Precision Magnetic Fields for Fundamental Neutron Symmetries

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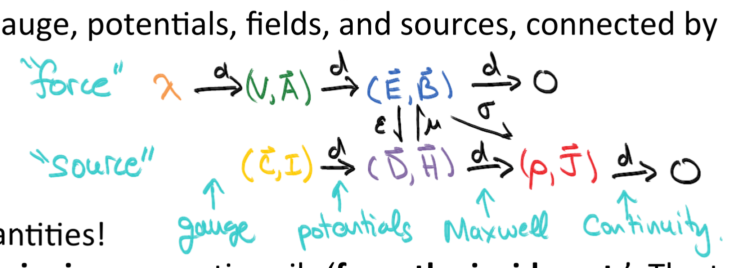
Abstract: *Because the neutron has only a single static moment--the magnetic dipole, virtually every fundamental neutron physics experiment relies on static and/or oscillating magnetic fields to manipulate the neutron spin and/or kinematics. The level of precision required for modern experiments places stringent constraints on field uniformity, fringes, geometry, magnetic materials, and shielding, which impose formidable challenges on magnet coil design.*

We describe new techniques being developed to design precision coils and construct them with an industrial robotic arm. These techniques allow direct calculation of coil windings subject to geometric and field constraints. The techniques are based on a new physical interpretation of the magnetic scalar potential, which lends itself to elegant illustrations of the mixed symmetry between electric and magnetic fields.

Introduction

The **magnetic scalar potential** is widely recognized as a \`practical tool' for designing permanent magnets and pole-tips. However its theoretical importance and practical usefulness as a \`true potential' is considered to be hindered by being only well-defined in current-free regions. Thus it is overshadowed by the magnetic vector potential ***A***, which is always well-defined. The purpose of this poster is two-fold:

a) to show the physical significance of the scalar potential as one of the **source potentials** (λ, I) as opposed to the dynamical potentials (V, ***A***). Ironically, this interpretation stems from currents, albeit ones on the surface. We demonstrate this in terms of the 'twisted symmetry' of electric and magnetic fields, illustrated in the following exact sequence of gauge, potentials, fields, and sources, connected by vector derivatives. Note The two scalar potentials represent *voltage* and *current*, the two most common electromagnetic quantities!



b) to illustrate its power **designing** magnetic coils '**from the inside out**.' The typical design process involves calculating the magnetic field from reasonably placed coils, and iteratively refining coil positions. In this poster, we advocate starting with field constraints as boundary conditions and calculating the exact coil geometry required to produce those fields.

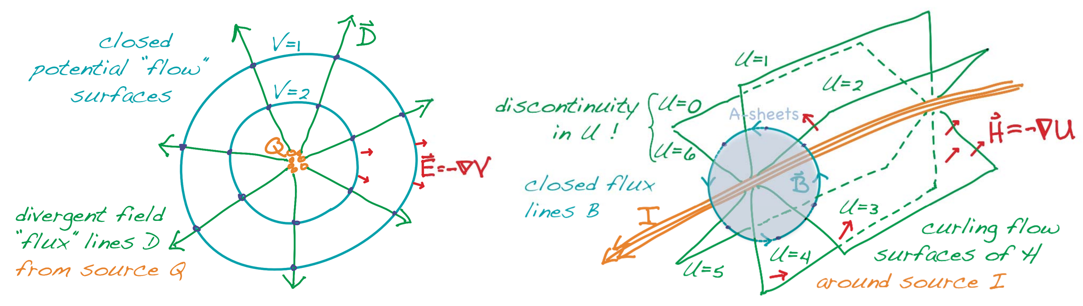
The Magnetic Scalar Potential

By the **Helmholtz** theorem, a field is characterized by its divergence and curl, which generate **flux** (surface integral) and **flow** (the line integral), respectively. In absence of sources, the flow from 'ground' to any other point is the **potential**.

