FROM RESEARCH TO INDUSTRY

FRINGE FIELD MODELING FOR LARGE APERTURE QUADRUPOLES

The HiLumi LHC Design Study is included in the High Luminosity LHC project and is partly funded by the European Commission within the Framework Programme 7 Capacities Specific Programme, Grant Agreement 284404

CEA - Saclay

BARBARA DALENA

IN COLLABORATION WITH:

O. GABOUEV, J. PAYET , A. CHANCÉ, CEA

R. DE MARIA, M. GIOVANNOZZI, CERN

D.R. BRETT, R. B. APPLEBY, MANCHESTER UNIV AND COCKCROFT INSTITUTE

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CONTENTS

- Motivation
- Computation of realistic symplectic transfer maps of charged particles
- Non linear fringe field transfer maps
- Conclusion & Outlook

MOTIVATION

INNER TRIPLET FRINGE FIELD EFFECT

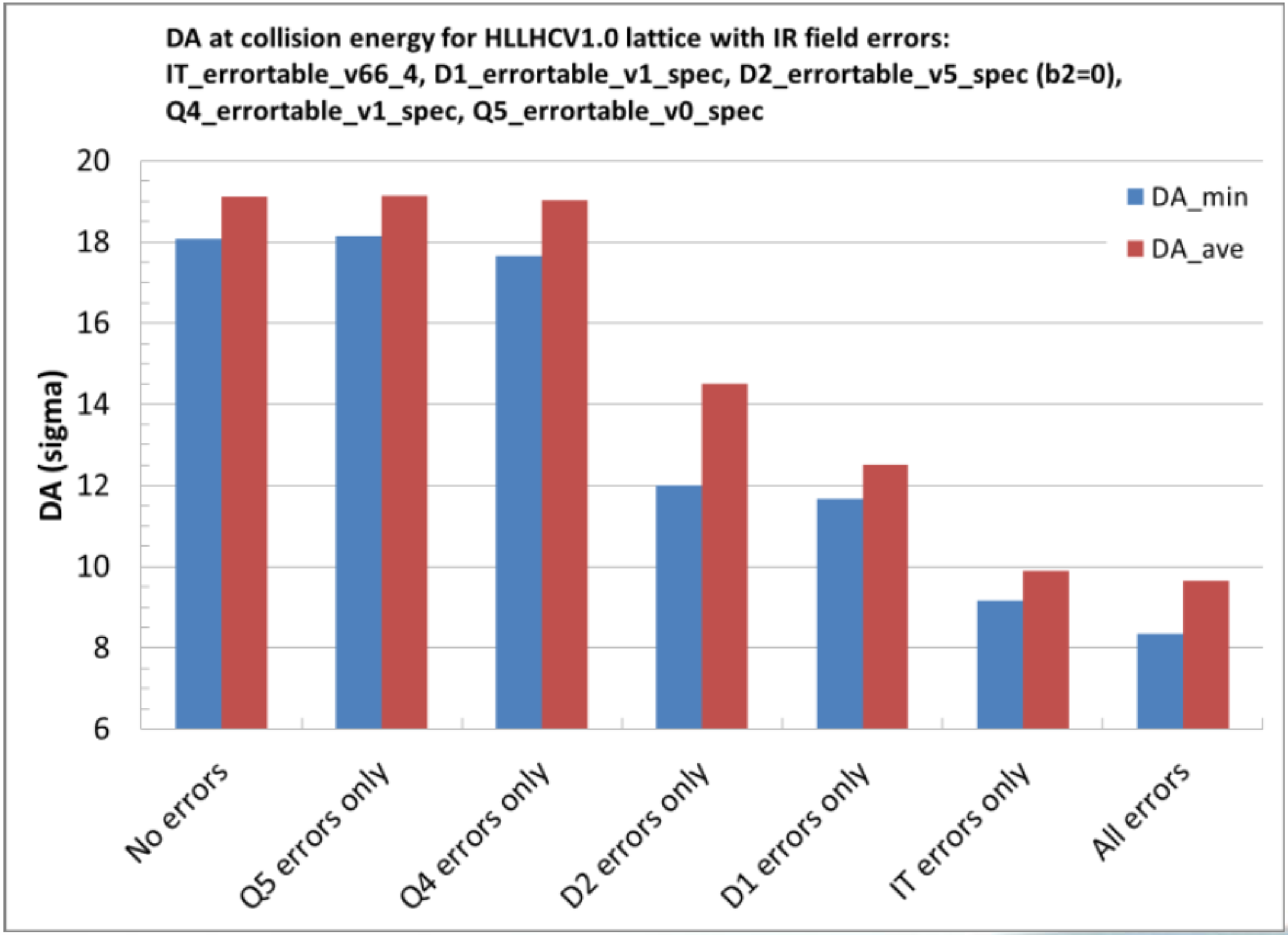
The HL-LHC project relies on large aperture magnets (due to increased beam sizes before the IP)
⇒ The beam is much more sensitive to non-linear perturbations in this region.

- the effect has been quantified by direct analytical estimates of detuning with amplitude and chromatic effects (**A. V. Bogomyagkov** et al. **WEPEA049 @ IPAC'13**).
  The effect of the fringe fields is small, nevertheless it cannot be completely neglected.

⇒ Quantify long term beam dynamics effects

Ultimate Goals
- Definition of **field quality** and **corrections**
- Provide **feedback** to the designers of magnets

Impact of "IT_errortable_v66_4" at collision compared to impact of the other IR magnets



