Fun with Formulas !

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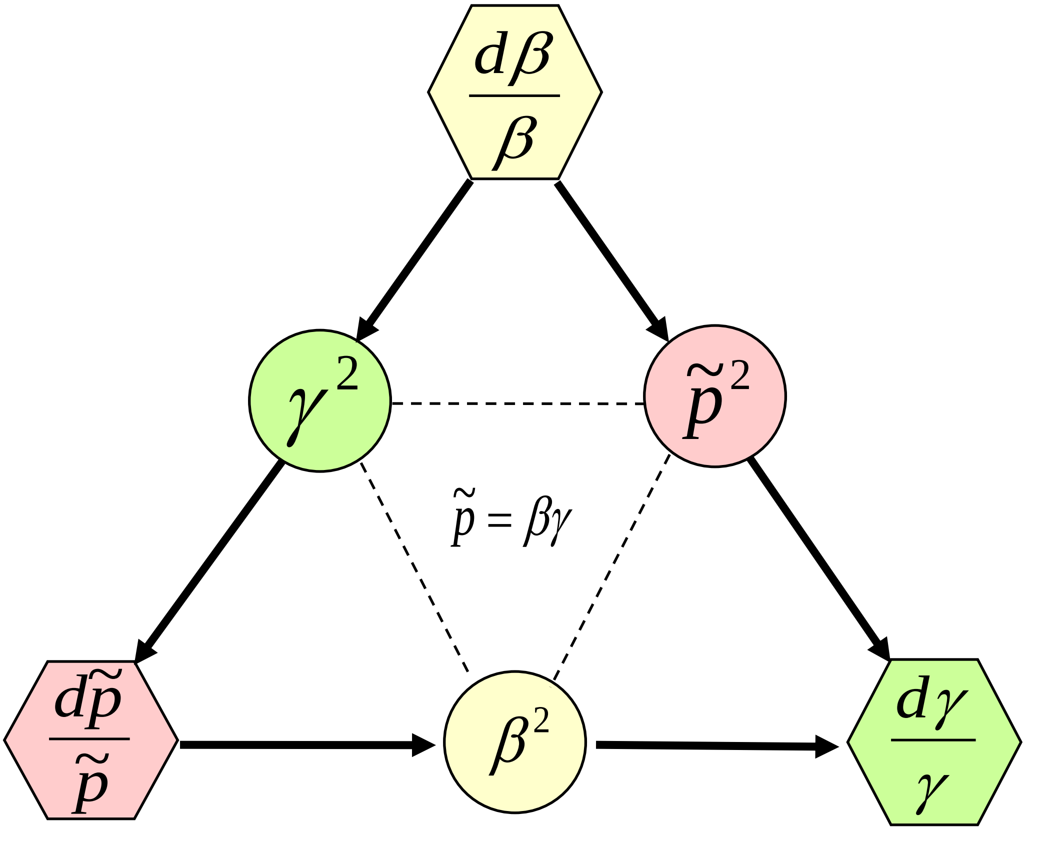

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Introduction

Formulas can be fun. They often can be made to look simple, transparent and thus beautiful (in the spirit of Einstein and Chandrasekhar).

This can be achieved with some simple rules and a few tricks of the trade.

In **Part 1** I present a sample of some simple formulas and tricks of the trade, collected during my career as a physicist.

This material was presented (but not published)

at a seminar talk given at the CERN Accelerator School on

‘’Synchrotron Radiation and Free Electron Lasers’’,

Brunnen, Switzerland, 2-9 July 2003 and upgraded since

several times.

Part 2 contains some general topics like "how to win money with unfair bets" , "the beauty of mathematic" and other curiosities connected with mathematics.

A pdf-file of all this can be found on the WEB with

www.google.ch: „Werner Joho PSI Fun“

Part 1
professional topics

- philosophy for formulas
- new interpretation of Ohm‘s law
- logarithmic derivatives
- the relativistic equations of Einstein , his triangle
- the magic triangle of relativistic logarithmic derivatives
- simple representation of phase space ellipses
- binomial curves everywhere;

  approximation of various functions: beam profiles,

  magnet fringe field ,flux of synchrotron radiation etc.
- **A**lternative **G**radient Focusing, constructed by hand
- design of beautiful tables with a Hamiltonian

philosophy for formulas

- simplify formulas, they should look „beautiful“
- formula should indicate the proper dimensions
- **use units of 1'000** (cm should not exist in formulas!)
- choose right scales for plots (e.g. logarithmic)

example:

$c = 3 \cdot 10^8$ m/s ?? better is:

$c =$ **0.3**$\cdot 10^9$ m/s or **300** m/μs or 0.3 mm/ps !!

for comparison of electric forces: $q \cdot \mathcal{E}$ (kV/mm)

with magnetic Lorentz force: $q \cdot v \cdot B = q \cdot \beta \cdot c \cdot B$

to deflect a particle with charge q:

$c = 300$ (kV/mm)/T !!

i.e. at v=c : 1T $\Leftrightarrow$ 300 kV/mm (=> out of reach!)

$\mu_0 = 4\pi\ 10^{-7}$ Vs/Am ?? better is:

$\mu_0 = 0.4\pi\ \mu$H/m $= 0.4\pi$ T/(kA/mm)

how to avoid akward numbers in electrodynamics

electron mass: $m = 9.11 \cdot 10^{-31}$ kg (?) => forget it !

use $mc^2 \equiv eU_o$, $U_o = .511$ MV

$\varepsilon_o = 8.854 \cdot 10^{-12}$ As/Vm (?) => forget it !

use $\mu_o \varepsilon_o = 1/c^2$ with $\mu_o = 0.4\,\pi\,\mu$H/m

introduce impedance Z_o :

$$\boxed{Z_o \equiv \frac{1}{4\pi\varepsilon_0 c} = \frac{\mu_0}{4\pi} c = 30\ \Omega} \qquad (\equiv 29.9792458\ \Omega)$$

Alfven current I_A for electrons (used in space charge calculations):

$I_A = 4\pi\varepsilon_o mc^3/e$ (?) => forget it ! (similarly for "perveance")

use instead "Ohms-law" : $I_A = U_o/Z_o = 511$ kV/30Ω = 17 kA

charged particle in a magnetic field B_o

=> Larmor-frequency $\boxed{\omega_o = \frac{e\,B_0}{m\,\gamma}}$

electron: $\frac{e}{m} = 1.76 \cdot 10^{11}$ C/kg (?) => forget it !

use $\frac{e}{2\pi\, m} = \frac{c^2}{2\pi\, U_0} =$ **28 GHz/T** (**15.25 MHz/T** for protons)

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use logarithmic derivatives !

Example 1: $$y = x^n$$

For a plot: take logarithmic scales

For the derivatives we have the relation

$$\frac{dy}{y} = n\frac{dx}{x}$$

=> 1% change in x gives n % change in y

e.g. a 1% change in radius gives

2% change in the surface and

3% change in the volume of a sphere

Example 2:

normalized relativistic parameter for

momentum, velocity and total energy:

$$\tilde{p} \equiv \beta\gamma$$

$$\frac{d\tilde{p}}{\tilde{p}} = \frac{d\beta}{\beta} + \frac{d\gamma}{\gamma}$$

Einstein triangle

in relativistic equations use normalized quantities (dimensionless) for velocity, energy and momentum on a **democratic** basis!

=> Einstein triangle and magic triangles for logarithmic derivatives

velocity: $v = \beta c$

total energy: $E = \gamma E_0$, $E_0 = mc^2 = eU_0$ (0.511 MeV for electron)

$\gamma = (1-\beta^2)^{-1/2}$,

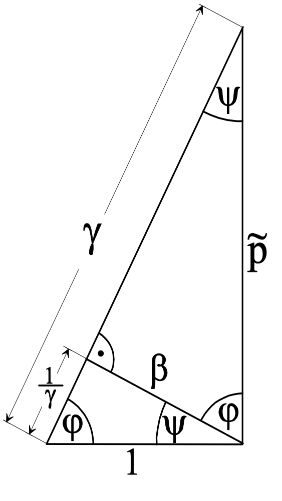
momentum: $p = \tilde{p}\ E_0/c = \tilde{p}\ mc$ $\boxed{\tilde{p} = \beta\gamma}$

"Pythagoras"- connection: $E^2 = E_0^2 + (pc)^2$

give the Einstein triangle and derivatives

$$\boxed{1 + \tilde{p}^2 = \gamma^2} \qquad \tilde{p}\, d\tilde{p} = \gamma\, d\gamma, \quad d\gamma = \beta\, d\tilde{p}$$

$$\boxed{\beta^2 + \frac{1}{\gamma^2} = 1} \qquad d\gamma = \beta\gamma^3 d\beta$$



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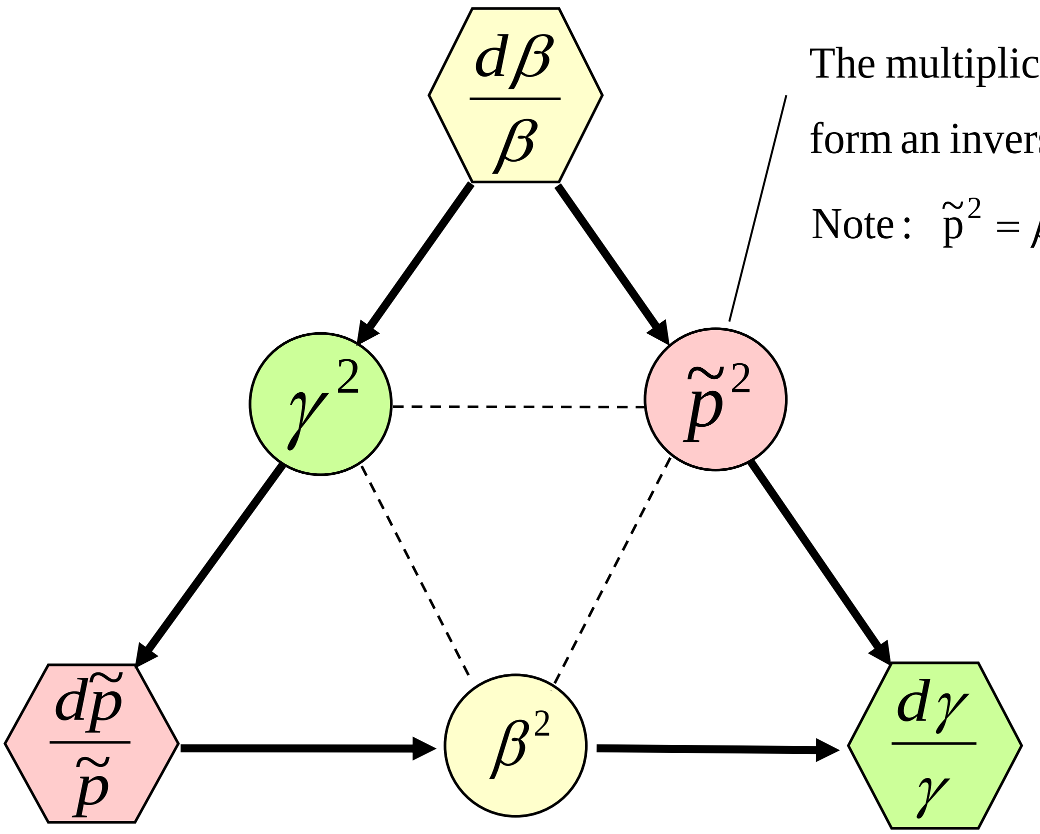
„Magic Triangles" (W.Joho) with logarithmic derivatives of relativistic parameter $\beta, \gamma, \tilde{p} \equiv \beta\gamma$

$$\boxed{\frac{d\tilde{p}}{\tilde{p}} = \frac{d\beta}{\beta} + \frac{d\gamma}{\gamma}}$$

$$\frac{d\tilde{p}}{\tilde{p}} = \gamma^2 \frac{d\beta}{\beta}, \quad \frac{d\gamma}{\gamma} = \tilde{p}^2 \frac{d\beta}{\beta}, \quad \frac{d\gamma}{\gamma} = \beta^2 \frac{d\tilde{p}}{\tilde{p}}$$

$\beta, \gamma, \tilde{p}$ are treated equally => democracy!



The multiplication factors form an inverse triangle

Note: $\tilde{p}^2 = \beta^2\gamma^2$

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„Magic Triangles” with logarithmic derivatives
for trigonometric functions
s≡sinφ, c≡cosφ, t≡tanφ

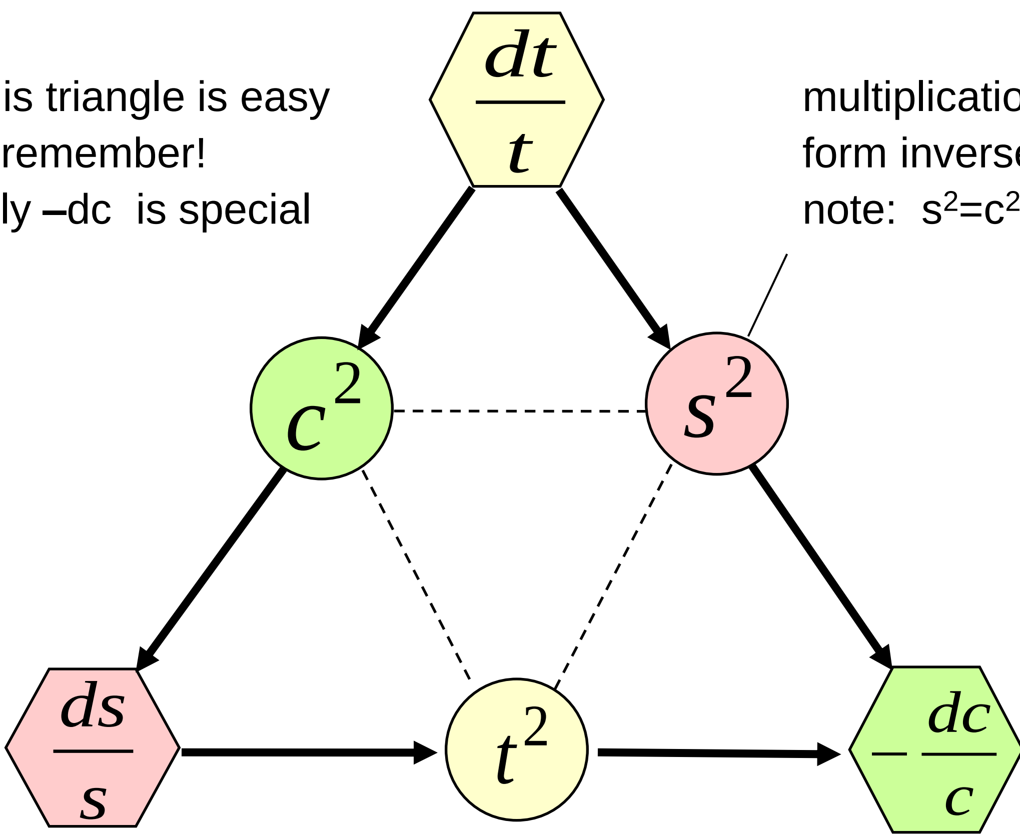
$$s^2 + c^2 = 1, \qquad t = \frac{s}{c}$$

$$\frac{ds}{s} = c^2 \frac{dt}{t}, \qquad -\frac{dc}{c} = s^2 \frac{dt}{t}, \qquad -\frac{dc}{c} = t^2 \frac{ds}{s}$$

s, c, t are treated equally => democracy!

This triangle is easy to remember!
only –dc is special

multiplication factors form inverse triangle;
note: $s^2 = c^2 \cdot t^2$



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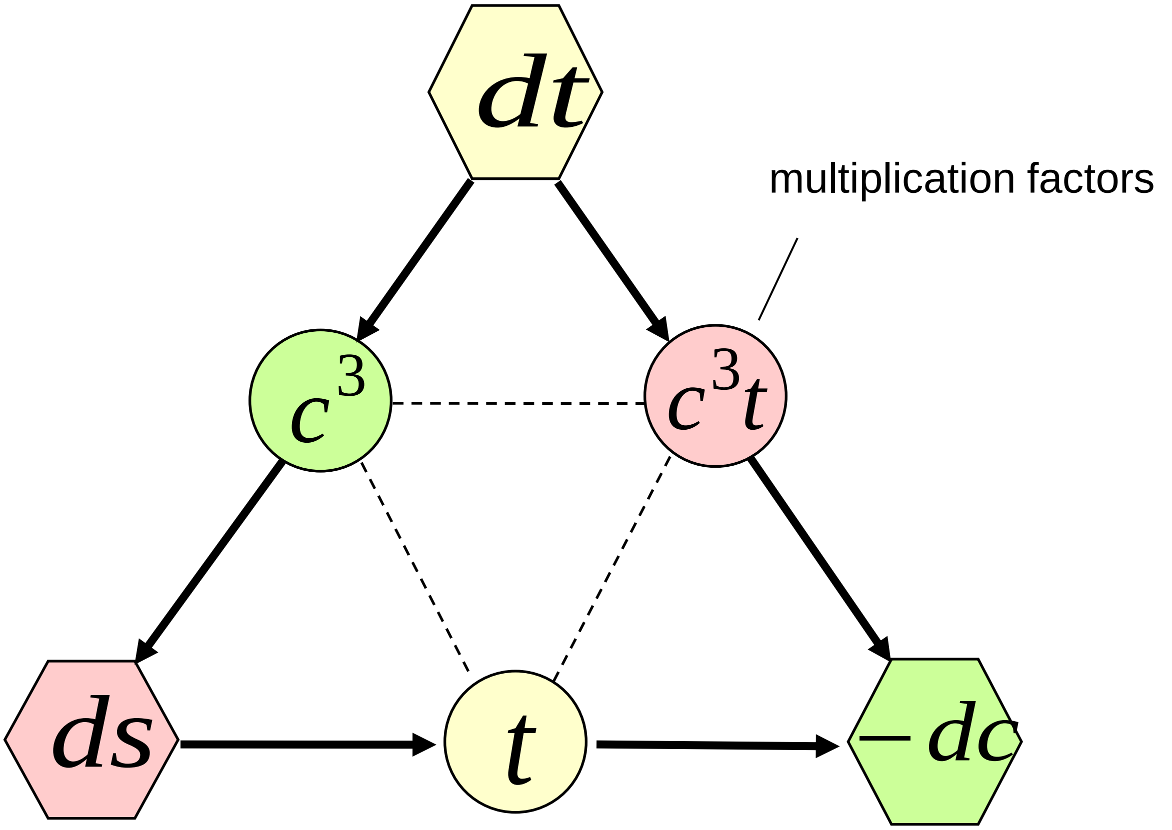
„Magic Triangles" with derivatives
for trigonometric functions
s≡sinφ, c≡cosφ, t≡tanφ

$$s^2 + c^2 = 1, \quad t = \frac{s}{c}$$

from the logarithmic derivatives it is easy to
get the relation between the normal derivatives:

$$ds = c^3 dt, \quad -dc = t\,ds\,, \quad -dc = t\,c^3 dt$$



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trigonometric functions for relativistic formula !

$$\gamma = \frac{1}{\sqrt{(1-\beta^2)}} \quad , \quad (\beta = \frac{v}{c})$$

$$if \quad \beta \equiv \sin\varphi, \quad then \quad \gamma = \frac{1}{\cos\varphi}$$

$$\tilde{p} \equiv \beta\gamma = \tan\varphi \qquad (\text{"momentum"})$$

the following table gives some easy reference values for

quick estimates or for calculations with a **pocket calculator**!

