Cyclotrons for Ions

W.Joho
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The mass unit of a nucleon is defined with: $m_0 = m(C_{12}^{6+})/12$

mass m and charge q with dimensionless quantities:

$m \equiv A\, m_0$, $q \equiv Z\, e$ (for protons: A=1.007)

Rest energy E_0 of a nucleon : $E_0 = m_0 c^2 = 931.5$ MeV (938.27 MeV for protons)

kinetic energy of an ion : (E/A)= energy/nucleon

For numerical calculations in cyclotrons only **2 basic numbers** are needed:

$c \equiv 299.792458$ m/μs

$U_0 \equiv (E_0/e) = 931.5$ MV (=> the unit charge e is not needed!)

It is helpful to use the following derived constants:

$$(1)\qquad \left(\frac{c}{2\pi}\right) = 47.71\,\mathrm{MHz}\cdot\mathrm{m}$$

$$(2)\qquad \left(\frac{e}{2\pi\, m_0}\right) = \frac{c^2}{2\pi\, U_0} = 15.356\,\mathrm{MHz/T}\quad \text{(reference for cyclotron frequency)}$$

$$(3)\qquad (B\rho)_0 \equiv \left(\frac{m_0 c}{e}\right) \equiv \frac{U_0}{c} = 3.107\,\mathrm{Tm}\quad \text{(reference for magnetic rigidity } B\rho\text{)}$$

$$(4)\qquad \xi \equiv \frac{e^2}{2m_0} \equiv e\frac{c^2}{2U_0} \equiv \frac{E_o}{2(B\rho)_0^2} = 48.24\,\frac{\mathrm{MeV}}{(\mathrm{Tm})^2}\quad \text{(reference for kinetic energy)}$$

$$\text{note}:\quad (1)/(2) = (3)\qquad (47.71/15.356 = 3.107)$$

$$(2)\times\pi = (4)\qquad (15.356\times\pi = 48.24)$$

For protons the corresponding constants are:

$$15.245\,\mathrm{MHz/T},\qquad 3.130\,\mathrm{Tm},\qquad 47.89\,\mathrm{MeV/(Tm)^2}$$

use dimensionless units for

velocity v, total Energy $E_{tot} = \gamma\, E_0$ and momentum p :

(5) $$\beta \equiv \frac{v}{c} \qquad \text{(can be parametrized as } \beta = \sin\varphi\text{)}$$

(6) $$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad (\gamma = \frac{1}{\cos\varphi})$$

(7) $$\tilde{p} \equiv \beta\gamma = \frac{p}{mc}, \qquad (\tilde{p}^2 = \gamma^2 - 1) \qquad (\tilde{p} = \tan\varphi)$$

The two basic equations for Cyclotrons are :

(8) $$p = q(B\rho)\ , \qquad (B\rho) = \text{magnetic rigidity}$$

(9) $$\omega_0 = (\frac{q}{m\gamma})\ \langle B \rangle\ , \qquad \text{Larmorfrequency}$$

For each energy exists an equilibrium orbit with circumference $L \equiv 2\pi R$ and revolution frequency ω_0. This defines the average radius R, the average magnetic field $< B > \equiv B_0(R)$ and the magnetic rigidity $(B\rho)$:

$$(10) \quad B_0(R) \equiv \frac{1}{2\pi R}\int B_z(s)ds$$

$$(11) \quad p = q(B\rho)$$

$$(12) \quad (B\rho) = B_0(R)\cdot R$$

since $\omega_0 = v/R$, $(v \equiv \beta c)$

=> <u>scaling laws</u> for cyclotrons :

$$13) \quad R = \beta R_\infty , \quad R_\infty \equiv \frac{c}{\omega_0}$$

$$14) \quad B_0(R) = B_{center}\,\gamma(R)$$

using reference value $(B\rho)_0$ gives the formula

connecting $(B\rho)$ and (E/A) :

$$(15) \qquad \tilde{p} = \left(\frac{Z}{A}\right)\frac{(B\rho)}{(B\rho)_0}$$

$$(16) \qquad \gamma = \sqrt{1+\tilde{p}^2}$$

$$(17) \qquad (E/A) = (\gamma - 1)\,E_0$$

The K_B - value of a cyclotron is given by the maximum

bending power $(B\rho)$ of an ion before extraction :

$$(18) \qquad (B\rho)_{max} = B_0(R_{max})\cdot R_{max}$$

$$(19) \qquad K_B \equiv \xi(B\rho)^2_{max}\,, \qquad \xi = 48.24\frac{MeV}{(Tm)^2}$$

for nonrelativistic energies :

$$(20) \qquad E = \frac{p^2}{2m}$$

$$(21) \qquad (E/A)_{nonrel} = \frac{1}{2}\left(\frac{q}{m}\right)^2 (B\rho)^2$$

and for the maximum energy :

$$(22) \qquad (\mathrm{E/A})_{\mathrm{max}} = \varepsilon \equiv \left(\frac{Z}{A}\right)^2 K_B$$

the relativistically correct maximum energy is :

$$(23) \qquad (\mathrm{E/A}) = (\gamma - 1)\,\mathrm{E}_0, \qquad \gamma = \sqrt{1 + \frac{2\varepsilon}{E_0}}$$

series expansion for low energies :

$$(24) \qquad (E/A) = \varepsilon\left[1 - \frac{1}{2}\left(\frac{\varepsilon}{E_0}\right) + \frac{1}{2}\left(\frac{\varepsilon}{E_0}\right)^2 - \ldots\right]$$

inverse formula for a given energy E/A :

$$(25) \qquad \gamma = 1 + \frac{(E/A)}{E_0}, \qquad \tilde{p} = \sqrt{\gamma^2 - 1}, \qquad \beta = \frac{\tilde{p}}{\gamma}$$

$$(26) \qquad \varepsilon = \frac{E_0}{2}\tilde{p}^2 = (E/A)\left[1 + \frac{(E/A)}{2E_0}\right]$$

$$(27) \qquad K_B = \varepsilon\left(\frac{A}{Z}\right)^2$$

$$(28) \qquad (B\rho) = \frac{A}{Z}\tilde{p}(B\rho)_0 = \sqrt{\frac{K_B}{\xi}} = \sqrt{\frac{K_B}{48.24MeV}}\,[Tm]$$

