

Theoretical study of muon capture in light nuclei

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Muon captures in light nuclei ($A \leq 3$)

- $\mu^- + p \rightarrow n + \nu_\mu$
- $\mu^- + d \rightarrow n + n + \nu_\mu$

- $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$ (70%)
- $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$ (20%)
- $\mu^- + {}^3\text{He} \rightarrow n + n + p + \nu_\mu$ (10%)

Muon capture on protons

$\Gamma(\mu - p)_{f=0} = 725 \text{ s}^{-1}$ [MUCAP Experiment (PSI)]
Extraction of the pseudoscalar form factor of the nucleon

$$j^\mu = \overline{u_p} \left[F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} - G_A(q^2) \gamma^\mu \gamma^5 - G_{PS}(q^2) \frac{q^\mu \gamma^5}{2M_N} \right] u_n$$

$$G_{PS}^{\text{expt}} = 8.06 \pm 0.48 \pm 0.28$$

[arXiv:1210.6545]

$$G_{PS}^{\chi\text{PT}} = 7.99 \pm 0.20$$

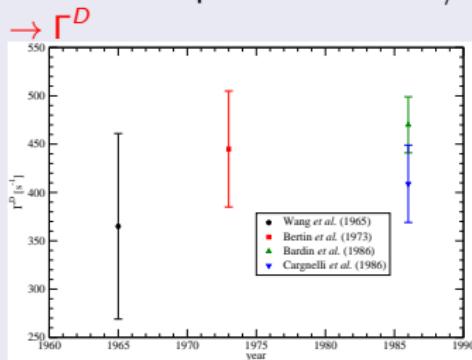
Kaiser, 2003

Muon capture on d and ${}^3\text{He}$

- important contributions from meson exchange currents
- consistent treatment of the strong and weak interactions



- Two hyperfine states: $f = 1/2$ and $3/2$
- Dominant capture from $f = 1/2$



- New measurement in progress:
MuSun (see next talk)



- Hyperfine states: $(f, f_z) = (1, \{\pm 1, 0\})$ and $(0, 0)$

$$\begin{aligned} \frac{d\Gamma}{d(\cos \theta)} &= \frac{1}{2} \Gamma_0 [1 + A_v P_v \cos \theta \\ &\quad + A_t P_t \left(\frac{3 \cos^2 \theta - 1}{2} \right) + A_\Delta P_\Delta] \\ P_v &= P_{1,1} - P_{1,-1} \\ P_t &= P_{1,1} + P_{1,-1} - 2P_{1,0} \\ P_\Delta &= 1 - 4P_{0,0} \end{aligned}$$

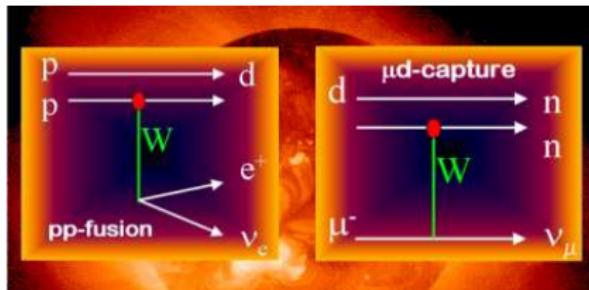
- Γ_0 = total capture rate, $A_{v,t,\Delta}$ = angular correlation parameters
- Ackerbauer et al., (1998):

$$\boxed{\Gamma_0 = 1496(4) \text{ s}^{-1}}$$
- Souder et al., (1998): $A_v = 0.63 \pm 0.09$ (stat.) $^{+0.11}_{-0.14}$ (syst.)

Relation with fusion reactions of astrophysical interest

Examples:

- $p + p \rightarrow {}^2\text{H} + e^+ + \nu_e$
- $p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e$



- Same transition operators as for muon capture
- MuSun experiment: critical test of our knowledge of the weak transitions in nuclei

Theory

- old: "Standard Nuclear Physics Approach" (SNPA)
 - based on $\pi-$, $\rho-$, ω -exchanges, Δ -excitation
 - not consistency between nuclear wave functions and transition operators
 - no clear way how to "improve" the accuracy
 - *hep* fusion → [Marcucci *et al.*, 2000]
- new: "Chiral Effective Field Theory" (χ EFT) approach