(with use of Pythagoras triangle with sides 3,4,5)

φ	**β = sinφ**	cosφ	**γ= 1/cosφ**	**p = βγ = tanφ**
30^0	0.5	0.87	1.15	0.58
36.9^0	**0.6**	**0.8**	1.25	0.75
45^0	0.71	0.71	1.41	1
53.1^0	**0.8**	**0.6**	1.67	1.33
60^0	0.87	0.5	2	1.71

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The relativity ellipse

$$\beta \equiv \frac{\mathrm{v}}{c} \equiv \sin\chi \qquad \text{(velocity)}$$

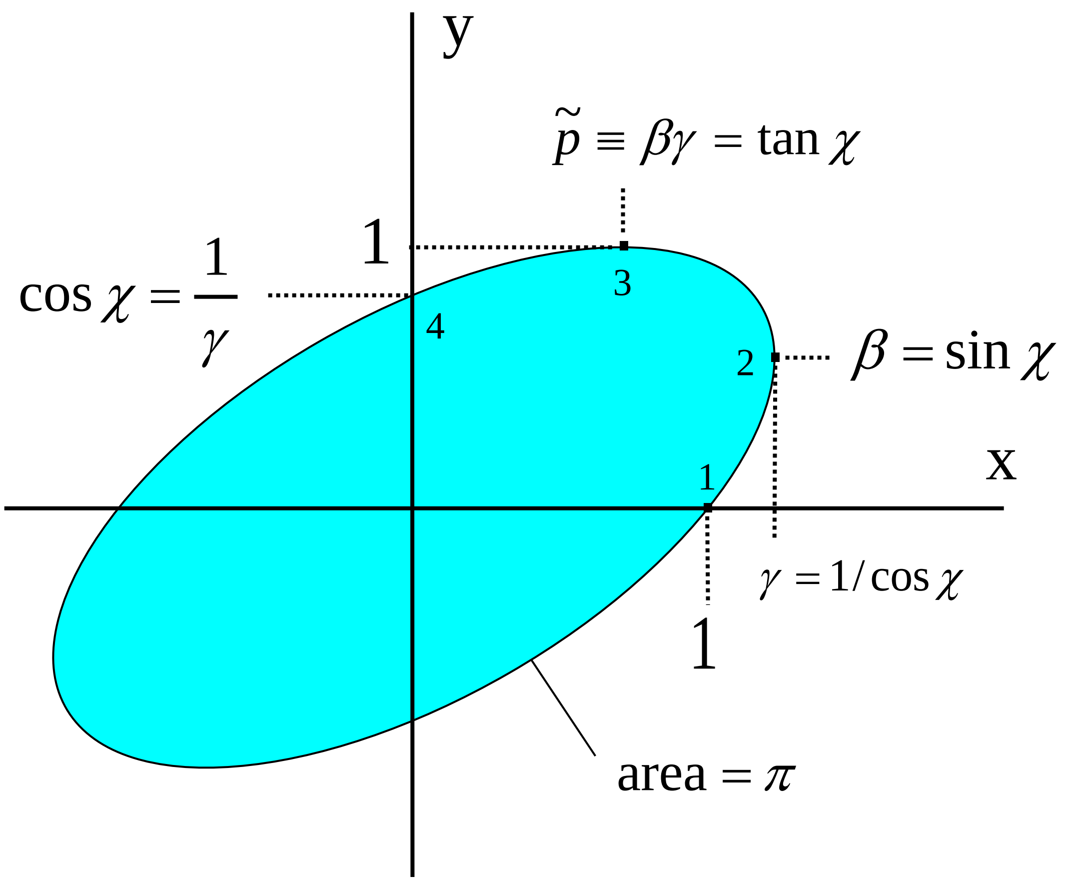
$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\cos\chi} \qquad \text{(total energy)}$$

$$\tilde{p} \equiv \beta\gamma = \tan\chi \qquad \text{(momentum)}, \qquad \gamma^2 = 1 + \tilde{p}^2$$

$$\boxed{\begin{aligned} x &= \gamma\cos\varphi \\ y &= \sin(\varphi+\chi) \end{aligned}}$$

$$\boxed{\begin{aligned} &0 \le \varphi \le 2\pi \\ &= \text{''}\textit{running parameter}\text{''} \end{aligned}}$$

$$(\text{or } x^2 - 2\tilde{p}\,xy + \gamma y^2 = 1)$$



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highly relativistic case

$\gamma >> 1$, $\beta \approx 1$ => use angle ψ

$\beta \equiv \cos\psi \approx 1 - \psi^2 / 2$

$1/\gamma \equiv \sin\psi \approx \psi$

$\tilde{p} \equiv \beta\gamma = 1/\tan\psi \approx 1/\psi$

$$\boxed{1-\beta \approx \frac{1}{2\gamma^2}}$$

a) race to the moon over distance L between electron and photon:

electron "looses" by

$\delta L = (1 - \beta)\, L \approx \frac{1}{2\gamma^2} L$

$$\boxed{\delta L \approx \frac{50m}{E^2[GeV]}}$$

SLS:	2.4 GeV,	δL = 8 m
ESRF:	6 GeV,	δL = 1.4 m
fastest electron at LEP II:	100 GeV,	δL = 5 mm

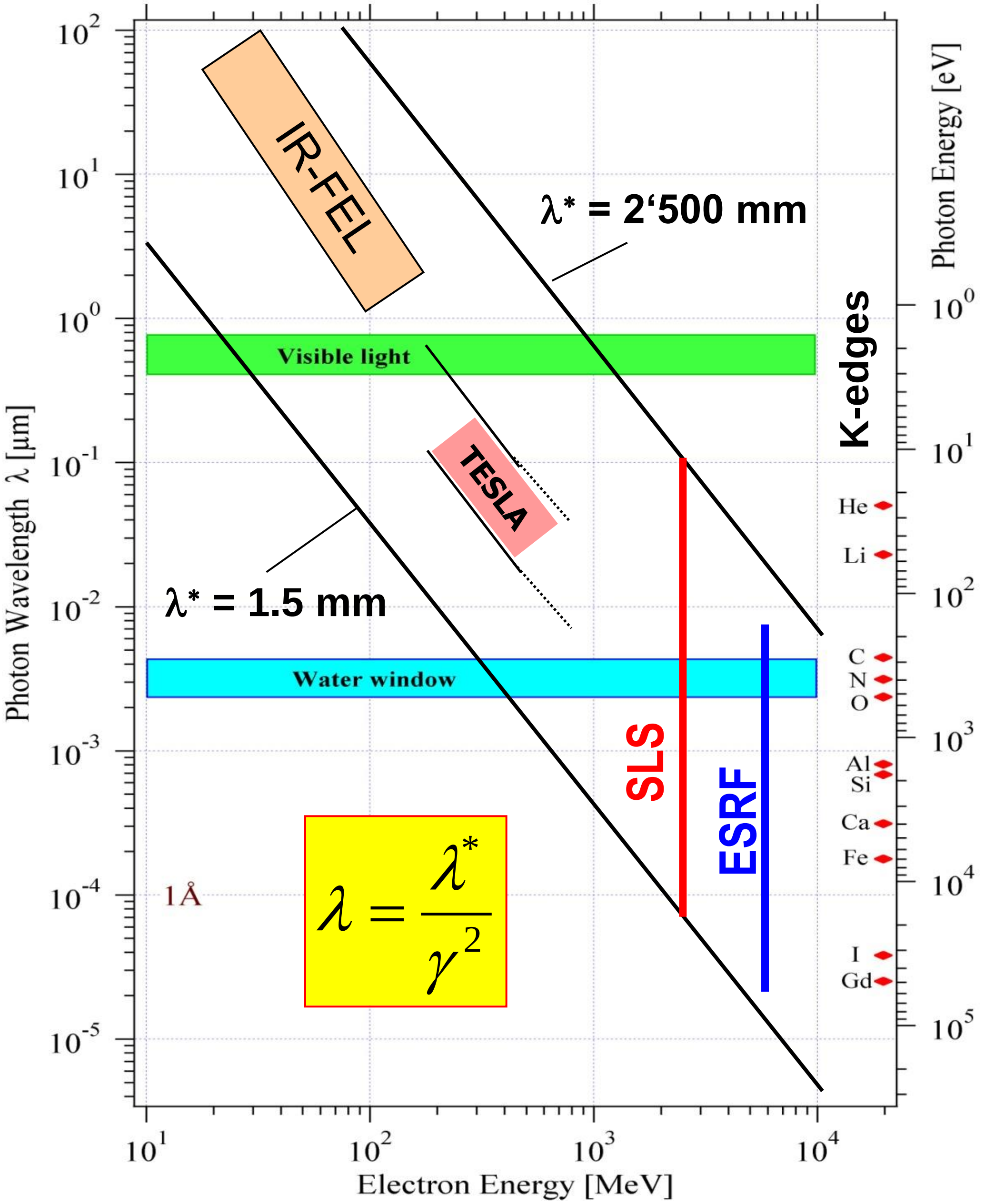
b) race over one undulator period λ_u : if electron is just one or n wavelengths λ behind photon (slippage) => positive interference

$$\boxed{\delta L = n\lambda = \frac{1}{2\gamma^2} \lambda_u \underbrace{(1 + K^2/2)}_{\text{detour due to slalom in B-field !}}}$$

$K = 0.0934\, B[T]\, \lambda_u\, [mm]$

Undulator Radiation

produced by an electron beam of energy E = γmc^2 (W.Joho, L.Rivkin 1993)



$$\lambda^* = \frac{\lambda_u}{2n}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right) \quad \text{(propertyof undulator)}$$

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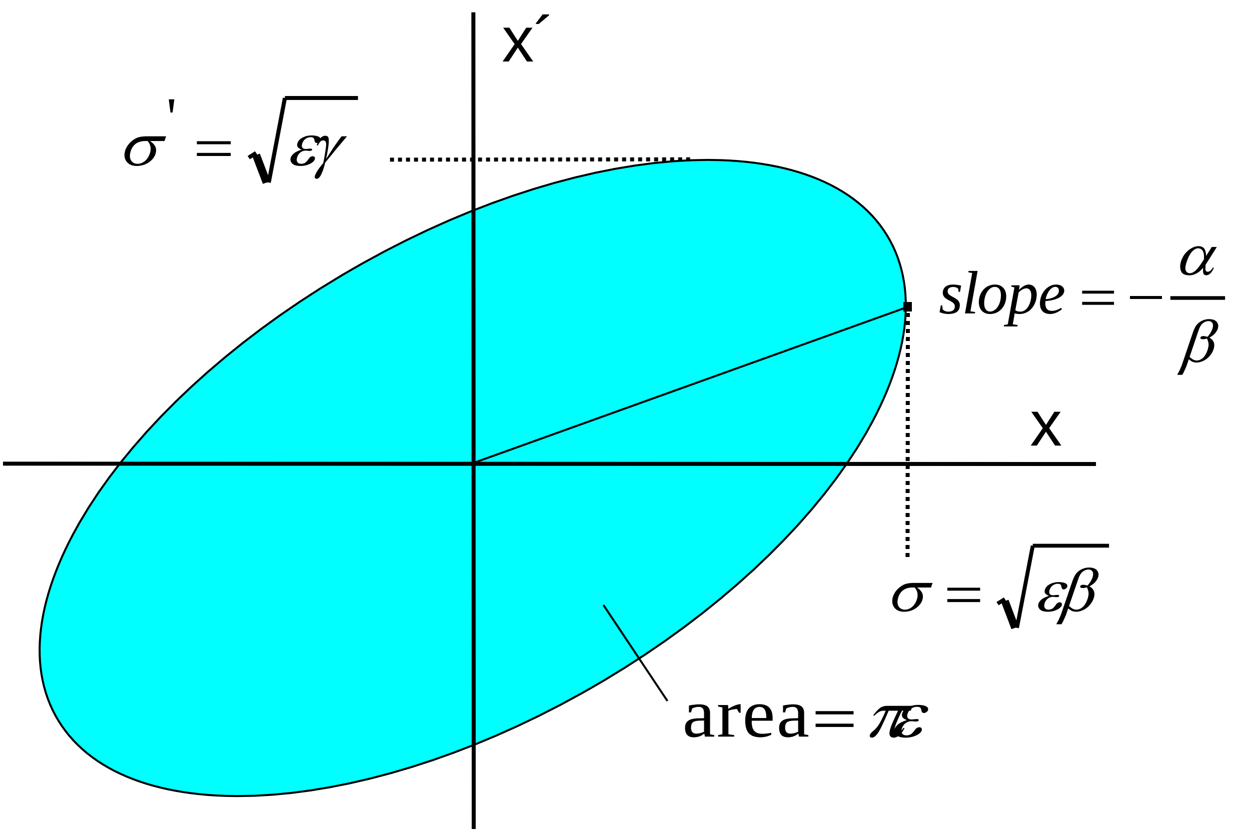
Courant-Snyder representation of the rms beam ellipse in phase space (x, x‘)

$$\boxed{\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2}$$

$\varepsilon^2 = \langle x^2\rangle\langle x'^2\rangle - \langle x\, x'\rangle^2$, (ε=emittance)

$\sigma^2 \equiv \langle x^2\rangle$, $\sigma'^2 \equiv \langle x'^2\rangle$

$\beta = \sigma^2/\varepsilon$, $\gamma = \sigma'^2/\varepsilon$, $\alpha = -\langle xx'\rangle/\varepsilon$, $1+\alpha^2 = \beta\gamma$



not easy to plot and akward to remember !

The **parametric representation** of the rms beam ellipse in phase space (x, x')

$x = \sigma \cos \varphi$

$x' = \sigma' \sin(\varphi + \chi)$

$0 \leq \varphi \leq 2\pi$ = "running parameter"

…but what is phase shift χ ?

$\sin \chi \equiv r_{12} \equiv \frac{<xx'>}{\sigma\sigma'}$ = correlation parameter

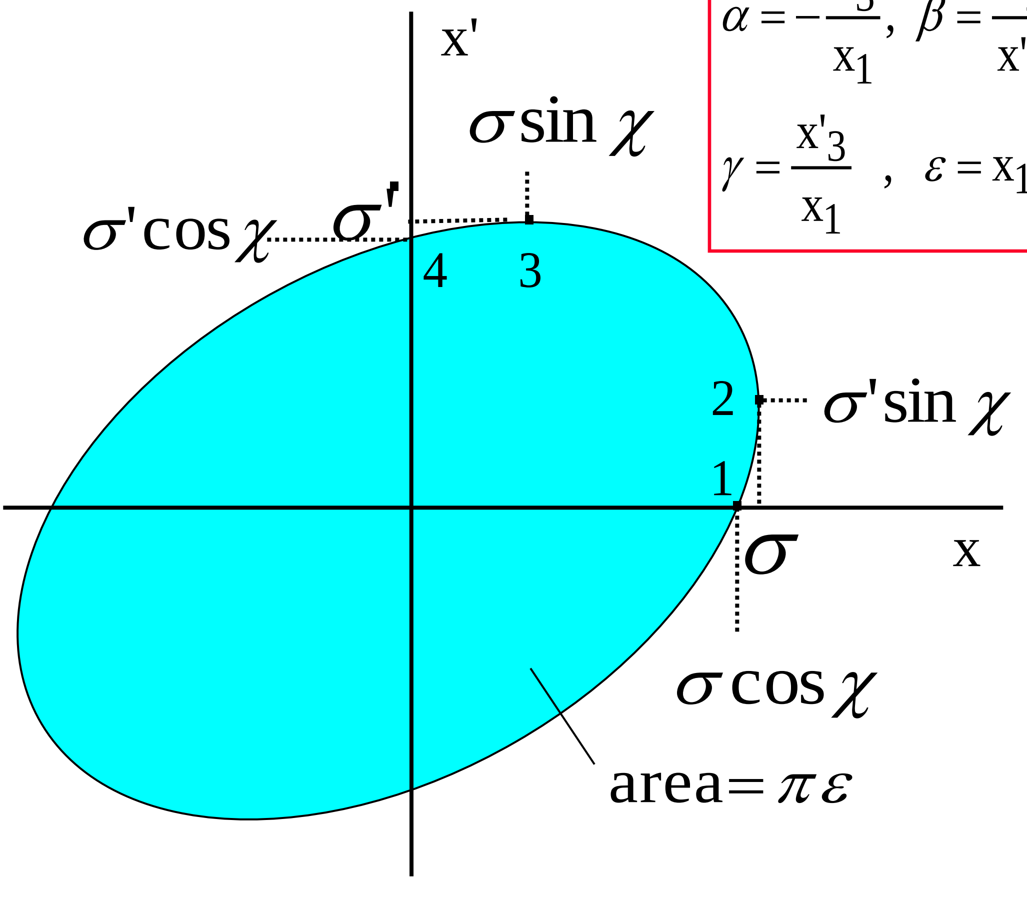
this representation of the rms-ellipse is easy to remember

and easy to plot! ($\chi = 0$ gives a circle)

connection with Courant-Snyder: **$\alpha = -\tan \chi$**

$$\alpha = -\frac{x_3}{x_1}, \quad \beta = \frac{x_2}{x'_4}$$

$$\gamma = \frac{x'_3}{x_1}, \quad \varepsilon = x_1 \, x'_3 = x_2 \, x'_4$$



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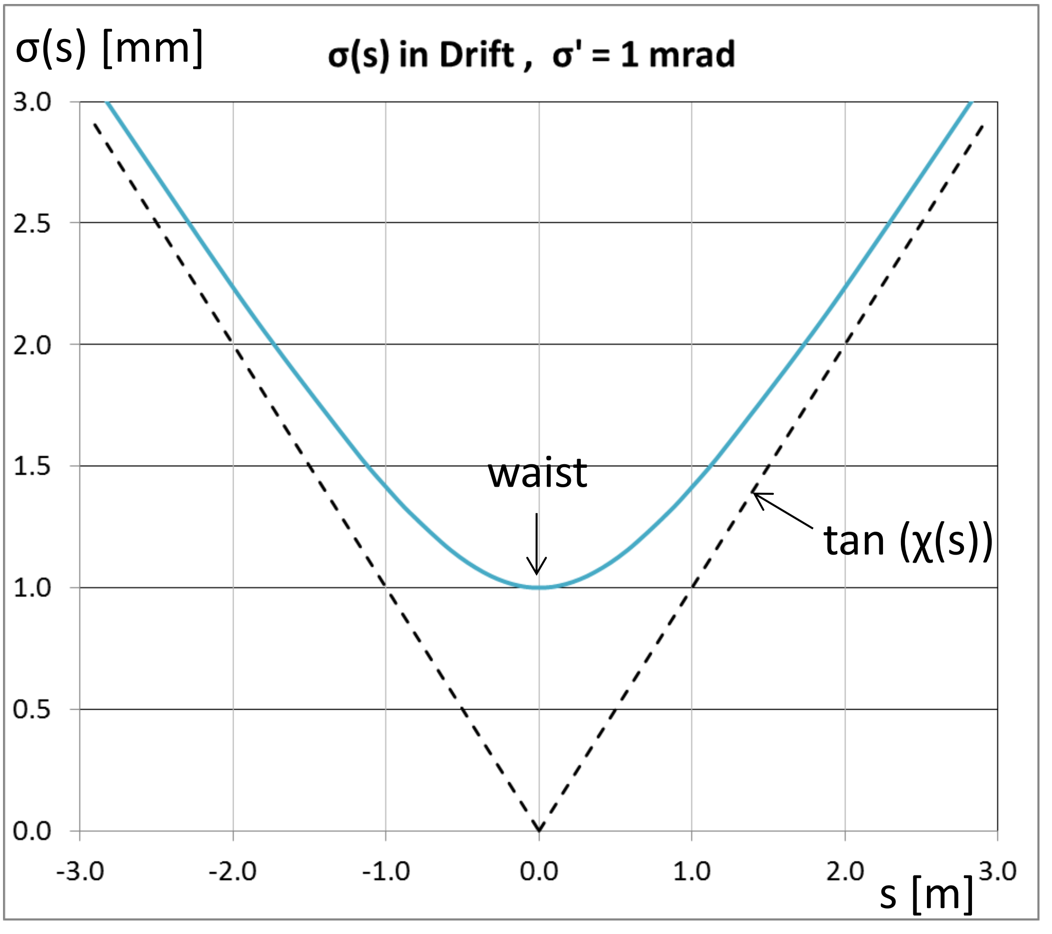
Beamsize in Drift



In waist at s = 0 :

$\sigma_0 = 1\mathrm{mm}\,,\quad \sigma' = 1\mathrm{mrad},\ \beta_0 = \dfrac{\sigma_0}{\sigma'} = 1m$

emittance $\varepsilon = \sigma_0 \sigma' = 1\mathrm{mm\,mrad}$

After drift s : $\tan\chi = \dfrac{s}{\beta_0}\ \ (= -\alpha)\,,\qquad \sigma(s) = \dfrac{\sigma_0}{\cos\chi}$

$\sigma^2 = \sigma_0^2 + \sigma'^2 s^2$

$\beta(s) = \sqrt{\varepsilon\sigma}$ or $\beta(s) = \beta_0 + \dfrac{s^2}{\beta_0}$

Dictionary for Beam Parameters

some useful quantities are easy to construct using σ, σ'

to get the right dimensions (mm, mrad, m) and using

as factors either sinχ *or cos*χ *(*χ *is 0 at a waist!)*

emittance:	$\varepsilon = \sigma\sigma' \cos\chi$	[mm mrad]
slope of envellope:	$d\sigma/ds = \sigma' \sin\chi$	[mrad]
virtual waist size:	$x_w = \sigma \cos\chi$	[mm]
β-function at virtual waist:	$\beta_{min} = (\sigma/\sigma') \cos\chi$	[m]
distance from virtual waist:	$L_w = (\sigma/\sigma') \sin\chi$ (= $\beta_{min} \tan\chi$)	[m]
phase advance from virtual waist:	$\psi = \chi$	

the dictionary between the 2 representations is:

$\alpha = -\tan\chi = -\dfrac{<xx'>}{\varepsilon}$ ($= -x_3/x_1 = -x_2'/x_4'$) [1]

$\beta = \dfrac{\sigma^2}{\varepsilon} = \dfrac{\sigma}{\sigma' \cos\chi}$ ($= x_2/x_4'$) [m]

$\gamma = \dfrac{\sigma'^2}{\varepsilon} = \dfrac{\sigma'}{\sigma \cos\chi}$ ($= x_3'/x_1$) [m^{-1}]

(as a check : $\beta\gamma = 1/\cos^2\chi = 1 + \tan^2\chi = 1 + \alpha^2$)

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Convolution of two ellipses

Example: **convolution** of the electron beam ellipse (x_1, x_1'), with parameter $\sigma_1, \sigma_1', \chi_1$ and the diffraction limited photon beam (x_2, x_2'), with parameter $\sigma_2, \sigma_2', \chi_2$ from an undulator.

