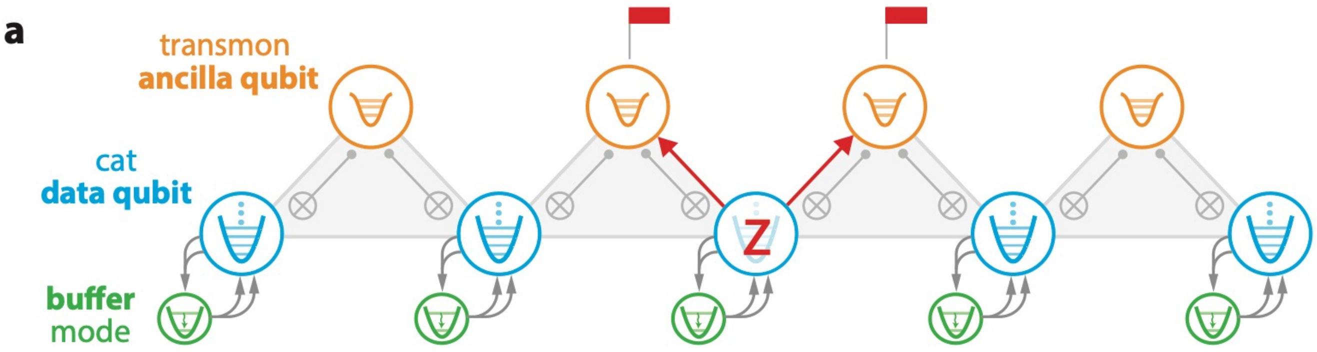
$$\hat{M}_0 = 1 - dt(i\hat{H} + \gamma\hat{a}^\dagger\hat{a}/2)$$

$$\hat{M}_1 = \sqrt{\gamma dt}\hat{a}$$

$$\sum_r \hat{M}_r^\dagger \hat{M}_r = \hat{1}$$

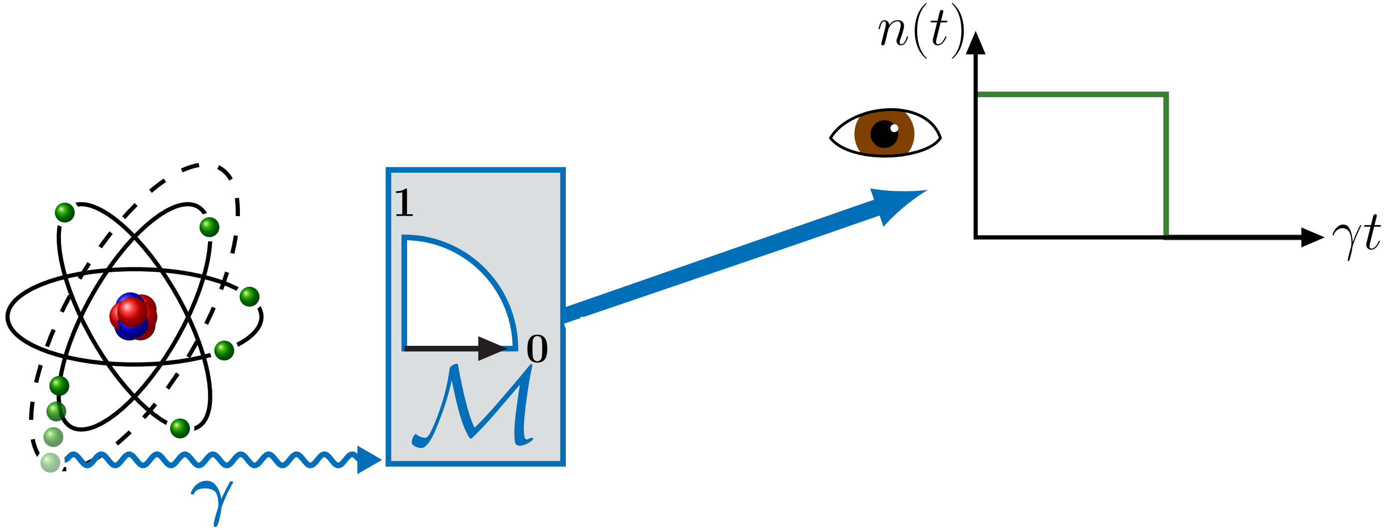


Google Quantum AI, arXiv, (2024)



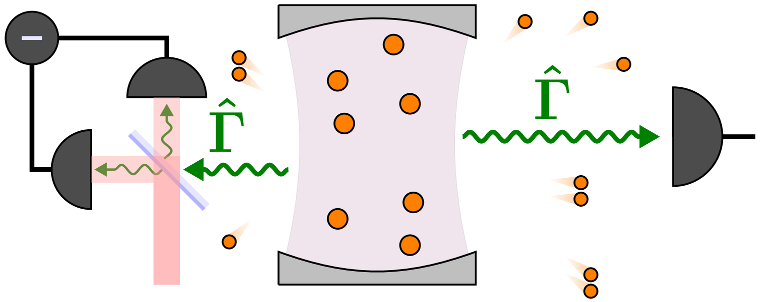
AWS, arXiv, (2024)

Trajectories vs master equation

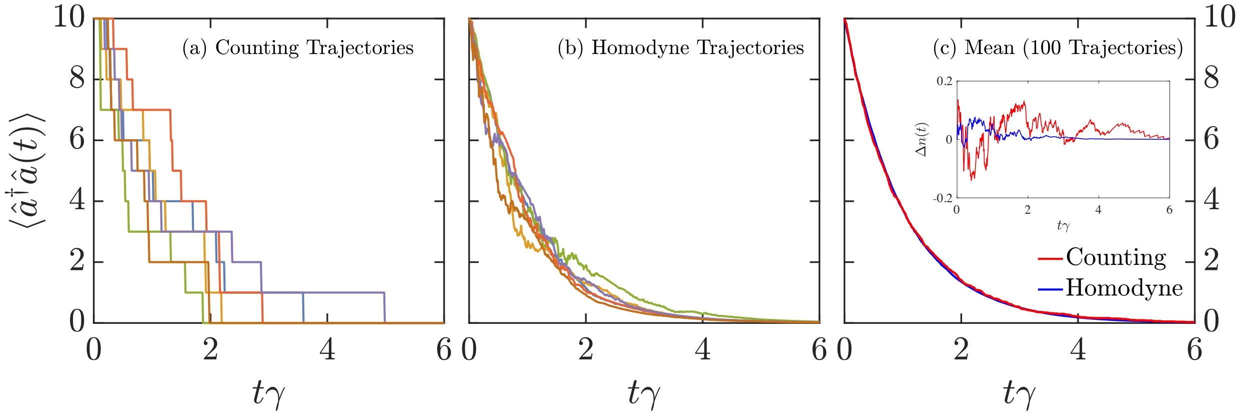


Quantum trajectory

Master equation

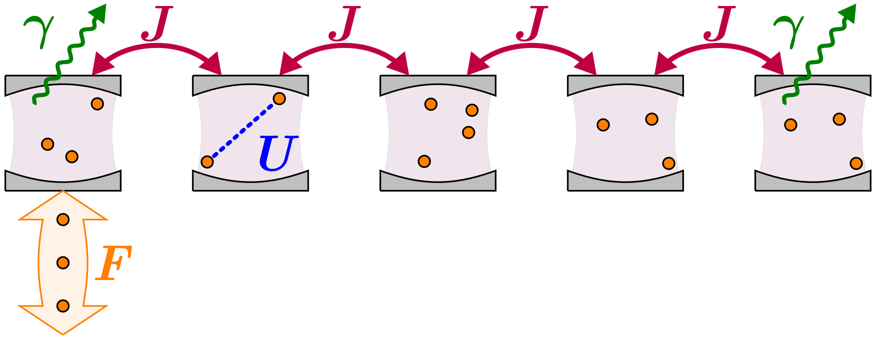


$$\kappa\mathcal{D}[\hat{a}]$$

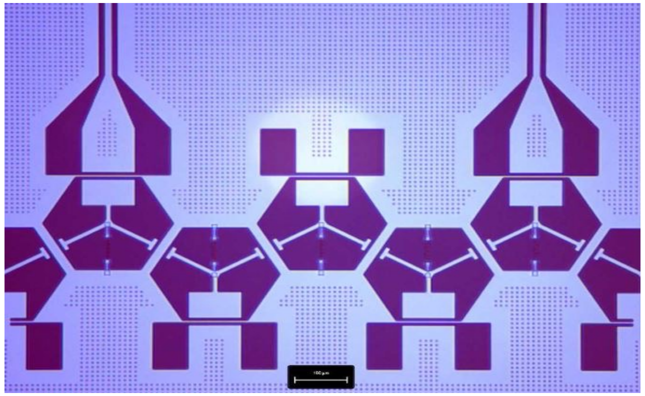


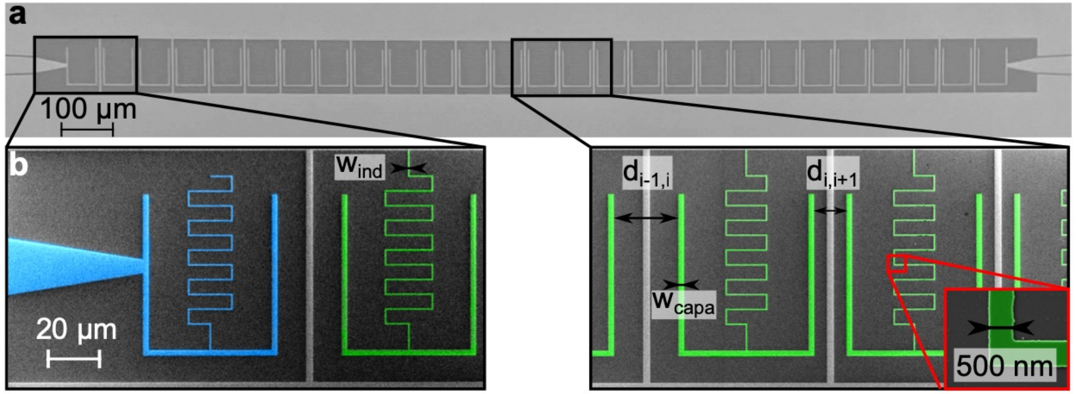
Driven-dissipative Hubbard model

$$\hat{H} = \sum_{j=1}^{N}\left(-\Delta \hat{a}_j^\dagger \hat{a}_j + \frac{1}{2}U\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j\right) - J\sum_{j=1}^{N-1}(\hat{a}_{j+1}^\dagger\hat{a}_j + \hat{a}_j^\dagger\hat{a}_{j+1}) + F(\hat{a}_1^\dagger + \hat{a}_1)$$

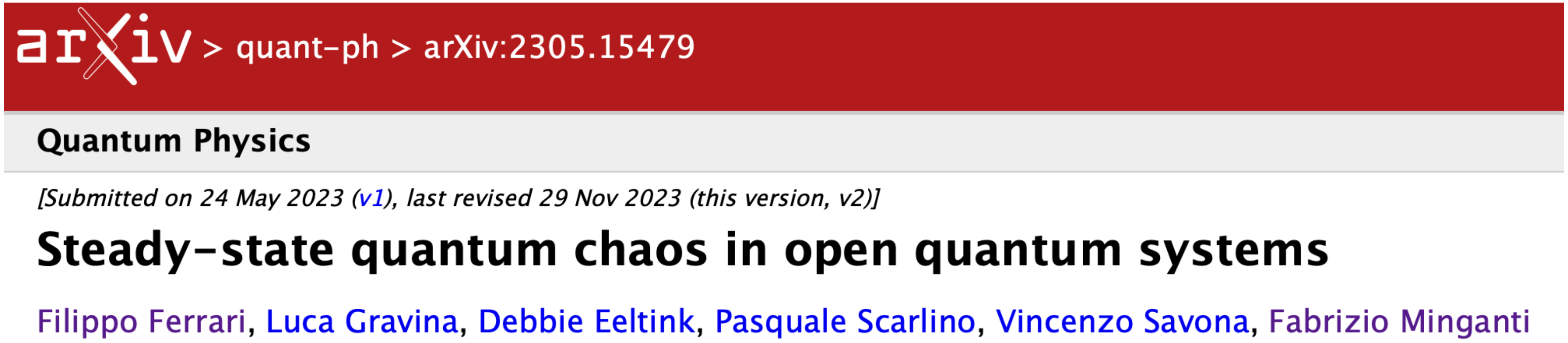


$$\frac{d\hat{\rho}}{dt} = -i[\hat{H},\hat{\rho}] + \gamma\mathcal{D}[\hat{a}_1] + \gamma\mathcal{D}[\hat{a}_N]$$





The ambiguity of dissipative chaos



> quant-ph > arXiv:2305.15479

Quantum Physics

[Submitted on 24 May 2023 (v1), last revised 29 Nov 2023 (this version, v2)]

