
Gravitational four-fermion interaction and dynamics of the early Universe

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According to common beliefs, present expansion of Universe is result of **Big Bang**. Quite popular idea is that this expansion had been preceded by compression with subsequent **Big Bounce**.

We analyze the assumption that **Big Bounce** is due to
gravitational four-fermion interaction

1. Interaction of fermions with gravity results, due to torsion, in four-fermion interaction (Kibble; Rodichev; Perez, Rovelli; Freidel, Minic, Takeuchi)

$$S_{ff} = \frac{3}{2} \pi G \frac{\gamma^2}{\gamma^2 + 1} \int d^4x e \left[\eta_{IJ} A^I A^J - \frac{\alpha}{\gamma} \eta_{IJ} (V^I A^J + A^I V^J) - \alpha^2 \eta_{IJ} V^I V^J \right].$$

This interaction, proportional to Newton constant G and to particle number density squared n^2 , gets essential and dominating on the Planck scale only.

We need energy-momentum tensor (EMT) $T_{\mu\nu}$ generated by this action. Metric tensor enters this action via $\sqrt{-g}$ only (!), thus

$$\frac{1}{2} \sqrt{-g} T_{\mu\nu} = \frac{\delta}{\delta g^{\mu\nu}} S_{ff}.$$

With identity

$$\frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial g^{\mu\nu}} = -\frac{1}{2} g_{\mu\nu},$$

we arrive at the following expression for EMT:

$$T_{\mu\nu} = -\frac{3\pi}{2} G \frac{\gamma^2}{\gamma^2 + 1} g_{\mu\nu} [\eta_{IJ} A^I A^J + \frac{\alpha}{\gamma} \eta_{IJ} (V^I A^J + A^I V^J) - \alpha^2 \eta_{IJ} V^I V^J].$$

Nonvanishing components of this expression, written in locally inertial frame, are energy density $T_{00} = \rho_{ff}$ and pressure $T_{11} = T_{22} = T_{33} = p_{ff}$. Thus, equation of state (EOS) is

$$\rho_{ff} = -p_{ff} = -\frac{\pi}{48} G \frac{\gamma^2}{\gamma^2 + 1} n^2 [(3 - 11 \zeta) - \alpha^2(60 - 28 \zeta)].$$

Here n is total density of fermions and antifermions, $\zeta = \langle \sigma_a \sigma_b \rangle$ is average value of product of corresponding σ -matrices, presumably universal for any a and b . With large number of sorts of fermions and antifermions, we neglect here for numerical reasons contributions of exchange and annihilation contributions, and the fact that if σ_a and σ_b refer to same particle, $\langle \sigma_a \sigma_b \rangle = 3$.

After averaging over momenta orientations, P -odd contributions of VA to ρ_{ff} and p vanish.

At last, there are no reasons to expect that $\zeta = \langle \sigma_a \sigma_b \rangle$ can survive under the extreme conditions. Thus, EOS simplifies to

$$\rho_{ff} = -p_{ff} = -\frac{\pi}{16} G \frac{\gamma^2}{\gamma^2 + 1} n^2 (1 - 20\alpha^2) .$$

From now on, energy density is rewritten as

$$\rho_{ff} = Gn^2\varepsilon ,$$

with

$$\varepsilon = -\frac{\pi}{16} \frac{\gamma^2}{\gamma^2 + 1} (1 - 20\alpha^2) .$$

2. Common matter on the Planck scale is ultrarelativistic. Its energy density is

$$\rho = \nu n^{4/3},$$

ν is numerical factor, $n^{1/3}$ is typical energy per particle.

Another factor n is total density of ultrarelativistic particles and antiparticles, fermions and bosons, contributing to this density. This factor exceeds fermion density n entering above four-fermion expressions. This difference is absorbed here by factor ν . In corresponding EMT, due to isotropy,

$$T_{0m} = T_{m0} = 0, \quad T_{11} = T_{22} = T_{33}.$$

Since $T_{\mu}^{\mu} = 0$,

$$T_{\nu}^{\nu} = \rho \operatorname{diag}(1, -1/3, -1/3, -1/3),$$

or $T_{\mu\nu} = \rho \operatorname{diag}(1, 1/3, 1/3, 1/3)$; thus $p = \rho/3$.

3. We assume that, even when EOS reduces to this one, Universe is homogeneous and isotropic, and is described by Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2 = dt^2 - a(t)^2 [dr^2 + f(r)(d\theta^2 + \sin^2 \theta d\phi^2)];$$

$f(r)$ depends on topology of Universe as a whole:

$$f(r) = r^2, \sin^2 r, \sinh^2 r$$

for spatial flat, closed, and open Universe, respectively.