Simple recipe:

add variances and correlations linearly

to form the combined ellipse (X,X') with parameter Σ, Σ', χ

$$\langle X^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle$$

$$\langle X'^2 \rangle = \langle x_1'^2 \rangle + \langle x_2'^2 \rangle$$

$$\langle XX' \rangle = \langle x_1 x_1' \rangle + \langle x_2 x_2' \rangle$$

or

$$\Sigma^2 = \sigma_1^2 + \sigma_2^2$$

$$\Sigma'^2 = \sigma_1'^2 + \sigma_2'^2$$

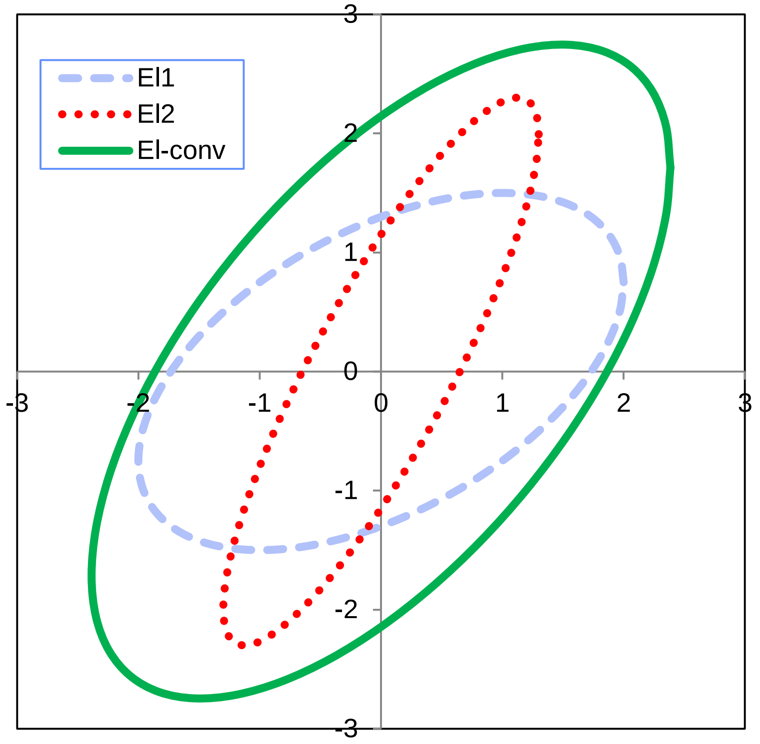
$$\Sigma\, \Sigma' \sin\chi = \sigma_1 \sigma_1' \sin\chi_1 + \sigma_2 \sigma_2' \sin\chi_2$$

the convoluted emittance is

$$\varepsilon = \Sigma\Sigma' \cos\chi \quad (\varepsilon \geq \varepsilon_1 + \varepsilon_2)$$

with the dictionary one can, if desired, transform these values back to the Courand-Snyder values α, β, γ .

Two Ellipses convoluted



Ellipses :

$$x = x_m \cos\varphi, \quad y = y_m \sin(\varphi + \chi), \quad (0 \le \varphi \le 2\pi)$$

$$\text{area} = \pi\varepsilon, \quad \varepsilon = x_m y_m \cos\chi$$

convolutedellipse :

$$x_m^2 = x_{m1}^2 + x_{m2}^2$$

$$y_m^2 = y_{m1}^2 + y_{m2}^2$$

$$x_m y_m \sin\chi = x_{m1} y_{m1} \sin\chi_1 + x_{m2} y_{m2} \sin\chi_2$$

	Xm	**Y**m	**χ**	**ε**
Ellipse 1	2.0	1.5	30^0	2.60
Ellipse 2	1.3	2.3	60^0	1.49
El-conv	2.39	2.75	38.6^0	5.12

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correlations x ⇔ y

example:

income and research for 50 US companies in 1976

(from journal „Physics Today“, march and september 1978)

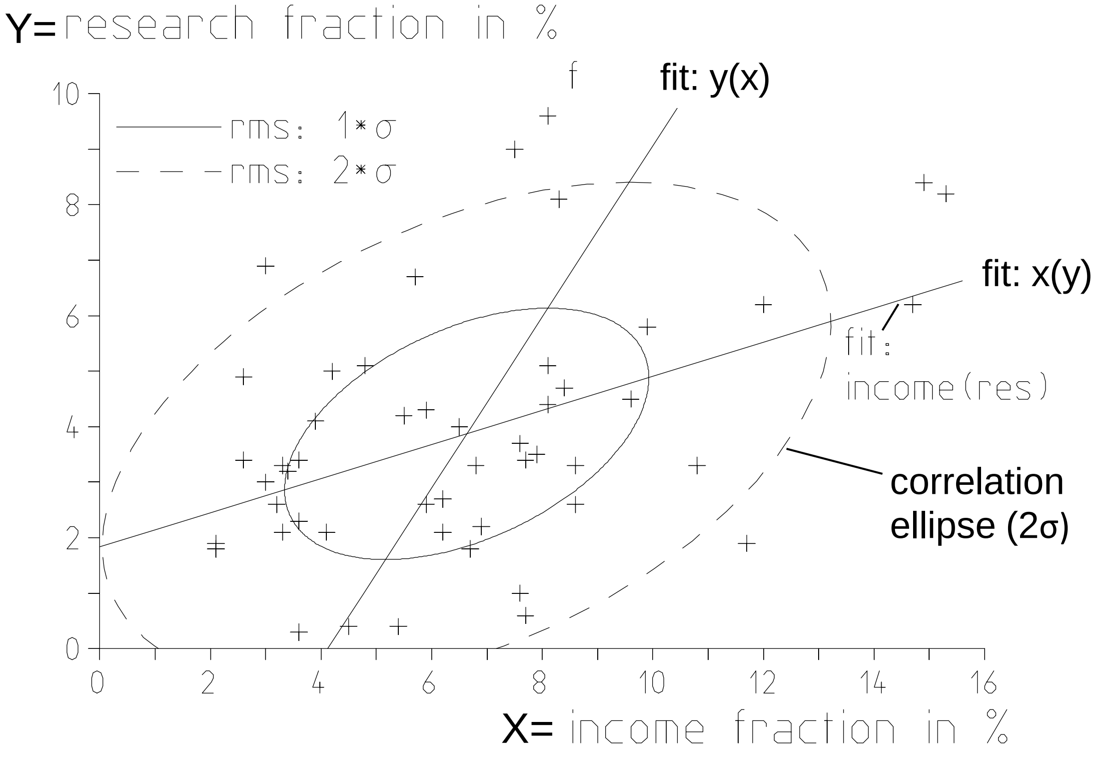
x = income / sales

y = research budget / sales

There are 3 possibilities to show a correlation:

1. linear fit of y(x) : income stimulates research !
2. linear fit of x(y) : research stimulates income !
3. correlation ellipse from $\langle x\ y \rangle$: high income ⇔ strong research

$$\langle x\ y\rangle/\sigma_x\ \sigma_y \equiv \sin\chi \approx 0.4$$



magnetic fringe field with binomial

$$B(x) \approx \frac{1}{(1+u^N)^{1/S}}, \quad u \equiv \frac{x}{x_L} \quad ,$$

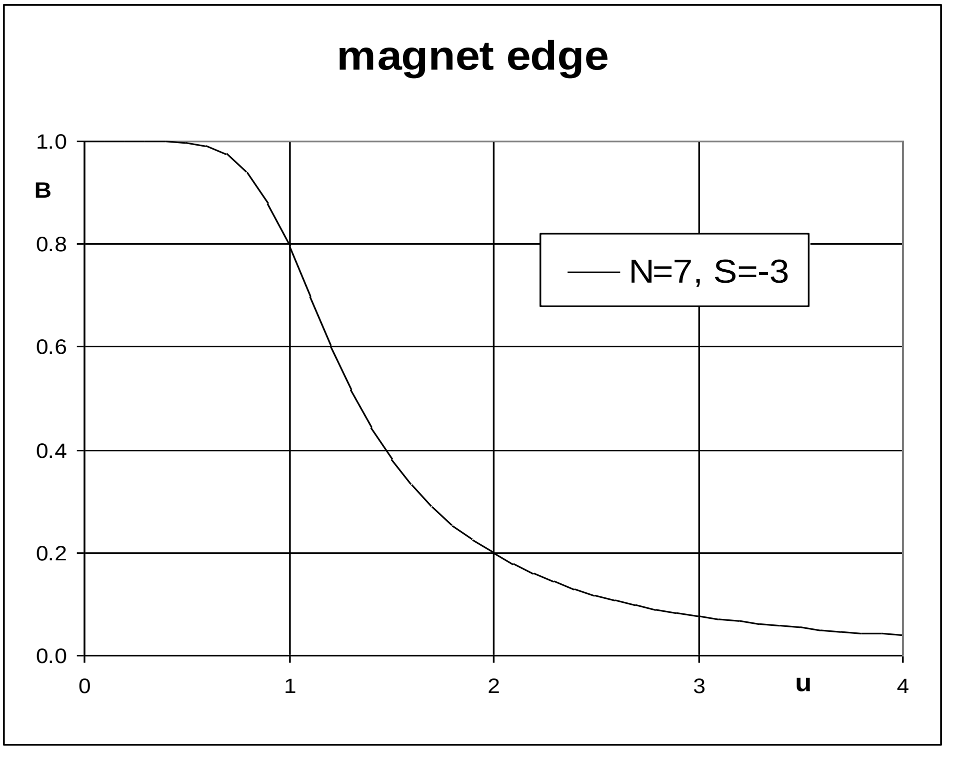
$$x_L \approx gap \quad , \quad N \approx 7 \quad , \quad S \approx 3 \quad ,$$

$$\Rightarrow 3 \quad \underline{free \quad parameter} \quad for \quad fit$$

$$origin \quad of \quad x \approx \quad at \quad x(80\% \, field) - gap$$

$$inverse:$$

$$u = \left(\frac{1}{B^S} - 1\right)^{1/N}$$



general binomial curves

$F(x) = F_{max}\, y(u)$, $u \equiv x/x_L$, $y(0) = 1$

3 general cases :

short range : $s > 0$

$$y = (1 - u^n)^{1/s} \quad , \; (0 \le u \le 1)$$

inverse

(interchange n and s !): $u = (1 - y^s)^{1/n}$, $(0 \le y \le 1)$

long range : $s < 0$

$$y = (1 + u^n)^{1/s} \quad , \; (0 \le u \le \infty)$$

inverse : $u = (y^s - 1)^{1/n}$, $(0 \le y \le 1)$

exponentials : (limit $s = 0$)

$$y = \exp(-u^n) \quad , \; (0 \le u \le \infty)$$

inverse : $u = (-\ln y)^{1/n}$, $(0 \le y \le 1)$

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properties of binomials

1. $y(u) = (1 \pm u^n)^{1/s}$ is monotonically decreasing, but transformations like $F(x) = A\, x^\alpha\, y(u)$ allow representations of functions which are not monotonic (e.g. Flux- or Brightness- curves)

2. inverse function exists : $y \Leftrightarrow u$, $n \Leftrightarrow s$

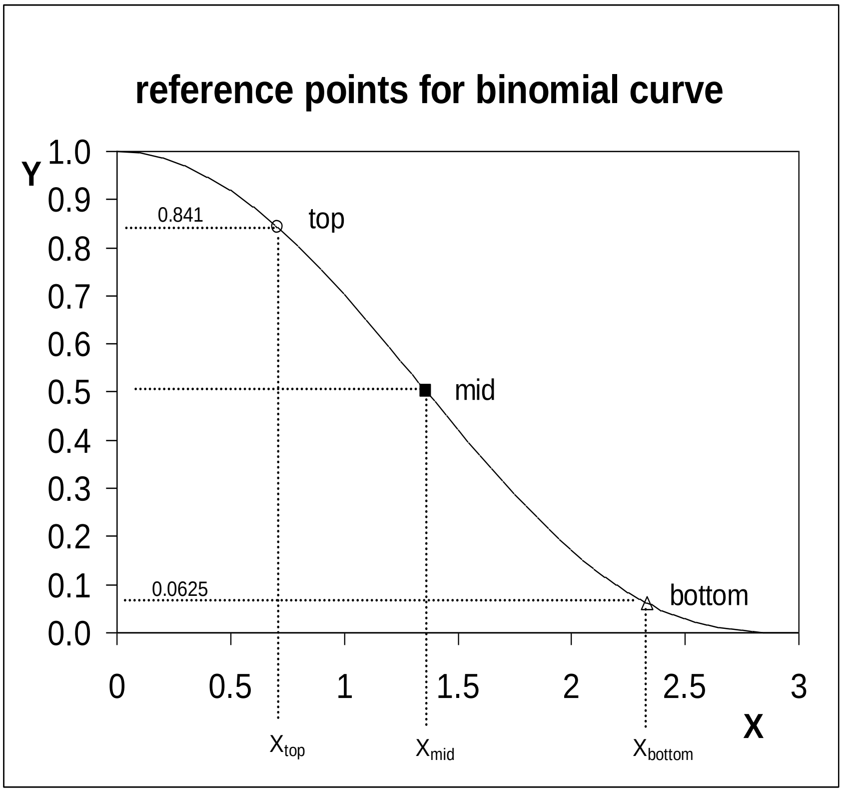
3. 4 free parameter:

    1. F_{max} gives **scaling in y**
    2. x_L gives **scaling in x** , (u=1)
    3. n determines **flatness** at x = 0
    4. s determines **tail** at large x

4. Fit to data using the 4 parameter with programs like MATLAB", "IGOR" or "Excel" (insert , name , define) + (extras , solver)

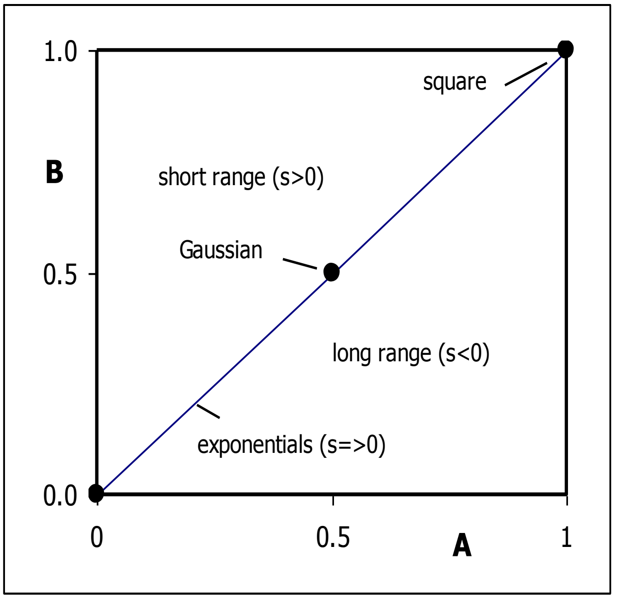
5. (A , B) – diagram allows rough estimates of parameter n and s

Classification of binomial curves



$$binomial: \quad y(x) = [1 - sign(s)(\frac{x}{x_L})^N]^{\frac{1}{S}}$$

any 3 reference points will give a fit for N, S and x_L. The chosen points top, mid and bottom allow a convenient classification in the (A,B)-Diagram



Classification of binomials in (A,B) - Diagram

$$y^{top} = 0.5^{1/4} = 0.841$$

$$y^{mid} = 0.5$$

$$y^{bottom} = 0.5^4 = 0.0625$$

$$A \equiv \frac{x_{top}}{x_{mid}}, \quad B \equiv \frac{x_{mid}}{x_{bottom}}$$

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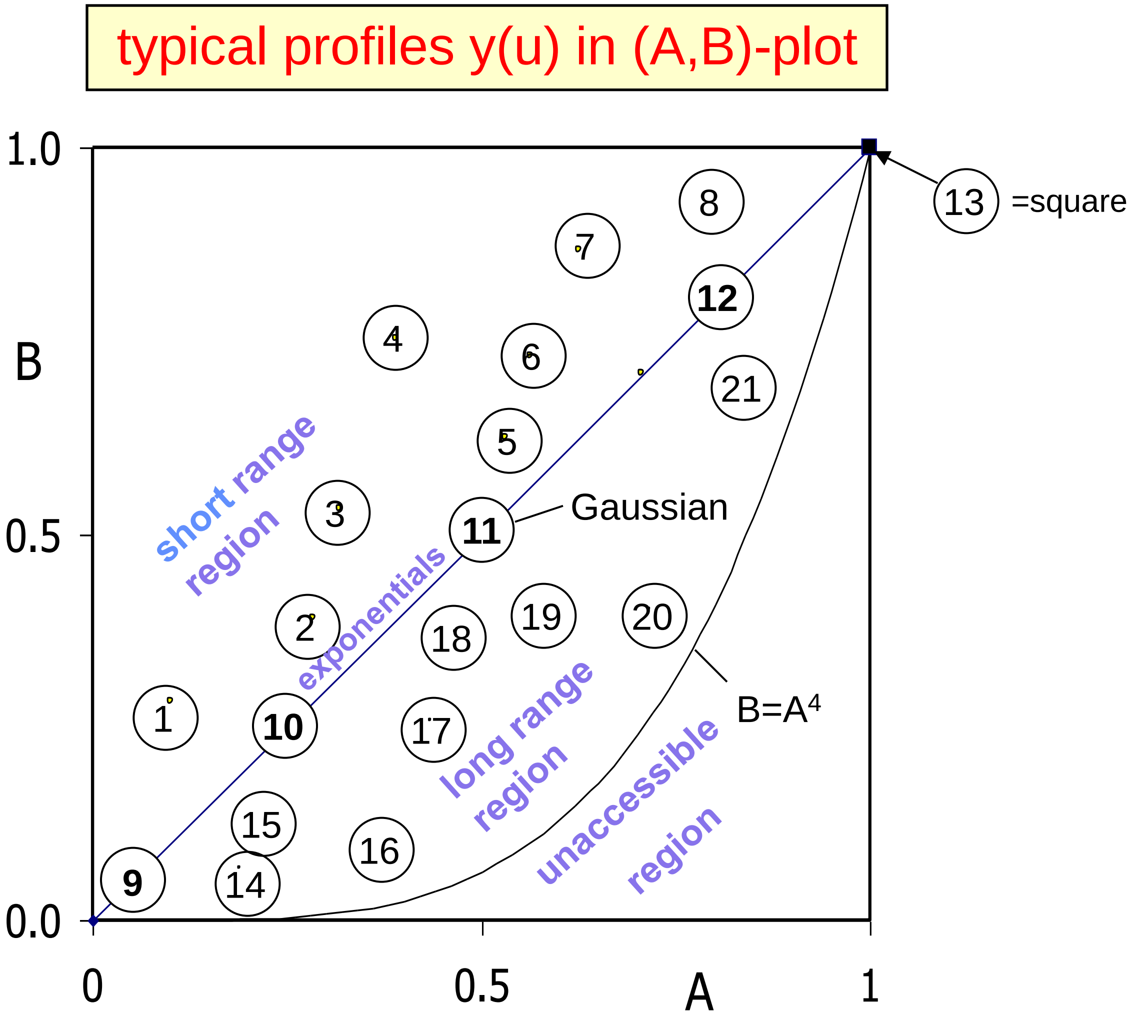

