Radiative Corrections for Elastic Scattering of Muons and Electrons on a Proton

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Plan of talk

Radiative corrections for elastic charge lepton scattering

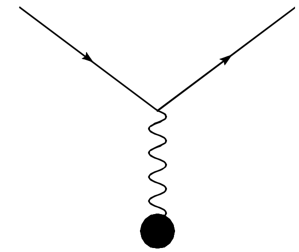
- Model-independent and model-dependent; soft and hard photons
- Refined bremsstrahlung calculations
- Role of the masses

Two-photon exchange effects

- Formulation of a problem
- Soft-photon exchange approximation and IR regularization
- Beyond soft-photon approximation

Summary

Elastic Nucleon Form Factors



- Based on one-photon exchange approximation

$$M_{fi} = M_{fi}^{1\gamma}$$

$$M_{fi}^{1\gamma} = e^2 \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(t)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p$$

- Two techniques to measure

$$\sigma = \sigma_0 (G_M{}^2 \tau + \varepsilon \cdot G_E{}^2) \quad : Rosenbluth \quad technique$$

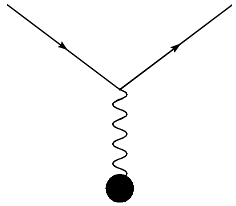
$$\frac{P_x}{P_z} = -\frac{A_x}{A_z} = -\frac{G_E \sqrt{\tau} \sqrt{2\varepsilon(1-\varepsilon)}}{G_M \ \tau \sqrt{1-\varepsilon^2}} \quad : Polarization \quad technique$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

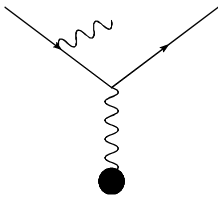
$$(P_y = 0)$$

Latter due to: Akhiezer, Rekalo; Arnold, Carlson, Gross

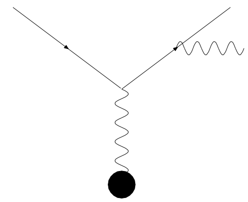
Basics of QED radiative corrections



(First) Born approximation

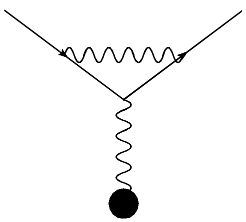


Initial-state radiation



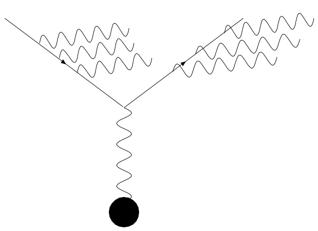
Final-state radiation

Cross section ~ dω/ω => integral diverges logarithmically: **IR catastrophe**



Vertex correction => cancels divergent terms; Schwinger (1949)
Assumed $Q^2/m_e^2 >> 1$

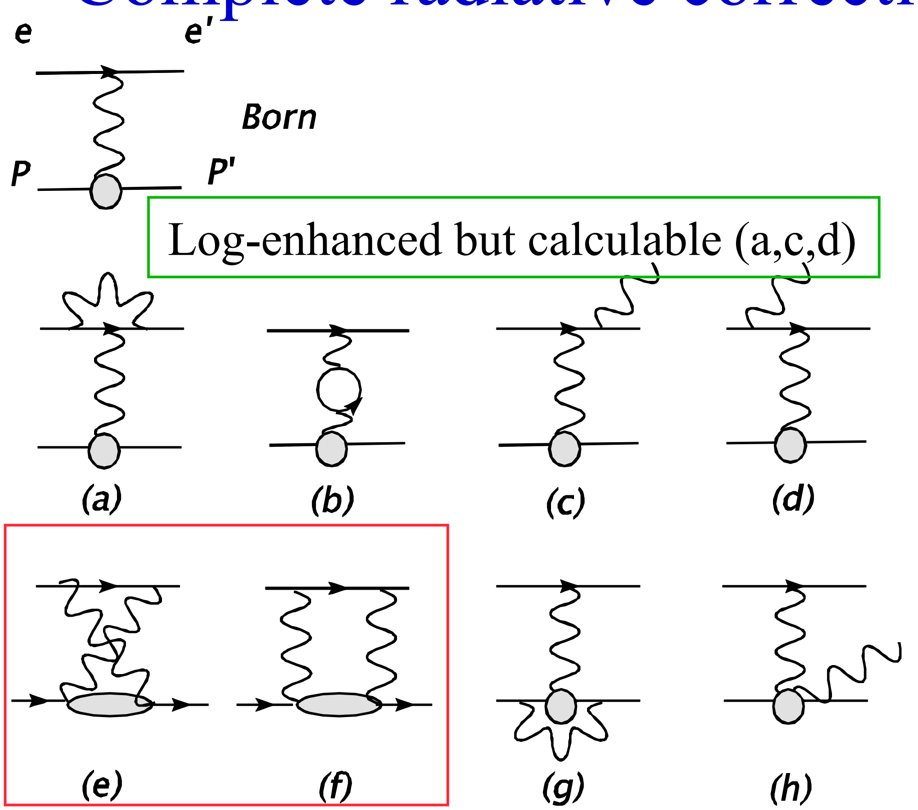
$$\sigma_{\exp} = (1+\delta)\sigma_{Born}, \quad \delta = \frac{-2\alpha}{\pi}\{(\ln\frac{E}{\Delta E} - \frac{13}{12})(\ln\frac{Q^2}{m_e^2} - 1) + \frac{17}{36} + \frac{1}{2}f(\theta)\}$$