- more contact with QCD
 - systematic and controlled expansion of nuclear potential/transition operators
 - [Weinberg, 1990], [Ordonez & Van Kolck, (1992)], [Meissner (1992)], [Park, Rho & Kubodera (1995)], ...
 - Used to study a large variety of nuclear processes
-
- Degrees of freedom at energy $Q > \Lambda$ integrated out
 - \mathcal{L} useful for processes of energy $Q \ll \Lambda$
 - $\Lambda \approx 500 \text{ MeV} - 700 \text{ MeV}$ (problem with the Δ)
 - \rightarrow organize the expansion in powers of Q/Λ (possible since chiral symmetry imposes **derivative couplings**)
 - “pionfull” χ EFT: degrees of freedom: pions and nucleons ($\Lambda \sim 500 \text{ MeV}$)
 - “pionless” χ EFT: degrees of freedom: nucleons ($\Lambda \leq \sim m_\pi \text{ MeV}$)

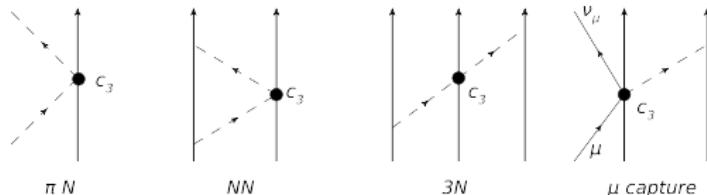
Low energy constants (LEC's)

- Include all possible Lagrangian terms contributing at a given Q^ν
- Each term is multiplied by a “coupling constant” → the LEC
- They encode our ignorance of what happens for $Q > \Lambda$
- There are (for a given ν) a finite number of LEC's**
 - Calculated using QCD **work in progress**
 - Fitted to some experimental data or extracted from some model
- The LEC's depend on Λ – the observables should not depend on it
- **the same LEC's enter the nuclear potentials & currents**

Two ingredients: nuclear wave functions + weak transition operators

Issue: consistency between strong/EM/weak interactions

Example: $\mathcal{L} = \dots + c_3 \bar{\Psi} u_\mu (u^\mu)^\dagger \Psi + \dots$ where $u_\mu = \frac{1}{f_\pi} \partial_\mu \vec{\pi} \cdot \vec{\tau} - \frac{G_F}{\sqrt{2}} \tau^+ J_\mu^\ell$



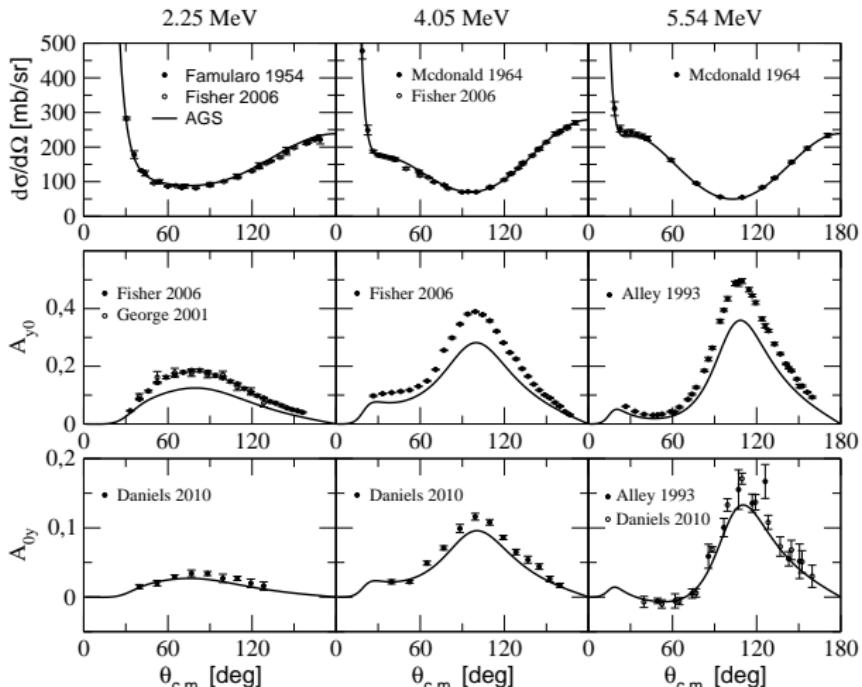
NN & 3N forces

	2N force	3N force	4N force
LO		—	—
NLO		—	—
N^2LO			—
N^3LO			

- NN force: N^3LO “N3LO” model – [Entem & Machleidt, 2003]
- 3N force: N^2LO “N2LO” model – [Navratil, 2007], [Marcucci *et al.*, 2012]
- In progress: 3N force at N^3LO & N^4LO [Krebs *et al.*, 2012]