1 $y = 1 - \sqrt{u}$

2 $y = (1-u)^2$ parabola, convex

3 $y = 1 - u$ triangle

4 $y = \sqrt{1-u}$

5 $y = (1-u^2)^2$ biquadratic

6 $y = 1 - u^2$ parabola, concave

7 $y = \sqrt{1-u^2}$ quarter circle

8 $y = \sqrt{1-u^4}$

9 $y = e^{-\sqrt{u}}$

10 $y = e^{-u}$ exponential decay

11 $y = e^{-u^2}$ Gaussian

12 $y = e^{-u^6}$

13 $y = 1$ square

14 $y = \frac{1}{1+u}$

15 $y = \frac{1}{(1+u)^2}$

16 $y = \frac{1}{\sqrt{1+u^2}}$

17 $y = \frac{1}{1+u^2}$ Lorentzian

18 $y = \frac{1}{(1+u^2)^2}$ bi-Lorentzian

19 $y = \frac{1}{1+u^3}$

20 $y = \frac{1}{(1+u^7)^{1/3}}$ magnetic fringe field

21 $y = \frac{1}{\sqrt{1+u^{12}}}$

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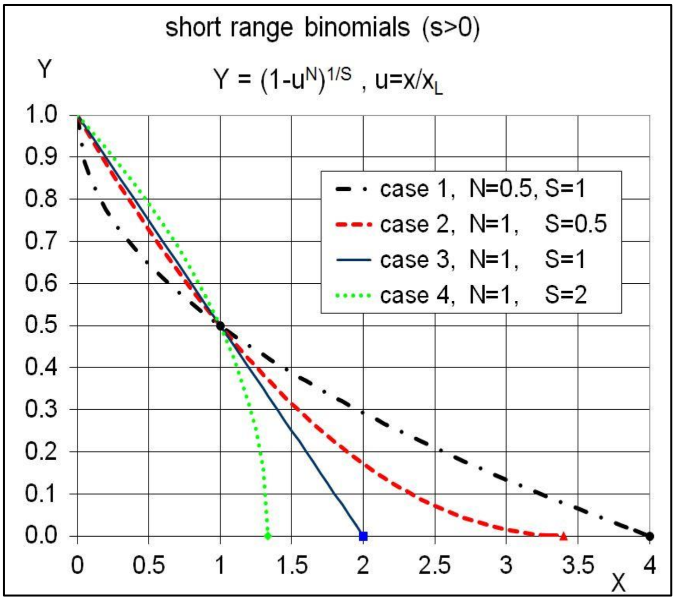
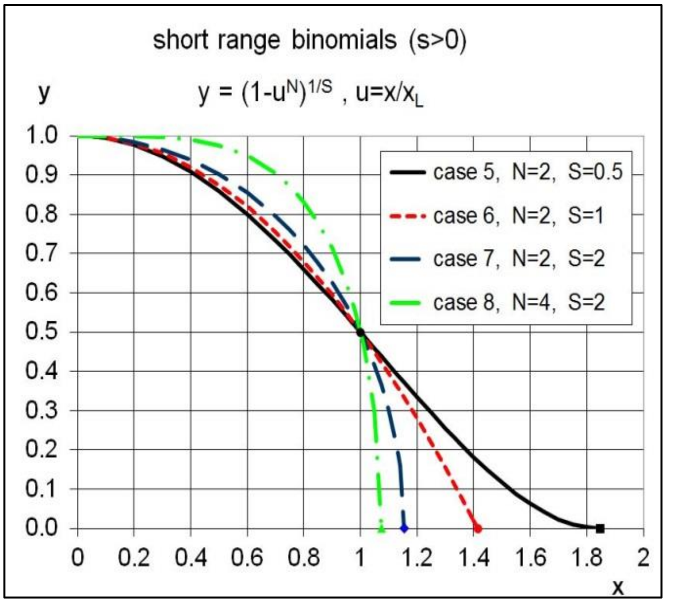
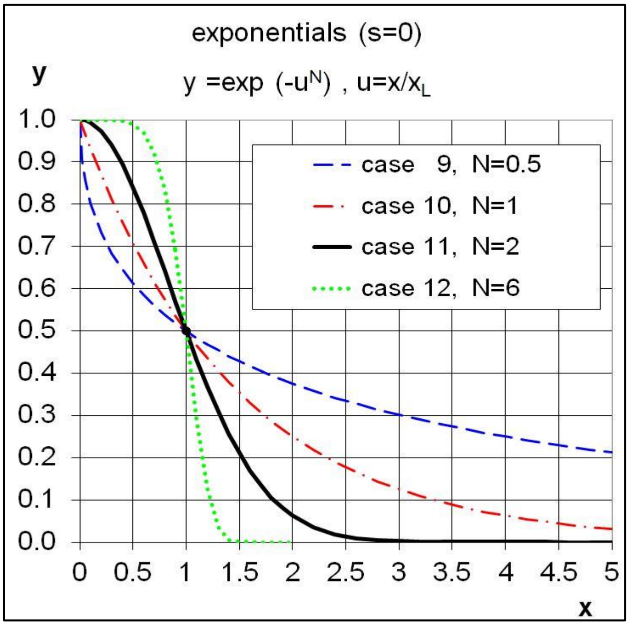
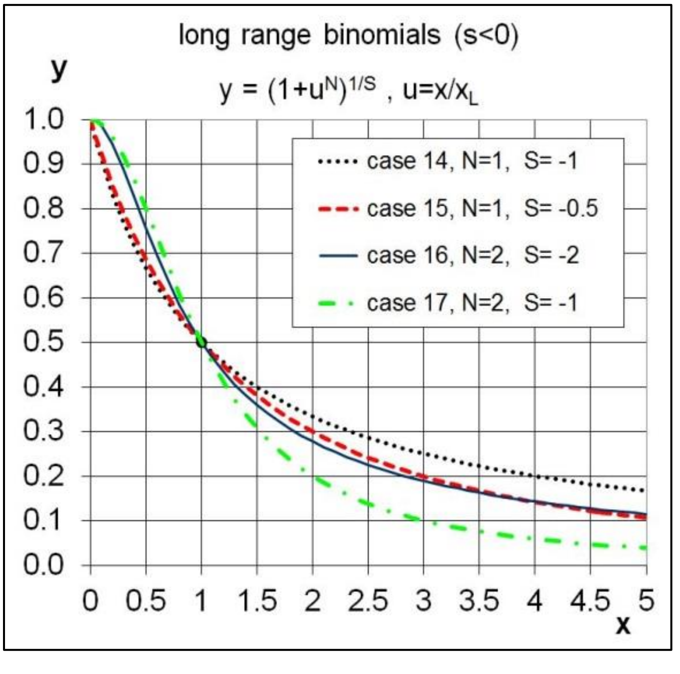
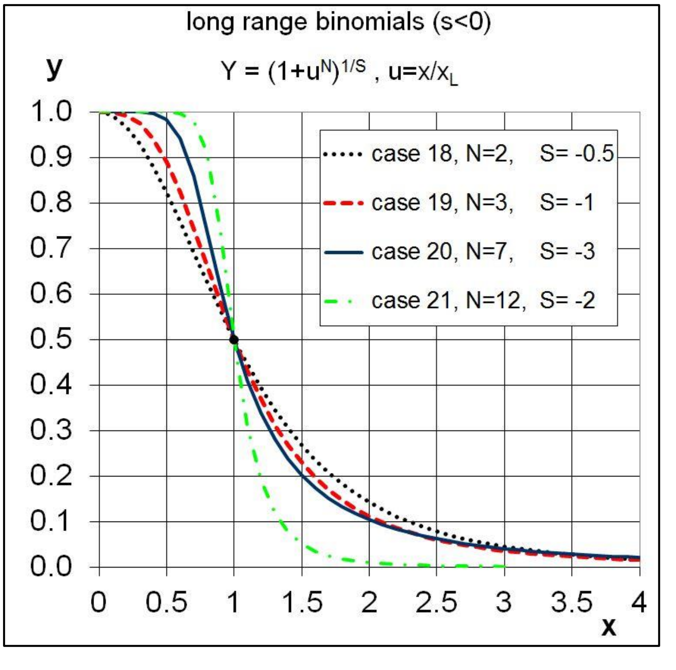




binomials & exponentials







Werner Joho

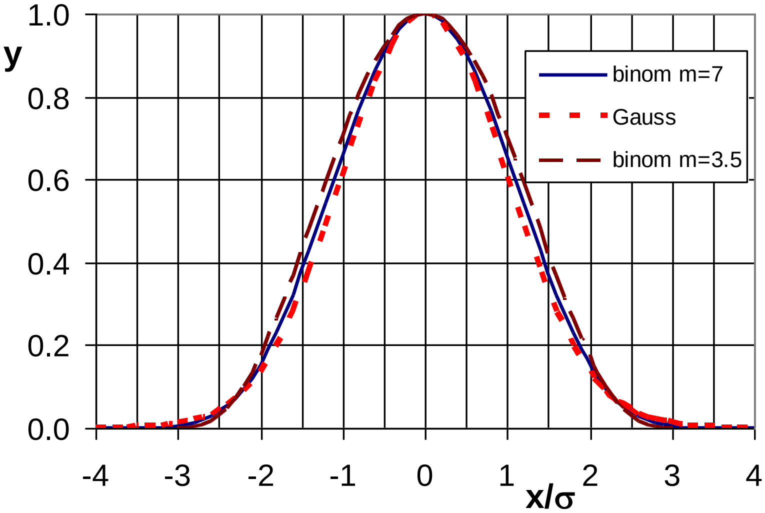
representations of beam profiles with binomials

1) Gaussian: $y = e^{-1/2(x/\sigma)^2}$

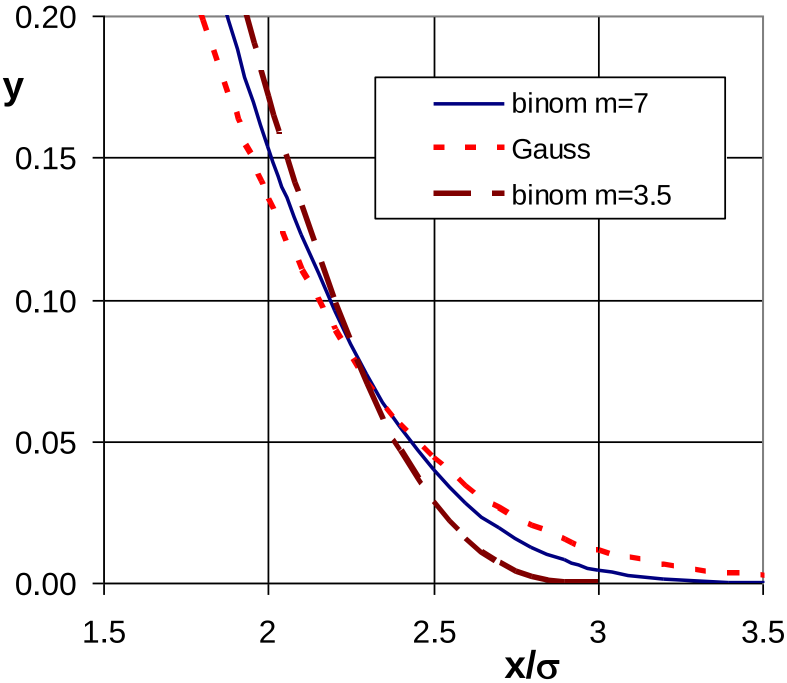
2) Binomials :

$$y = [(1-(\frac{x}{x_L})^2]^{m-1/2}, \quad x \le x_L \,, \quad x_L = \sqrt{2(m+1)}\,\sigma$$

clipped tails at x_L (e.g. m=3.5 => x_L= 3σ ; m=7 => x_L= 4σ)



Profiles



Tails of Profiles

(full width at 10% level :

≈ 4.4 σ for

large range of m)

Werner Joho

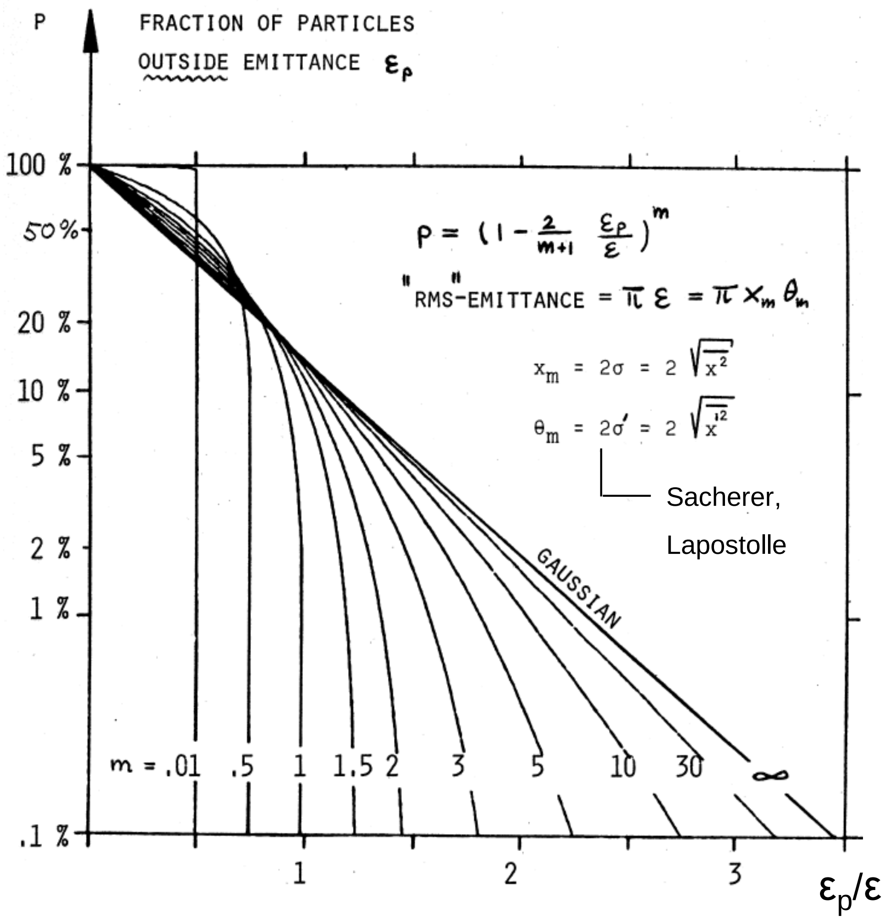
clipped binomial phase space densities

$$\rho(x, x') = (1 - a^2)^{m-1}$$

$$\left(a^2 \equiv u^2 + v^2 \leq 1, \quad u \equiv \frac{x}{x_L}, \quad v \equiv \frac{x'}{x'_L}\right)$$

projected profile : $y(x) = (1 - u^2)^{m-1/2}$

we get again a binomial with the exponent reduced by 1/2



big trick:

plot fraction which is **outside** of ellipse!

for m≥1.5 the curves have a crossing point at $\varepsilon_p \approx \varepsilon$ and p ≈ 13%; i.e. ca. 87% of all particles are inside an ellipse with emittance ε=(2σ)·(2σ‘) , independent of m.

For a Gaussian distribution we have p=exp(-2ε_p/ε), which gives a straight line in this diagram (m=∞).

W.Joho, 1980
PSI report TM 11-4

Flux-Spectrum of Synchrotron Radiation from Bending Magnet with Field B

$$G_1(x) = x\int_x^{\infty} K_{5/3}(x')\,dx'$$

$$x \equiv \frac{\varepsilon}{\varepsilon_c},\ \ \varepsilon = photon\,energy,\ \ \varepsilon_c = critical\ \ photon\,energy$$

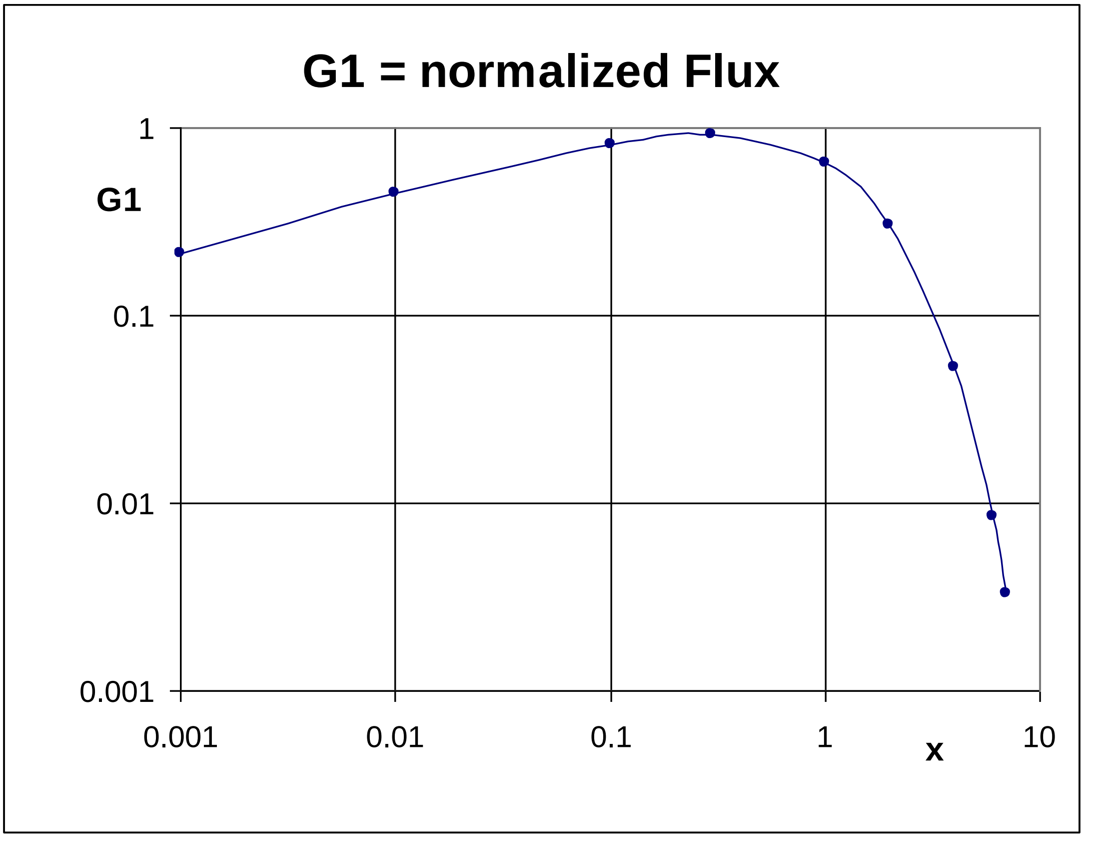
$$\varepsilon_c = 665eV \cdot E^2[GeV] \cdot B[T]$$

Fit with $G_1(x) = A\, x^{1/3}\, g(x)$,

$$\boxed{g(x) = [(1-(\frac{x}{x_L})^N]^{\frac{1}{S}}}$$

fit of binomial g(x) with 8 data points to ±1.5%:

$A = 2.11$, $N = 0.848$, $x_L = 28.17$, $S = 0.0513$



Werner Joho

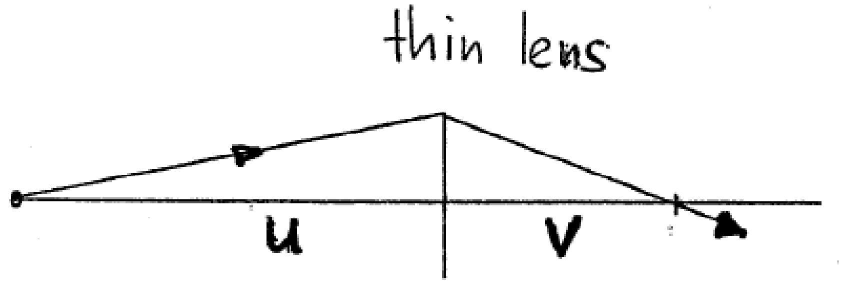
Graphical solution of the lens equation

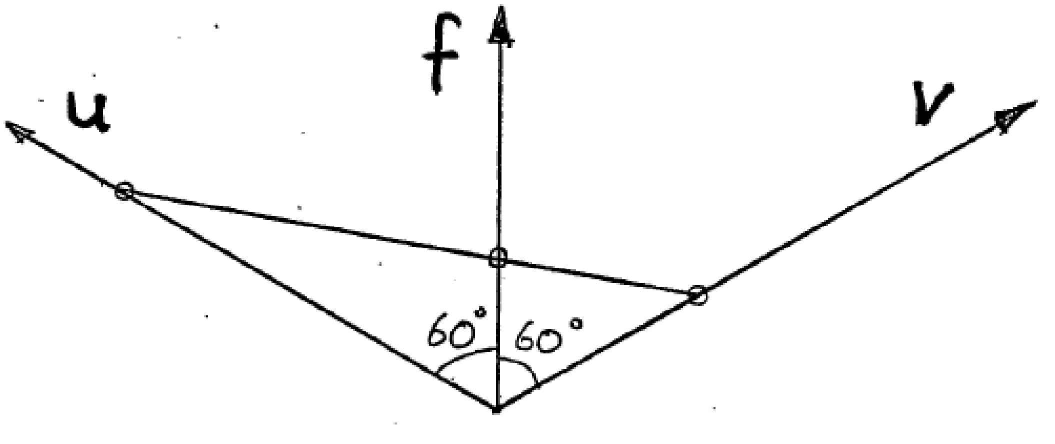
the lens equation (Newton) solved for a
thin lens with focal length f

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

symmetric case:

$u = v = 2f$





same graph for resistances in parallel,

capacitances in series etc.!

