Using Gradient Flow to Renormalise Matrix Elements for Meson Mixing and Lifetimes

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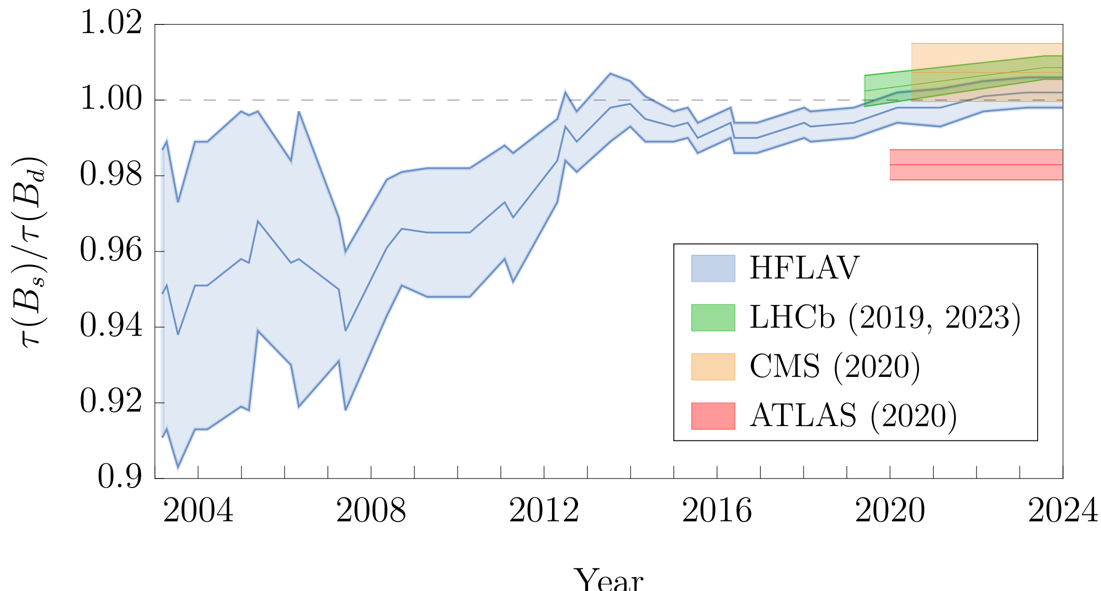
Introduction

- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**

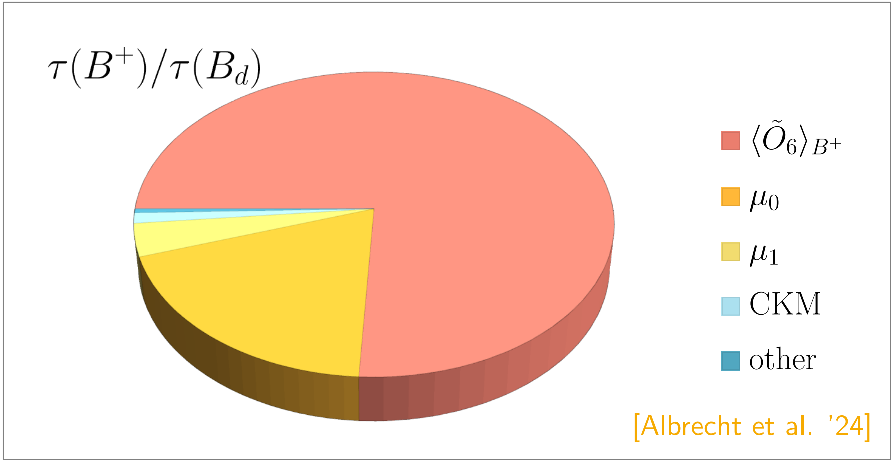


[Albrecht et al. '24]

- For B lifetimes and mixing, we use the **Heavy Quark Expansion**

$$\Gamma(H_Q) = \Gamma_3\langle\mathcal{O}_3\rangle + \Gamma_5\frac{\langle\mathcal{O}_5\rangle}{m_Q^2} + \Gamma_6\frac{\langle\mathcal{O}_6\rangle}{m_Q^3} + \ldots + 16\pi^2\left[\tilde{\Gamma}_6\frac{\langle\tilde{\mathcal{O}}_6\rangle}{m_Q^3} + \tilde{\Gamma}_7\frac{\langle\tilde{\mathcal{O}}_7\rangle}{m_Q^4} + \ldots\right]$$

- $\langle\tilde{\mathcal{O}}_6\rangle$ are leading uncertainties for both B lifetimes and mixing



[Albrecht et al. '24]

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups ➡ precision increasing
  - ➥ Preliminary $\Delta K = 2$ for Kaon mixing with gradient flow [Suzuki et al. '20], [Taniguchi, Lattice '19]
- $\Delta B = 0$ ➡ exploratory studies from ∼20 years ago
  - ➥ Contributions from statistically-noisy diagrams
  - ➥ Mixing with lower dimension operators in renormalisation

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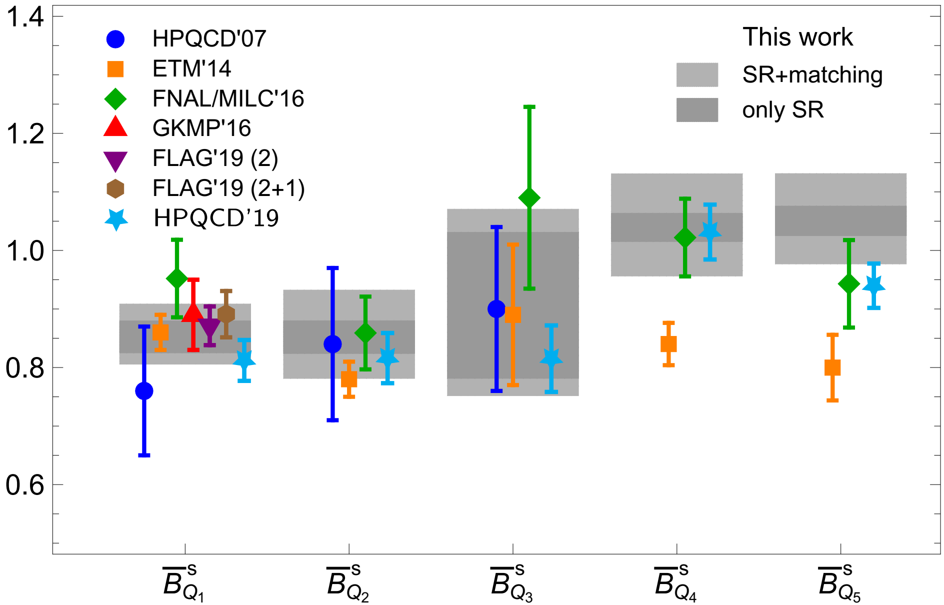
New Developments:

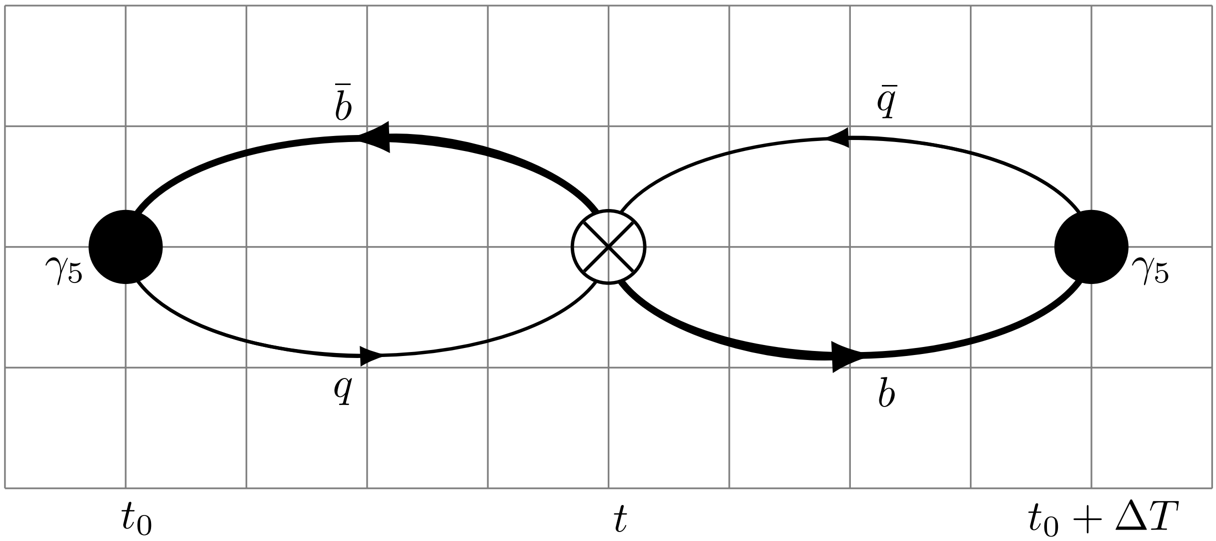
- [Lin, Detmold, Meinel '22] ➡ spectator effects in b hadrons
  - Focus on lifetime ratios for both B mesons and Λ_b baryon
  - Isospin breaking, $\langle B|\mathcal{O}^d - \mathcal{O}^u|B\rangle$
  - Position-space renormalisation + perturbative matching to $\overline{\mathrm{MS}}$
- This work; [Black et al. '23], [Black et al. '24]
  - Goal is individual $\Delta B = 0$ matrix elements for B mesons
  - Non-perturbative gradient flow renormalisation
  - Perturbative matching to $\overline{\mathrm{MS}}$ in short-flow-time expansion

