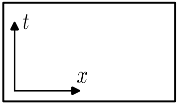
Information dynamics in inflationary spacetime

Xiangyu Cao (CNRS/ENS Paris)

October 3, 2024
Workshop on Quantum Chaos, Bernoulli Center

Quantum chaos playgrounds

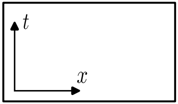
Closed



- Experiments possible
- Lattice: RMT, ETH
- Gravity: black holes
- QFT?
- Information perspective: scrambling/encoding as mechanism of irreversibility

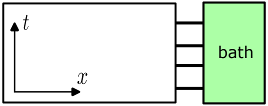
Quantum chaos playgrounds

Closed



- Experiments possible
- Lattice: RMT, ETH
- Gravity: black holes
- QFT?
- Information perspective: scrambling/encoding as mechanism of irreversibility

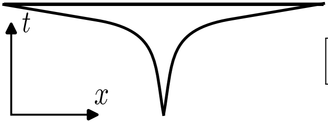
Open



- Experimentally Important
- Non-Hermitian RMT
- Use Bath to probe chaos
- Information flow to the bath: “phases of information” and sharp transitions

Why inflation?

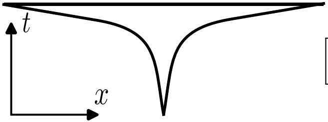
Unitary dynamics incorporating more and more degrees of freedom



$x \propto \exp(t)$

Why inflation?

Unitary dynamics incorporating more and more degrees of freedom

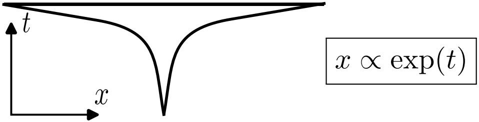


$$\boxed{x \propto \exp(t)}$$

- Cosmologically relevant. (Experiments: moveable qubits)

Why inflation?

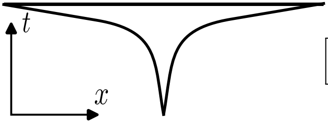
Unitary dynamics incorporating more and more degrees of freedom



- Cosmologically relevant. (Experiments: moveable qubits)
- Efficient simulation of lattice models.

Why inflation?

Unitary dynamics incorporating more and more degrees of freedom

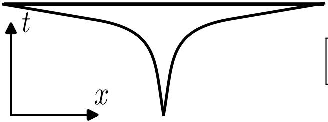


$$x \propto \exp(t)$$

- Cosmologically relevant. (Experiments: moveable qubits)
- Efficient simulation of lattice models.
- Generates scale invariance.

Why inflation?

Unitary dynamics incorporating more and more degrees of freedom



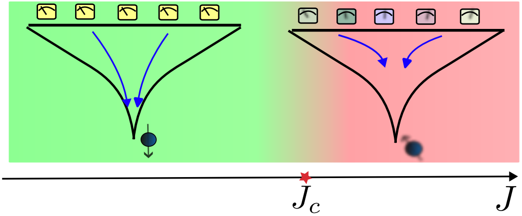
$x \propto \exp(t)$

- Cosmologically relevant. (Experiments: moveable qubits)
- Efficient simulation of lattice models.
- Generates scale invariance.
- **Information propagation**

Analoguous but different: QFT/error correction on AdS

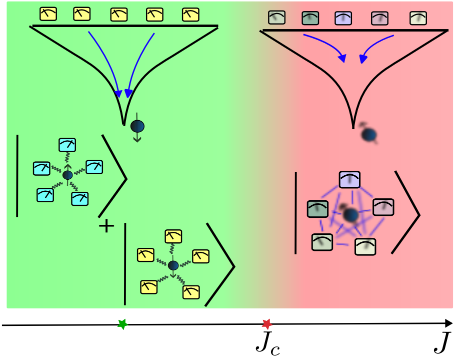
Information propagation through inflation

"Can we learn about the big bang?"



- Two distinct phases of information, in which later time observation can/cannot reveal initially injected information.
- We can go through a sharp "**encoding transition**" between the phases by tuning a knob.