M. Giovannozzi @ 4th HiLumi meeting

INTEGRATED FIELD HARMONICS

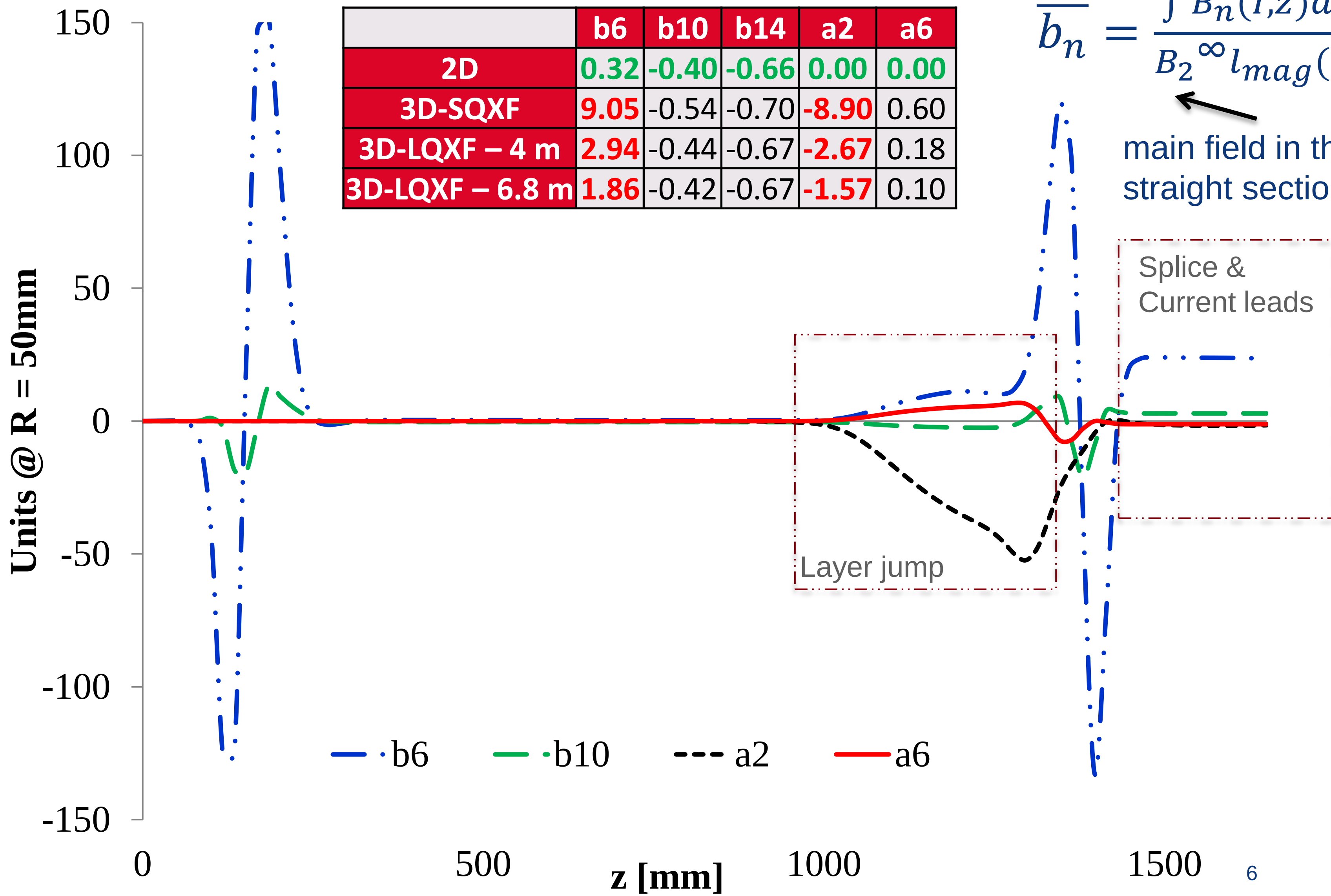
By S. Izquierdo Bermudez

G. Ambrosio @ 4th HiLumi meeting

	b6	b10	b14	a2	a6
2D	0.32	-0.40	-0.66	0.00	0.00
3D-SQXF	9.05	-0.54	-0.70	-8.90	0.60
3D-LQXF – 4 m	2.94	-0.44	-0.67	-2.67	0.18
3D-LQXF – 6.8 m	1.86	-0.42	-0.67	-1.57	0.10

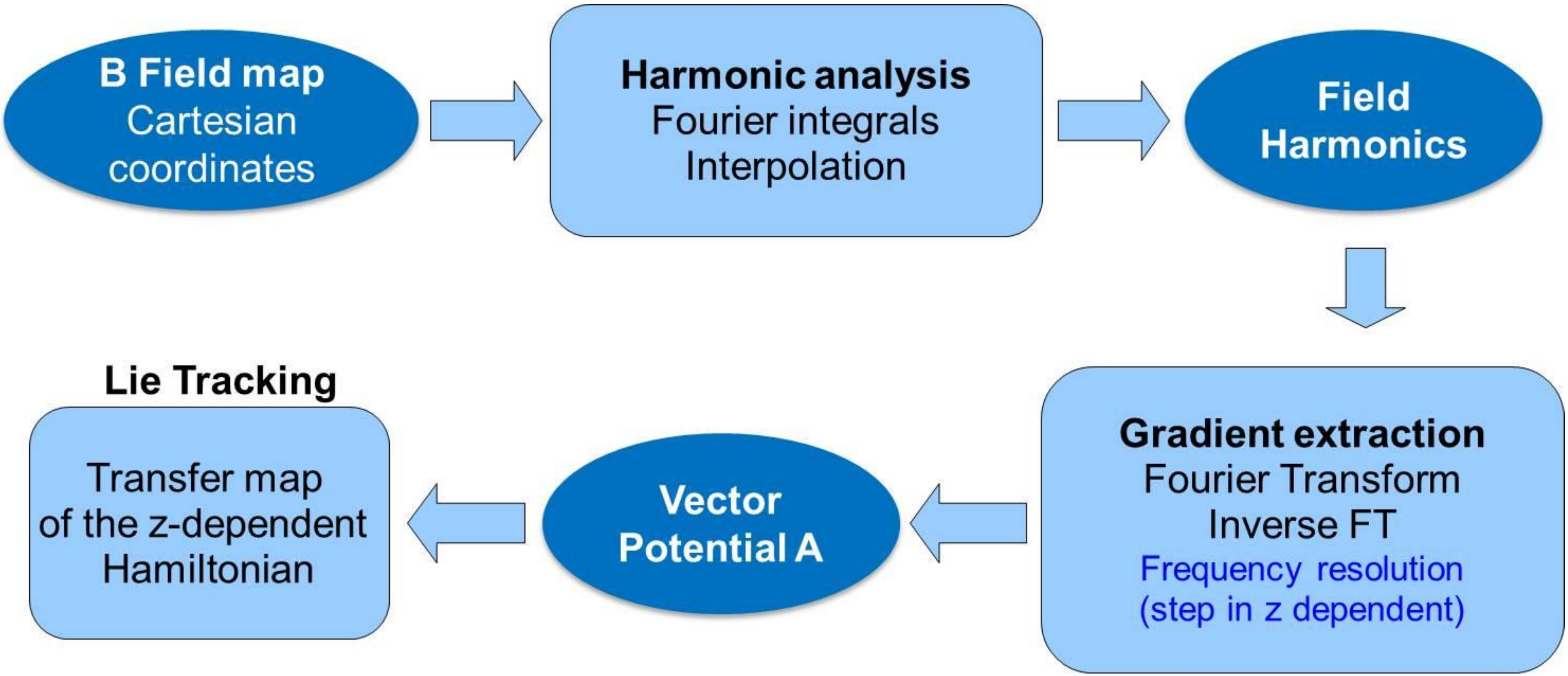
$$\overline{b_n} = \frac{\int B_n(I,z)dz}{B_2{}^{\infty} l_{mag}(I)}$$

