Understanding magnetic order and disorder

Tom Lancaster
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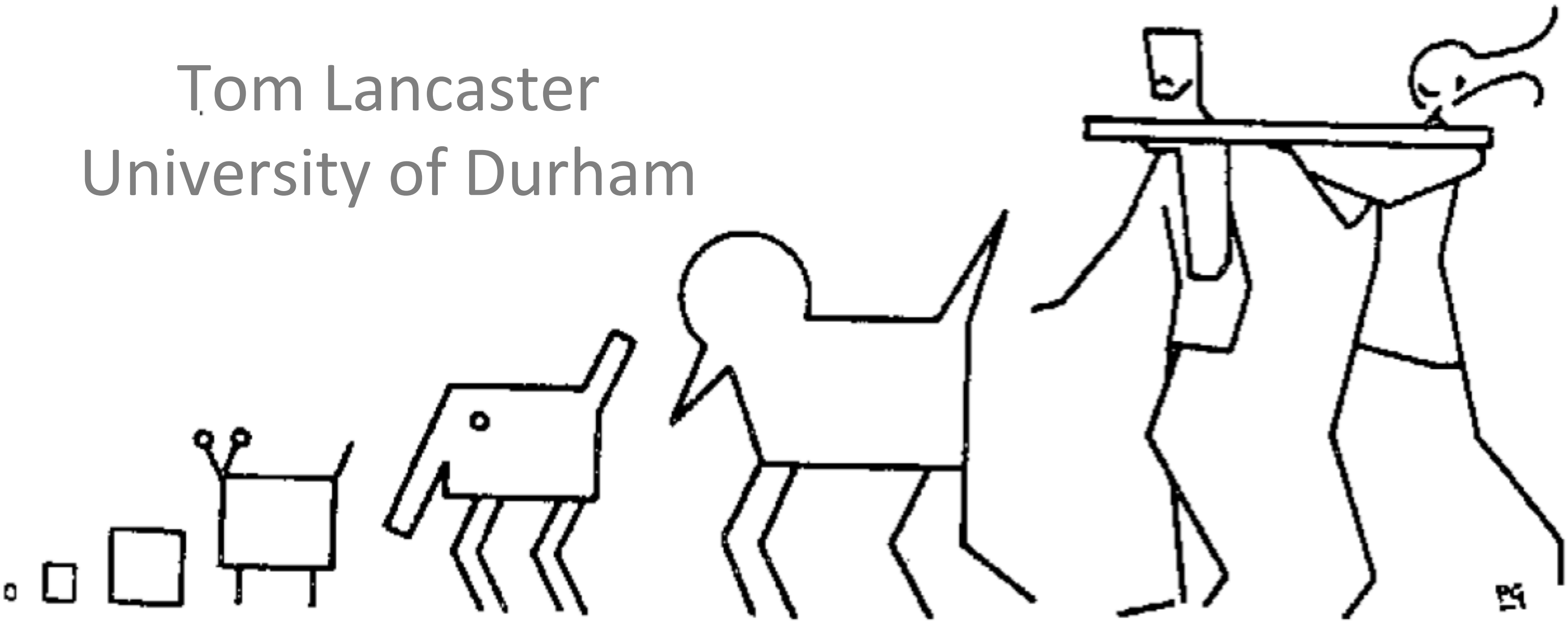


Various animals attempting to follow a scaling law.

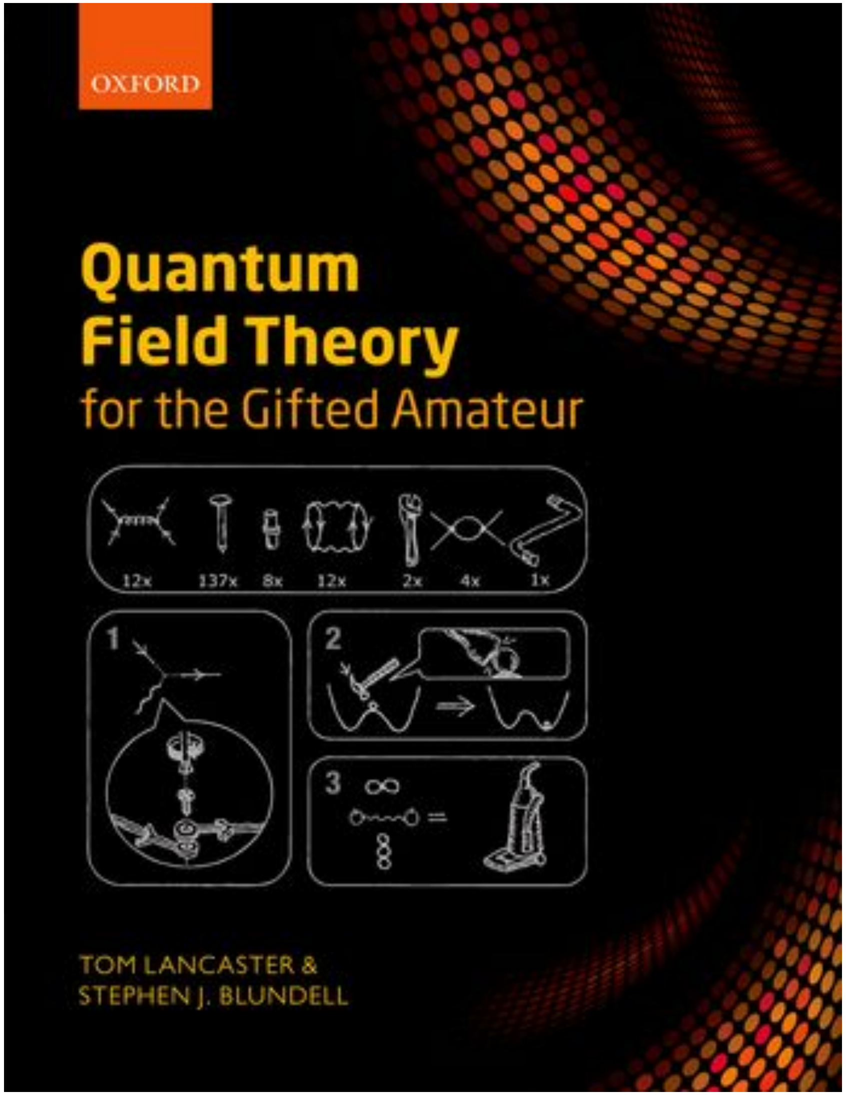
Outline

- Static magnetism and Landau's symmetry breaking
- The role of dimensionality
- An introduction to the renormalization group
- Notions of quantum disorder

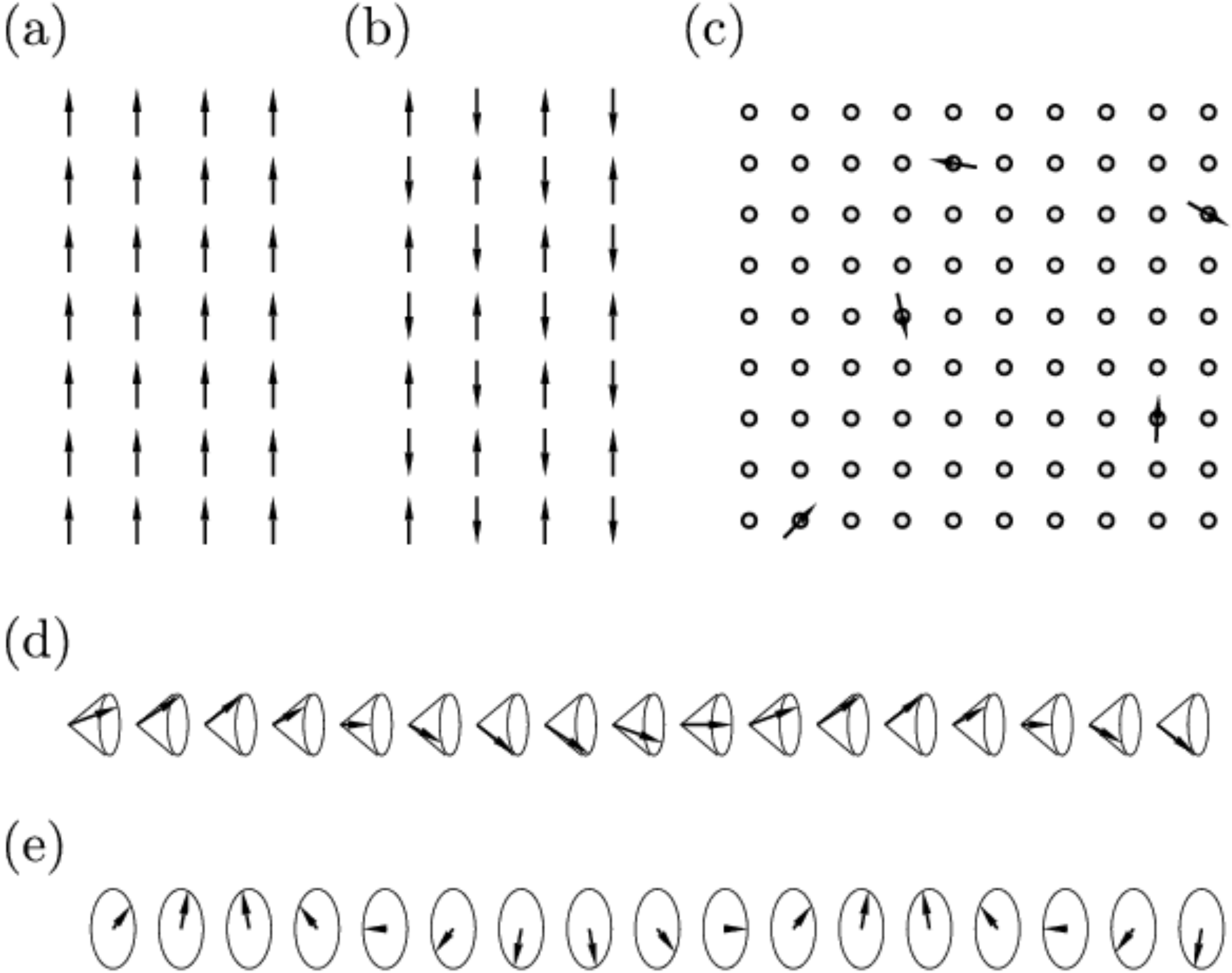
This new book may be of interest

For more details:

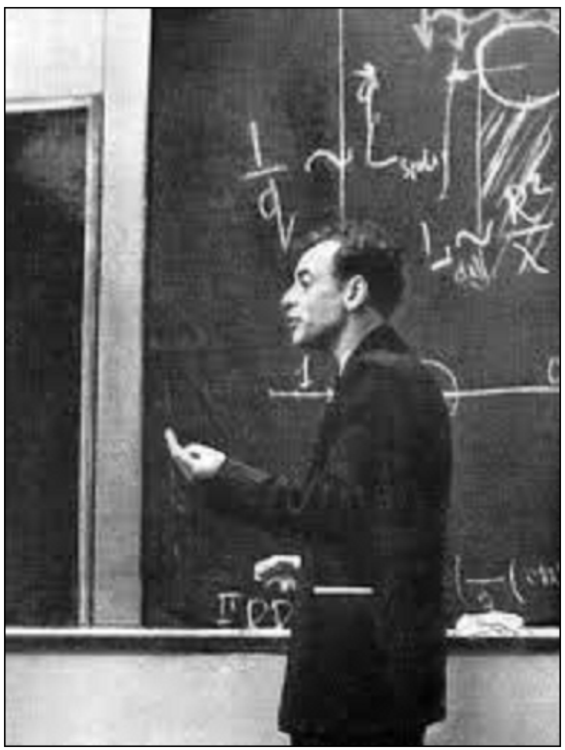
- Speak to me afterwards
- Email: tom.lancaster@durham.ac.uk
- Twitter: @exconsul



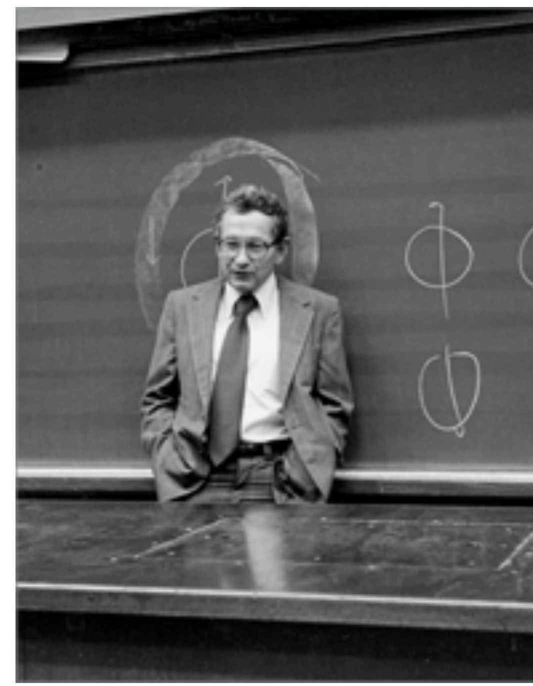
The many faces of magnetism



How do we understand the occurrence of magnetic order?