Focusing Limit K_F :

For light ions with a high (Z/A) ratio, the vertical focusing becomes critical and requires large spiral angles for the magnet sectors.

There exists an energy limit, which depends linearly on (Z/A) and can be lower than the bending limit.

Formula (22) for the non relativistic energy limit ε is then replaced by :

$$(29) \qquad (E/A)_{max} = \varepsilon_F \equiv \left(\frac{Z}{A}\right) K_F$$

and the relativistically correct maximum energy is given as in (23)

$$(30) \qquad (E/A) = (\gamma - 1)\, E_0, \qquad \gamma = \sqrt{1 + \frac{2\varepsilon_F}{E_0}}$$

cyclotron frequency ω_0 :

(31) $$\omega_0 \equiv 2\pi\nu_0 = \frac{q}{m}\frac{B_0(R)}{\gamma(R)}$$

(32) $$\nu_0 = \frac{Z}{A}\left(\frac{e}{2\pi m_0}\right)\frac{B_0(R)}{\gamma(R)} = \frac{Z}{A}\frac{B_0(R)}{\gamma(R)} 15.356\, MHz/T$$

the RF-frequency can be a <u>harmonic</u> h of the revolution frequency ν_0 :

(33) $$\nu_{RF} \equiv h\nu_0$$

the reference value R_∞ for the scaling of the radius is then :

(34) $$R_\infty \equiv \frac{c}{\omega_0} = \frac{(c/2\pi)\cdot h}{\nu_{RF}} = h\frac{47.71\,\mathrm{MHz}\cdot\mathrm{m}}{\nu_{RF}}$$

$$\mathrm{R} = \beta\,\mathrm{R}_\infty$$

to have low losses at extraction it is important to have a large turn separation dR/dn

$$(35) \qquad \frac{dR}{dn} = \frac{\gamma}{\gamma+1} R \frac{E_G}{(E/A)} \frac{f^2}{\nu_r^2}, \qquad E_G = \frac{Z}{A} e\hat{V}$$

$$\hat{V} = \text{peak voltage/turn}, \quad \nu_r = \text{radial tune}, \quad f = \frac{\nu_r(isochr.)}{\gamma} \quad (\approx 1.2 \text{ in ring cyclotrons})$$

This shows the advantage of Ring Cyclotrons with a large radius R:

- space for many high voltage cavities => large $\hat{V}$
- the turn separation scales with R

there are two further effects, which can enhance dR/dn:

1. ν_r drops in the radial fringe field region

   => this drop is fast in a warm magnet with a narrow gap (tolerable phase slip)

2. If ν_r is around 1.4-1.6 at extraction, one can double dR/dn

   between the last 2 turns with excentric injection

   and preserving the induced amplitude with single turn extraction

Example :

RIKEN Superconducting Ring Cyclotron SRC :

Uranium : A = 238, Z = 86, E/A = 345 MeV/N

$$(25) \quad \gamma = 1 + \frac{(E/A)}{E_0} = 1.37\,, \quad \tilde{p} = 0.94\,, \quad \beta = 0.68$$

$$(26) \quad \varepsilon = \frac{E_0}{2}\tilde{p}^2 = (E/A)\left[1 + \frac{(E/A)}{2E_0}\right] = 1.185\,(E/A) = 409\,MeV/N$$

$$(27) \quad K_B = \varepsilon\left(\frac{A}{Z}\right)^2 = 409\left(\frac{238}{86}\right)^2 = 3'130\,MeV$$

$$(28) \quad (B\rho) = \sqrt{\frac{K_B}{48MeV}} = \sqrt{\frac{3130}{48}} = 8.06\,Tm$$

Magnet weight W of a cyclotron: => Plot by W.Joho
("Modern Trends in Cyclotrons", CERN Accelerator School, Aarhus Denmark 1986):

$$W = W_1 (K_B / 1000 MeV)^{3/2} = W_2 (B\rho)^3 \qquad (36)$$

warm magnets: $W_1 \approx$ 5'000 t , $W_2 \approx$ 50 t

But PSI Ring Cyclotron with 590 MeV protons, K_B=775 MeV, Bρ=4Tm,

B_{max}=2.1T weighs only 2'000 t; => W_2 = 31 t .

sc-magnets: only a few operating cyclotrons in 1986. These magnets were about a factor 12 lighter than warm magnets => W_1=380 t , W_2=4 t

Today more emphasis on low stray fields => more iron in the yoke.

Examples:

1) COMET Cyclotron PSI, 250 MeV protons, Bρ=2.4Tm , K_B=283 MeV, B_{max}=5T

W=90 t , => W_2 = 6 t

2) SRC RIKEN, 345 MeV/N , Bρ=8.06 Tm , B_{max}=4T , W=8'300 t , => W_2 = 16 t

Thus here the weight advantage over the warm PSI Ringcyclotron is only a factor of 2.