main field in the straight section

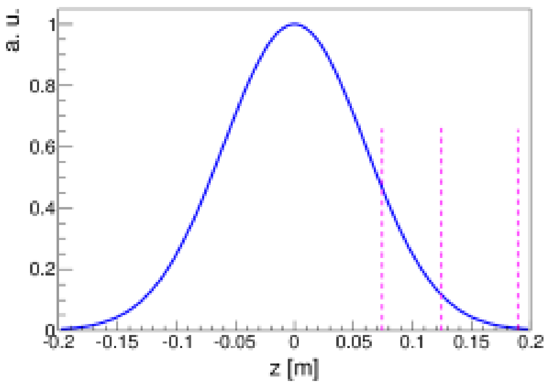


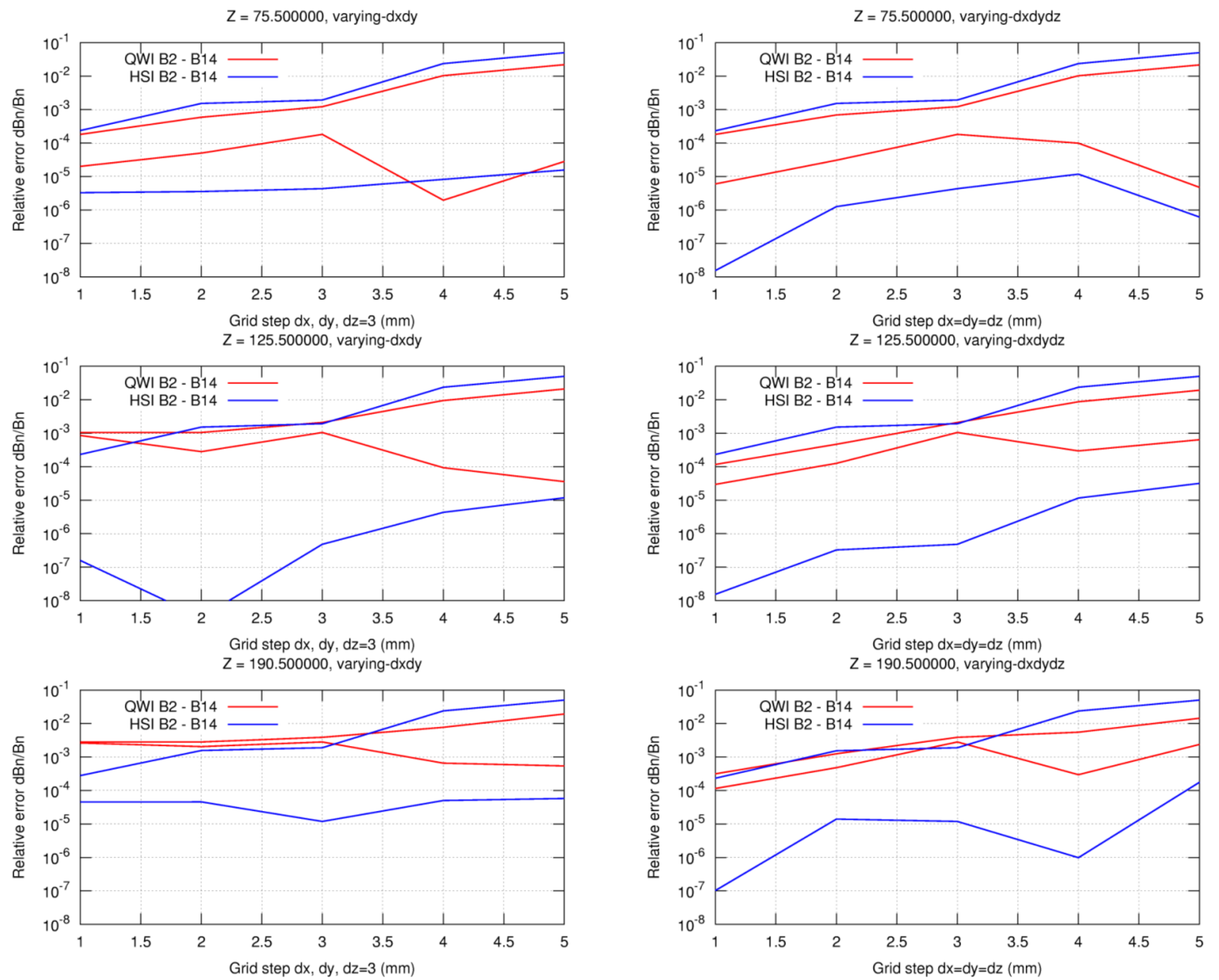
COMPUTATION OF REALISTIC TRANSFER MAPS OF CHARGED PARTICLES

REALISTIC TRANSFER MAP COMPUTATION: SCHEME OF THE METHOD



HARMONICS: ACCURACY STUDIES (ANALYTIC FIELD)

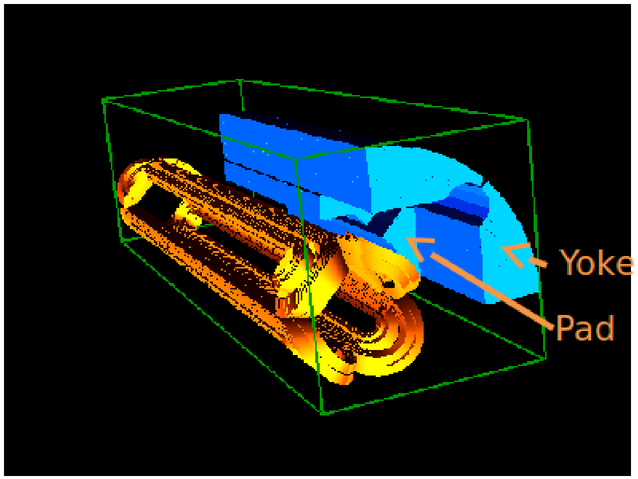




- Hermite Spline Interpolator (HSI) better precision for low harmonics than QWI (Quadratic Weight Interpolator)
- map step of 3 mm for high slope and for low field regions