Lev Landau (1908-1968)

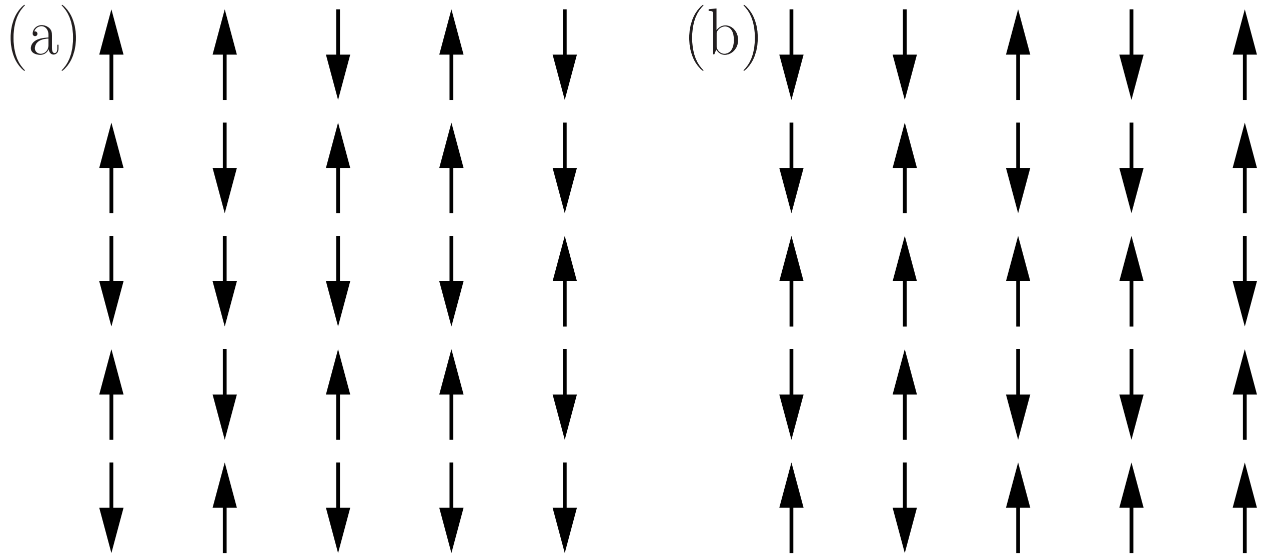


Philip Anderson (1923-)

Broken symmetry is a cornerstone of CMP

Consider a magnet

$T > T_c$

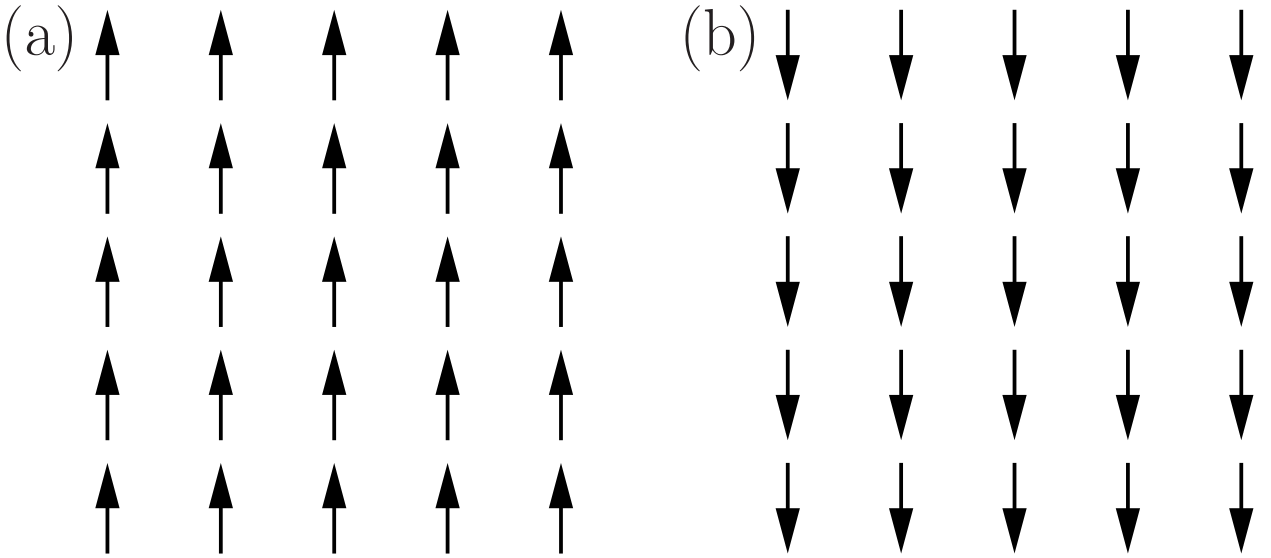


These magnets are the same

Broken symmetry is a cornerstone of CMP

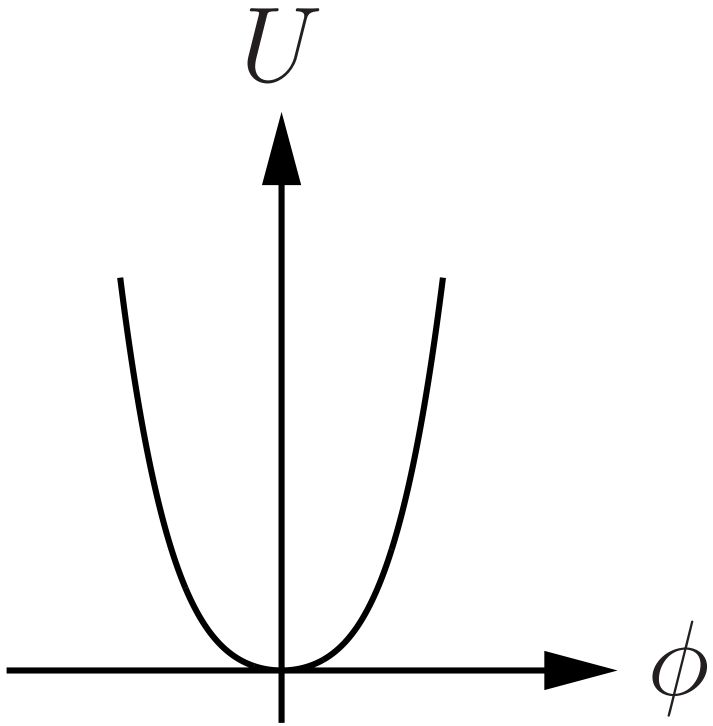
Consider a magnet

$T < T_c$

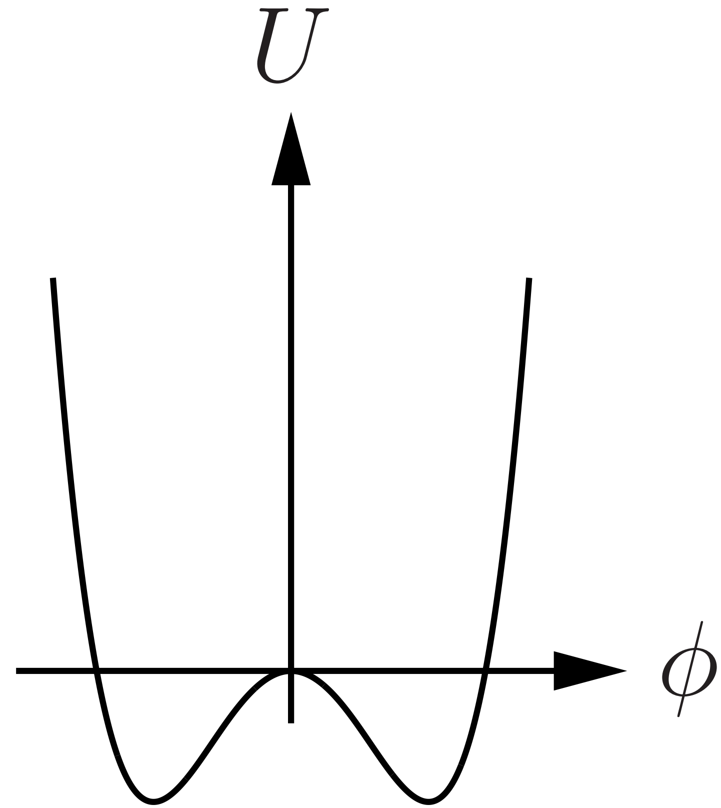


These magnets are different

This has a simple mathematical description

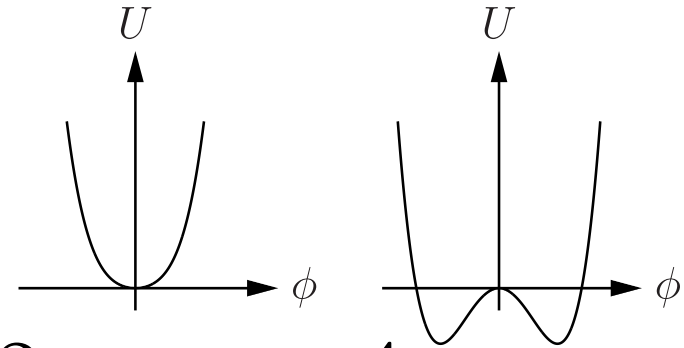


$T > T_c$



$T < T_c$

Landau mean-field theory



$$F = F_0 + a(T - T_c)M^2 + bM^4 + \ldots$$

$$\frac{\partial F}{\partial M} = 2a(T - T_c)M + 4bM^3 = 0$$

$$M_0^2 = -\frac{a(T - T_c)}{2b}$$

Minima in the free energy may be identified

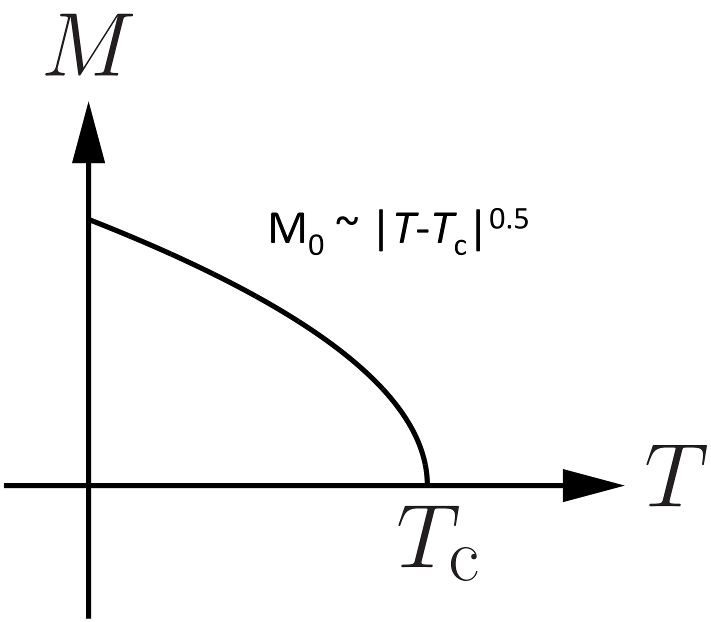
$$M_0 = \left[\frac{a(T_c - T)}{2b}\right]^{\frac{1}{2}} \qquad T < T_c$$

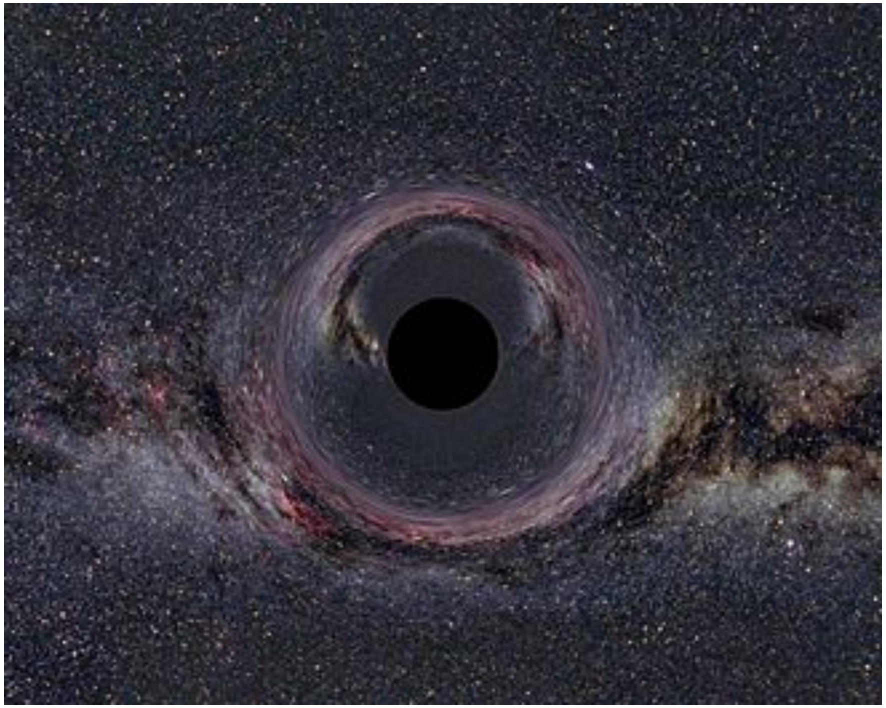
$$= 0 \qquad T > T_c$$

The 4-fold way of broken symmetry

- Phase transitions

Mathematical singularity at T_c





Like a black hole

Critical exponents

t = *T*-*T*$_c$

- Heat capacity: $C \sim |t|^{-\alpha}$,
- Magnetization: $M \sim (-t)^{\beta}$, for $B \to 0$, $T < T_{\mathrm{c}}$,
- Magnetic susceptibility: $\chi \sim |t|^{-\gamma}$,
- Field dependence of χ at $T = T_{\mathrm{c}}$: $\chi \sim |B|^{1/\delta}$,
- Correlation length: $\xi \sim |t|^{-\nu}$,
- The correlation function $G(r)$ behaves like

$$G(r) \sim \left\{ \begin{array}{cc} \frac{1}{|r|^{d-2+\eta}} & |r| \ll \xi \\ \mathrm{e}^{-\frac{|r|}{\xi}} & |r| \gg \xi, \end{array} \right\}$$

where r is distance and d is the dimensionality of the system.

