P H Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery

The Gradient Flow extended to the Standard Model

Janosch Borgulat, Jonas Kohnen, Robert Harlander

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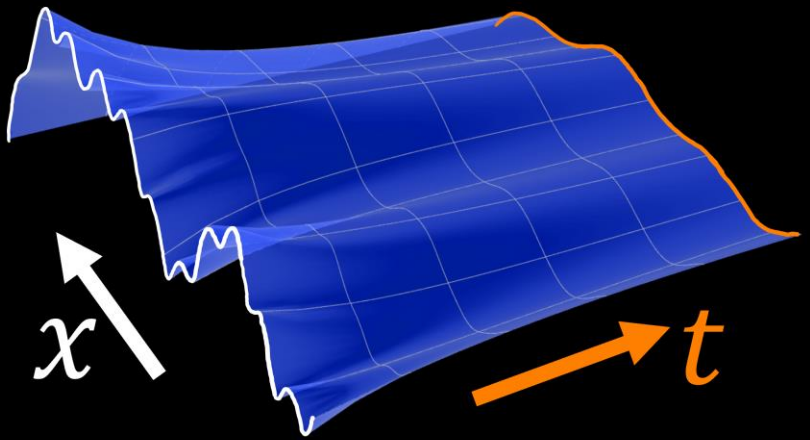
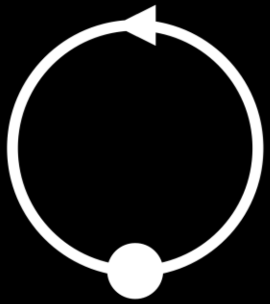
Gradient Flow in QCD

Why is it useful in perturbation theory?



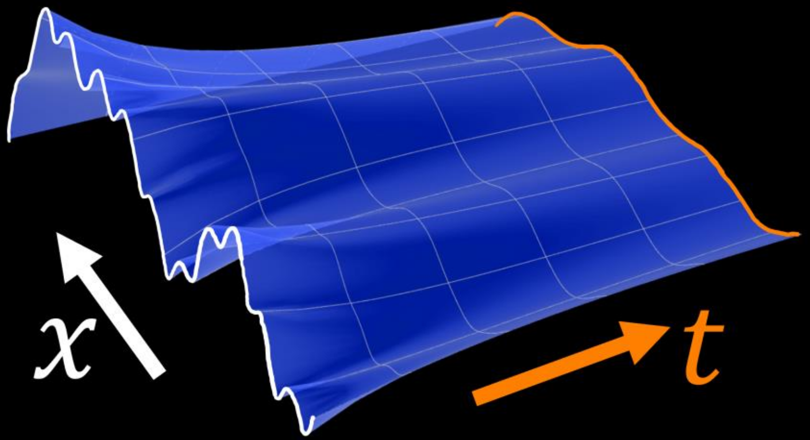
$$\partial_t \chi(t,x) = \mathcal{D}^2(t)\chi(t,x)$$

$$\partial_t \chi(t,x) = \partial^2 \chi(t,x) \quad \Rightarrow \quad \boxed{\hat{\chi}(t,p) = \hat{\psi}(p)e^{-tp^2}}$$



$$\sim \int \frac{\mathrm{d}^D p}{(4\pi)^2} e^{-tp^2}$$ *= fin.*

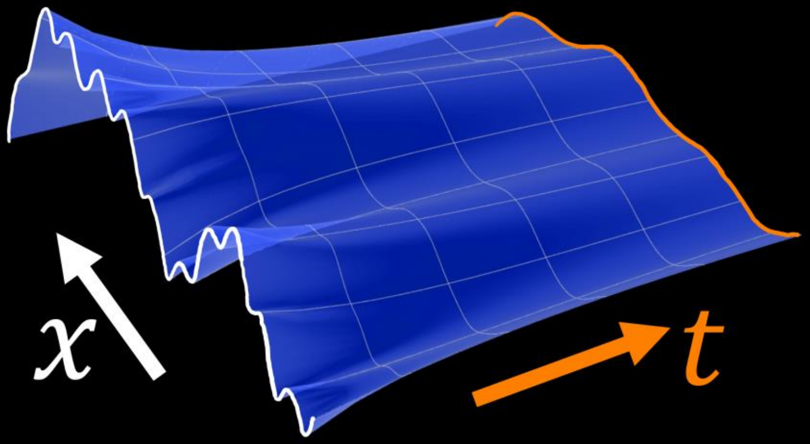
Gradient Flow in QCD



Why is it useful in perturbation theory?

❖ Composite operators are UV finite!

Gradient Flow in QCD



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- Relate to physical ones: SFTX

$$\tilde{\mathcal{O}}(t, x) = \zeta(t)\mathcal{O}(x)$$

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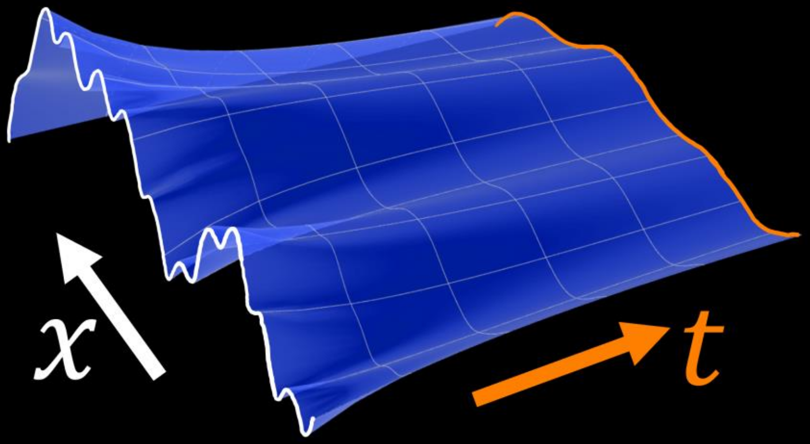
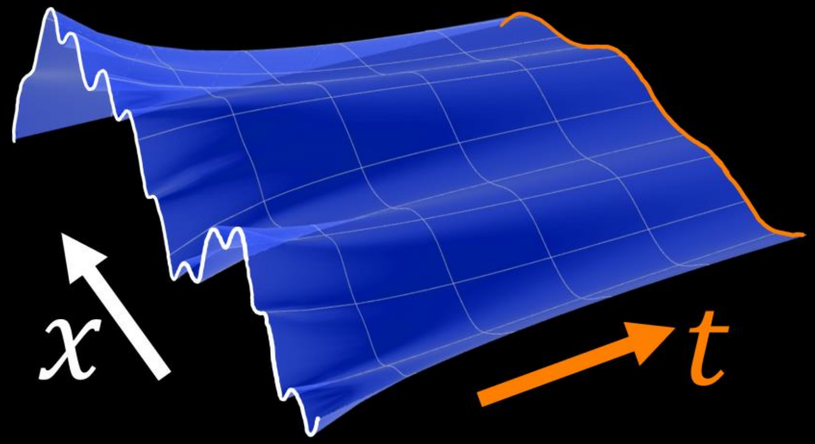


