Heavy Quark(onium) transport with GF on the lattice

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Based on collaboration with:
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In various combinations

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Zürich,
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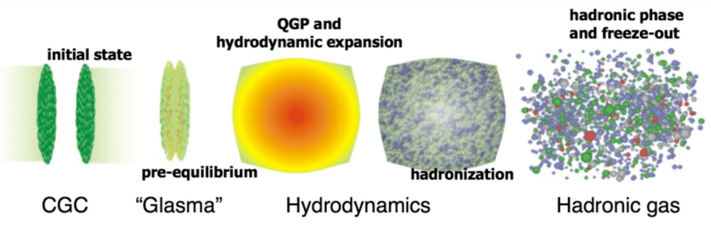
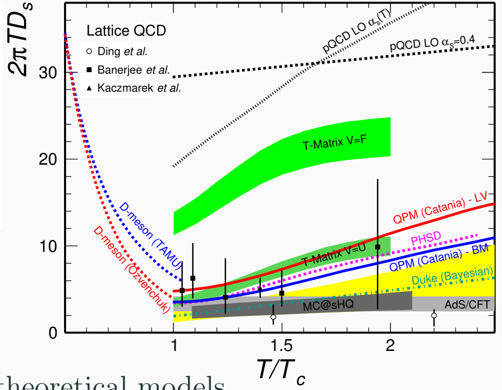
Motivation





- Nuclear modification factor R_{AA} and elliptic flow v_2 described by spatial diffusion coefficient D_x
- Varying results for temperature dependence across theoretical models
- Transport coefficients are input for quarkonium production models to describe heavy-ion collision physics and quark gluon plasma

Figure: S. Bass (left) X. Dong CIPANP (2018) (right)

Heavy Quark diffusion: Langevin perspective

- Heavy quark energy changes only little when colliding with medium

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
  → Brownian motion; Langevin dynamics can be used

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT}p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

Svetitsky 88; Mustafa et.al.97; Moore & Teaney 05; Rapp & van Hees 05

Associated Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i}[p_i \eta_D(p) f_Q(p,t)] + \frac{\partial^2}{\partial p_i \partial_j}[\kappa_{ij}(p) f_Q(p,t)]$$

- Single coefficient κ gives access to multiple interesting quantities:

$D_s = 2T^2/\kappa$	$\eta_D = \kappa/(2MT)$	$\tau_Q = \eta_D^{-1}$
Spatial diffusion	Drag coefficient	Relaxation time

HQET picture

- Expand the force in $1/M$

$$\mathcal{F}^{i} = \phi^{\dagger}\left[-gE^{i} + \frac{[D^{i}, D^{2} + c_{b} g\sigma\cdot B]}{2M} + \cdots\right]\phi$$

- Note also Lorentz force

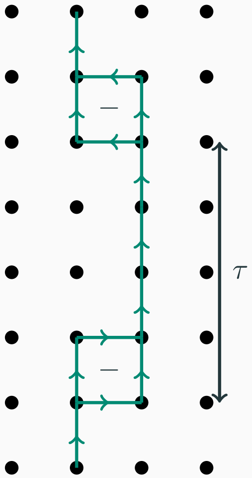
$$F(t) = \dot{p} = q\left(E + v \times B\right)\right)(t)$$

- Switch to Euclidean space correlation function:

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,0)]\rangle},$$

$$G_B(\tau) = \sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re}\,\mathrm{Tr}\,U(1/T,0)\rangle}$$

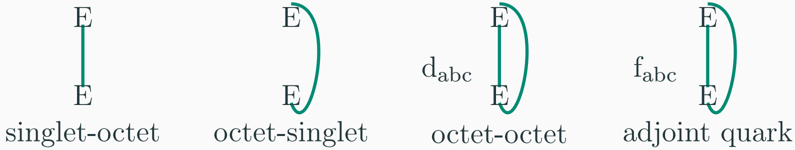
$$\kappa_{E,B} = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega) \qquad G_{E,B}(\tau) = \int_0^\infty \frac{d\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$



Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)
A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Heavy Quarkonium Diffusion

- Quarkonium in medium can be described by Limbland equation by using pNRQCD and open quantum systems
- Three possible interactions and adjoint quark Brambilla et.al.TUM-EFT 191/24