Exponents don't rely on any length scale in the system

Critical exponents for mean field theory

	α	β	γ	δ	ν	η
$\epsilon = 0$ ($d = 4$, mean field)	0	$\frac{1}{2}$	1	3	$\frac{1}{2}$	0

We will return to critical exponents a little later

The 4-fold way of broken symmetry

- Phase transitions

  Mathematical singularity at T_c

- Rigidity

  order transmits forces

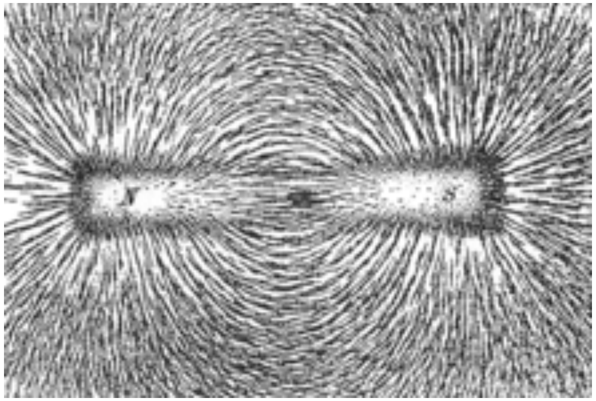
- New excitations

  New particle spectrum

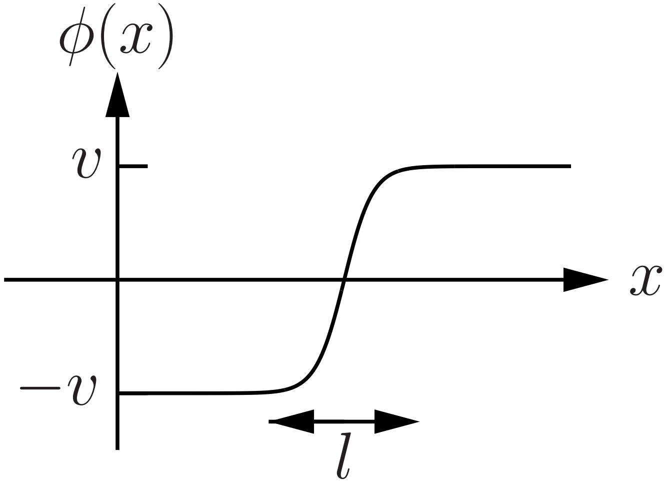
- Defects

  Walls that separate different order in different places

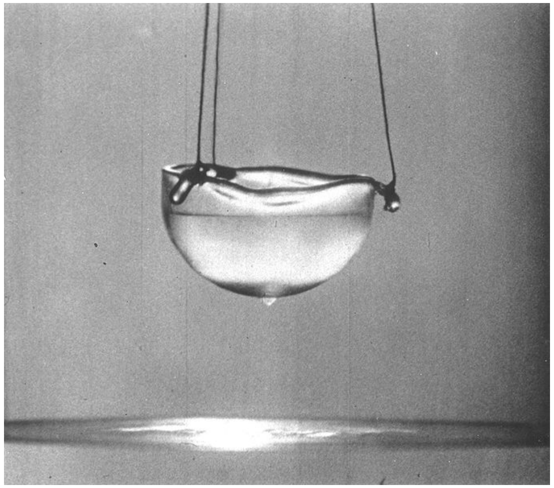
The magnet



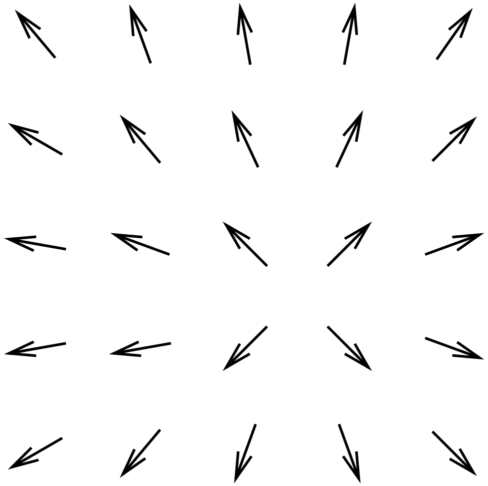
- Order parameter *M*
- Rigidity: permanent magnetism
- Excitations: magnon particles
- Defects: domain walls



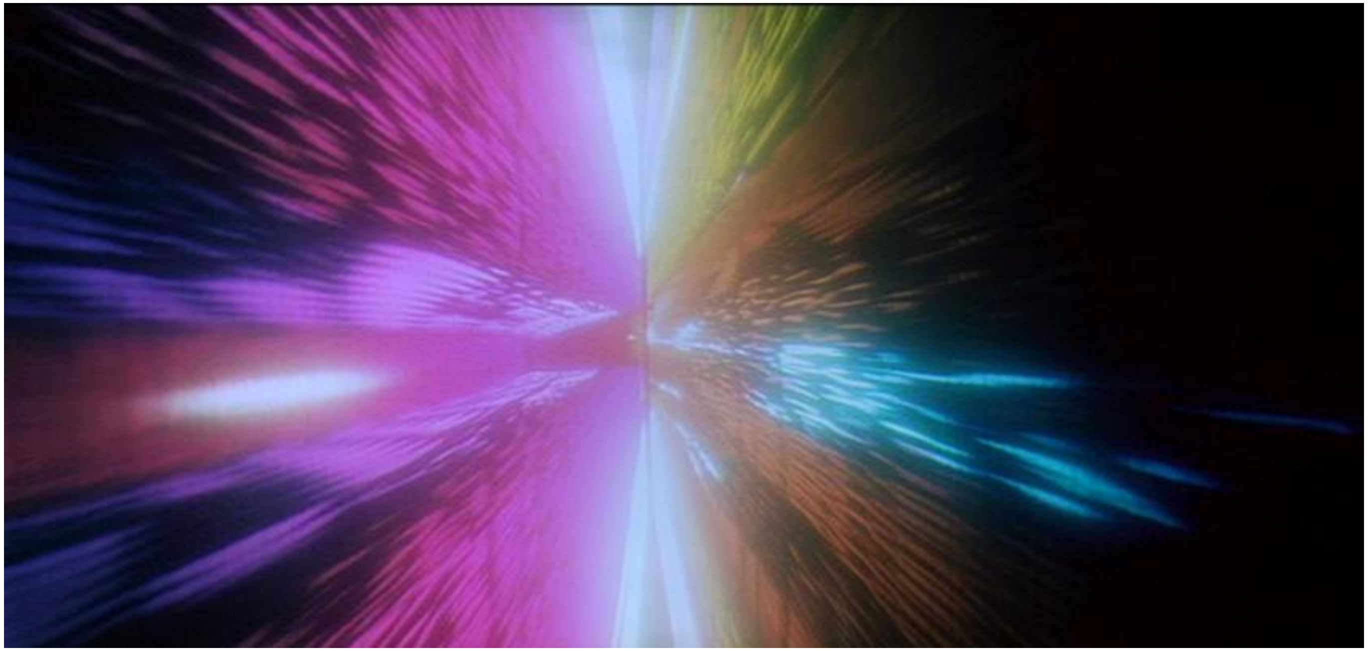
Superfluids and superconductors

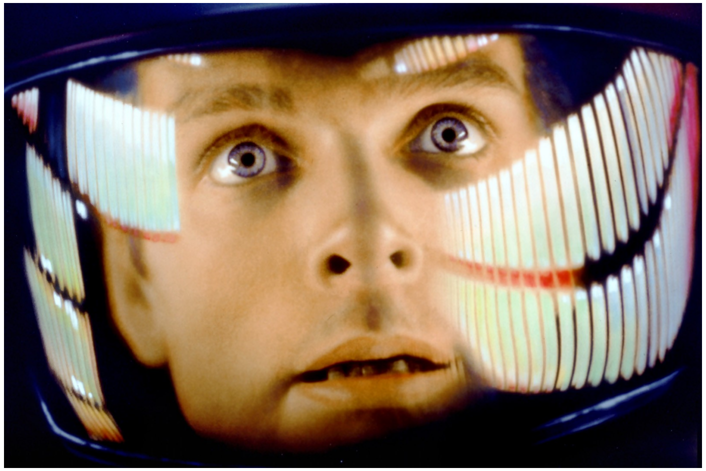


- Order parameter: $<\Psi(x)>$
- Rigidity: the supercurrent
- Excitations: Bogolons (via the Higgs mechanism in a SC)
- Defects: vortices

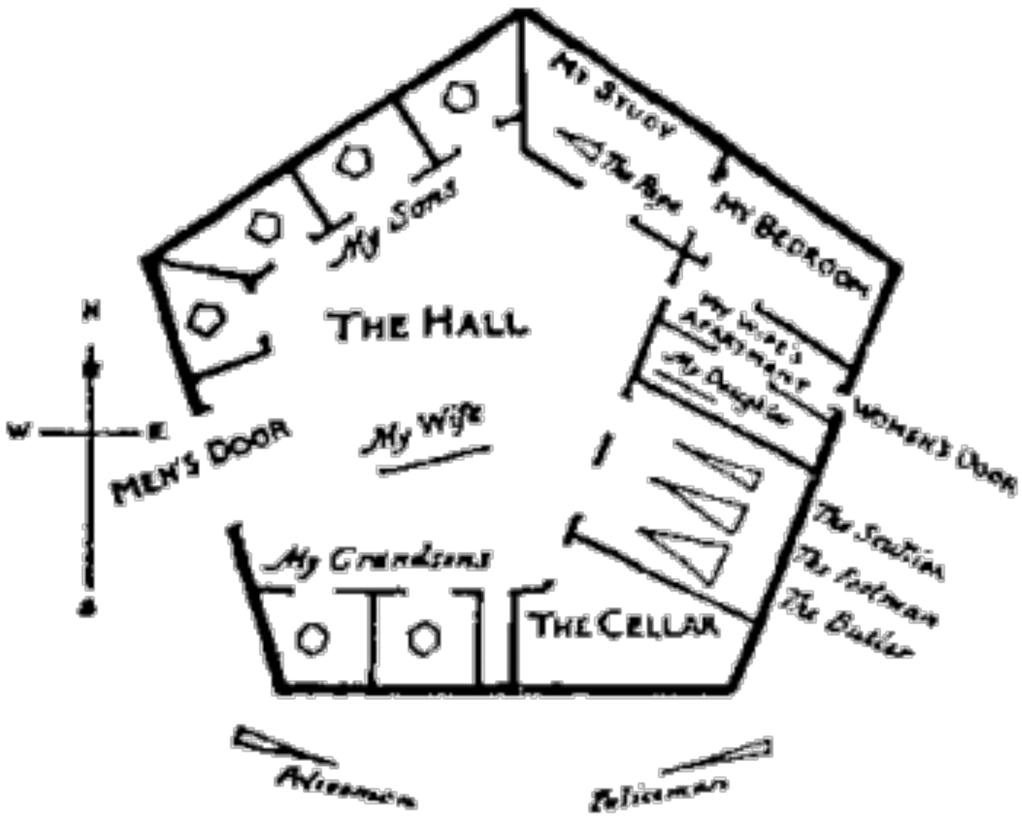


Dimensionality in magnetism



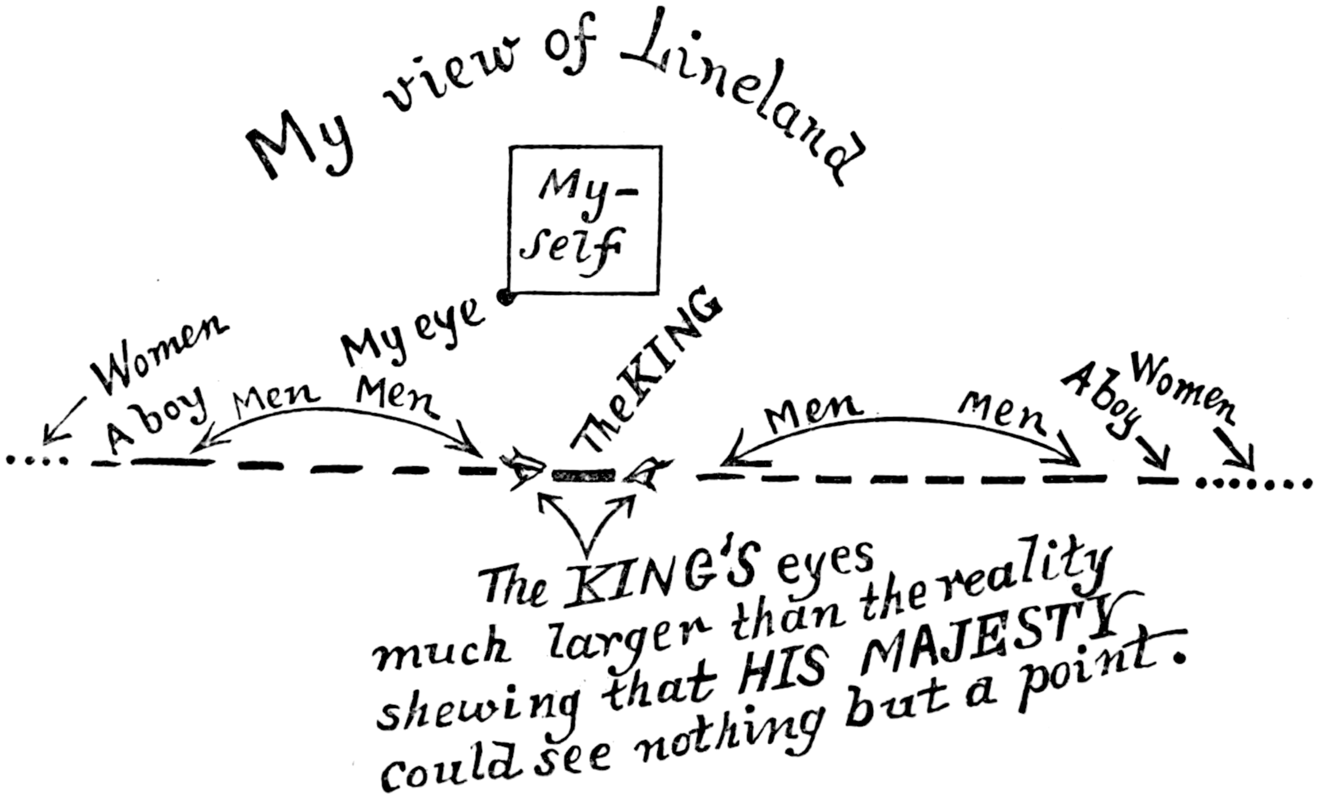


Dimensionality determines the behaviour of the magnet (or universe)