Werner Joho

AG-focusing
(by hand!)

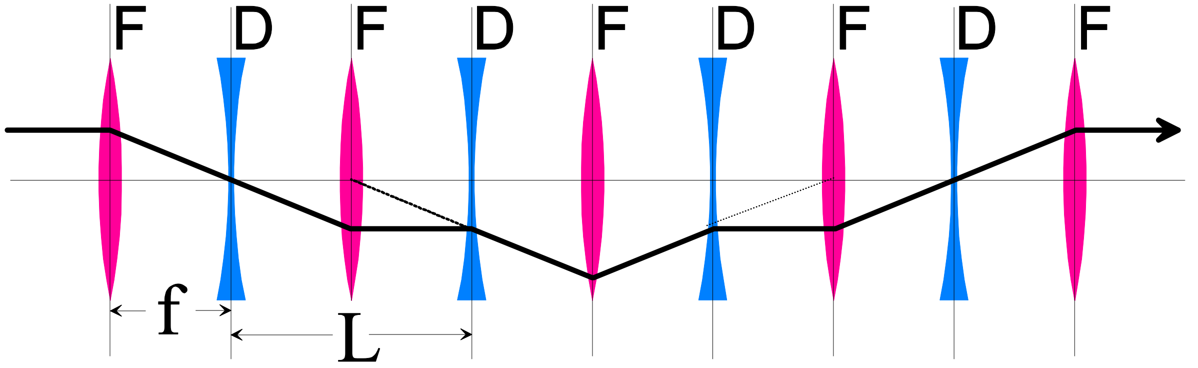
simple example of alternative gradient focusing:

$\Rightarrow$ FODO-lattice with thin lenses (focal length f)

if L = 2f => construction is possible by hand !

it takes 6 periods to get a 360^0-oscillation

i.e. the phase advance/period is $\psi = 60^0$



exact solution with transfer matrices gives

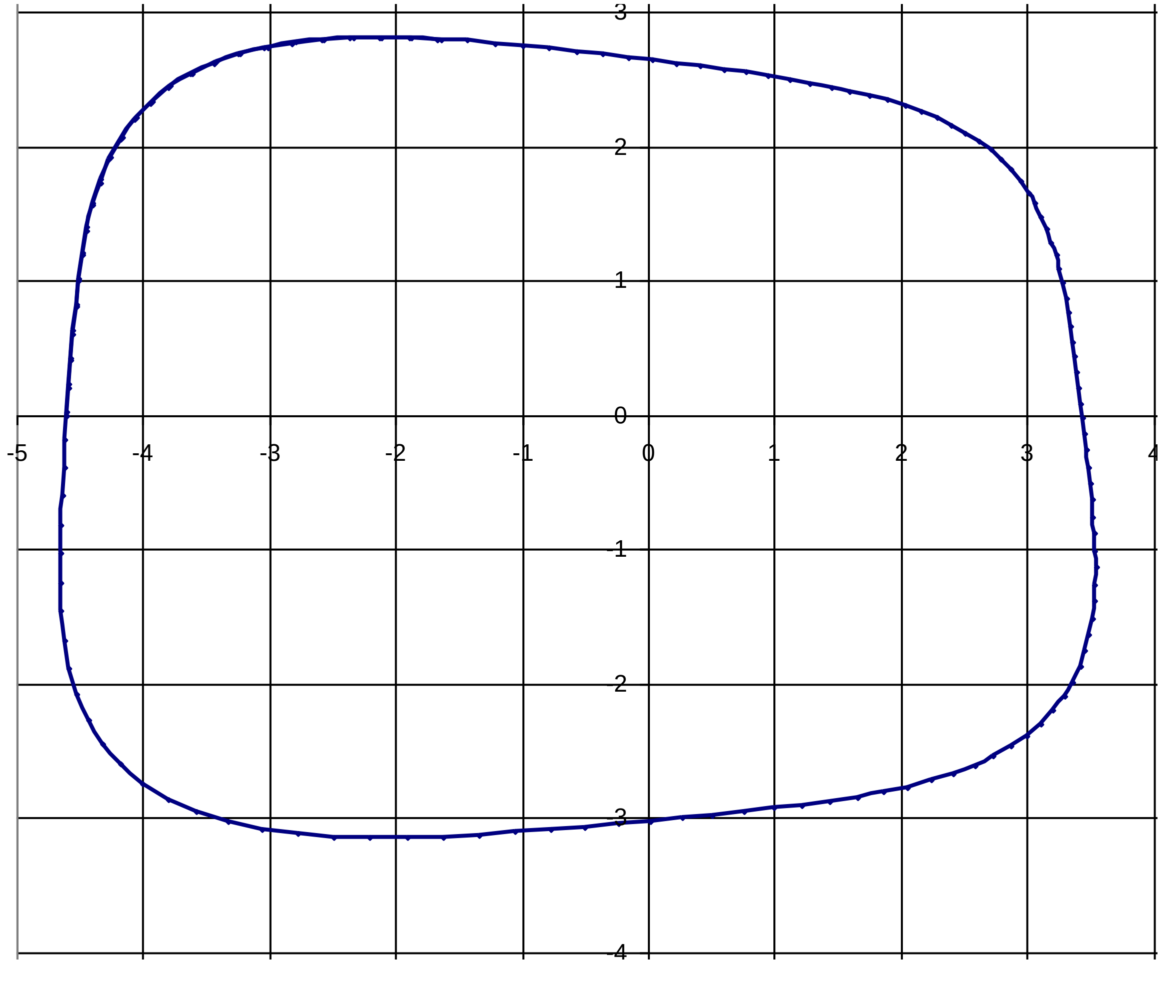
$$\boxed{\sin\frac{\psi}{2} = \frac{L}{4f}}$$

for L = 2f => $\psi = 60^0$ (graphic example)

for L = 4f => $\psi = 180^0$ (instability !)

„Hamiltonian Table“

You want to construct a nice table for your living room with this shape?



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„Hamiltonian“ Plots

Try to plot a curve F(x,y)=const.=c ,

where F(x,y) can be quite complex like

$$F(x,y) = (\frac{x}{a})^n + (\frac{y}{b})^n + S_x x + S_y y$$
$$+ G_x \exp[-(\frac{x}{L_x})^2] + G_y \exp[-(\frac{y}{L_y})^2]$$

One method is to choose a certain x and then solve

the corresponding nonlinear equation for y with iterative methods.

Then repeat this procedure for the next choice of x.

A more elegant method is the following:

1) Pretend that the function f(x, y) is a Hamiltonian H(x(t), y(t))
2) Solve the equations of motion (e.g. with an Excel-sheet using the Euler-Cauchy integration or the Runge-Kutta method).

$$\dot{x} = \frac{\partial H}{\partial y}, \quad \dot{y} = -\frac{\partial H}{\partial x},$$

To get some initial conditions one can choose e.g. $y_0 = 0$

and then solve $f(x_0, 0) = c$ to get the corresponding x_0.

The beauty of this method:

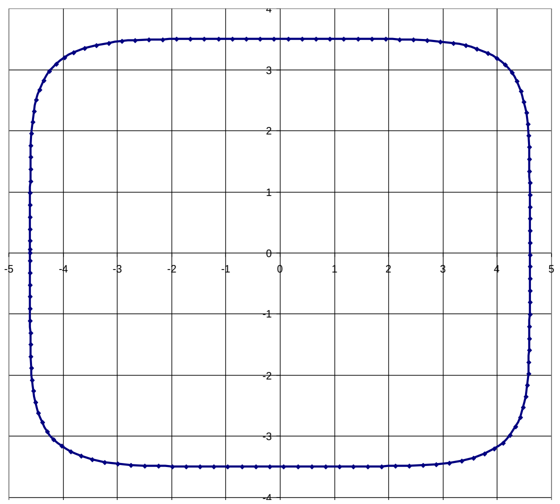
For a given integration step dt the points x(t), y(t) move smoothly

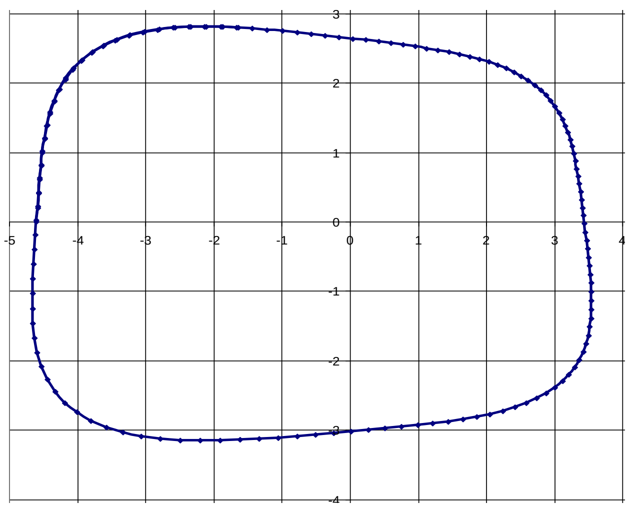
along the desired function F(x, y).

Werner Joho

Example of „Hamiltonian“ Plots with F(x,y)=const.

$$F(x,y)=(\frac{x}{a})^n+(\frac{y}{b})^n+S_x x+S_y y+G_x \exp[-(\frac{x}{L_x})^2]+G_y \exp[-(\frac{y}{L_y})^2]$$





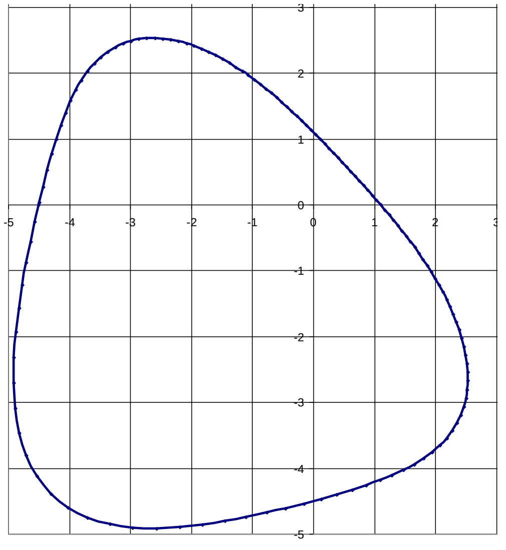
the corresponding parameters for these shapes are:

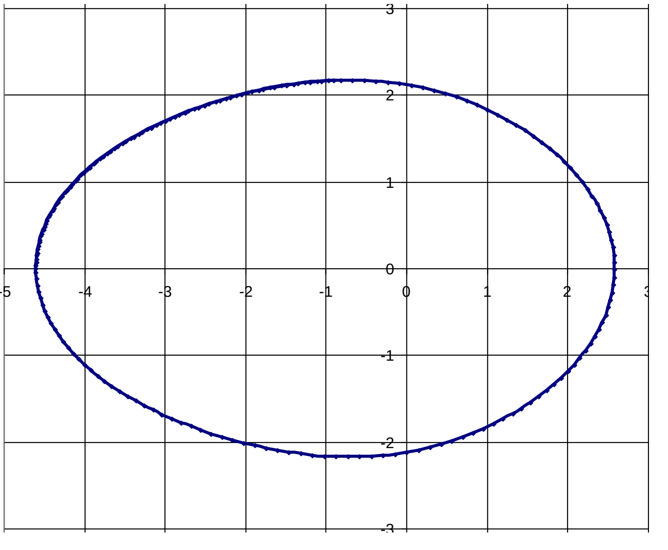
n:	6	4
a:	4.6	4
b:	3.5	2.8
S_x	0	0.15
S_y	0	0.1
G_x	0	0
G_y	0	0
(X_0, Y_0):	(-4.6, 0)	(-4.6, 0)

Werner Joho

more „Hamiltonian“ Plots with F(x,y)=const.

$$F(x,y) = (\frac{x}{a})^n + (\frac{y}{b})^n + S_x x + S_y y + G_x \exp[-(\frac{x}{L_x})^2] + G_y \exp[-(\frac{y}{L_y})^2]$$





the corresponding parameters for these shapes are:

n:	4	2
a:	3.8	3.8
b:	3.8	2.1
S_x	0.35	0.15
S_y	0.35	0
G_x	0	0.25
G_y	0	0
L_x	-	2.5
L_y	-	-
(X_0, Y_0):	(-4.5. 0)	(-4.6, 0)

This shape is similar to a famous table constructed by Isamu Noguchi 1944

Werner Joho

Part 2
mathematical curiosities

Math can be fun and beautiful. One can even make money with it by calculating the odds for some bets. Here are a few tricks of the trade, which I collected during my career as a physicist.

- money winning bets
- cyclic winning
- the quadratic birthday gifts of two brothers
- the impossible equation
- Euler beauties
- squaring made easy
- harmonic Tattoo Formations
- the mystery of the 1'001 arabian nights
- the magic number for exponential growth
- William Tell against the state
- birth and mortality rates, 1000 balls at birth
- Friday the 13th
- how many stars for everyone of us ?
- the power of $E=mc^2$

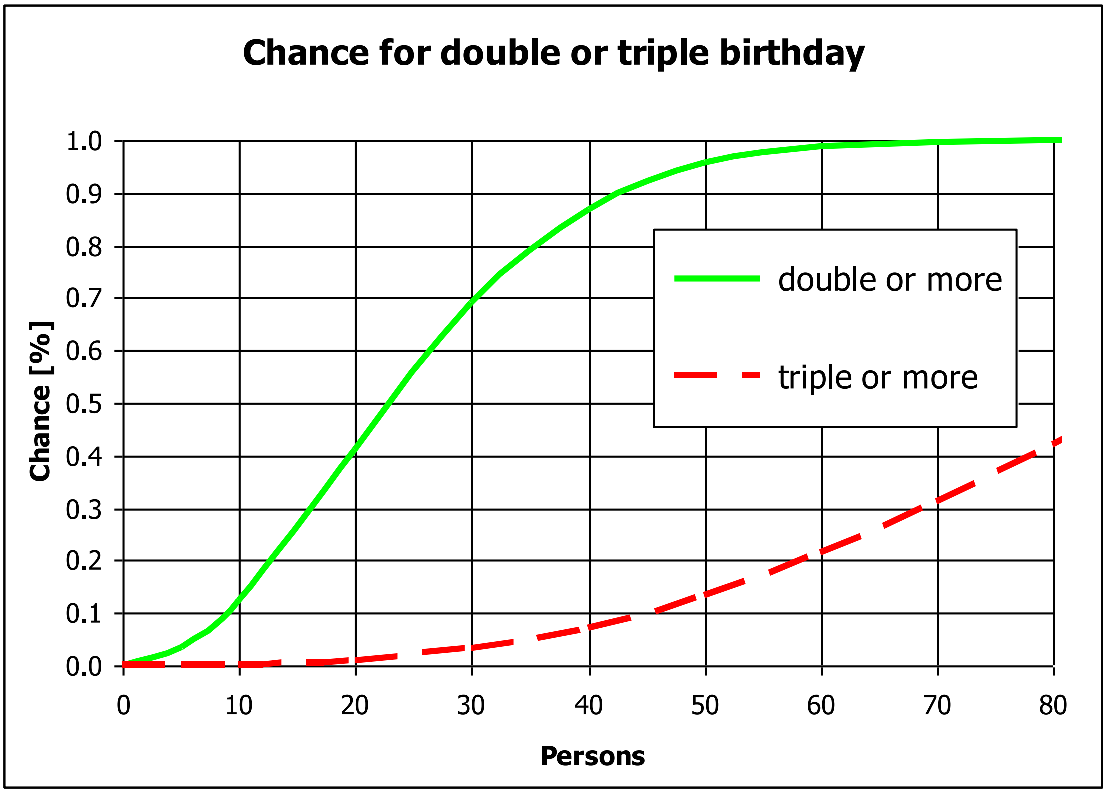
Bet for a common birthday in a group

what is the chance, that in a group of n persons, 2 people have the same birthday (disregarding the year of birth and the 29th february)?

$$\text{probability} \quad w = 1 - \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \ldots \cdot \frac{(366-n)}{365}$$

with 23 people the chance is already 50%, with 40 people it is 87% and with 80 people a double coincidence is a „sure bet“ (99.95%)

… and a triple coincidence has a 42% chance!



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winning money with statistics !

throw simultaneously 6 dices:

If all 6 dices show different numbers
(1, 2, 3, 4, 5, 6)

I give you **30 times** your betting sum ;

is this a fair bet?

No! **The chance for winning is only 1.5%** !!

to be fair, I should offer you 65 times

your betting sum!

$$(p = \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} = \frac{5!}{6^5} = \frac{5}{324} = 1.54\%)$$

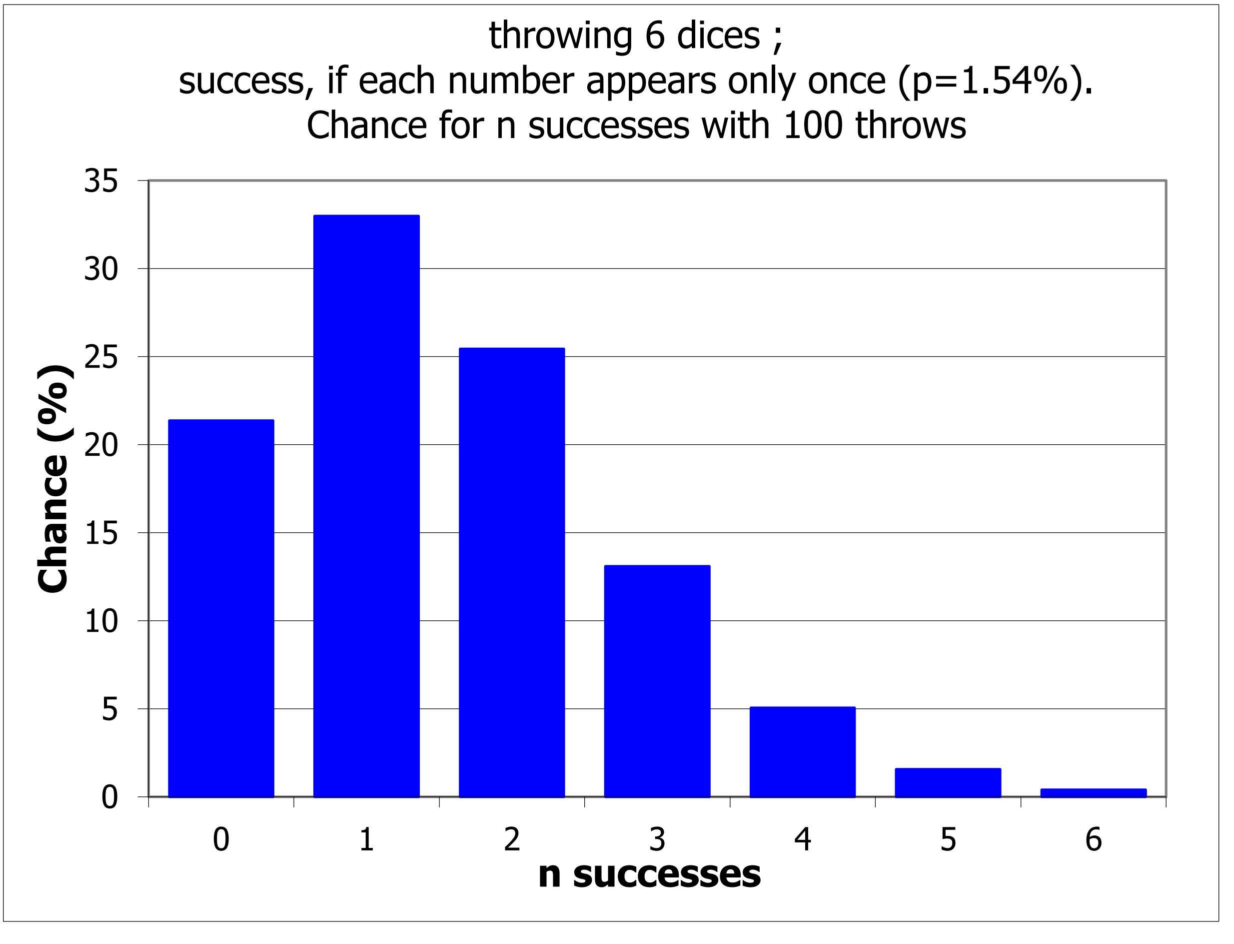
I perform this betting game often at weddings.
Everybody is encouraged to throw dices many times.
All the wins go the wedding couple!

