MuEDM superconducting solenoid: magnetic pre-design (Phase II)

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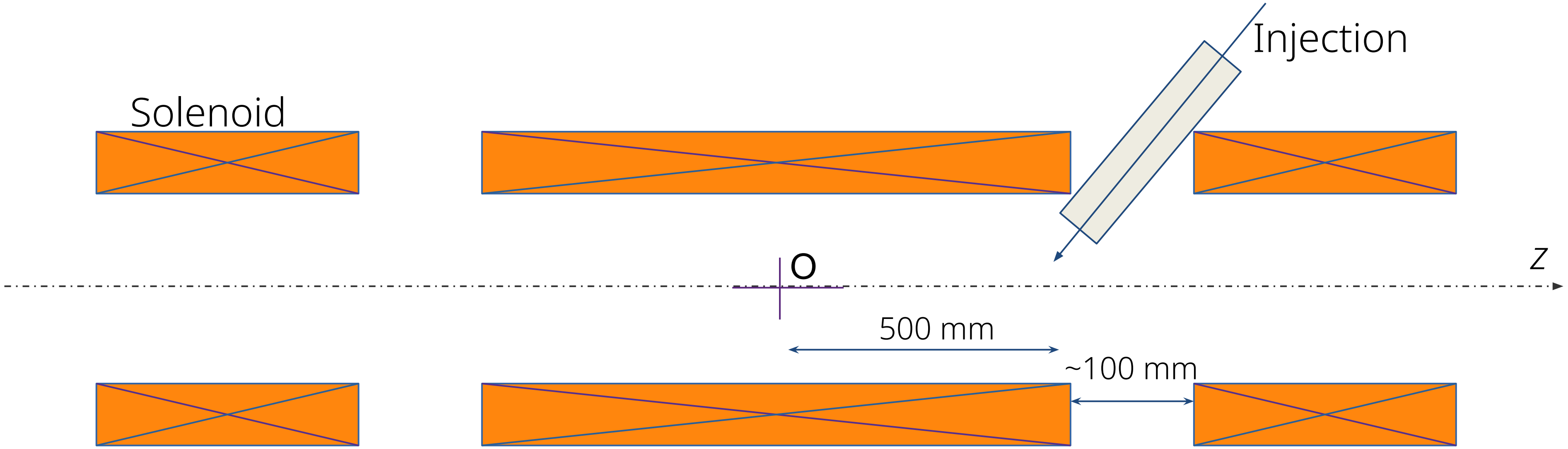
Overview

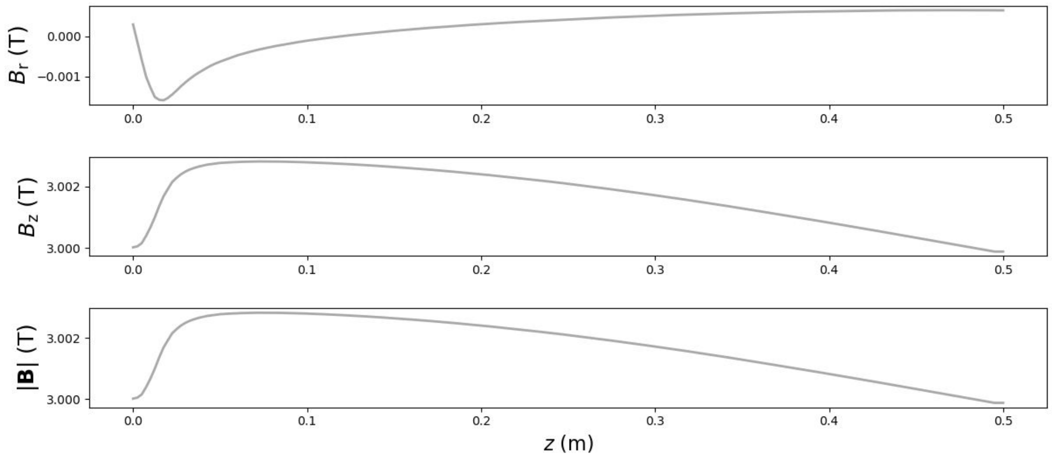
Pre-design of a superconducting solenoid magnet

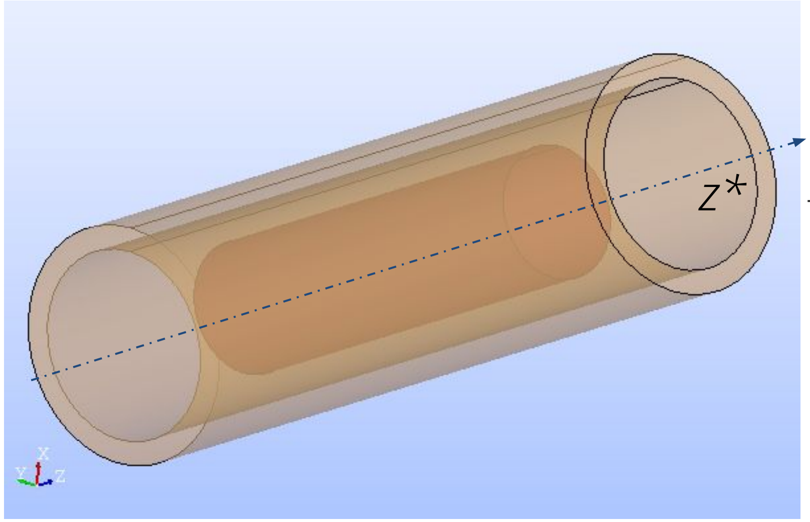
Find the optimum distribution of current densities across the magnet given magnetic field requirements