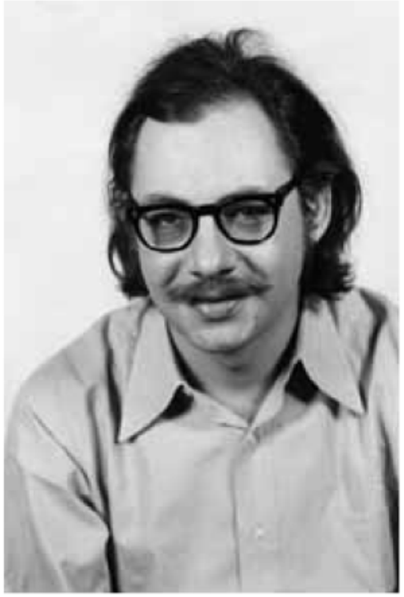
In two dimensions: Flatland

Dimensionality determines the behaviour of the magnet or universe



In one dimension: Lineland

Models of low dimensional magnetism



	$D=1$, Ising	$D=2$, XY	$D=3$, Heisenberg
$d=1$	no order	no order	no order
$d=2$	order	no order	no order
$d=3$	order	order	order

$D=$ Dimension of the spins

$d=$dimension of the lattice

Coleman-Mermin-Wagner theorem forbids breaking a continuous symmetry for $d=2$

We can describe the physics with a deceptively simple looking equation

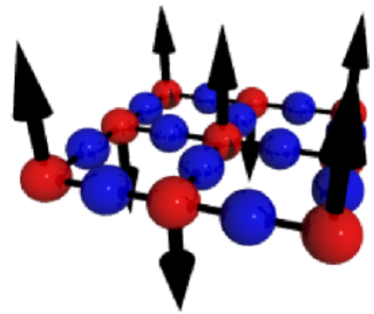
$$H = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Anderson *Basic notions in Condensed Matter Physics*

Models of low dimensional magnetism

	$D=1$, Ising	$D=2$, XY	$D=3$, Heisenberg
$d=1$	no order	no order	no order
$d=2$	order $T_c = 2.269J$	order$^{\mathrm{T}}$ $T_{\mathrm{KT}} = 0.353J$	order? $T=0$
$d=3$	order	order	order

2D S=$1/2$ Heisenberg antiferromagnet



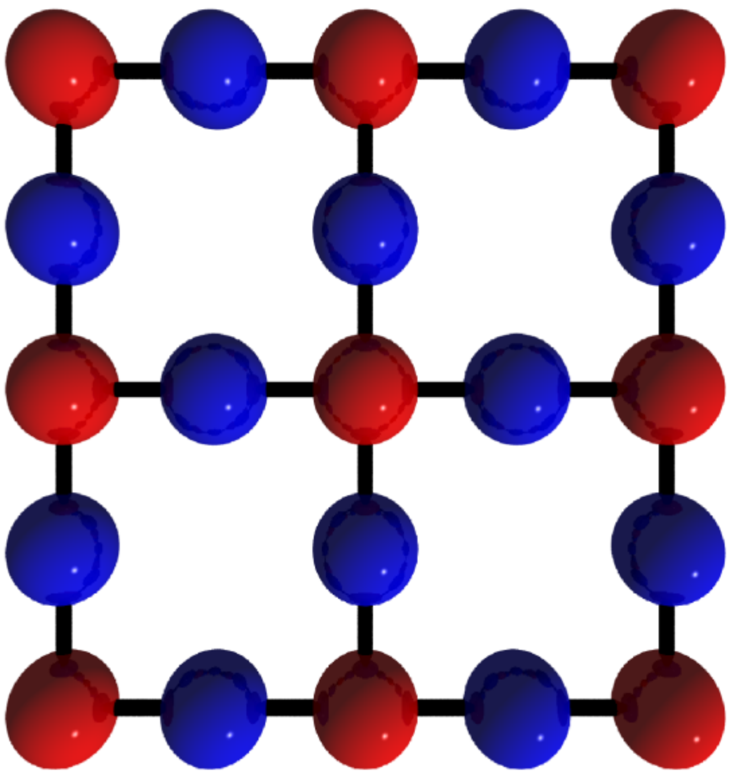
$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

- Its only parameter is J
- It lacks an exact solution
- No order for T>0, but good evidence of AFM order at T=0
- Quantum fluctuations reduce the magnetization to $m^{\dagger}$=0.62
- Excitations are spin waves, well defined at T=0
- Correlation length varies as $\xi(T) = Ce^{2\pi\rho_s/T}$

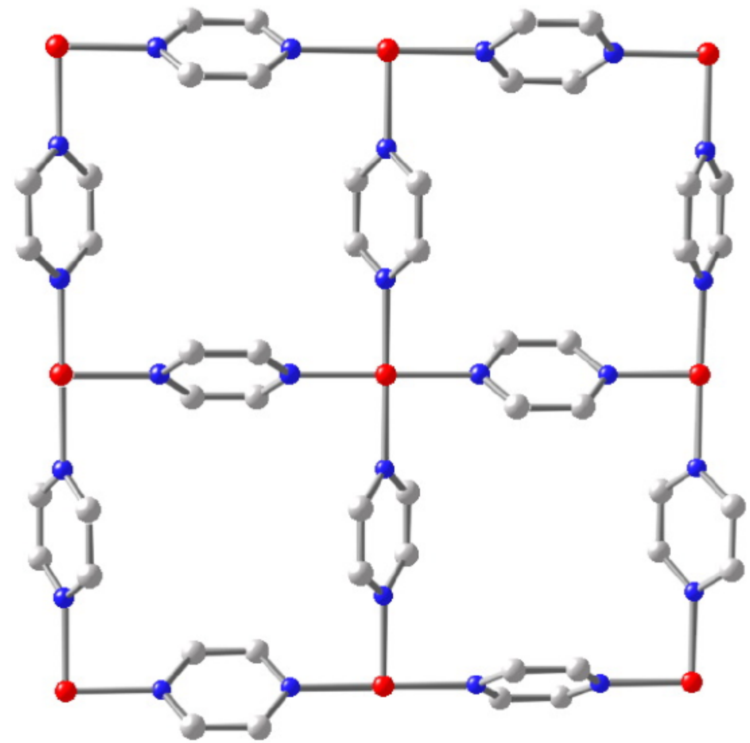
E. Manousakis, Rev. Mod. Phys. **63**, 1 (1991)

2D S=1/2 Quantum Heisenberg Antiferromagnet

S=1/2 Cu^{2+} $3d^9$ on a square lattice



CuO_2 planes



$[Cu(pyz)_2]^{2+}$ planes

$$\boxed{H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j}$$

2D S=1/2 Quantum Heisenberg Antiferromagnet

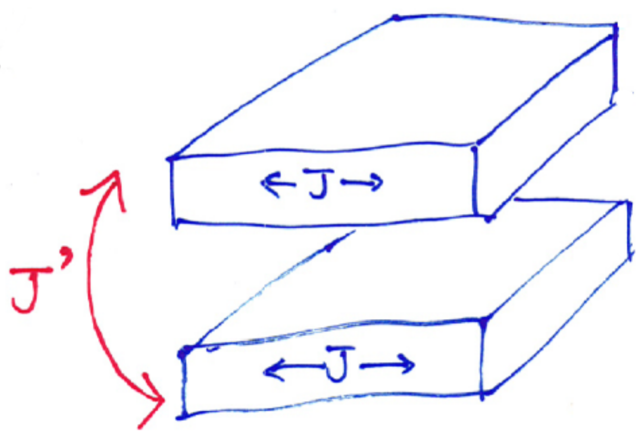
in real life (almost)

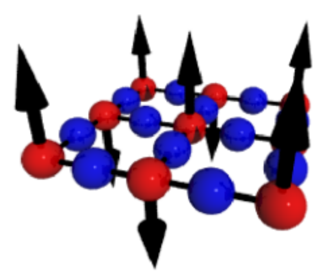
$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

becomes

$$H = J \sum_{\langle ij \rangle} \left[S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right] + J' \sum_{ii'} \mathbf{S}_i \cdot \mathbf{S}_{i'}$$

- J' is the interlayer coupling (the material isn't really 2D)
- Thermodynamic properties are controlled by J'
- J' causes the material to order at T_N>0



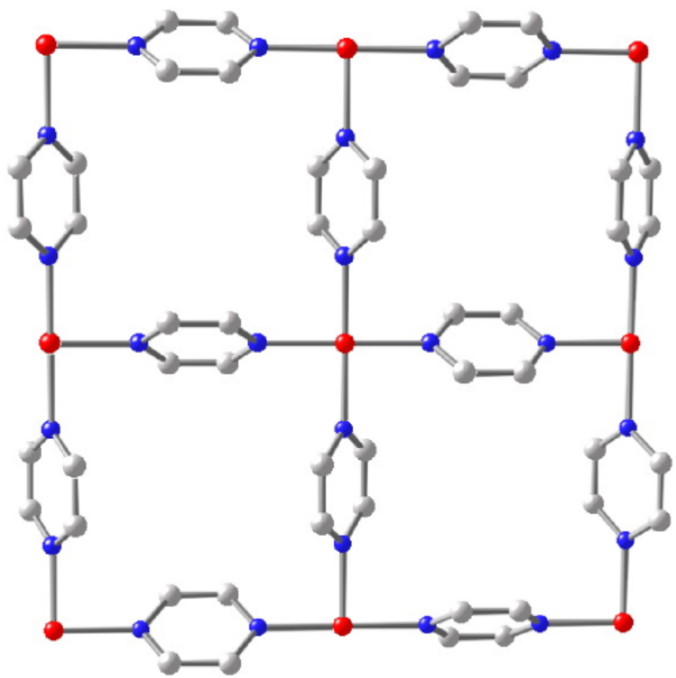


$[Cu(pyz)_2HF_2]X_2$ (X=BF_4, ClO_4, PF_6, AsF_6, SbF_6)

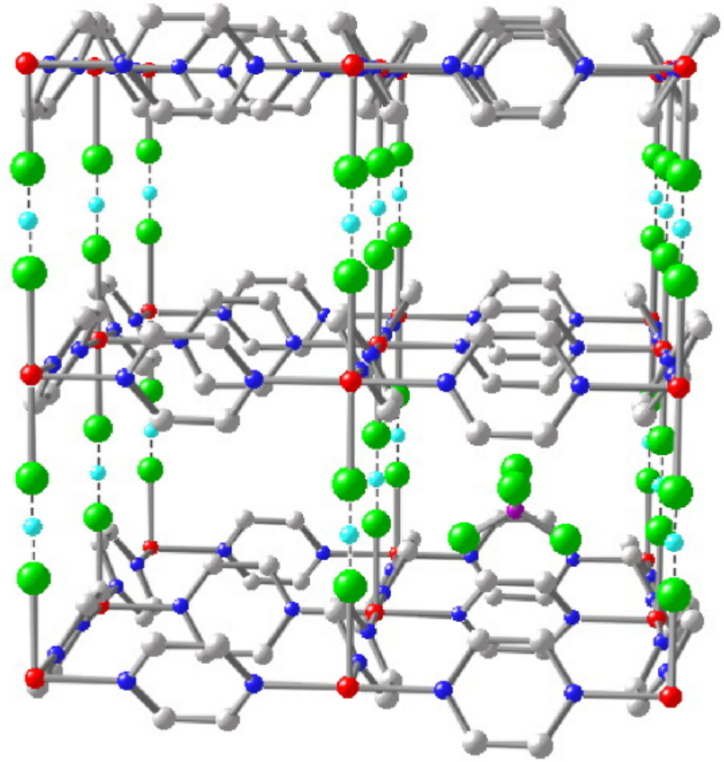
Highly tunable, self-assembled nanostructures with 2D character

First coordination polymer containing the HF_2^- ion

(strongest known hydrogen bond!)



2D square lattice of Cu^{2+} $S=1/2$ spins

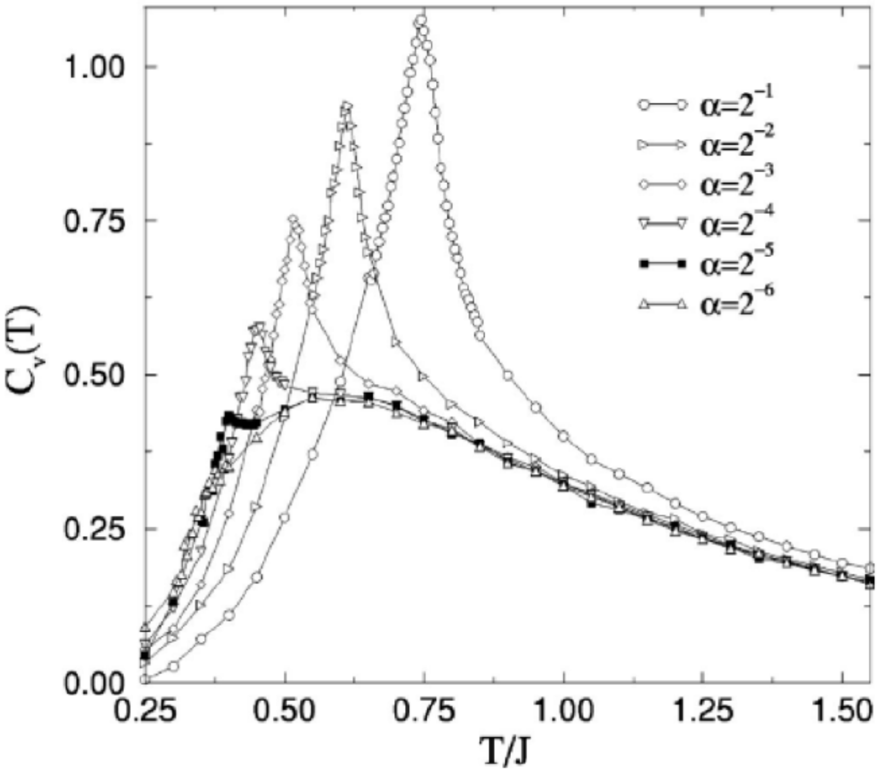


Linked by HF_2^- to form 3D structure

(with X anions in the cubes)

Problem: conventional techniques aren’t good at detecting magnetic order in low-dimensional systems

The problem with finding T_N in low-d systems



Stochastic series QMC simulations say

The anomaly in C_v decreases with decreasing α

This is due to correlations above T_N (ΔS at T_N is therefore reduced)