Multiple soft-photon emission: solved by exponentiation, Yennie-Frautschi-Suura (YFS), 1961

$$(1+\delta) \rightarrow e^{\delta}$$

Complete radiative correction in $O(\alpha_{em})$



Radiative Corrections:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

•Meister&Yennie; Mo&Tsai

•Further work by Bardin&Shumeiko; Maximon&Tjon; AA, Akushevich, Merenkov;

•Guichon&Vanderhaeghen' 03:
Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% ...

Main issue: Corrections dependent on nucleon structure

Model calculations:

•Blunden, Melnitchouk,Tjon, Phys.Rev.Lett.**91**:142304,2003

•Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004

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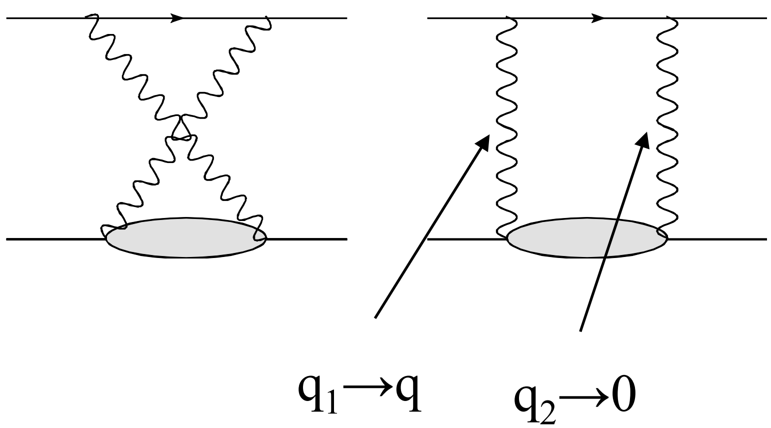
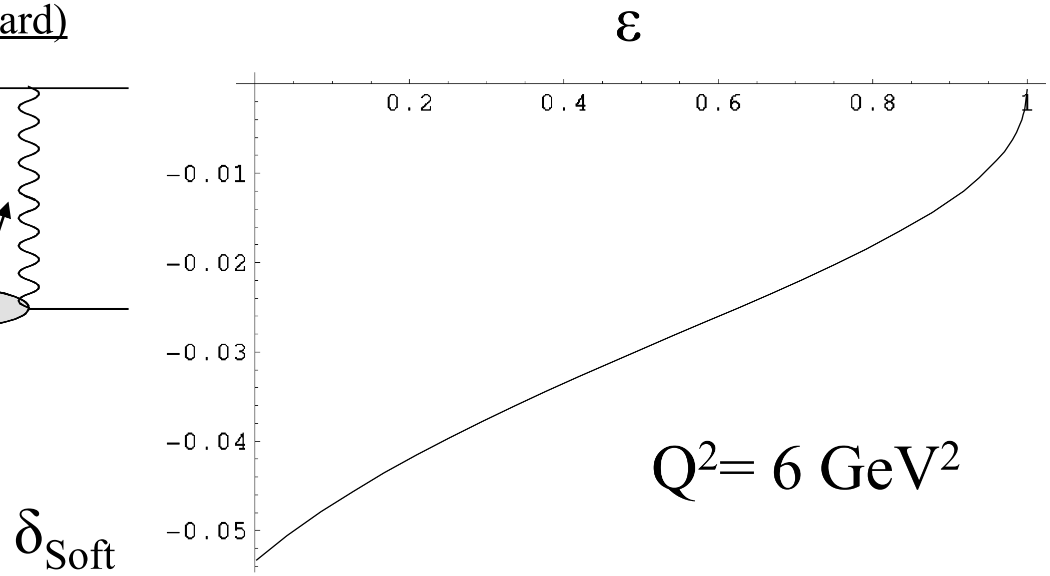
Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon (k→0); similar to Coulomb corrections at low Q^2
- Grammer &Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis for elastic ep**
  - Also done for pion electroproduction in AA, Aleksejevs, Barkanova, arXiv:1207.1767 (there we also used Passarino-Veltman parameterization of loop integrals, inclusion of lepton masses is straightforward)





Lepton mass is not essential for TPE calculation;
Two-photon effect below 1% for lower energies and $Q^2 < 0.1 GeV^2$