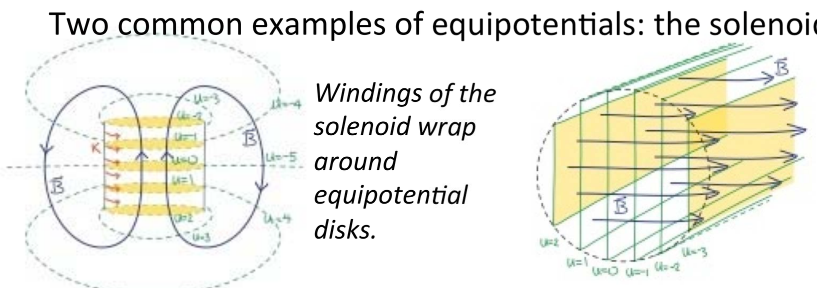
Corresponding boundary conditions are well-defined even with surface currents, which are absent in the electric potential. These currents give physical significance to the magnetic scalar potential, in that ***every equipotential surface is terminated on the boundary by a surface current 'wire'. The surface current I flowing between any two equipotentials equals the potential difference ΔU.*** Given this interpretation, the logical notation for the scalar potential is *I(x,y,z).*

	FLUX	FLOW
• Field Equations	$\Phi_B \equiv \int \boldsymbol{B}\cdot d\boldsymbol{a}$	$\mathcal{E}_H \equiv \int \boldsymbol{H}\cdot d\boldsymbol{l} = \Delta U$
field	$\nabla\cdot\boldsymbol{B} = 0$	$\nabla\times\boldsymbol{H} = \boldsymbol{J}$
potential	$-\nabla\cdot\mu\nabla U = 0$	$\boldsymbol{H} = -\nabla U \quad \{\boldsymbol{J}=\boldsymbol{0}\}$
• Boundary conditions		
field	$\hat{\boldsymbol{n}}\cdot\Delta\boldsymbol{B} = 0$	$\hat{\boldsymbol{n}}\times\Delta\boldsymbol{H} = \boldsymbol{K}$
potential	$\Delta(\mu\partial U/\partial n) = 0$	$U_2 - U_1 = \Delta I$

An intuitive picture of the scalar potential as a source potential comes by interpreting Ampere's law analogous to Gauss' law. Just as electric **flux lines** stem from charge, magnetic equipotential '**flow sheets**' emanate from current (wires). In contrast, electric equipotentials and magnetic flux lines are both closed (the twisted symmetry of E&M).



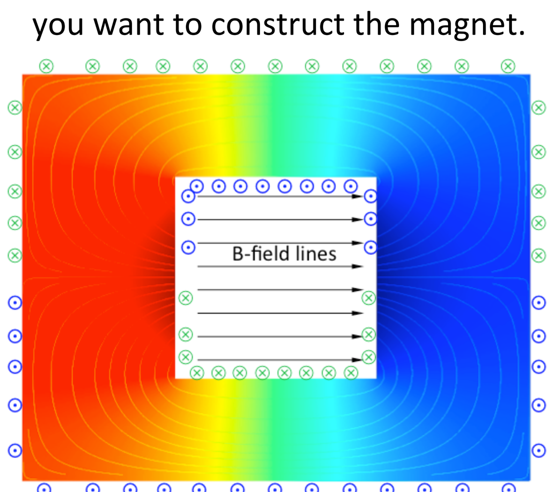
Two common examples of equipotentials: the solenoid and cos(θ) coil.

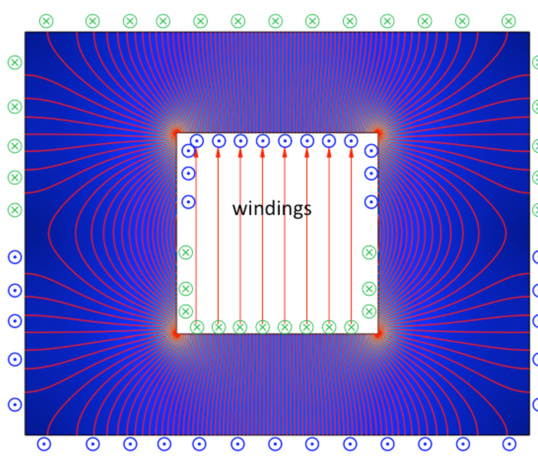


Technique for coil design

In a nutshell: **A)** solve Laplace's equation (with FEA software tools) for the scalar potential using **flux** boundary conditions determined by field constraints on the surface where you want to construct the magnet.