My contribution are the payments for eventual wins,
which I pay out of my own pocket!

Werner Joho

A game for wedding parties

I perform this betting game with the 6 dices often at weddings. Everybody is encouraged to throw the dices many times, for 1 Fr. a throw. All the wins go the wedding couple!
My contribution to the couple are the payments for eventual wins, which I pay out of my own pocket! Here are the chances, that I have to pay n times the 30 Fr. (= success for the thrower!) with 100 throws, according to Poisson statistics.
My average contribution is 100x30Fr./65Fr. = 46 Fr.
With a fair payment of 65 Fr. my contribution would be 100 Fr., the same amount as the income from 100 throws.



Werner Joho

Throwing a 6 in 3 trials

What is the chance to throw with a dice at least once a 6 with 3 trials?

The chance for a 6 in the first throw is 1/6.

But the chance for at least a 6 in 3 trials is not

3 x 1/6 = ½. It is less than 50%:

$$P = 1 - (\frac{5}{6})^3 = \frac{91}{216} = 42\%$$

To believe this: it is obvious, that after 6 trials the chance is not 6 x 1/6 = 100% , but only

$$P = 1 - (\frac{5}{6})^6 = 66.5\%$$

Quiz with 3 Questions

In this quiz (named “time is money”), which is often presented on a Swiss radio station , one has to answer 3 questions about a total of 9 events. In each of them one has to put given events in the right order of time.

In the first question one has to decide which one of 2 events happened first. In the second question we have to place in order 3 events like e.g.

1. Hillary and Sherpa Tensing climb onto Mount Everest.
2. Germany wins for the first time the world championship in soccer
3. Outbreak of the war in Korea

In the third and final task one has to classify 4 events.

Lets suppose, that we have no clue when all these 9 events happened.

What is then the chance to answer all 3 questions correctly, simply by guessing?

For the 1.question the probability is 1/2, for the second one it is $1/3 \cdot 1/2 = 1/6$ and for the third one just $1/4 \cdot 1/3 \cdot 1/2 = 1/24$.

Multiplied together the final probability is only 1/288 = 0.35% to answer all 3 questions correctly!

Werner Joho

an unfair betting offer

pull 4 cards from a bridge set (52 cards).

If you get from each set (heart, diamond, spade, club)

exactly one card, I pay you **5 times** your betting sum.

Is this fair?

No! **Your winning chance is only 10.5%** !!

$$\text{probability} \;\; p = \frac{39}{51} \cdot \frac{26}{50} \cdot \frac{13}{49} = 0.105$$

Werner Joho

Chance for twice the same card from 2 bridge sets

I take from a set of bridge cards (52) a subset of 6 cards. My opponent does he same from another full set. I win the bet, if we have a common card. What is my chance of winning?

$$p_6 = 1 - \frac{46}{52} \cdot \frac{45}{51} \cdot \frac{44}{50} \cdot \frac{43}{49} \cdot \frac{42}{48} \cdot \frac{41}{47} = 54\,\%$$

If my opponent draws 6 cards, but I draw 7 cards, my winning chances are increased to:

$$p_{67} = 1 - \frac{45}{52} \cdot \frac{44}{51} \cdot \frac{43}{50} \cdot \frac{42}{49} \cdot \frac{41}{48} \cdot \frac{40}{47} = 60\%$$

With 7 cards each my chances are getting unfair!

$$p_7 = 1 - \frac{45}{52} \cdot \frac{44}{51} \cdot \frac{43}{50} \cdot \frac{42}{49} \cdot \frac{41}{48} \cdot \frac{40}{47} \cdot \frac{39}{46} = 66\%$$

Werner Joho

Winning in the alphabet game

Choose from the 25 letters of the alphabet (J=I) **5 different letters** and write them on a sheet of paper, which you hide from me.

I do exactly the same**.**

Then we compare our letters.

If we have a common letter,

I win the betting sum. Otherwise you win it.

How big is the chance that you win?

Chance for my opponent to win

$$p_5 = \frac{20}{25} \cdot \frac{19}{24} \cdot \frac{18}{23} \cdot \frac{17}{22} \cdot \frac{16}{21} = 29\,\%$$

Play simultaneously against a whole group!

This accelerates your wins.

In the first round your chance for winning is

increased due to a psycholocical effect:

Your opponents tend to choose letters like (Q, X, Y..!)

This game is only somewhat fair, if both parties
choose **4 letters** => chance for opponent to win is

$$p_4 = \frac{21}{25} \cdot \frac{20}{24} \cdot \frac{19}{23} \cdot \frac{18}{22} = 47\,\%$$

Cyclic winning with 4 dices from Martin Gardner

Martin Gardner in the "Scientific American" of Dec 1970 (110-114):

There are 4 specially prepared dices, each showing a different arrangement of points from 0 to 6. I let you choose first one of these dices. Then I make my choice, depending on your choice!

Then we throw our dices. The higher point showing wins.

On the long run my chances of winning are 2:1,

no matter which dice you choose **first**!

Here are the 4 dices:

Dice	points showing	point average	
A	4 4 4 4 0 0	2.67	
B	3 3 3 3 3 3	3.0	(no need to throw it!)
C	6 6 2 2 2 2	3.33	
D	5 5 5 1 1 1	3.0	

Dice A beats Dice B with 24 wins and 12 losses
Dice B beats Dice C with 24 wins and 12 losses
Dice C beats Dice D with 24 wins and 12 losses
Dice D beats Dice A with 24 wins and 12 losses

The interesting point is, that we have a
cyclic winning pattern: There is no best dice!

Werner Joho

Cyclic winning for wrestling teams

I modified the Martin Gardner puzzle from 4 dices to 4 wrestling teams

In each team there are 6 wrestlers. Each wrestler has a rating for his strength; the stronger wrestler has a higher rating and always wins his match against a weaker wrestler.

In a team competition every wrestler fights against every opponent from the other team. There are thus 36 matches.

The following 4 teams take part in the competitions:

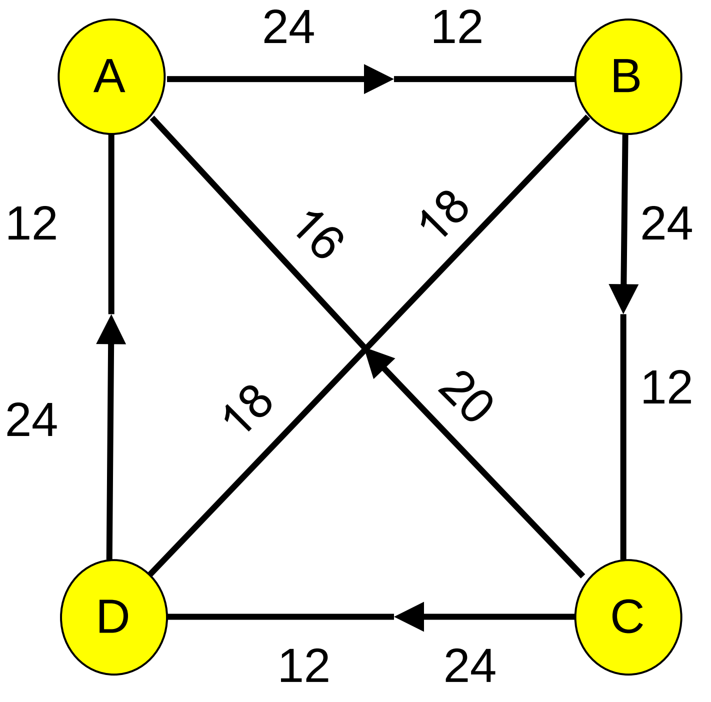
Team	individual strengths						average strength
A	17	17	17	17	6	6	13.33
B	13	13	13	13	13	13	13.0
C	20	20	8	8	8	8	12.0
D	18	18	18	7	7	7	12.5

Team A beats team B with 24 wins and 12 losses
Team B beats team C with 24 wins and 12 losses
Team C beats team D with 24 wins and 12 losses
Team D beats team A with 24 wins and 12 losses

Again we have a cyclic winning pattern!

Werner Joho

illustrated in a graph:



In a **round robin** competition each team plays the other three teams. Each team gets 2 points for a win, and 1 point for a draw.

The final score will look like this:

Result	Team	wins	losses	draws	points	team strength
1.	C	2	1	0	4	12.0
2a.	D	1	1	1	3	12.5
2b.	B	1	1	1	3	13.0
4.	A	1	2	0	2	13.3

The result in a round robin is thus just opposite to the average team strength:

the „weakest" team wins, the „strongest" team looses!

With a play-off or cup system we have 3 cases:

Case 1.

Semifinals:

A – B , A wins 24:12

C – D , C wins 24:12

Final:

A – C , **C wins** 20:16

Case 2.

Semifinals:

A – C , C wins 20:16

B – D , draw 18:18, D wins Tie break between the 2 top wrestlers

Final:

C – D , **C wins** 24:12

Case 3.

Semifinals:

A – D , D wins 24:12

B – C , B wins 24:12

Final:

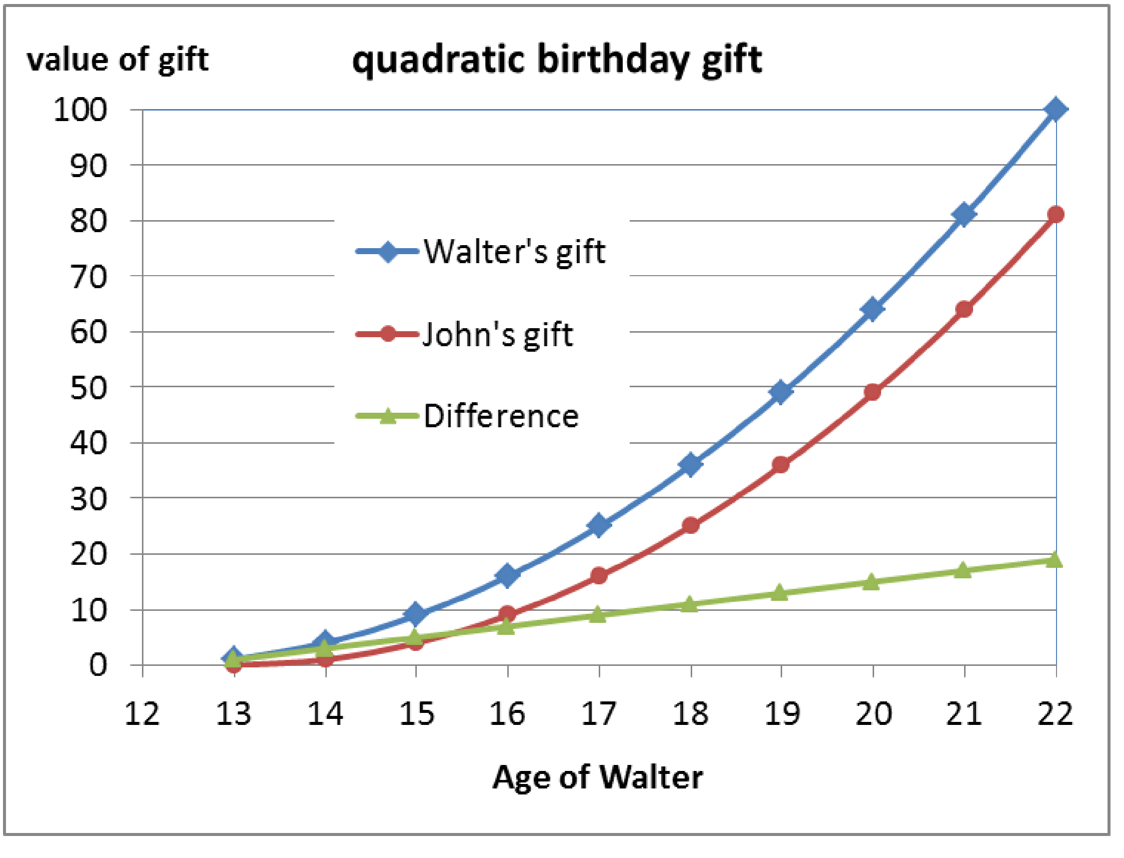
B – D , draw 18:18, **D wins** Tie break between the 2 top wrestlers

Again: the „weakest" team C (lowest average strength) wins 2 of the 3 possible playoffs !

Werner Joho

Quadratic birthday gifts for two brothers

Walter and John are brothers. Walter is 13 years old. His younger brother is 12 years old, but the brighter one of the two. John proposes to his brother the following scheme for their birthday gifts:
Let's give each other gifts, which increase quadratically in value with each year. We start this year with a gift for 1 Fr. Next year the gift has a value of 4 Fr., the following year it is worth 9 Fr. and so on. Why don't you start the series this year with 1 Fr. Since I am 1 year younger than you, I will start with my series next year with 1Fr. Is this OK with you? Walter agrees, and does not realize that each year the difference between their gifts increases linearly for the rest of their lives!



Werner Joho

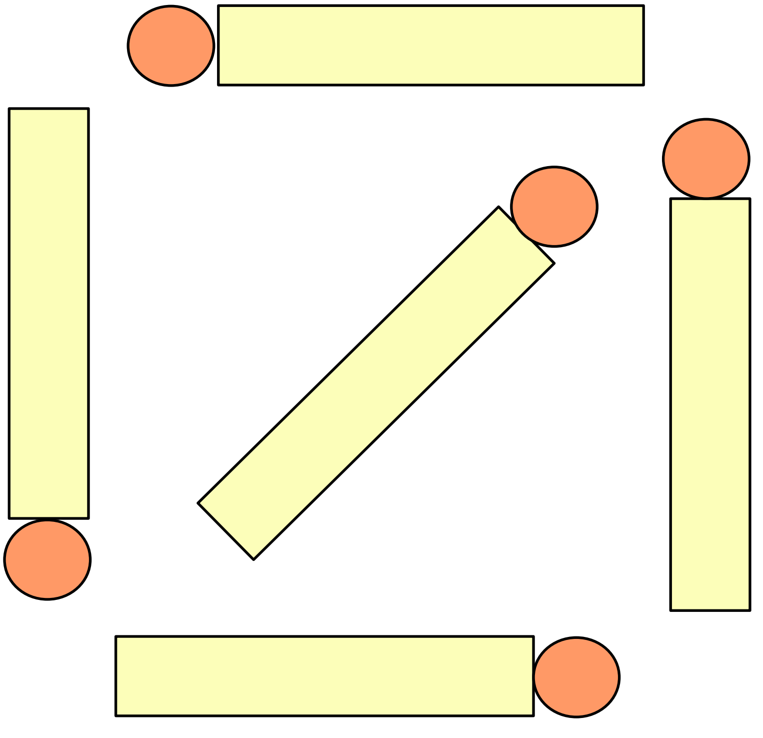
The impossible equation !?

$$X - 3 + 2 = X$$

X = 5 matches

1. Remove 3 matches

2. then add 2 matches to get the same picture !?



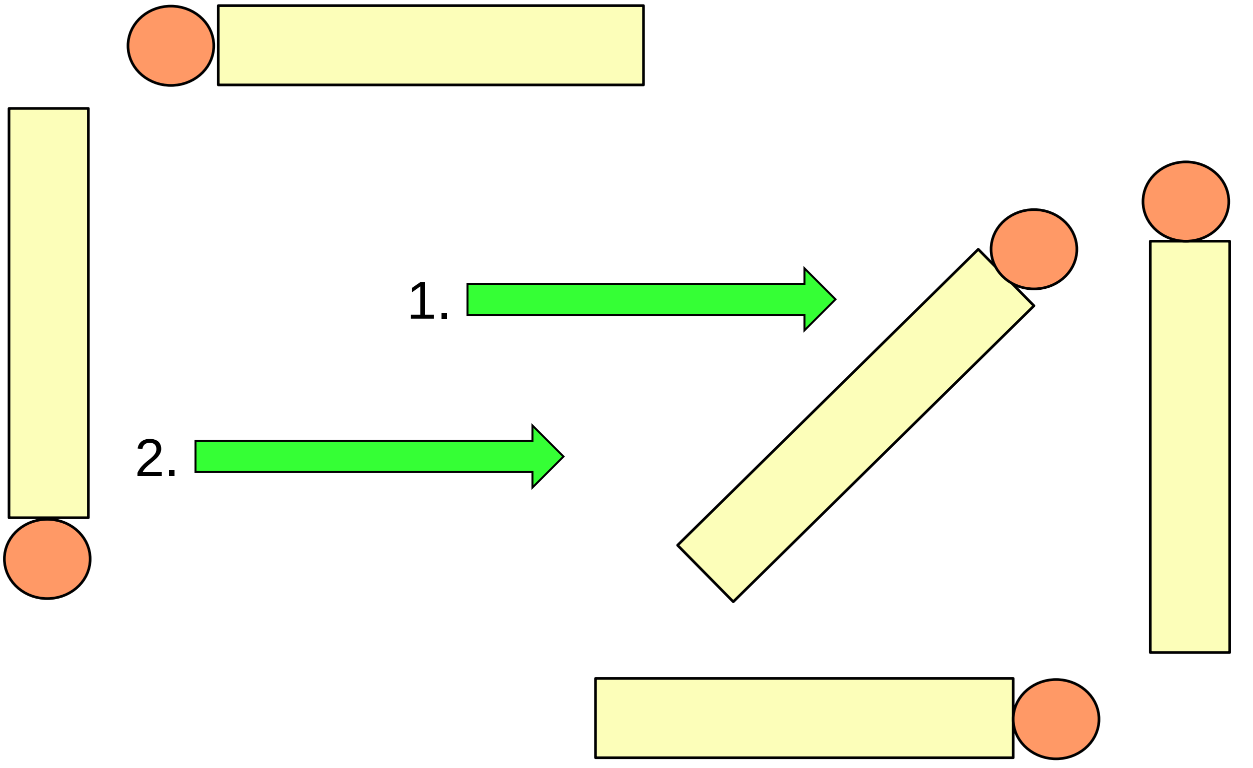
warning: this problem can drive people crazy!

(think a while, before you see solution on next page)

1. remove 3 matches

2. add 2 matches

to get the same picture !



π , the fast way

$$\frac{355}{113}$$

$$\pi = 3.141592\,6$$

$$\frac{355}{113} = 3.141592\,9$$

A real beauty

$$e^{i\pi} = -1$$

This beautiful formula from Euler (1707-1783)

contains **2 fundamental numbers** (e, π)

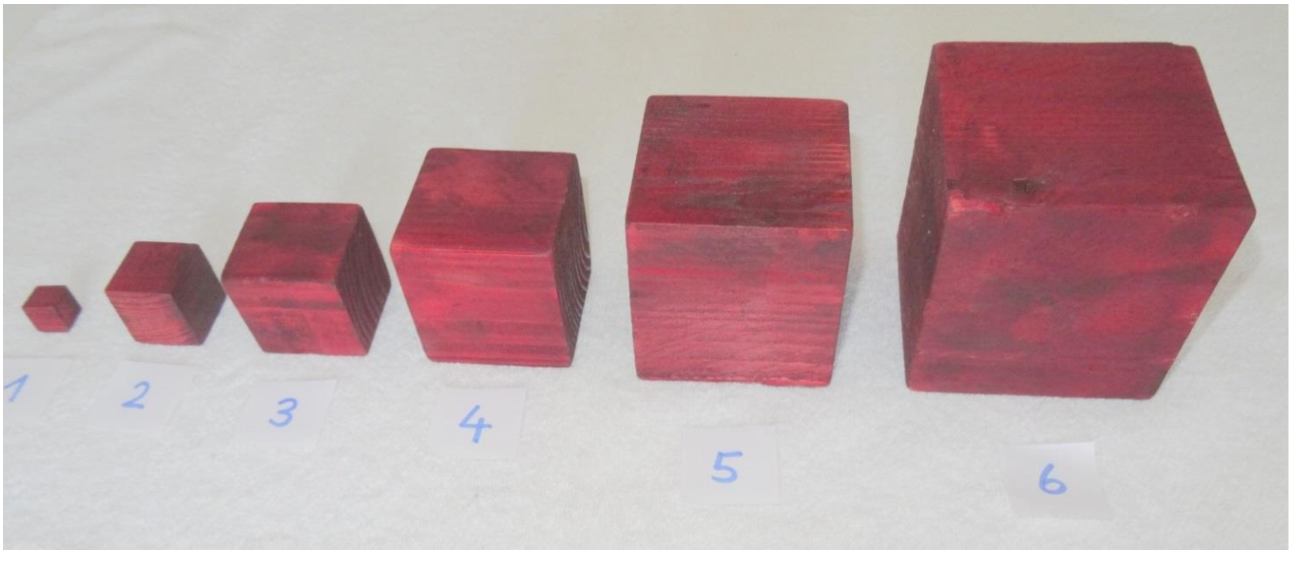
plus **3 important inventions** in mathematics:

1. the equal sign = (replaced the words: „**is equal to**“)
   invented in 1557 by the Welsh mathematician
   Robert Recorde who quoted:
   „**no two things can be more equal than these two bars**“
2. the negative numbers
3. the imaginary numbers with the unit i

Euler himself invented the symbols for e, π and i, as well as the symbols Σ for a sum and f(x) for functions.

