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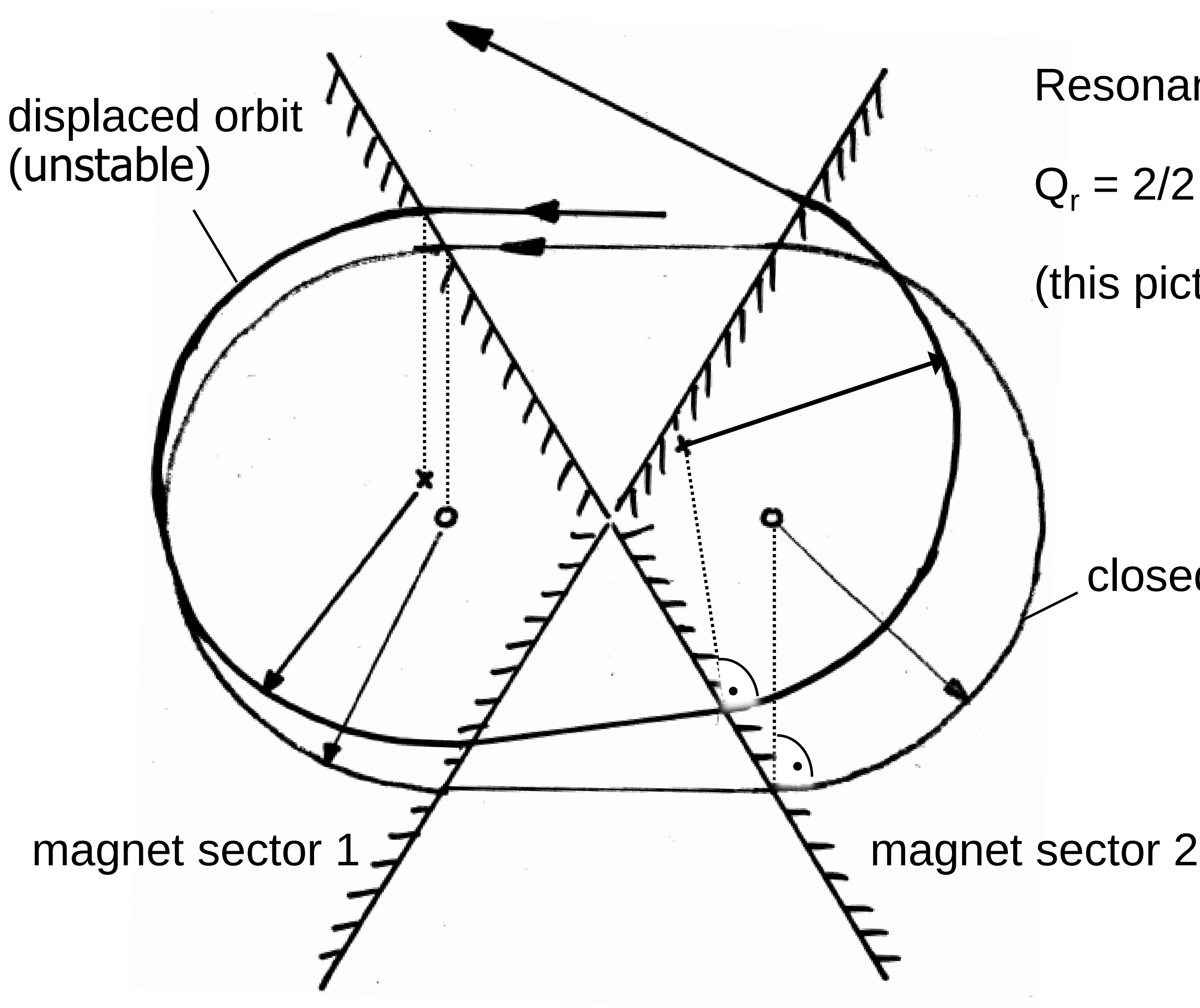
cyclotron specials

Werner Joho

Paul Scherrer Institute

Villigen, Switzerland

unstable 2-sector Cyclotron

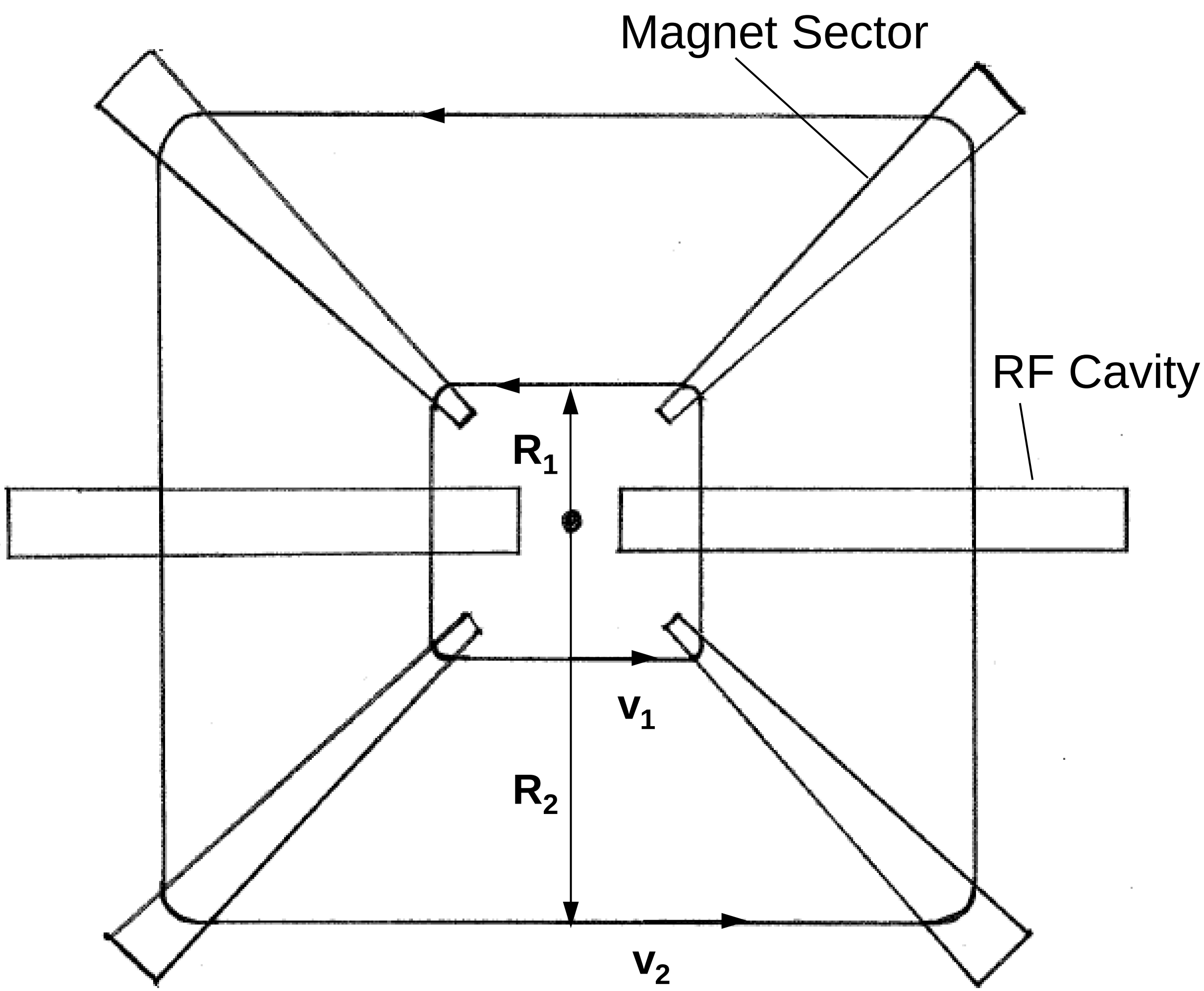


Resonance of horiz. focusing frequency:

$Q_r = 2/2$ (π-mode stopband)

(this picture can be constructed by hand!)

„Square Cyclotron“



isochronous condition :

constant revolution time,

independent of energy

$$\Rightarrow \quad \boxed{\frac{v_1}{v_2} = \frac{R_1}{R_2}}$$

the magnet sectors in this hypothetical example are too narrow and would lead to vertical overfocusing !

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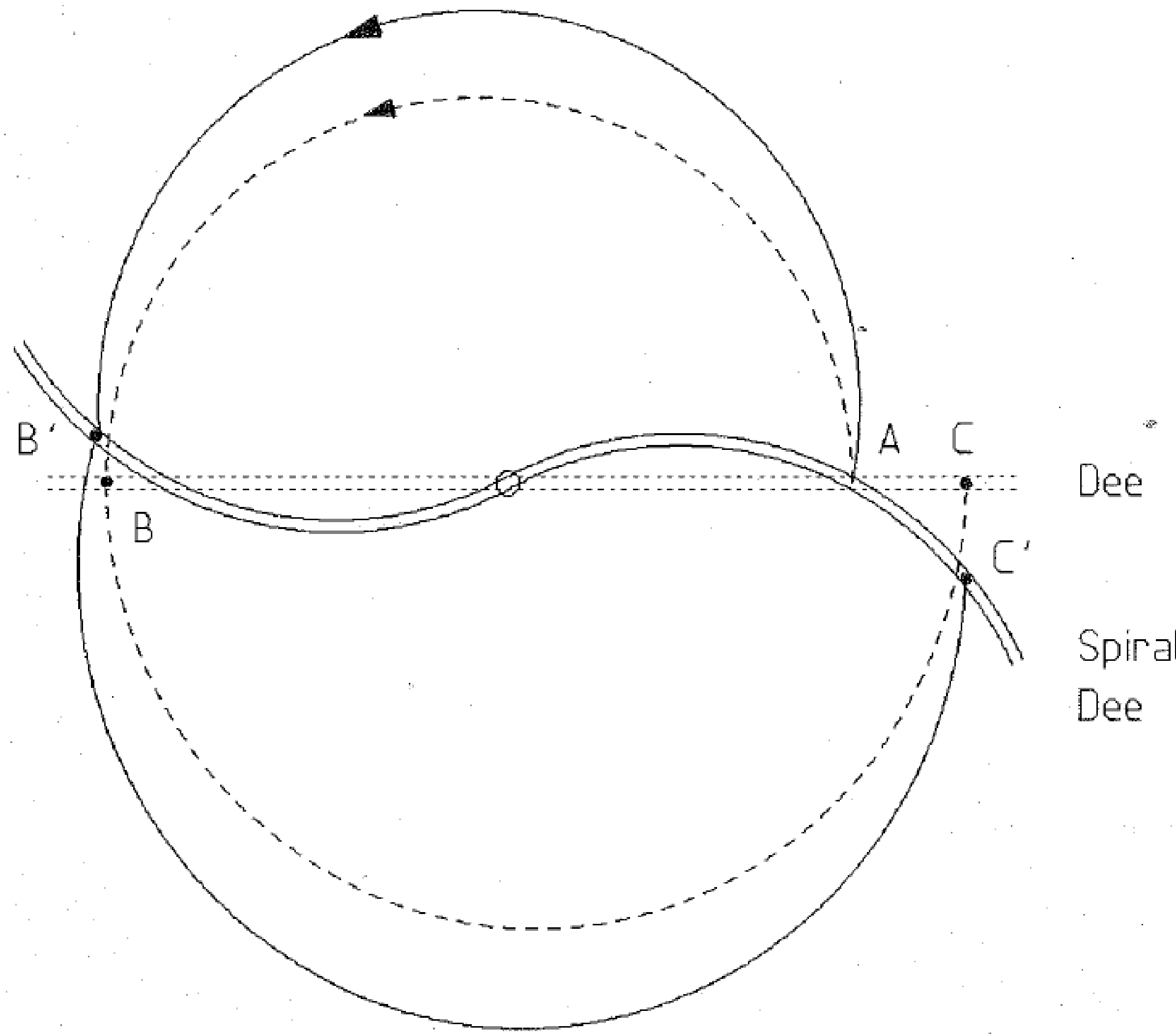
spiral dee ?