Operators — Current Status

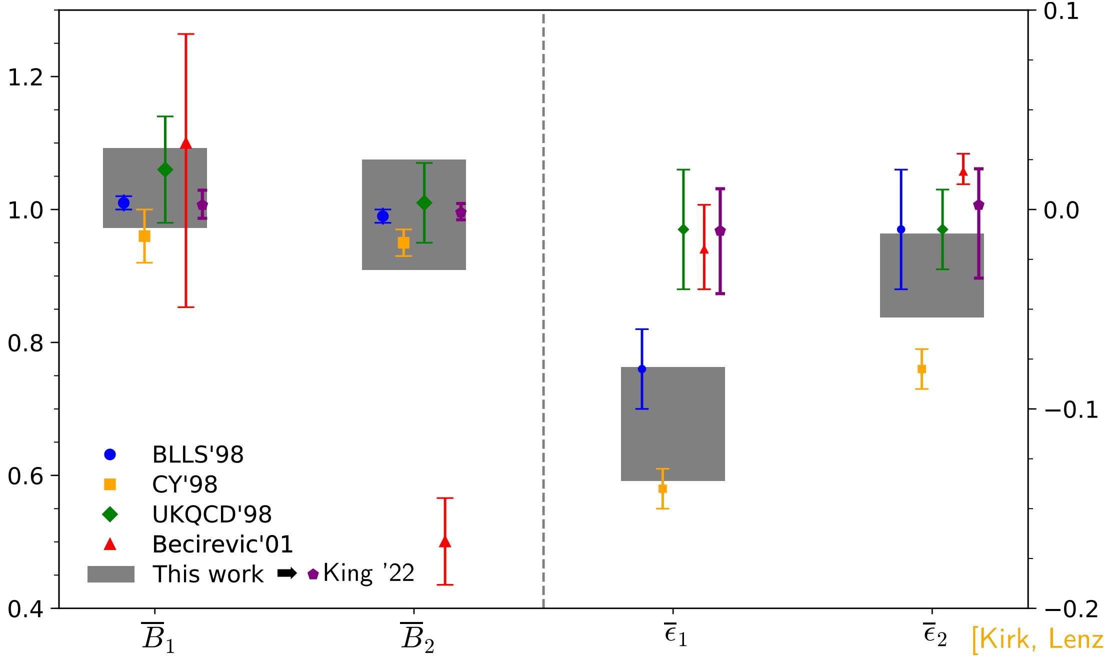


[King '22]

- $\Delta B = 2$ Bag parameters well-studied on the lattice and with QCD sum rules
- See also ongoing work by RBC/UKQCD and JLQCD [Boyle et al '21] [Tsang, Lattice '23]
- Dimension-7 matrix elements calculated for first time [HPQCD '19]



B_q ➡ $\langle Q_6 \rangle$ ➡ $\bar{B}_q$



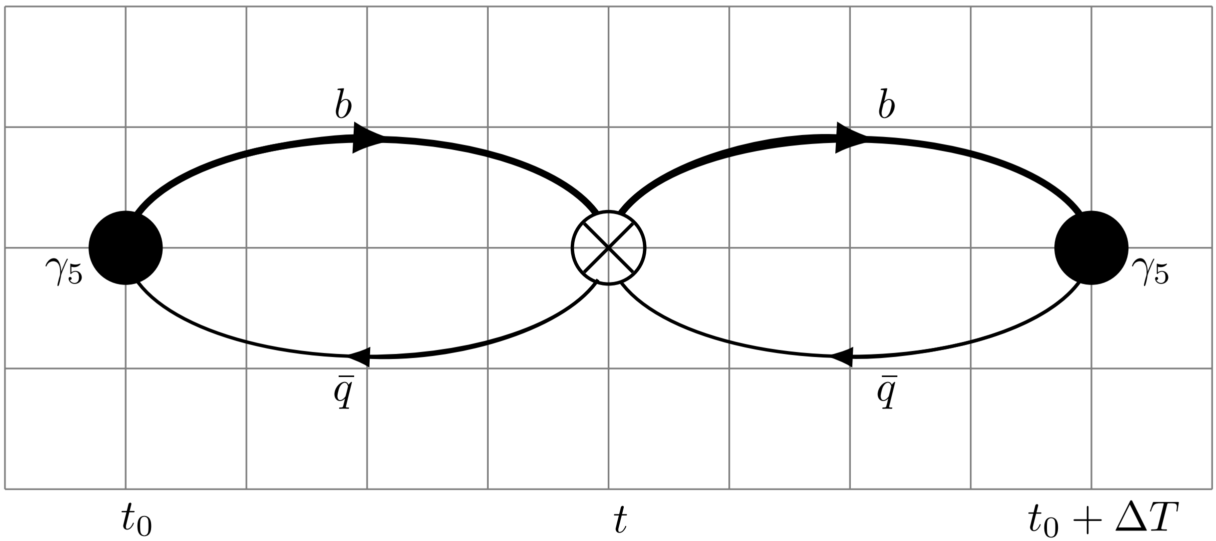
[Kirk, Lenz, Rauh '17]

➤ Sum rules results taken in HQET limit

➤ No complete unquenched lattice simulations to date!

Why?

- Start of calculation follows similar to operators for neutral meson mixing
  - ➥ Well-established on lattice!



B_q ➡ $\langle \mathcal{O}_6 \rangle$ ➡ B_q

- Start of calculation follows similar to operators for neutral meson mixing
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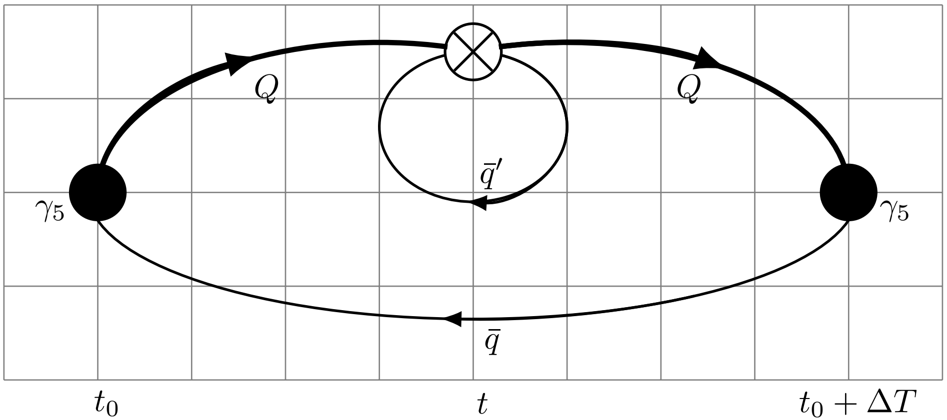
But

- Gluon-disconnected diagrams
- 'Eye' diagrams

→ Statistically very noisy

➥ Modern computing speed and algorithms can help! ✔

Not yet included in lattice simulation or matching



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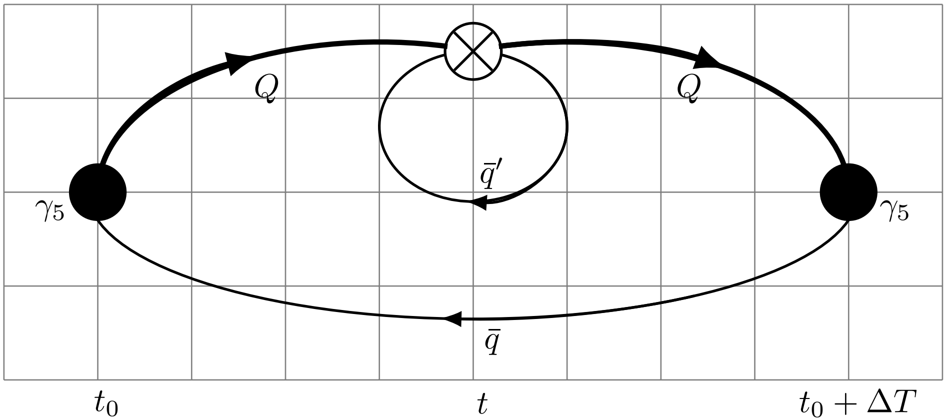


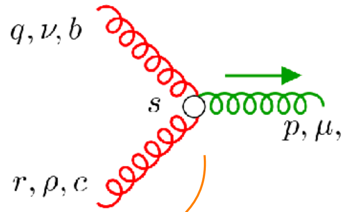
- ➤ Mixing with lower-dimensional operators in renormalisation
  - ➥ Power divergent ➥ Notoriously challenging

→ **How can we tackle this?**

Gradient Flow

- Well-studied for e.g. energy-momentum tensor [Makino, Suzuki '14] [Harlander, Kluth, Lange '18]
- Re-express effective Hamiltonian in terms of 'flowed' operators:

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau)\widetilde{\mathcal{O}}_n(\tau).$$



new Feynman diagrams

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

matching matrix calculated perturbatively

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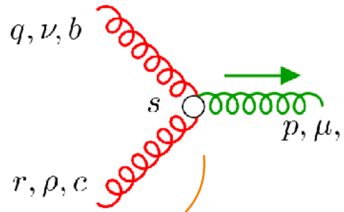

new Feynman diagrams

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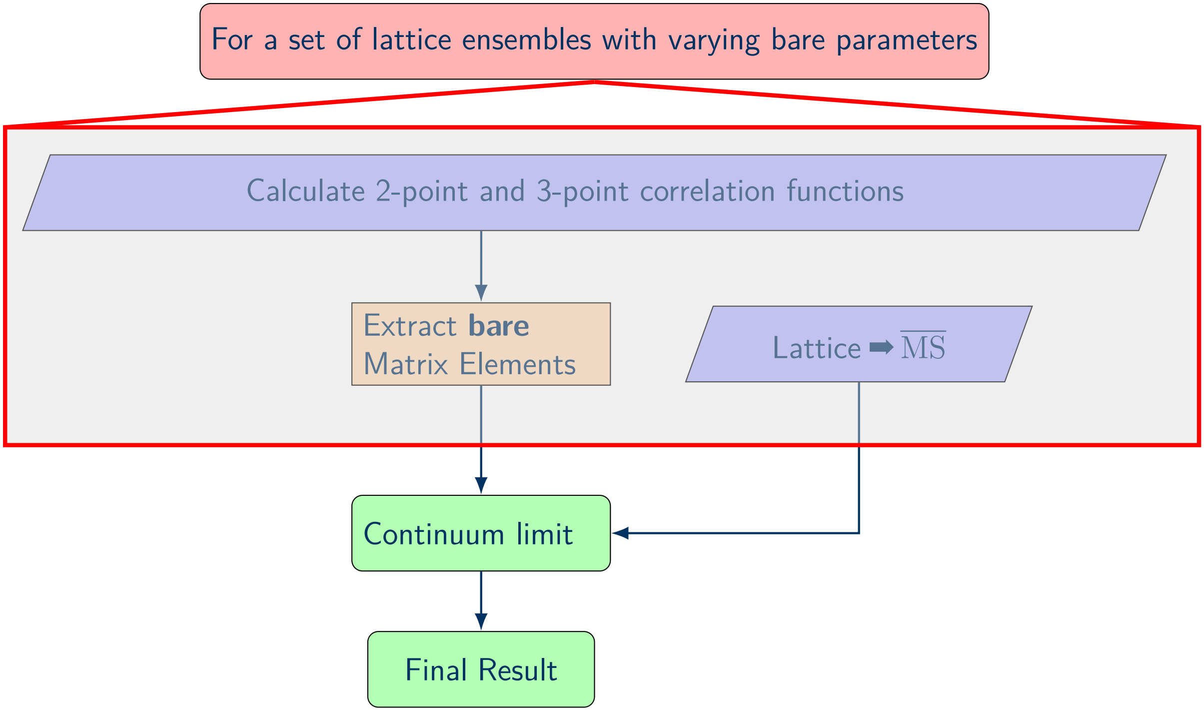
$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

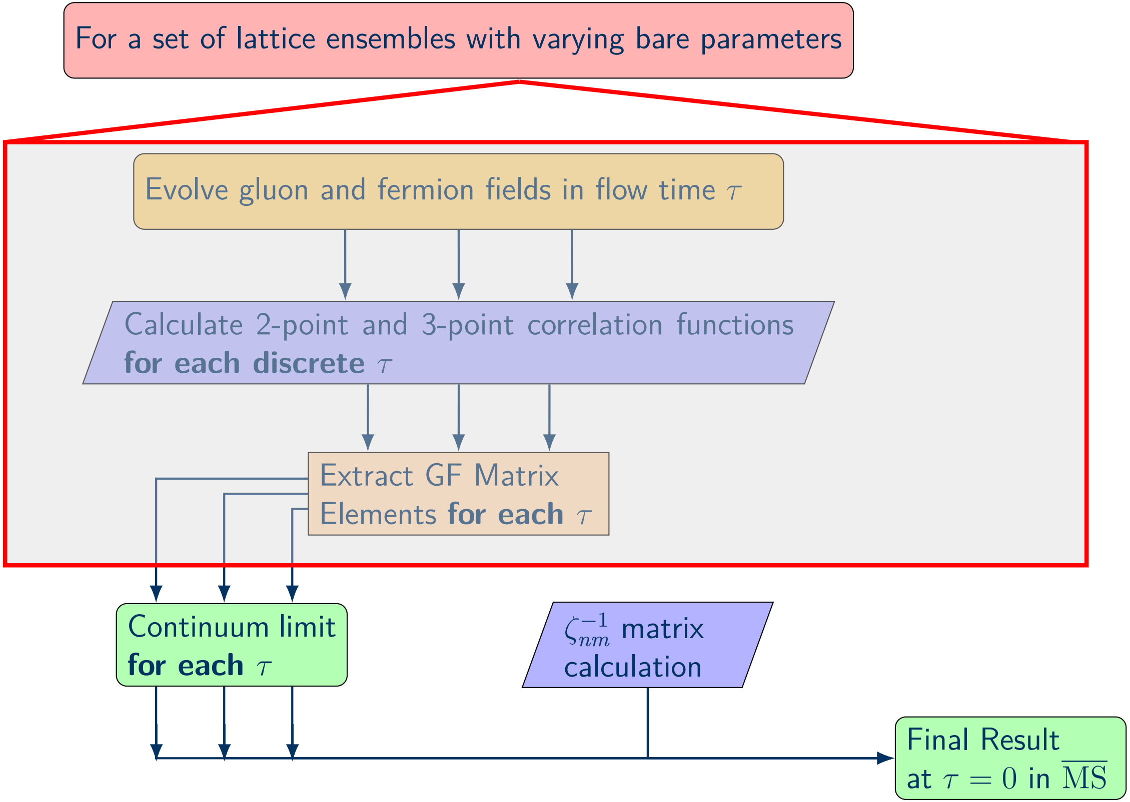
'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

matching matrix calculated perturbatively

$$\sum_n \zeta^{-1}_{nm}(\mu, \tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Matrix element $\langle\mathcal{O}_m\rangle(\mu)$ in $\overline{\text{MS}}$ found in $\tau \to 0$ limit ➡ 'window' problem
  - ➥ large systematic effects at very small flow times
  - ➥ large flow time dominated by operators $\propto O(\tau)$


For a set of lattice ensembles with varying bare parameters
Calculate 2-point and 3-point correlation functions
Extract **bare** Matrix Elements
Lattice ➡ $\overline{\mathrm{MS}}$
Continuum limit
Final Result



For a set of lattice ensembles with varying bare parameters
Evolve gluon and fermion fields in flow time τ
Calculate 2-point and 3-point correlation functions
for each discrete τ
Extract GF Matrix
Elements **for each** τ
Continuum limit
for each τ
ζ_{nm}^{-1} matrix
calculation
Final Result
at $\tau = 0$ in $\overline{\text{MS}}$

Lattice Details

- We use RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles

	L	T	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	M_π/MeV	srcs × N_{conf}
C1	24	64	1.7848	0.005	0.040	340	32 × 101
C2	24	64	1.7848	0.010	0.040	433	32 × 101
M1	32	64	2.3833	0.004	0.030	302	32 × 79
M2	32	64	2.3833	0.006	0.030	362	32 × 89
M3	32	64	2.3833	0.008	0.030	411	32 × 68
F1S	48	96	2.785	0.002144	0.02144	267	24 × 98

[Allton et al. '08]
[Aoki et al. '10]
[Blum et al. '14]
[Boyle et al. '17]

- For strange quarks tuned to physical value, $am_q \ll 1$ ✔
- For heavy b quarks, $am_q > 1$ ➡ large discretisation effects ✗
  - ➥ manageable for physical c quarks instead
  - ➥ stout-smeared Möbius DWF [Cho et. al '15]
- Exploratory setup using physical charm and strange quarks
  - ➥ $\Delta B = 0, 2$ ➡ $\Delta Q = 0, 2$, for generic heavy quark Q

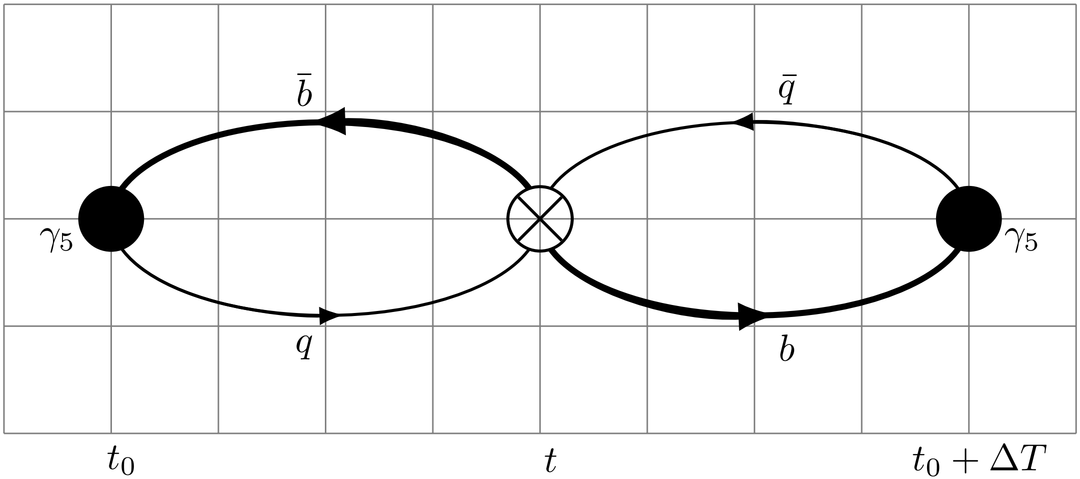
1. Complete exploratory studies in simplified setup without additional extrapolations
   - ➥ test case for gradient flow renormalisation and short-flow-time expansion procedure
   - ➥ simulate physical charm and strange quarks
2. Use $\Delta Q = 2$ matrix elements for further validation of method
   - ➥ neutral charm-strange meson ➡ proxy to short-distance D^0 mixing ($+$ spectator effects)
3. Pioneer $\Delta Q = 0$ matrix element calculation for lifetime differences
4. Run full-scale simulations for B meson mixing and lifetimes
   - ➥ simulate at multiple charm-like masses to extrapolate to b
   - ➥ consider both light and strange spectators
5. Tackle additional contributions for absolute lifetimes
   - ➥ 'eye' diagrams