Werner Joho

The Euler cubes



Here are 6 cubes with side lengths of

1, 2, 3, 4, 5 and 6 units.

Build, with the help of these cubes,

two piles, which have the same volume.

Solution: see next page!

Werner Joho

Beautiful formula

A nice one from Euler is:

$$3^3 + 4^3 + 5^3 = 6^3$$

from Ramanujan comes:

$$9^3 + 10^3 = 1^3 + 12^3 = 1'729$$

computers disproved 1966 a ‘theorem’ by Euler:

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5$$

I figured out myself the nice formula below; but it is mentioned already by the hungarian mathematician George Polya

$$\boxed{1^3 + 2^3 + 3^3 + \ldots n^3 = (1 + 2 + 3 + \ldots + n)^2}$$

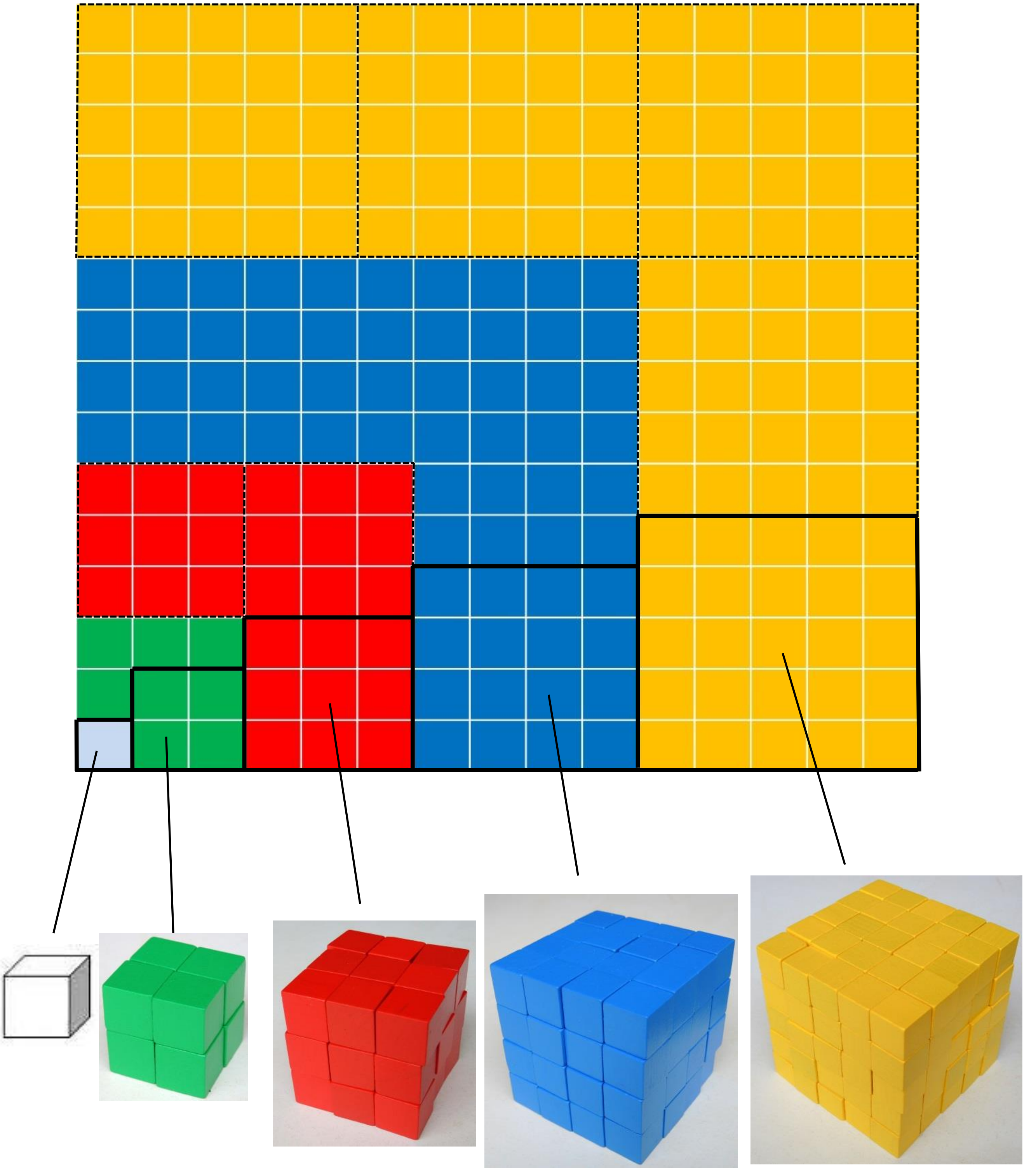
$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\frac{(n+1)^2 n^2}{4} \qquad\qquad [\frac{(n+1)n}{2}]^2$$

Werner Joho

Example with arrangement of 225 single cubes to a 15x15 square or to cubes of sides 1, 2, 3, 4 and 5

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1+2+3+4+5)^2 = 225$$



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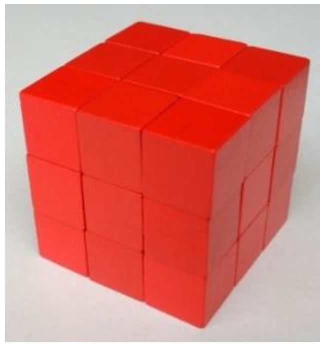
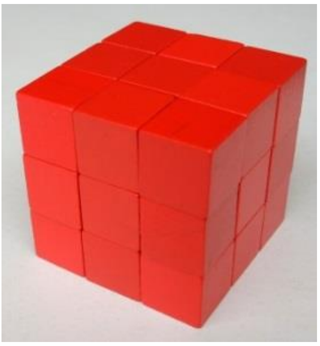
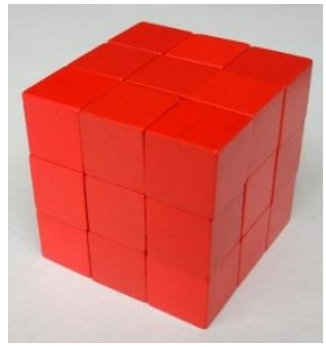
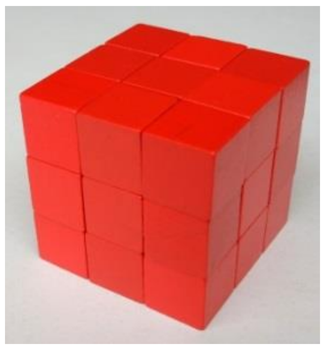
Euler: $3^3 + 4^3 + 5^3 = 6^3$

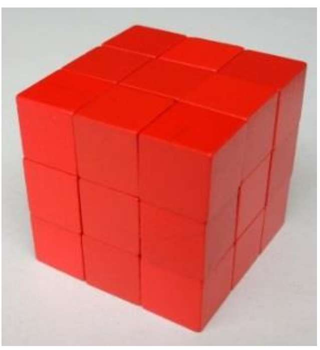
"reloaded" : $7 \text{ x } 3^3 = 4^3 + 5^3 = 189$

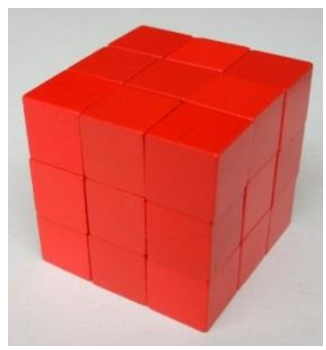


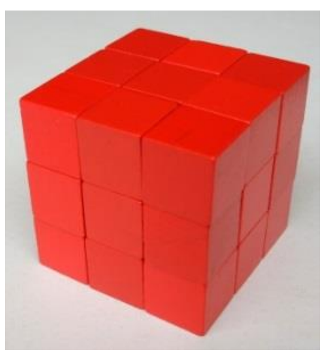












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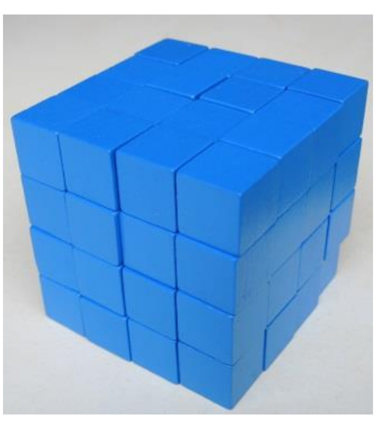

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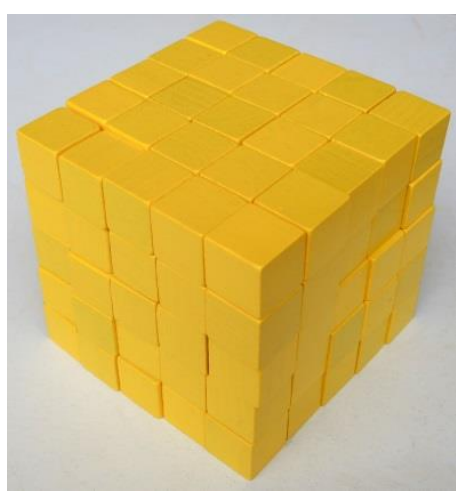
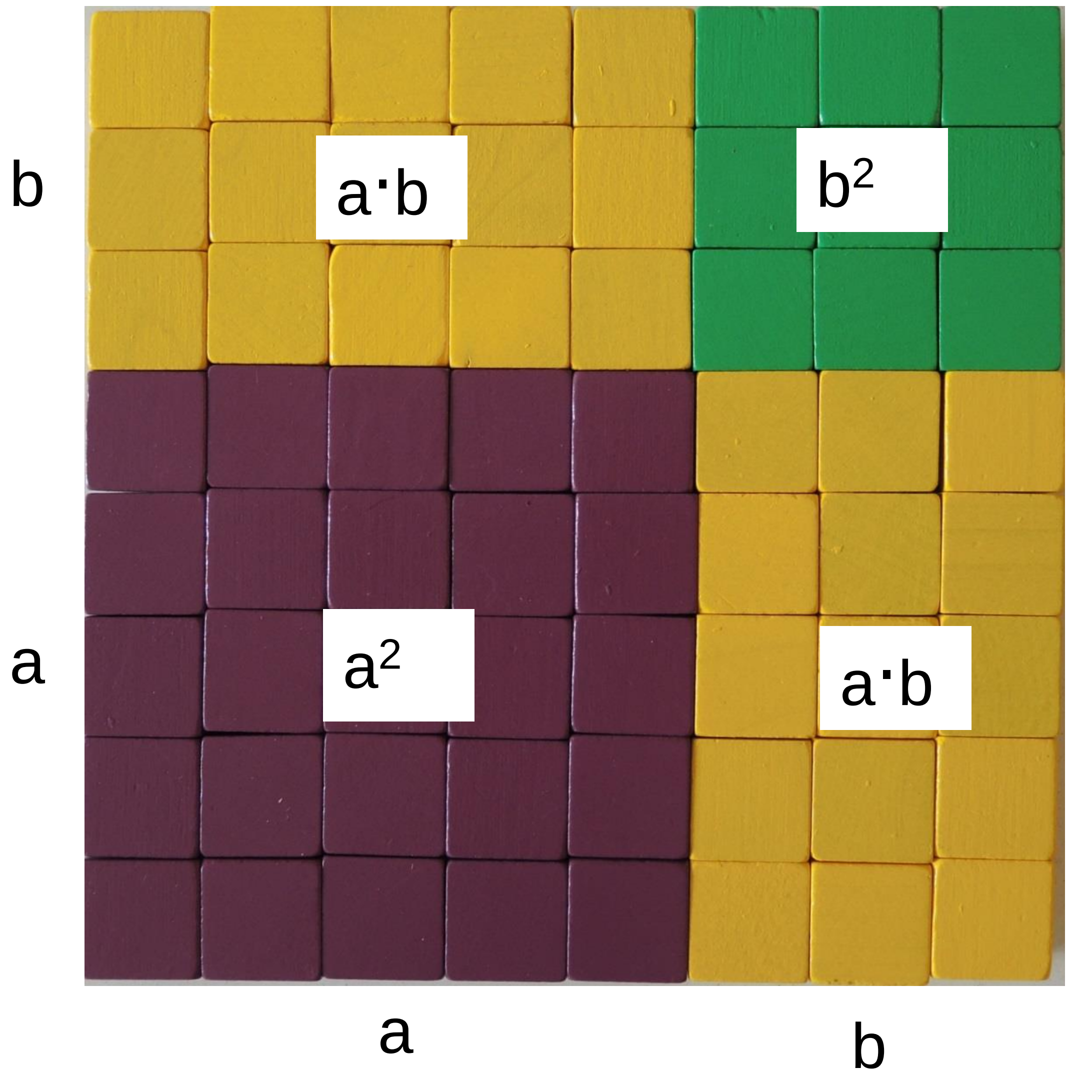


$(a+b)^2$ illustrated graphically



$$(a+b)^2 = a^2 + b^2 + 2 \cdot a \cdot b$$

example: $a=5$, $b= 3$

$$(5 + 3)^2 = 5 \cdot 5 + 3 \cdot 3 + 2 \cdot 5 \cdot 3$$
$$= 25 + 9 + 30 = 8^2 = 64$$

Werner Joho

3.5 x 3.5 in 3.5 seconds

Recipe:

$(n-\frac{1}{2})\ (n+\frac{1}{2}) = n^2 - \frac{1}{4}$

$$\boxed{n^2 = (n-\tfrac{1}{2})\ (n+\tfrac{1}{2}) + \tfrac{1}{4}}$$

for n = half integer:

n	n^2
2.5	$2 \cdot 3 + 0.25 = 6.25$
3.5	$3 \cdot 4 + 0.25 = 12.25$
4.5	$4 \cdot 5 + 0.25 = 20.25$
5.5	$5 \cdot 6 + 0.25 = 30.25$
6.5	$6 \cdot 7 + 0.25 = 42.25$
7.5	$7 \cdot 8 + 0.25 = 56.25$
…	
11.5	$11 \cdot 12 + 0.25 = 132.25$

Squaring numbers around 50 and 100

1) $x \approx 50$, $x \equiv 50 + n$

$x^2 = 50^2 + n \cdot 100 + n^2$

$\mathbf{x^2 = (x-25) \cdot 100 + n^2 = (25+n) \cdot 100 + n^2}$

e.g. $n = -2$, $x = 48$

$48^2 = 2'300 + 4 = 2'304$

this trick was used by the famous physicists Richard Feynmann and Hans Bethe

2) $x \approx 100$, $x \equiv 100 + n$

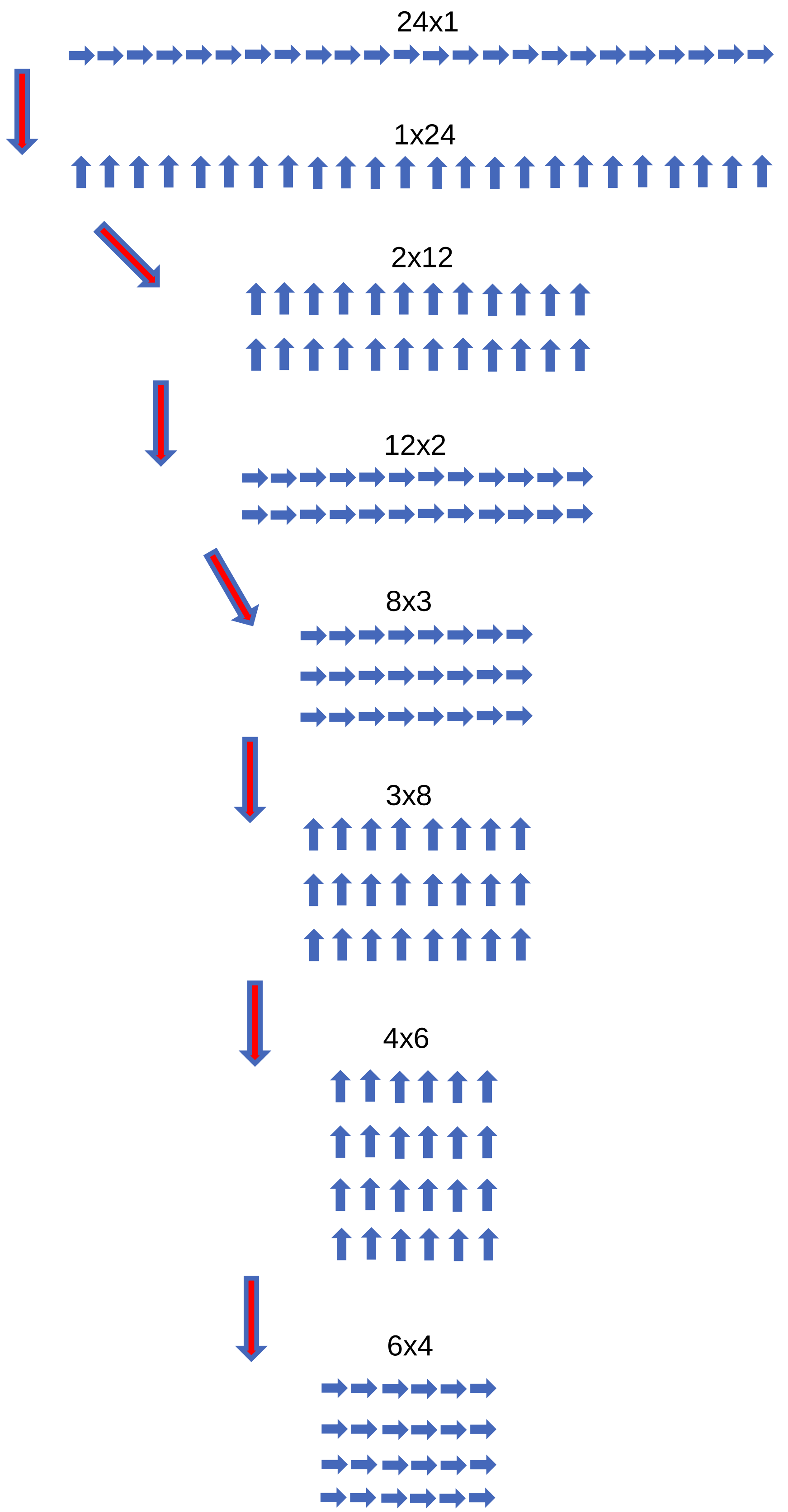
$\mathbf{x^2 = 100^2 + 2n \cdot 100 + n^2}$

e.g. $n = 4$, $x = 104$

$104^2 = 10'00 + 8 \cdot 100 + 16 = 10'816$

harmonic Tattoo Formations

8 rectangular formations with 24 people



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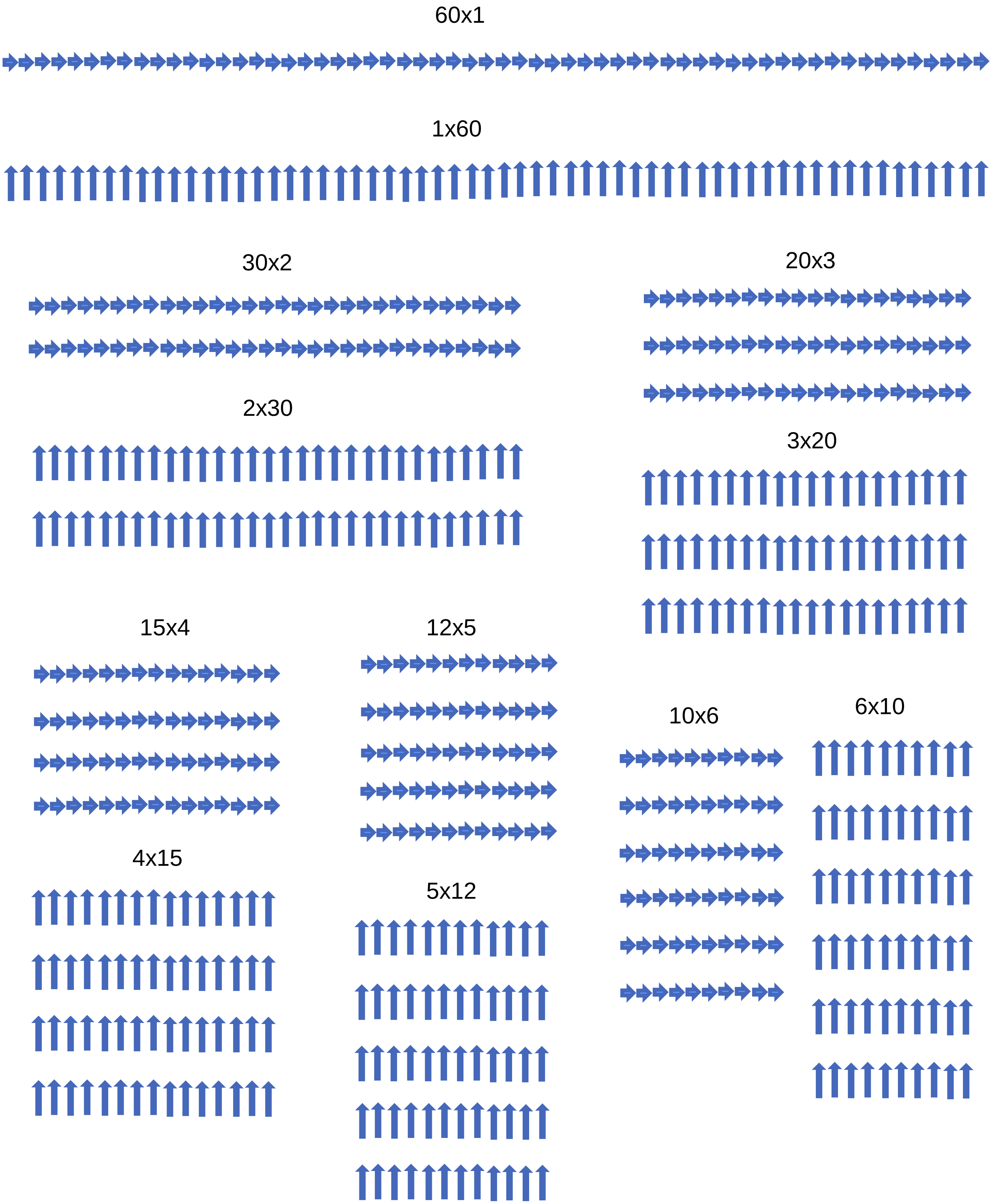
harmonic Tattoo Formations