In a classical cyclotron with a radially decreasing field, the revolution frequency decreases with energy.

The "clever" idea of compensating this with a Dee, which spirals towards the oncoming particles, does not work!

There is no effect on isochronism due to the radial kick at the Dee!



Center Region of TRIUMF Cyclotron

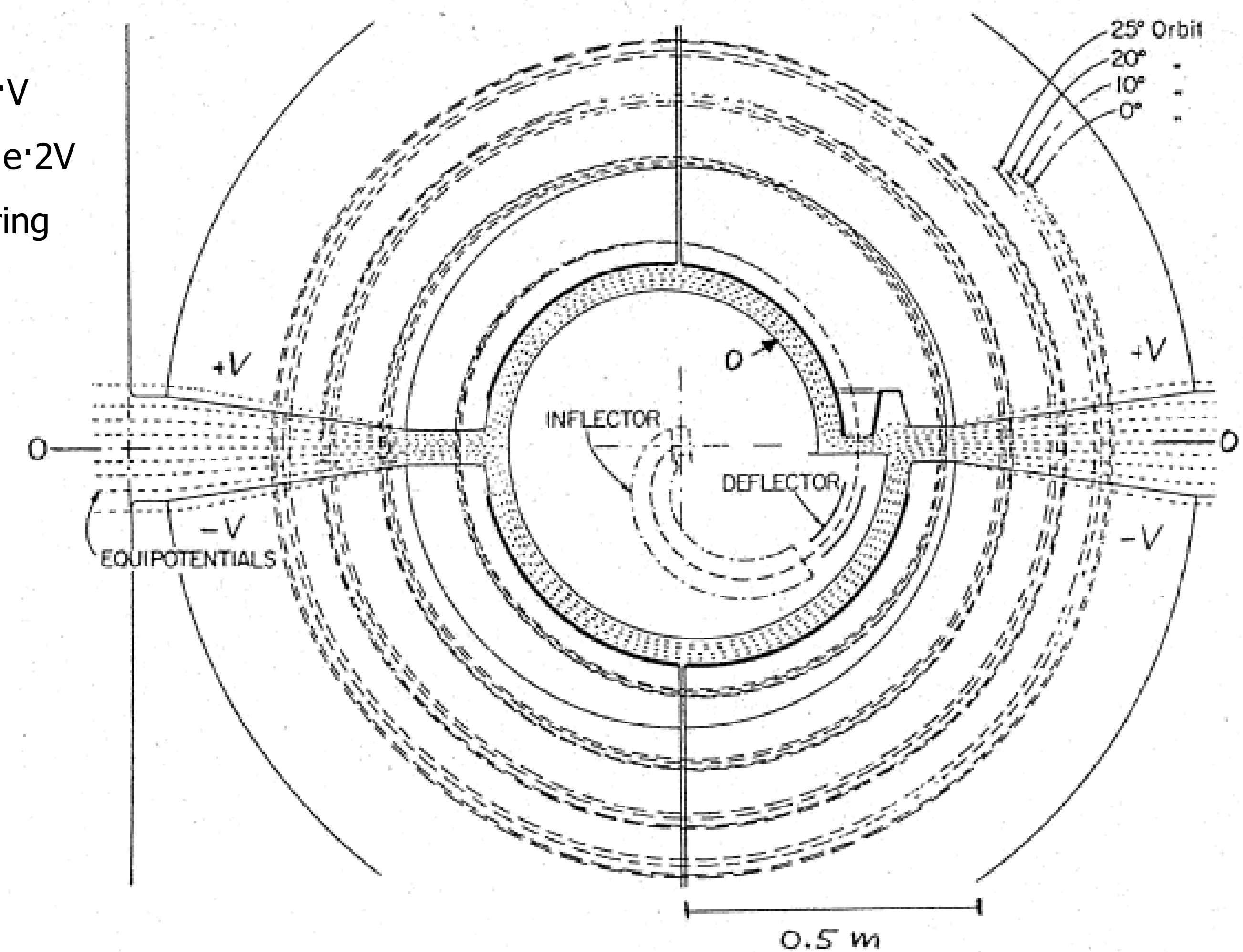
energy gain at first gap: e·V

on successive Dee crossings: e·2V

this gives ideal beam centering

for different RF-phases

(G.Dutto 1971)



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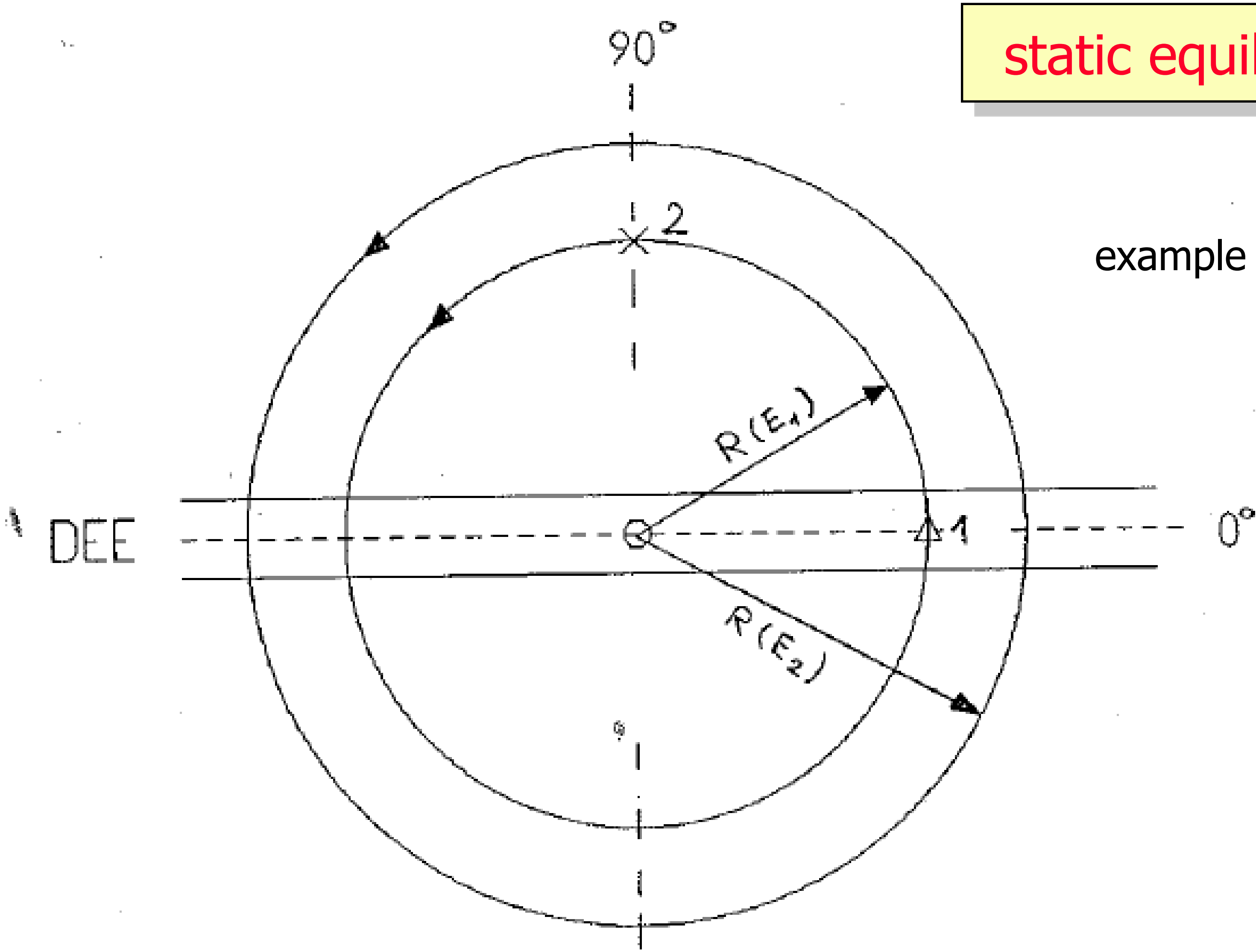
static equilibrium orbits

example with 180^0 Dee



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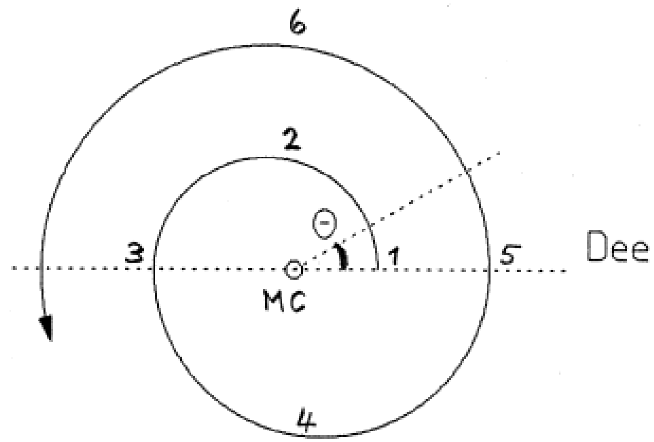
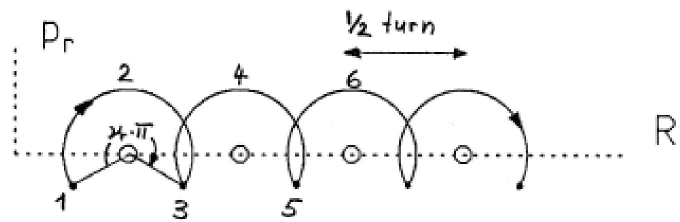
accelerated orbit