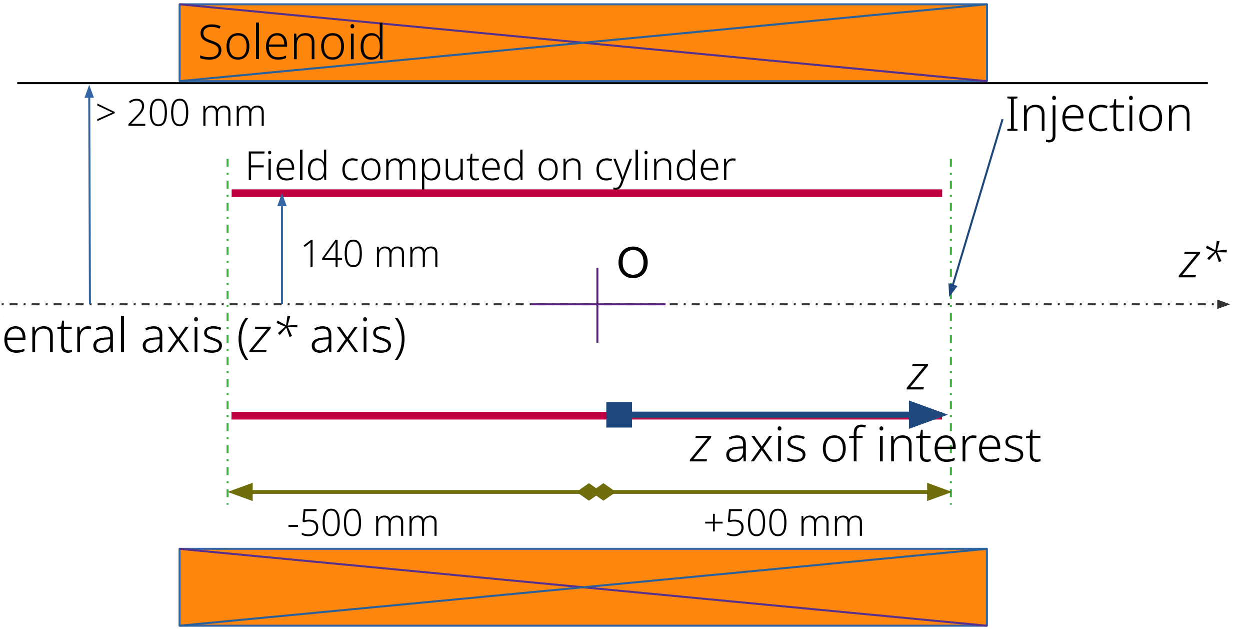


Spatial and magnetic specifications

- Cylinder radius: R_{cl} = $140^{\pm 8}$ mm
- $B(R_{cl},0) = 3^{\pm 0.3}$ T
- Magnetic design spanning ±500 mm around the center (covering the injection point)
- $\Delta B = B_{max} - B_{min}$ < 1‰ over ±500 mm
- Inner radius of solenoid: > 200 mm
- No ferromagnetic material