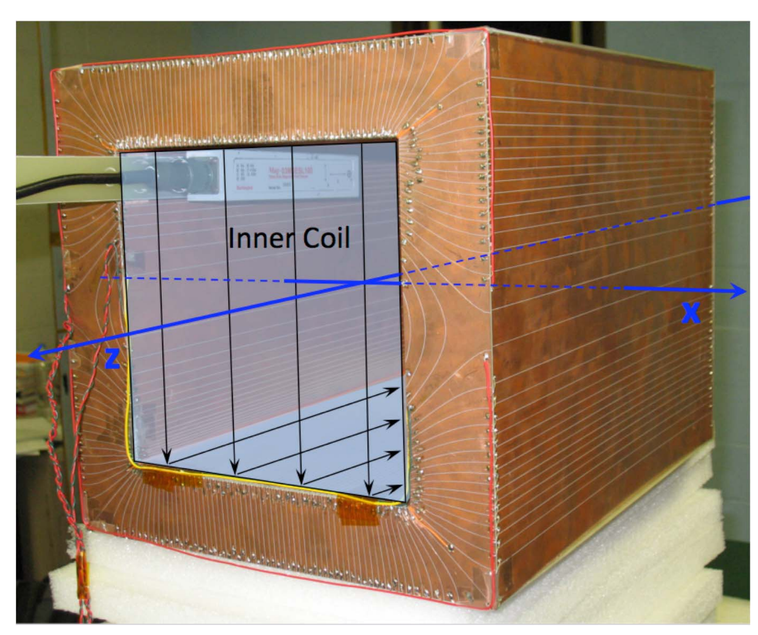
B) Wind wires along each equipotential contour of U on the surface (**flow** boundary conditions). The coils can be wound in series if ΔU is the same between each contour.



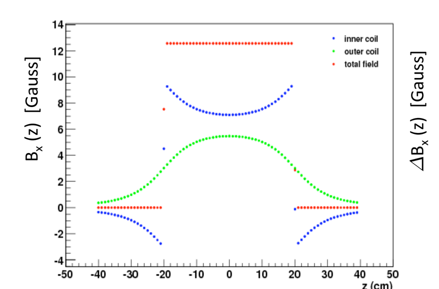


Prototype surface current coil

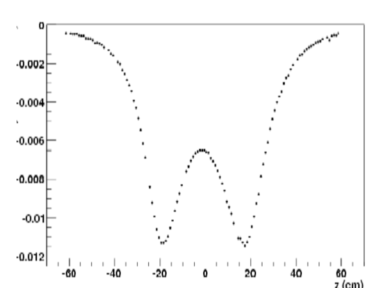
Double-Square-Cosine-Theta-Coil (DSCTC)



To test the technique, we designed a guide-field coil to have a constant horizontal field inside with zero flux leakage on the outside. Both an inner coil and outer 'flux return' coil were needed for the combined field (red) to satisfy these constraints. We assembled the outer coil by soldering flat printed circuit boards (PCB) together and connecting each trace at the corner with soldered leads. The resulting field agreed to 1% on the z-axis, even with crude construction.