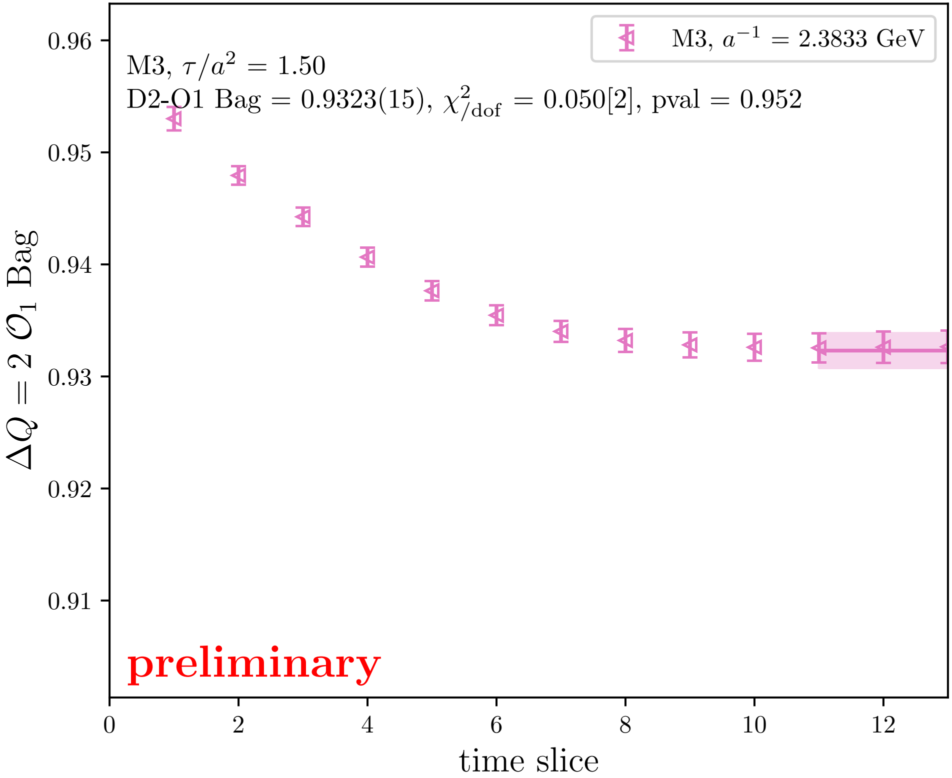
Analysis and Results

- Three-point correlation function:

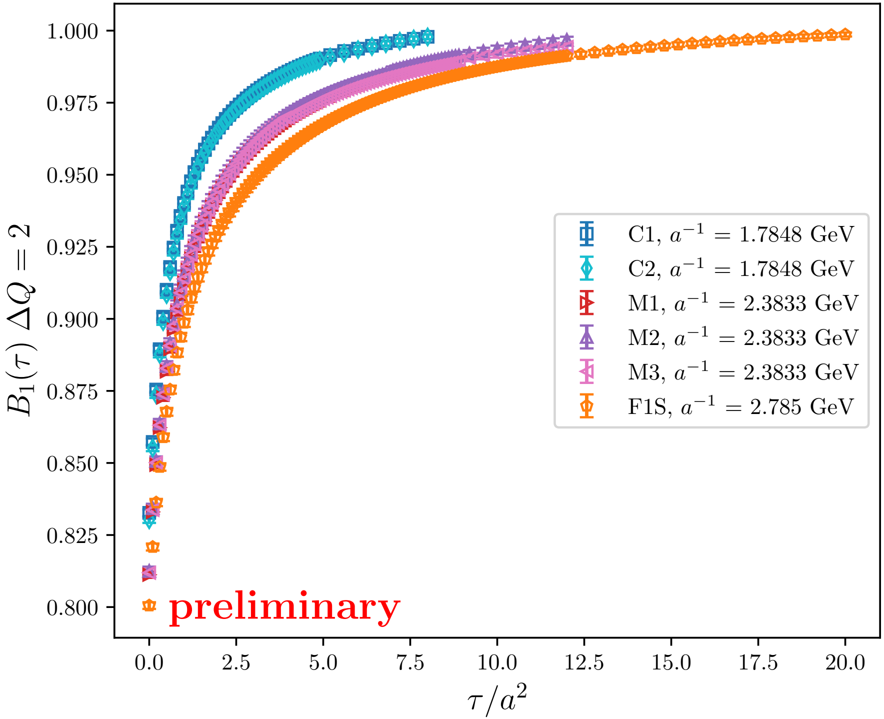
$$C^{3\text{pt}}_{\mathcal{Q}_i}(t, \Delta T, \tau) = \sum_{n,n'} \frac{\langle P_n | Q_i | \bar{P}_{n'} \rangle(\tau)}{4 M_n M_{n'}} e^{-(\Delta T - t) M_n} e^{-t M_{n'}} \underset{t_0 \ll t \ll t_0 + \Delta T}{\Longrightarrow} \frac{\langle P \rangle^2}{4 M^2} \langle Q_i \rangle(\tau) e^{-\Delta T M}$$

- Normalise with two-point correlation functions ➜ $\mathcal{B}_i = \frac{\langle Q_i \rangle}{\eta_i m^2 f^2}$
- Measure along positive flow time τ

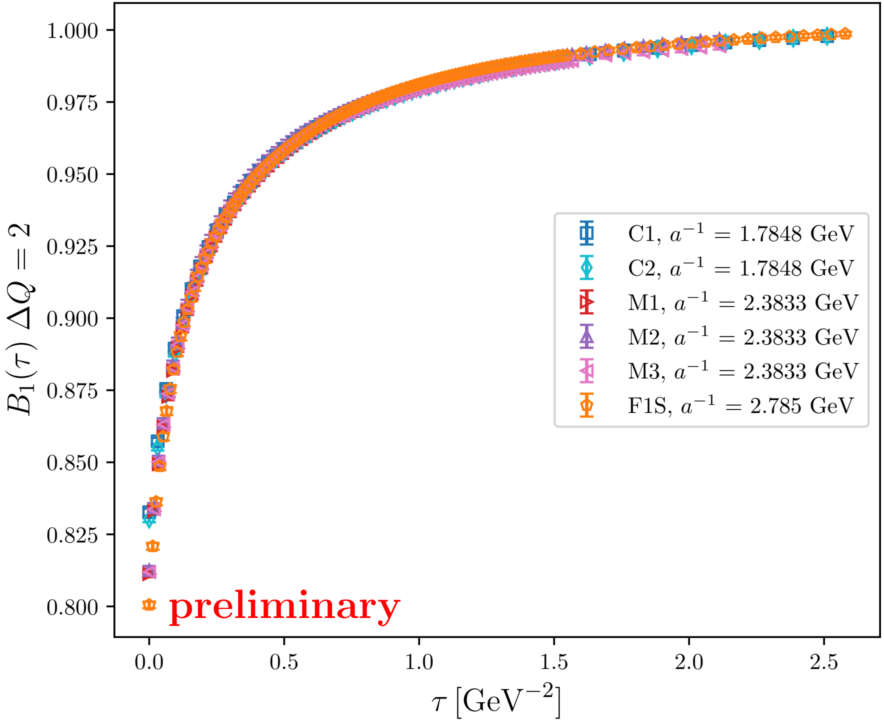




- Symmetric signal ‘folded’ across time extent
- Single correlator fit at each flow time



- operator is renormalised in 'GF' scheme as it is evolved along flow time
- data at same lattice spacing overlap ➡ no light sea quark effects



➤ different lattice spacings overlap in physical flow time ➡ mild continuum limit

- Combine with perturbative matching at NNLO (+ higher logs) [Harlander, Lange '22] [Borgulat et al. '23]