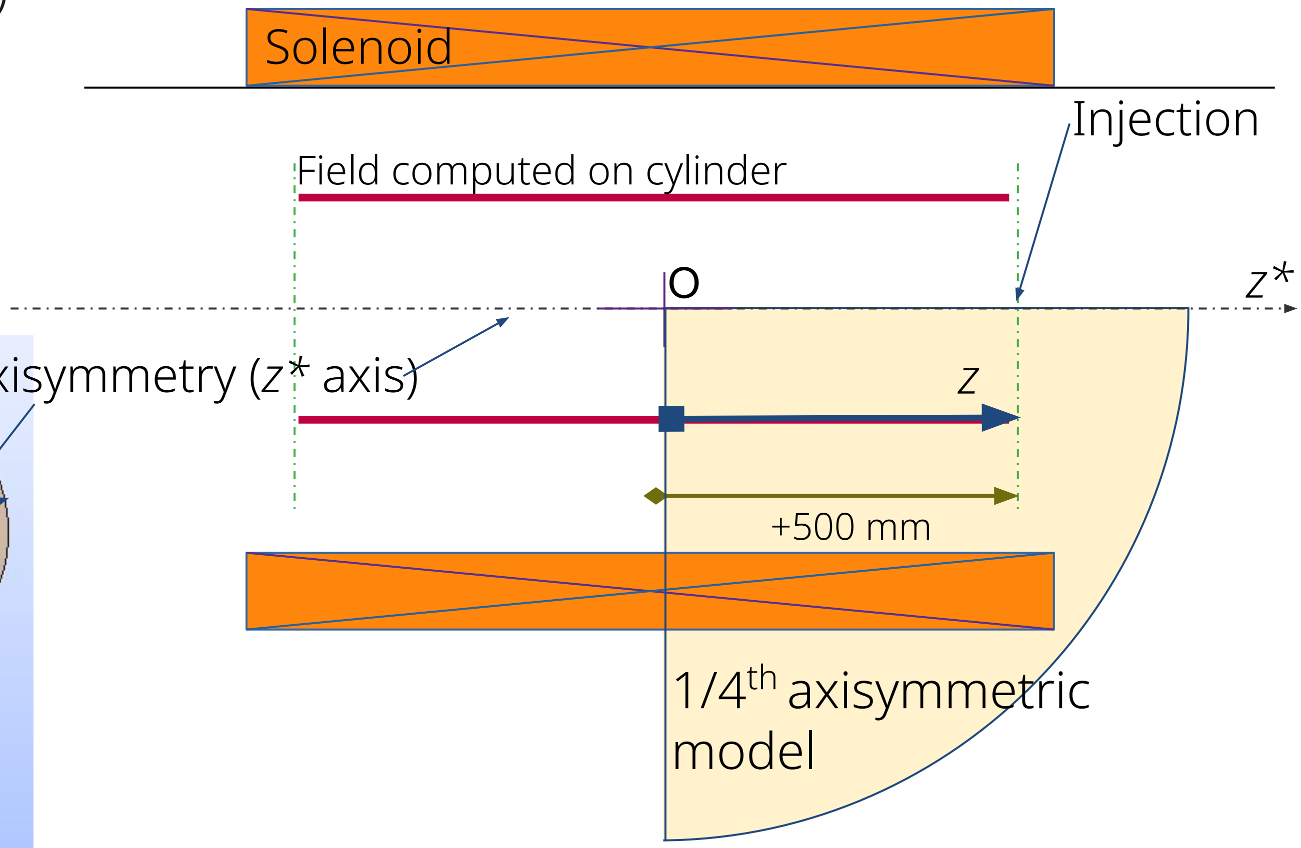


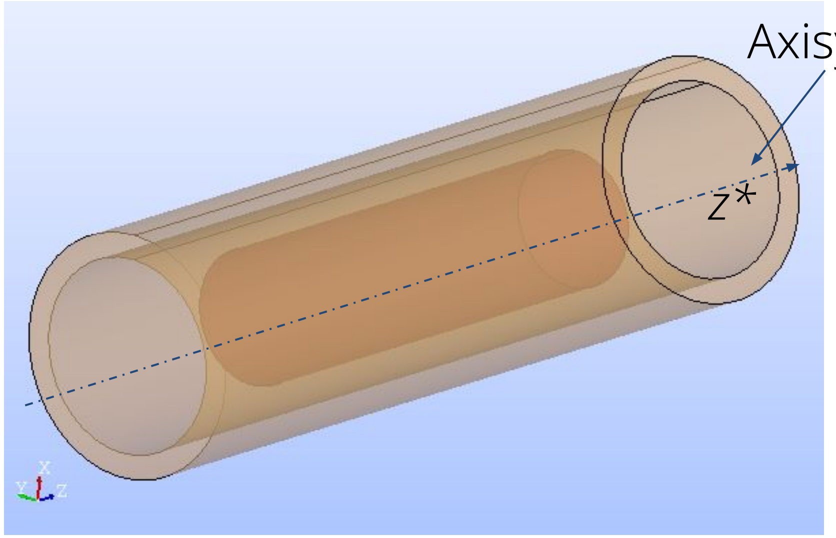


Axisymmetric model

<u>Basic assumptions</u>:

- Axisymmetry
- Continuous magnet (solenoid as a block, no injection aperture)





Tools

Not successful

1) Two optimization tools: Genetic Algorithm (GA) and Particle Swarm Optimization (PSO)

- GA: pygad library (https://pygad.readthedocs.io/en/latest/)
- PSO: pyswarms library (https://pyswarms.readthedocs.io/en/latest/)

2) Inverse problem (Singular Value Decomposition and regularization)

For 1) and 2), we used Finite Element Model (FEM) to solve the field on the region of interest:

- Gmsh (https://gmsh.info/) to build the geometry and the mesh
- GeTDP (https://getdp.info/) to solve the magnetic field over the mesh