$$\tilde{\mathcal{O}}(t,x) = \zeta(t)\mathcal{O}(x)$$

$$\tilde{\mathcal{O}}(t) = \zeta^{B}(t,\epsilon)\mathcal{O}^{B}(\epsilon) = \underbrace{\zeta^{B}(t,\epsilon)Z^{-1}(\epsilon)}_{\zeta^{R}(t)\overset{!}{=}\text{fin.}}\mathcal{O}^{R}$$

$$\Rightarrow \boxed{Z(\epsilon)}$$

Gradient Flow in QCD

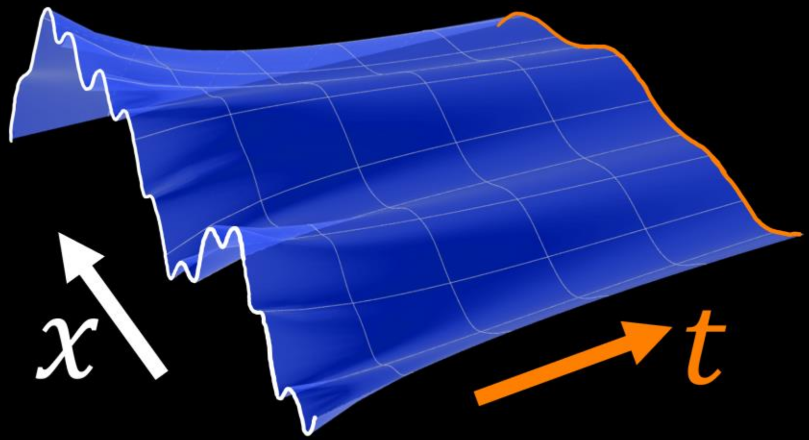


Why is it useful in perturbation theory?

- Composite operators are UV finite!
- Relate to physical ones: SFTX
- $Z(\epsilon)$ cancels IR divergences

$$0 = \frac{1}{\epsilon_{UV}} + \frac{1}{\epsilon_{IR}} \sim \int \mathrm{d}^D k \, \frac{1}{(k^2)^n} \quad \xrightarrow{\text{flow}} \quad \int \mathrm{d}^D k \, \frac{e^{-tk^2}}{(k^2)^n} \sim \frac{1}{\epsilon_{IR}} = -\frac{1}{\epsilon_{UV}}$$

Gradient Flow in QCD



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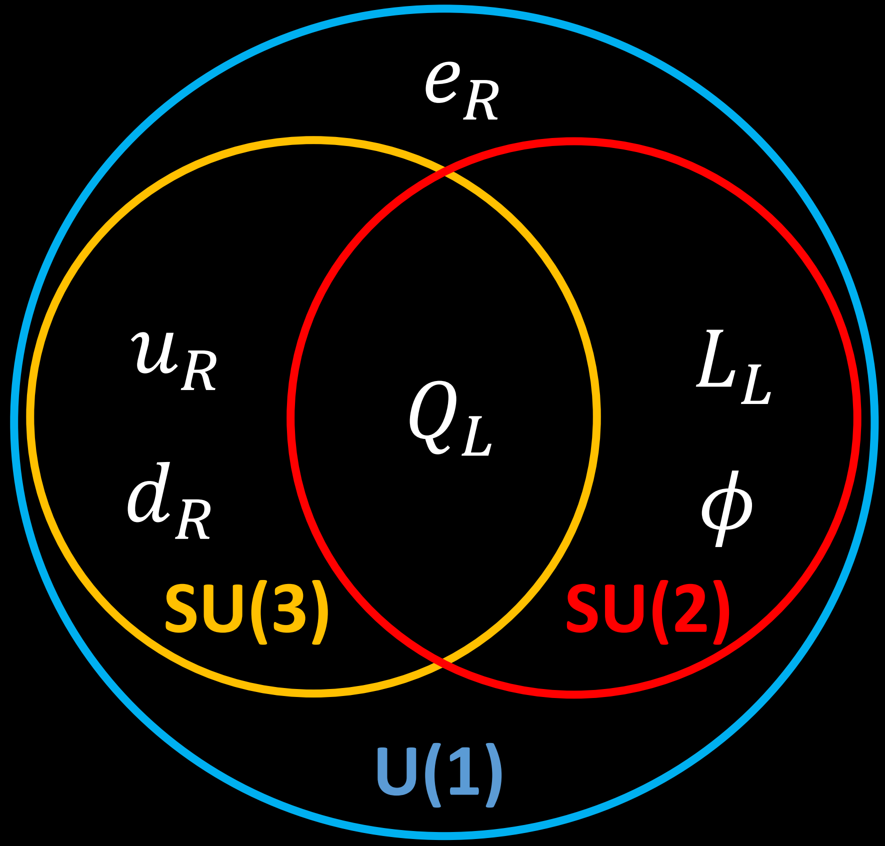
Concept not limited to QCD → What about other theories, like SM?