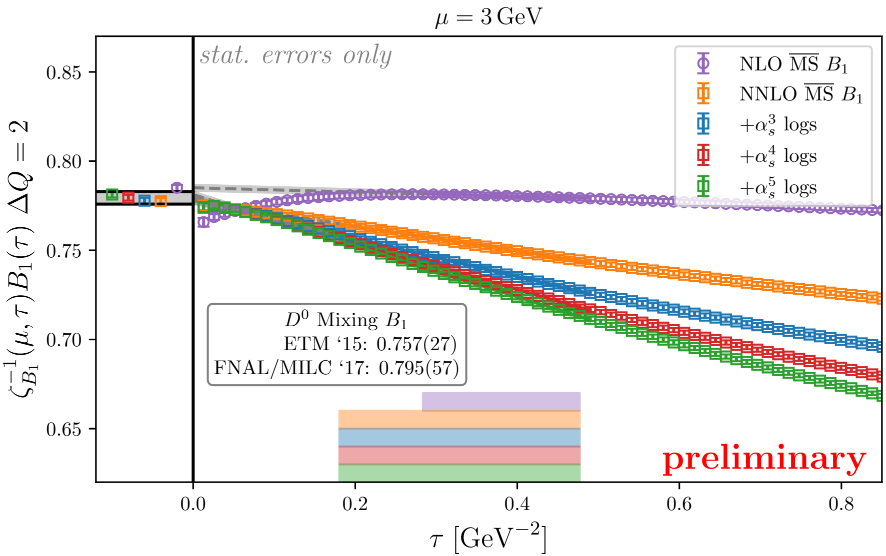
- Consistency with literature

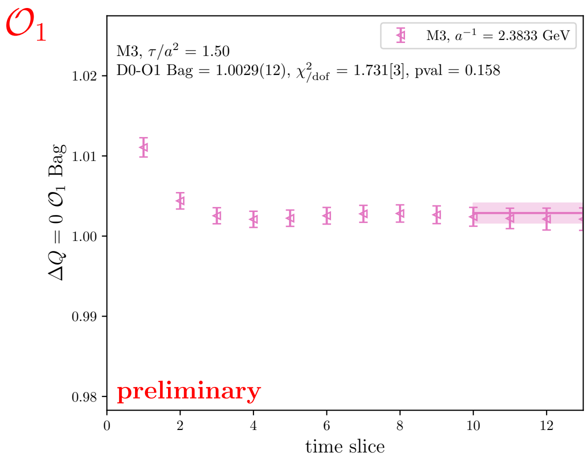
- Different perturbative orders show agreement
  - systematic errors needed for meaningful comparison

- Preliminary value

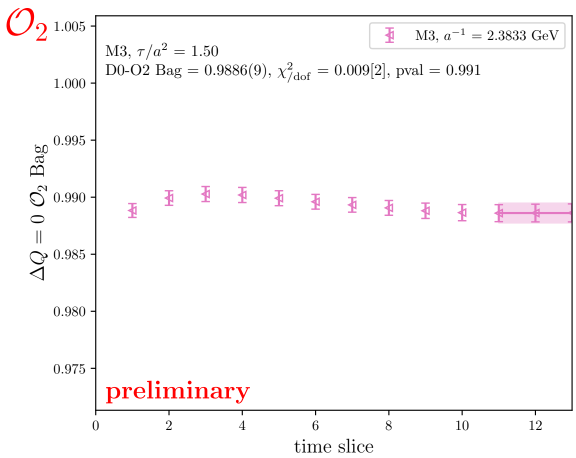
  0.779(4)

  - take spread at NNLO and higher

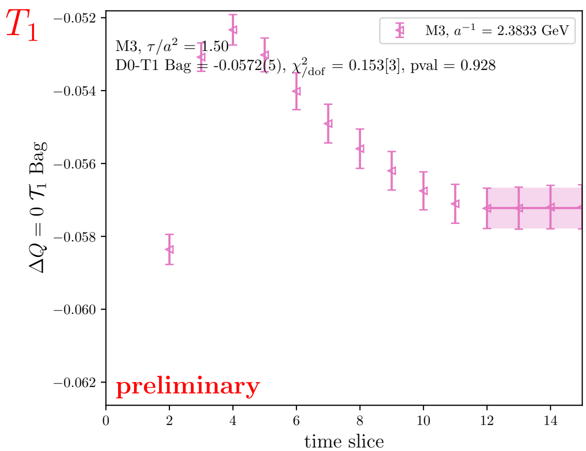




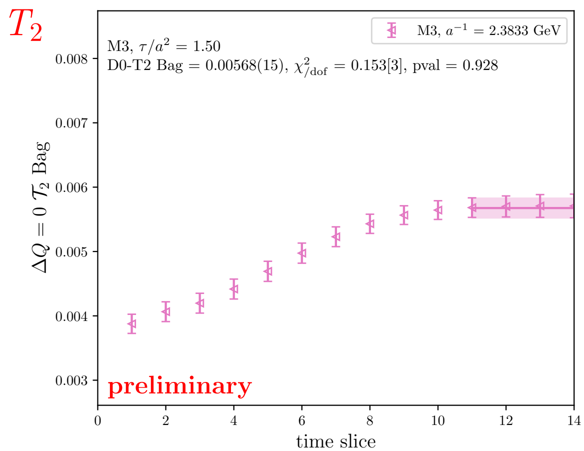
$\mathcal{O}_1$
M3, a^{-1} = 2.3833 GeV
M3, τ/a^2 = 1.50
D0-O1 Bag = 1.0029(12), $\chi^2_{/\mathrm{dof}}$ = 1.731[3], pval = 0.158
$\Delta Q = 0$ $\mathcal{O}_1$ Bag
time slice
preliminary



$\mathcal{O}_2$
M3, a^{-1} = 2.3833 GeV
M3, τ/a^2 = 1.50
D0-O2 Bag = 0.9886(9), $\chi^2_{/\mathrm{dof}}$ = 0.009[2], pval = 0.991
$\Delta Q = 0$ $\mathcal{O}_2$ Bag
time slice
preliminary

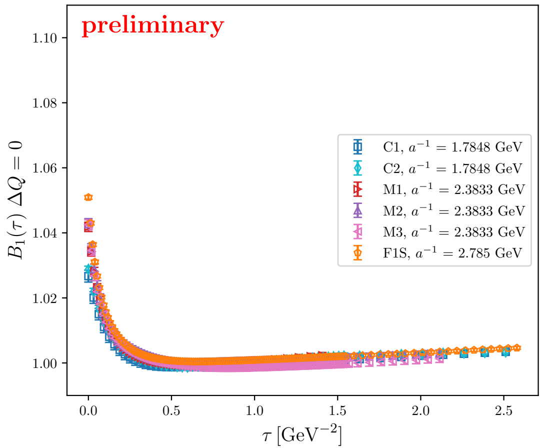


T_1
M3, a^{-1} = 2.3833 GeV
M3, τ/a^2 = 1.50
D0-T1 Bag = -0.0572(5), $\chi^2_{/\mathrm{dof}}$ = 0.153[3], pval = 0.928
$\Delta Q = 0$ $\mathcal{T}_1$ Bag
time slice
preliminary

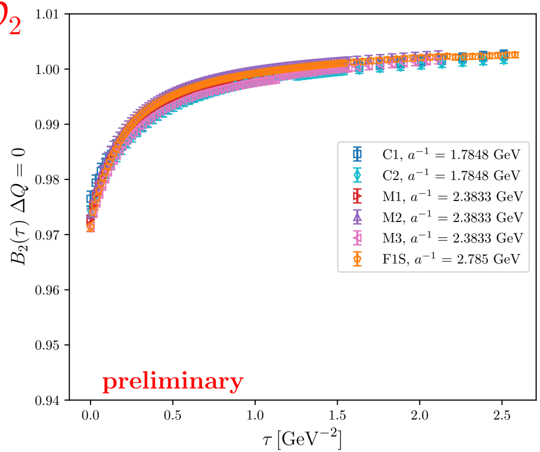


T_2
M3, a^{-1} = 2.3833 GeV
M3, τ/a^2 = 1.50
D0-T2 Bag = 0.00568(15), $\chi^2_{/\mathrm{dof}}$ = 0.153[3], pval = 0.928
$\Delta Q = 0$ $\mathcal{T}_2$ Bag
time slice
preliminary