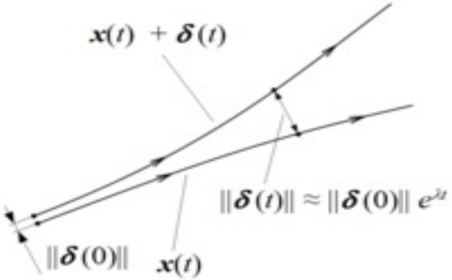
Steady-state quantum chaos in open quantum systems

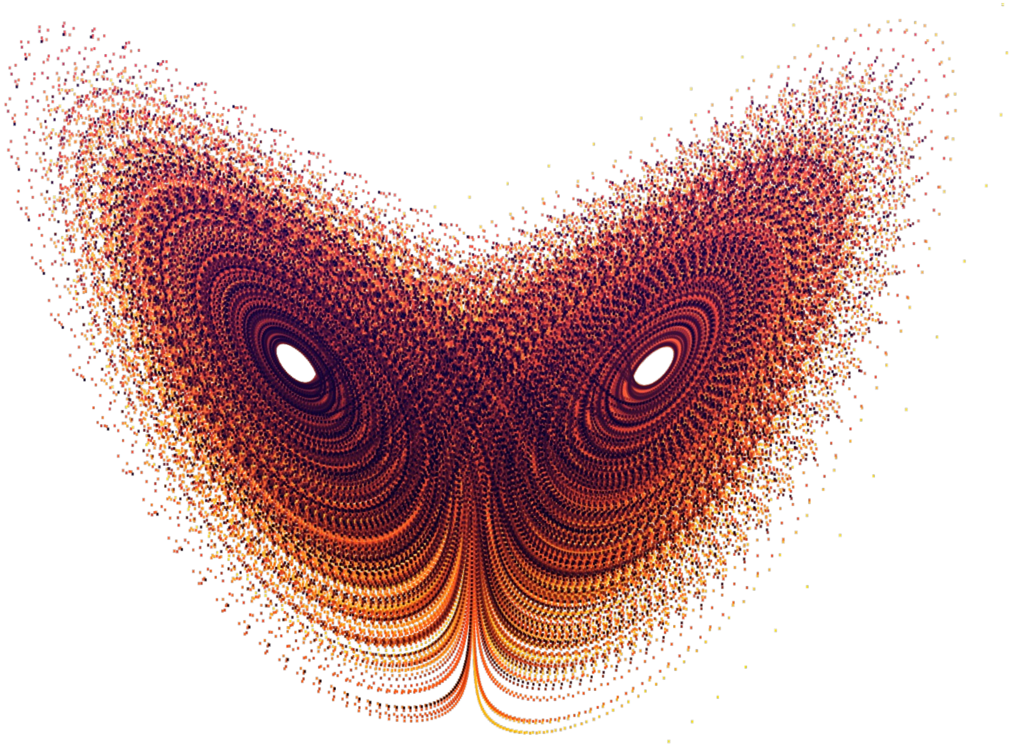
Filippo Ferrari, Luca Gravina, Debbie Eeltink, Pasquale Scarlino, Vincenzo Savona, Fabrizio Minganti

Classical systems

Classical picture of chaos: extreme sensitivity to initial conditions

$$\delta x(t) = e^{\Lambda t}\delta x(0)$$





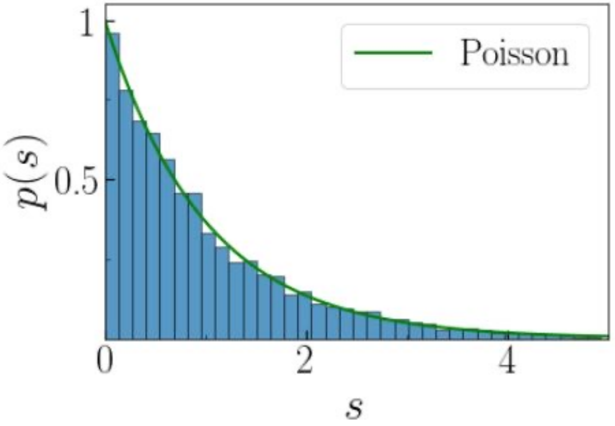
[Lorenz, AMS Journal, **20**, 130 (1963)]

Quantum systems

Integrable: **independent** eigenvalues ⇒ Behavior of **uncorrelated random variables**

⇓

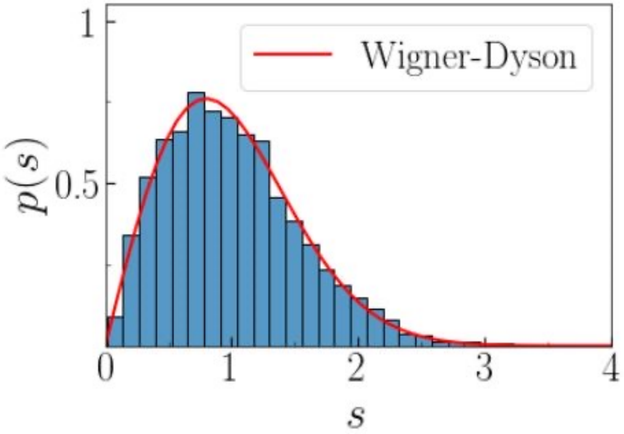
We look to the spacings: $s_j = E_{j+1} - E_j$



Main signature of quantum integrability: **level clustering**

[Berry and Tabor, Proceedings RS, **356**, 375 (1977)]

Chaotic system: **correlated** eigenvalues ⇒ Energy levels are rigid



Main signature of quantum chaos: **level repulsion**

[Bohigas *et al.*, Phys. Rev. Lett., 52, 1 (1984)]

The Liouvillian

We introduce the **Liouvillian superoperator**

$$\begin{cases} \partial_t \hat{\rho}(t) = \mathcal{L}\hat{\rho}(t) = -\mathrm{i}[\hat{H}, \hat{\rho}(t)] + \sum_\mu \kappa_\mu \mathcal{D}\left[\hat{J}_\mu\right] \hat{\rho}(t) \\ \mathcal{D}\left[\hat{J}_\mu\right] \cdot = \hat{J}_\mu \cdot \hat{J}_\mu^\dagger - \frac{\hat{J}_\mu^\dagger \hat{J}_\mu}{2} \cdot - \cdot \frac{\hat{J}_\mu^\dagger \hat{J}_\mu}{2} \end{cases}$$

Formal solution:

$$\hat{\rho}(t) = e^{\mathcal{L}t} \hat{\rho}(0)$$

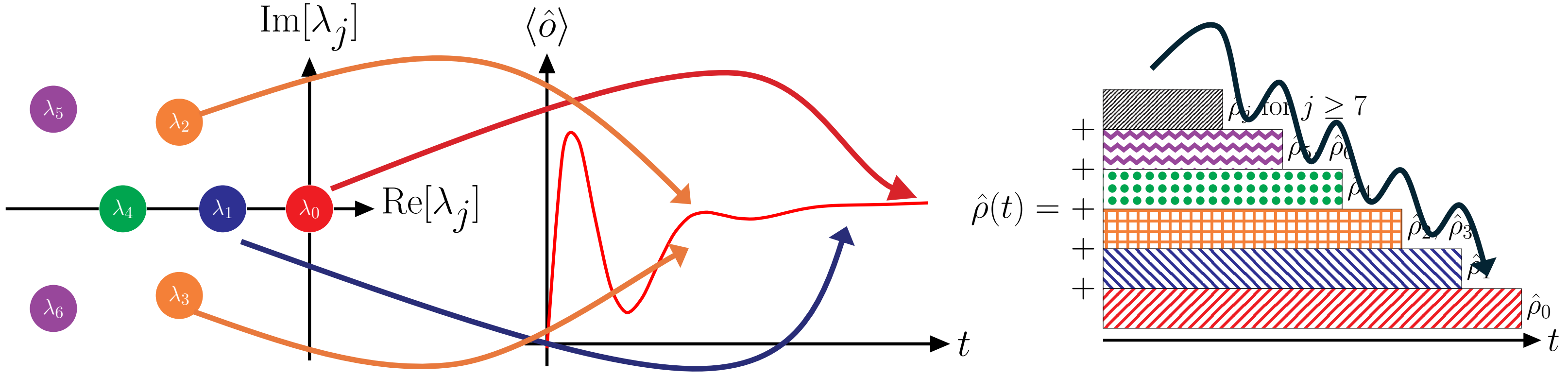
Steady state

$$\partial_t \hat{\rho}_{ss} = \mathcal{L}\hat{\rho}_{ss} = 0$$

$$\hat{\rho}_{ss} = \lim_{t\to\infty} \hat{\rho}(t) = \lim_{t\to\infty} e^{\mathcal{L}t} \hat{\rho}(0)$$

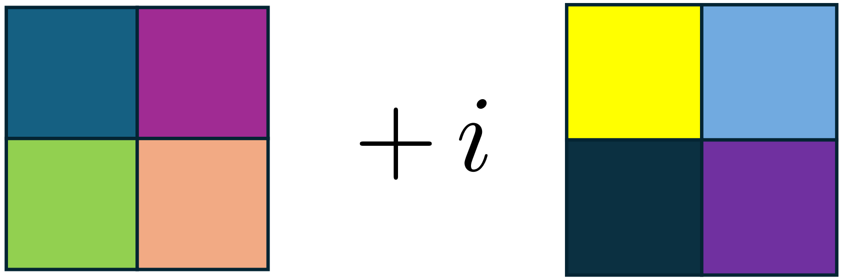
$$\mathcal{L}\hat{\rho}_i = \lambda_i \hat{\rho}_i$$

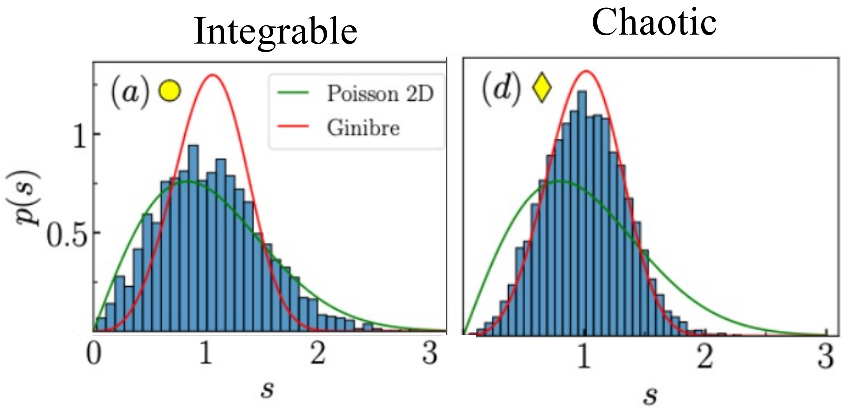
$$\mathcal{L} \neq \mathcal{L}^\dagger \qquad \lambda_i \in \mathbb{C}$$



Dissipative quantum systems

Dissipative quantum chaos: **correlated Liouvillian** eigenvalues ⇨ Spacings between **complex** eigenvalues $s_j = |\lambda_j - \lambda_j^{NN}|$



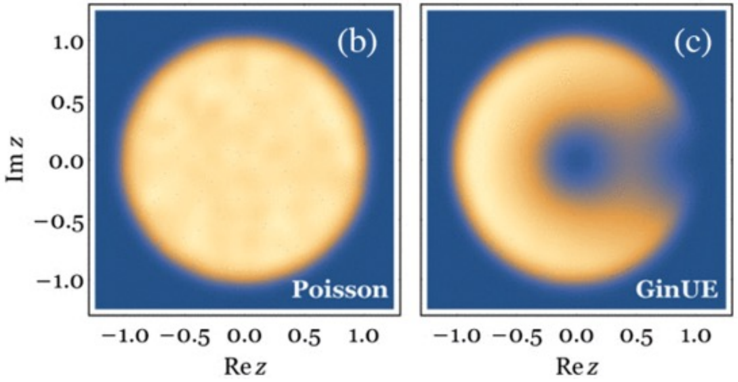


We can also look to the **Complex Spacing Ratios**

$$z_j = \frac{\lambda_j^{NN} - \lambda_j}{\lambda_j^{NNN} - \lambda_j} = r_j e^{i\theta_j}$$

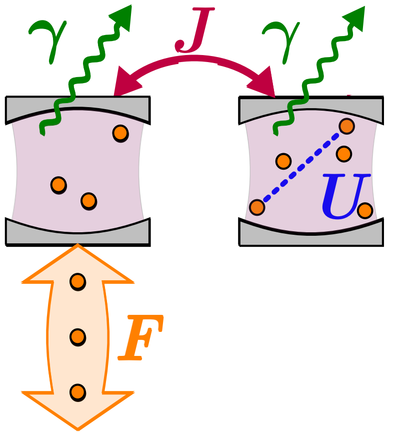
$-\langle\cos(\theta)\rangle = 0 \rightarrow$ **integrability**

$-\langle\cos(\theta)\rangle = 0.24 \rightarrow$ **chaos**



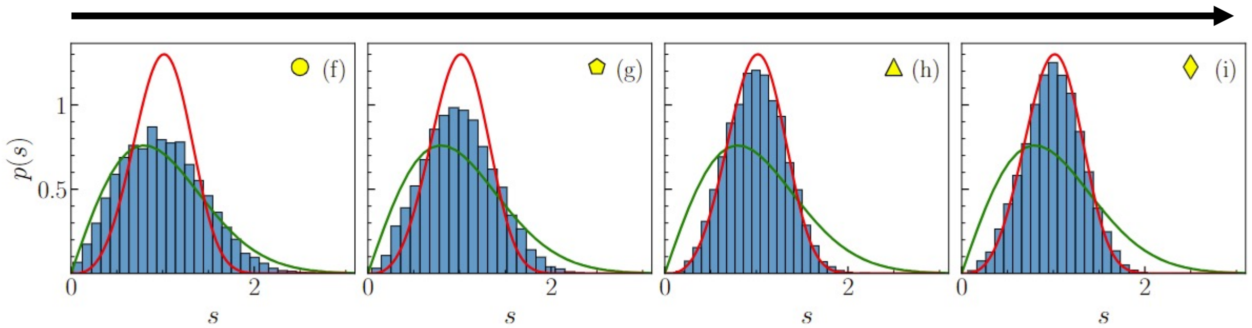
[Grobe *et al*, PRL, **61**, 1899 (1988)]
[Akemann *et al.*, PRL, **123**, 254101 (2019)]
[Sà *et al.*, PRX, **10**, 021019 (2020)]

Chaos as a transient (?)