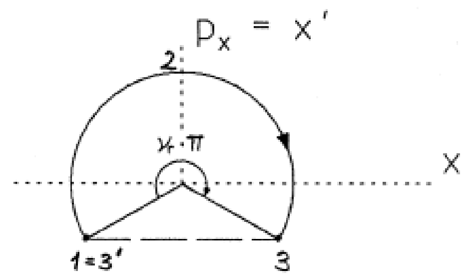
example with 180^0 Dee



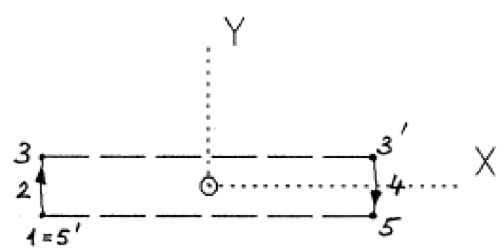
real space



phase space (R, p_r)



phase space (x, p_x)



orbit center coordinates

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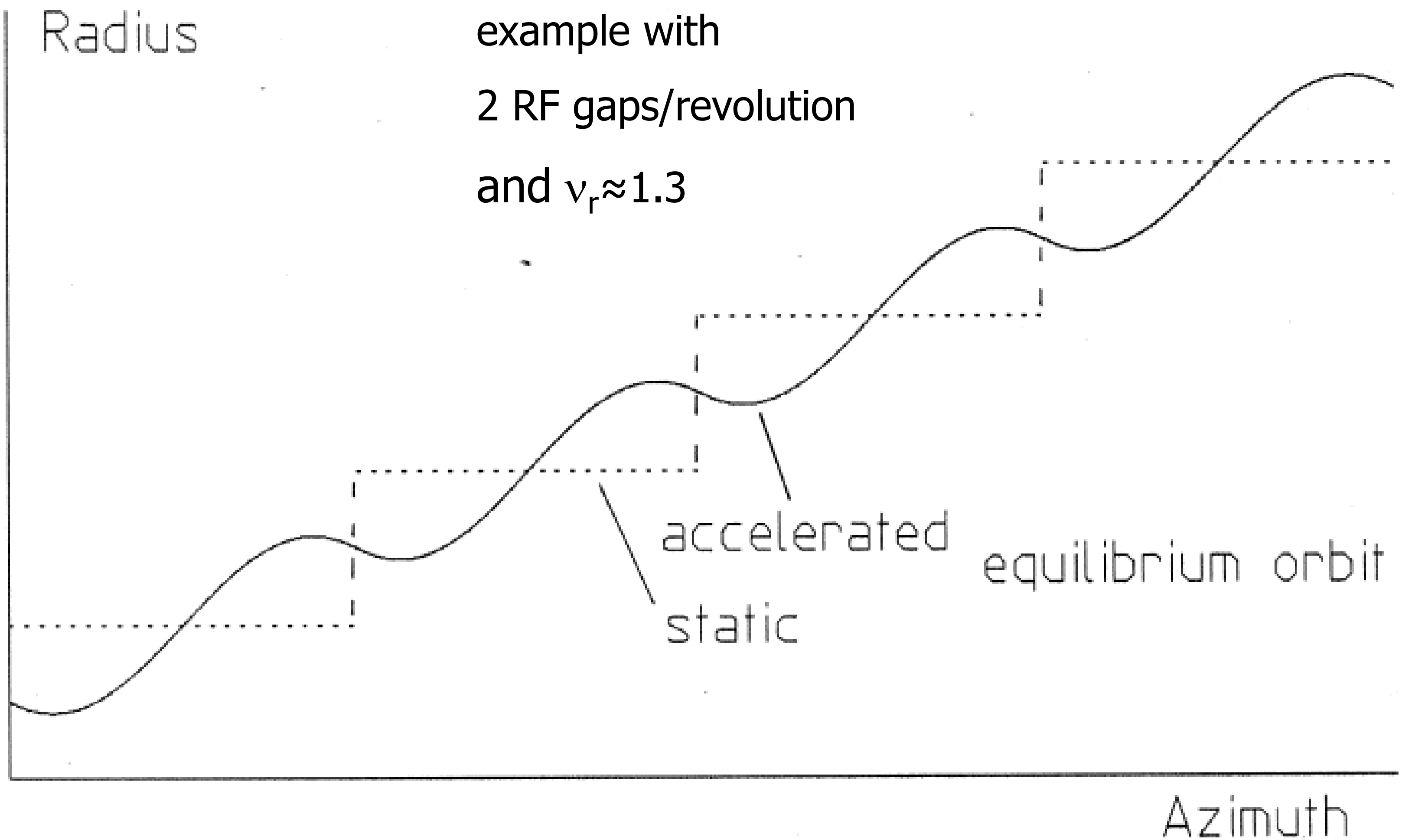
equilibrium orbits in cyclotrons

example with

2 RF gaps/revolution

and $\nu_r \approx 1.3$



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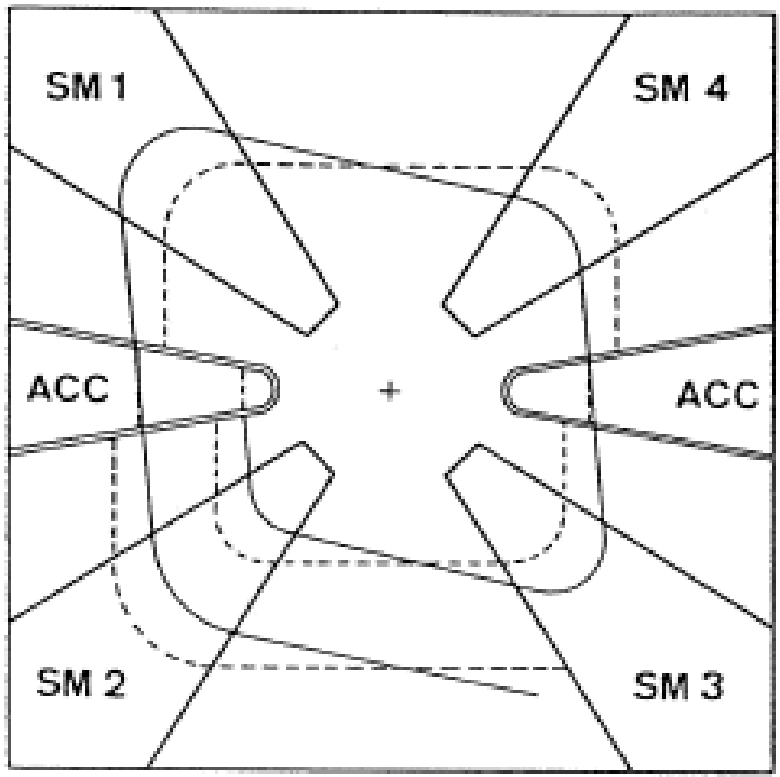
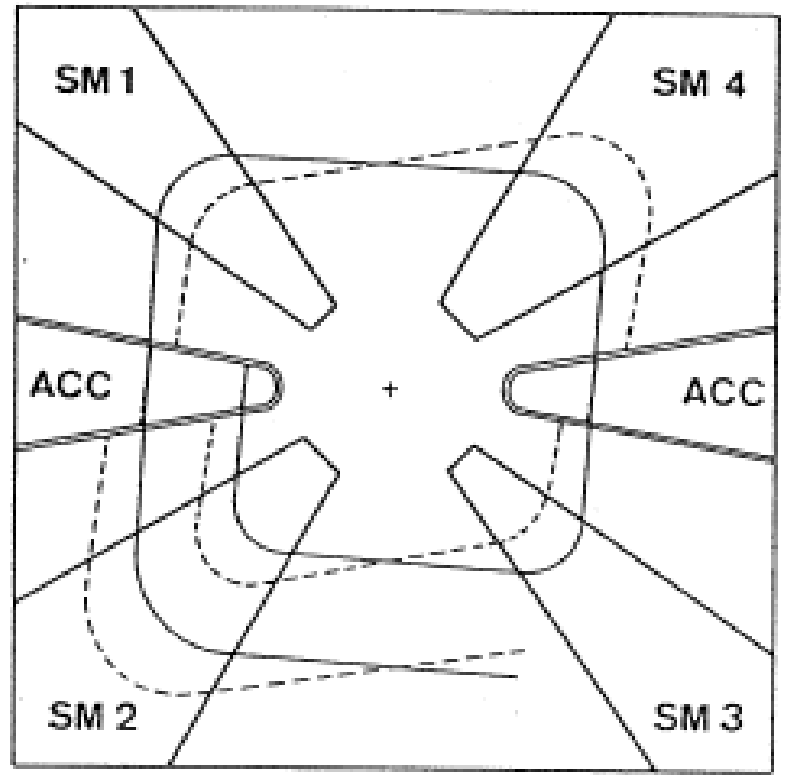
Regularization of Orbits in Injector II



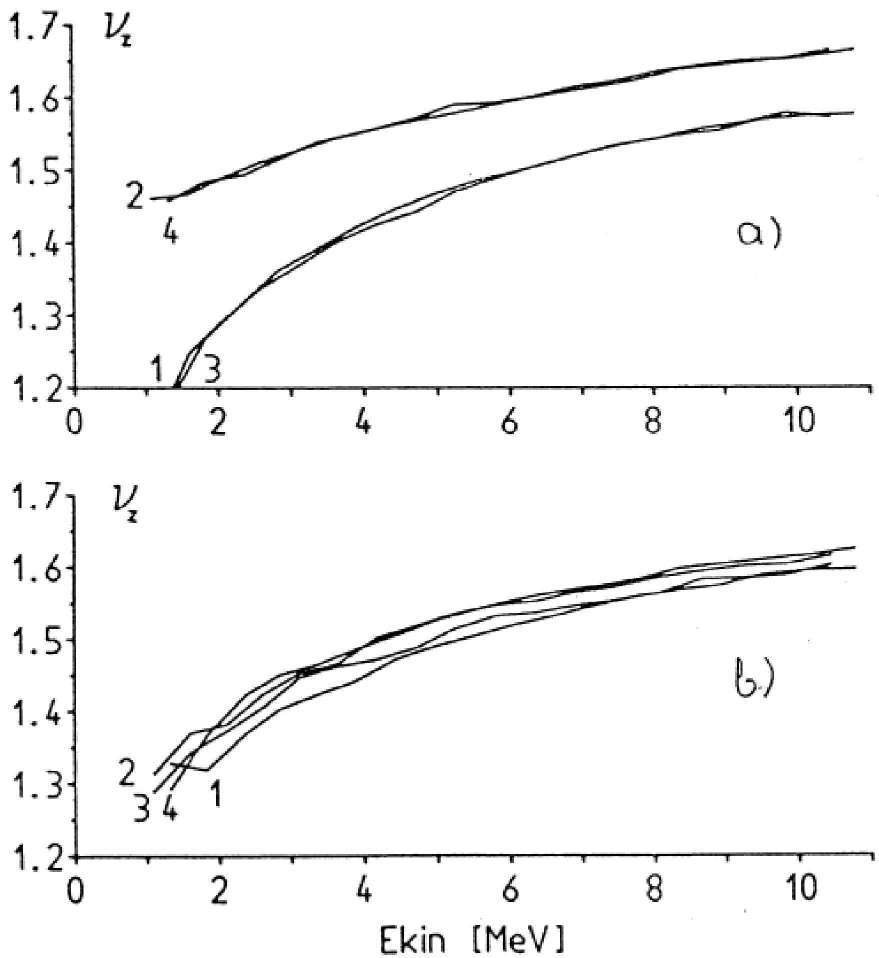
all 4 sectors have the same width => **accelerated** orbit has only 2-fold symmetry



sectors 1 and 3 are narrower than 2 and 4 => accelerated orbit has now 4-fold symmetry