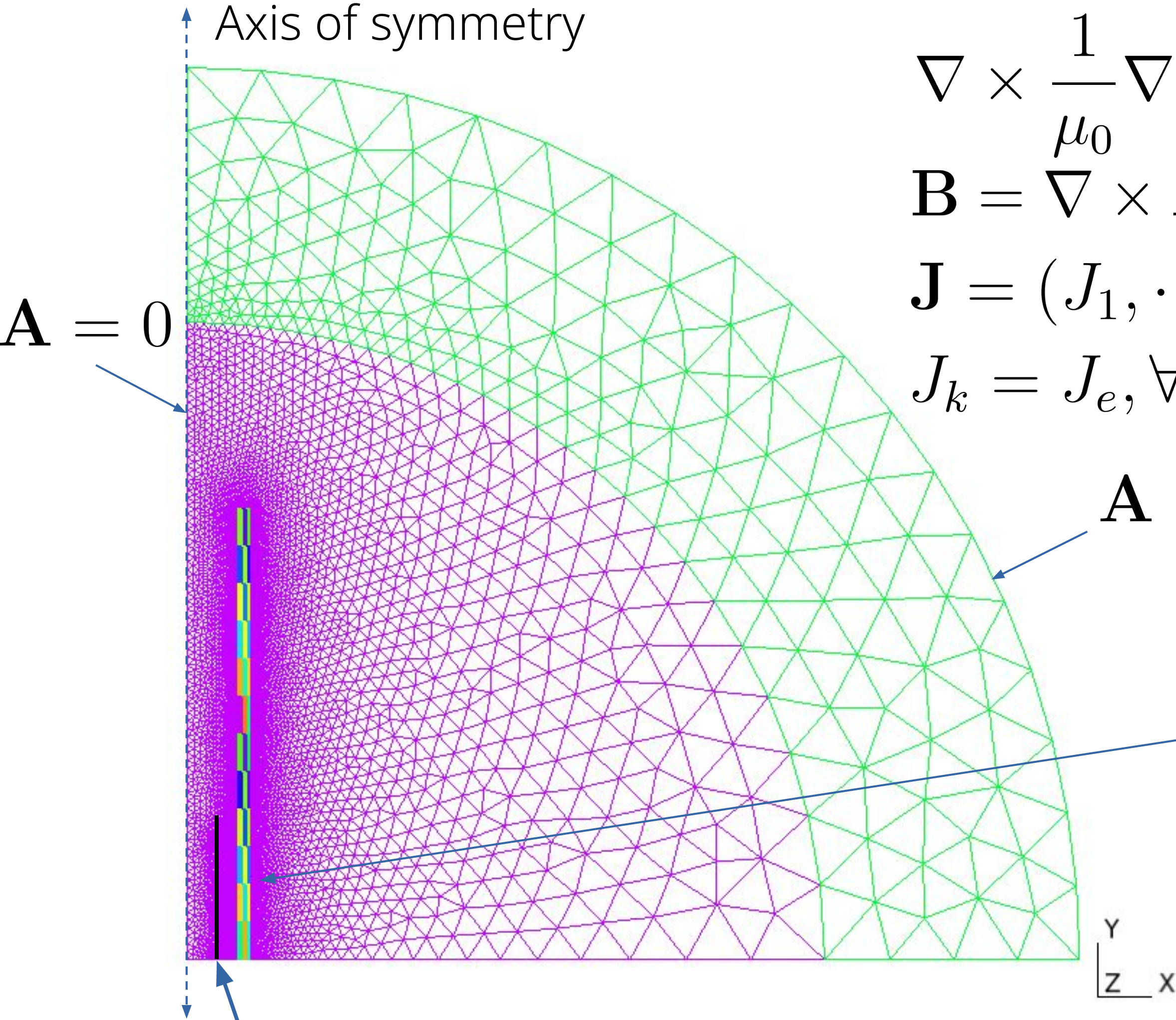
FEM

$$\nabla \times \frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{J} = 0$$

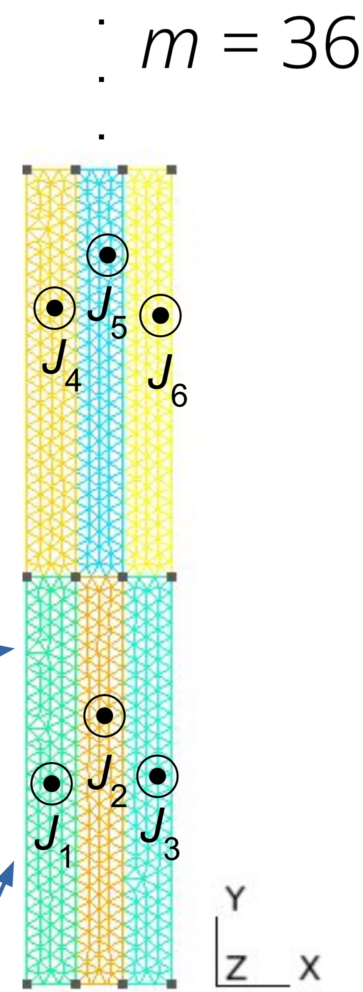
$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{J} = (J_1, \cdots, J_m)$$

$$J_k = J_e, \forall k = \{1, \cdots, m\}$$



Field interpolated over line of interest divided into n-1 segments (140 mm off axis)