Extension to SM

$$\mathcal{L}_{\text{SM}} = \bar{Q}_L \not{D} Q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{l}_L \not{D} l_L + \bar{e}_R \not{D} e_R$$
$$- \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$
~~$+ \bar{Q}_L Y_u \tilde{\phi} u_R + \bar{Q}_L Y_d \phi d_R + \bar{l}_L Y_e \phi e_R + \text{h.c.}$~~
$$+ (D_\mu \phi)^\dagger (D^\mu \phi) + m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$
$$+ \mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{gf}}$$

First step: Unbroken phase, no Yukawas

❖ Reuse QCD for SU(3) and SU(2) sectors



Extension to SM

$$\mathcal{L}_{\rm SM} = \bar{Q}_L \not{D} Q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{l}_L \not{D} l_L + \bar{e}_R \not{D} e_R$$
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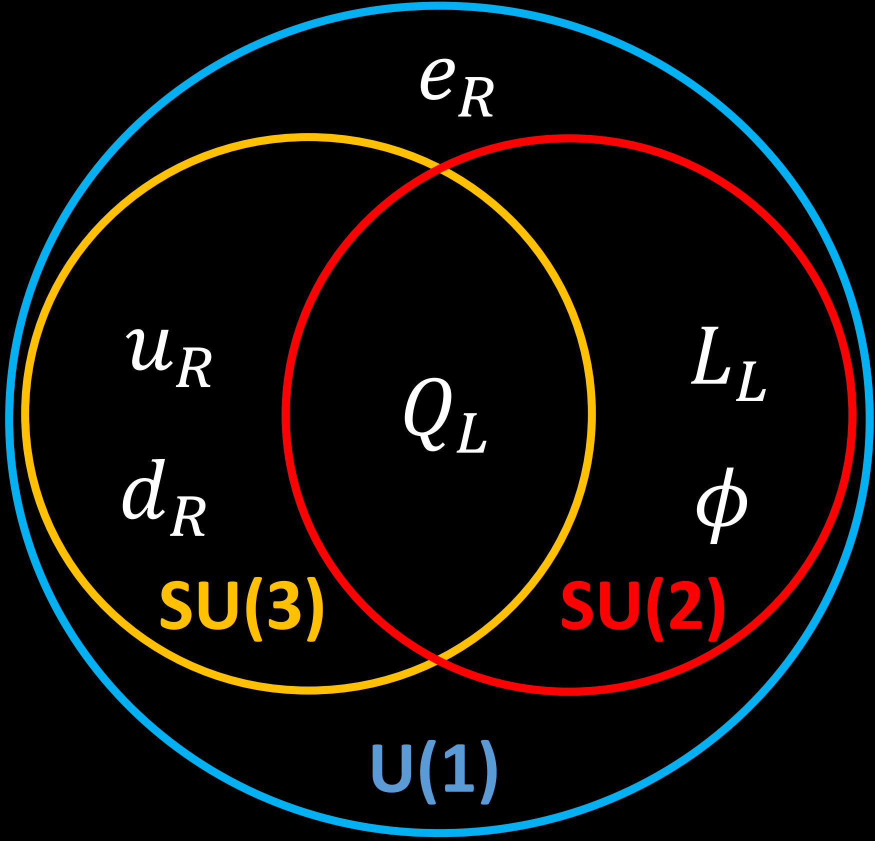
- Reuse QCD for SU(3) and SU(2) sectors
- U(1) linear → no flow lines

$$\partial_t \mathcal{B}_\mu = \left[\partial^2 \delta_{\mu\nu} + (\kappa - 1) \partial_\mu \partial_\nu \right] \mathcal{B}_\nu$$



Extension to SM

$$\begin{aligned}\mathcal{L}_{\text{SM}} = &\bar{Q}_L \not{D} Q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{l}_L \not{D} l_L + \bar{e}_R \not{D} e_R \\ &- \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a\end{aligned}$$

~~$+ \bar{Q}_L Y_u \tilde{\phi} u_R + \bar{Q}_L Y_d \phi d_R + \bar{l}_L Y_e \phi e_R + \text{h.c.}$~~

$$\begin{aligned}&+ (D_\mu \phi)^\dagger (D^\mu \phi) + m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \\ &+ \mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{gf}}\end{aligned}$$

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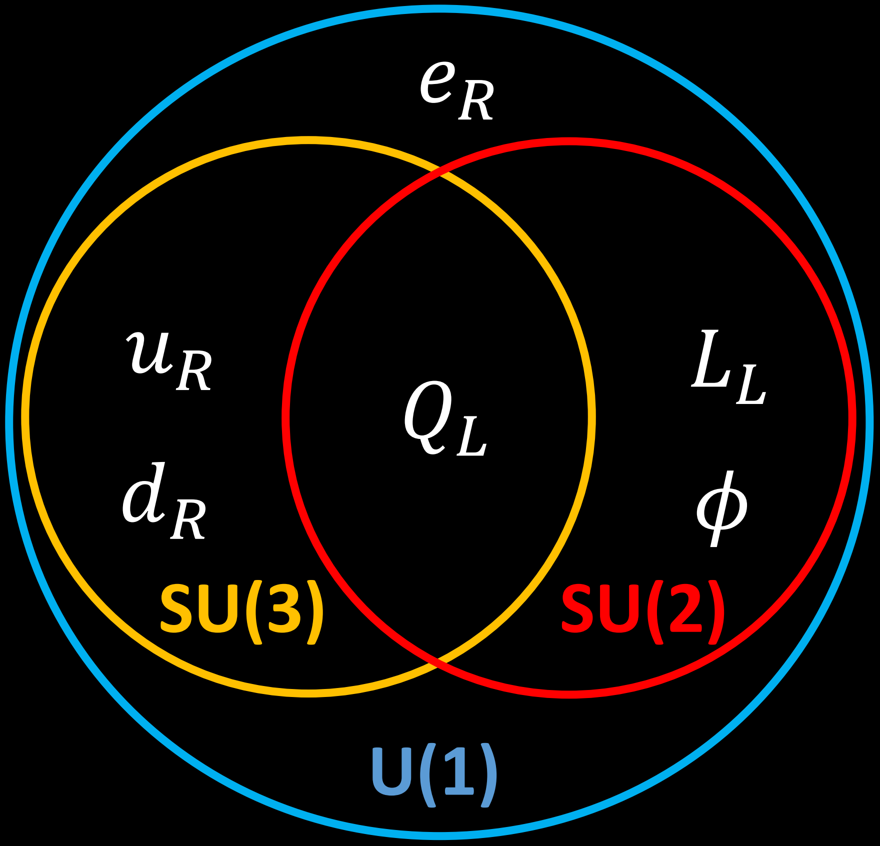
Nontrivial changes:

- Scalar sector
- Mixed terms from multiple gauge groups



Overview

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SFTX: $\varphi^\dagger D_\mu \varphi$

Scalar Noether current

$\rightarrow \quad Z_\varphi$

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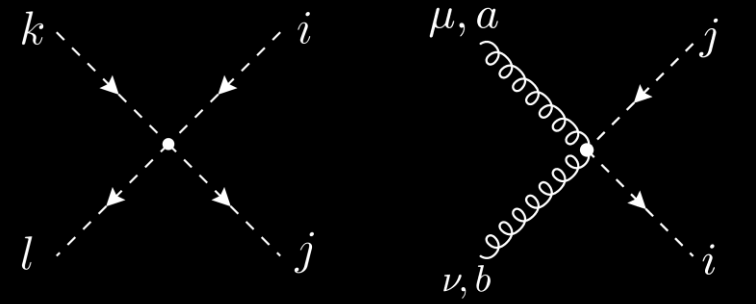
GF scheme for gauge couplings

Scalar Sector

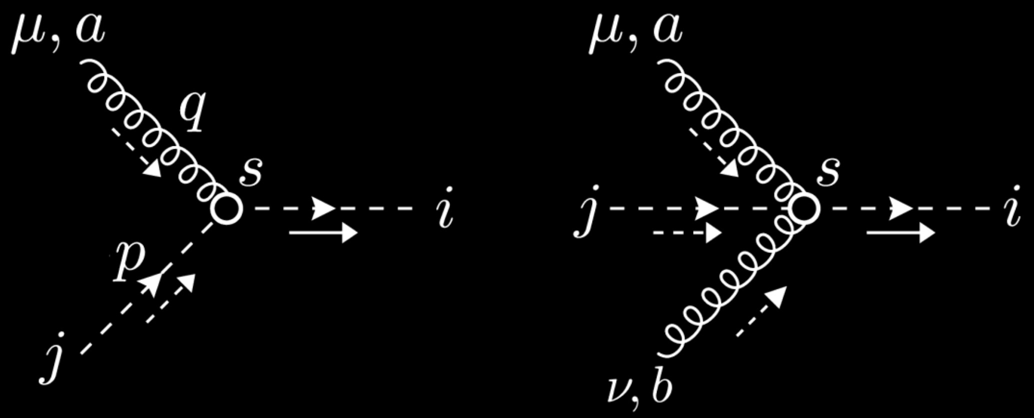
[JB, Harlander, Kohnen 2025]

$$\mathcal{L}_\phi = (D_\mu^F \phi)^\dagger (D_\mu^F \phi) - \frac{\lambda_\mathrm{B}}{4} (\phi^\dagger \phi)^2$$



Flow equation:

$$0 = \partial_t \varphi - \Delta\varphi + g_\mathrm{B} \kappa \partial_\mu B_\mu^a t^a \varphi$$



Two-loop gradient-flow renormalization of scalar QCD

Janosch Borgulat, Nils Felten, Robert V. Harlander, and Jonas T. Kohnen

TTK, RWTH Aachen University, 52056 Aachen, Germany

Abstract

The gradient-flow formalism is applied to a non-Abelian gauge theory with scalar and fermionic particles, dubbed "scalar QCD". It is shown that the flowed scalar quark requires a field renormalization, albeit only beyond the one-loop level. A pseudo-physical, t-dependent renormalization scheme is defined, and the corresponding renormalization constant is evaluated at the two-loop level. We also calculate the gluon action density as well as the condensate of the scalar and the fermionic quark at three-loop level in this theory. The results validate the consistency of the gradient-flow formalism in this theory and provide a further step towards applying the gradient-flow formalism to the full Standard Model.

arXiv:2501.07150

Scalar Sector

[JB, Harlander, Kohnen 2025]

Flowed scalar renormalization

Use SFTX of Noether current!

$$Z_\varphi \, \varphi^\dagger(t) D_\mu(t) \varphi(t) \; = \; d(t) \, \phi^\dagger D_\mu \phi \, + \, O(t)$$

Scalar Sector

[JB, Harlander, Kohnen 2025]

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Use SFTX of Noether current!

❖ Scalar Noether current is finite

$$Z_\varphi \, \varphi^\dagger(t) D_\mu(t) \varphi(t) \; = \; d(t) \boxed{\phi^\dagger D_\mu \phi} + O(t)$$

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[JB, Harlander, Kohnen 2025]

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Use SFTX of Noether current!

- ❖ Scalar Noether current is finite
- ❖ Flowed operators are finite

$$Z_\varphi \boxed{\varphi^\dagger(t) D_\mu(t) \varphi(t)} = d(t) \boxed{\phi^\dagger D_\mu \phi} + O(t)$$

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[JB, Harlander, Kohnen 2025]

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Use SFTX of Noether current!

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$$Z_\varphi \boxed{\varphi^\dagger(t) D_\mu(t) \varphi(t)} = d(t) \boxed{\phi^\dagger D_\mu \phi} + O(t)$$

- SFTX coefficient is finite! → Obtained Z_φ through NNLO (analytic)

Scalar Sector

[JB, Harlander, Kohnen 2025]

Flowed scalar renormalization

How do we calculate $d(t)$?