Comparison of 4N calculations [PRC 84, 054010 (2011)]

HH = expansion on a complete basis + Kohn variational principle
FY & AGS = solution of the Faddeev-Yakubovsky or AGS equations



$p - {}^3\text{He}$ elastic scattering

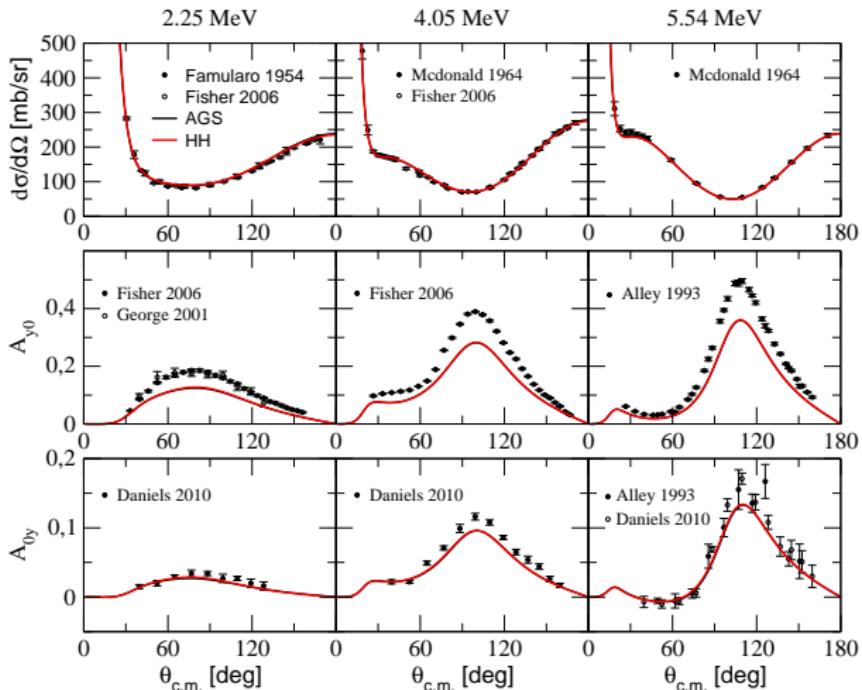
N3LO potential

AGS = Deltuva & Fonseca

FY = Lazauskas & Carbonell

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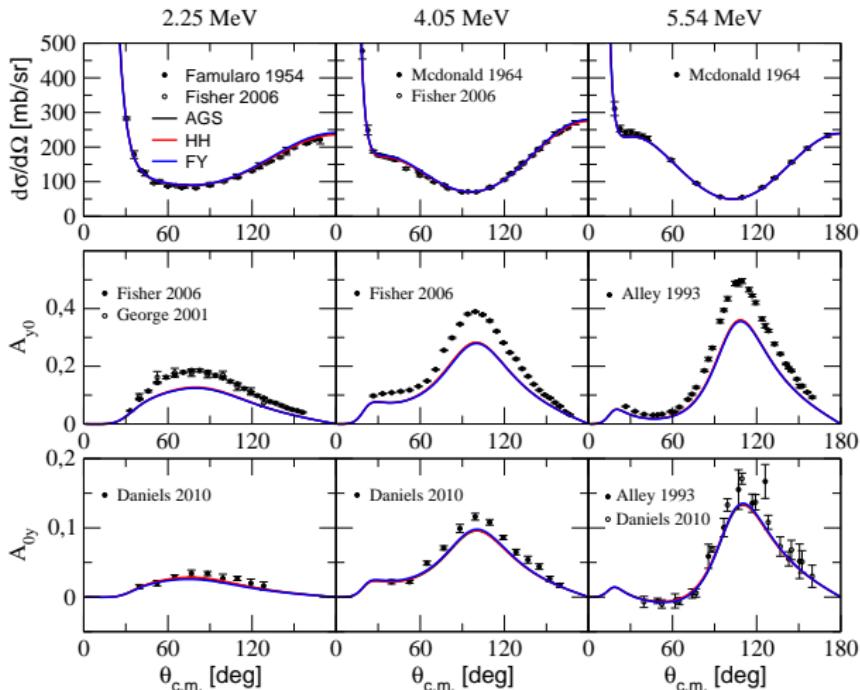
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Transition operators