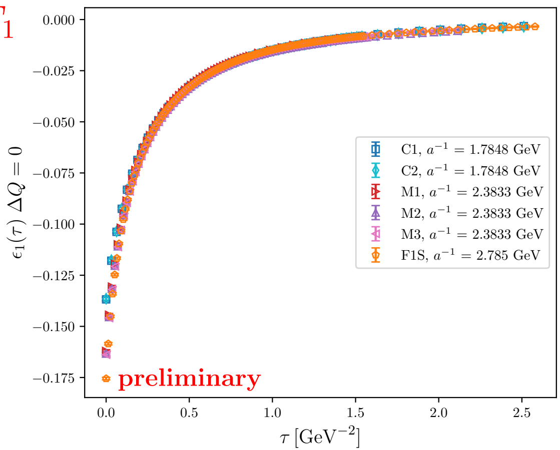
$\mathcal{O}_1$



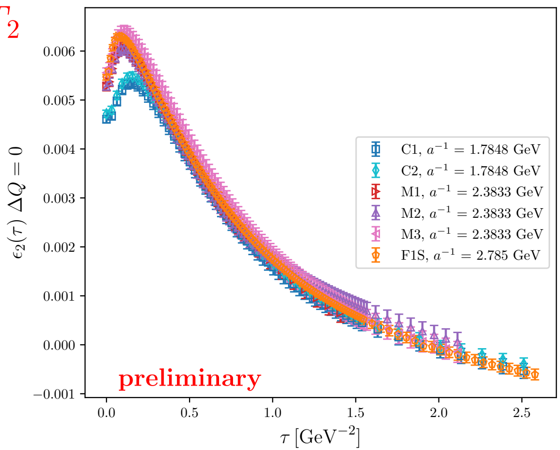
$\mathcal{O}_2$



T_1



T_2



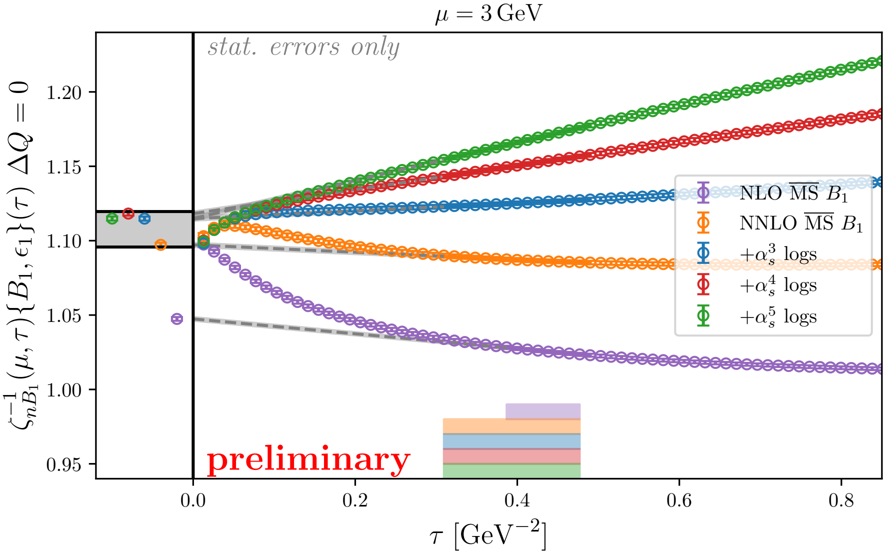
- Combine with perturbative matching at NNLO (+ higher logs) [Harlander, Lange '22] [Borgulat et al. '23]

- Increasing perturbative orders show convergence

- Preliminary value

  1.107(12)

  ➥ take spread at NNLO and higher



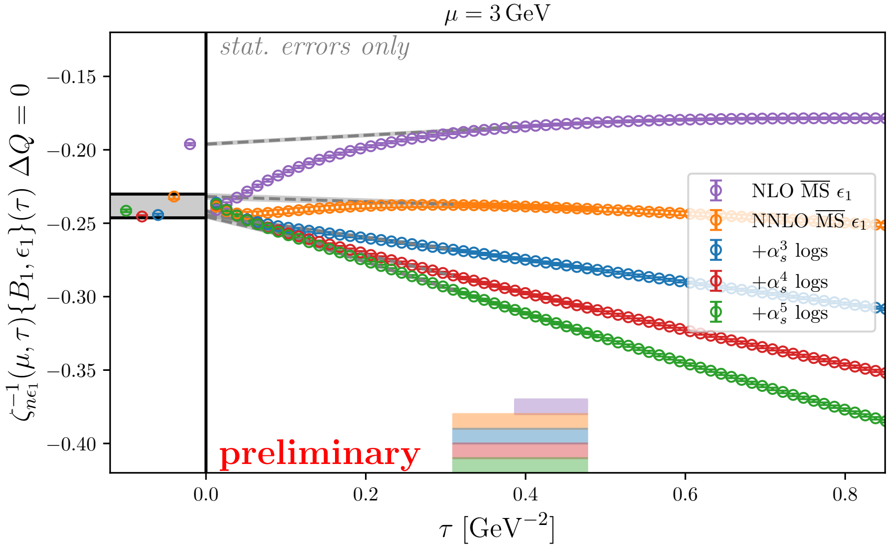
➤ Combine with perturbative matching at NNLO (+ higher logs) [Harlander, Lange '22] [Borgulat et al. '23]

➤ Increasing perturbative orders show convergence

➤ Preliminary value

$-0.238(8)$

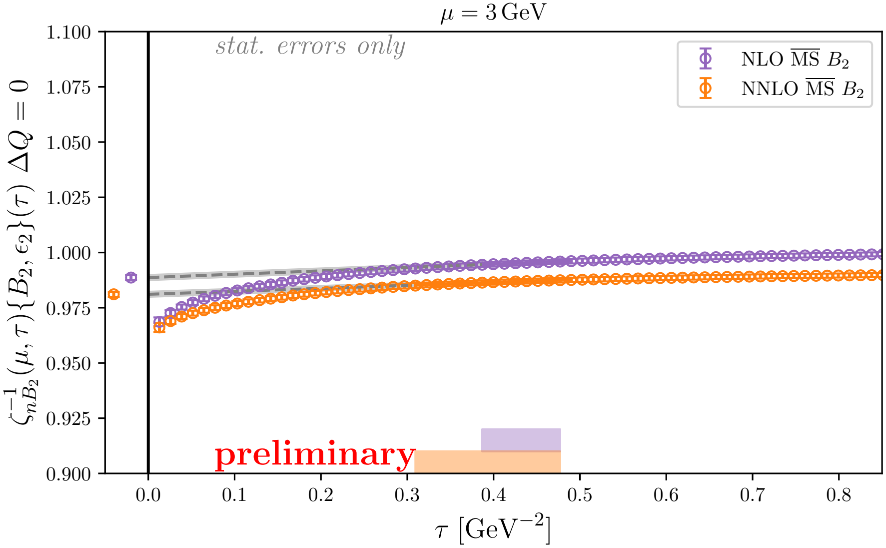
➥ take spread at NNLO and higher



- Combine with perturbative matching at NNLO ➜ see Jonas' talk
  - ➥ higher logs in progress

- Preliminary value @ NNLO

  0.981(1)

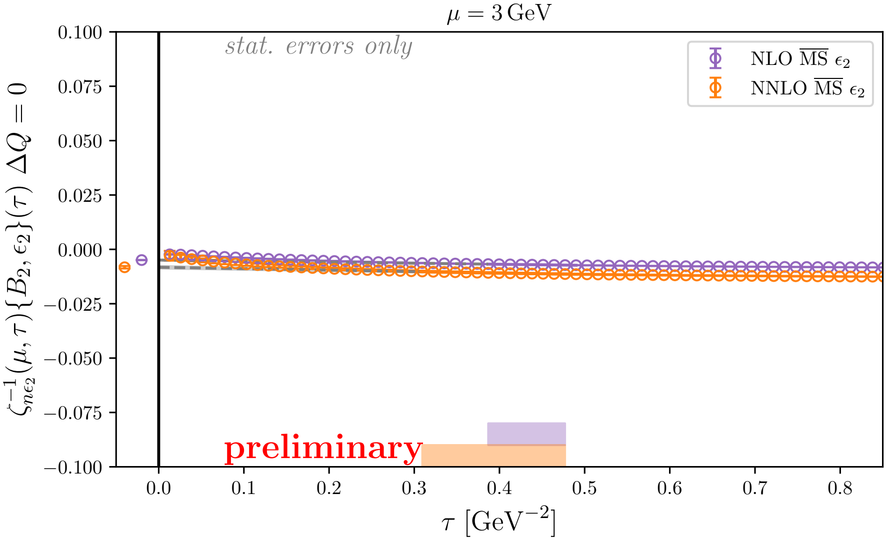


➤ Combine with perturbative matching at NNLO ➜ see Jonas' talk

➥ higher logs in progress

➤ Preliminary value @ NNLO

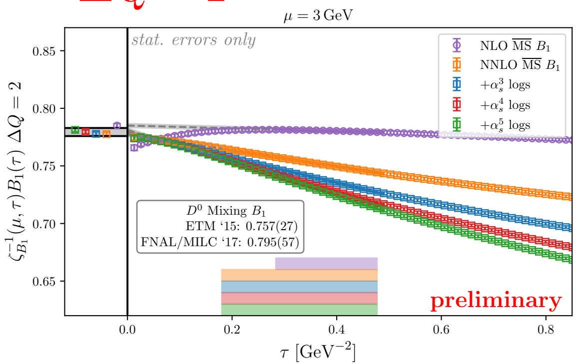
$-0.008(6)$



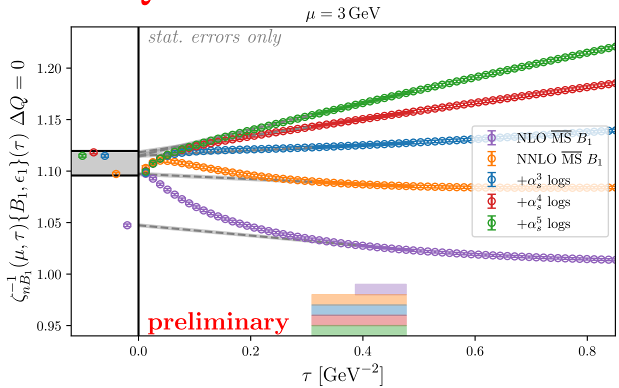
Summary and Outlook

- $\Delta B = 0$ four-quark matrix elements are strongly-desired quantities
  - ➥ Standard renormalisation introduces mixing with operators of lower mass dimension
  - ➥ We aim to use the fermionic gradient flow as a non-perturbative renormalisation procedure
- We calculate $\Delta Q = 2$ matrix elements as a test case for the short-flow-time expansion
  - ➥ preliminary results consistent with literature
- Analysis of $\Delta Q = 0$ matrix elements for lifetime differences progressing with promise