Information propagation: a universe and a lab



The transition is also between a measurement apparatus and a quantum scrambler/encoder.

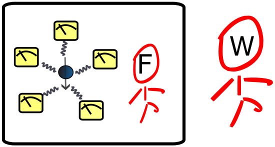
There is also another more subtle "purification" transition.

Plan

- Background: Wigner's friend, classical objectivity, . . .
- Setup: defining the phases
- Toy models on expanding trees [Ferté, XC, PRL+PRA 24]
- General "Harris criterion" for encoding transition (beyond trees). [XC, to be written up]
- News & perspective

Wigner's friend scenario [Wigner 1961]

Wigner observes his friend measuring a qubit in a lab:



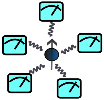
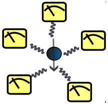
Friend: Born's rule

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \longrightarrow$$

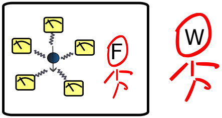
$$\begin{cases} |\uparrow\rangle & \text{with prob. } |\alpha|^2 \\ |\downarrow\rangle & \text{with prob. } |\beta|^2 \end{cases}$$

Wigner: unitary evolution

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\text{app.}\rangle \rightarrow$$

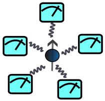
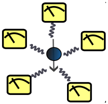
$\alpha \Big|$  $\Big\rangle + \beta \Big|$  $\Big\rangle$

Classical objectivity [Zurek from 2000's]



For the friend, the measurement outcome is *objective*: It is retrievable from multiple records (computer, notebook, . . .) and can be agreed upon by many observers.

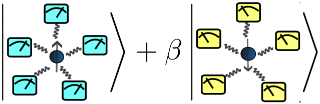
Wigner can attest the emergence of objectivity from the multi-partite correlation established by the dynamics:

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \to \alpha \left| \right.$$  $$\left. \right\rangle + \beta \left| \right.$$  $$\left. \right\rangle . \qquad (1)$$

Classical objectivity *vs* encoding

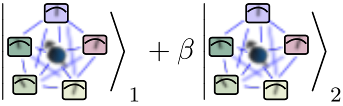
The emergence of objectivity

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \rightarrow \alpha \left| \cdots \right\rangle + \beta \left| \cdots \right\rangle \tag{2}$$



is qualitatively different from quantum thermalisation:

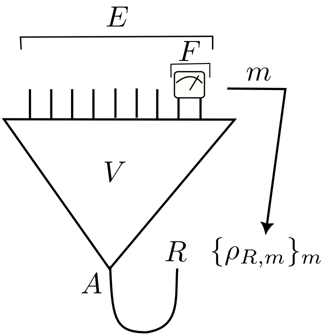
$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \rightarrow \alpha \left| \cdots \right\rangle_1 + \beta \left| \cdots \right\rangle_2 \tag{3}$$



The two terms on the RHS are orthogonal but are locally indistinguishable! The initial information is *encoded*.

Goal: Interpolate between objectivity and encoding.

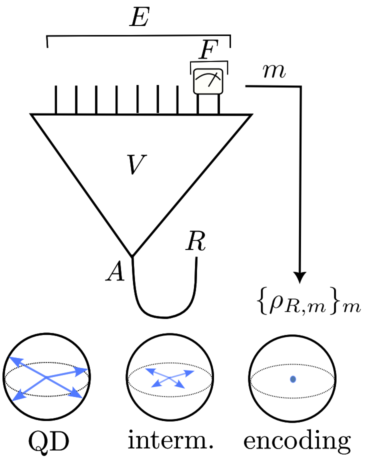
General setup



- Start with an EPR pair AR between the input A (to be measured) and a reference R (record of input).
- Interaction between A and the lab results in output bits E: $V : \mathcal{H}_A \to \mathcal{H}_E, V^\dagger V = I$.
- Can we learn the input (disentangle R) by measuring a **small fraction** F of the output?