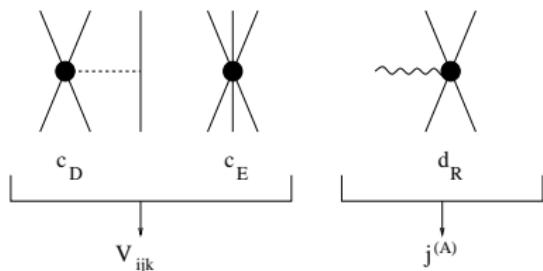
$$\langle \Psi_f | H_I | \Psi_i \rangle = \frac{G_F}{\sqrt{2}} [V^\mu - A^\mu] J_\mu^I$$

$$V^\mu = (\rho_V, \mathbf{J}_V)$$
$$A^\mu = (\rho_A, \mathbf{J}_A)$$

	ρ_V	J_V	ρ_A	J_A
LO				
NLO				
N2LO				
N3LO				
N4LO				

- ρ_V and \mathbf{J}_V : N⁴LO [Park *et al.*, 1996], [Koelling *et al.*, 2009-2011], [Pastore, MV, *et al.*, 2009,2011]
- ρ_A and \mathbf{J}_A : N³LO [Park *et al.*, 2003] (N⁴LO in progress)

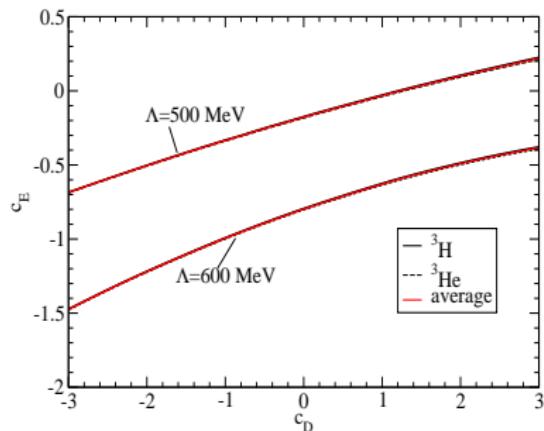
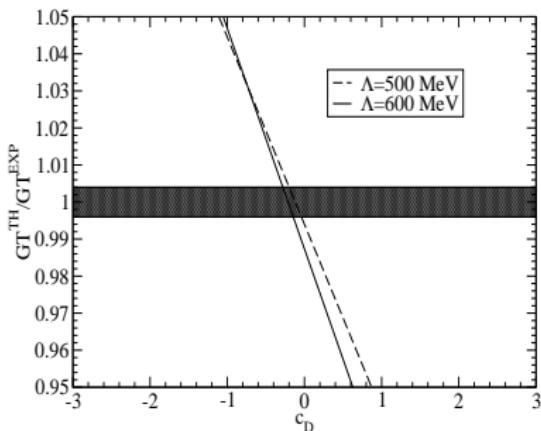
Example: the LEC d_R



$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N(c_3 + 2c_4) + \frac{1}{6}$$

Gardestig and Phillips, PRL **96**, 232301 (2006)
 Gazit *et al.*, PRL **103**, 102502 (2009)

fit c_D (d_R) to GT_{exp} and c_E to $B(A=3)$ (using the N3LO/N2LO model)
 $\Rightarrow \{c_D; c_E\}_{MAX}$ and $\{c_D; c_E\}_{MIN}$



Remaining LEC's: g_{4S} and g_{4V} in the vector current \Rightarrow fit to the $A = 3$ magnetic moments

	$\{c_D; c_E\}$	g_{4S}	g_{4V}
$\Lambda = 500 \text{ MeV}$	$\{-0.20; -0.208\}$	0.207 ± 0.007	0.765 ± 0.004
	$\{-0.04; -0.184\}$	0.200 ± 0.007	0.771 ± 0.004
$\Lambda = 600 \text{ MeV}$	$\{-0.32; -0.857\}$	0.146 ± 0.008	0.585 ± 0.004
	$\{-0.19; -0.833\}$	0.145 ± 0.008	0.590 ± 0.004

Radiative corrections¹ ARE included

¹ Czarnecki *et al.*, PRL **99**, 032003 (2007)