3D MAGNETIC FIELD DATA

Inner triplet prototype magnet for HL-LHC



Courtesy of CERN magnet group

Gradient 140 T/m, □ = 150 mm

QXF: Symmetric Return end

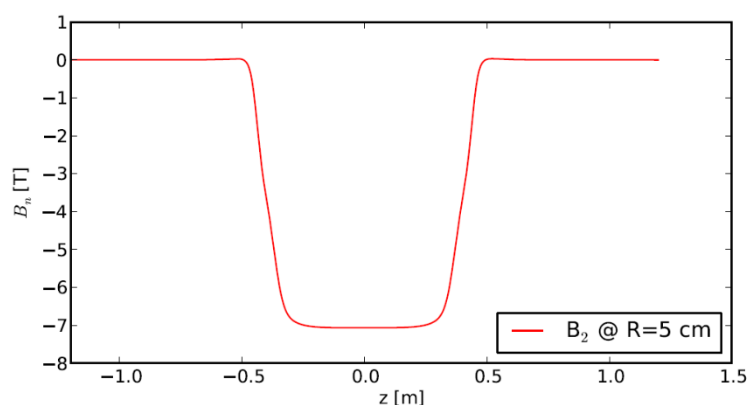
z=[0,487.5] mm: Magnetic yoke and pad

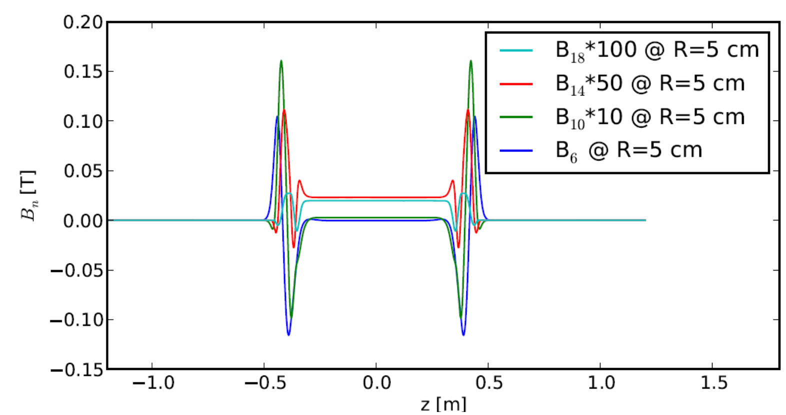
z=[487.5,7125] mm: Magnetic yoke, non-magnetic pad

Data

Bx, By, Bz in a Cartesian grid:

- x = 0:75:3 mm
- y = 0:75:3 mm
- z = 100:1300:3 mm





VECTOR POTENTIAL IN CARTESIAN COORDINATES

- The three components of the quadrupole vector potential can be written as expansions of normal (s) and skew (c) multipoles
- Each of the multipole can be expanded in terms of **homogenous polynomials** in ***x,y*** and ***z*-dependent coefficients** $C_{m,\alpha}^{[n]}(z)$ (called **generalized gradients**)

$$A_x = \sum_{m=1}^{\infty} A_x^{m,s} - A_x^{m,c}$$

$$A_y = \sum_{m=1}^{\infty} A_y^{m,s} - A_y^{m,c}$$

$$A_z = \sum_{m=1}^{\infty} A_z^{m,s} - A_z^{m,c}$$

normal multipole

$$\alpha = s \Rightarrow \mathrm{C} = \Re$$

skew multipole

$$\alpha = c \Rightarrow \mathrm{C} = \Im$$

$$A_x^{m,\alpha} = -\frac{1}{m} x \, \mathbb{C}\left[(x+iy)^m\right] \sum_{l=0}^{\infty} \frac{(-1)^l m!}{2^{2l} l! (l+m)!} C_{m,\alpha}^{[2l+1]}(z) \left(x^2+y^2\right)^l$$

$$A_y^{m,\alpha} = -\frac{1}{m} y \, \mathbb{C}\left[(x+iy)^m\right] \sum_{l=0}^{\infty} \frac{(-1)^l m!}{2^{2l} l! (l+m)!} C_{m,\alpha}^{[2l+1]}(z) \left(x^2+y^2\right)^l$$

$$A_z^{m,\alpha} = \frac{1}{m} \mathbb{C}\left[(x+iy)^m\right] \sum_{l=0}^{\infty} \frac{(-1)^l m! (2l+m)}{2^{2l} l! (l+m)!} C_{m,\alpha}^{[2l]}(z) \left(x^2+y^2\right)^l$$

$$\vec{B} = \nabla \psi$$

$$\nabla^2 \psi = 0$$

$$\nabla \times \vec{A} = \nabla \psi$$

Reference:
A. J. Dragt, www.physics.umd.edu/dsat , C. E. Mitchell and A. J. Dragt Phys. Rev. ST AB 13, 064001, 2010

GENERALIZED GRADIENTS

For normal multipoles

$$C_m^{[l]}(z) = \frac{i^l}{2^m m! \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{ikz} k^{m+l-1}}{I'_m(kR)} \tilde{B}_m(R,k) dk$$

$$\nabla \times \vec{A} = \vec{B}$$

$$\mathcal{F}(f^{(n)})(k) = (ik)^n \mathcal{F}(f)(k)$$

where: $I'_m(kR)$ is the derivative of the modified Bessel function

$$\tilde{B}_m(R,k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\mathrm{ikz}} B_m(R,z) dz$$

$$B_r(R,\phi,z) = \sum_{m=1}^{\infty} B_m(R,z)\sin(m\phi) + A_m(R,z)\cos(m\phi)$$