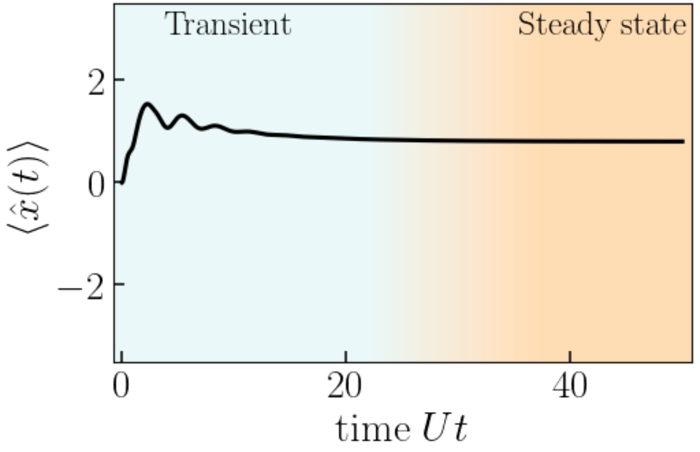


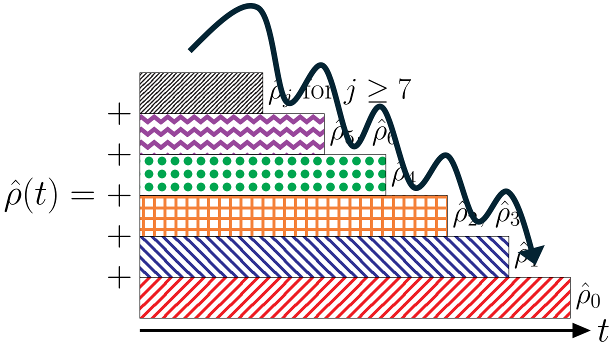
$$\hat{H} = \sum_{j=1}^{2}\left(-\Delta \hat{a}_j^\dagger \hat{a}_j + \frac{1}{2}U\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j\right) - J\left(\hat{a}_2^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_2\right) + F(\hat{a}_1^\dagger + \hat{a}_1)$$

$$\frac{\partial\hat{\rho}}{\partial t} = -i[\hat{H},\hat{\rho}] + \gamma\sum_{j=1}^{2}\left(\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \frac{1}{2}\{\hat{a}_j^\dagger\hat{a}_j,\hat{\rho}\}\right)$$



Increasing drive





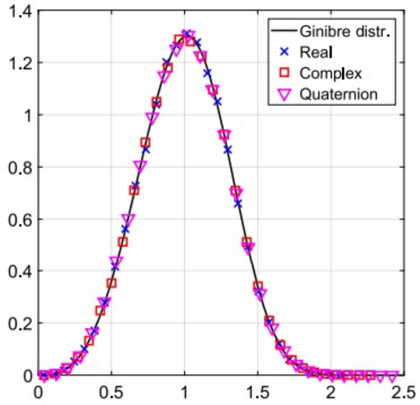
- Chaos depends on the **initial condition**
- Chaos is a **transient phenomenon**

Q: Can dissipative chaos be defined in the steady state?

Testing the waters

Can we detect **steady-state and transient chaos** with current well-established criteria?

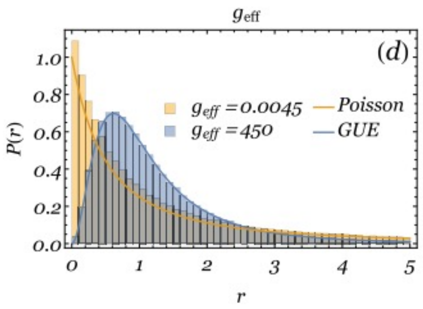
1. Statistics of Liouvillian eigenvalues



[Akemann *et al.,* Phys. Rev. Lett., **123**, 254101 (2019)]

2. Steady-state density matrix analysis

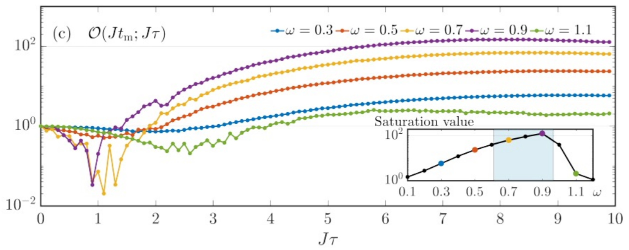
$$\hat{H}_{\text{ent}} = -\log(\hat{\rho}_{\text{ss}})$$



[Sà *et al.,* JPhA, **53,** 305303 (2020)]
[Sà *et al.,* PRB, **102,** 134310 (2020)]

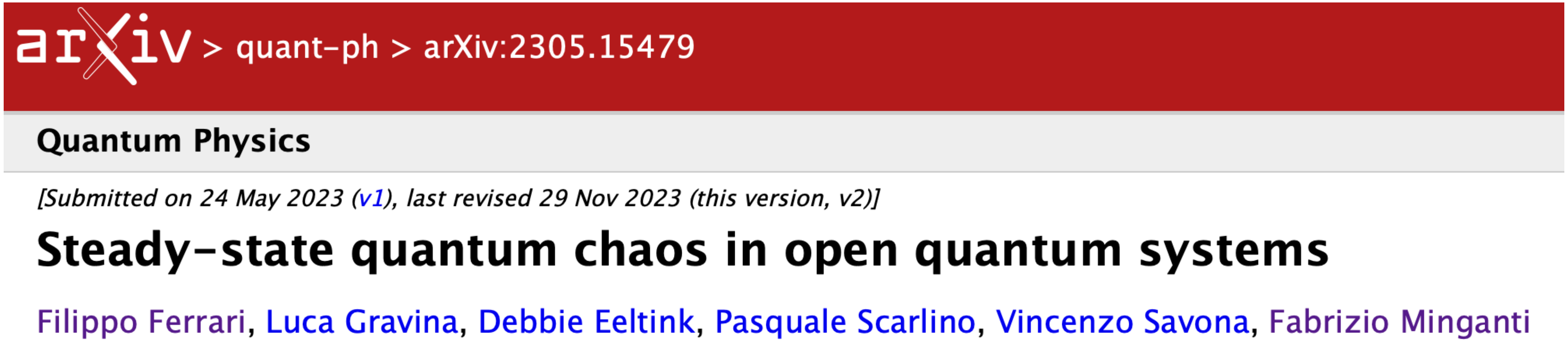
3. Out-of-time order correlators

$$O(t,\tau) = -\langle[\hat{Q}(t+\tau), \hat{P}(t)]^2\rangle$$



[Dahan *et al*., npj Quantum Information, **8**, 14 (2022)]

Spectral statistics of quantum trajectories



arXiv > quant-ph > arXiv:2305.15479

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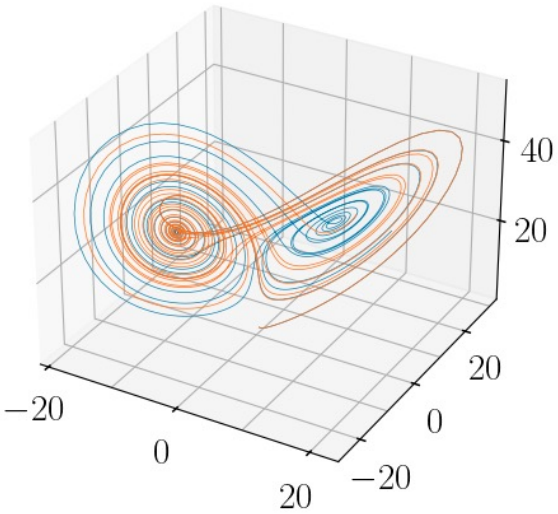
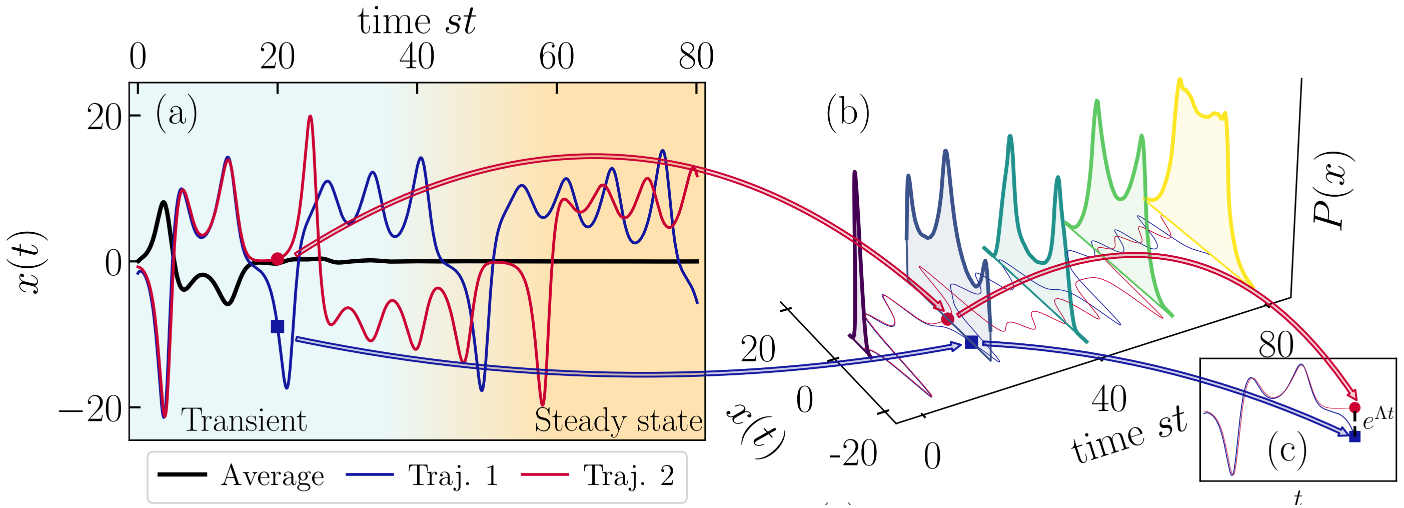
One step back…

$$\frac{\partial x}{\partial t} = \sigma(y - x),$$

$$\frac{\partial y}{\partial t} = x(\rho - z) - y,$$

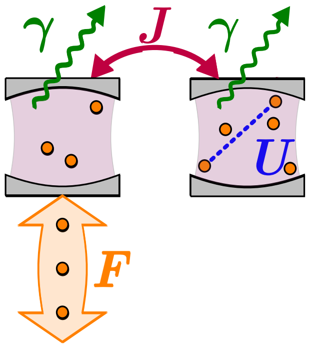
$$\frac{\partial z}{\partial t} = xy - \beta z$$