Other measurements made difficult by the small magnetic moment in anisotropic systems

P Sengupta *et al.* Phys Rev B **68** 94423 (2003)

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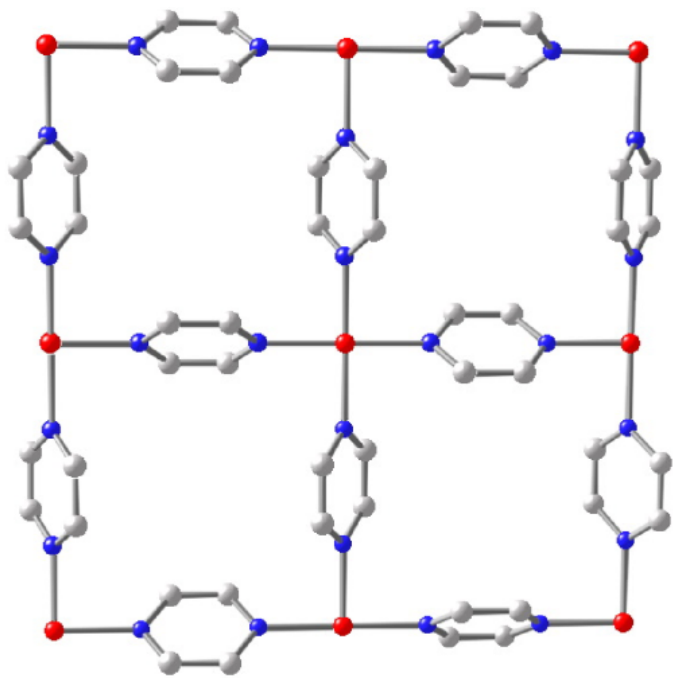

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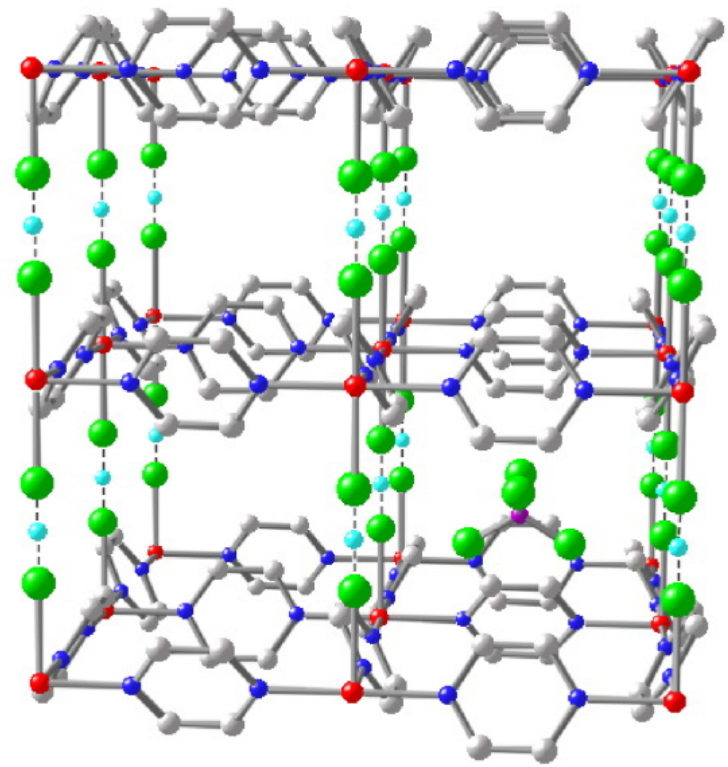
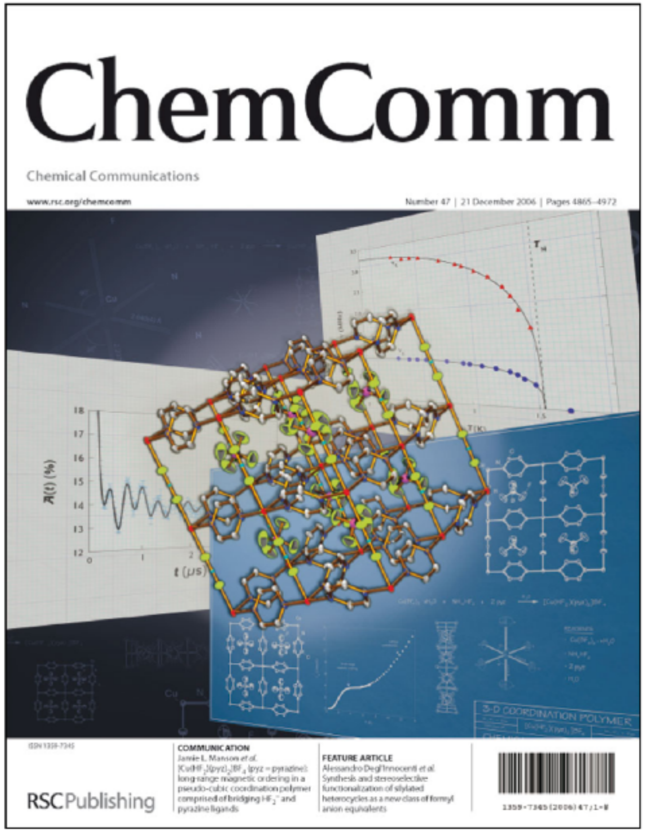


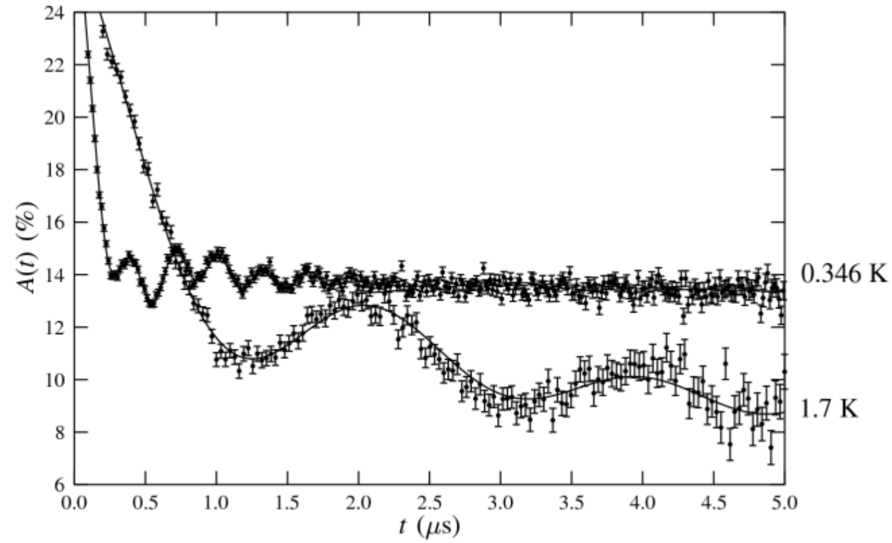
Linked by HF_2^- to form 3D structure
(with X anions in the cubes)

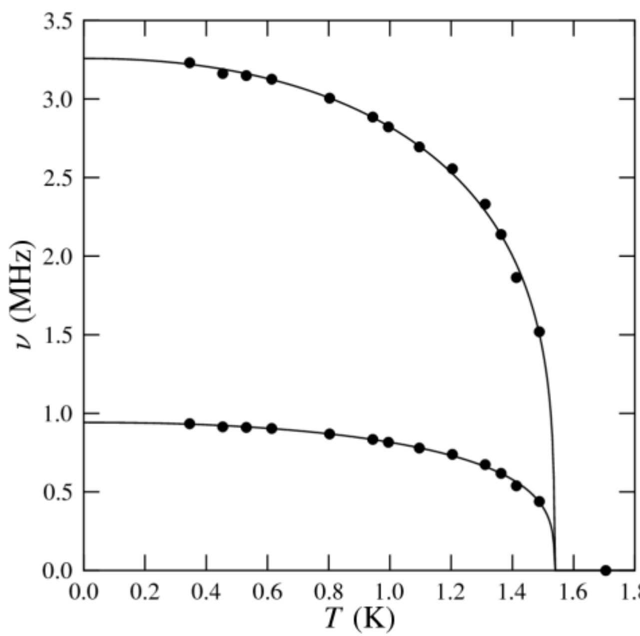
$$\mu$$

The muon

$[Cu(pyz)_2HF_2]BF_4$







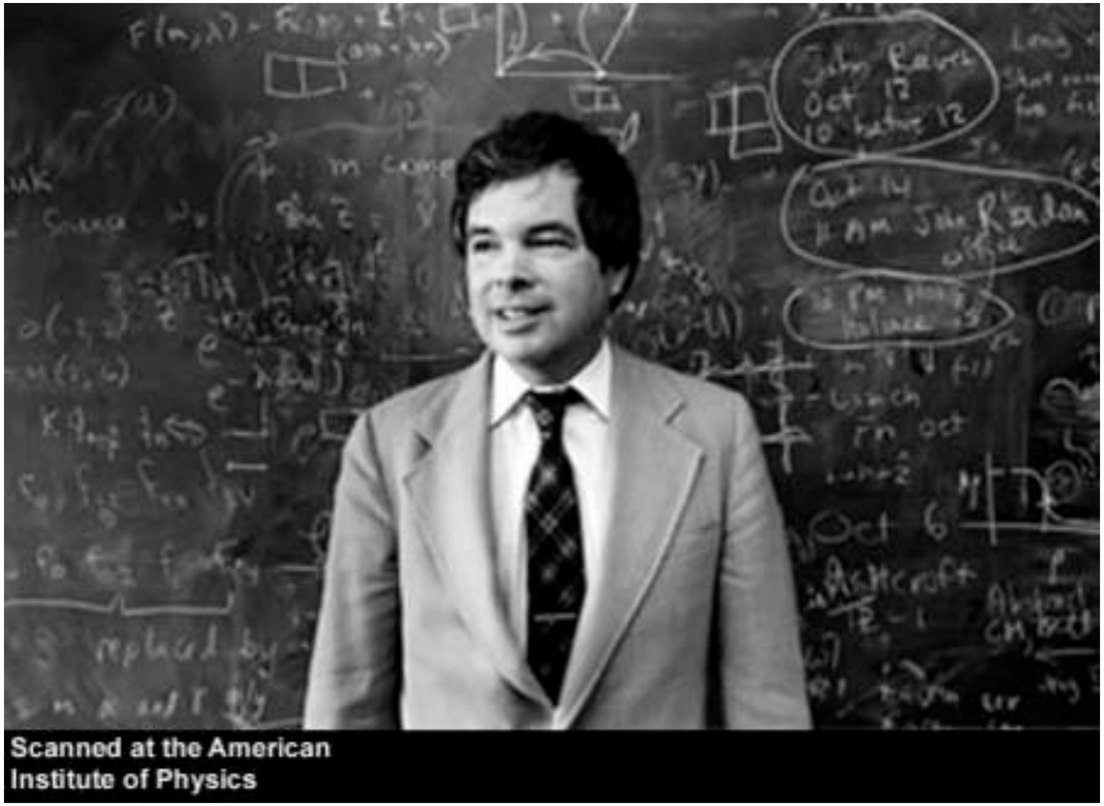
Magnetic order below T_{N}=1.54 K

Slow oscillations above 1.54 K aren't due to magnetic order...

Chem. Comm. 4894 (2006)

How do we systematically deal with scales in magnetism?

Renormalization group



Kenneth Wilson
(1936 - 2013)

Explains how more is different

The RG allows us to understand how more is different

- Physics is all about length scales. We only care about one limit for a given problem

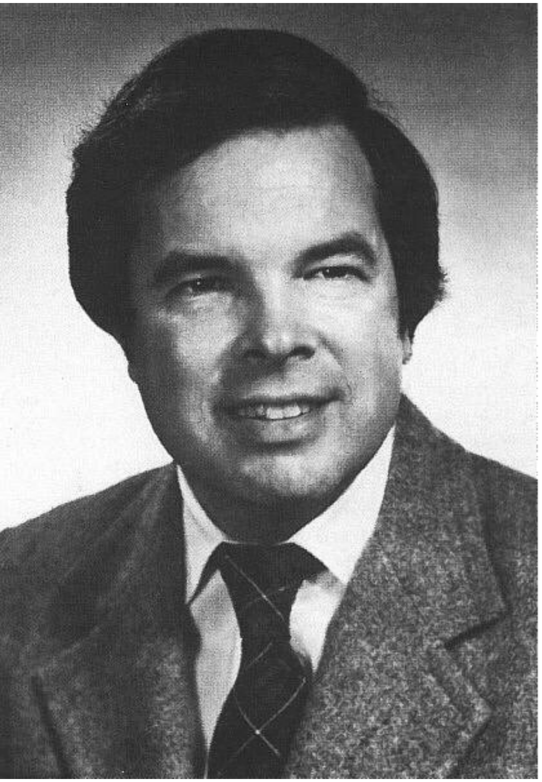
- The RG tells us how the physics changes as we alter the length scale of interest