Scalar Sector

[JB, Harlander, Kohnen 2025]

Flowed scalar renormalization

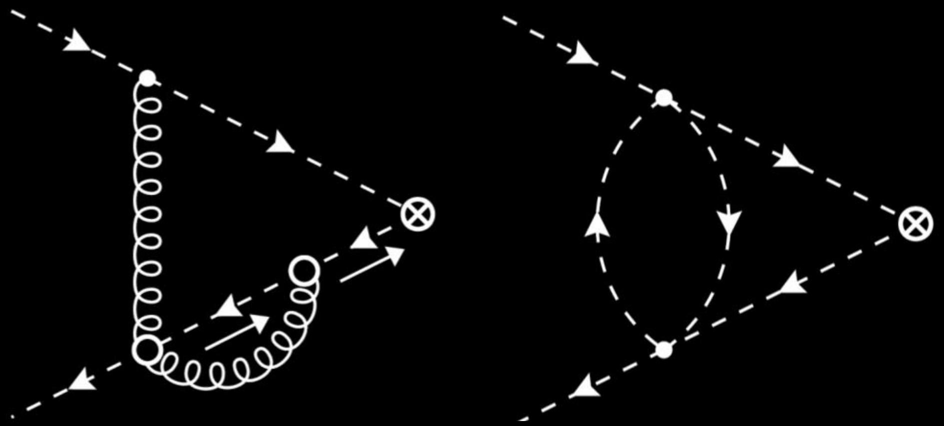
How do we calculate $d(t)$?

General SFTX: $\tilde{\mathcal{O}}(t) = \boxed{\zeta^B(t,\epsilon)}\mathcal{O}^B(\epsilon)$

?

Scalar Sector

[JB, Harlander, Kohnen 2025]



Flowed scalar renormalization

How do we calculate $d(t)$?

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Method of Projectors

[Gorishnii, Larin, Tkachov 1983; Gorishnii, Larin 1987]

Only tree-level contributions at t = 0

$$P[X] \sim \Pi\left(\frac{\partial}{\partial p_i^\nu}, \frac{\partial}{\partial m}\right)\langle\phi(p_1)\psi(p_2)\ldots|X|0\rangle\Big|_{p_i=m=0}$$

Scalar Sector

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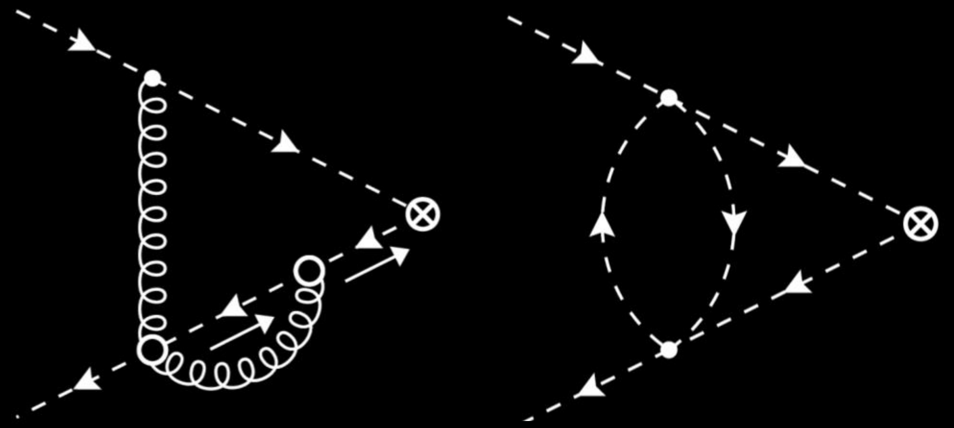

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$$P_i[\mathcal{O}_j] = \delta_{ij} \quad \Rightarrow \quad \zeta^B_{ij}(t,\epsilon) = P_j[\tilde{\mathcal{O}}_i(t)]$$

Higher orders for t > 0

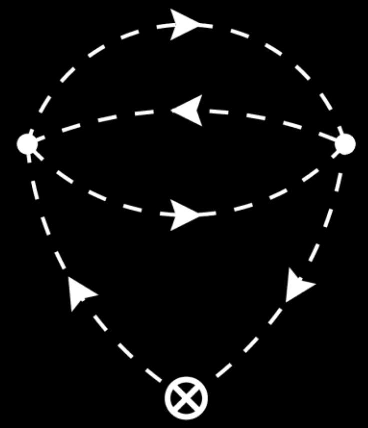
Scalar Sector

[JB, Harlander, Kohnen 2025]

Check: SFTX of scalar bilinear

$$Z_\varphi \varphi^\dagger(t)\varphi(t) = \frac{1}{t} c_0(t)\mathbb{1} + c_1(t) Z_{\phi^\dagger\phi}\phi^\dagger\phi$$

- NNLO results: $c_1(t)$ (2-loop), $c_0(t)$ (3-loop)
- Renormalized by Z_φ
- Corrected $Z_{\phi^\dagger\phi}$ from literature [Gorishnii, Kataev, Larin 1987]



Overview

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Scalar Noether current

✓ $\rightarrow \; Z_\varphi$

$\langle \bar{\chi} \not{D} \chi \rangle$

Ringed scheme

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Fermion vector currents

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$\langle F_{\mu\nu} F_{\mu\nu} \rangle$

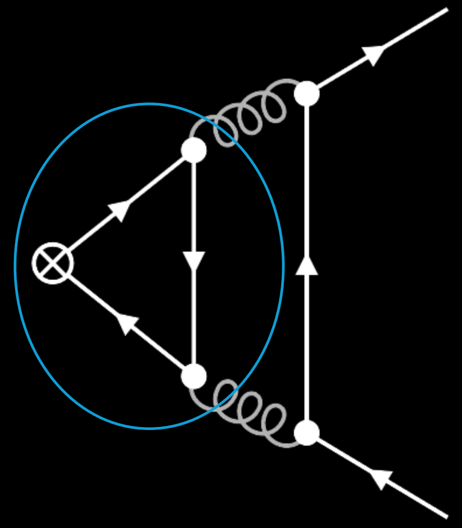
GF scheme for gauge couplings

Fermionic Sector

see also [JB, Harlander, Kohnen, Lange 2023]

Flowed fermion renormalization

- Vector currents $\bar{\psi}\gamma^{\mu}\psi$ finite
- Singlet diagrams require γ_5-scheme