Zurek's "Quantum Darwinism" (QD) idea: objectivity is accessibility from small fractions.

Phases of information



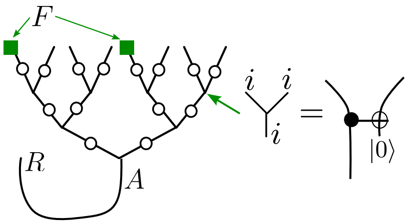
The lab is in ...

- a **QD** phase if $\rho_{R,m}$ is pure (with probability $\to 1$)
- An **encoding** phase $\rho_{R,m}$ is maximally mixed.
- An **intermediate** phase if otherwise.

Two possible transitions: encoding and purification.

Toy models exhibiting the transitions

"Mean-field" model defined on a dynamically expanding tree:

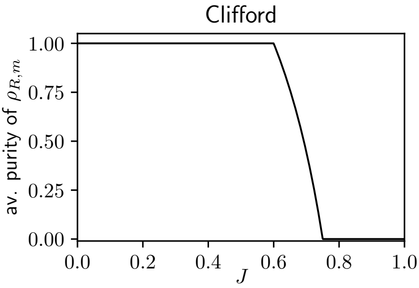


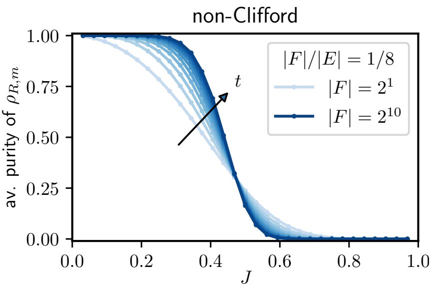
Two similar variants:

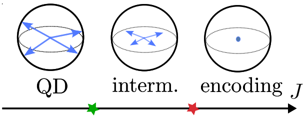
- Random Clifford: $\circ$ is random one-site Clifford with probability J and identity otherwise. Exactly solvable.
- Deterministic: $\circ = \exp(-iJ\sigma^y\pi/4)$. Numerically tractable.

$J = 0$: GHZ state, perfect information broadcast.

Phase diagram

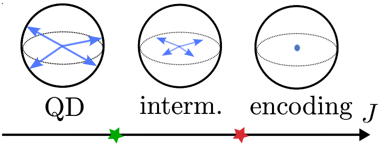






$$\text{purity} := \mathrm{Tr}[\rho^2] - 1 = \begin{cases} 1 & \text{pure state} \\ 0 & \text{maximally mixed state} \end{cases}$$

Remarks

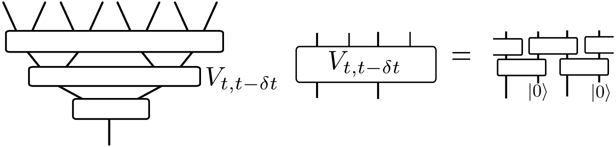


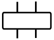
- The ensemble $\{\rho_{R,m}\}_m$ (weighed by Born's rule) becomes independent of the relative fraction size $|F|/|E|$ in the thermodynamic limit $|E| \to \infty$.
- Objectivity emerges in both QD and intermediate phases, in which the detector is working with nonzero efficiency.

Next: focus on the encoding transition.

Theory for the encoding transition

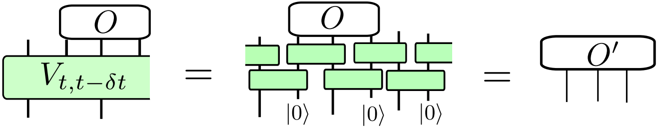
Consider inflationary dynamics in discrete space-time, also known as “MERA”. For definitiveness, focus on the following geometry:

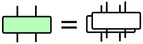


Here  = unitary, and $\delta = \ln 2$ so $L \propto e^t$.

Main point: Such dynamics generates scale invariant states, and allows to define scaling operator and scaling dimension. [Vidal *et al*]

Operator dynamics and coarse-graining

$V^{\dagger}_{t,t-\delta t} O V_{t,t-\delta t} =$ 

where  $U \otimes U^*$.