↓

Fields Harmonics

Reference:
M. Venturini, A.J. Dragt, NIM A 427, p.387,1999

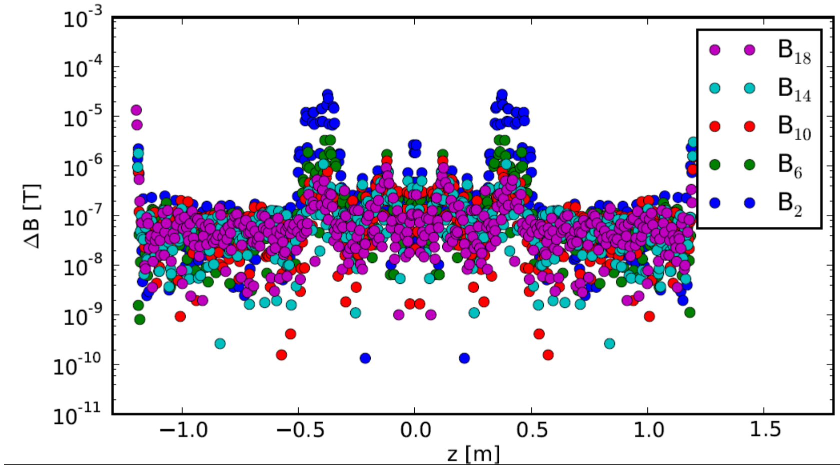
GRADIENTS EXTRACTION

- Numerical computation of Fourier integrals using Filon-spline formula*: spline interpolation of data

$$C_m^{[l]}(z) = \frac{i^l}{2^m m! \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{ikz} k^{m+l-1}}{I'_m(kR)} \tilde{B}_m(R,k) dk$$

- Comparison between harmonics from harmonic analysis and harmonics reconstructed from the gradient sum

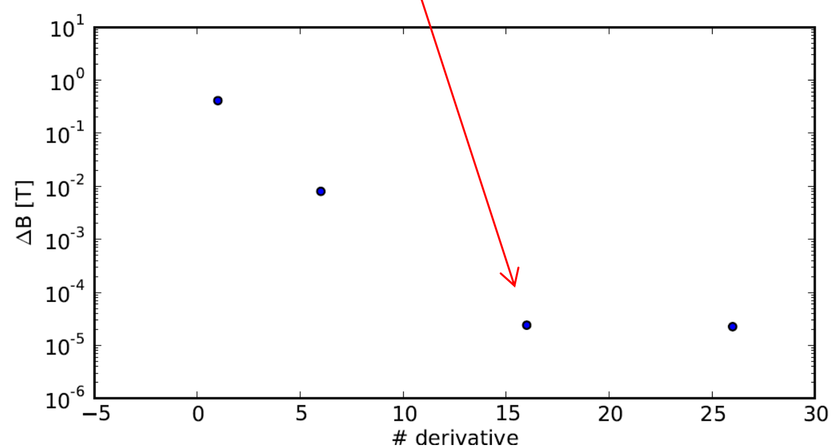
$$B_m(R,z) = \sum_{n=0}^{\infty} (m+2l) \frac{(-1)^l m!}{4^l l!(m+l)!} R^{m+2l-1} C_m^{[2l]}(z)$$



- Parameters
  - Longitudinal step
  - Map length
  - Frequency step
  - Number of gradient derivatives

Fringe field slope reconstitution

Gibbs oscillations (at boundaries)



*E., Catmull, and R., Rom "A class of local interpolating splines", Computer Aided Geometric Design, R. E. Barnhill and R. F. Reisenfeld, Eds. Academic Press, New York, 1974, pp. 317–326.

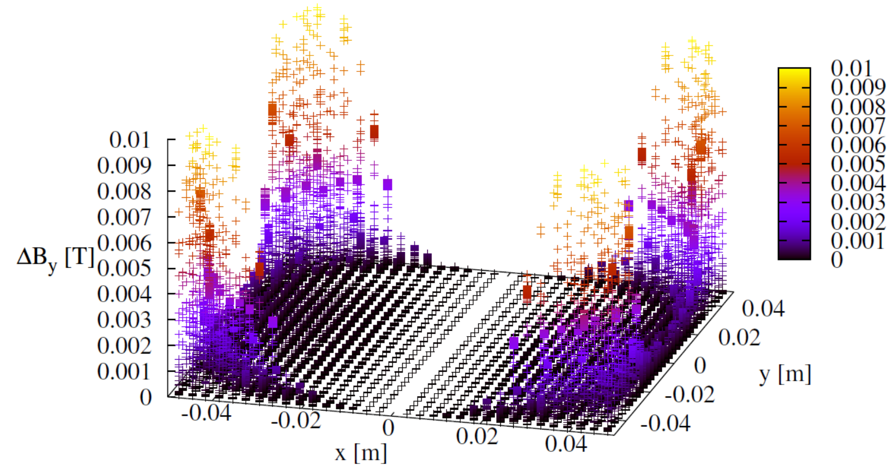
B. Einarsson, "Numerical computation of Fourier integrals with cubic splines", 1968.