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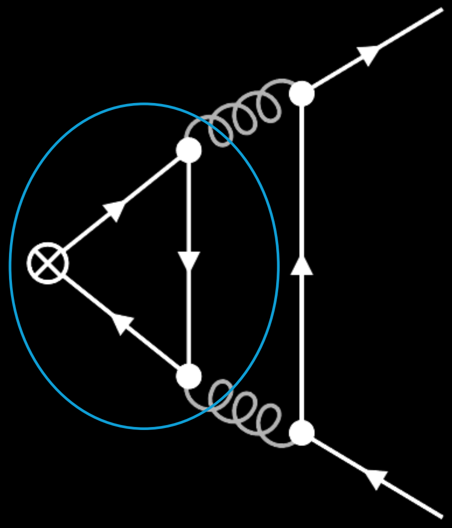
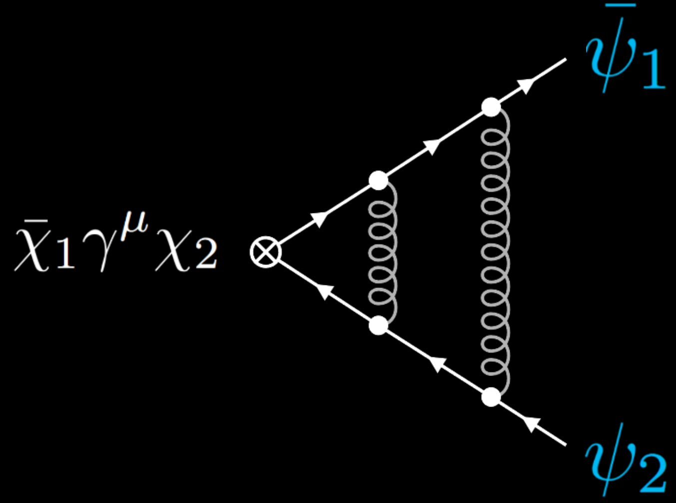


Flowed fermion renormalization

- Vector currents $\bar{\psi}\gamma^\mu\psi$ finite
- Singlet diagrams require γ_5-scheme
- Solution: Off-diagonal currents (non-singlet diagrams)

$$Z_\chi \, \bar{\chi}_1(t)\gamma^\mu\chi_2(t) \; = \; v_\chi(t) \; \bar{\psi}_1\gamma^\mu\psi_2$$



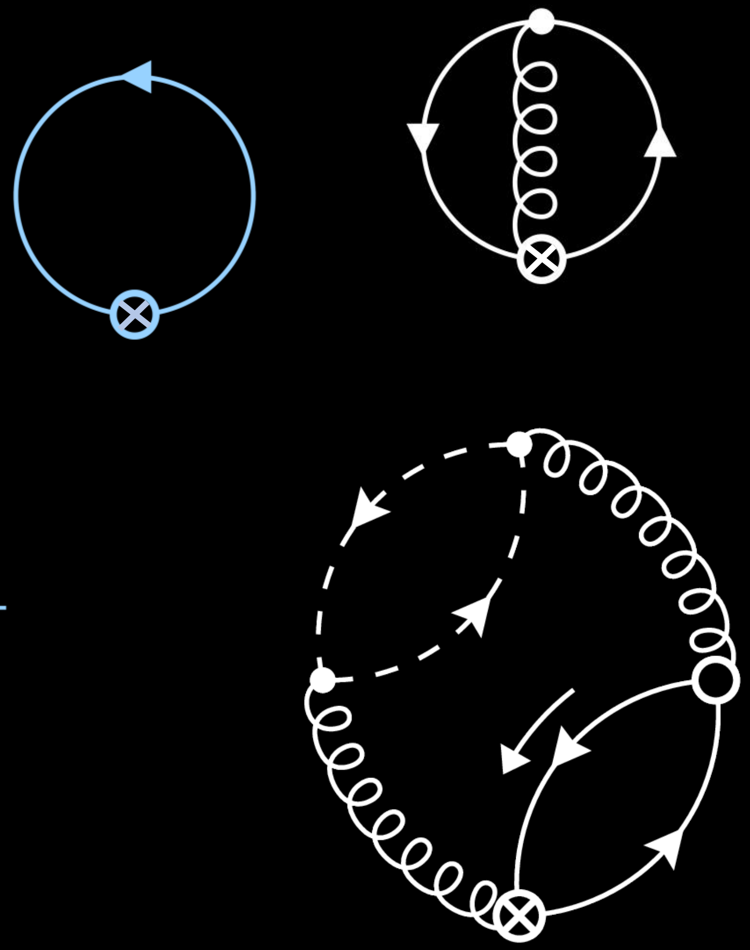
- Obtained Z_χ through NNLO (2-loop)

Fermionic Sector

see also [JB, Harlander, Kohnen 2025]

Check: VEVs of fermion FKOs

❖ QCD: FKO used for ringed scheme



$$Z_\chi \, \langle \overline{\chi}(t) \not{D} \chi(t) \rangle = - \, \zeta_\chi^{-1}(t) \, \frac{N n_\chi}{(4\pi t)^2}$$

❖ Checked Z_χ through NNLO (3-loop)

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$\langle F_{\mu\nu} F_{\mu\nu} \rangle$

GF scheme for gauge couplings

Boson Action Densities

see also [JB, Harlander, Kohnen 2025]

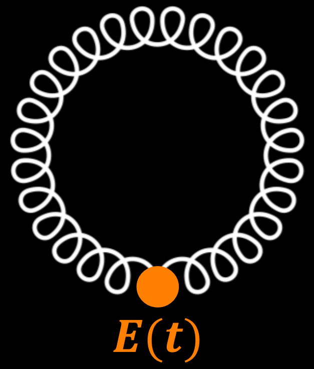
GF scheme for gauge couplings [Lüscher 2010; Lüscher 2014; Harlander, Neumann 2016]