- Heisenberg picture goes back in time, so space shrinks, and operator growth saturates.
- $O \mapsto O' = V^{\dagger} O V$ implements the operator coarse-graining.
- Diagonalisation gives the spectrum of scaling operators with scaling dimensions $\{\Delta\}$

$$V^{\dagger}_{t,t-\delta t} O_{\Delta} V_{t,t-\delta t} = e^{-\Delta \delta t} O. \quad (4)$$

Analytical result 1

"Harris criterion"

An expanding dynamics is in the encoding phase if and only if all scaling dimensions (except for the identity operator) are large enough:

$$\forall\, \Delta > d/2 \tag{5}$$

where d is the space dimension.

Analytical result 1

"Harris criterion"

An expanding dynamics is in the encoding phase if and only if all scaling dimensions (except for the identity operator) are large enough:

$$\forall \Delta > d/2 \tag{5}$$

where d is the space dimension.

- $\Delta_{\text{lightest}} = d/2$ allows to locate J_c.

Analytical result 1

"Harris criterion"

An expanding dynamics is in the encoding phase if and only if all scaling dimensions (except for the identity operator) are large enough:

$$\forall \Delta > d/2 \tag{5}$$

where d is the space dimension.

- $\Delta_{\text{lightest}} = d/2$ allows to locate J_c.
- Same criterion (due to Harris) determines the relevance of a disordered perturbation.

Analytical result 1

"Harris criterion"

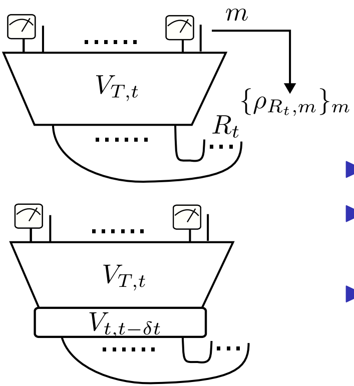
An expanding dynamics is in the encoding phase if and only if all scaling dimensions (except for the identity operator) are large enough:

$$\forall \Delta > d/2 \tag{5}$$

where d is the space dimension.

- $\Delta_{\text{lightest}} = d/2$ allows to locate J_c.
- Same criterion (due to Harris) determines the relevance of a disordered perturbation.
- Same criterion determines whether weak measurement on a CFT ground state affects its long-distance correlations. [Garratt, Weinstein, Altman] [Patil, Ludwig]

Argument for the criterion



- Consider RG flow ($t \to t - dt$) of the ensemble $\{\rho_{R_t,m}\}$

$$\rho \propto e^{\sum_\Delta \int u_\Delta(x) O_\Delta(x) d^d x}, \; u_\Delta \ll 1 \quad (6)$$

near the encoding transition.

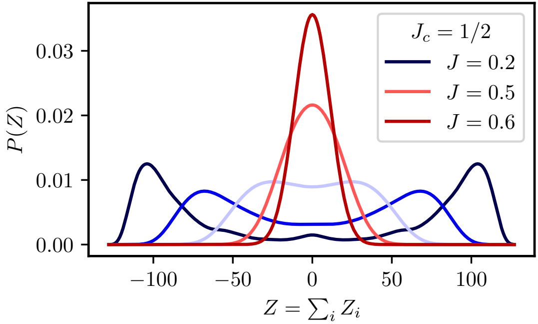
- Sum over m: $\langle \rho \rangle \propto \mathbf{1} \Rightarrow \langle u_\Delta \rangle = 0$.
- Causality $\Rightarrow u_\Delta(x)$ short-range correlated.
- Equivalent to the RG of random perturbation (on a hypersurface):

$$\frac{d\left\langle u_\Delta^2 \right\rangle}{dt} = (d - 2\Delta)\langle u_\Delta^2 \rangle + \dots \quad (7)$$

Analytical result 2

Full counting and Gaussianity

The above Harris criterion applies as well if we only measure the full counting statistics $\sum_{x \in F} O(x)$, where O is a generic operator. In the encoding phase, the statistics tends to Gaussian.