The solenoid is divided in 36 sub-coils (3x12) having current densities J

Target field

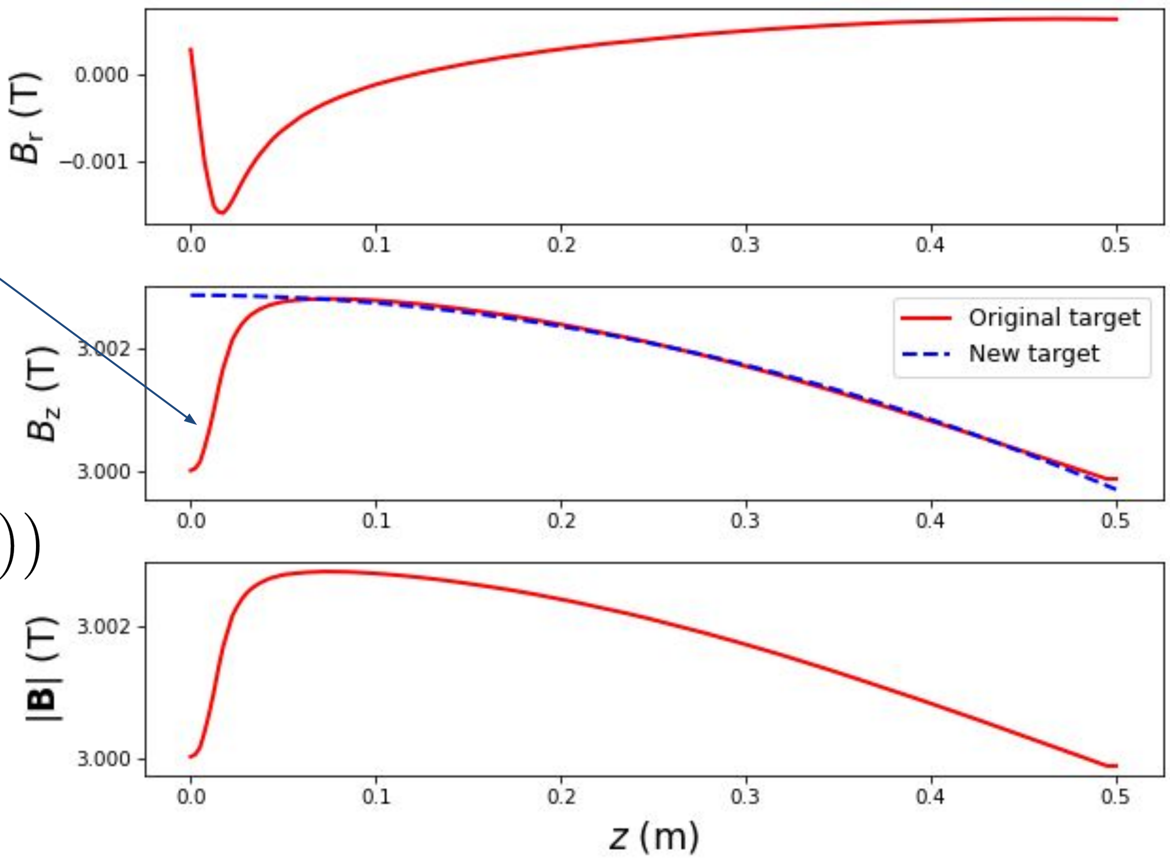
<u>Problematic</u>: providing $B_r(z)$ and $B_z(z)$ find the current density in each sub-coil to fit the specifications

To be dealt later

Objective

$$\mathbf{B}_t = (B_t(z_1), \cdots, B_t(z_n))$$

$$\mathbf{z} = (z_1, \cdots, z_n)$$



Inverse problem

Try a different approach based on an inverse problem benefitting from the linearity of the problem

$$\hat{B}_c \mathbf{J} = \mathbf{B}_t$$

$$\boxed{\mathbf{J}_s = \hat{B}_c^{\dashv} \mathbf{B}_t}$$

Singular matrix of computed B_z field

Vector of current densities

Vector of B_z target

Pseudo inverse

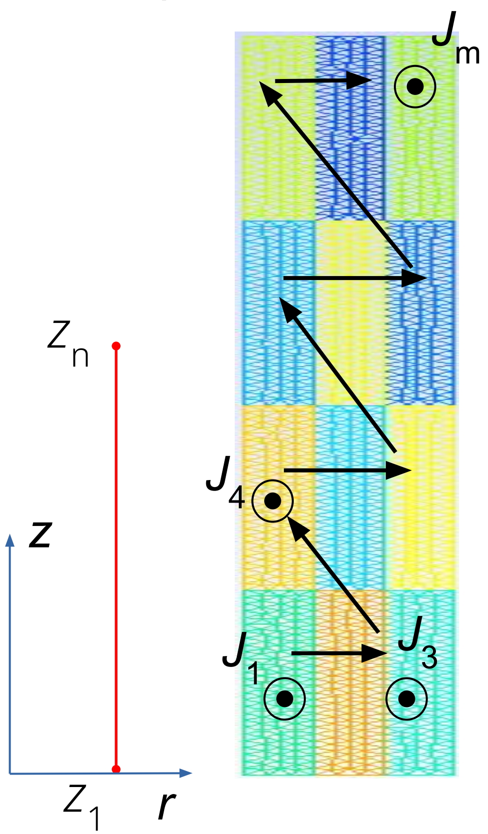
Computation of the "pseudo inverse" of $\hat{B}_c$:

Singular value decomposition and regularization on new target yielding a **good first** solution

Construction of B_z field matrix

The matrix is built considering the independent contribution of each sub-coil k to the B_z field on the line of interest (red line) divided into n-1 segments

m = 36 (3x12)
n = 200
J_e = 1e6 A/m^2



$$\begin{array}{cc} & \begin{array}{ccccc} J_1 & J_2 & & J_m & \end{array} \\ \hat{B}_c = & \begin{pmatrix} B_{1,1} & B_{1,2} & \cdots & B_{1,m} & \\ B_{2,1} & B_{2,2} & \cdots & B_{2,m} & \\ \cdot & \cdot & \cdots & & \cdot \\ \cdot & \cdot & \cdots & & \cdot \\ \cdot & \cdot & \cdots & & \cdot \\ B_{n,1} & B_{n,2} & \cdots & B_{n,m} & \end{pmatrix} \begin{array}{c} z_1 \\ z_2 \\ \\ \\ \\ z_n \end{array} \end{array}$$

SVD and regularization

Singular value decomposition (SVD):

$$\hat{B}_c = U\Sigma V^T$$

Regularization, tolerance ε = 1e-9:

Maximum SV

$$\Sigma^{\dashv} = \left(\Sigma^T\Sigma + \epsilon\lambda_{max} I_{m\times m}\right)^{-1}\Sigma^T$$