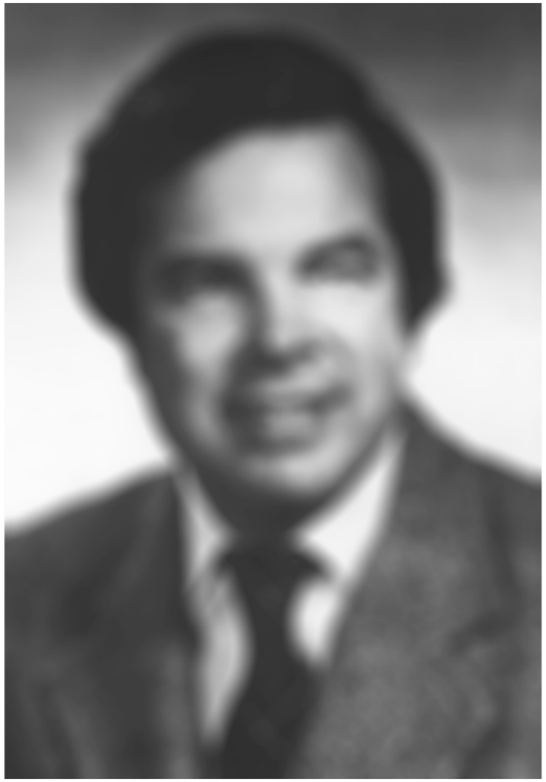
- It works by telling us about *coupling constants*

$$F = F_0 + aM^2 + bM^4 + ...$$

In CMP we measure long length scales

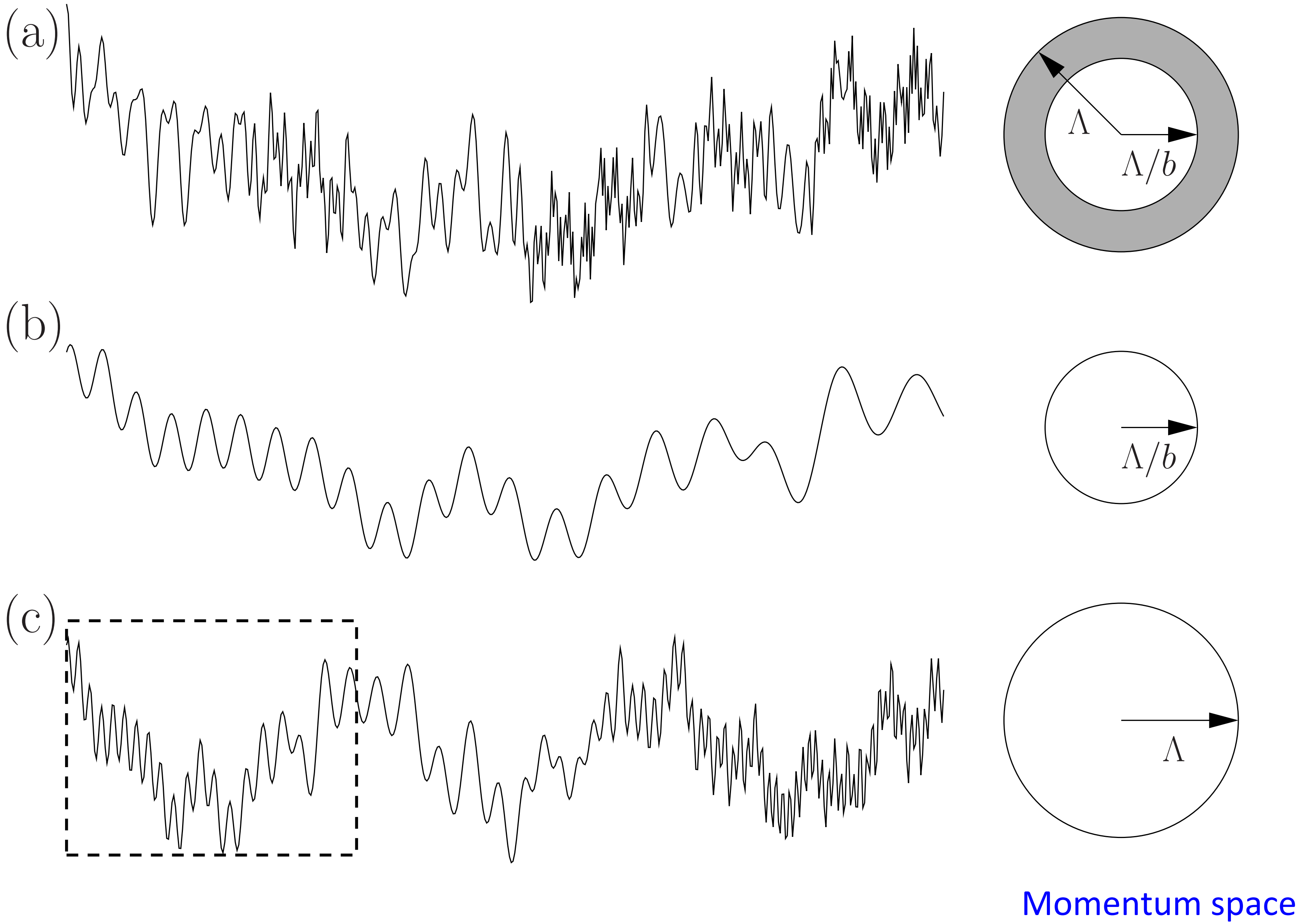
This corresponds to losing small detail



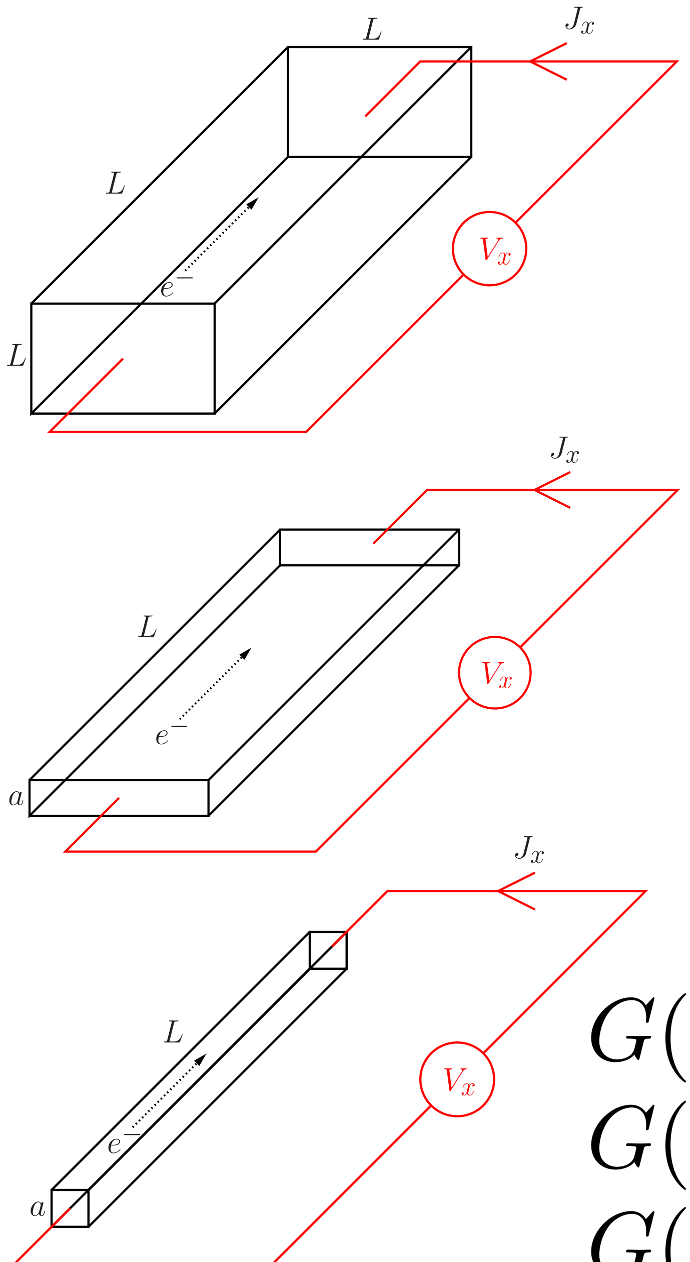


Imagine losing your glasses…

The technical description of the process (in pictures)



Anderson localization



d=3

d=2

d=1

Consider conductance (i.e. 1/*R*) in three, two and one dimensions

$$G(L) = \sigma L \quad \text{3 dimensions}$$
$$G(L) = \sigma a \quad \text{2 dimensions}$$
$$G(L) = \sigma a / L \quad \text{1 dimension}$$

Anderson localization

Conclusion: for Ohmic conductors we expect:

$$G(L) \propto L^{d-2}$$

In contrast, for insulators:

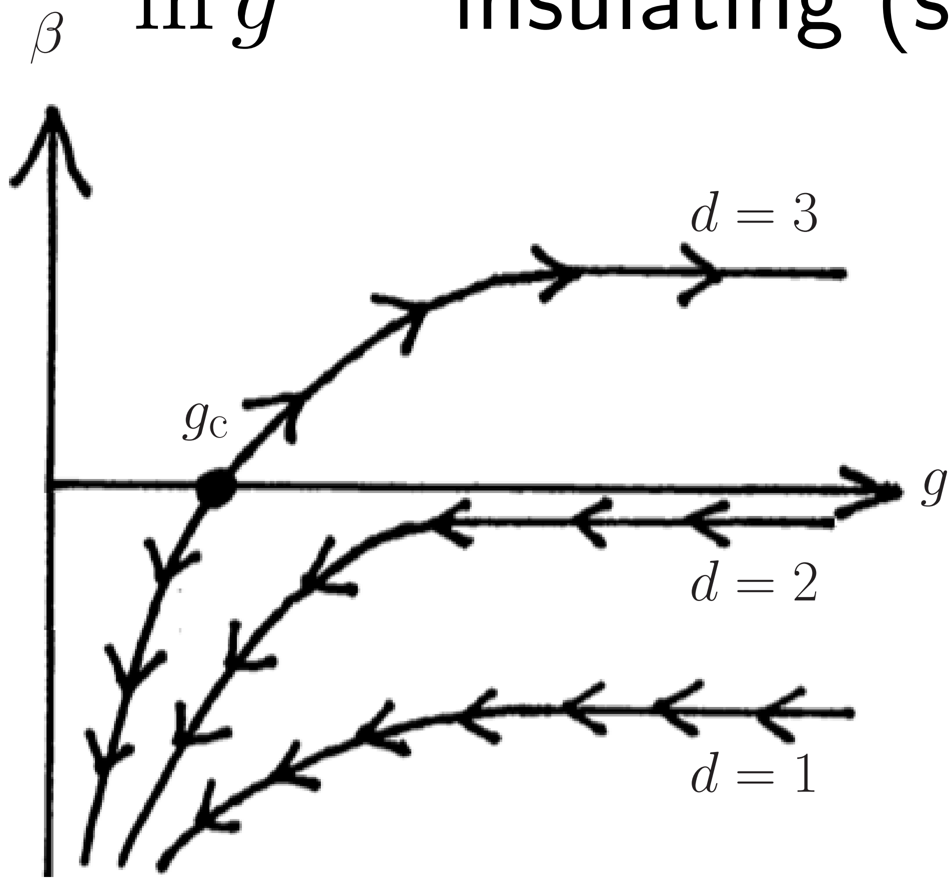
$$G(L) \propto \mathrm{e}^{-L/\xi}$$

Redefine $g = \hbar G/e^2$ and determine ‘flow’:

$$\beta(g) = \frac{\mathrm{d}\ln g}{\mathrm{d}\ln L} = \frac{L}{g}\frac{\mathrm{d}g}{\mathrm{d}L}$$

Conclusion

$$\beta \approx \begin{cases} (d-2) & \text{metallic (large } g\text{)} \\ \ln g & \text{insulating (small } g\text{)} \end{cases}$$



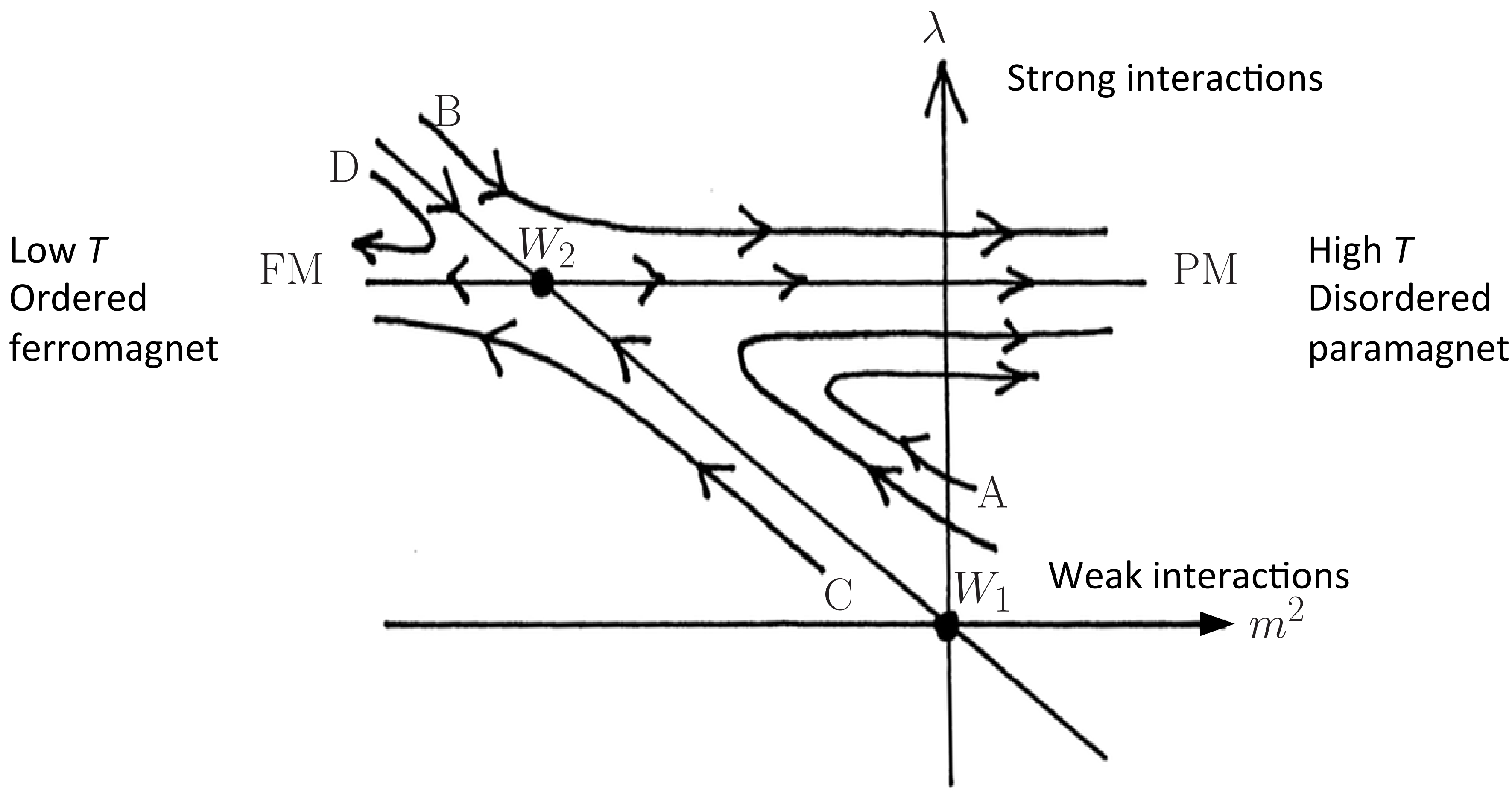
2 and 1 dimensional metals are insulators. d=3 metals have a fixed point ('mobility edge')