$\Delta Q = 2$



$\Delta Q = 0$



- Complete exploratory work with physical charm-strange meson
  - GF$\rightarrow \overline{\mathrm{MS}}$ analysis for all dimension-six $\Delta Q = 2$ operators
  - GF$\rightarrow \overline{\mathrm{MS}}$ analysis for dimension-six $\Delta Q = 0$ operators (connected pieces) ➜ lifetime differences
  - Fully-correlated analysis to be done
- Perturbative matching needed for complete $\Delta B = 2$ basis
  - higher logs still needed for all $\Delta B = 0$ operators
- Complete full-scale simulations for B meson mixing and lifetimes
  - multiple heavier-than-charm masses ➡ extrapolate to physical b mass
  - consider both strange and light spectators
- Eye diagrams needed for absolute lifetime operators
  - to be included in both lattice simulations and perturbative matching calculations

Backup Slides

- Mass difference of neutral mesons $\Delta M_q\,(q = d, s)$ governed by $\Delta B = 2$ four-quark operators
- General BSM basis has 5 dimension-six operators

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \qquad \langle Q_1^q \rangle = \langle \bar{B}_q | Q_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q$$

$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \qquad \langle Q_2^q \rangle = \langle \bar{B}_q | Q_2^q | B_q \rangle = \frac{-5 M_{B_q}^2}{3(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$Q_3^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1-\gamma_5) q^\alpha, \qquad \langle Q_3^q \rangle = \langle \bar{B}_q | Q_3^q | B_q \rangle = \frac{M_{B_q}^2}{3(m_b + m_q)^2} f_{B_q}^2 M_{B_q}^2 B_3^q,$$

$$Q_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \qquad \langle Q_4^q \rangle = \langle \bar{B}_q | Q_4^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{(m_b + m_q)^2} + \frac{1}{3} \right] f_{B_q}^2 M_{B_q}^2 B_4^q,$$

$$Q_5^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1+\gamma_5) q^\alpha, \qquad \langle Q_5^q \rangle = \langle \bar{B}_q | Q_5^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{3(m_b + m_q)^2} + 1 \right] f_{B_q}^2 M_{B_q}^2 B_5^q.$$

- In the **SM**, only Q_1^q contributes to ΔM
- Matrix elements parameterised in terms of **decay constant** f_{B_q} and **bag parameters** B_i^q

- For lifetimes, the dimension-6 $\Delta B = 0$ operators are:

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle Q_1^q \rangle = \langle B_q | Q_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \mathcal{B}_1^q,$$

$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle Q_2^q \rangle = \langle B_q | Q_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \mathcal{B}_2^q,$$

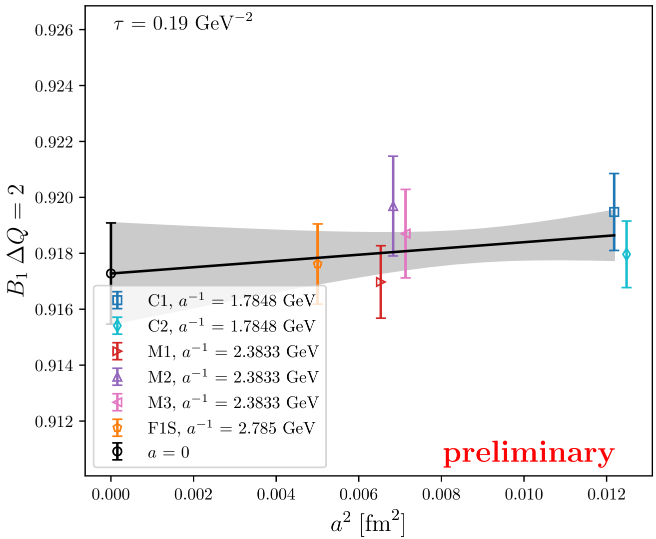
$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma \gamma_\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

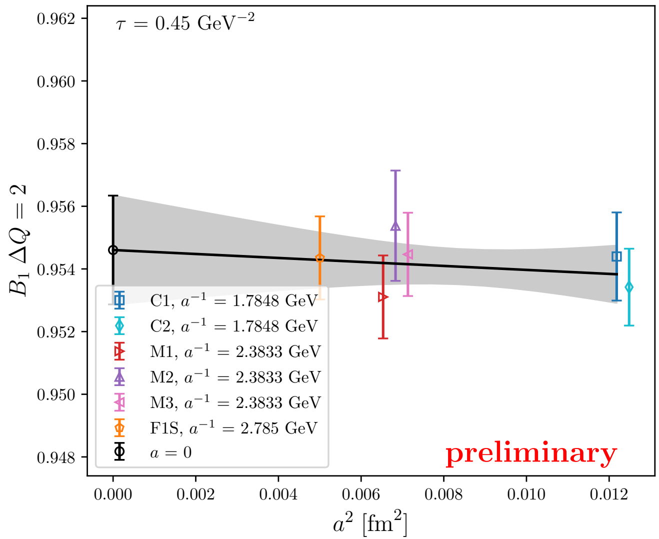
$$T_2^q = \bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

- For simplicity of computation, we want these to be colour-singlet operators:

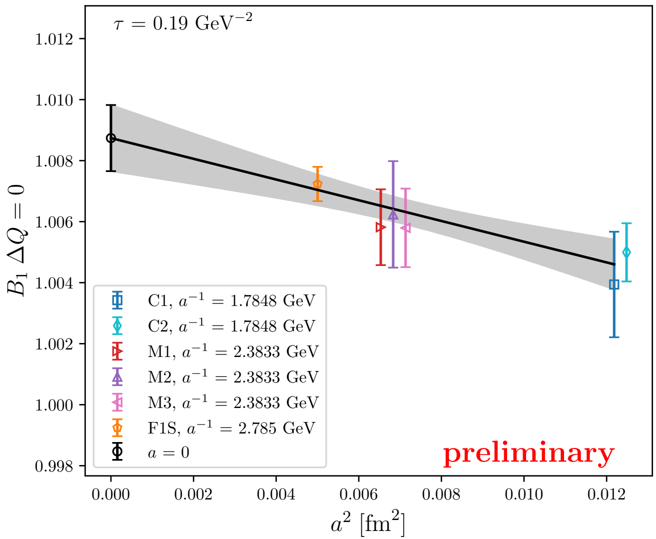
$$\begin{aligned}
\mathcal{Q}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta \\
\mathcal{Q}_2 &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1+\gamma_5) b^\beta) \\
\tau_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \\
\tau_2 &= \bar{b}^\alpha \gamma_\mu (1+\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta
\end{aligned}$$

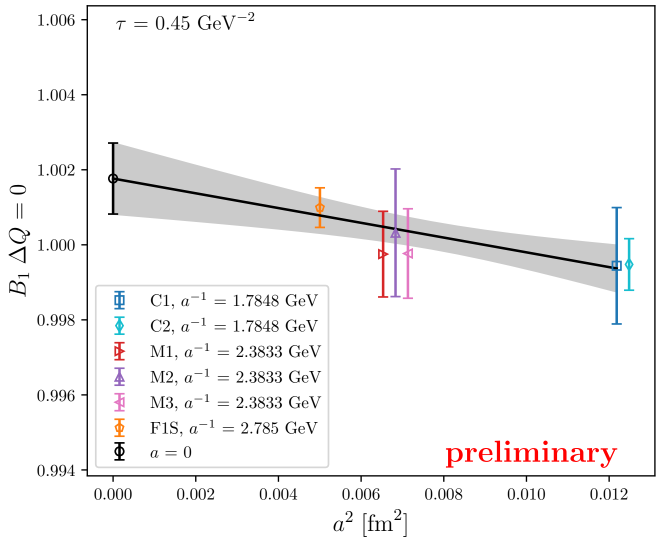
$$\begin{pmatrix} Q_1^+ \\ Q_2^+ \\ T_1^+ \\ T_2^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2N_c} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2N_c} & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \tau_1^+ \\ \tau_2^+ \end{pmatrix}$$

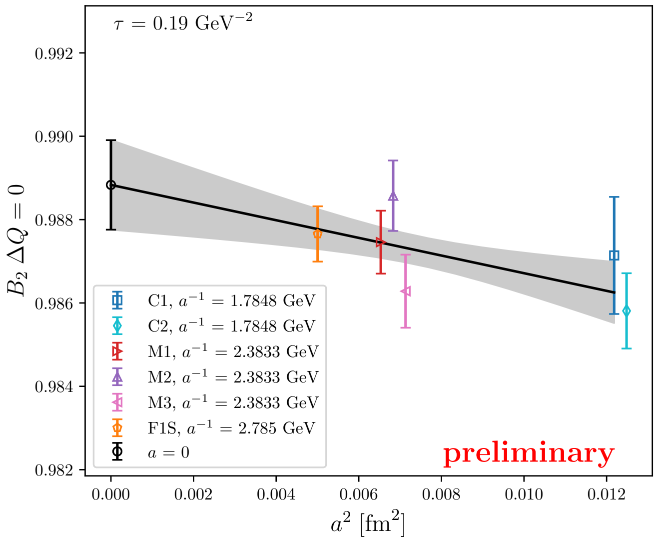

$\tau = 0.19$ GeV^{-2}
$B_1 \; \Delta Q = 2$
a^2 [fm^2]
C1, $a^{-1} = 1.7848$ GeV
C2, $a^{-1} = 1.7848$ GeV
M1, $a^{-1} = 2.3833$ GeV
M2, $a^{-1} = 2.3833$ GeV
M3, $a^{-1} = 2.3833$ GeV
F1S, $a^{-1} = 2.785$ GeV
$a = 0$
preliminary