W.Joho 2011

Regularization of Focusing Tunes



all 4 sectors have the same width

=> **accelerated** orbit has only 2-fold symmetry

sectors 1 and 3 are narrower than 2 and 4

=> accelerated orbit has now 4-fold symmetry

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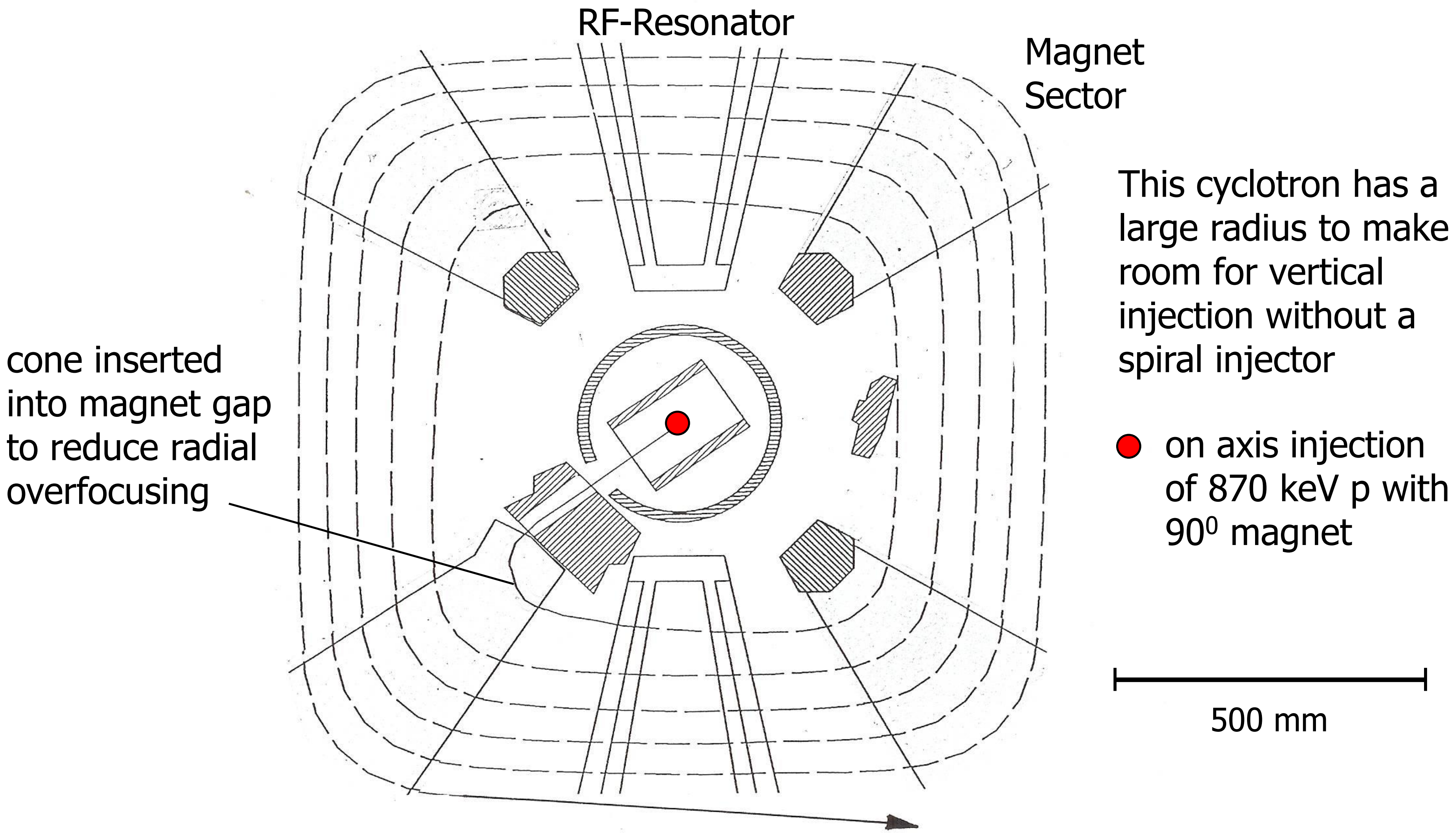
Injector II, Central Region



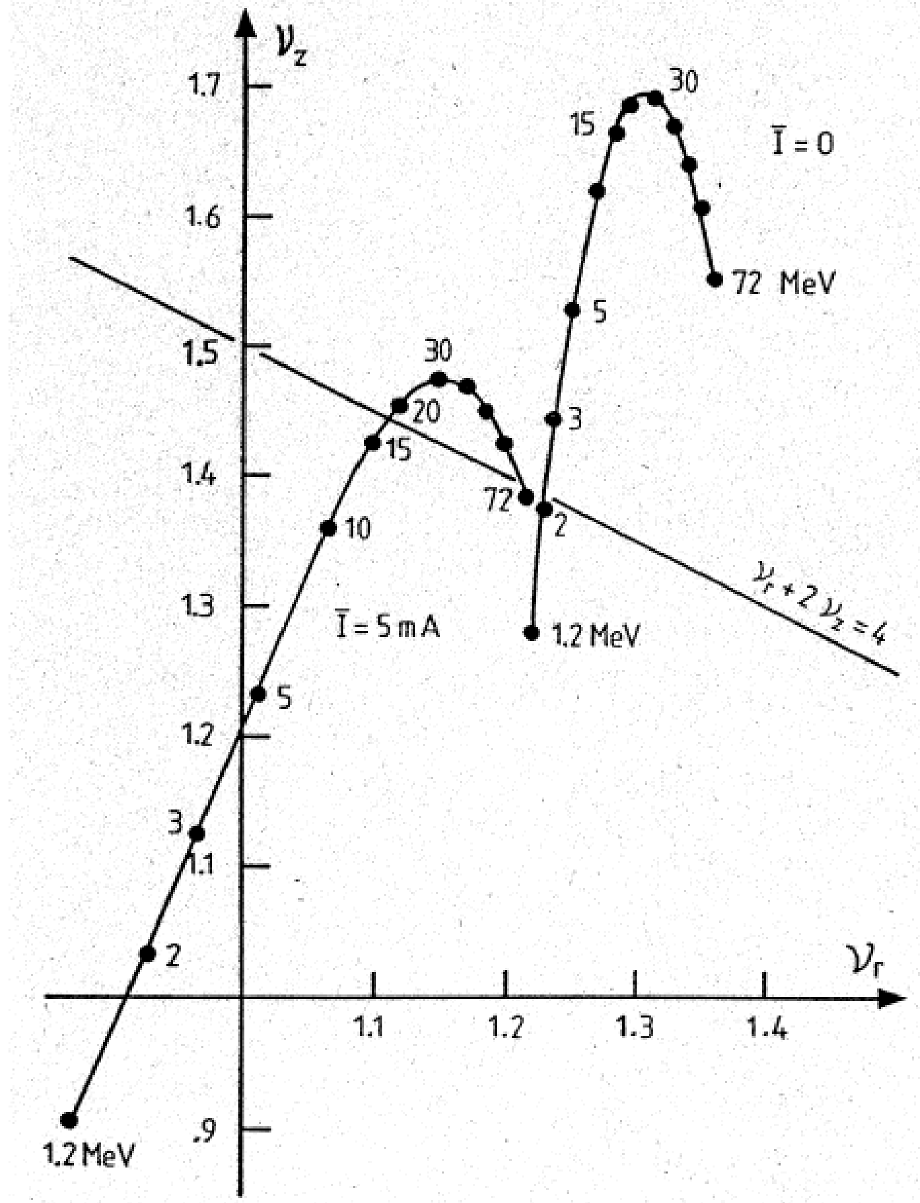
This cyclotron has a large radius to make room for vertical injection without a spiral injector

- on axis injection of 870 keV p with 90^0 magnet

Focusing tunes Injector II

Due to the strong vertical focusing in a sector cyclotron the current limit due to the space charge tune shift is a few mA in Injector II

Resonances can be crossed very fast with high RF voltages



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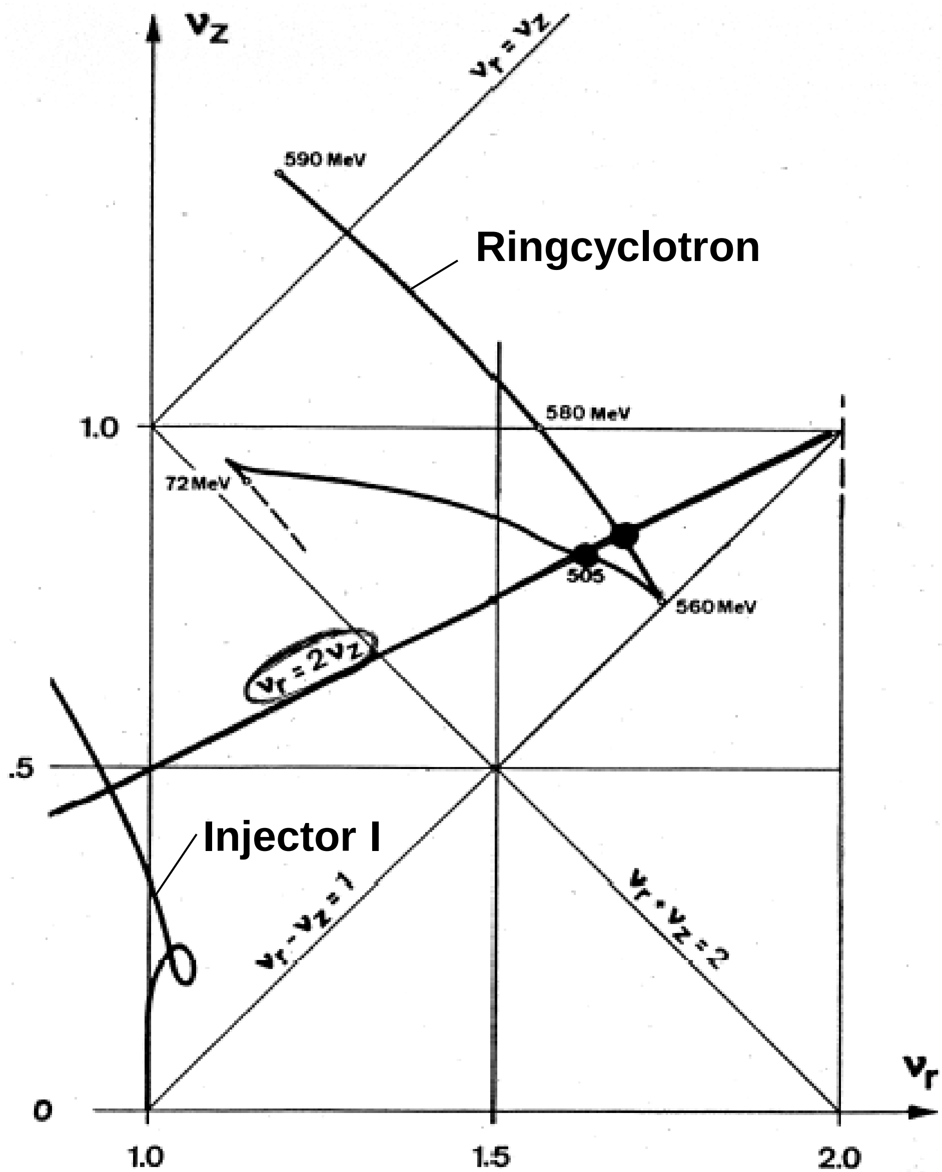
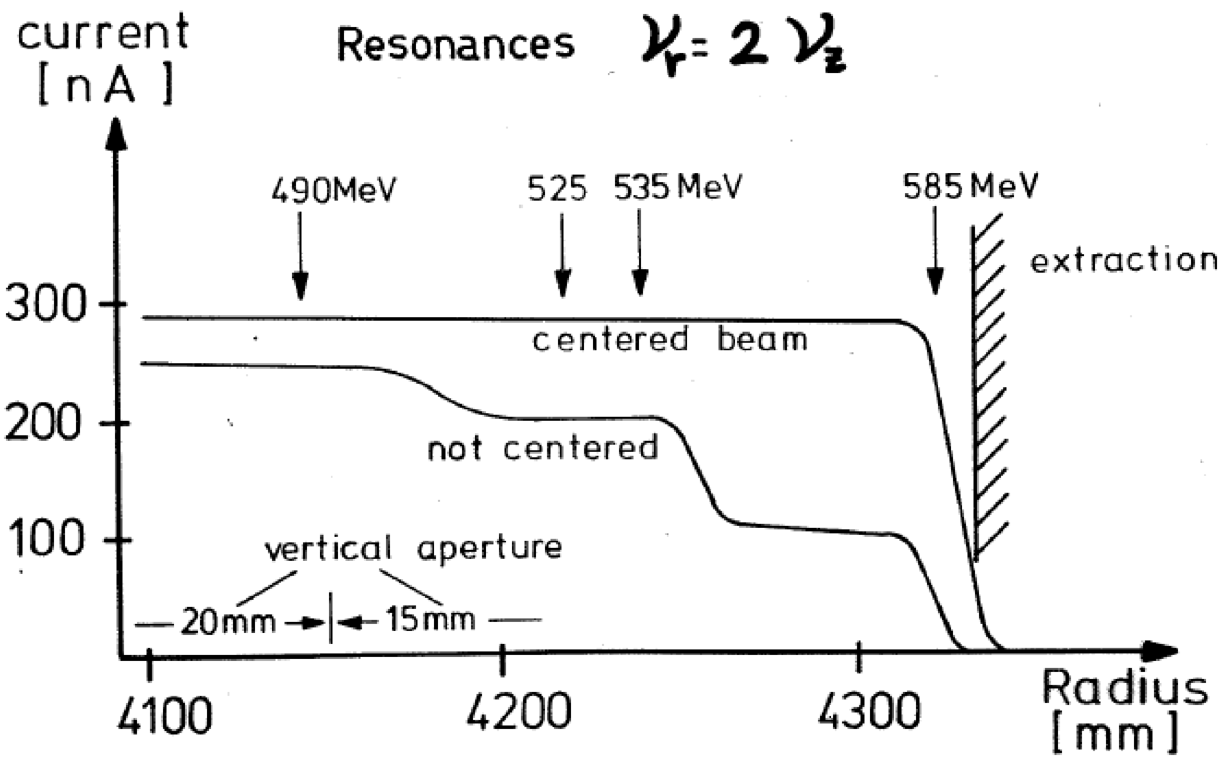
Resonance Diagram of focusing frequencies