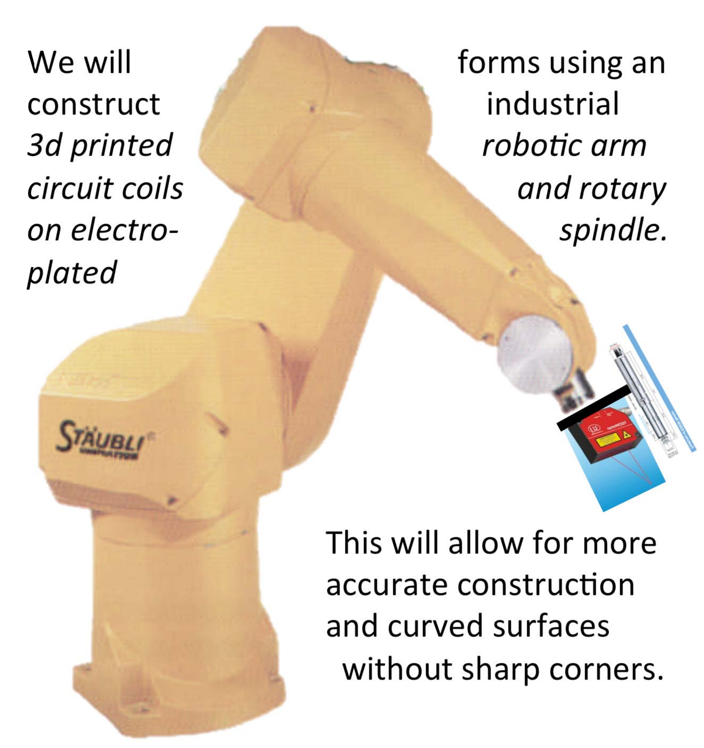
Calculated field along z-axis



Residual of measured field.

Construction

We will construct *3d printed circuit coils on electro-plated* forms using an *industrial robotic arm and rotary spindle.*



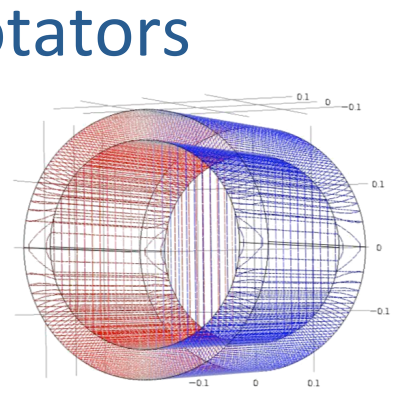
This will allow for more accurate construction and curved surfaces without sharp corners.

Examples of specialty coils designed using the scalar potential

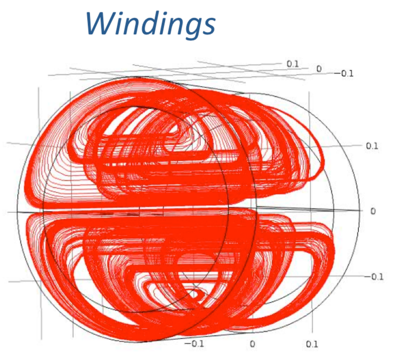
Resonant Spin Rotators

The same technique works to design RF coils as TEM wave guides. This longitudinal spin flipper for the FnPB cold neutron guide is a cos(θ) coil with transverse fields and an outer flux return layer. The advantages over a solenoid coil are:

1) It rotates both longitudinal and transverse spins
2) It has no fringes in the neutron beam line
3) It is completely self-shielding (no RF leakage)
4) The magnetic field, inductance, other electrical characteristics can be calculated analytically