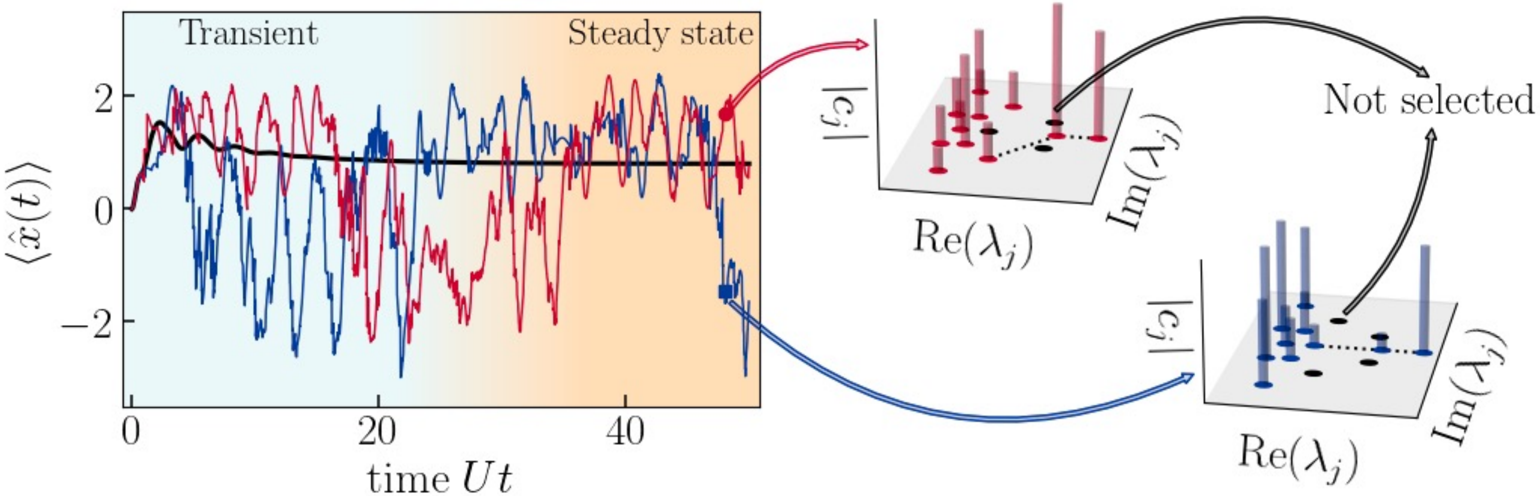




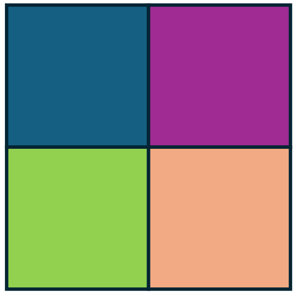
Averaging makes it ambiguous to define chaos

Chaos via unraveling

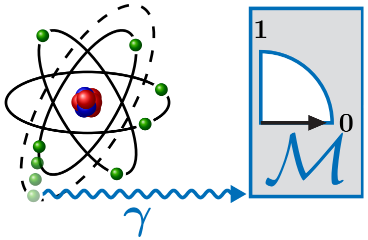




+

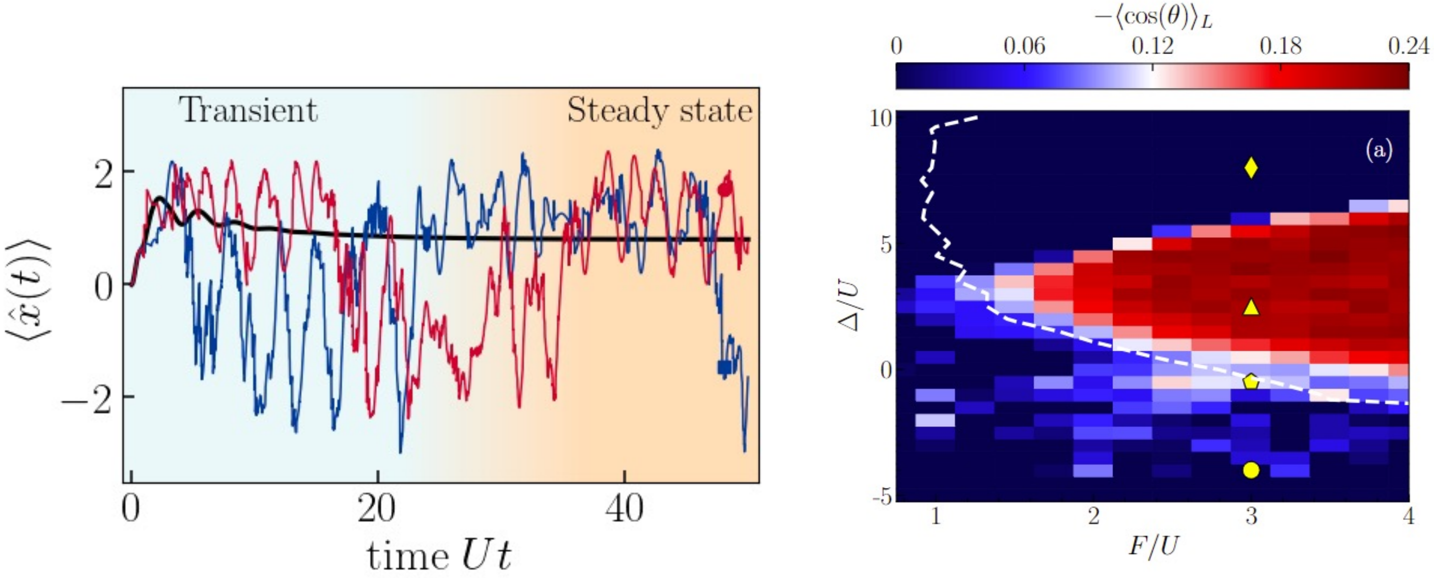


Quantum trajectory



Master equation

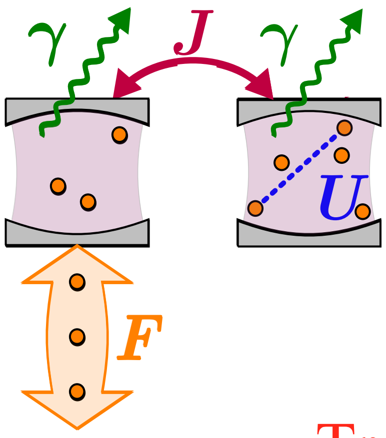
Applying the criterion



RMT only on those eigenvalues that are activated by quantum trajectories

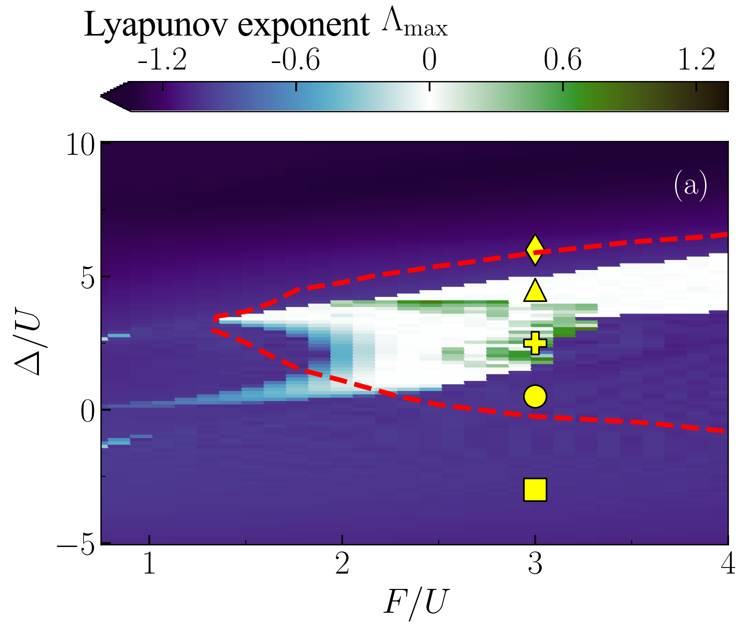
BGS conjecture

Correspondence between classical and quantum chaos



$$\frac{\partial}{\partial t}\alpha_1 = \left(i\Delta - \tfrac{1}{2}\gamma\right)\alpha_1 - iU\left|\alpha_1\right|^2\alpha_1 + iJ\alpha_2 - iF,$$
$$\frac{\partial}{\partial t}\alpha_2 = \left(i\Delta - \tfrac{1}{2}\gamma\right)\alpha_2 - iU\left|\alpha_2\right|^2\alpha_2 + iJ\alpha_1.$$

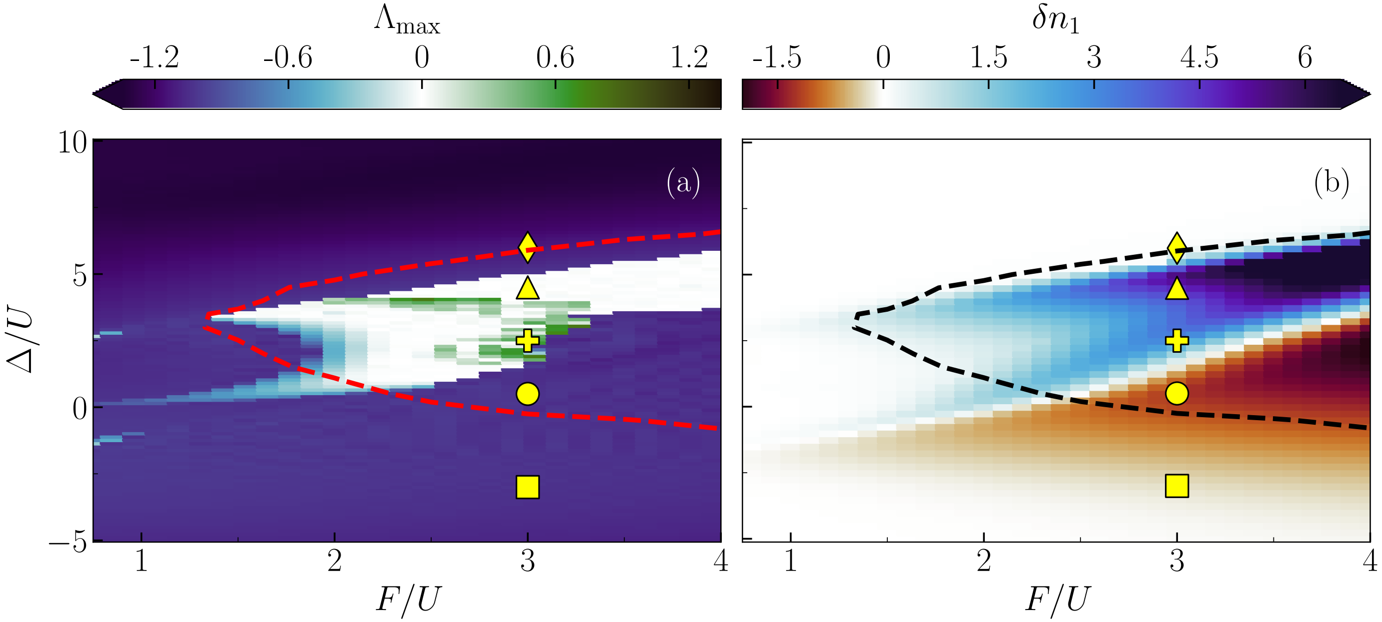
Treat fields as C-numbers



No chaos in the classical system

Measurement

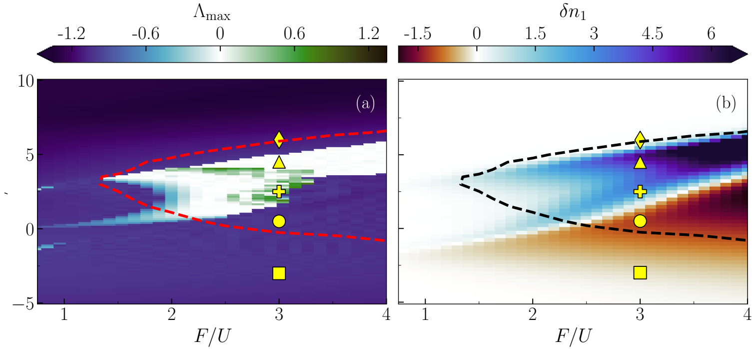
Q: Why the lack of correspondence?