- Each process described by two parameters κ_{xx} and γ_{xx}
- κ_{so} is related to the thermal width and describes heavy quarkonium diffusion
- γ_{so} is related to the mass shift $\gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega}$

Brambilla et.al.PRD96 (2017), Brambilla et.al.PRD97 (2018), Brambilla et.al.PRD100 (2019), Eller et.al.PRD99 (2019), Scheihing-Hitschfeld & Yao PRD108 (2023), V.L. Lattice2023

Heavy Quarkonium Diffusion on the lattice

- Euclidean correlators similar to HQ-case, but with adjoint Wilson line
- $\kappa_{\rm so}$ and $\kappa_{\rm os}$ given by

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\langle \mathrm{Re\,Tr}\,[gE_i(\tau,0)\Phi(\tau,0)gE_i(0,0)]\rangle = -\frac{2}{3}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)U(\tau,0)E_i(0)U(0,\tau)]\rangle\,,$$

- Separating $\kappa_{\rm so}$ and $\kappa_{\rm os}$ on lattice still work in progress
- $\kappa_{\rm oo}$ similar to HQ-diffusion

$$G_{EE}^{\rm oct} \equiv \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \Phi^{A}_{xa}(N_T,t)d_{abc}E^{i,c}(t)\Phi^{A}_{bz}(t,0)d_{zxg}E^{i,g}(0)\rangle$$

$$= \frac{-1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)P(\tau)^\dagger]\mathrm{Tr}\,[E_i(0)P(0)] + \mathrm{Tr}\,[E_i(\tau)U(\tau,1/T)E_i(0)U(0,\tau)]\mathrm{Tr}\,[P(0)] - \frac{4}{3}\mathrm{Tr}\,[E_i(\tau)U(\tau,0)E_i(0)U(0,\tau)] + \mathrm{h.c.}$$

- Also related: Diffusion of an adjoint static quark

$$G_{EE}^{\rm symm} \equiv \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \Phi^{A}_{xa}(N_T,t)f_{abc}E^{i,c}(t)\Phi^{A}_{bz}(t,0)f_{zxg}E^{i,g}(0)\rangle$$

$$= \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)P(\tau)^\dagger]\mathrm{Tr}\,[E_i(0)P(0)] - \mathrm{Tr}\,[E_i(\tau)U(\tau,1/T)E_i(0)U(0,\tau)]\mathrm{Tr}\,[P(0)] + \mathrm{h.c.}$$

Theory papers in preparation: Brambilla et.al.2025 (TUM-EFT 191/42, 190/24)

Gradient flow

- Smears the noisy Wilson lines
- Generalizes to unquenched (this talk pure gauge)
- Renormalizes gauge independent observables
  - Discretization of $F_{\mu\nu}$ involves lattice only renormalization
  - If flow time larger than the discretization $\Rightarrow$ point-like, renormalized

    See talk from Julian
- Avoid overlap $\sqrt{8\tau_f} < \tau/2$
- Careful with divergences
  - BB-correlator has finite anomalous dimension $\Rightarrow \sim \ln(8\tau_f\mu^2)$

    See talk from Xiangpeng
  - Adjoint correlators have finite Wilson lies with divergence $\sim 1/\sqrt{8\tau_f}$

Strategy for $G_{E,B}^{fund}$

- At zero flowtime all out operators have same LO contribution (up to color factors)

$$\frac{G_{E,B}^{LO}}{g^2 C_F} \equiv f(\tau) = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right]$$

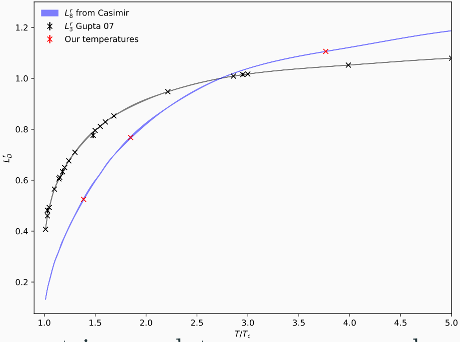
- Tree-level improve by matching to lattice perturbation theory (currently at zero flowtime)
- 1-loop GF behavior of $G_{E,B}^{fund}$ known for some discretizations
  L. Altenkort et.al.2023, de la Cruz et.al.2024
- Model the spectral function by connecting known IR and UV behavior with ansatz:

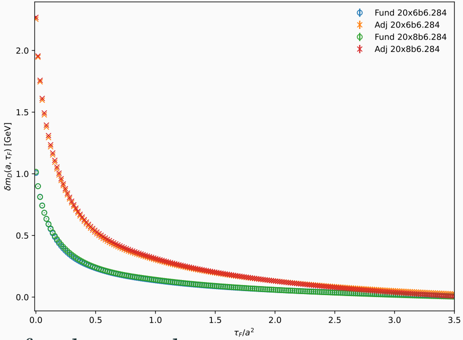
$$\rho_{E,B}^{IR}(\omega) = \frac{\kappa\omega}{2T}, \quad \rho_E^{UV}(\omega) = \frac{g^2 C_F \omega^3}{6\pi}\left\{1 + \frac{g^2}{(4\pi)^2}\left[N_c\left(\frac{11}{3}\ln\frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{2\pi^2}{3}\right)\right]\right\}$$

$$\rho_B^{UV}(\omega, \tau_F) = Z_{flow}\frac{g^2(\mu)\omega^3}{6\pi}(1 + g^2(\mu)(\beta_0 - \gamma_0)\ln(\mu^2/(A\omega^2)) + g^2(\mu)\gamma_0\ln(8\tau_F\mu^2)$$

- Set scale such that NLO UV contribution vanishes

Adjoint correlators strategy



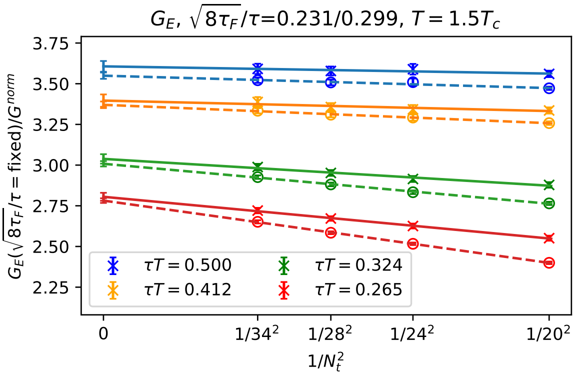


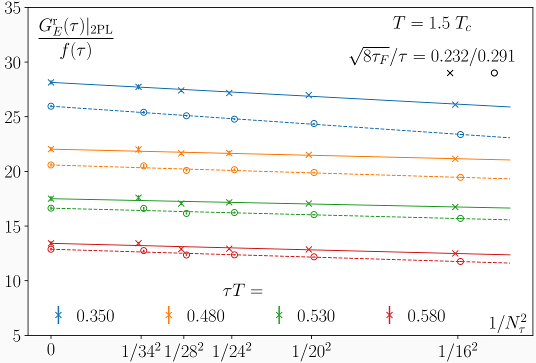
- For symmetric correlators: same procedure as fundamental
- Non-symmetric correlator: no normalization w.r.t Polyakov loop

  $\Rightarrow$ Wilson line has divergence $\exp(\delta m \tau)$ with $\delta m \propto 1/\sqrt{8\tau_F}$
- Use renormalized Polyakov loops from Gupta et.al.PRD77 2008

$$L_8^r = e^{\delta m(\tau_F)/T} L_8(\tau_F)\,, \quad G_E^r(\tau,\tau_F) = e^{\delta m(\tau_F)\tau} G_E(\tau,\tau_F) = \left(\frac{L_8^r}{L_8(\tau_F)}\right)^{\tau T} G_E(\tau,\tau_F)$$