Windings



B-field lines

Analytic Scalar Potentials in Three Regions

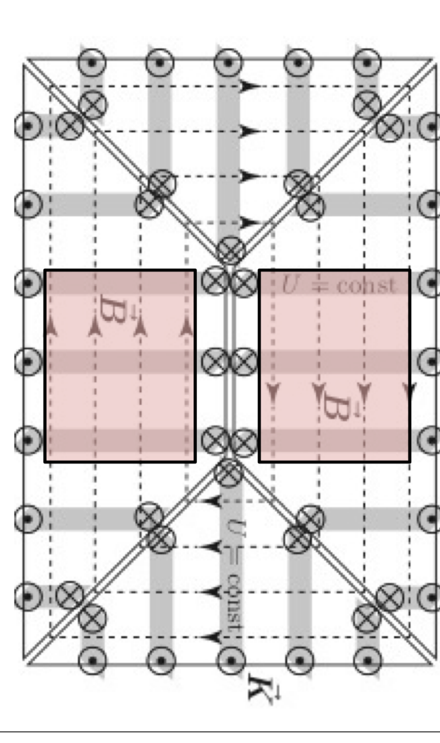
Inner Region: $U = -H_0 s\cos\phi$

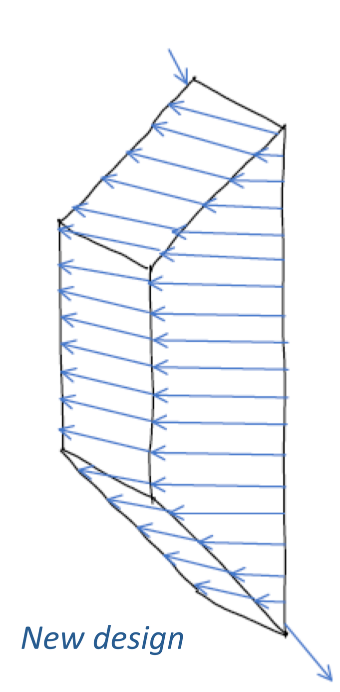
Mid Region: $U = \left(\frac{a^2}{A^2-a^2}\right) H_0\cos\phi\,(s + A^2 s^{-1})$

Outer Region: $U = 0$

Previous π-coil

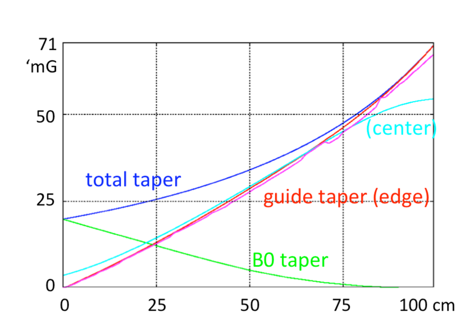
DC π-coil: The PV neutron spin rotation experiment required a fringeless uniform DC magnetic field in the center of the zero-field spin precession chamber. This new design creates rectangular field lines by kinking them at each corner with surface currents. It is made of four trapezoids each with square coils.