In the Ring Cyclotron the coupling resonance $\nu_r = 2\nu_z$ is crossed twice before extraction

In the Injector I the resonance $\nu_r = 1$ is used to enhance the extraction efficiency

coupling resonance

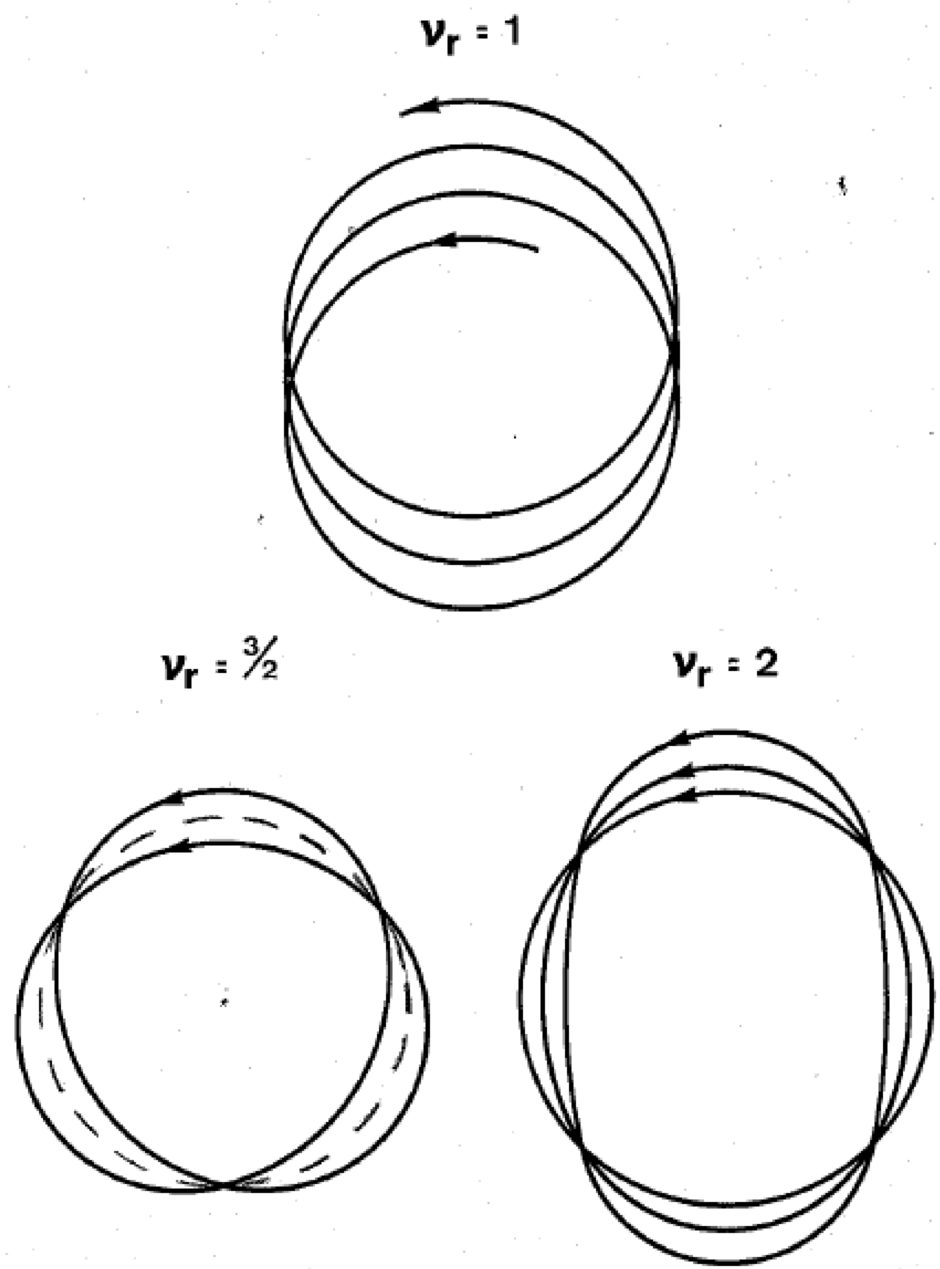


Ring cyclotron 590 MeV p

a large horizontal oscillation is transformed into a large vertical one at the coupling resonance $\nu_r = 2\nu_z$

This can lead to beam losses

orbit patterns for different resonances



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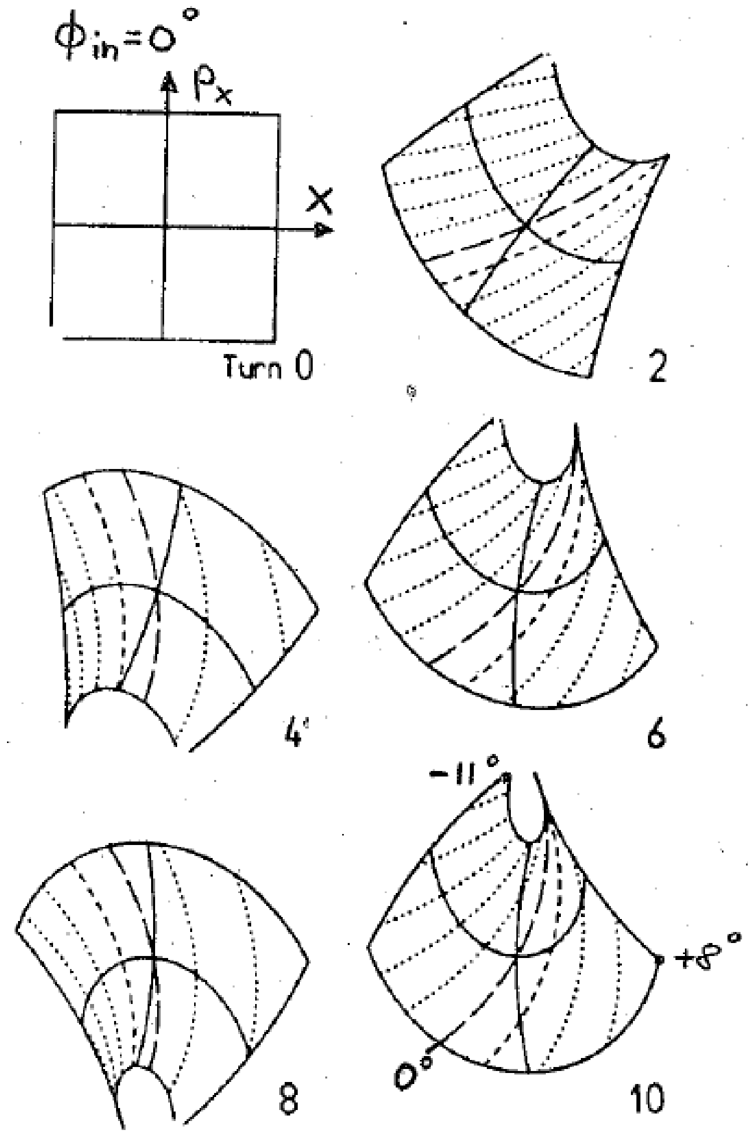
radial-longitudinal coupling not compensated

If all particles start with the same RF phase, then an initially nice radial phase area gets distorted due to different path lengths

(calculations by Stefan Adam)