Continuum limit

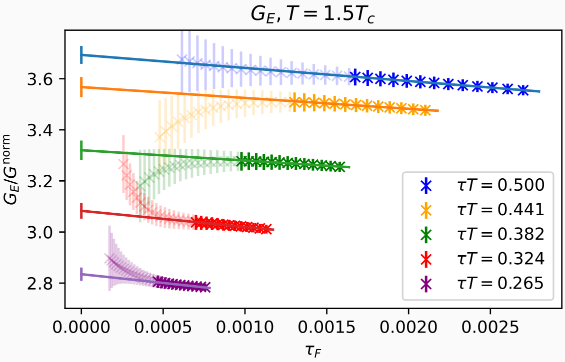




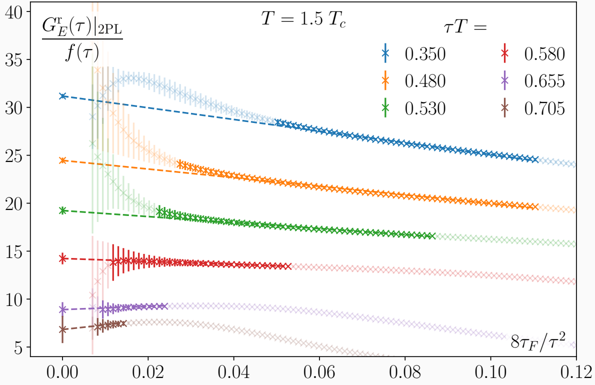
$\mathrm{G_E^{fund}}$ $\mathrm{G_E^{r}}$

- Good limits for valid ranges of τT and τ_F for both symmetric and asymmetric correlators
- Spatial volume scaling negligible

Flow time dependence of G_E



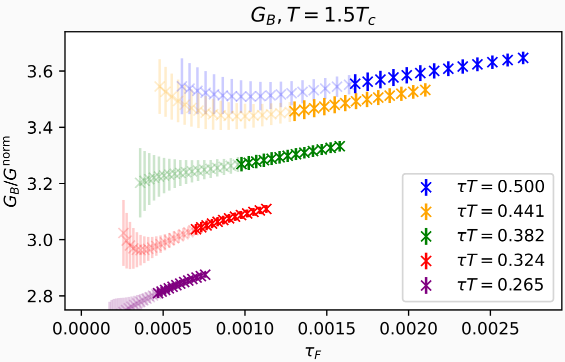
G_E^{fund}



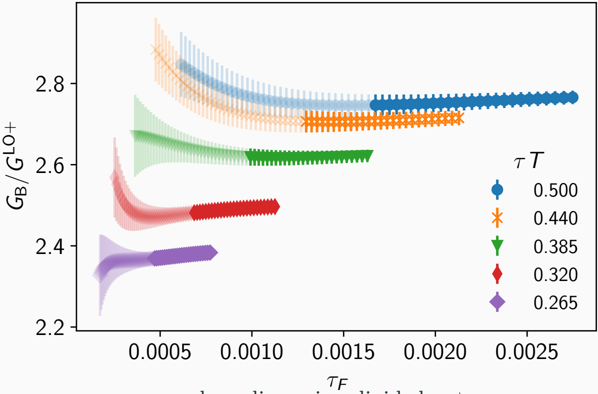
G_E^r

- Good linear flow time limits within valid range for both correlators

Flow time dependence of G_B



anomalous dimension

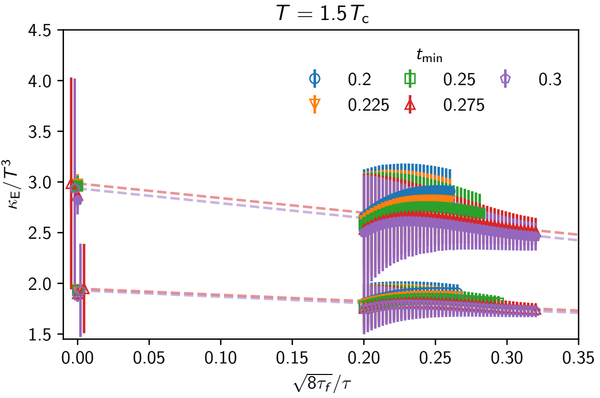


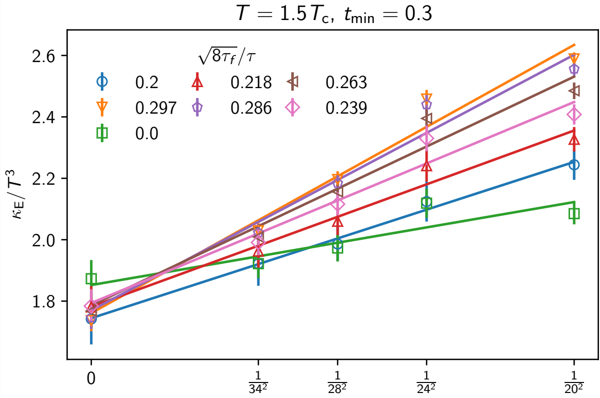
anomalous dimension divided out

- G_E and G_B scale differently w.r.r flow time
- Removing the anomalous dimension removes most flow time dependence

$$G_B^{\text{flow,UV}}(\tau, \tau_F) = (1 + \gamma_0 g^2 \ln(\mu\sqrt{8\tau_F}))^2 Z_{\text{flow}} G_B^{\overline{\text{MS}},\text{UV}}(\tau, \mu) + h_0 \cdot (\tau_F/\tau)\,,$$