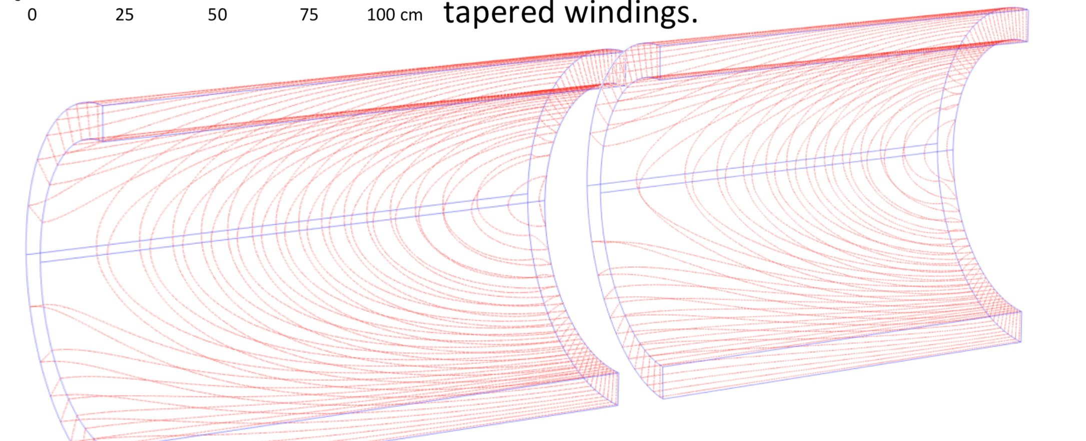


New design

Tapered Guide Field Coils



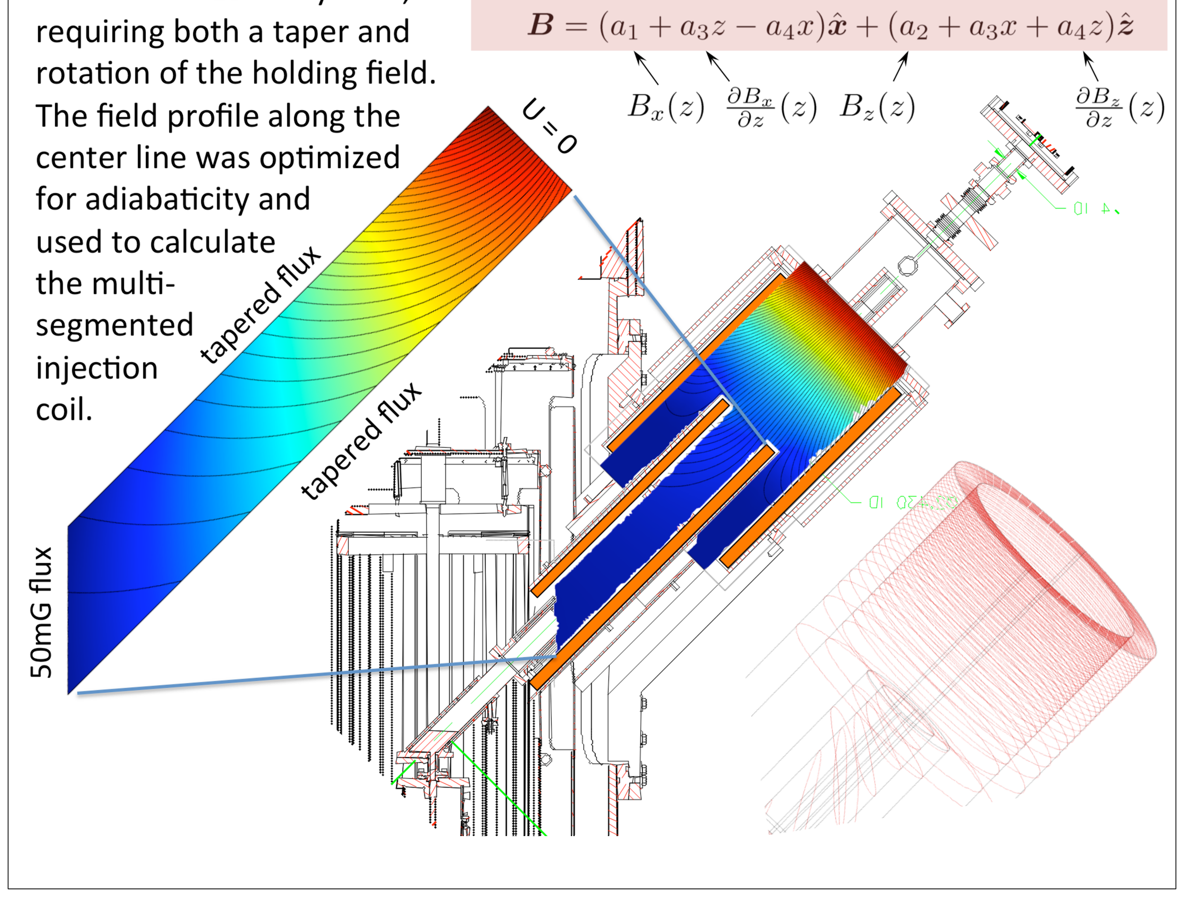
In the nEDM experiment, the holding field must taper from 5 G in the neutron guides to 40 mG in the measurement cell without depolarizing the neutrons. The optimal taper was calculated and input as flux boundary conditions to calculate the tapered windings.



^{3}He Injection Field: a polarized atomic beam is injected at 45° into the EDM cryostat, requiring both a taper and rotation of the holding field. The field profile along the center line was optimized for adiabaticity and used to calculate the multi-segmented injection coil.