Results: $\Gamma^D(\mu^- + d)$

SNPA(AV18)	1S_0	3P_0	3P_1	3P_2	1D_2	3F_2	Total
$g_A=1.2654(42)$	246.6(7)	20.1	46.7	71.6	4.5	0.9	390.4(7)
$g_A=1.2695(29)$	246.8(5)	20.1	46.8	71.8	4.5	0.9	390.9(7)
χ EFT (N3LO)	1S_0	3P_0	3P_1	3P_2	1D_2	3F_2	Total
IA – $\Lambda = 500$ MeV	238.8	21.1	44.0	72.4	4.4	0.9	381.7
IA – $\Lambda = 600$ MeV	238.7	20.9	43.8	72.0	4.4	0.9	380.8
FULL – $\Lambda = 500$ MeV	254(1)	20.5	46.8	72.1	4.4	0.9	399(1)
FULL – $\Lambda = 600$ MeV	255(1)	20.3	46.6	71.6	4.4	0.9	399(1)

$$\Gamma^D = 399(3) \text{ s}^{-1}$$

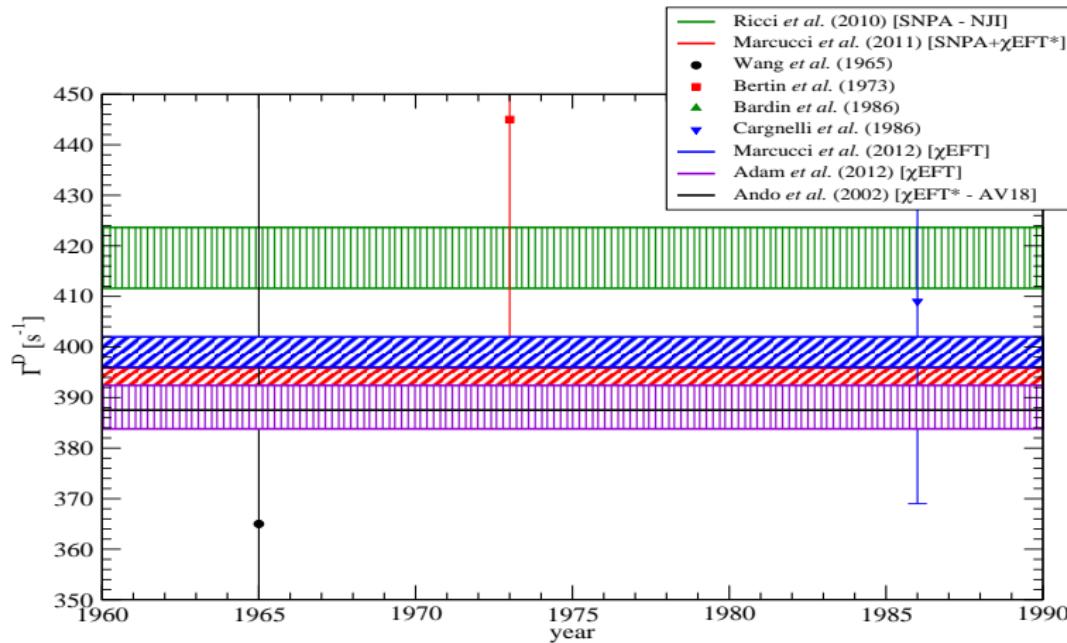
(conservative assumption)

Theoretical “error”: from the uncertainties in c_D, c_E, g_{4S}, g_{4V}

Calculation performed assuming $G_{PS} = G_{PS}^{\chi\text{PT}} = 7.99 \pm 0.20$

[Marcucci *et al.*, PRC **83**, 014002 (2011), PRL **108**, 052502 (2012)]

Comparison with data and previous calculations



Results: $\Gamma_0(\mu^- + {}^3\text{He})$

SNPA(AV18/UIX)	Γ_0
$g_A=1.2654(42)$	1486(8)
$g_A=1.2695(29)$	1486(5)
$\chi\text{EFT}(\text{N3LO}/\text{N2LO})$	Γ_0
IA - $\Lambda = 500$ MeV	1362
IA - $\Lambda = 600$ MeV	1360
FULL - $\Lambda = 500$ MeV	1488(9)
FULL - $\Lambda = 600$ MeV	1499(9)

[Marcucci *et al.*, PRL 108, 052502 (2012)]