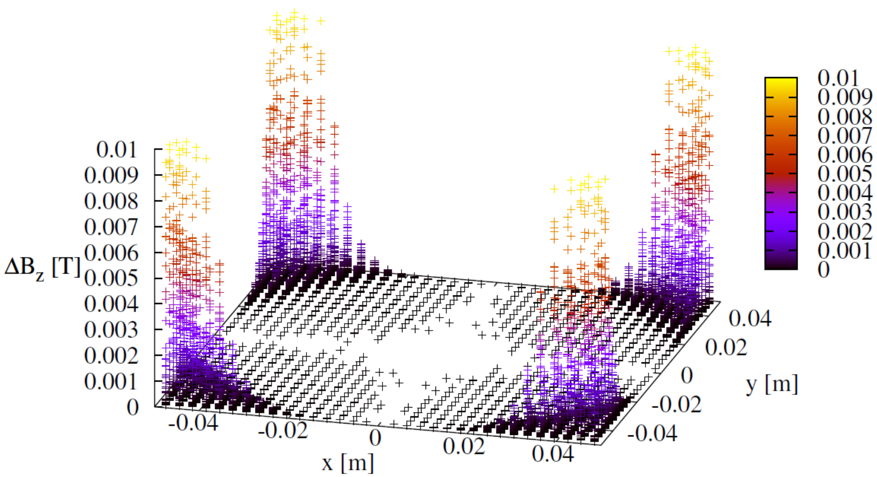
FIELD RECONSTRUCTION

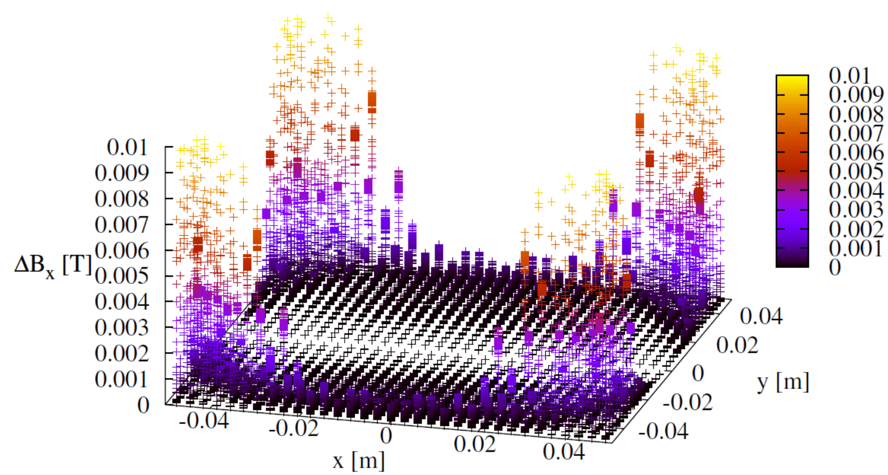
Outside the radius of the Harmonic Analysis the quality of field reconstruction is not good.

Need to:

- use a radius as larger as possible, without loosing homogeneity of the field
- study alternative field fitting procedures







SYMPLECTIC INTEGRATOR OF Z-DEPENDENT HAMILTONIAN

Equivalent paraxial **Hamiltonian** in the extended phase space:

$$K(x,p_x,y,p_y,\delta,l,z,p_z;\sigma)\approx-\delta+\frac{(p_x-a_x)^2}{2(1+\delta)}+\frac{(p_y-a_y)^2}{2(1+\delta)}-a_z+p_z$$

$a_{x,y,z}\equiv a_{x,y,z}(x,y,z)=\dfrac{qA_{x,y,z}(x,y,z)}{P_0c}$ scaled vector potential

(z,p_z) 4th canonical pairs

$d\sigma=dz$ independent variable

The solution of the equation of motion for this Hamiltonian using Lie algebra formalism is (**Transfer Map** or **Lie Map**):

$$M(\sigma)=\exp(-\sigma:K:)$$

The transfer map ***M(σ)*** can be replaced by a product of symplectic maps which approximates it (**symplectic integrator**).

Reference:
Y. Wu, E. Forest and D. S. Robin, Phys. Rev. E 68, 046502, 2003

SECOND ORDER APPROXIMATION OF THE MAP

K split as:

- $K_1 = p_z - \delta$
- $K_2 = -a_z$
- $K_3 = \left(\frac{(p_x - a_x)^2}{2(1+\delta)}\right)$
- $K_4 = \left(\frac{(p_y - a_y)^2}{2(1+\delta)}\right)$

The second order approximation of the Lie Map is:

$$\mathcal{M}_2(\Delta\sigma) = exp(: -\frac{\Delta\sigma}{2}(p_z - \delta) :) exp(: \frac{\Delta\sigma}{2} a_z :) exp(: -\int a_x\, dx :) exp(: -\frac{\Delta\sigma}{2} \frac{(p_x)^2}{2(1+\delta)} :)$$
$$exp(: \int a_x\, dx :) exp(: -\int a_y\, dy :) exp(: -\Delta\sigma \frac{(p_y)^2}{2(1+\delta)} :) exp(: \int a_y\, dy :) exp(: -\int a_x\, dx :)$$
$$exp(: -\frac{\Delta\sigma}{2} \left(\frac{(p_x)^2}{2(1+\delta)}\right) :) exp(: \int a_x\, dx :) exp(: \frac{\Delta\sigma}{2} a_z :) exp(: -\frac{\Delta\sigma}{2}(p_z - \delta) :)$$

Explicit dependence on z

using

$$exp(: -\Delta\sigma K_4 :) = exp(: -\Delta\sigma \left(\frac{(p_y - a_y)^2}{2(1+\delta)}\right) :)$$
$$= \exp\left(: -\int a_y\, dy :\right) exp\left(: -\Delta\sigma \frac{(p_y)^2}{2(1+\delta)} :\right) exp\left(: \int a_y\, dy :\right)$$

Reference:
Y. Wu, E. Forest and D. S. Robin, Phys. Rev. E 68, 046502, 2003

EXPLICIT TRANSFORMATION OF PHASE SPACE VARIABLES

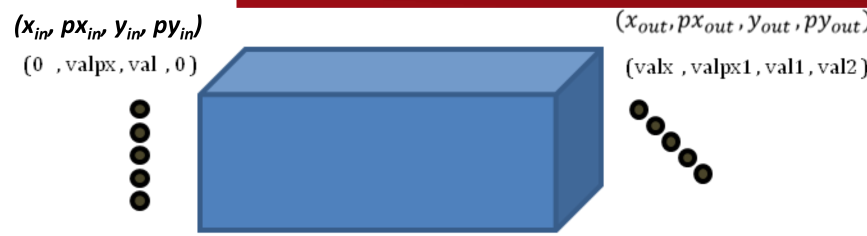
	K_1	K_2	K_3			K_4		
	$-\frac{\Delta\sigma}{2}(p_z-\delta)$	$\frac{\Delta\sigma}{2}a_z$	$-\int a_x dx$	$-\frac{\Delta\sigma}{2}\frac{(p_x)^2}{2(1+\delta)}$	$\int a_x dx$	$-\int a_y dy$	$-\Delta\sigma\frac{(p_y)^2}{2(1+\delta)}$	$\int a_y dy$
x				$+\frac{p_x\Delta\sigma}{2(1+\delta)}$				
p_x		$+\frac{\partial a_z}{\partial x}\frac{\Delta\sigma}{2}$	$-a_x$		$+a_x$	$-\int\frac{\partial a_y}{\partial x}dy$		$+\int\frac{\partial a_y}{\partial x}dy$
y							$+\frac{p_y\Delta\sigma}{(1+\delta)}$	
p_y		$+\frac{\partial a_z}{\partial y}\frac{\Delta\sigma}{2}$	$-\int\frac{\partial a_x}{\partial y}dx$		$+\int\frac{\partial a_x}{\partial y}dx$	$-a_y$		$+a_y$
l	$-\frac{\Delta\sigma}{2}$			$-\frac{(p_x)^2}{4(1+\delta)^2}\Delta\sigma$			$-\frac{(p_y)^2}{2(1+\delta)^2}\Delta\sigma$	
δ								
z	$+\frac{\Delta\sigma}{2}$							
p_z		$+\frac{\partial a_z}{\partial z}\frac{\Delta\sigma}{2}$	$-\int\frac{\partial a_x}{\partial z}dx$		$+\int\frac{\partial a_x}{\partial z}dx$	$-\int\frac{\partial a_y}{\partial z}dy$		$+\int\frac{\partial a_y}{\partial z}dy$

The second half of iterations for K_1, K_2 and K_3 are not reported in the table.