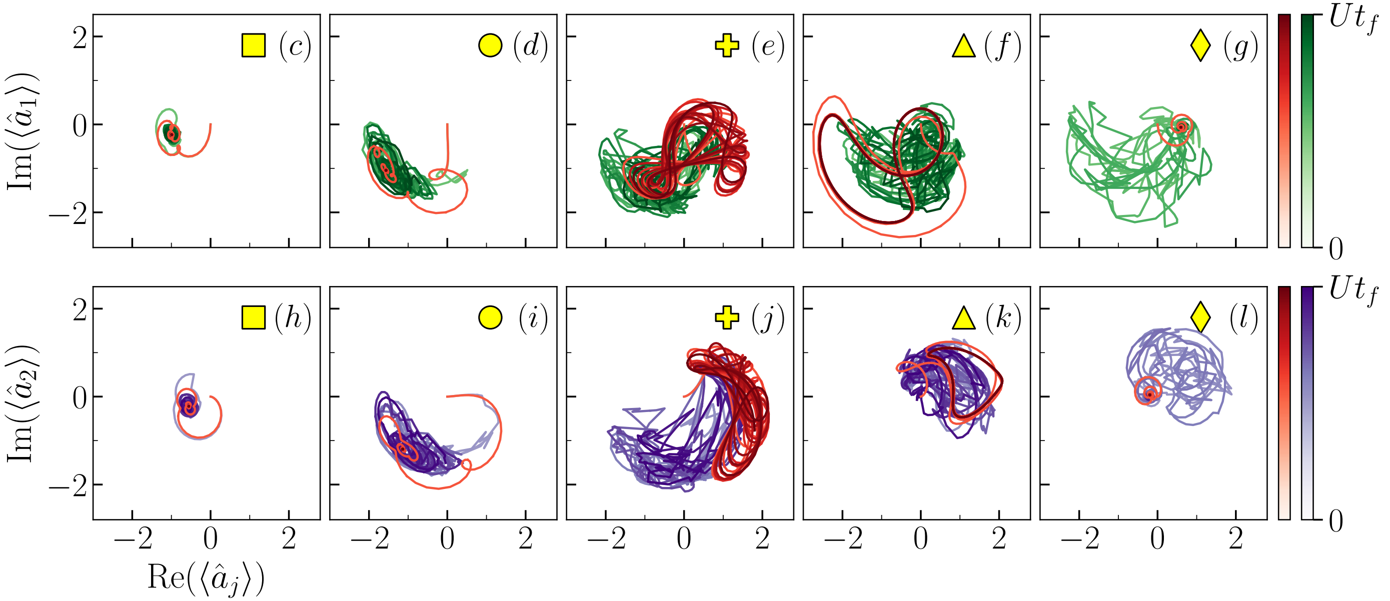


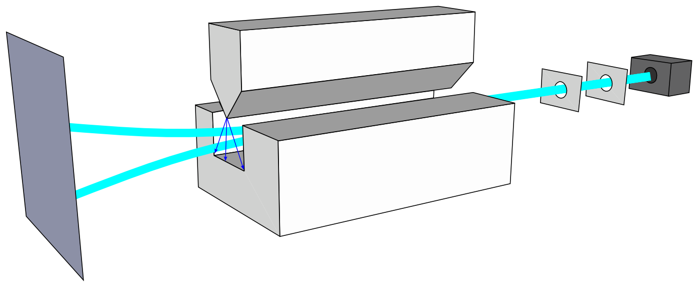
$$\delta n = (\Delta n)^2 - \langle \hat{n} \rangle_{\text{ss}}$$

Measurement

Q: Why the lack of correspondence?



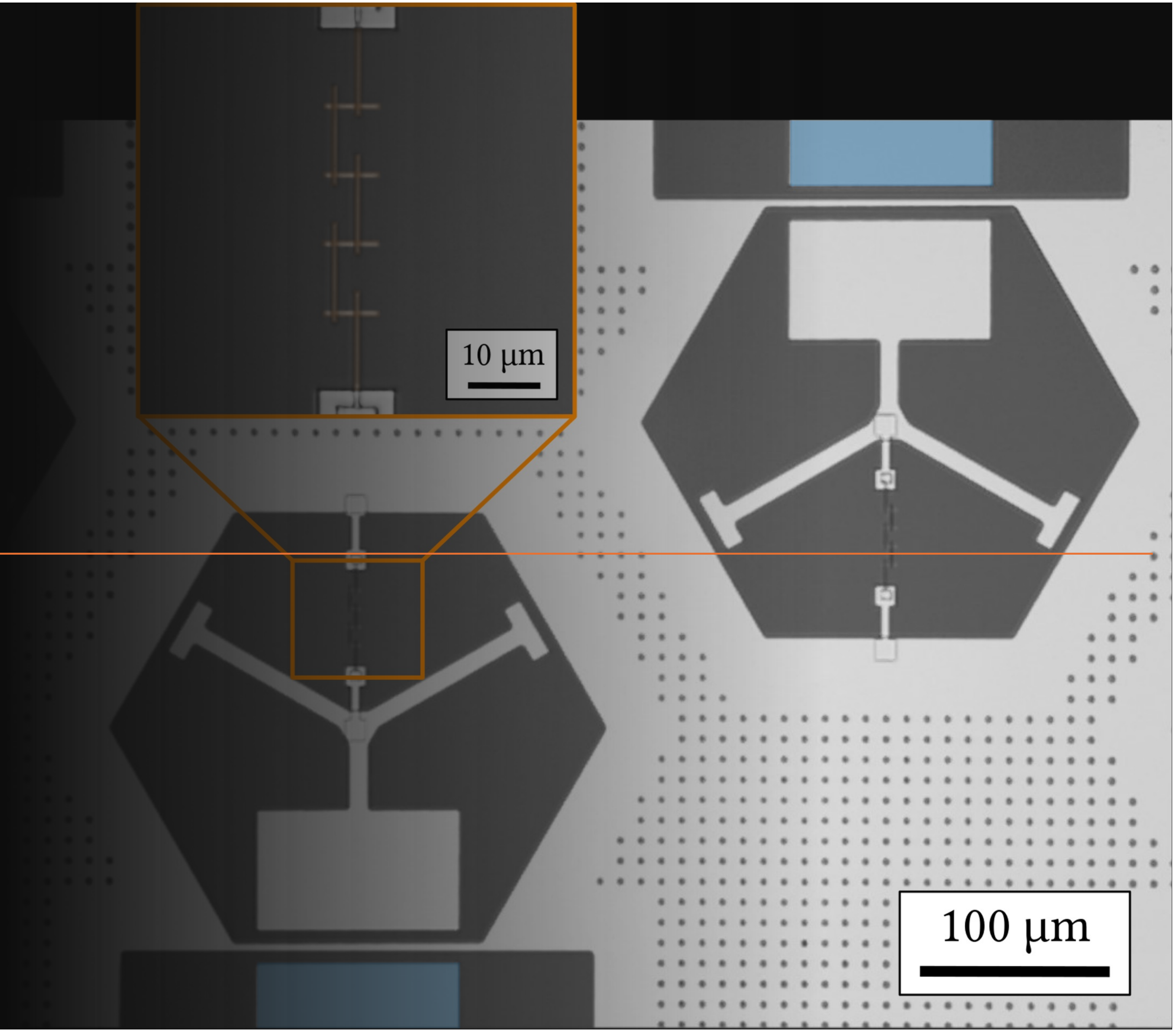




Quantum jumps **re-set** the dynamics: **Zeno-like effect**

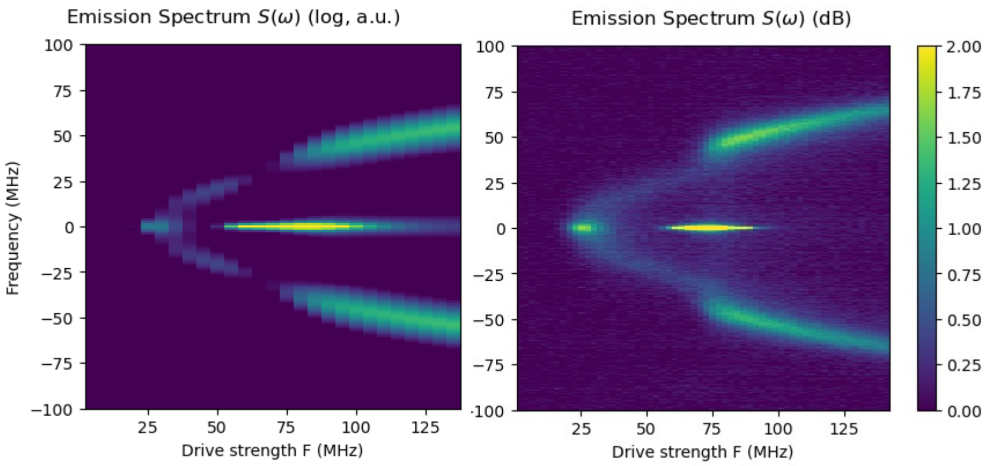
… a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery.

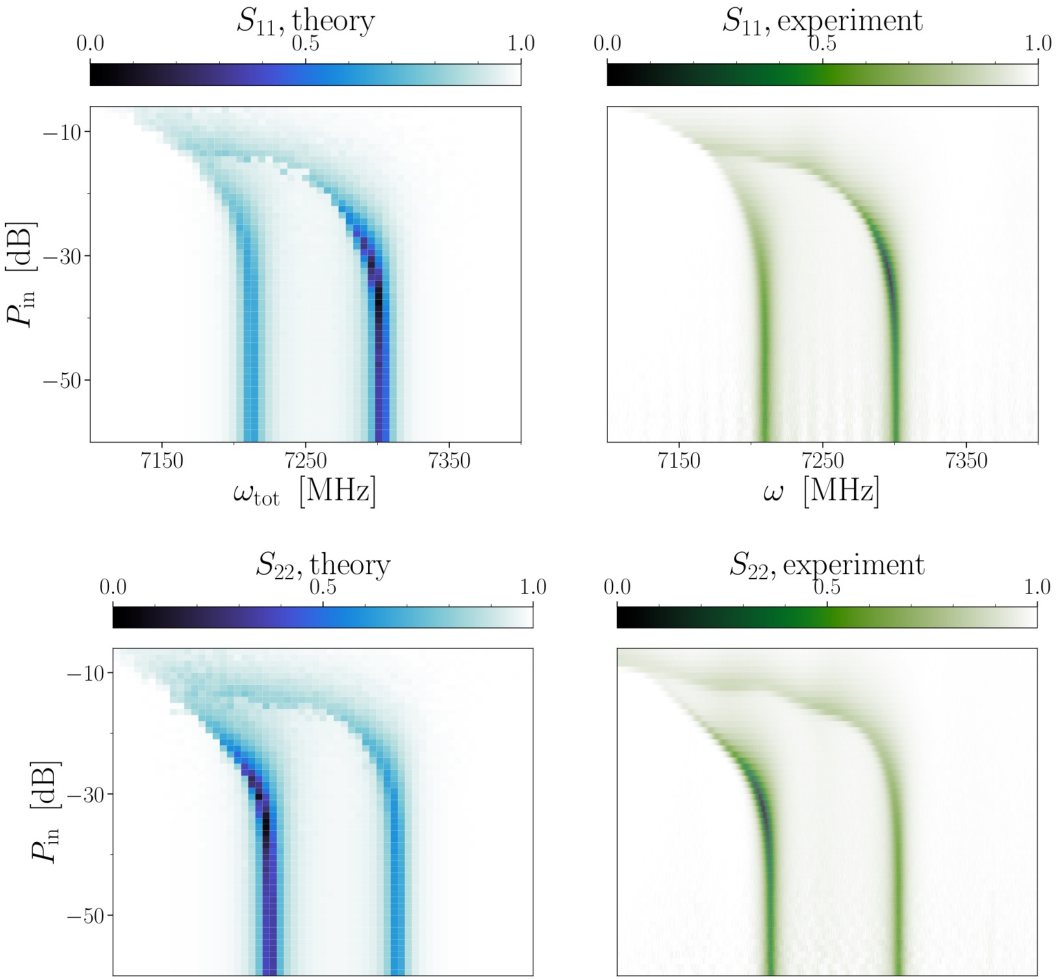
Experimental verifications



Experiment 1 (vs theory)

Objective: measure signature of chaos in the Bose-Hubbard dimer

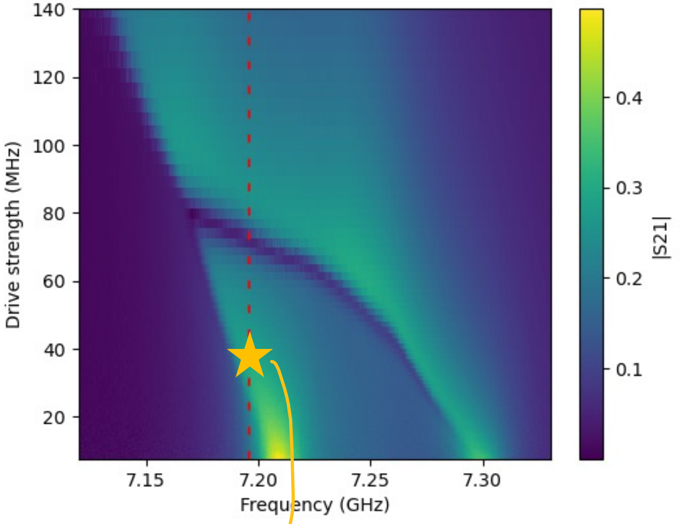




So far, good agreement experiment/theory

➔ Measure next signature = quantum trajectories

Towards trajectory reconstruction



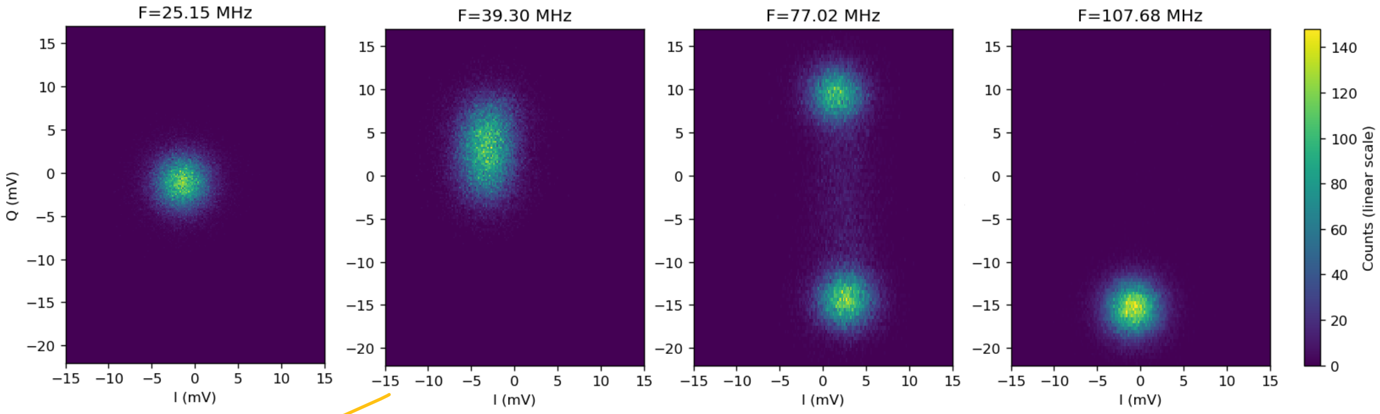
Chaos?