Renormalization group and magnetism

$$S_{\mathrm{E}} = \int \mathrm{d}^d x \left[\frac{1}{2}(\boldsymbol{\nabla}\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \right]$$

The Landau-Ginzburg model of magnetism

$$m^2 = a(T - T_{\mathrm{c}})$$



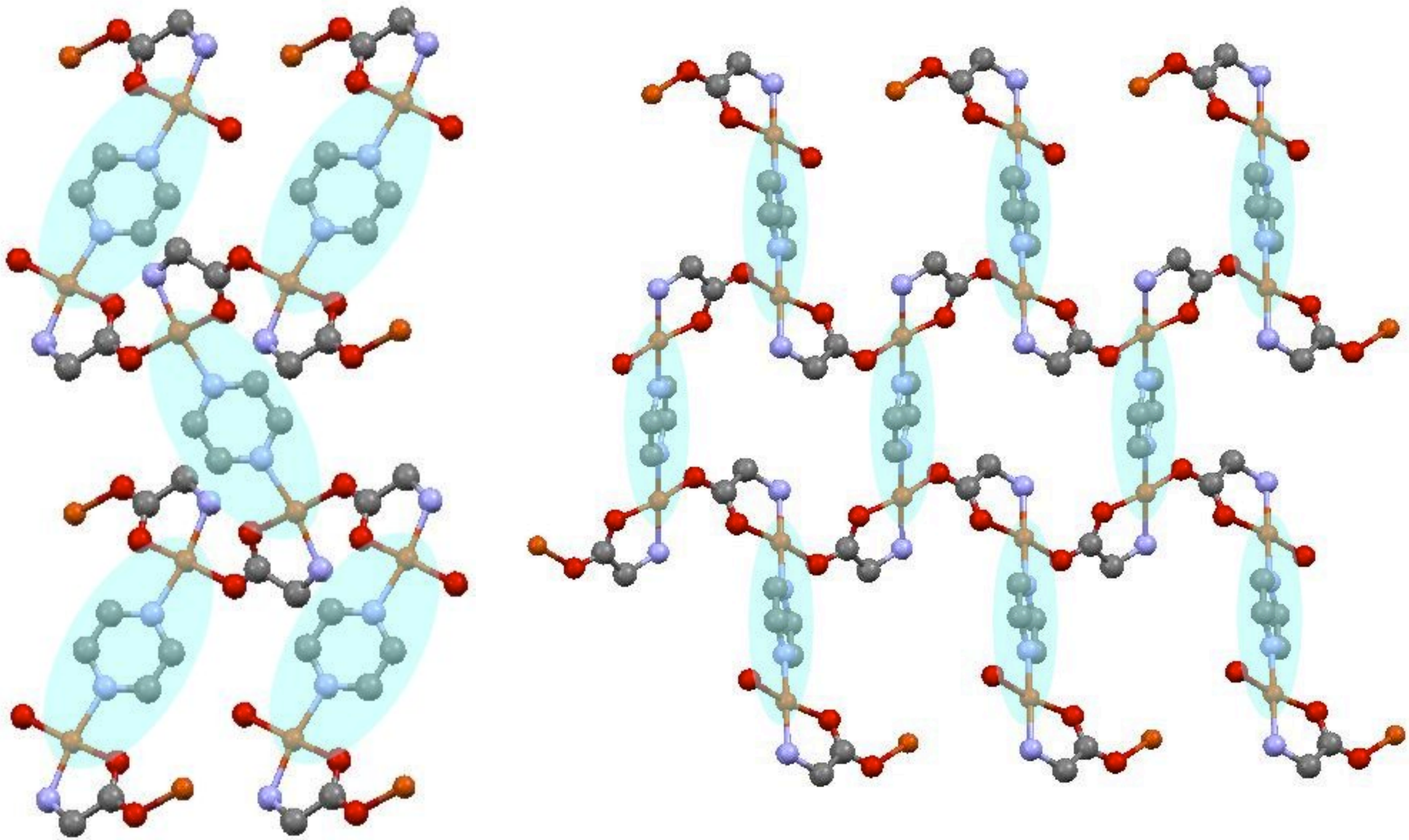
RG allows us to derive the critical exponents from the flow

	α	β	γ	δ	ν	η
$\epsilon = 0$ ($d = 4$, mean field)	0	$\frac{1}{2}$	1	3	$\frac{1}{2}$	0
$\epsilon = 1$ $O(\epsilon)$	0.167	0.333	1.167	4	0.583	0
$\epsilon = 1$ $O(\epsilon^2)$	0.077	0.340	1.244	4.462	0.626	0.019
3D Ising	0.110	0.327	1.237	4.789	0.630	0.036
2D Ising (exact)	0	$\frac{1}{8}$	$\frac{7}{4}$	15	1	$\frac{1}{4}$

Quantum disorder and quantum magnetism

A dimer-based molecular magnet

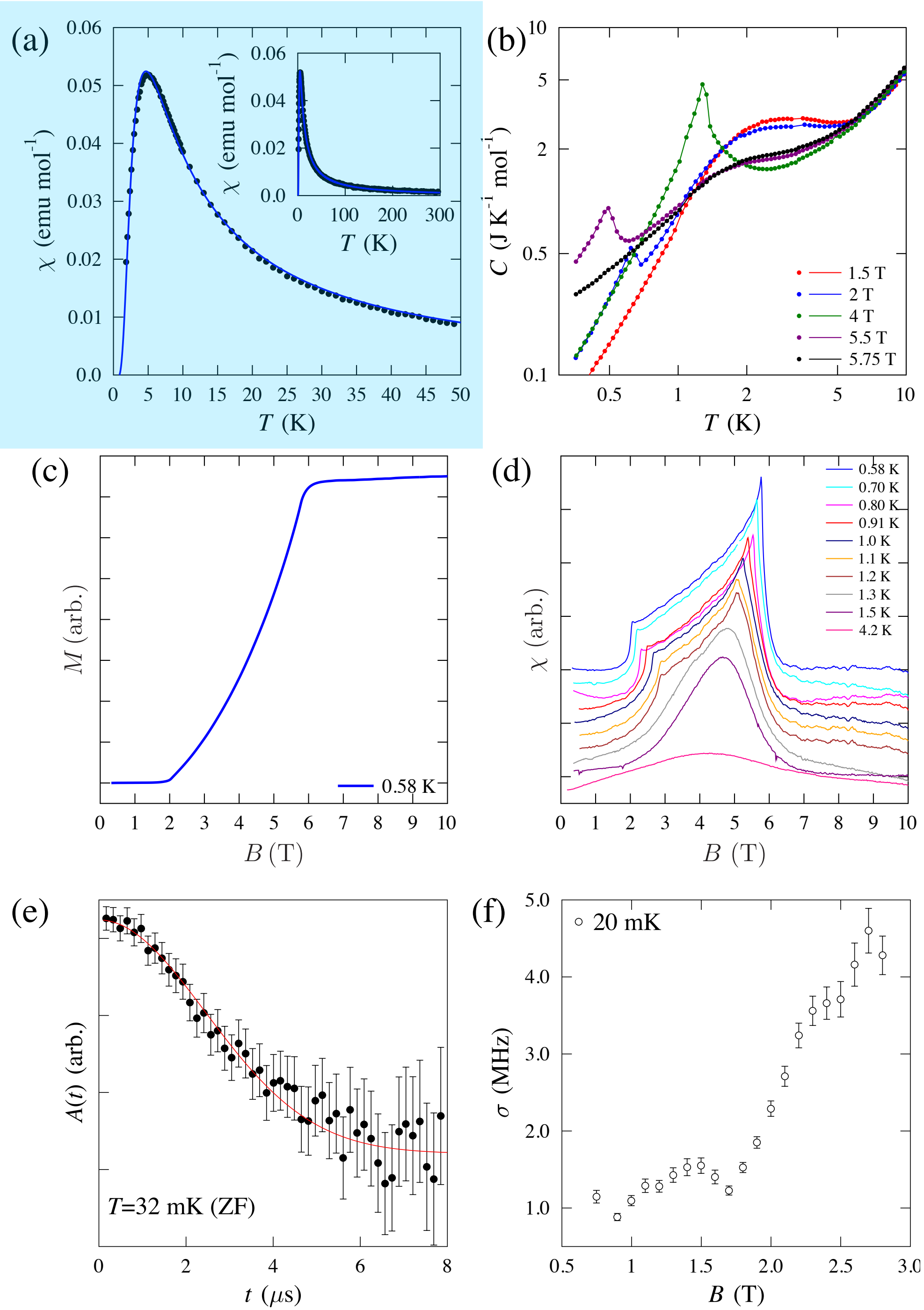
$[Cu(gly)(pyz)](ClO_4)$



Phys. Rev. Lett. 112, 207201 (2014)

[Cu(gly)(pyz)](ClO_4)

Bleaney-Bowers
Susceptibility: *J*=7.5 K



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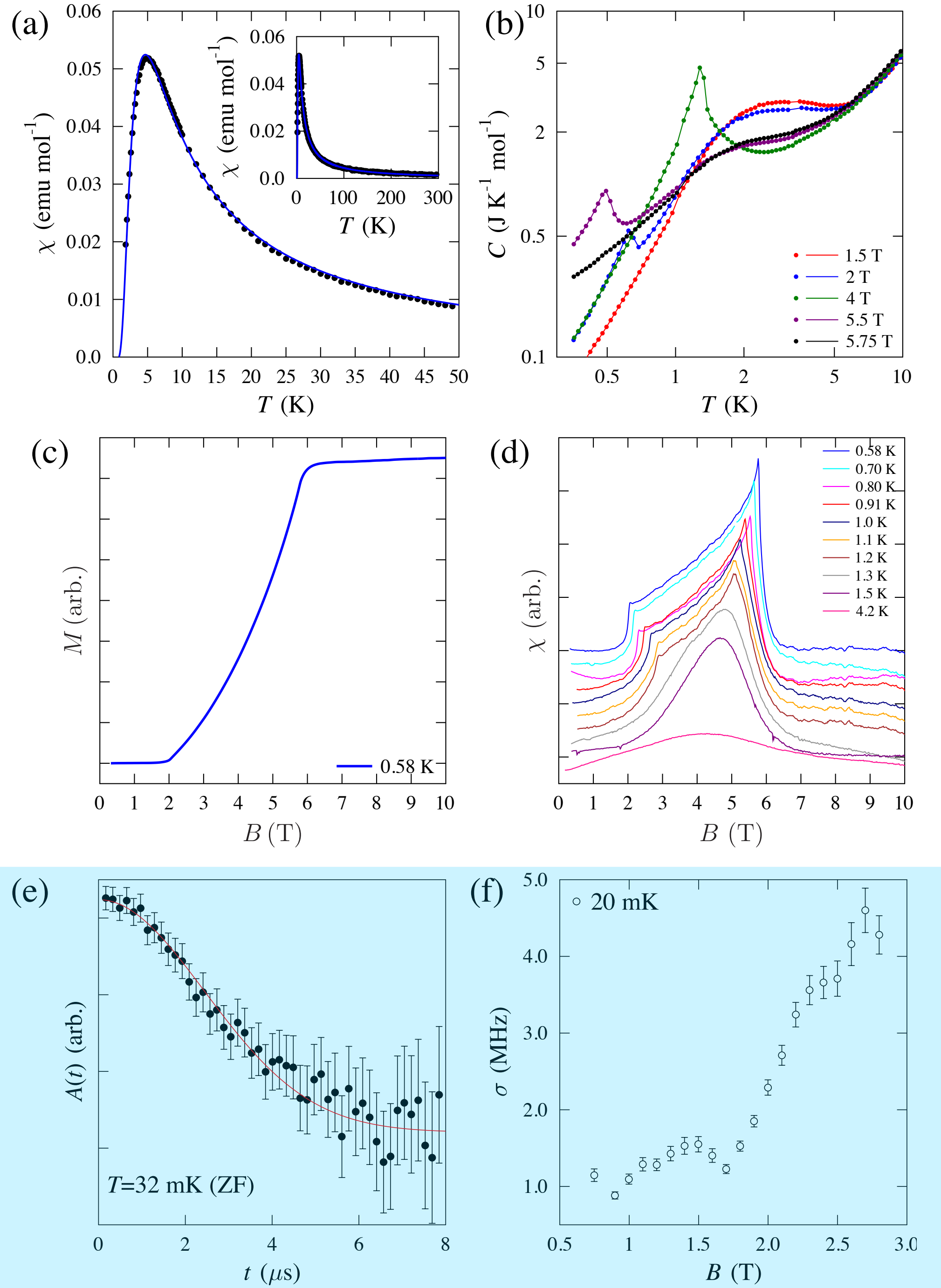
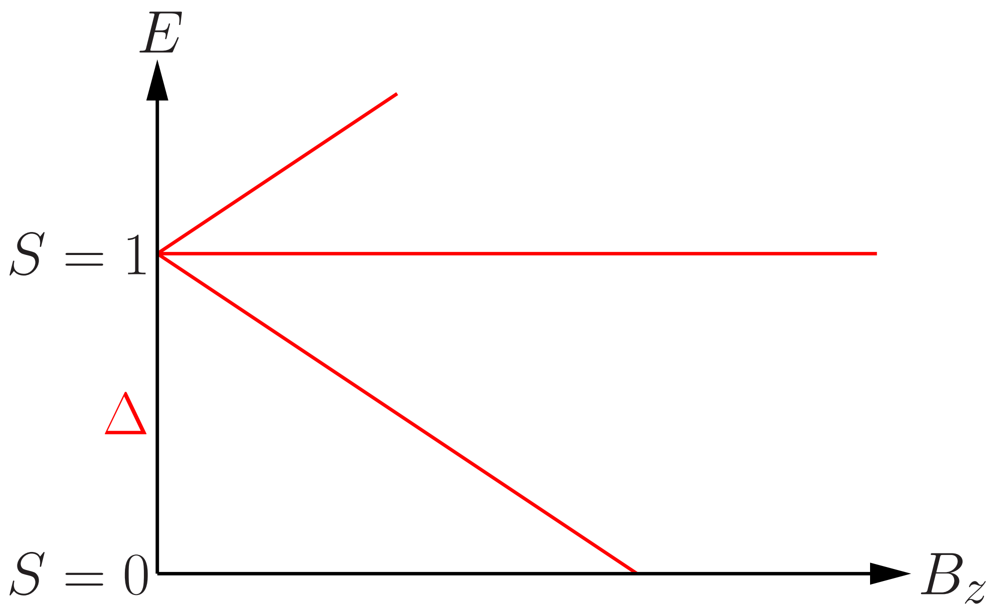
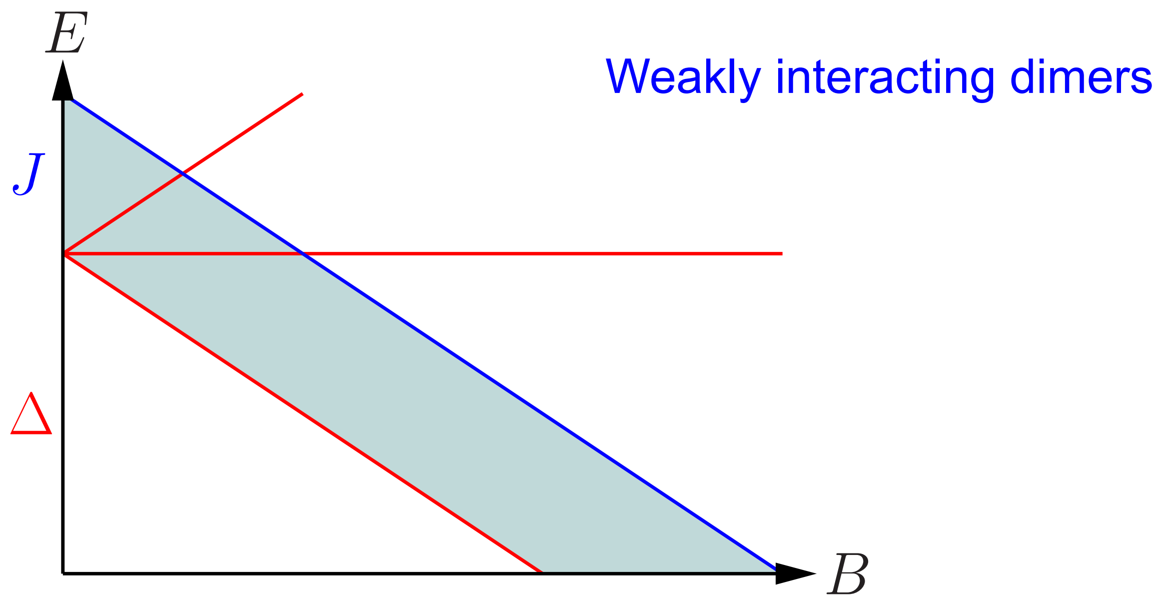


