Light muonic atoms in search of new interactions at the Compton wavelength scale

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Proton charge radius puzzle

- global fit to H and D spectrum: $r_p = 0.8758(77) \text{ fm}$ (CODATA 2010)
- $e - p$ scattering: $r_p = 0.8791(79)$ (Bernauer, 2010)
- from muonic hydrogen: $r_p = 0.84089(39) \text{ fm}$ (PSI, 2010, 2012)

There is no any accepted explanation for this discrepancy, so far.
Proton charge radius puzzle

- Is it obvious that the Standard Model predicts the same \( e - p \) and \( \mu - p \) interaction at the 1fm scale?

- If \( e - p \) experiments and \( \mu H \) theory are correct, the plausible solution of this puzzle is the additional interaction at the 1 fm or the electron Compton wavelength scales.

How it can be verified?

Let us say few words about \( \mu H \) theory, why is it so reliable.
energy levels of $\mu H$

$E_L = 202.1$ meV

$E_{FS} = 8.4$ meV

$E_{HFS} (2S_{1/2}) = 22.7$ meV

$E_{HFS} (2P_{1/2}) = 8.0$ meV

$E_{HFS} (2P_{3/2}) = 3.4$ meV

$\Delta = 0.1$ meV
\( \mu H \) energy levels

- \( \mu H \) is essentially a nonrelativistic atomic system
- muon and proton are treated on the same footing
- \( m_\mu/m_e = 206.768 \Rightarrow \beta = m_e/(\mu \alpha) = 0.737 \) the ratio of the Bohr radius to the electron Compton wavelength
- the electron vacuum polarization dominates the Lamb shift in muonic hydrogen
Theory of $\mu H$ energy levels

- nonrelativistic Hamiltonian $H_0 = \frac{p^2}{2 m_\mu} + \frac{p^2}{2 m_p} - \frac{\alpha}{r}$

- and the nonrelativistic energy $E_0 = -\frac{m_r \alpha^2}{2 n^2}$

- the evp dominates the Lamb shift

\[ E_L = \int d^3 r \ V_{vp}(r) (\rho_{2P} - \rho_{2S}) = 205.0073 \text{ meV} \]

\[ \text{without finite size} = 206.0336(5) \text{ meV} \]

- important corrections: second order, two-loop vacuum polarization, and the muon self-energy

- other corrections are much smaller than the discrepancy of 0.3 meV.
Breit-Pauli Hamiltonian

\[ H_{BP} = H_0 + \delta H_{BP} \]

\[ \delta H_{BP} = -\frac{p^4}{8 m_\mu^3} - \frac{p^4}{8 m_p^3} - \frac{\alpha}{2 m_\mu m_p} p^i \left( \frac{\delta_{ij}}{r} + \frac{r^i r^j}{r^3} \right) p^j \]

\[ + \frac{2 \pi \alpha}{3} \left( \langle r_p^2 \rangle + \frac{3}{4 m_\mu^2} + \frac{3}{4 m_p^2} \right) \delta^3(r) \]

\[ + \frac{2 \pi \alpha}{3 m_\mu m_p} g_\mu g_p \vec{s}_\mu \cdot \vec{s}_p \delta^3(r) - \frac{\alpha}{4 m_\mu m_p} g_\mu g_p \frac{s_\mu^i s_p^j}{r^3} \left( \delta_{ij} - 3 \frac{r^i r^j}{r^2} \right) \]

\[ + \frac{\alpha}{2 r^3} \vec{r} \times \vec{p} \left[ \vec{s}_\mu \left( \frac{g_\mu}{m_\mu m_p} + \frac{(g_\mu - 1)}{m_\mu^2} \right) + \vec{s}_p \left( \frac{g_p}{m_\mu m_p} + \frac{(g_p - 1)}{m_p^2} \right) \right] \]
Leading relativistic correction

\[ \delta_{\text{rel}} E_L = \langle 2P_{1/2} | \delta H_{BP} | 2P_{1/2} \rangle - \langle 2S_{1/2} | \delta H_{BP} | 2S_{1/2} \rangle = \frac{\alpha^4 m_r^3}{48 m_p^2} = 0.05747 \text{ meV} \]

- valid for an arbitrary mass ratio
- quite small and higher order relativistic corrections are negligible
Leading vacuum polarization

\[ V_{vp}(r) = -\frac{Z \alpha}{r} \frac{\alpha}{\pi} \int_{4}^{\infty} \frac{d(q^2)}{q^2} e^{-m_e q r} u(q^2) \]

\[ u(q^2) = \frac{1}{3} \sqrt{1 - \frac{4}{q^2}} \left( 1 + \frac{2}{q^2} \right) \]

\[ \delta_{vp} E_L = \langle 2P_{1/2} | V_{vp} | 2P_{1/2} \rangle - \langle 2S_{1/2} | V_{vp} | 2S_{1/2} \rangle = 205.0073 \text{ meV} \]