❖ QCD:

$$E(t) = \frac{1}{4}\mathcal{G}^a_{\mu\nu}(t)\mathcal{G}^a_{\mu\nu}(t)$$

$$\langle t^2 E(t)\rangle = \frac{3\alpha_s}{4\pi}\frac{N_\mathrm{A}}{8} + \mathcal{O}(\alpha_s^2)$$

$$\alpha_s = \frac{32\pi}{3N_\mathrm{A}}\langle t^2 E(t)\rangle + \ldots$$



$$\sim \int \frac{\mathrm{d}^D p}{(4\pi)^2} e^{-tp^2} \neq 0$$

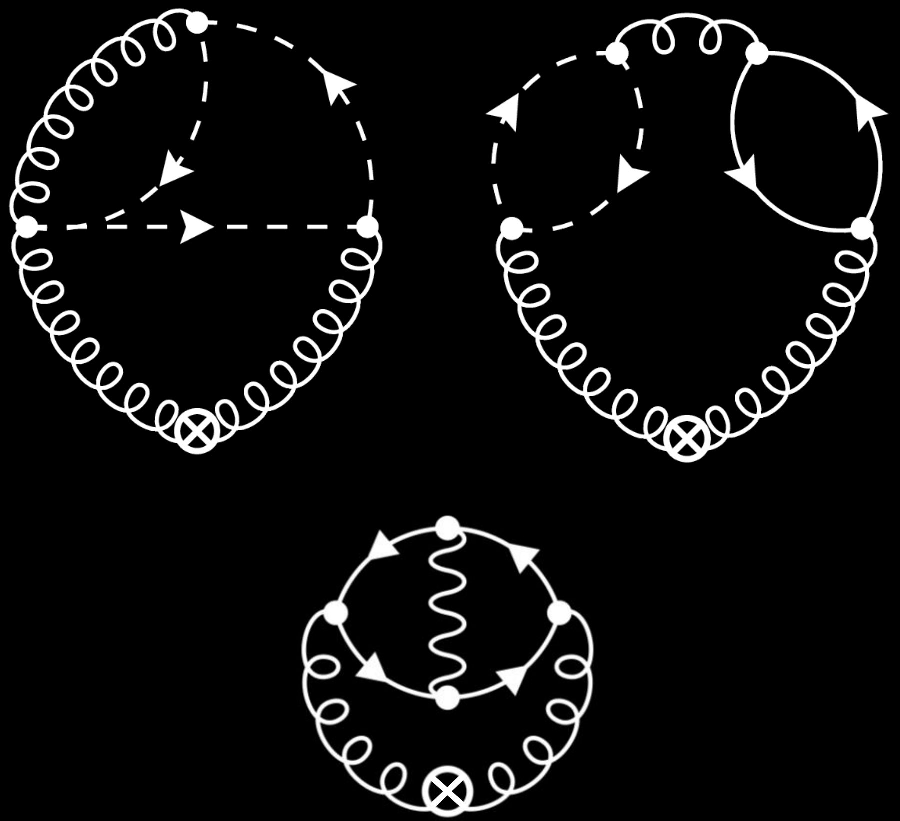
Boson Action Densities

see also [JB, Harlander, Kohnen 2025]

GF scheme for gauge couplings

- SM:

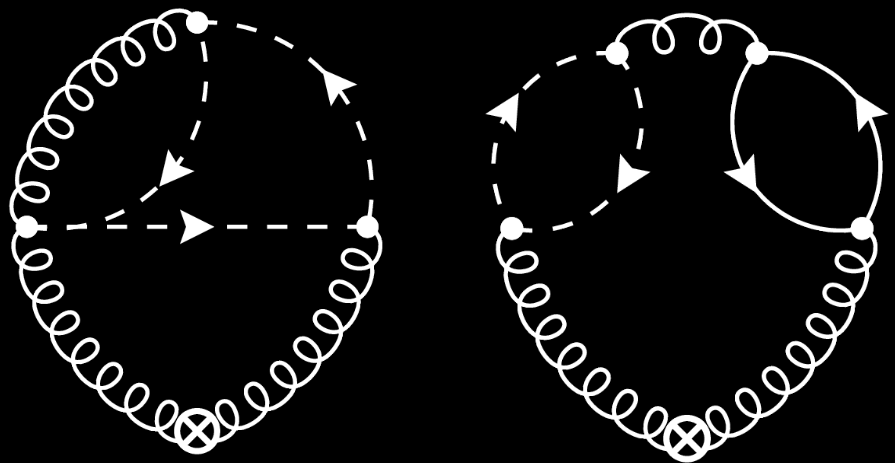
$$\alpha_1^{\mathrm{GF}} \sim E_1(t) = Z_1 \langle \mathcal{B}_{\mu\nu} \mathcal{B}_{\mu\nu} \rangle$$
$$\alpha_2^{\mathrm{GF}} \sim E_2(t) = Z_2 \langle \mathcal{W}^a_{\mu\nu} \mathcal{W}^a_{\mu\nu} \rangle$$
$$\alpha_3^{\mathrm{GF}} \sim E_3(t) = Z_3 \langle \mathcal{G}^a_{\mu\nu} \mathcal{G}^a_{\mu\nu} \rangle$$



- $E_i(t)$ obtained through NNLO (3-loop)
- Check: Z_i agree with SM values (NLO)

Boson Action Densities

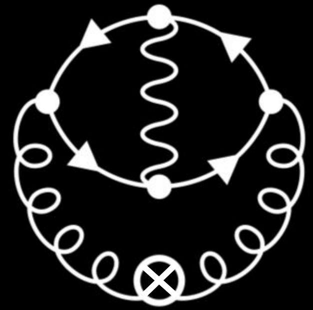
see also [JB, Harlander, Kohnen 2025]



GF scheme for gauge couplings

❖ Example result (preliminary):

$$16\pi^2 t^2 E_2(t_\mu) = \frac{9}{2} + a_2\left(\frac{203}{16} - n_G + \frac{33}{2}\log 2 - \frac{27}{4}\log 3\right)$$
$$+ a_1 a_2\left(0.7770 - 0.4593\, n_G\right) + a_2 a_3\left(1.378\, n_G\right)$$
$$+ a_2^2\left(27.48 - 16.50\, n_G + 0.5447\, n_G^2\right)$$



$$t_\mu = \frac{1}{8\pi\mu^2}$$

Boson Action Densities

see also [JB, Harlander, Kohnen 2025]