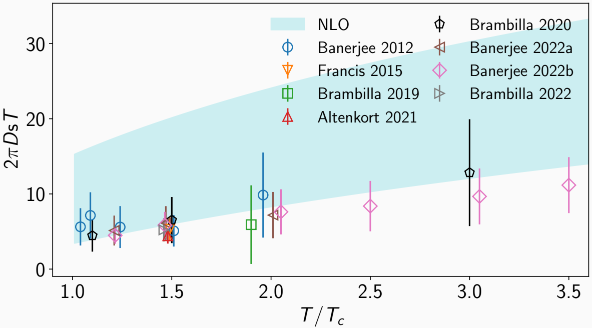
Flow time κ and order of limits

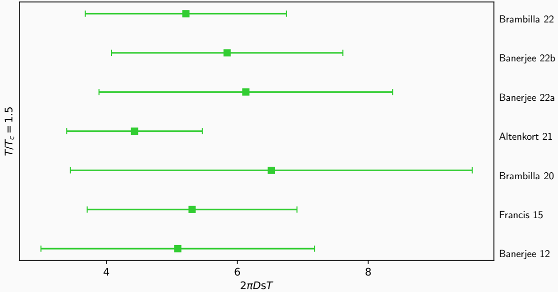




- Very little dependence on flow time
- Ordering of Continuum limit, zero flow time limit, and spectral function inversion doesn't seem to matter much

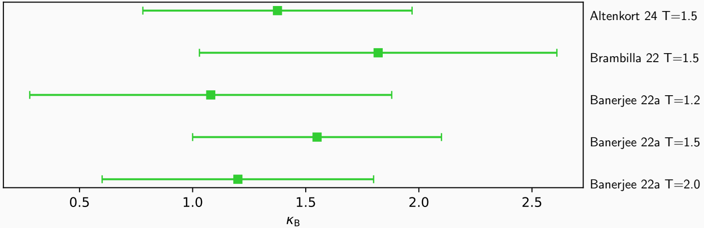
κ_{E} results





- Results of different groups agree very well
- Error dominated by the systematics from the $\rho(\omega)$ inversion
- Matches well with NLO perturbation theory

κ_B Results

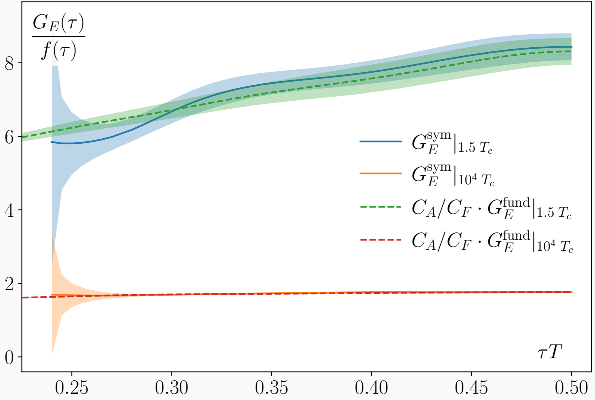


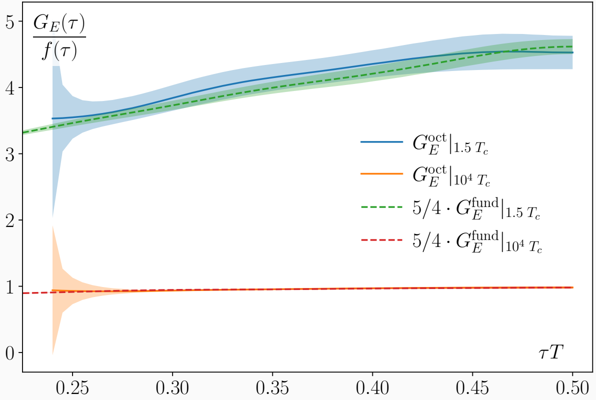
- Good agreement between existing results, close to κ_E
- Minor temperature dependence in the current measured range