Pseudo-inverse:

$$\hat{B}_c^{\dashv} = V\Sigma^{\dashv}U^T$$

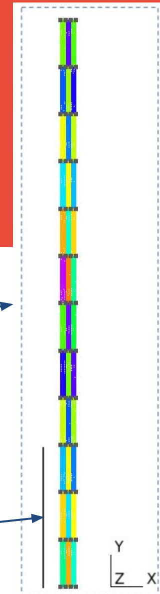
Solution:

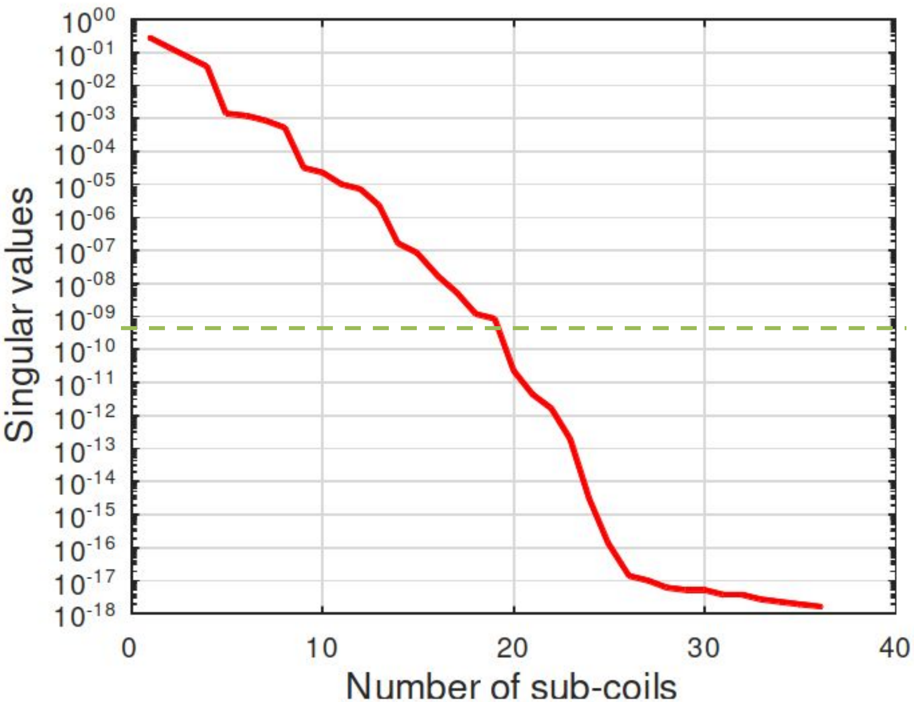
$$\mathbf{J}_s = \hat{B}_c^{\dashv}\mathbf{B}_t$$