Vacuum

Chaos?

Bistability

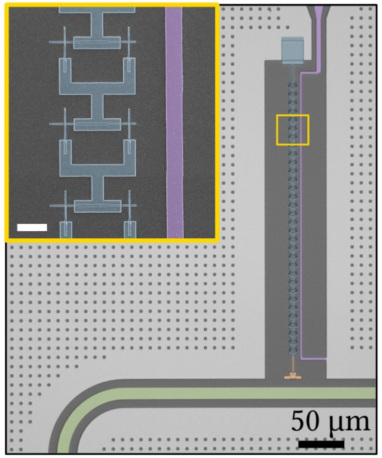
High photon-
number state



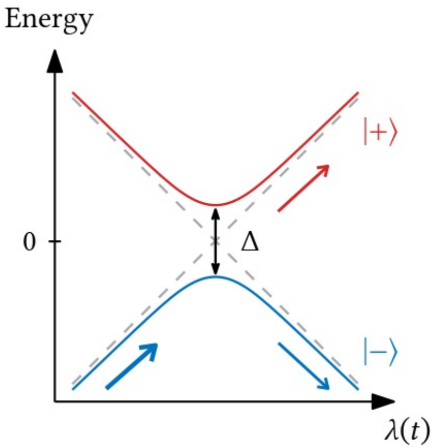
First hint of distorted state in IQ plane

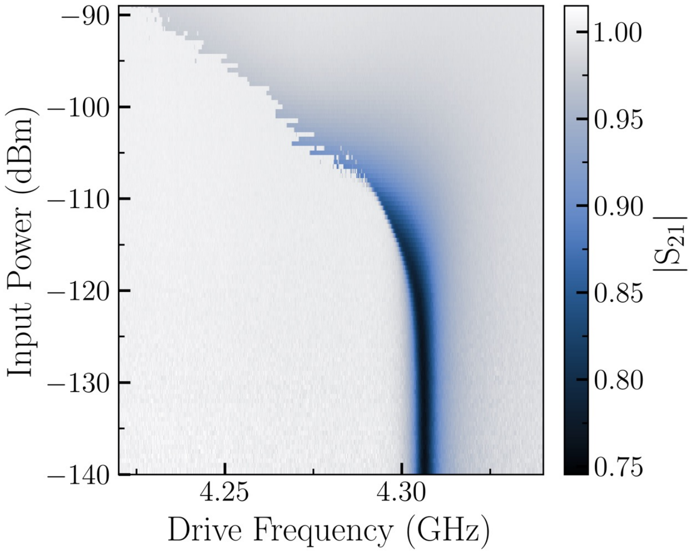
➔few photon-state, **need to reduce added noise!**

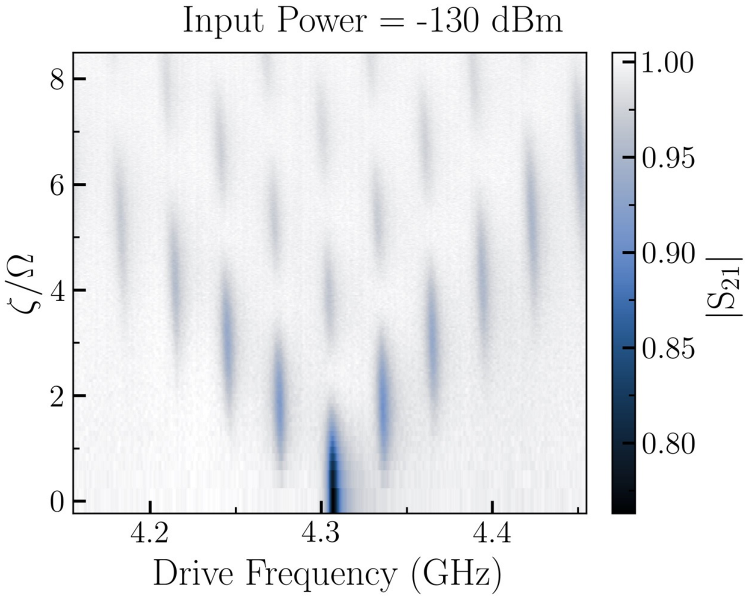
Experiment 2: Floquet dynamics



$$\hat{H} \simeq (\Delta - \bar{m}\Omega)\hat{a}^\dagger\hat{a} + \chi\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + F J_{\bar{m}}\left(\frac{\zeta}{\Omega}\right)(\hat{a}+\hat{a}^\dagger)$$