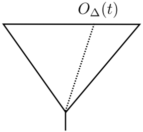
Another argument for the criterion

Let the input be in state $|s\rangle$. Can we infer it by measuring

$$\mathcal{O}(t) = \int O_\Delta(t,\mathbf{r}) d^d\mathbf{r}?$$

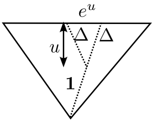
Compare signal and noise:

$$\underbrace{\langle \mathcal{O}(t)\rangle_s}_{\text{signal}} =$$



$$\sim e^{td}e^{-\Delta t}$$

$$\gtrless \underbrace{\left\langle \mathcal{O}(t)^2 \right\rangle}_{\text{noise}} =$$

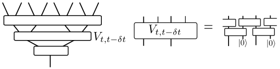


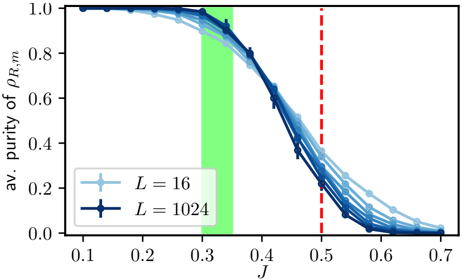
$$\sim C_{\Delta\Delta}^{\mathbf{1}} e^{td} \int_{u=0}^{t} e^{du-2\Delta u} du$$

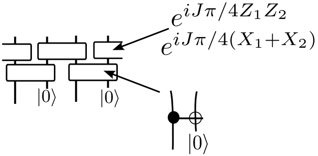
$\Delta > d/2 \implies$ OPE happens at $u=0$: nontrivial operator cannot reach input. (HIgher moments satisfy Wick theorem.)

Recent progress: Numerics beyond trees

Backward evolution method: exact direct sampling of measurement results with space/time cost $O(L)$.

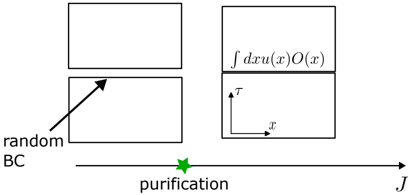


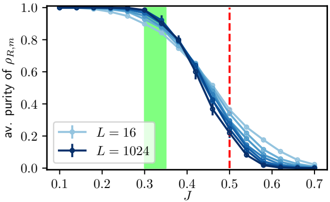




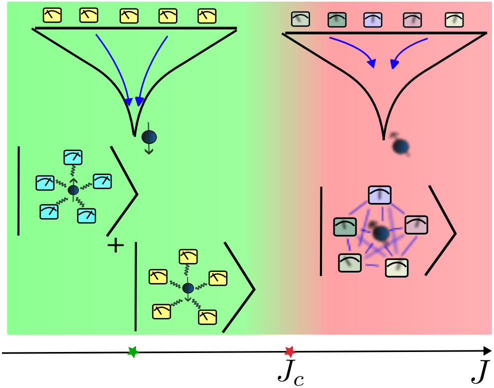
Challenge: purification transition

Vague intuition: a strong "impurity" (co-dimension 1) perturbation tears spacetime (bra and ket) apart, creating random boundary conditions.



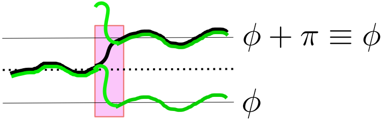


Fleshing out the theory using BCFT?


+
J_c
J

Preliminary example

In a Tomogana-Luttinger liquid (TLL) ground state, where we measure $\cos(2\phi)$, $\sin(2\phi)$, purification can fail from prolification of following defects:



$$S = \frac{1}{2\pi K} \int (\nabla \phi)^2 dx d\tau$$

Free energy cost $\propto 1/(4K) \ln L$.
For an expanding dynamics generating TLL,

$$K_c^{\text{purification}} = \frac{1}{4}, K_c^{\text{encoding}} = \frac{1}{2} \text{ [Garratt \textit{et al}],}$$