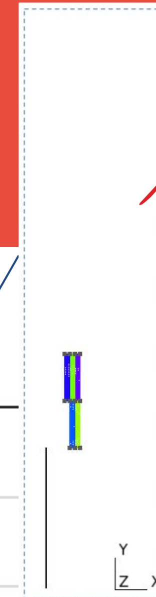
Results: SVD

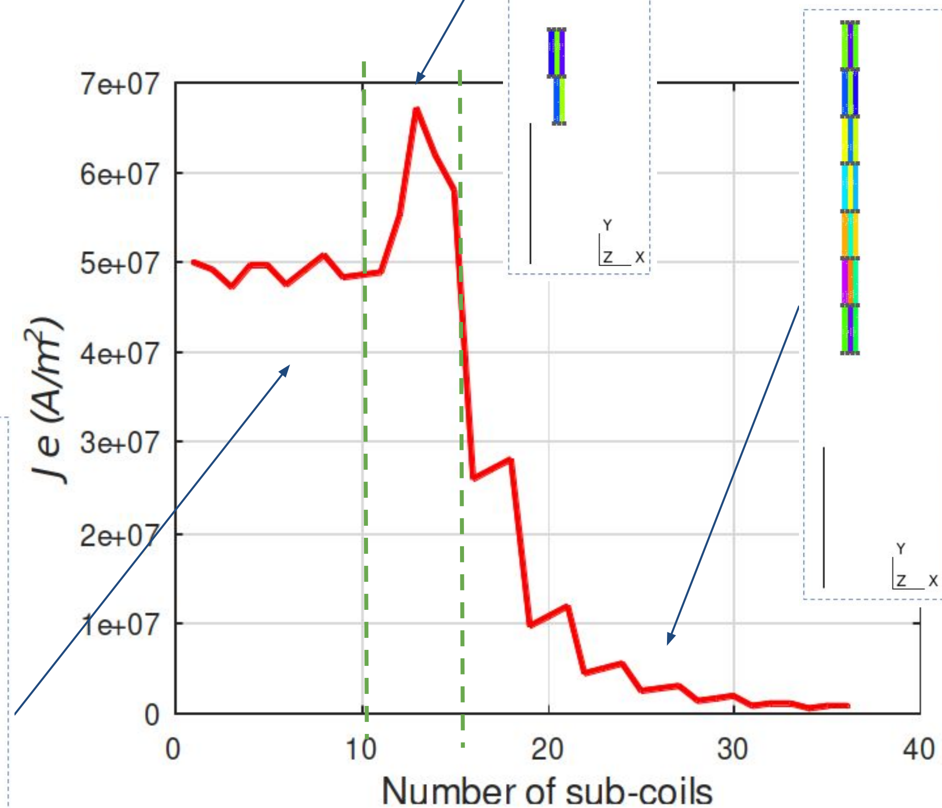
All of the sub-coils

Line of interest

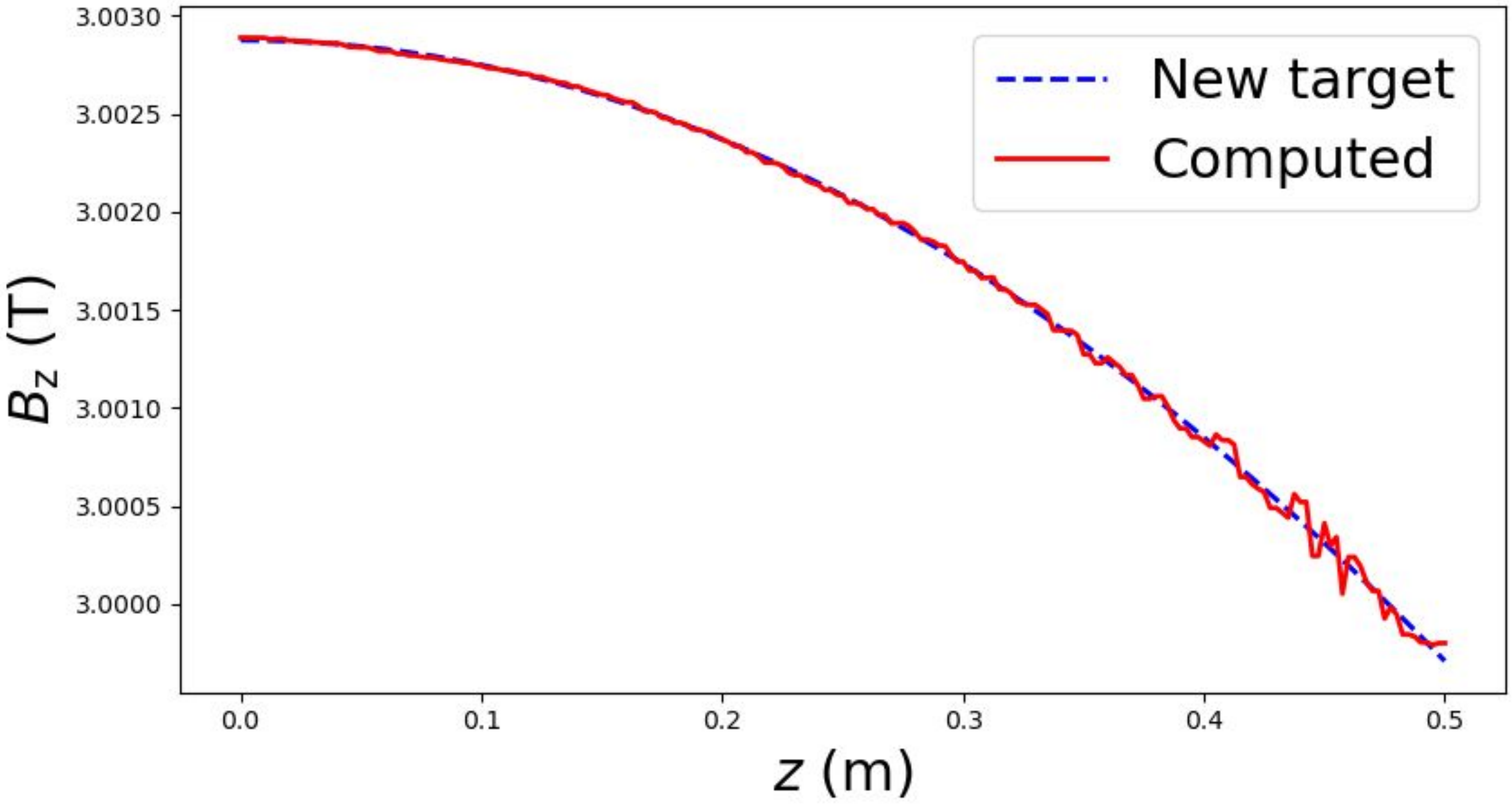






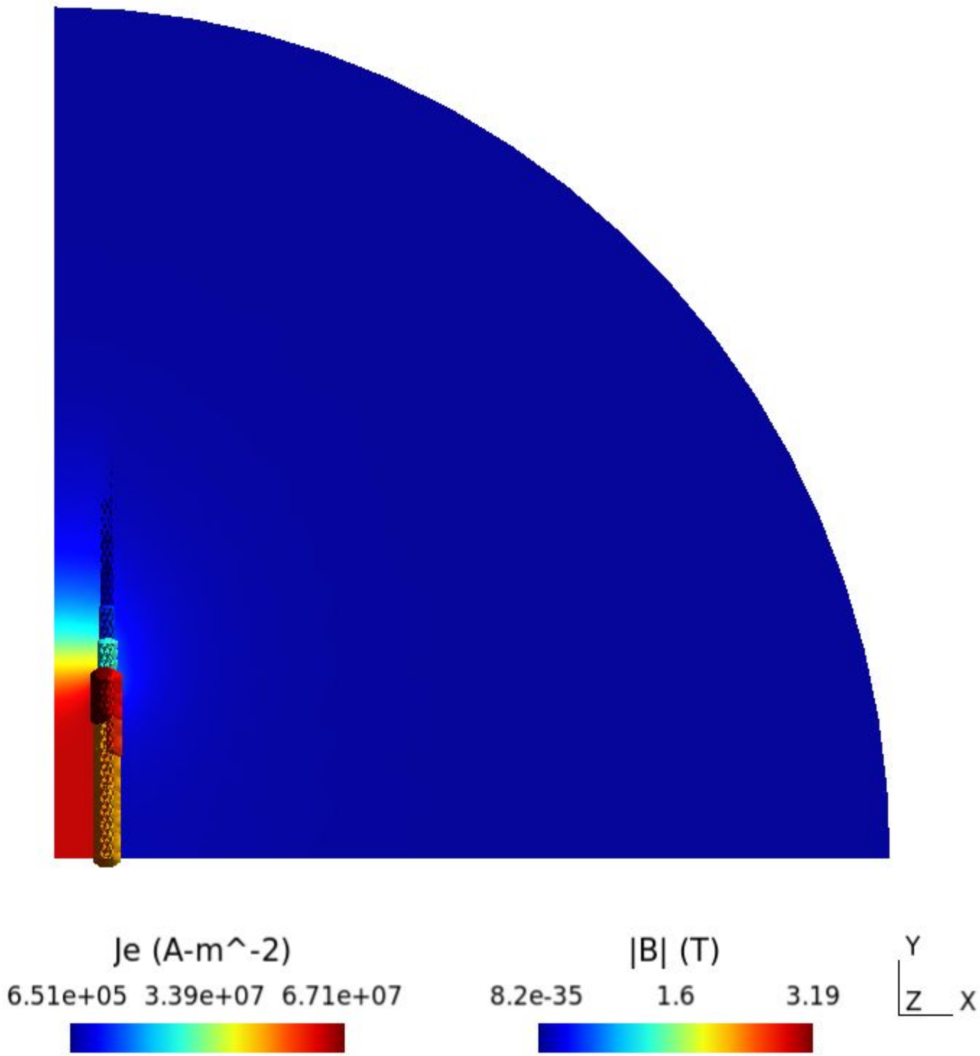


Results: SVD

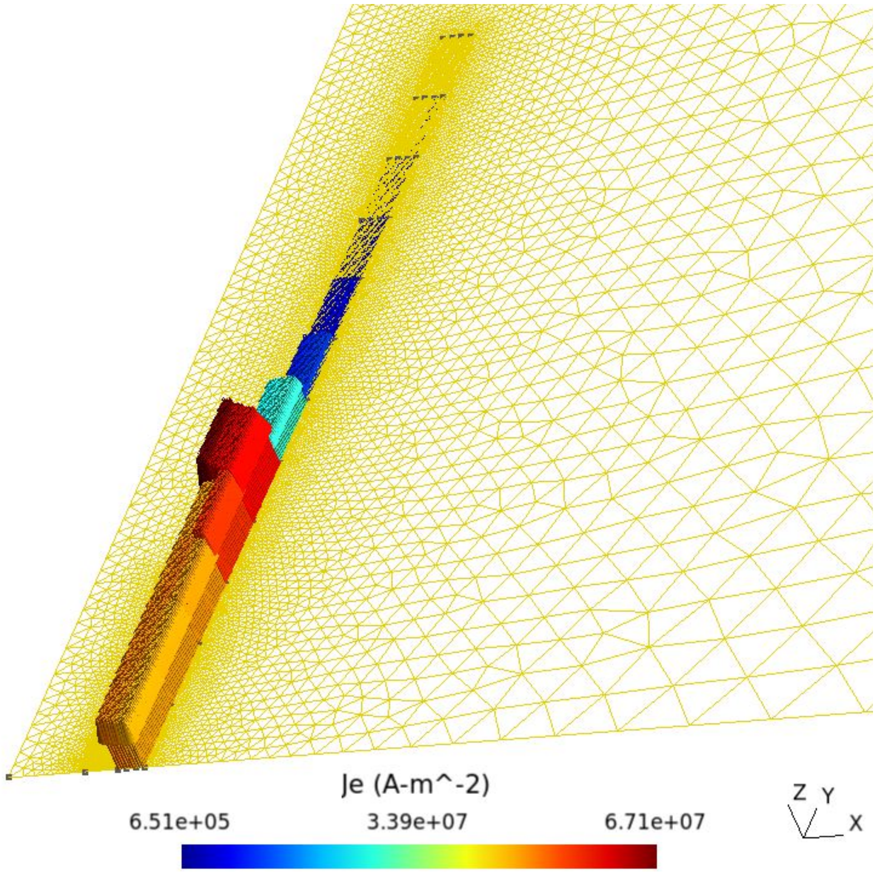


3.0030
3.0025
3.0020
3.0015
3.0010
3.0005
3.0000
B_z (T)
0.0
0.1
0.2
0.3
0.4
0.5
z (m)
New target
Computed

Results: SVD



Je (A-m^-2)
6.51e+05 3.39e+07 6.71e+07
|B| (T)
8.2e-35 1.6 3.19
Y
Z X



Je (A-m^-2)
6.51e+05 3.39e+07 6.71e+07
Z Y
X

Summary

Inverse problem using SVD and regularization -> first **good solution** based on new target (smoother B_z field)

<u>Next steps</u>:

1) Improve regularization
2) Converge on the target fields B_z and B_r
3) Include injection space (~100 mm)
4) Optimize blocks for a better current distribution
5) Iterate optimization with engineering design