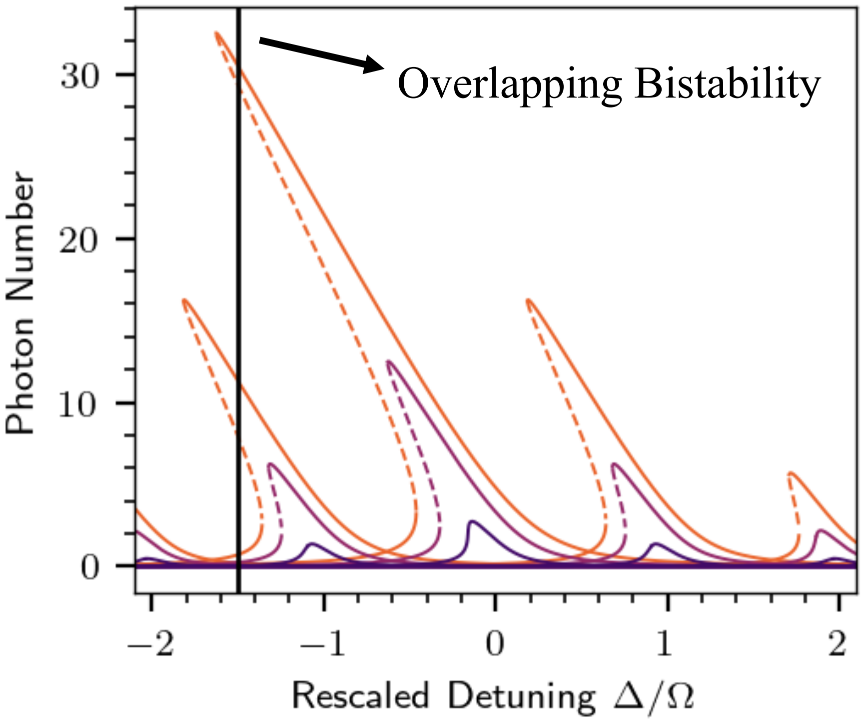


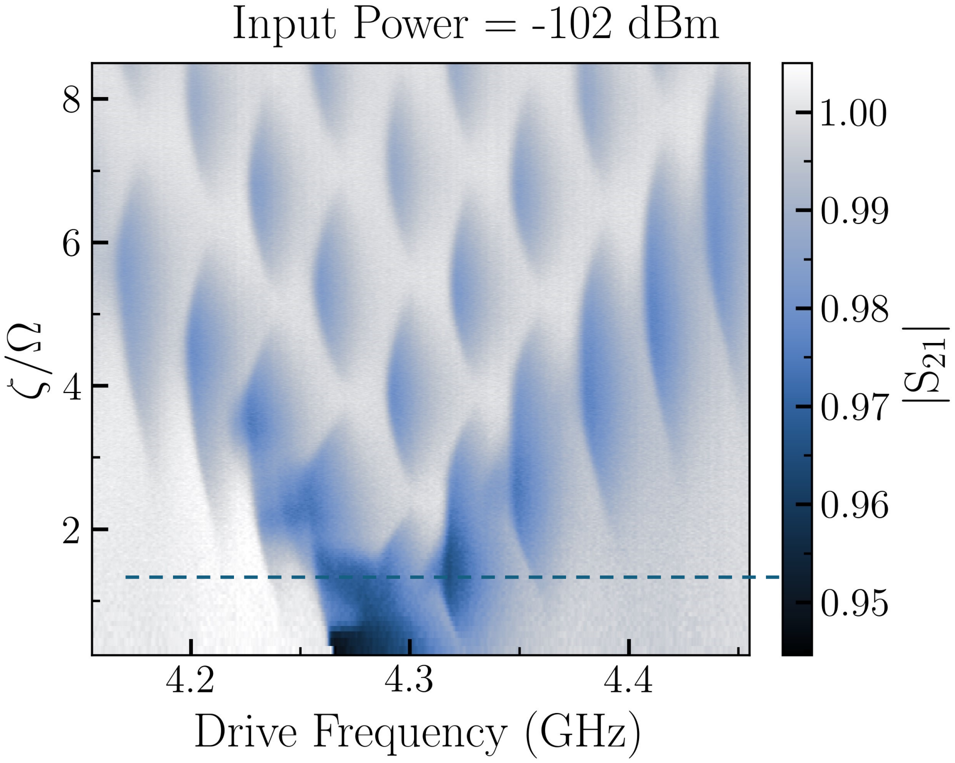




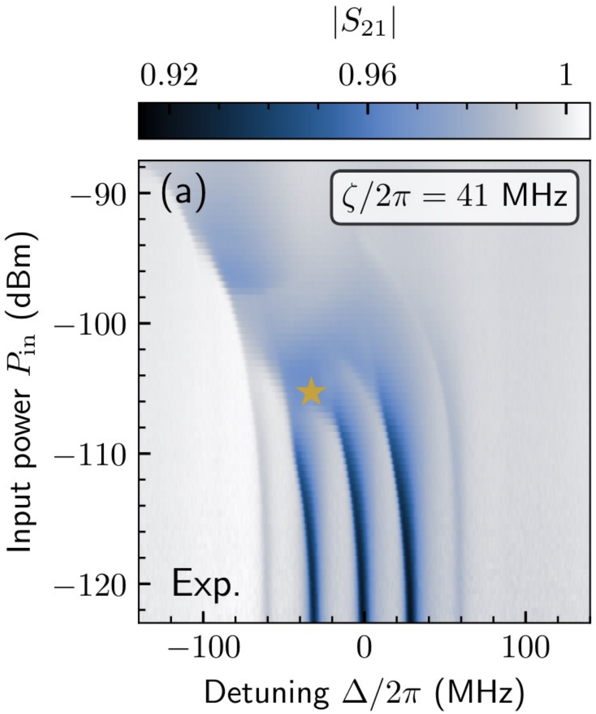
Experiment 2: Synthetic dimensions

Semiclassical simulation for **increasing drive** strengths *F*

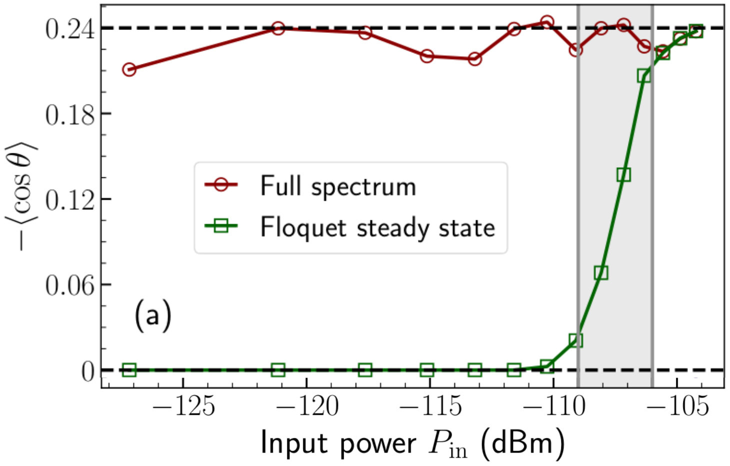




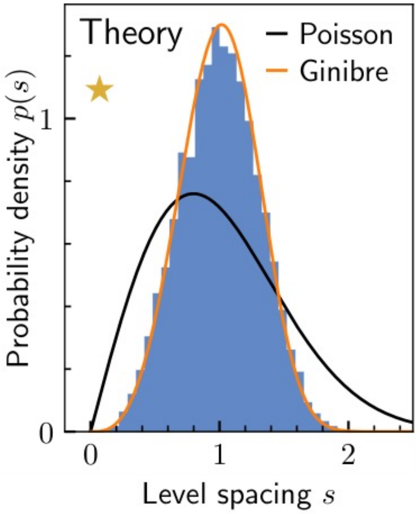
Experiment 2: Chaos



- Complex level spacing ratio



- Level spacing distribution



➔ Quantum Chaos signatures coincide with merging of Floquet states


arXiv > quant-ph > arXiv:2404.10051

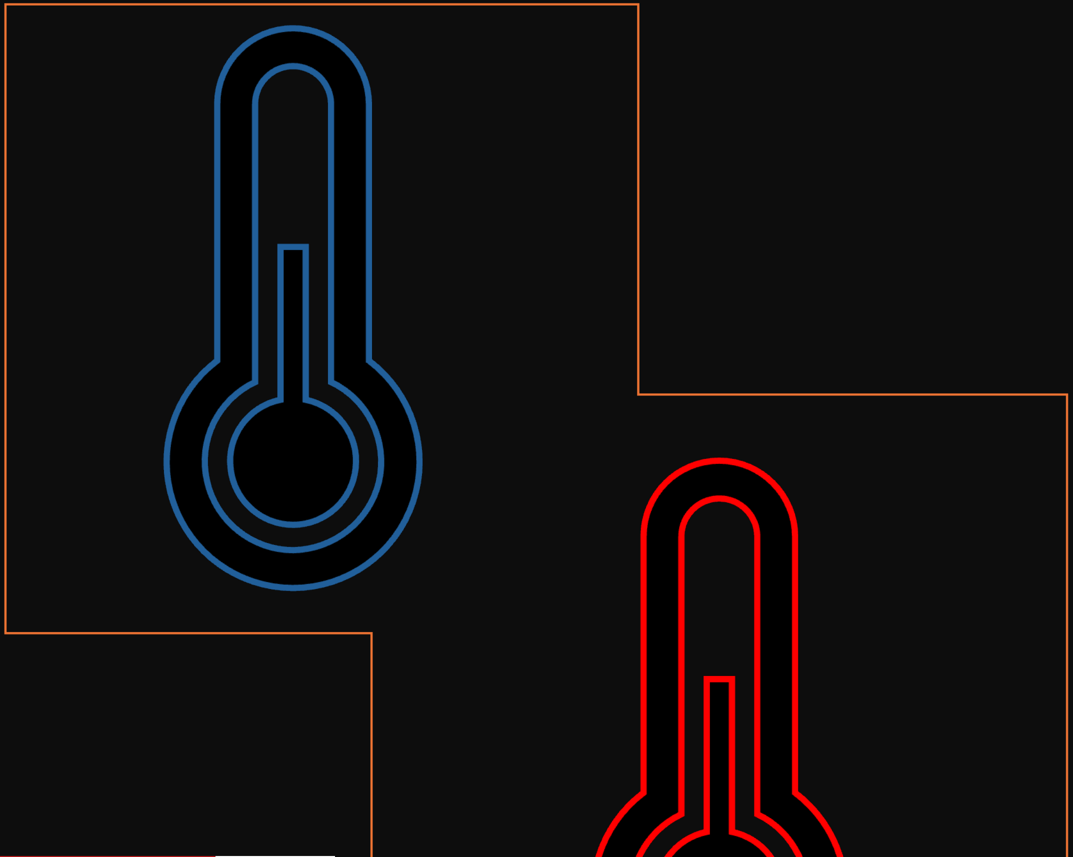
Quantum Physics

[Submitted on 15 Apr 2024]

Landau-Zener without a Qubit: Unveiling Multiphoton Interference, Synthetic Floquet Dimensions, and Dissipative Quantum Chaos

Leo Peyruchat, Fabrizio Minganti, Marco Scigliuzzo, Filippo Ferrari, Vincent Jouanny, Franco Nori, Vincenzo Savona, Pasquale Scarlino

(Spatial pre-) Thermalization



Quantum Physics

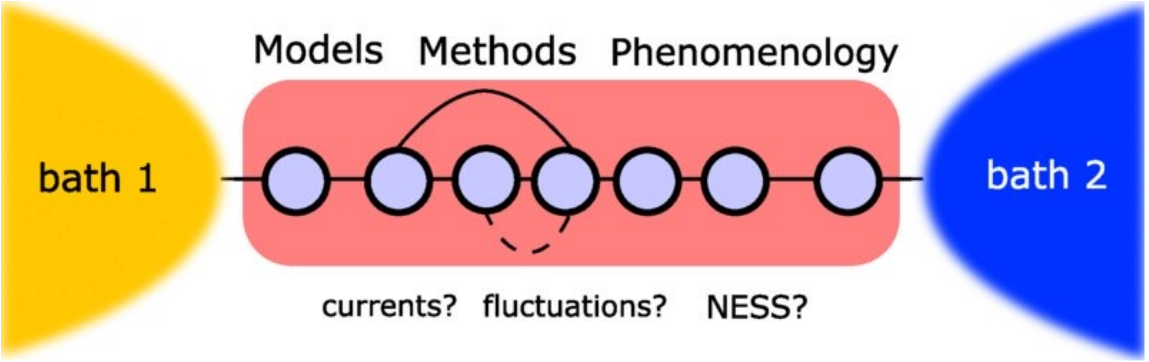
[Submitted on 18 Sep 2024]

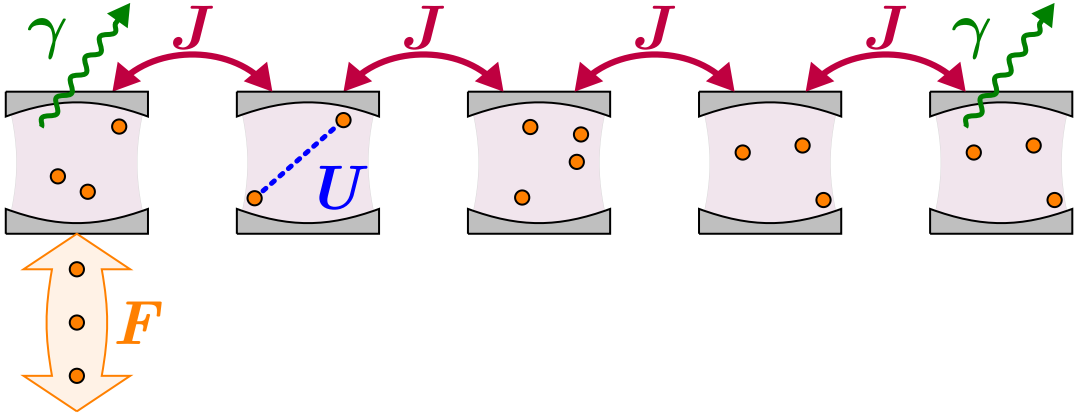
Chaos and spatial prethermalization in driven-dissipative bosonic chains

Filippo Ferrari, Fabrizio Minganti, Camille Aron, Vincenzo Savona

What did we learn?

- Chaos can emerge in the **steady state**;
- Chaotic features present in **quantum trajectories**;
- Combination of **Hamiltonian and dissipative** effects.





- Hamiltonian bulk vs dissipative edge;
- Effect of drive [no U(1) symmetry];
- Bosons vs spin.

- Exponentially large Hilbert space;
- Quantum + Open + Transport.

Truncated Wigner Approximation

The TWA

Stochastic trajectory calculations based on the truncated Wigner approximation

[K. Vogel and H. Risken, PRA 39, 4675 (1989)]

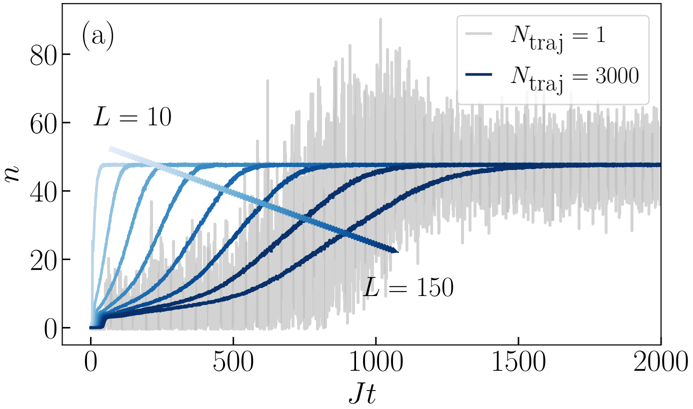
$$\hat{\rho} \longrightarrow W(\vec{\alpha}) = W(\alpha_1, \alpha_2 \ldots \alpha_N) = \frac{1}{\pi^N} \int \prod_{j=1}^{N} d^2\xi_j \; e^{\alpha_j \xi_j^* - \alpha_j^* \xi_j} \; \mathrm{Tr}\left[\hat{\rho}\; e^{\xi_j \hat{a}_j^\dagger - \xi_j^* \hat{a}_j}\right]$$

Fokker Planck

$$\partial_t \hat{\rho} \longrightarrow \partial_t W(\vec{\alpha}) = \partial_{\vec{\alpha}}\, A\, W(\vec{\alpha}) + \partial_{\vec{\alpha}}^2 \frac{D}{2} W(\vec{\alpha}) + \cancel{\mathcal{O}\left(U \partial_{\vec{\alpha}}^3 W(\vec{\alpha})\right)}$$

$$\dot{\alpha_j} = \left[-i(\Delta - U(|\alpha_j|^2 - 1) - \gamma/2)\right] \alpha_j - iJ \sum_{j'} \alpha_{j'} + iF + \sqrt{\gamma/2}\, \chi(t)$$

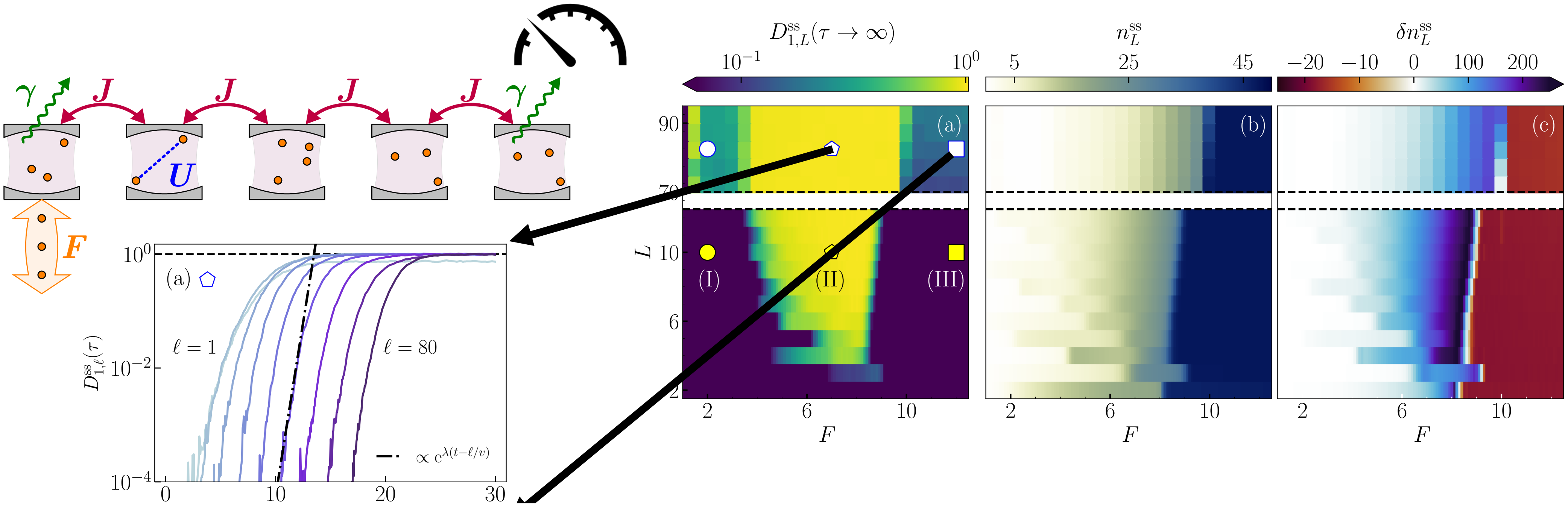
Langevin



$$D_{k,\ell}(t,\tau) := 1 - \left\langle \cos\left[\phi_\ell^{(a)}(t+\tau) - \phi_\ell^{(b)}(t+\tau)\right]\right\rangle$$