NON LINEAR FRINGE FIELD EFFECT

MODEL COMPARISON (1/2)

Tracking procedure:



Forest Hard Edge model*

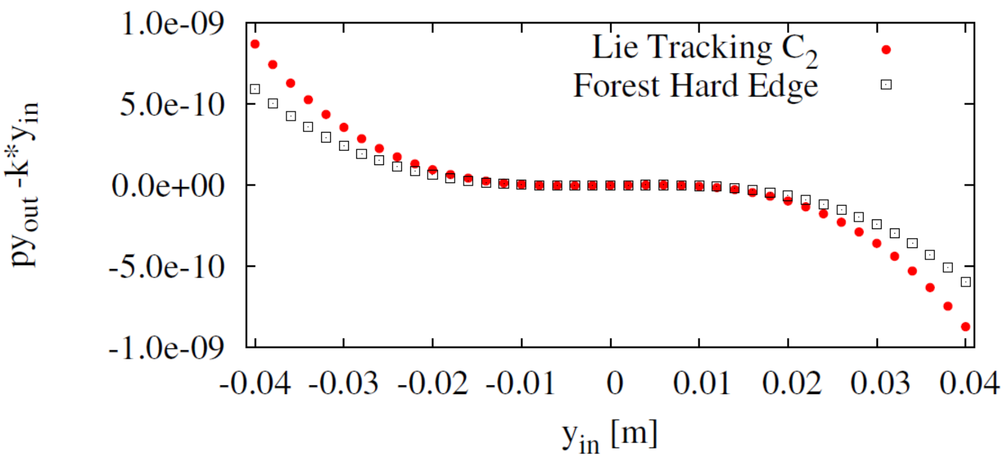
Rotation of -45°

Skew Hard edge kicks:

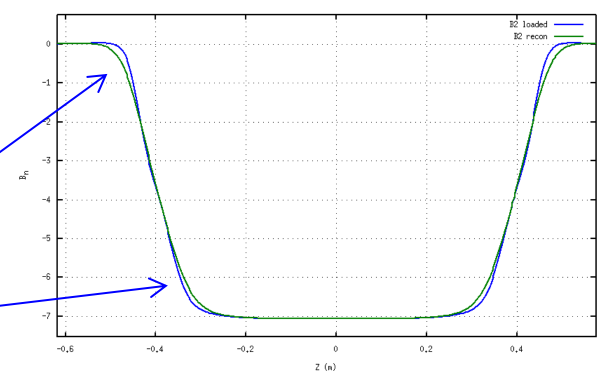
$$\Delta x = \frac{-k_0}{6}\frac{y^3}{1+\delta}$$

$$\Delta p_x = \frac{k_0}{6}\left[\frac{3p_y x^2}{1+\delta}\right]$$

Rotation of 45°



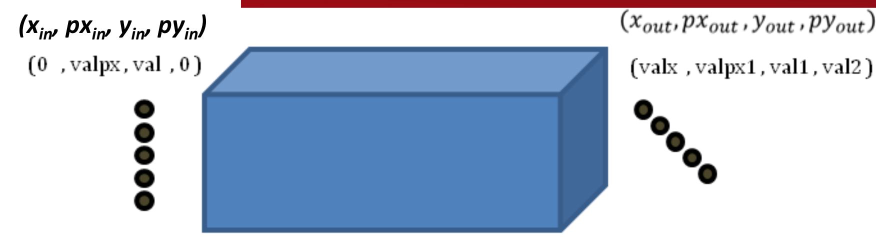
First order derivative of the generalized gradient is not enough to describe the fringe field of this quadrupole



*É. Forest and J. Milutinovic, Nuclear Instruments and Methods in Physics Research A269 (1988) 474-482

MODEL COMPARISON (2/2)

Tracking procedure:



4th order integrator

$$\frac{d^2\vec{r}}{ds^2} = \frac{1}{B\rho}\frac{d\vec{r}}{ds} \times \vec{B}$$

integration step

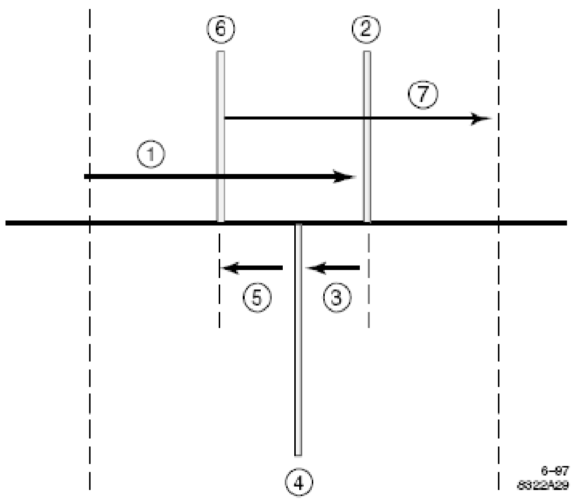
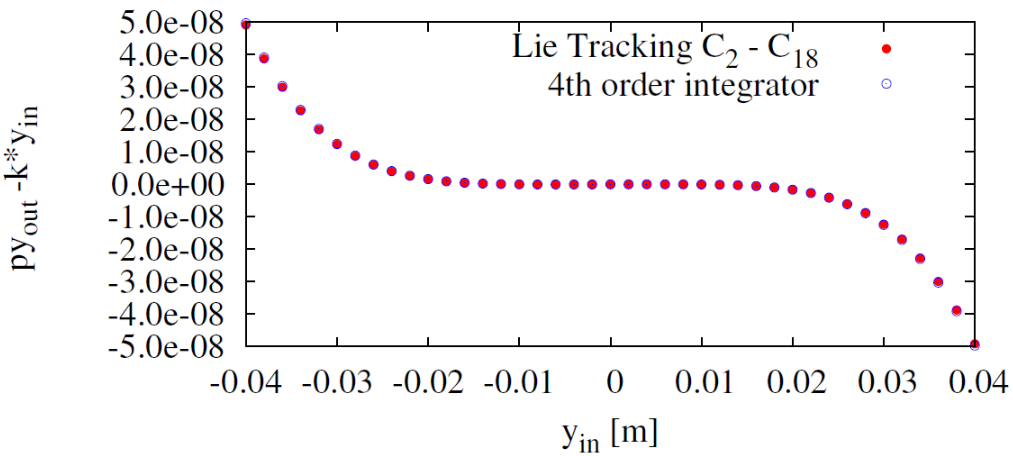


Figure 7.3: Seven steps in the 4-th order symplectic integration.