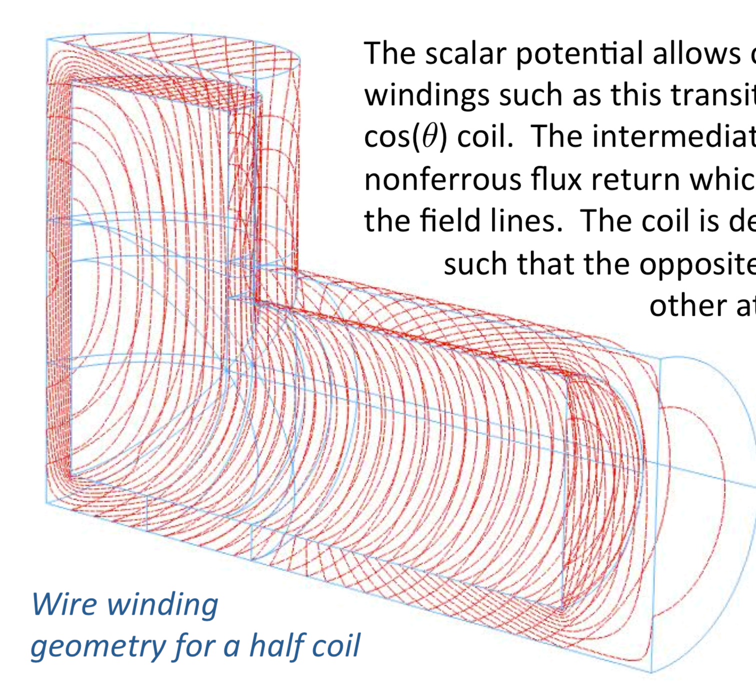
Quadrupole field approximation

$$-\Phi_m = a_1 x + a_2 z + a_3 xz + \tfrac{1}{2}a_4(z^2 - x^2)$$
$$\boldsymbol{B} = (a_1 + a_3 z - a_4 x)\hat{\boldsymbol{x}} + (a_2 + a_3 x + a_4 z)\hat{\boldsymbol{z}}$$



Clamshell Cancellation Coil

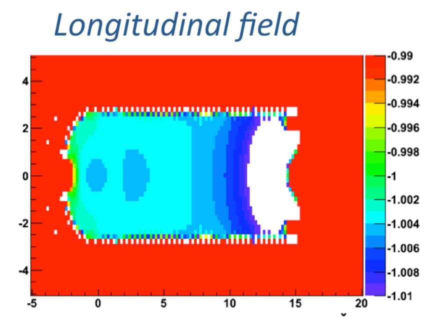
The scalar potential allows calculation of complex windings such as this transition from a solenoid to a cos(θ) coil. The intermediate layer serves as nonferrous flux return which hermetically contains the field lines. The coil is designed with two halves such that the opposite currents cancel each other at the interface.



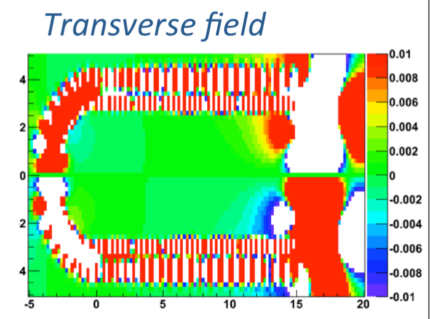
Wire winding geometry for a half coil

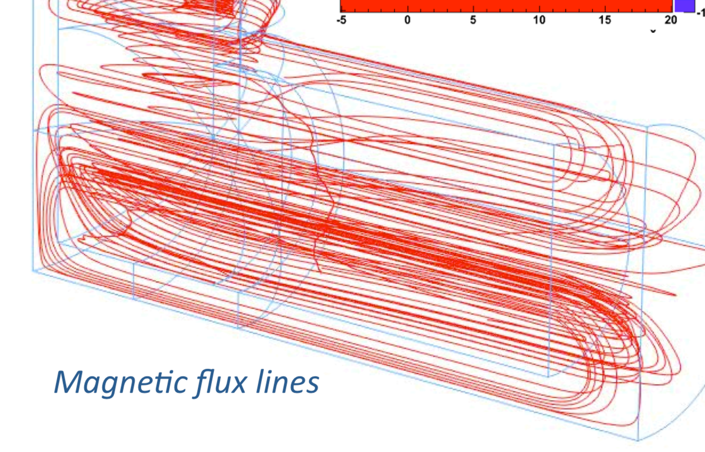
This winding is the algebraic sum of an inner and outer coil calculated in the corresponding inner and outer regions.

Longitudinal field



Transverse field





Magnetic flux lines

With 50 turns, the calculated field agrees with the design goal to 0.5% in almost the whole interior region. Even better field uniformity could be attained using wide traces instead of wires