$\tau = 0.45$ GeV^{-2}
$B_1 \; \Delta Q = 2$
a^2 [fm^2]
C1, $a^{-1} = 1.7848$ GeV
C2, $a^{-1} = 1.7848$ GeV
M1, $a^{-1} = 2.3833$ GeV
M2, $a^{-1} = 2.3833$ GeV
M3, $a^{-1} = 2.3833$ GeV
F1S, $a^{-1} = 2.785$ GeV
$a = 0$
preliminary

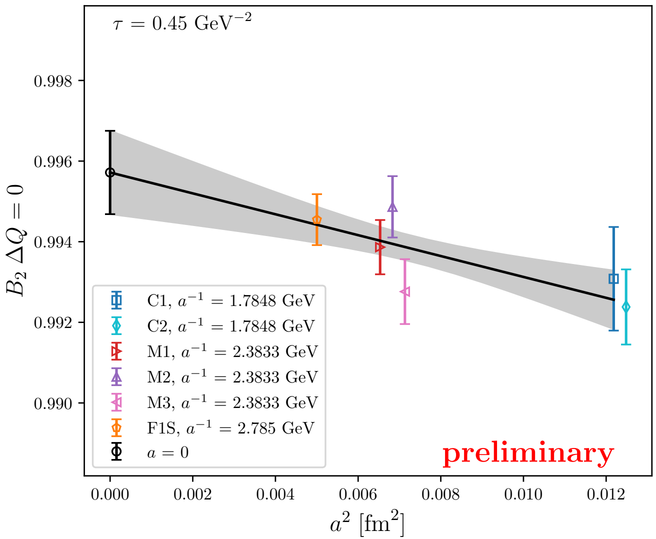
Lifetimes $\mathcal{O}_1$ Continuum Limit



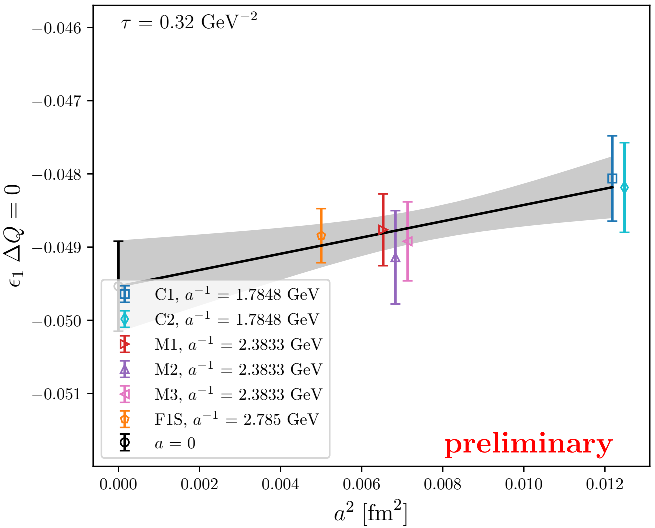




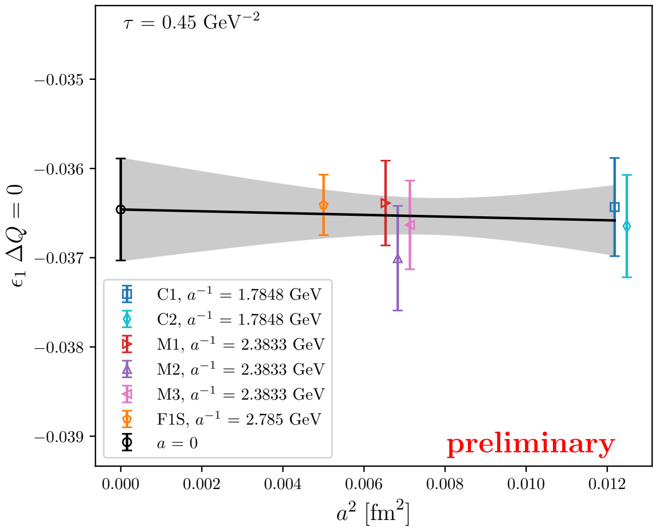
τ = 0.19 GeV^{-2}
B_2 $\Delta Q = 0$
a^2 [fm^2]
0.982
0.984
0.986
0.988
0.990
0.992
0.000
0.002
0.004
0.006
0.008
0.010
0.012
C1, a^{-1} = 1.7848 GeV
C2, a^{-1} = 1.7848 GeV
M1, a^{-1} = 2.3833 GeV
M2, a^{-1} = 2.3833 GeV
M3, a^{-1} = 2.3833 GeV
F1S, a^{-1} = 2.785 GeV
$a = 0$
preliminary



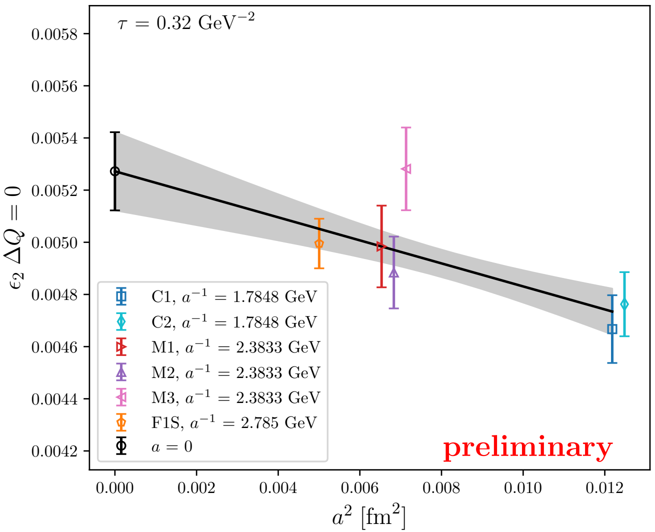
τ = 0.45 GeV^{-2}
B_2 $\Delta Q = 0$
a^2 [fm^2]
0.990
0.992
0.994
0.996
0.998
0.000
0.002
0.004
0.006
0.008
0.010
0.012
C1, a^{-1} = 1.7848 GeV
C2, a^{-1} = 1.7848 GeV
M1, a^{-1} = 2.3833 GeV
M2, a^{-1} = 2.3833 GeV
M3, a^{-1} = 2.3833 GeV
F1S, a^{-1} = 2.785 GeV
$a = 0$
preliminary

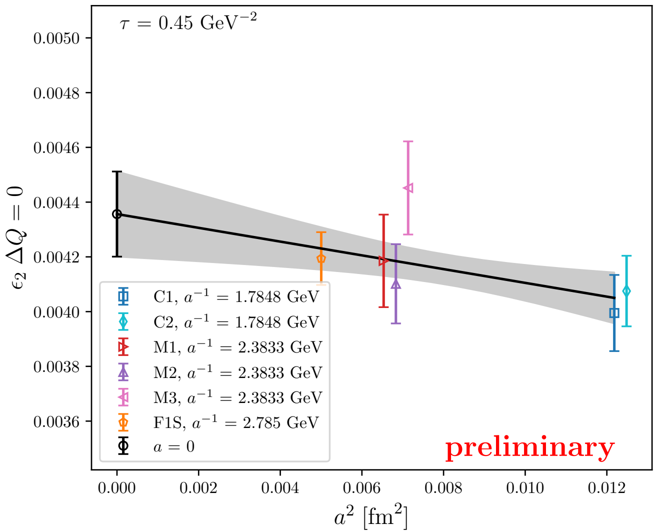


$\tau = 0.32$ GeV^{-2}
$\epsilon_1 \; \Delta Q = 0$
a^2 [fm^2]
C1, $a^{-1} = 1.7848$ GeV
C2, $a^{-1} = 1.7848$ GeV
M1, $a^{-1} = 2.3833$ GeV
M2, $a^{-1} = 2.3833$ GeV
M3, $a^{-1} = 2.3833$ GeV
F1S, $a^{-1} = 2.785$ GeV
$a = 0$
preliminary



$\tau = 0.45$ GeV^{-2}
$\epsilon_1 \; \Delta Q = 0$
a^2 [fm^2]
C1, $a^{-1} = 1.7848$ GeV
C2, $a^{-1} = 1.7848$ GeV
M1, $a^{-1} = 2.3833$ GeV
M2, $a^{-1} = 2.3833$ GeV
M3, $a^{-1} = 2.3833$ GeV
F1S, $a^{-1} = 2.785$ GeV
$a = 0$
preliminary


$\tau = 0.32$ GeV^{-2}
$\epsilon_2\ \Delta Q = 0$
a^2 [fm^2]
C1, $a^{-1} = 1.7848$ GeV
C2, $a^{-1} = 1.7848$ GeV
M1, $a^{-1} = 2.3833$ GeV
M2, $a^{-1} = 2.3833$ GeV
M3, $a^{-1} = 2.3833$ GeV
F1S, $a^{-1} = 2.785$ GeV
$a = 0$
preliminary


$\tau = 0.45$ GeV^{-2}
$\epsilon_2\ \Delta Q = 0$
a^2 [fm^2]
C1, $a^{-1} = 1.7848$ GeV
C2, $a^{-1} = 1.7848$ GeV
M1, $a^{-1} = 2.3833$ GeV
M2, $a^{-1} = 2.3833$ GeV
M3, $a^{-1} = 2.3833$ GeV
F1S, $a^{-1} = 2.785$ GeV
$a = 0$
preliminary