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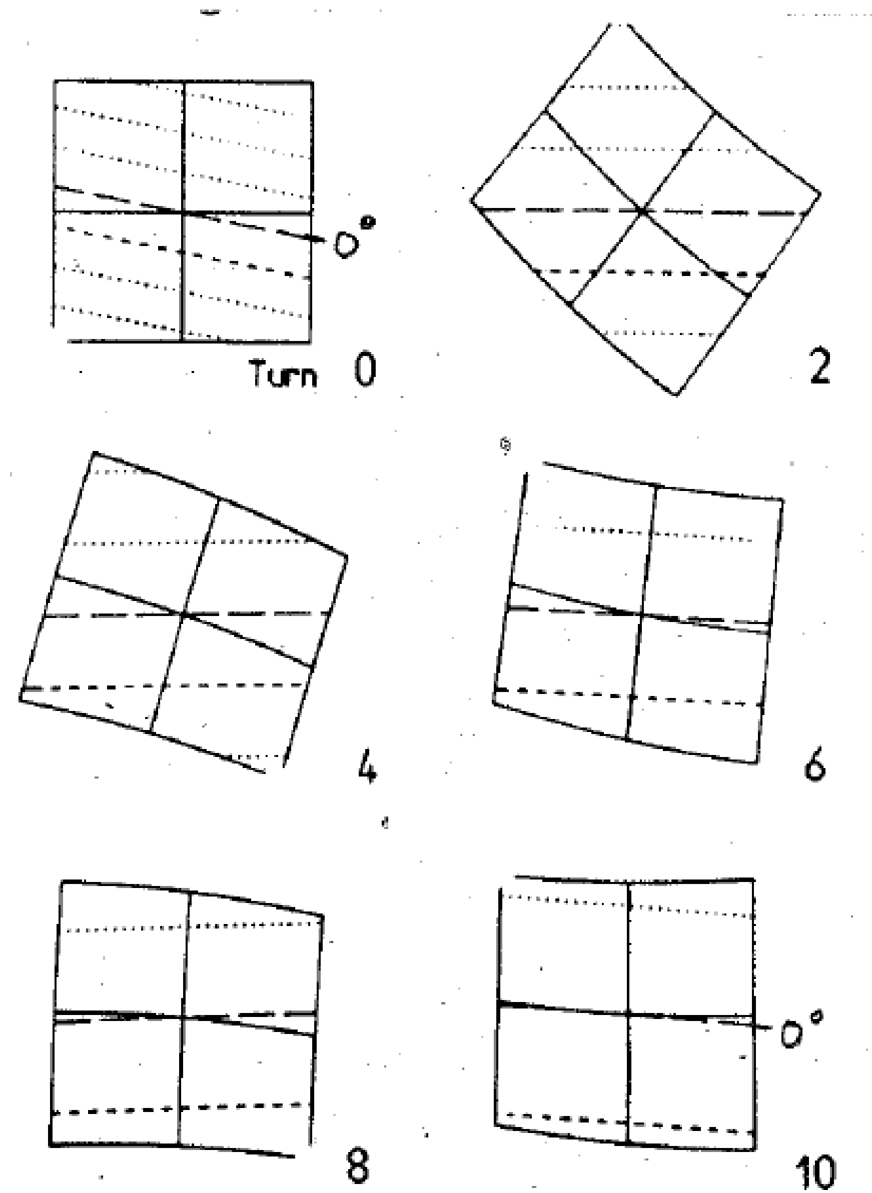
radial-longitudinal couplin compensated

If particles at different positions in the radial phase space (x, x‵) start with adjusted RF phases, distortion of the phase space area can be avoided

(calculations by Srefan Adam)



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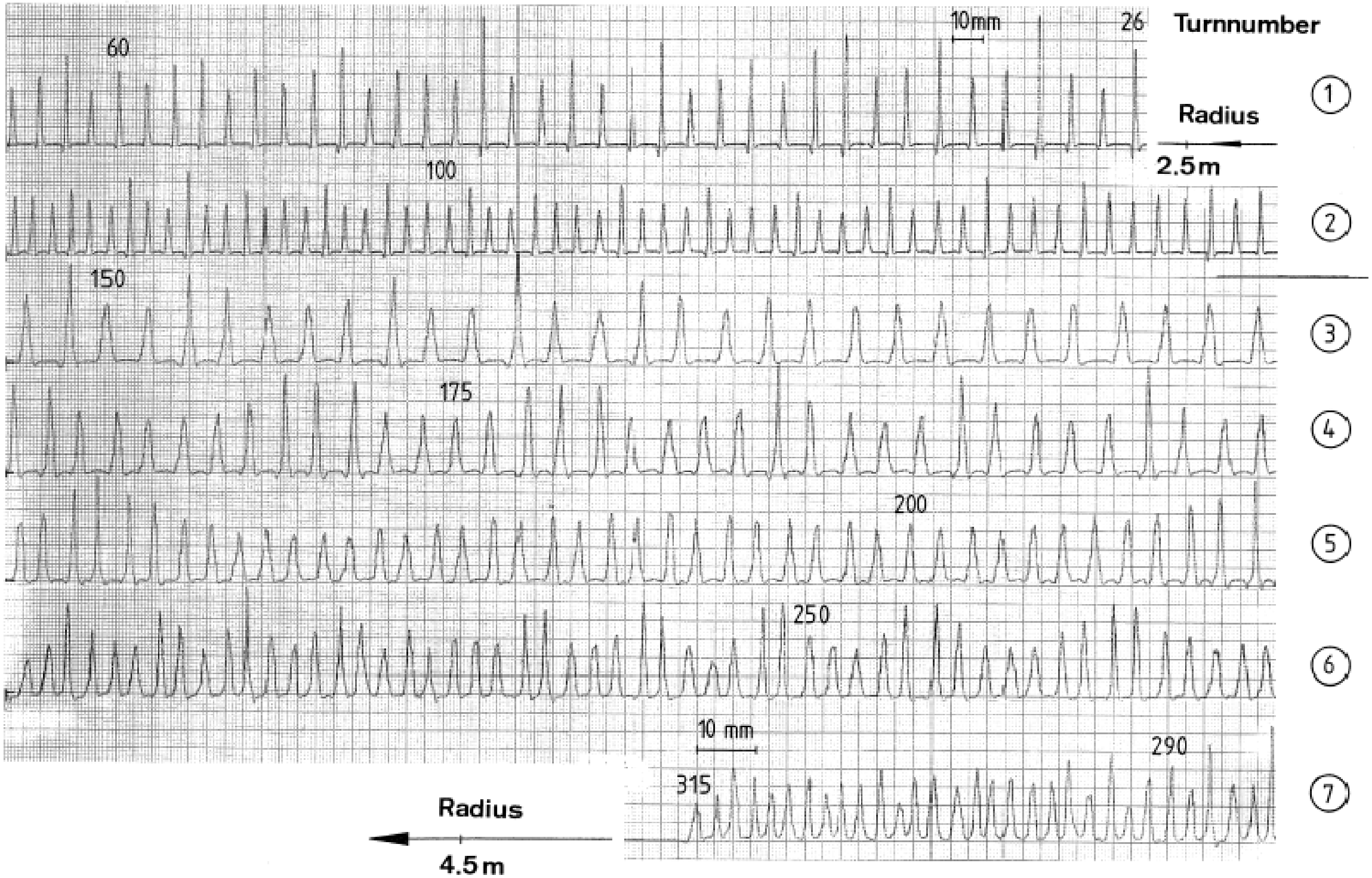
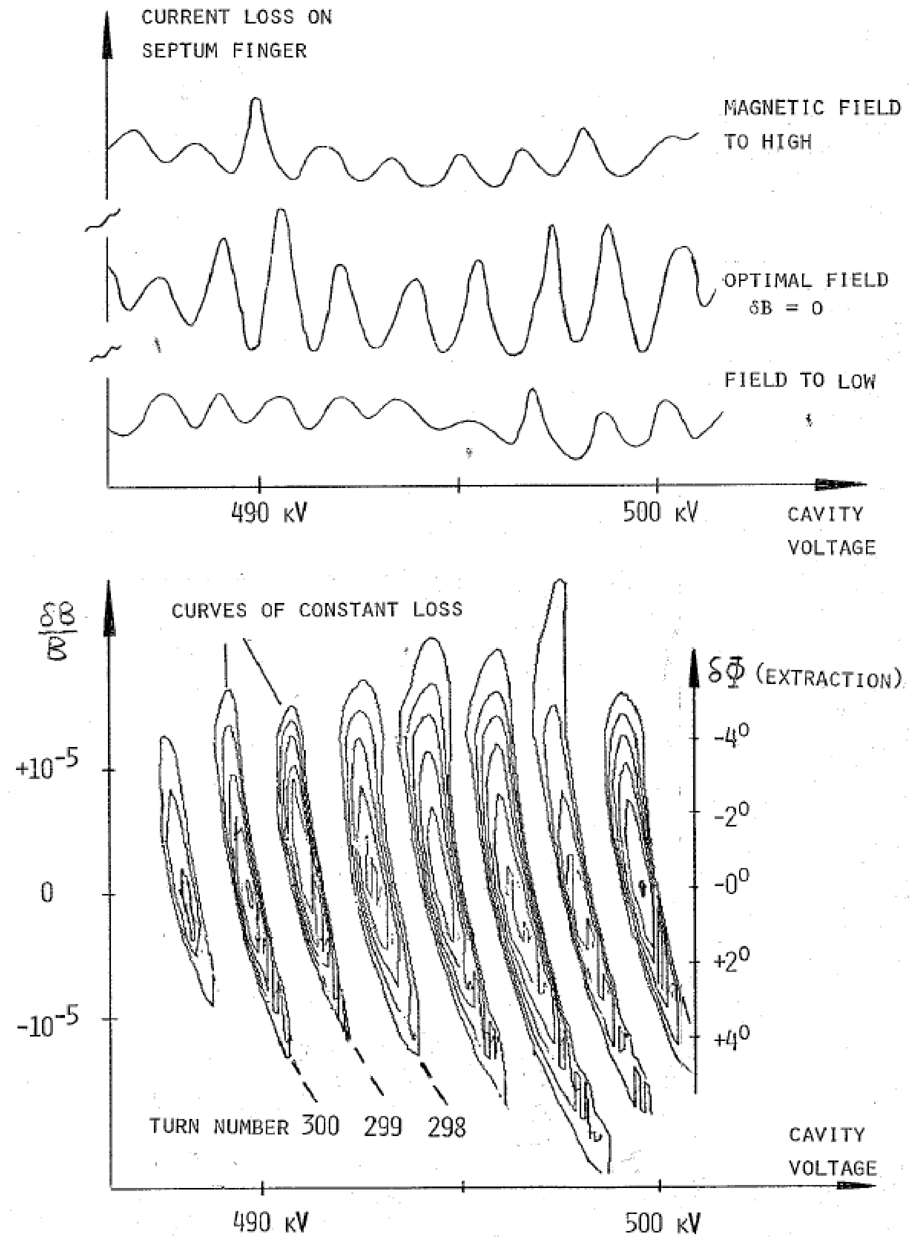
Ring Cyclotron (3.12.1980)
turns 26-315, 100-590 MeV



cavity scan

plotting extraction losses by varying the cavity voltage at different magnetic field levels displays the valleys for minimum losses