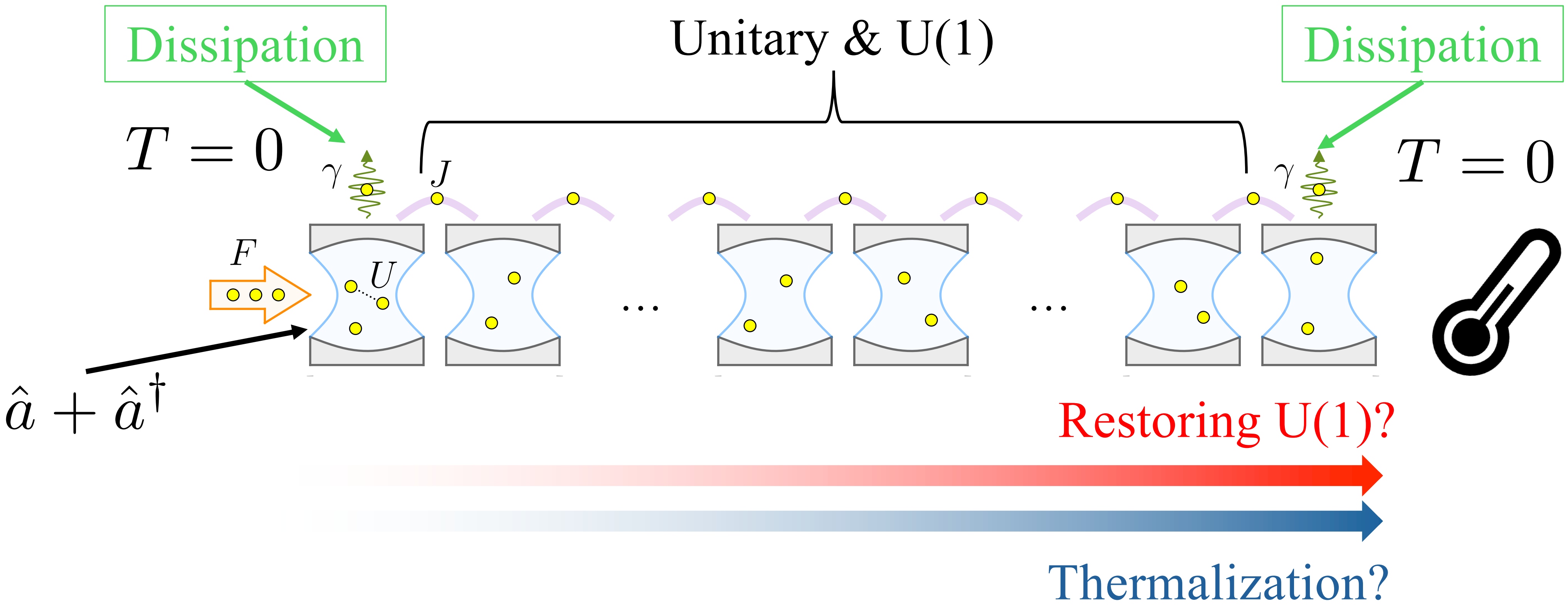
$$\lim_{t\to\infty} D_{k,\ell}(t,\tau) = D_{k,\ell}^{\mathrm{ss}}(\tau)$$

The TWA



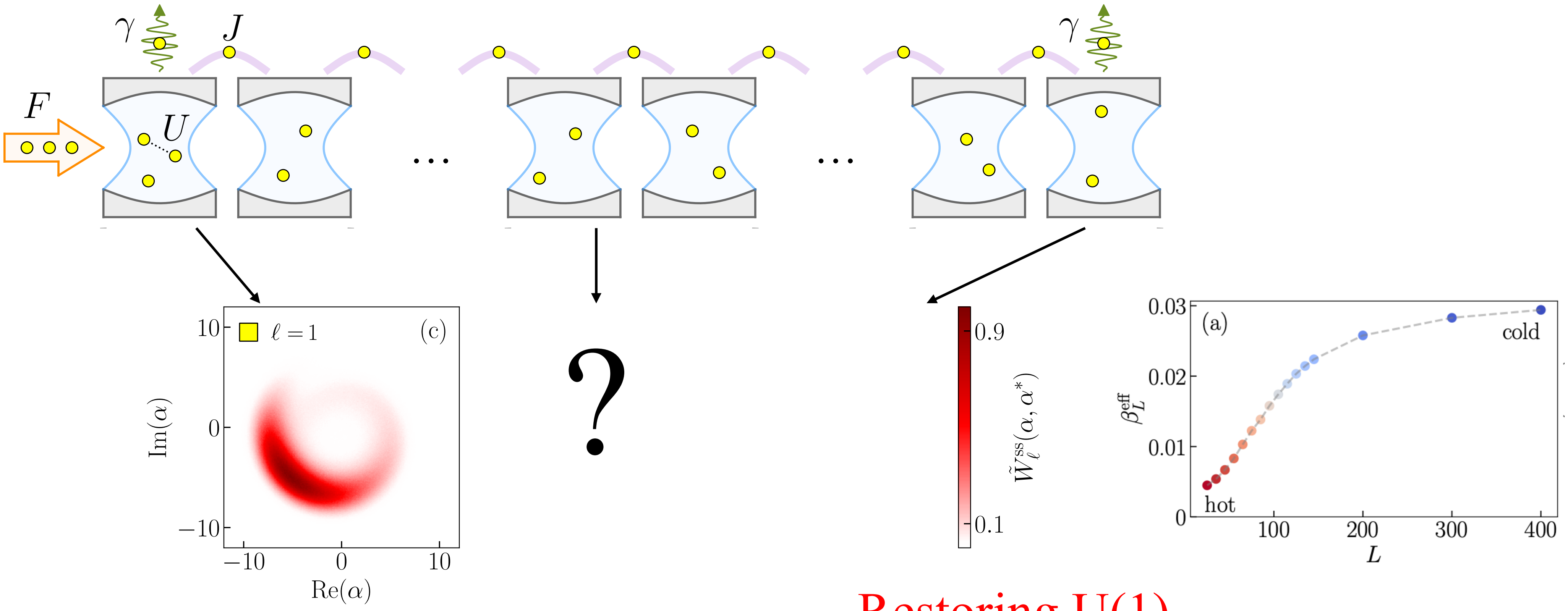
γ
J
J
J
J
γ
U
F
$D^{\rm ss}_{1,L}(\tau\to\infty)$
10^{-1}
10^{0}
$n^{\rm ss}_L$
5
25
45
$\delta n^{\rm ss}_L$
-20
-10
0
100
200
(a)
(b)
(c)
90
70
10
6
2
L
(I)
(II)
(III)
2
6
10
F
10^0
(a)
$\ell = 1$
$\ell = 80$
$D^{\rm ss}_{1,\ell}(\tau)$
10^{-2}
10^{-4}
$\propto e^{\lambda(t-\ell/v)}$
0
10
20
30

Thermalization



How can we capture these phenomena? → Non-linear model with gain and saturation

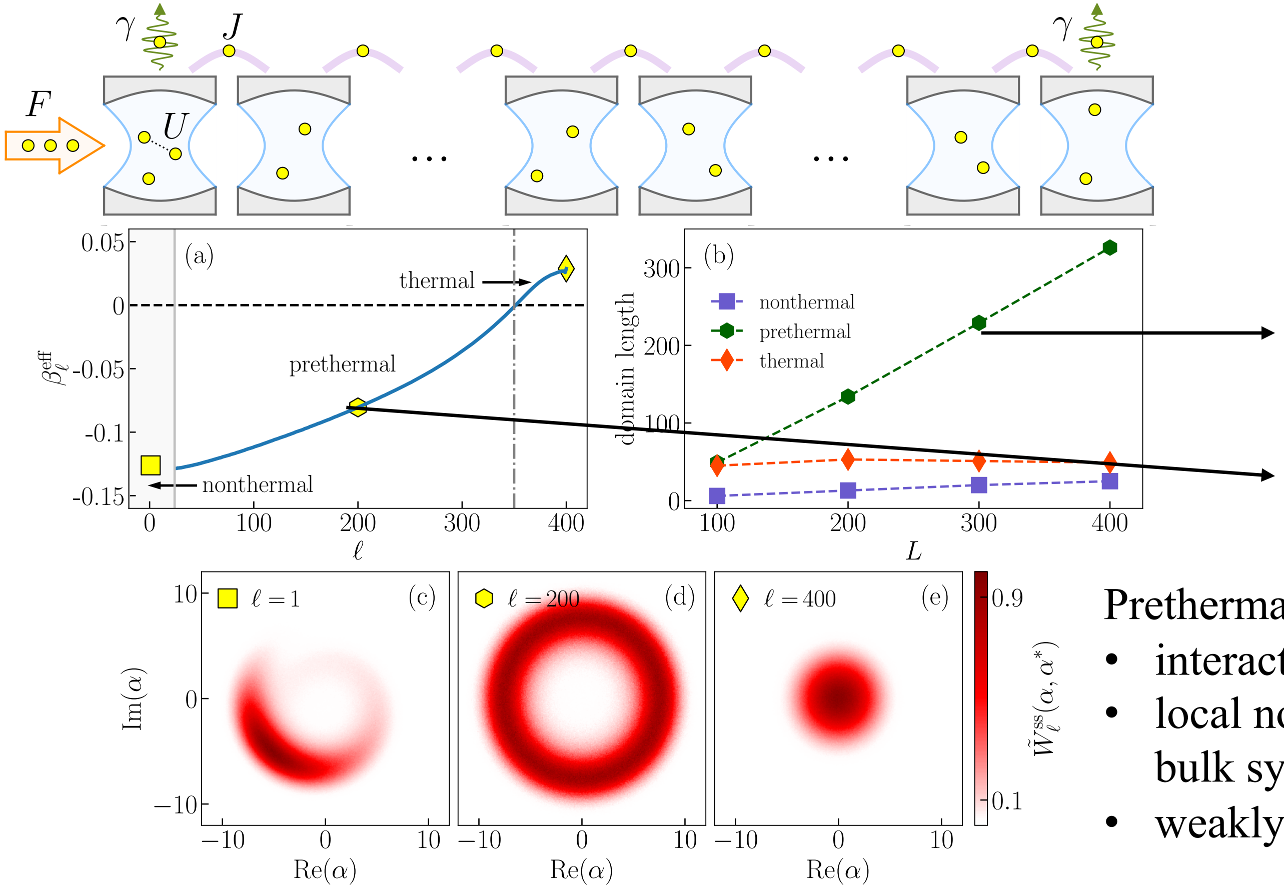
Temperature along the chain



Restoring U(1)

Thermalization

Spatial pre-thermalization



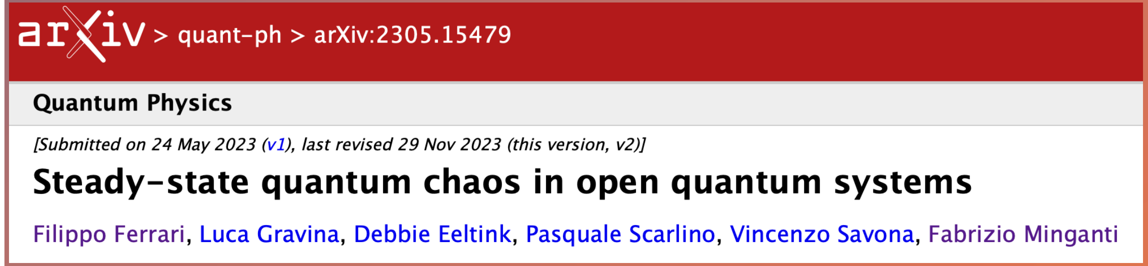
Extended phase

Negative T

Prethermal chaotic phase depends on:

- interacting bulk and conserved charge
- local nonequilibrium drive that "breaking" the bulk symmetry;
- weakly-symmetric dissipation channel.

Summing up



arXiv > quant-ph > arXiv:2305.15479

Quantum Physics

[Submitted on 24 May 2023 (v1), last revised 29 Nov 2023 (this version, v2)]

Steady-state quantum chaos in open quantum systems

Filippo Ferrari, Luca Gravina, Debbie Eeltink, Pasquale Scarlino, Vincenzo Savona, Fabrizio Minganti

arXiv > quant-ph > arXiv:2409.12225

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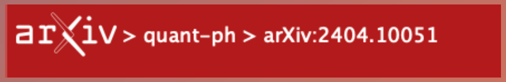
Summing up

arXiv > quant-ph > arXiv:2305.15479

arXiv > quant-ph > arXiv:2409.12225



arXiv > quant-ph > arXiv:2404.10051

- Chaos open quantum system as a persistent phenomenon;
- Experimental signatures in SC devices;
- Thermalization phenomena in extended lattice systems.