$$\kappa_{tot} \simeq \kappa_E + \frac{2}{3}\langle v^2 \rangle \kappa_B$$

- Using $\langle v^2 \rangle$ from (Petreczky et.al. Eur. Phys. J. C62 (2009))
- $\langle v^2_{charm} \rangle \simeq 0.51$ and $\langle v^2_{bottom} \rangle \simeq 0.3$, we get that the mass suppressed effects on the heavy quark diffusion coefficient is 34% and 20% for the charm and bottom quarks respectively.

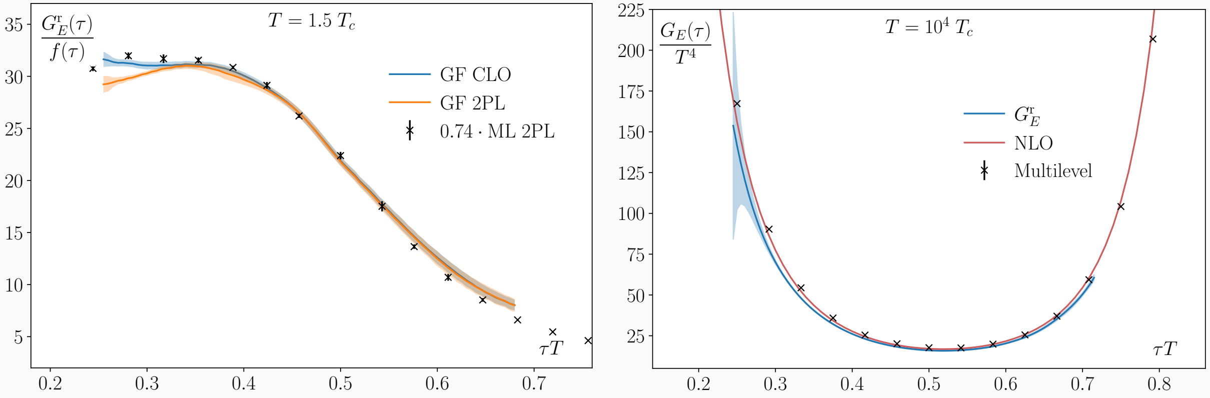
κ_{oo} and adjoint heavy quarks





- Left: adjoint quark, Right: quarkonium
- For symmetric correlators we observe expected (Casimir) scaling nonperturbatively
- These results translate from $\mathrm{G_E}$ to κ trivially

κ_{so} and κ_{os}



- Asymmetric correlator on lattice relates to both κ_{so} and κ_{os}
- Spectral reconstruction still pending
- At high temperatures, excellent agreement with the perturbation theory

NLO result: N. Brambilla, P. Panayiotou, S. Säppi, A. Vairo: in preparation TUM-EFT 190/24

Conclusions and Future prospects

- Gradient flow helps with extraction of heavy quark diffusion
- We can measure the correlators required for quarkonium diffusion
- Symmetric correlators, and hence κ, match within errors up to color factors
- Spectral functions for quarkonium still require more work
- Work for zero temperature correlators in progress
- Magnetic version of quarkonium correlators also in progress

Conclusions and Future prospects

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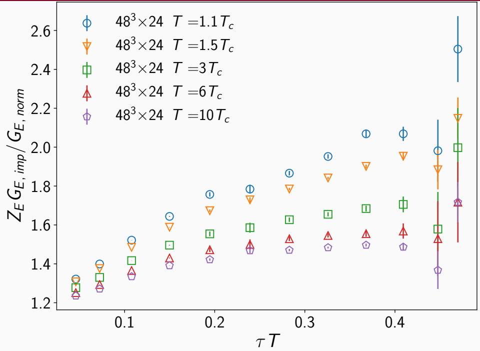
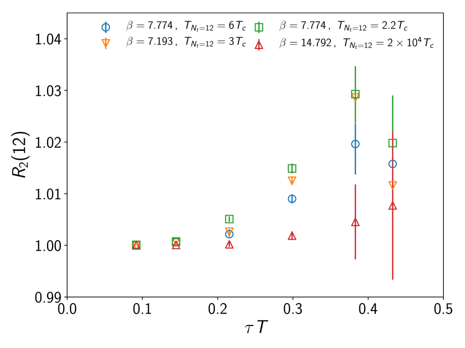
Thank you for your attention!