12 rectangular formations with 60 people



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mathematical beauties

9 * 123’456’789 + 10 = **1’111’111’111**

888’888’888 : 9 = **98’765’432**

111’111’111 * 111’111’111 =

1234**567898765**4321

11’111’111’111 = 21’649*513’239

prime numbers

Test of multiplication skills

- Take a 3-digit number e.g. **569**
- multiply it with **7**
- then multiply the result with **11**
- and this result finally with **13**

Result :

$$7 \cdot 11 \cdot 13 \cdot \mathbf{569} = \mathbf{1'001} \cdot \mathbf{569} = \mathbf{569'569}$$

Werner Joho

The mystery of 1’001 arabian nights

The secret behind the 1’001 arabian fairy tales is finally resolved:

Sheherazade, the daughter of the visier,
avoided with her fairy tales for 1’001 nights
her execution by the king Sharyar.

But why 1’001? Since **1’001 = 7 x 13 x 11** ,
this means first a period of **7** days, then **13** weeks (=1 season), when Sheherazade found out she was pregnant and thus safe, and finally **11** seasons!

These **11** seasons are composed of:

3 seasons: first pregnancy

1 season: recovery

3 seasons: second pregnancy

1 season: recovery

3 seasons: third pregnancy

Werner Joho

exponential growth with compound interest

With an interest rate of 2% it takes 35 years to double the income (50 years without compound interest)

How can we get this result very quickly?

For a quick estimate of exponential growth one uses:

$$e^7 \approx 2^{10} \approx 10^3$$

The **magic number** is thus **70 years** ($70 \approx 100 \ln 2$):
With an interest rate of p(%) it takes T_2 years to double an initial capital investment C_0.

$$T_2 = 70 \text{ years}/p(\%)$$

To have an increase by a factor of 1‘000 ($\approx 2^{10}$) it takes T_{1000} years:

$T_{1000} = 10\ T_2 = 700$ years/p(%)

Saving account of William Tell

700 years ago William Tell opened a bank account with 1 Fr.

Tell decided, that always the oldest child inherites this account.

With an interest rate of p % this account is worth today **10^{3p}** Fr.

=> With a **3% interest** this amounts to **10^9 = 1 Billion Fr**.

But 1/3 of the annual interest always went to the state for

taxes. Thus the net interest rate is only **2%**

=> The sole inheritor of Tell's account owns today "only"

10^6 = 1 Million Fr.

Where are the remaining 999 Millions?

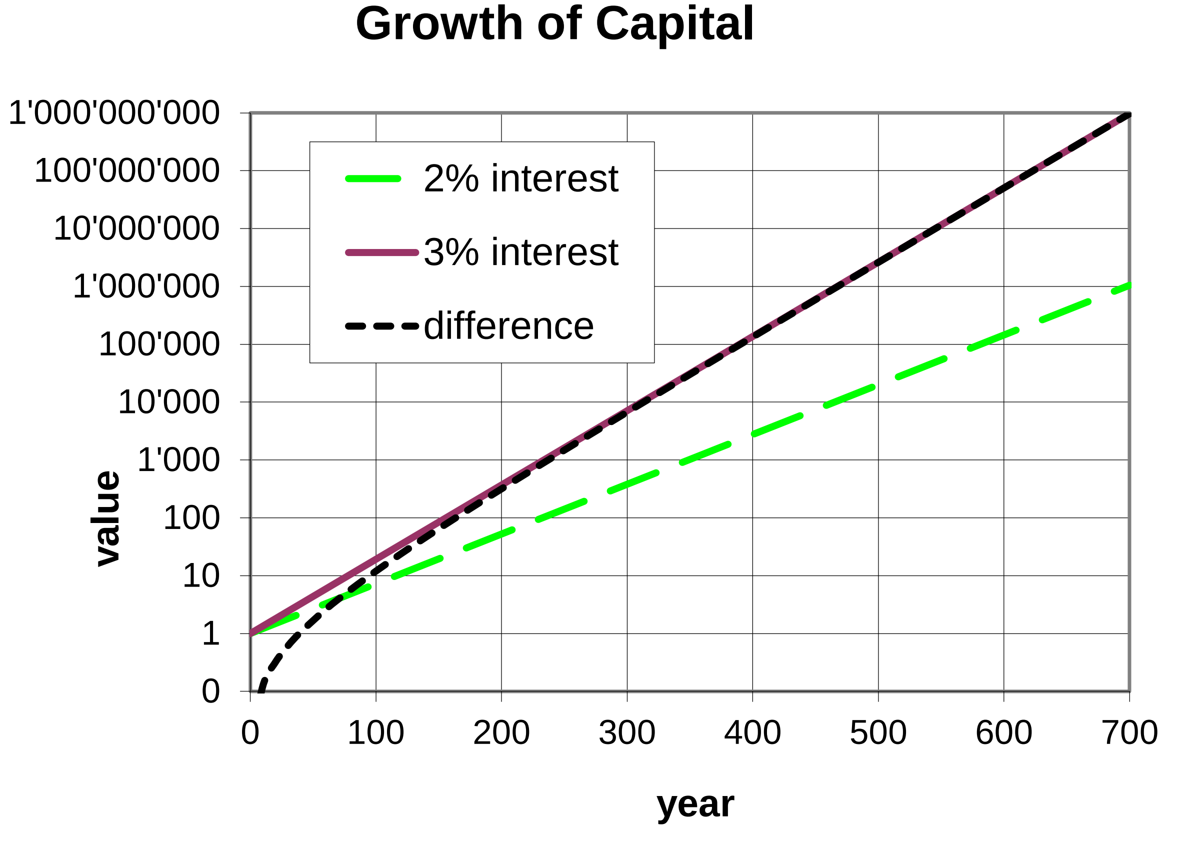
Werner Joho

The state takes it all !

It is hard to believe, but the remaining **999 Million Fr.** belong **to the state!**

Because the state does not pay taxes to himself and gets the full 3% interest on his income from taxes.

After one year the state owns only 0.01 Fr. from the income tax of Tell. But after 70 years the state owns already 4 Fr. , the same amount as William Tell. But now the difference between the exponential growth of 2% and 3% comes fully into play!



Werner Joho

treacherous predictions !

The following formula was constructed by the Swiss Physicist Leonard Euler:

$$P(n) = n(n-1)+41$$

Believe it or not, but for n=1, 2, 3….up to 40
this formula gives a **prime number** !
It fails the first time at n=41, where
P(41)=41*41=1'681
(It then fails further at n=42, 45, 50, 57, 66 etc.)

If you see a series of numbers: 2, 4, 6, 8, 10, 12, …
created by a formula F(n), for n=1, 2, 3, ...6
you probably guess, that the next term is 14 !?

Now give me the number **Y**, the year you were born.
I give you below a formula F(n), where the next term
in the series (for n=7) is not 14, but exactly **Y** !

$$F(n)=2n+ (Y-14)(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)/6!$$

Werner Joho

Phoning "the speed of light" to Greenland

Up to 1983 the speed of light (= c) was determined by physical measurements.

But in this year, the "Urmeter" was given up as the standard unit for length.

Instead the speed of light was fixed as a constant:

$$C = 299'792'458 \ \text{m/s}$$

I tried to phone "the speed of light" as a phone number to Greenland, since 299 is the country code for Greenland. But this number does not exist (yet).

In 2011 I wrote a letter to Mr Jens Frederiksen, responsible for the telephone services in Greenland.
I suggested, that he should install such a number, and that a person phoning this numer would get e.g. some information about science projects in Greenland.
I did not get an answer.
Maybe somebody else is more successful?!

In Switzerland there is no area code with 029,
but maybe in some other countries?

Werner Joho

Birth and Mortality Rate world wide

How many babies are born each second?

and how many people die each second?

Simple model:

- there are 7.4 billion people on earth
- average lifetime 70 years

$\Rightarrow$ average “exchange rate” for **constant** population:

$7.4 \cdot 10^9 / 70$ = 100 million people/year. But with growth rate of 1.1%/year: increase of population by **80** million people/year (=> doubling time = 64 years)

$\Rightarrow$ **birth rate ≈ 140 million/year** (100+½·**80**) **≈ 4.5 babies/s**

$\Rightarrow$ **death rate ≈ 60 million/year** (100-½·**80**) **≈ 2 deaths/s**

=> 2.5 additional people on earth every second

actual numbers in 2012 were:

≈ 136 million births/year, ≈ 56 million deaths/year

(≈ 20 % from hunger!)

a human street around the world

All people living on earth are standing in rows on a

50m wide and 40’000 km long street around the earth

$\Rightarrow$ 7.2 Billion people/40’000 km = 180 persons/m

assume a difference of 0.5m between 2 rows

=> 90 people on each row

during an average lifetime of **80 years**

the people move once around the world

=> velocity = 500km/year = 1.4km/day = **1m/min**

Like in a queue: one small step every 30s !

Werner Joho

special dates in your life

You lived

100’000 hours after about 11 years and 9 months

1’000 weeks after about 19 years and 2 months

10’000 days after about 27 years and 5 months

1 billion seconds after about 31 years and 9 months

and: if you are young,

there are worldwide about **350’000 persons**

who were born on the same day as you!

about **250’000 persons** if you were born around 1940

Werner Joho

1’000 birthday balls

Today about 7 billion people are living on earth.

Before you were born, you could draw one single ball from a bowl with 1’000 balls. This ball determined where you were born. Each ball represents 7 million people. As an example: I myself has chosen the only ball, which was marked with "Switzerland"!

The majority of these 1’000 balls, namely about 600, represent a country in Asia. For Africa there are 150 balls. The rest is distributed to the other continents: Europe incl. Russia 110, Latin America 80,
North America 50 and Australia 6.

If you are unhappy with your life, would you rather have another chance?

Probably not. If can you read this article you are already privileged, because on 500 of the original balls was a note saying:
You will have no access to sanitary installations and you will not earn more than 3$ per day !
On 140 balls is written: **you will always be hungry.**

Your new gender will be chosen by a toss of a coin.
There is thus a 50% chance, that it will change!

Werner Joho

Friday the 13th

Are you superstitious ?

What is **N**, the number of those days

you did experience so far ?

Simple estimate:

- each year has 12 times a 13th
- the chances for a Friday are 1/7

$\Rightarrow$ **N ≈ your age x 12/7**

$\Rightarrow$ at age 58 you survived

about 100 times a Friday the 13th !!

Werner Joho

Heart ⇔ Motor

What is more reliable,
your heart or the motor of your car ?

Heart	2.5 Billion heart beats
Motor	0.5 Billion cycles

Assumptions:

- a car can make about 200’000 km with an average speed of 50km/h
  => runs for about 4’000 h or 240’000 min.
- the motor runs at an average of 2’000 cycles/min

=> the motor makes about **$0.5 \cdot 10^9$ cycles**,

If you live 80 years, your heart has made about

$2.5 \cdot 10^9$ heart beats (non stop!)

=> your heart will make about a factor 5 more cycles than the motor of a car!!

Werner Joho

Stars and Galaxies in the Universe

In our galaxy, the milky way, there are approximately

$400 \cdot 10^9$ stars.

… and in the universe there are an estimated

$200 \cdot 10^9$ galaxies.

To put these numbers in relation with the people on earth:

Each person on earth can give a personal name to

50 stars in our galaxy

… and can give personal names to

25 Galaxies in our universe !!

Werner Joho

How much longer does humanity exist?

Hypothesis: till every person who ever lived gets his own star in our galaxy!

=> until about **400 billion people** lived on earth**.**

According to the old testament (Exodus 32; 13) Moses was swearing to Abraham, Isaac and Israel: “**I will make your descendants as numerous as the stars in the sky**”.

About 100 billion people have lived so far. Lets suppose, that the population grows till it reaches about 30 billion people in the year 2300 and stays constant after that.

Then in the year 2700 about 400 billion people have ever lived and each of them can own now a star.

=> Humanity ends after another 700 years !

Werner Joho

$E = mc^2$

The Swiss physicist Paul Scherrer gave a beautiful analogy (mentioned by Max Frisch in „Stiller") for this famous Einstein equation:

„This energy E is deposited on a blocked bank account"!

$E = mc^2$ gives an enormous energy density

- 1kg ⇔ 10^{17} J = 25 TWh ,
  energy consumed in 5 weeks in Switzerland !
- Fission of U_{235} – nucleus with Neutrons
  => chain reaction
- **1 kg** natural-Uranium => **7 g** fissionable Isotopes U_{235} ,
  => **7 mg** can be converted into kinetic energy
  = 175 MWh , equivalent to **20'000 lt Oil**
- => enough energy to put a 30 t lorry
  into a satellite orbit !

Is Antimatter the Energy Solution ?

Visit of a sattelite from a galaxy with Antimatter :

10 ton of antimatter

1. with controlled annihilations:

   energy supply for the whole earth for 3 years !

2. with uncontrolled annihilations:

   energy of 30 Million Hiroshima bombs !!

Theorem on Salary

Dilbert's Theorem on salary states, that Engineers, Teachers, Programmers and Scientists can never earn as much as business executives and sales people.

Mathematical proof (unknown source):

Postulate 1: Knowledge = Power (Knowledge is Power)

Postulate 2: Time = Money (Time is Money)

Postulate 3: Power = Work/Time (law of physics)

$\Rightarrow$ Knowledge = Work/Time (since Knowledge = Power)

$\Rightarrow$ Knowledge = Work/Money (since Time = Money)

Solving for Money we get:

Money = Work/Knowledge

=> As Knowledge approaches zero, Money approaches infinity, regardless of the amount of work you do.

Conclusion:

The Less you Know, the More you Make!

No risk, no fun !

The author enjoys some "risky walking" in the pre-alps of Appenzell in Switzerland (2009)

Werner Joho

Herman

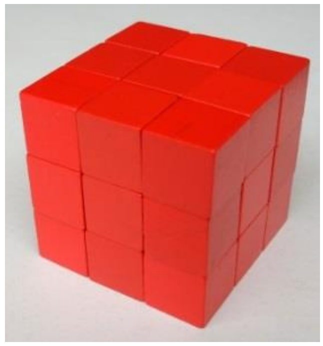
The last word belongs to the cartoonist

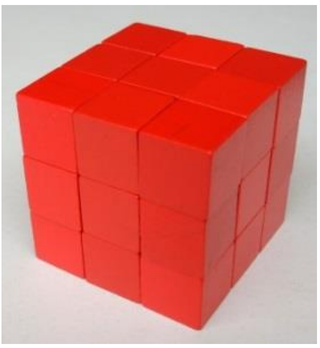
Jim Unger (1937-2012) with his figure "Herman"

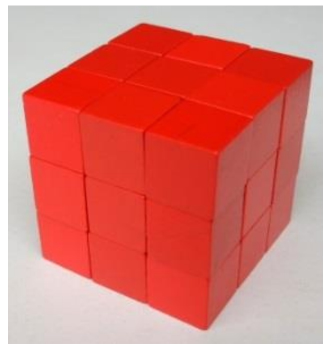
What is he ever going to need to know about algebra? Stick to this country!

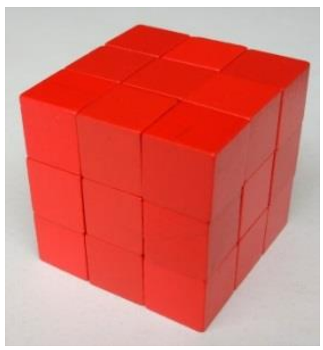
Euler: $3^3 + 4^3 + 5^3 = 6^3$

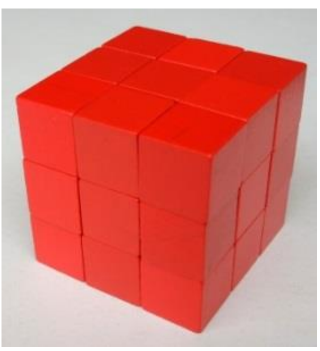
"reloaded" : $7 \times 3^3 = 4^3 + 5^3 = 189$

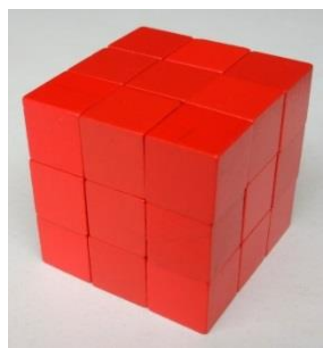


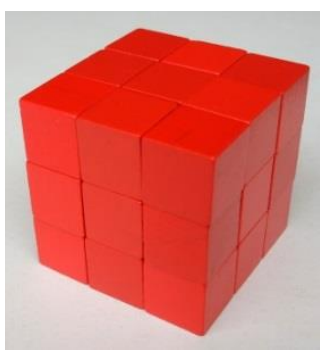












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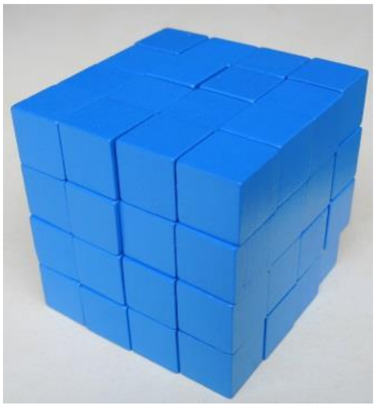

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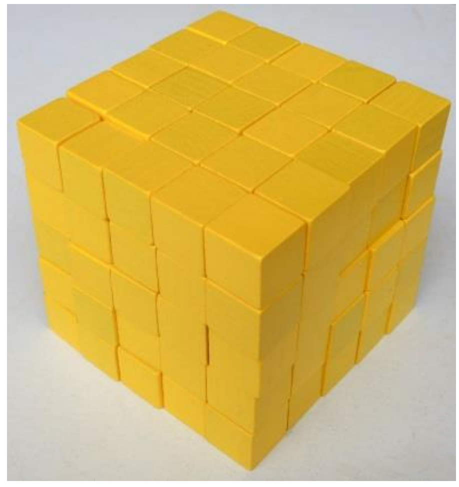

Varia

Werner Joho

5 Fridays, 5 Saturdays, 5 Sundays in January

Sometimes I get a mail from people claiming,

that it happens very seldom, that there are

5 Fridays, 5 Saturdays and 5 Sundays in **January**

This is a joke! because this happens

each time when the 1.January is a Friday.

Thus on average about once in 7 years.

The last times in 2010 and in 2016.

Werner Joho