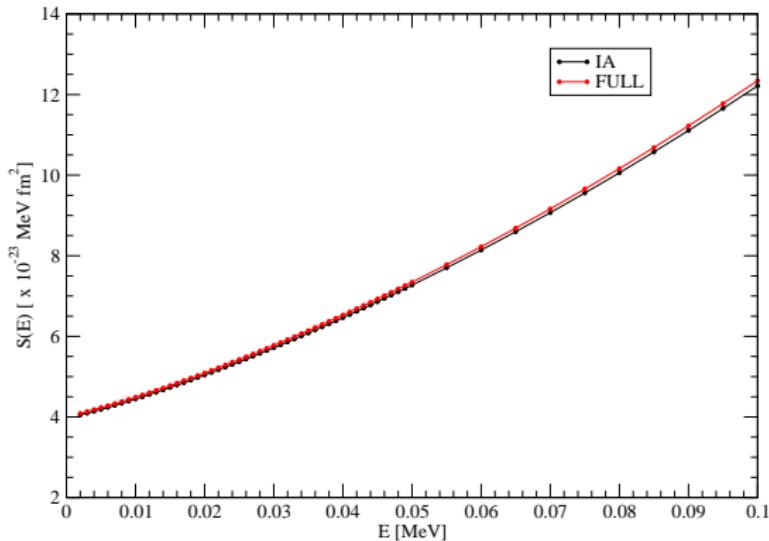
$$\Gamma_0 = 1494(13) \text{ s}^{-1}$$

$$\text{vs. } \Gamma_0(\text{exp}) = 1496(4) \text{ s}^{-1}$$

- [Gazit, PLB 666, 472 (2008)] $\chi\text{EFT}^* - \text{AV18/UIX} \rightarrow 1499(16) \text{ s}^{-1}$
- If G_{PS} is left free $\Rightarrow G_{PS} = 8.2 \pm 0.7$ vs. $G_{PS}^{\chi\text{PT}} = 7.99 \pm 0.20$
- MUCAP experiment ($\mu^- - p$ capture) $G_{PS}^{\chi\text{expt}} = 8.06 \pm 0.48 \pm 0.28$
[arXiv:1210.6545]

$p + p \rightarrow d + e^+ + \nu_e$ astrophysical factor (1)

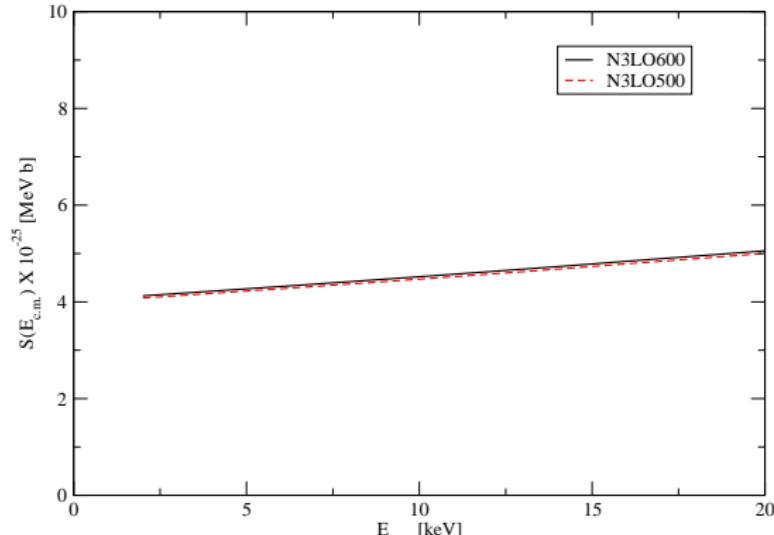
Effect of two-body currents



Calculation with N3LO - only the 1S_0 wave

$$p + p \rightarrow d + e^+ + \nu_e$$
 astrophysical factor (2)

Dependence on Λ



Calculation with N3LO - only the 1S_0 wave

Conclusions and outlook

- New theoretical study of muon capture in χ EFT
- $\Gamma_0(\mu^- + {}^3\text{He})$: nice agreement theory vs. experiment
- $\Gamma^D(\mu^- + d)$:
 - some **discrepancies** among different theoretical works
 - more accurate experimental results → **MuSun**
- New refined calculation of the pp fusion up to 100 keV
- In the future:
 - χ EFT → $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$
 $\mu^- + {}^3\text{He} \rightarrow n + n + p + \nu_\mu$
 - χ EFT → reactions of astrophysical interest
 - $p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e$
 - $p + d \rightarrow {}^3\text{He} + \gamma$
 - $d + d \rightarrow {}^4\text{He} + \gamma$
 -
- in progress: j^A, ρ^A derived at N⁴LO [Pastore, MV, et al.]

Collaborators

- L. Girlanda - *INFN & Salento University, Lecce (Italy)*
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- M. Piarulli & A. Barone (PhD students) - *ODU, Norfolk (VA, USA)*
- A. Kievsky & L. E. Marcucci - *INFN & Pisa University, Pisa (Italy)*
- F. Spadoni (graduate student) & D. Cartisano (undergraduate student) *Pisa University, Pisa (Italy)*

Thank you for your attention!