A. Chao Lectures

E. Forest and R. D. Ruth, Physica D 43, 105, 1990

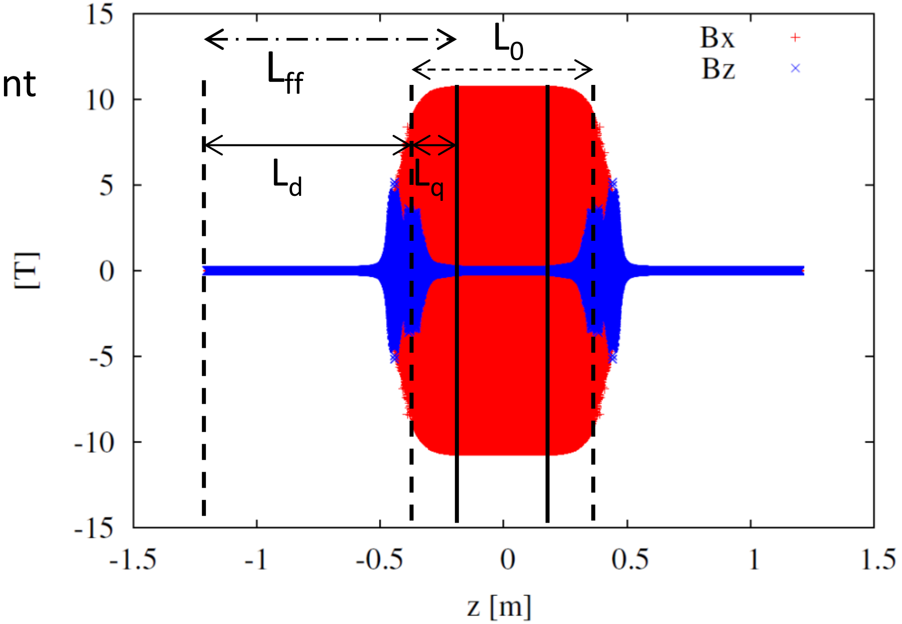


DISCUSSION

Pros:

I. Possibilities to control the field harmonics used in the simulations. Each field component can be switched on and off easily in the calculation of the generalized gradients.

II. Lie Tracking (***I(L$_{ff}$)***) of fringe field region only



$$D(-L_d)I(L_{ff})Q^{-1}(L_q)Q(L_0)Q^{-1}(L_q)I(L_{ff})D(-L_d)^*$$

Cons:

slow with respect to multipole kicks
(need 100-200 steps for each fringe field)

*E. Forest, Beam Dynamics: A New Attitude and Framework, harwood academic publishers, 1998

CONCLUSION

- The method to compute a transfer map of a z-dependent Hamiltonian using 3D magnetic field data has been implemented
- It has been validated with a 4th order symplectic integrator using directly the 3D magnetic field data in a single quadrupole
- The comparison with analytical leading order fringe field model by Forest-Milutinovic shows a discrepancy at large particle amplitudes due to the higher order derivatives needed to describe the fringe field shape

OUTLOOK

- Study the impact of realistic fringe field on the long term beam dynamics
  $\Rightarrow$ integration of the method in Sixtrack
- frequency map analysis (A. Wolski)
- improve the fitting of the 3D magnetic field map

REFERENCES

Y. Wu, E. Forest and D. S. Robin, Phys. Rev. E 68, 046502 (2003)

A. J. Dragt, www.physics.umd.edu/dsat

M. Venturini, A.J. Dragt, NIM A 427, 387 (1999)

C.E. Mitchell and A. J. Dragt, Phys. Rev. ST AB 13, 064001 (2010)

É. Forest and J. Milutinovic, Nucl. Instr. and Meth. A 269, 474 (1988)

E. Forest and R. D. Ruth, Physica D 43, 105 (1990)

E. Forest, “Beam Dynamics A New Attitude and Framework”, Harwood publisher

B. Dalena et al. TUPRO002, IPAC’14

SPARES

MONOMIAL REPRESENTATION OF THE VECTOR POTENTIAL

$$A_x = \sum_m \sum_l \sum_{p=0:2:m} \sum_{q=0}^{l} -\frac{1}{m} \frac{(-1)^l m!}{2^{2l} l!(l+m)!} \binom{m}{p} \binom{l}{q} \underline{C_{m,\alpha}^{[2l+1]}}(z) i^p x^{m-p+2l-2q+1} y^{p+2q}$$

$$A_y = \sum_m \sum_l \sum_{p=0:2:m} \sum_{q=0}^{l} -\frac{1}{m} \frac{(-1)^l m!}{2^{2l} l!(l+m)!} \binom{m}{p} \binom{l}{q} \underline{C_{m,\alpha}^{[2l+1]}}(z) i^p x^{m-p+2l-2q} y^{p+2q+1}$$

$$A_z = \sum_m \sum_l \sum_{p=0:2:m} \sum_{q=0}^{l} \frac{1}{m} \frac{(-1)^l m!(2l+m)}{2^{2l} l!(l+m)!} \binom{m}{p} \binom{l}{q} \underline{C_{m,\alpha}^{[2l]}}(z) i^p x^{m-p+2l-2q} y^{p+2q}$$

generalized gradients

with $[(x+iy)^m] = \sum_{p=0}^{m} \binom{m}{p} x^{m-p}(iy)^p = \sum_{p=0:2:m} \binom{m}{p} x^{m-p}(iy)^p + \sum_{p=1:2:m} \binom{m}{p} x^{m-p}(iy)^p$

$$(x^2+y^2)^l = \sum_{q=0}^{l} \binom{l}{q} x^{2l-2q} y^{2q}$$

References:
A. J. Dragt, www.physics.umd.edu/dsat