Einstein equations for FLRW metric reduce now to

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_{ff} + \rho), \quad \frac{\ddot{a}}{a} = \frac{8\pi G}{3} (\rho_{ff} - \rho);$$

here k equals 0, 1, and -1 for spatial flat, closed, and open Universe, respectively. Observational data strongly favor the idea that our Universe is spatial flat, i.e. that $k = 0$. Above equations are supplemented by covariant continuity equation:

$$\dot{\rho}_{ff} + \dot{\rho} + 4\frac{\dot{a}}{a}\rho = 0.$$

4. Solution of FLRW equations. With substitution

$$a(t) = a_0 \exp(f(t)) , \quad (1)$$

continuity equation is satisfied identically. Two other equations result in

$$\frac{8\pi G}{3} (\rho_{ff} + \rho) = \dot{f}^2, \quad \frac{8\pi G}{3} \rho = -\frac{1}{2} \ddot{f}.$$

Differentiating first of them and combining with second, we arrive at

$$f = -\frac{3}{4\nu} G \varepsilon n^{2/3} - \frac{1}{3} \ln n ,$$

and thus

$$a(t) = a_0 \exp(f(t)) \sim n^{-1/3} \exp\left\{-\frac{3}{4\nu} G \varepsilon n^{2/3}\right\}.$$

1) $\varepsilon > 0$. In compression, at first, both factors shrink to zero. Rewrite previous equations as

$$\dot{a} = -\sqrt{\frac{8\pi G}{3}} a \sqrt{\rho_{ff} + \rho}, \quad \ddot{a} = \frac{8\pi G}{3} a (\rho_{ff} - \rho).$$

At first, when $\rho_{ff} \ll \rho$, both \dot{a} and \ddot{a} are negative, Universe shrinks with acceleration. At $\rho_{ff} = \rho$ acceleration \ddot{a} changes sign, while \dot{a} remains negative, and compression of Universe decelerates.

Rather tedious calculations demonstrate that

1. it takes finite time for a to shrink to zero,
2. \dot{a} and \ddot{a} also vanish at the same moment.

Repulsive GFFI does not stop the collapse, but only reduces its rate.

The asymptotic behavior of $a(t)$ is

$$a(t) \sim (t_1 - t) \exp \left(-\frac{9 \varepsilon^2 G}{128 \pi \nu^3} \frac{1}{(t_1 - t)^2} \right),$$

t_1 is the moment of the collapse for $\varepsilon > 0$.

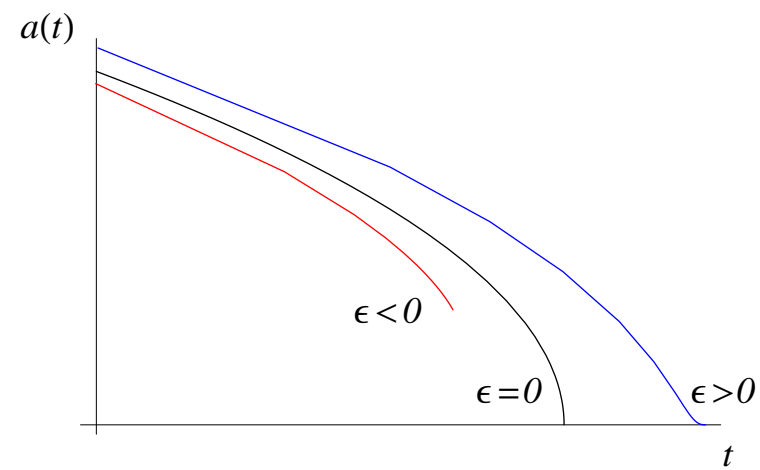


Figure 1: Dependence of scale factor on time for compression

For negative ε , situation is different. Here right-hand side of relation

$$a(t) \sim \frac{1}{\sqrt{|\xi(t)|}} e^{\frac{3}{4}|\xi(t)|}, \quad \xi(t) = \frac{\rho_{ff}}{\rho} = \frac{G\varepsilon}{\nu} n^{2/3}.$$

has minimum at $|\xi_m| = 2/3$, i.e., $a(t)$ cannot decrease further. However, the compression rate \dot{a} at this point does not vanish and remains finite. In a sense, the situation here resembles that in the standard cosmology with ultrarelativistic particles: therein $a(t) \sim \sqrt{t_0 - t} \rightarrow 0$ for $t \rightarrow t_0$ (t_0 is the moment of the collapse in this case), though at this point \dot{a} is not finite, but tends to infinity.

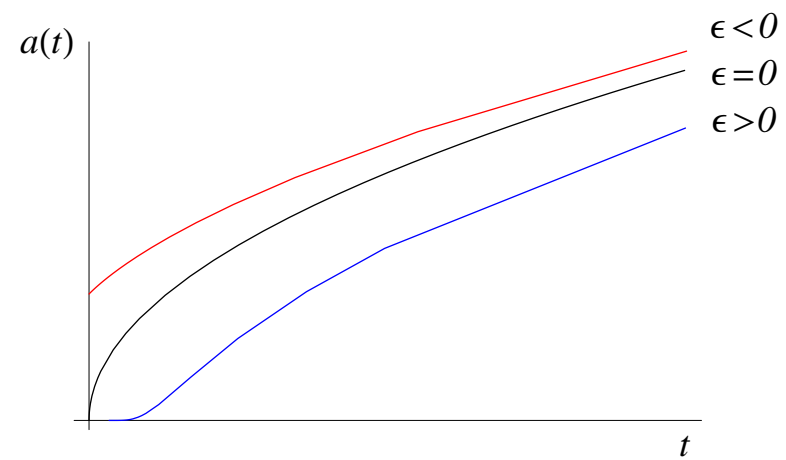


Figure 2: Dependence of scale factor on time for expansion

Gravitational four-fermion interaction does not result in Big Bounce.