- the dominating part of the muonic hydrogen Lamb shift
- the expectation value is taken with nonrelativistic wave function
- the muon-proton mass ratio \( \eta \) is included exactly
Higher order vacuum polarization

- second order $V_{vp}$: $\delta E_L = 0.1509$ meV
- two-loop vp: $\delta E_L = 1.5081$ meV
- three-loop vp: $\delta E_L = 0.0053$ meV
- hadronic vp: $\delta E_L = 0.0112(4)$ meV

Muonic vp is included later together with the self-energy

Is there any further correction related to vp?
$\delta E_L = -0.0009$ meV

- significant cancellation between diagrams

- S.G. Karshenboim et al., arXiv:1005.4880
Small corrections

- relativistic correction to $\nu p$

\[
\delta_{\nu p, \text{rel}} E_L = \langle \delta_{\nu p} H_{BP} \rangle + 2 \langle V_{\nu p} \frac{1}{(E - H)'} H_{BP} \rangle
= 0.01876 \text{ meV}.
\]

If one used the Dirac equation in the infinite nuclear mass limit, the obtained result would be 0.021 meV.

- muon self-energy and muon $\nu p$: $\delta E_L = -0.6677 \text{ meV}$
- muon self-energy combined with evp: $\delta E_L = -0.0025 \text{ meV}$
- pure recoil corrections of order $\alpha^5$: $\delta E_{LS} = -0.0450 \text{ meV}$
Summary of theoretical predictions

\[
\Delta E_{LS} = 206.0336(15) - 5.2275(10) r_p^2 + \Delta E_{TPE}
\]
\[
\Delta E_{FS} = 8.3521 \text{ meV}
\]
\[
\Delta E_{2S_{1/2}}^{HFS} = 22.8089(51) \text{ meV, (exp. value)}
\]
\[
\Delta E_{2P_{1/2}}^{HFS} = 7.9644 \text{ meV}
\]
\[
\Delta E_{2P_{3/2}}^{HFS} = 3.3926 \text{ meV}
\]
\[
\Delta = 0.1446 \text{ meV}
\]

where \(\Delta E_{TPE} = 0.0351(20) \text{ meV}\) is a proton structure dependent two-photon exchange contribution, on the next slide...
Nuclear structure effects

- if nuclear excitation energy is much larger than the atomic energy, the two-photon exchange scattering amplitude gives the dominating correction

- the total proton structure contribution $\delta E_L = 0.0351(20)$ meV is much too small to explain the discrepancy, but its calculation is not very certain [Carlson, Vanderhaeghen, 2011]
Lepton-proton interaction at the 1 fm scale

- Question: How to test universality of the lepton-proton interaction?
- Answer: compare $e - p$ with $\mu - p$ scattering: MUSE project, old $\mu - p$ Brookhaven scattering data (1969) are not conclusive
- Answer: $\mu^4$He and $\mu^3$He measurements, if discrepancy persists, is should be parametrized by

$$\delta E = (Z \delta r_p^2 + (A - Z) \delta r_n^2) \frac{2\delta l_0}{3 n^3} Z^3 \alpha^4 \mu^3$$

Determination of $r_N$ from muonic atoms spectra requires an accurate calculation of the nuclear polarizability correction, not necessarily easy task (comments on recent calculations)
Lamb shift in $\mu H$

Fifth force at the Compton wavelength scale

- $\mu H$ is very sensitive to the electron vacuum polarisation:
  for $H$
  \[
  \frac{E_{\text{discrepancy}}}{E_{\text{Uehling}}} = \frac{94 \text{ kHz}}{216,676 \text{ kHz}} = 0.00043
  \]
  for $\mu H$
  \[
  \frac{E_{\text{discrepancy}}}{E_{\text{Uehling}}} = \frac{0.31 \text{ meV}}{205.0073 \text{ meV}} = 0.0015
  \]
- This means that a small modification of $V_{vp}$ may explain discrepancy as the change in $\mu H$ is 4 times larger than in $H$.
- Are there any measurements sensitive to 5th force at the electron Compton wavelength?
**fifth force: Salumbides, Ubachs, Korobov (2013)**

- 

\[
\begin{align*}
    &\text{dd}\mu^+ \\
    &\bar{p}^4\text{He}^+ \quad n=36\rightarrow 34 \\
    &\bar{p}^4\text{He}^+ \quad n=33\rightarrow 31 \\
    &\bar{p}^3\text{He}^+ \quad n=35\rightarrow 33 \\
    &\text{HD}^+ \\
\end{align*}
\]
rovibrational spectra are sensitive to the fifth force at the Compton wavelength down to 1fm

the strong interaction shift is negligible for higher rotational levels

one can determine the $r$-dependence

theoretical spectra can be obtained with the required accuracy