Backup slides

Raw (normalized*, tree-level improved) lattice data



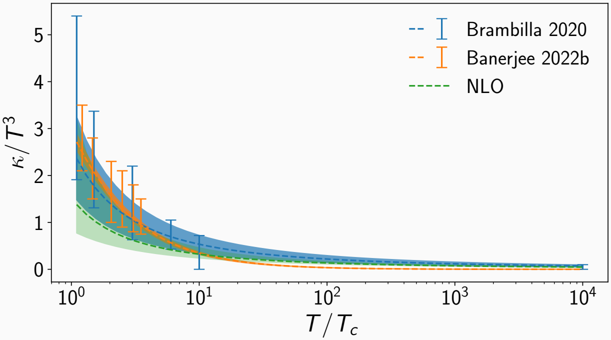


- Normalize and improve the data with LO perturbation theory
- Thermal effects dominant in forming the shape

$$R_2(N_t) = \frac{G_E(N_t,\beta)}{G_E^{norm}(N_t)} \Big/ \frac{G_E(2N_t,\beta)}{G_E^{norm}(2N_t)}\,.$$

κ_E temperature dependence

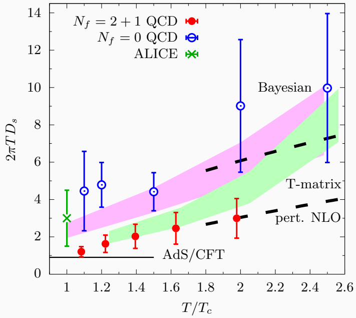
- Can fit temperature dependence

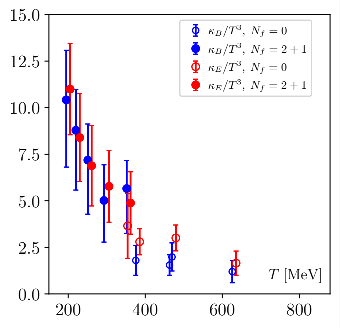


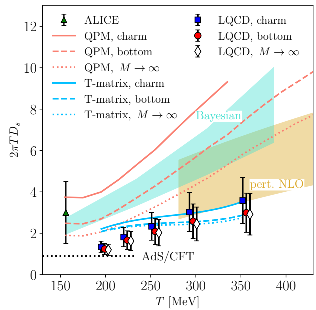
$$\kappa = \frac{g^4 C_F N_c}{18\pi}\left[\ln\frac{2T}{m_E} + \xi + C\frac{m_E}{T}\right] T^3$$ Brambilla 2020

$$2\pi T D_s = \alpha + \gamma\left(\frac{T}{T_c} - 1\right)$$ Banerjee 2022b

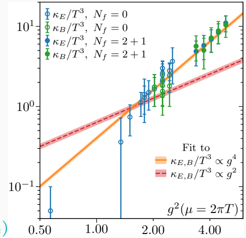
- Good agreement between the different approaches







- Most studies have been in pure gauge
- Recent results from HOTQCD
- Main difference to pure gauge is different T_c



HOTQCD: Phys.Rev.Lett. 130 (2023), Phys.Rev.Lett. 132 (2024), Phys.Rev.D 109 (2024)

γ

- Only existing determination uses mass shifts for rough estimate
- More recent similar analysis gets $\gamma \sim 0$ (see Tom's talk)
- Euclidean correlators for Quarkonium will allow measurement of γ
- Need to subtract zero temperature contribution
- Promising results on zero temperature measurements
- Combination of zero and finite T still in progress
- stay tuned

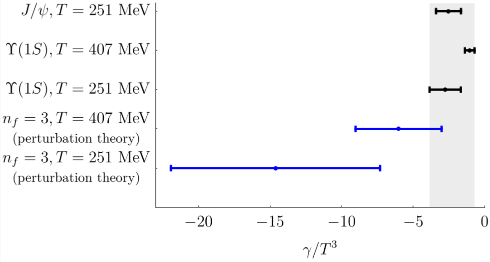


Figure: Brambilla et.al.Phys.Rev.D 100 (2019)