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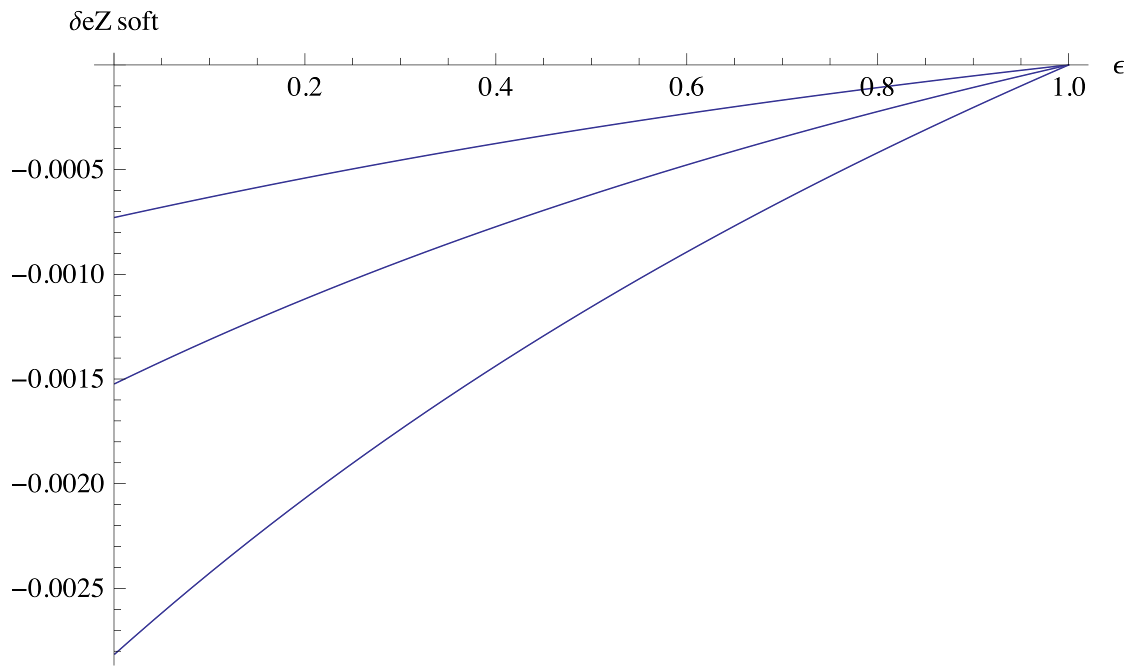
Soft TPE for MUSE

- Soft TPE correction for ep scattering using Mo-Tsai formalism for three fixed electron energies: 115 MeV/c (top), 153 MeV/c (middle) 210 MeV/c (bottom curve); no large logs involved, muon correction is about the same: 0.25% max.



Hard Bremsstrahlung

- Need to include radiative lepton tensor in a complete form:

AA et al, **Phys.Rev. D64 (2001) 113009; PLB 514, 269 (2001*):*** terms ~ k emitted photon momentum) usually neglected in rad.correction calculations, but can lead to ~1% effect for Rosenbluth slope at high Q^2

$$L^r{}_{\mu\nu} = -\frac{1}{2} Tr(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1 + \gamma_5 \hat{\xi}_e)(\hat{k}_1 + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2}\right)\gamma_\mu - \frac{\gamma_\mu \hat{k} \gamma_\alpha}{2k \cdot k_1} - \frac{\gamma_\alpha \hat{k} \gamma_\mu}{2k \cdot k_2}$$

$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2}\right)\gamma_\nu - \frac{\gamma_\alpha \hat{k} \gamma_\nu}{2k \cdot k_1} - \frac{\gamma_\nu \hat{k} \gamma_\alpha}{2k \cdot k_2}$$

Lepton Mass Effects

- Standard approximations keep the lepton mass in the logarithms but neglect it in power terms. May be justified in the ultrarelativistic case and Q^2>>(lepton mass)^2
- Most of analysis codes use exact mass dependence for hard brem, but use above approximations for the "soft" part of brem correction
- Revised approach is required that will NOT result in new theoretical uncertainties
- New rad.correction codes no longer use peaking approximation (justified for relatively small lepton masses)
- Formalism and Monte-Carlo generators can be adapted for this analysis (ELRADGEN; MASCARAD, etc)

Coulomb and Two-Photon Corrections

- Coulomb correction calculations are well justified at lower energies and Q2 considered in this experiment
- Hard two-photon exchange (TPE) contributions cannot be calculated with the same level of precision as the other contributions.
- Two-photon exchange is independent on the lepton mass in an ultra-relativistic case.
- Issue: For energies ~ mass TPE amplitude is described by 6 independent generalized form factors; but experimental data on TPE are for ultrarelativistic electrons, hence independent info on 3 other form factors will be missing.
- Theoretical models show the trend that TPE has a smaller effect at lower Q^2 . The reason is that "hard" TPE amplitudes do not have a $1/Q^2$ Coulomb singularity, as opposed to the Born amplitude.

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Conclusions

- Radiative corrections are standard and well-established in electron scattering, care must be taken in this experiment that the radiative correction calculations are correctly implemented without invalid approximations.
- Parts of the radiative corrections are expected to be suppressed for muons due to the larger muon mass. Two-photon exchange corrections are generally expected to be small, and should be similar for electrons and muons. However, two-photon exchange remains more poorly understood than one would like.
- We evaluate systematic uncertainty from radiative corrections at 0.5% level for both muon and electron cross sections, mostly arising from uncertainty in two-photon exchange effects