example for a centered beam



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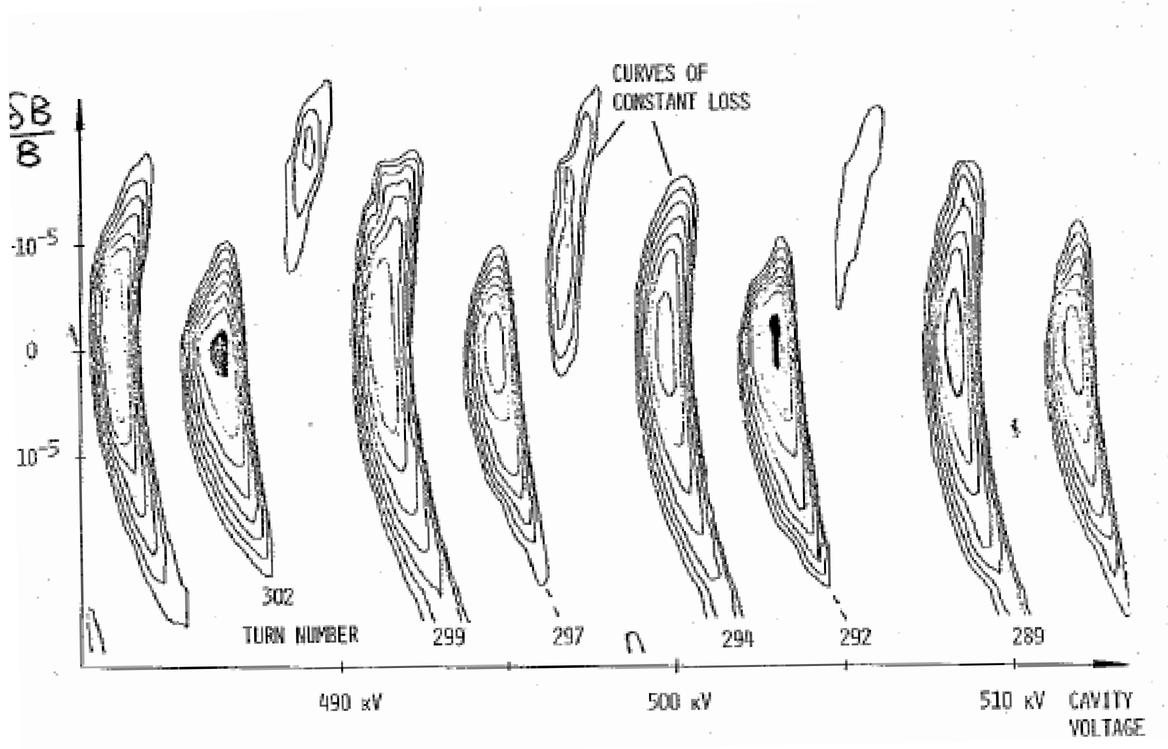
cavity scan

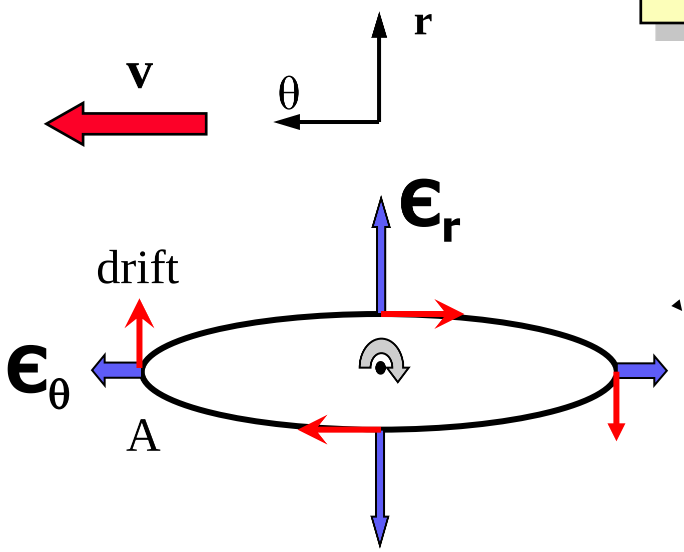
plotting extraction losses by varying the cavity voltage at different magnetic field levels displays the valleys for minimum losses

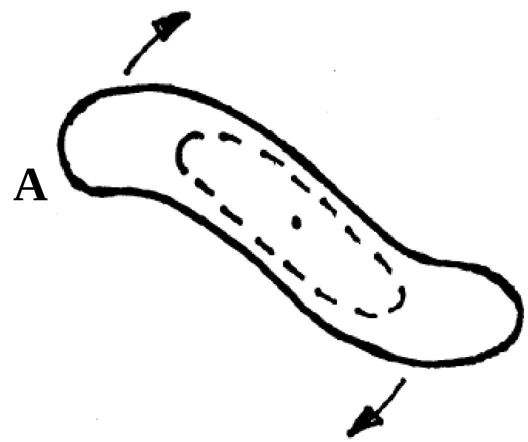


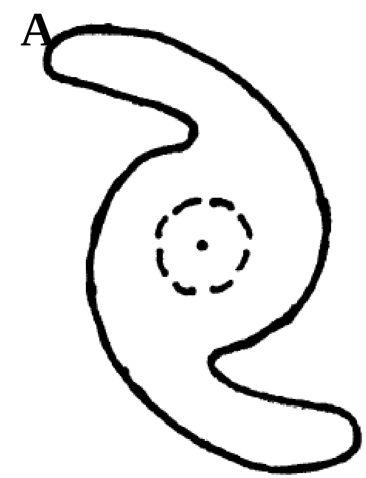
example for an eccentrically injected beam

the losses are very low for turn numbers separated by 5 turns, due to an average tune of Q_x=1.4

Longitudinal Space Charge in a Cyclotron Beam







Particle at position A:

=> gains additional energy from space charge forces

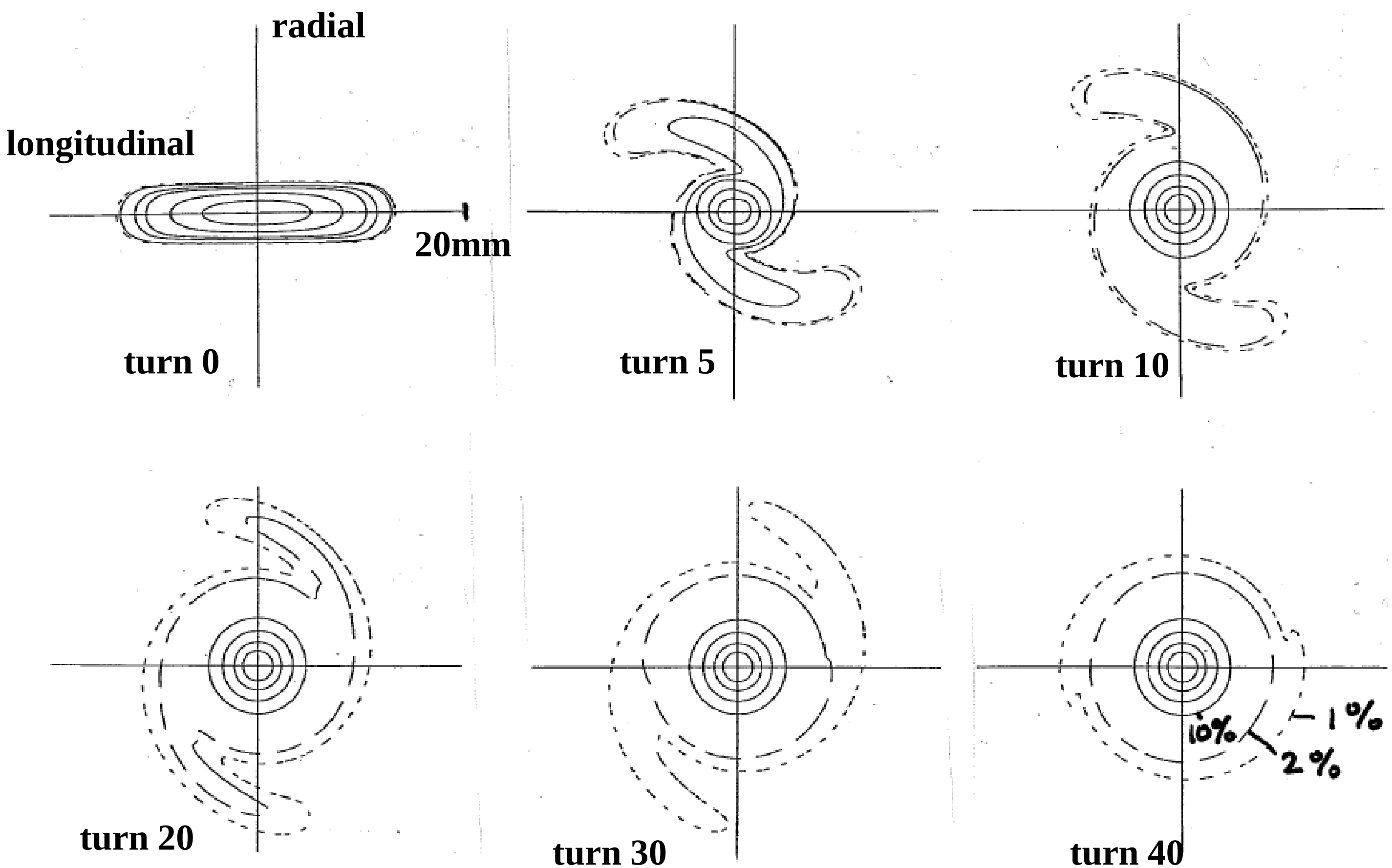
=> moves to higher radius due to isochronous condition

=> rotation of the bunch

=> nonlinearities produce spiral shaped halos

=> production of a rotating sphere (spaghetti effect)

Longitudinal Space Charge in Injector II



Simulation of a 1mA beam, circulating in Injector II at 3 MeV for 40 turns without acceleration.

The core stabilizes faster than the halos; rotating sphere produces phase mixing (calculations by S.Adam)

Aristocracy ⇔ Democracy

Synchrotrons

Linacs

democratic:

a particle oscillates between head and tail

(phase focusing)

Cyclotrons

aristocratic :

a particle „born ahead“ stays ahead !

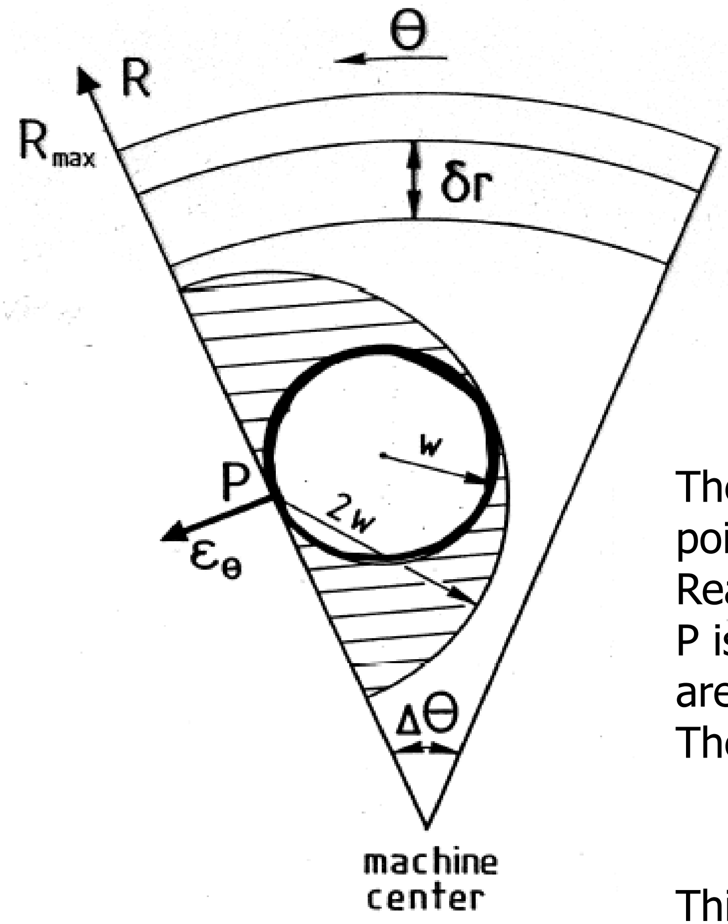
(isochronism)

but **at high intensity**

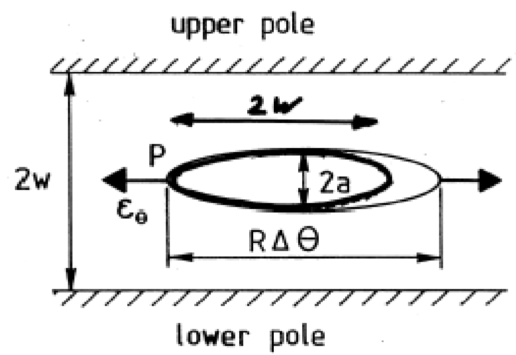
a cyclotron becomes **democratic** !!