No order in ZF down to 30 mK

Isolated dimers

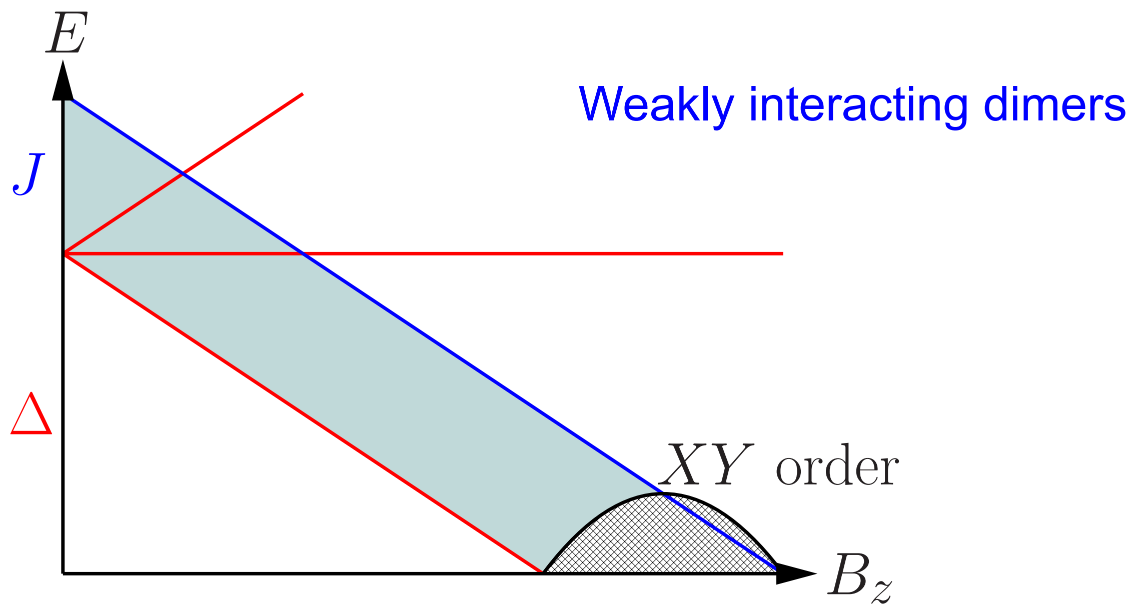


Weakly coupled dimers



In an idealized case we expect a quantum phase transition to *XY* magnetic order

Weakly coupled dimers



In an idealized case we expect a quantum phase transition to *XY* magnetic order

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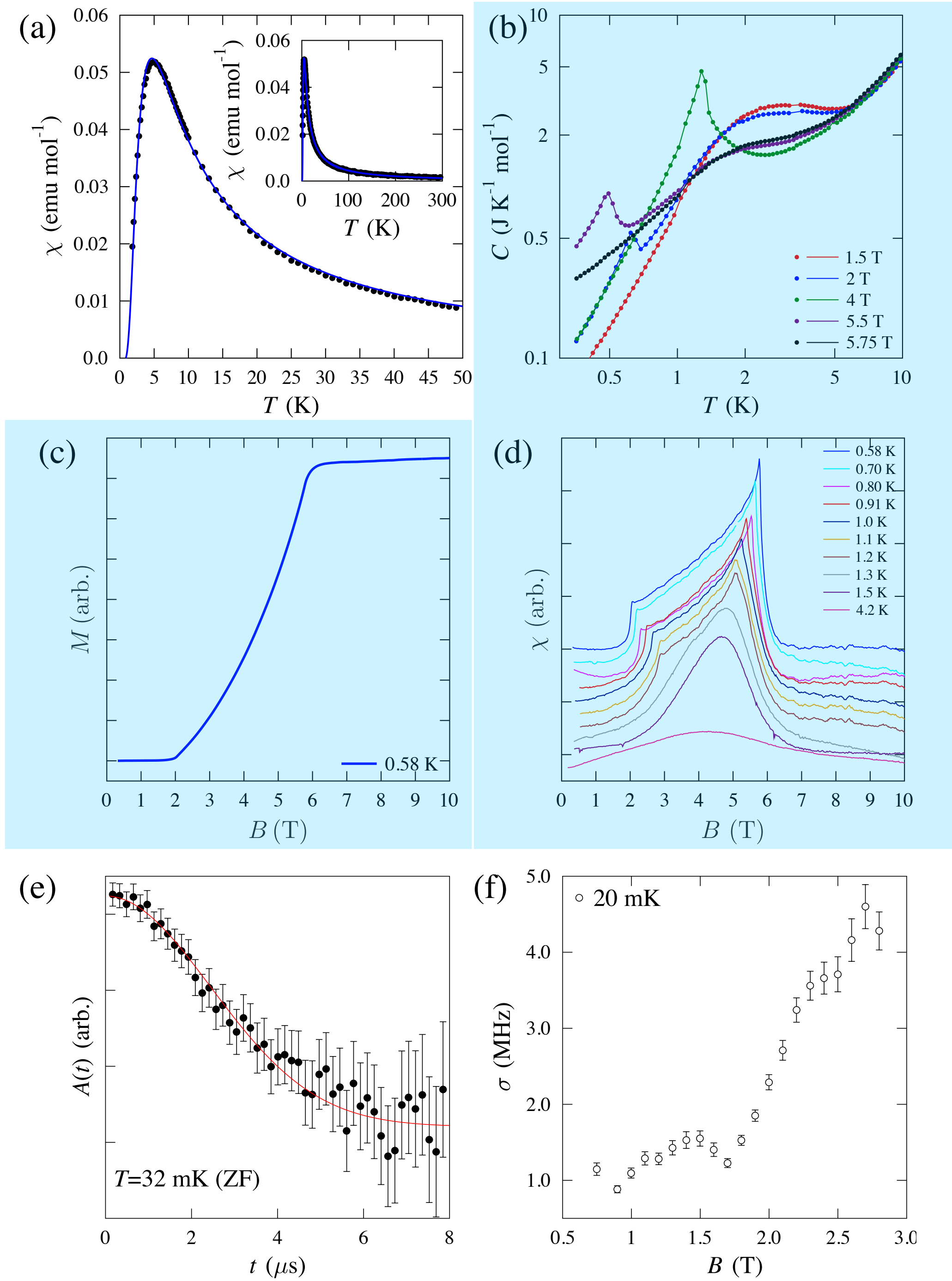
Bleaney-Bowers Susceptibility: J=7.5 K

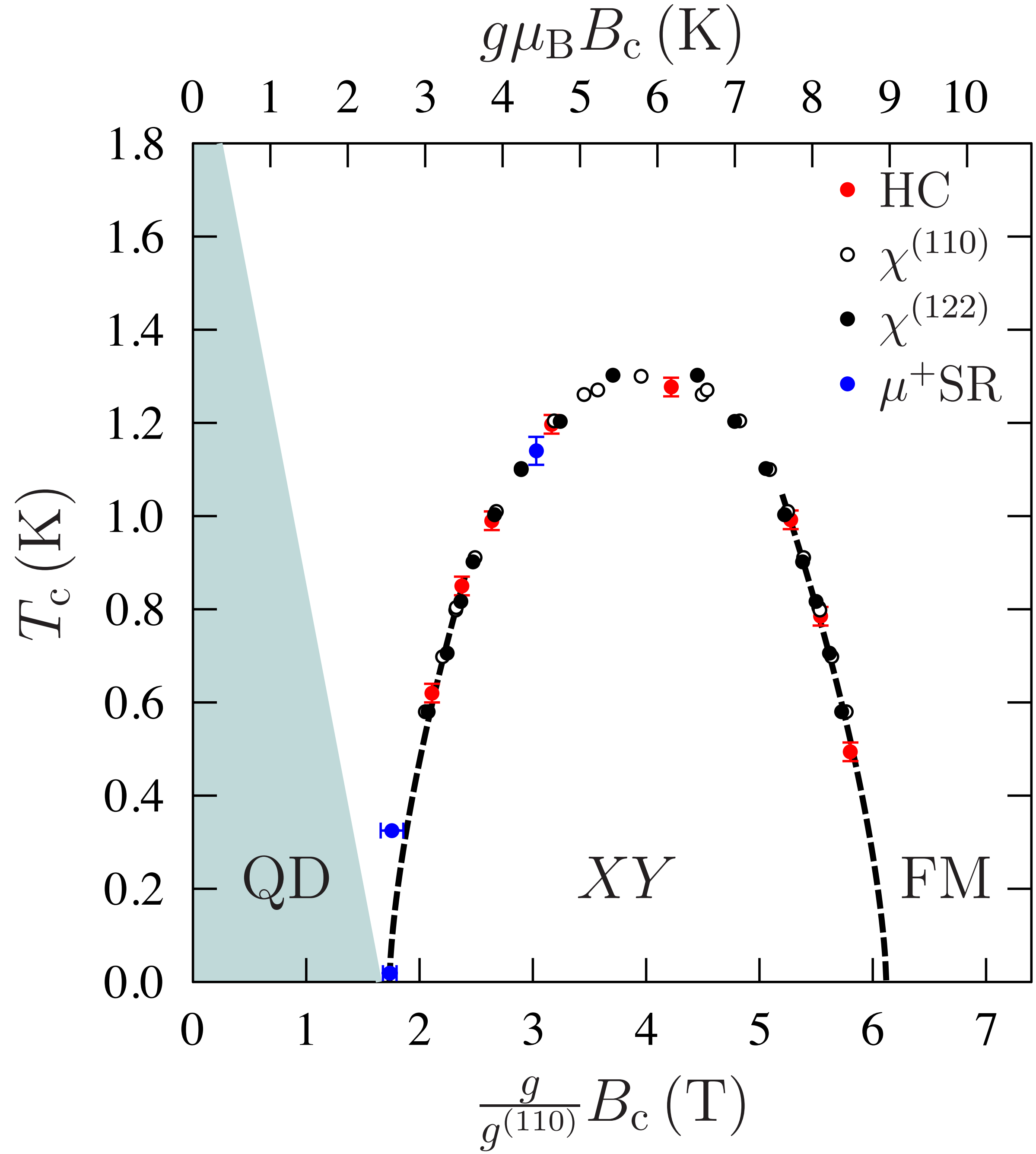
Two set of transitions in applied field



[Cu(gly)(pyz)](ClO_4)

Consistent with a FM Coupled dimers

Suggests J = 7.3 K and J' = 3.3 K



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Conclusions

- Static magnetism may be understood via symmetry breaking and Landau's mean field theory
- Dimensionality is crucial in determining the properties of a system
- The renormalization group allows us to probe the behaviour at length scales of interest
- Quantum dimers are building blocks of disordered states

This new book may be of interest

For more details:

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