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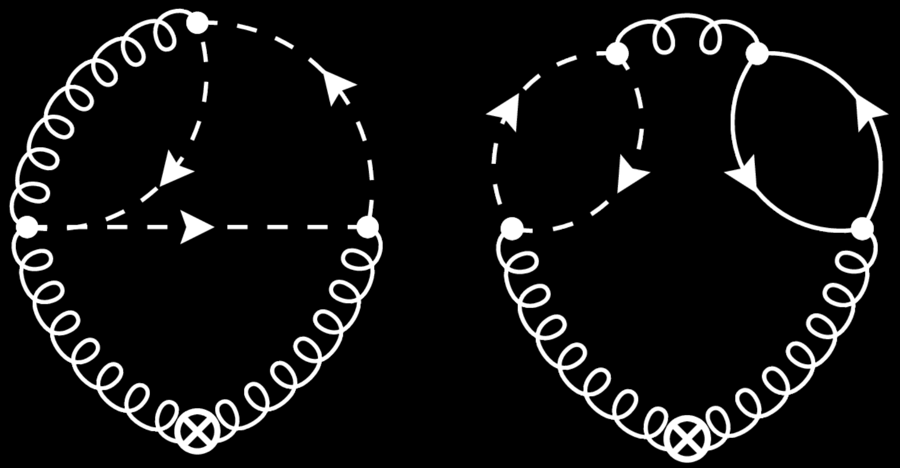

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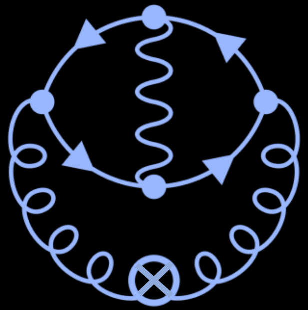

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✓ $\rightarrow \quad Z_\varphi$

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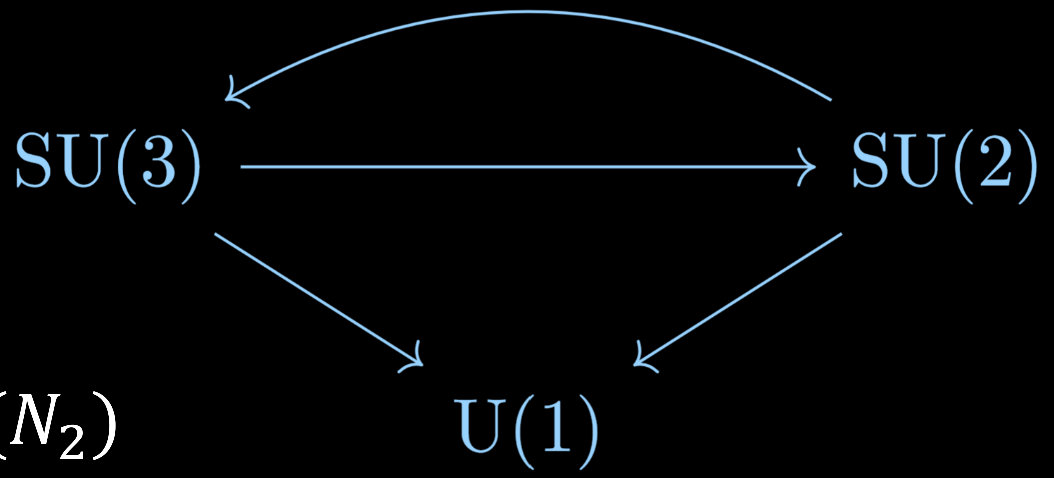
Ringed scheme
✓

SFTX: $\bar{\chi}\gamma^\mu\chi$
Fermion vector currents
✓ $\rightarrow \quad Z_\chi$

$\langle F_{\mu\nu} F_{\mu\nu} \rangle$
GF scheme for gauge couplings
✓

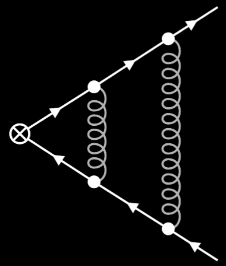
Additional Checks



- We actually use $\mathrm{U}(1) \times SU(N_1) \times SU(N_2)$
- We reproduced earlier flowed QCD results
- Except for VEVs, we use general R_ξ-gauge

Automation

$$\int_{p,k}\int_{[0,1]^3} \mathrm{d}^3u\, u^c \frac{e^{-\left[P_1(u)p^2+P_2(u)k^2+P_3(u)(p+k)^2\right]t}}{(p^2)^{a_1}(k^2)^{a_2}((p+k)^2)^{a_3}}$$



≤ 4125 diagrams

208 flowed integrals

qgraf → **tapir** → **exp** → **form**

- qgraf: Generate Diagrams
- tapir: Insert abstract Feynman rules
- exp: Implement topology
- form: Feynman Rules Integral Identification

Generate IBP relations

[Tkachov 1981; Chetyrkin, Tkachov 1981; Artz, Harlander, Lange, Neumann, Prausa 2019]

Reduction with **kira+FireFly**

[Maierhöfer, Usovitsch, Uwer 2018; Klappert *et al.* 2021; Klappert, Lange 2020; Klappert, Klein, Lange 2021]

→ **6 master integrals**

3-loop: Numerical evaluation using ftint & pySecDec

[Harlander et al. 2024; Borowka et al. 2015, 2018, 2019; Heinrich et al. 2024]

Conclusion & Outlook

SM Results (unbroken phase, vanishing Yukawa couplings)

✓ Z_φ & Z_χ through NNLO for all fermions & Higgs

✓ Flowed Noether currents and ringed scheme

✓ Flowed bosonic action densities

✓ Agreement with SM & flowed QCD results

✓ All results independent of R_ξ gauge

***Next steps:** Yukawas, calculate higher-dimensional operators ...*