(space charge mixes phases)

longitudinal Space Charge Fields in a Cyclotron



top view



side view

Disc-Model

(W.Joho, Int. Cyclotron Conf. Caen 1981)

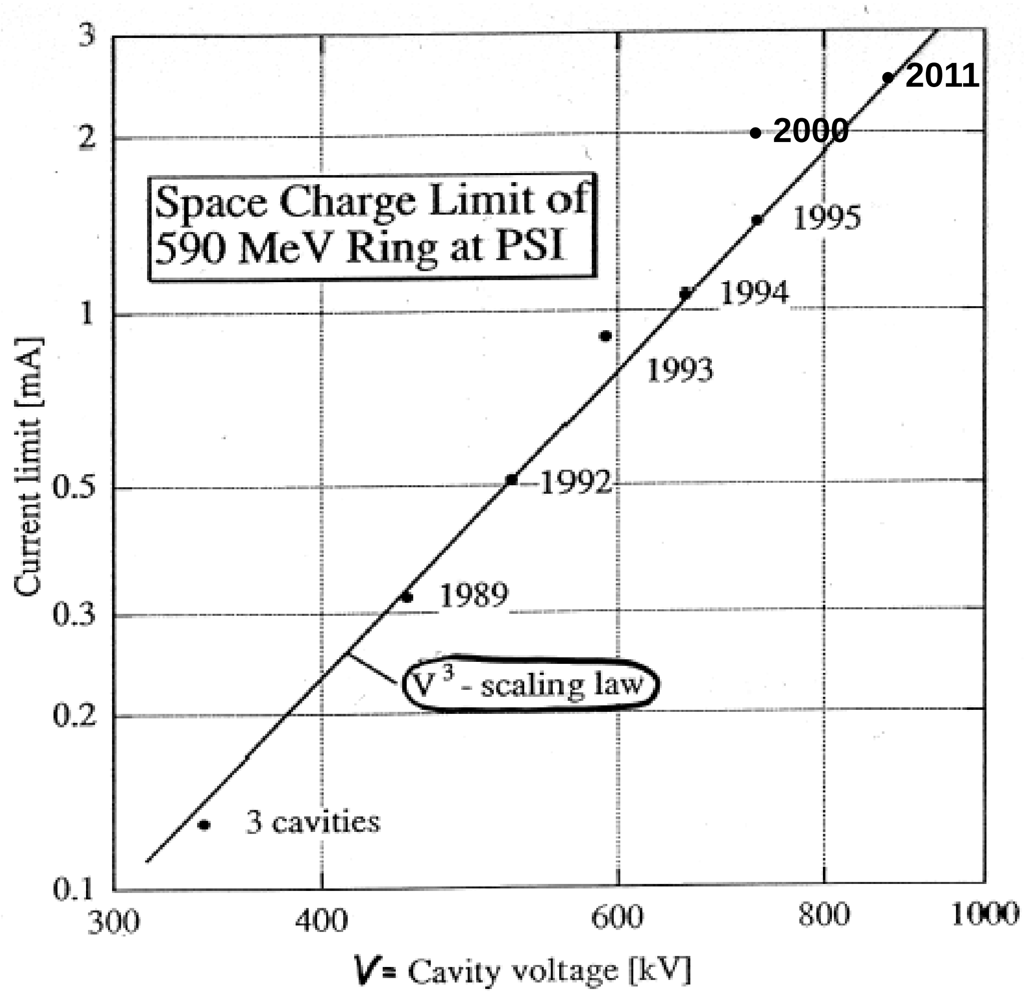
circulating protons fill a cake-like piece with azimuthal extension Δθ. Neighbouring orbits are assumed to overlap radially.

The azimuthal electric field at the edge of the „piece of cake" at point P is approximated by the calculable field of a disc with radius w. Reasoning: the charge of the protons outside of the half circle around P is screened by the upper and lower poles and protons in the hashed area give only a small contribution to the azimuthal field ϵ_θ.
The proton at P gains through ϵ_θ an additional energy/turn:

$$dE/dn = 2\pi R\, \epsilon_\theta$$

This simple model predicts, that the intensity limit from longitudinal space charge forces increases with **V^3** !!
(V=cavity voltage/turn)

Current Limit in Ring Cyclotron



Longitudinal space charge forces

increase the energy spread

=> higher extraction losses

=> limit on beam current

Remedy:

higher voltage V on the RF cavities

=> lower turn number n (V·n = const.)

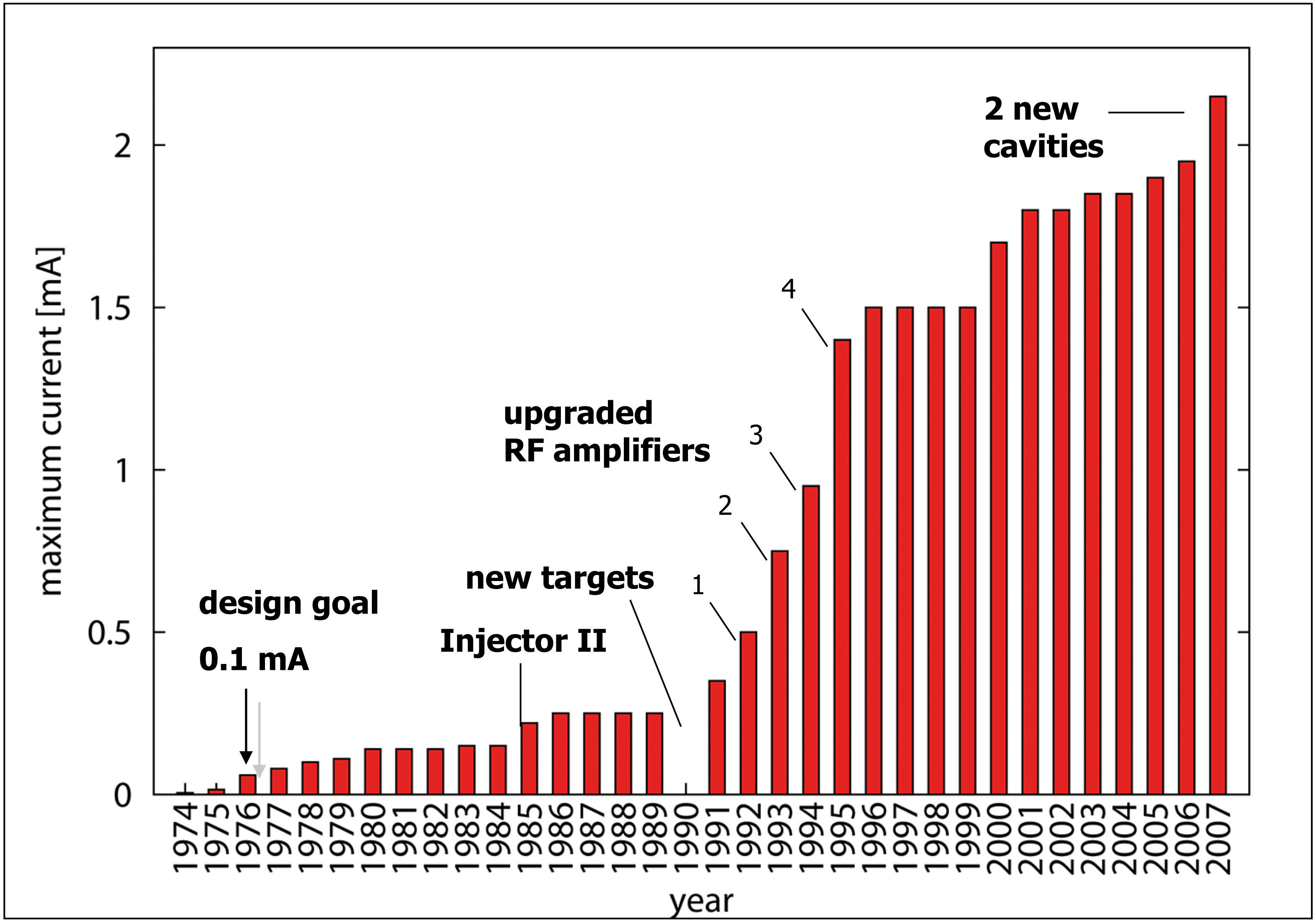
current limit ~ V^3 !

There are 3 effects, each giving a factor V(~1/n):

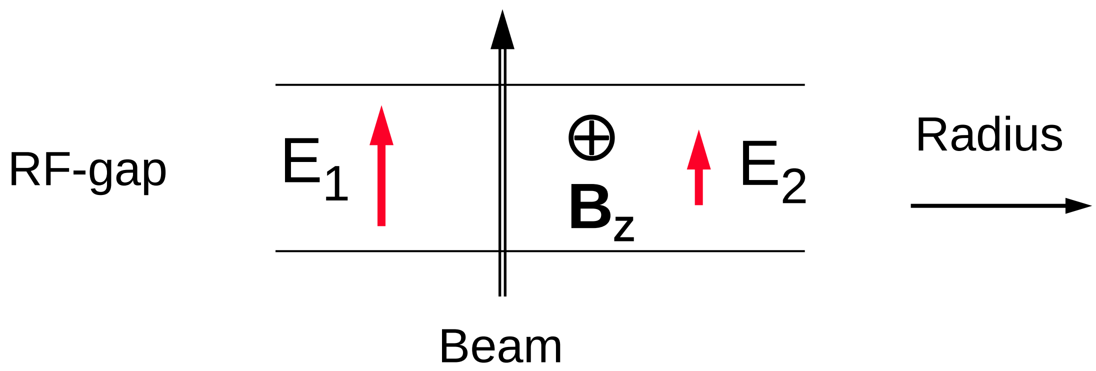
1) beam charge density ~ n
2) total path length in the cyclotron ~ n
3) turn separation ~ V

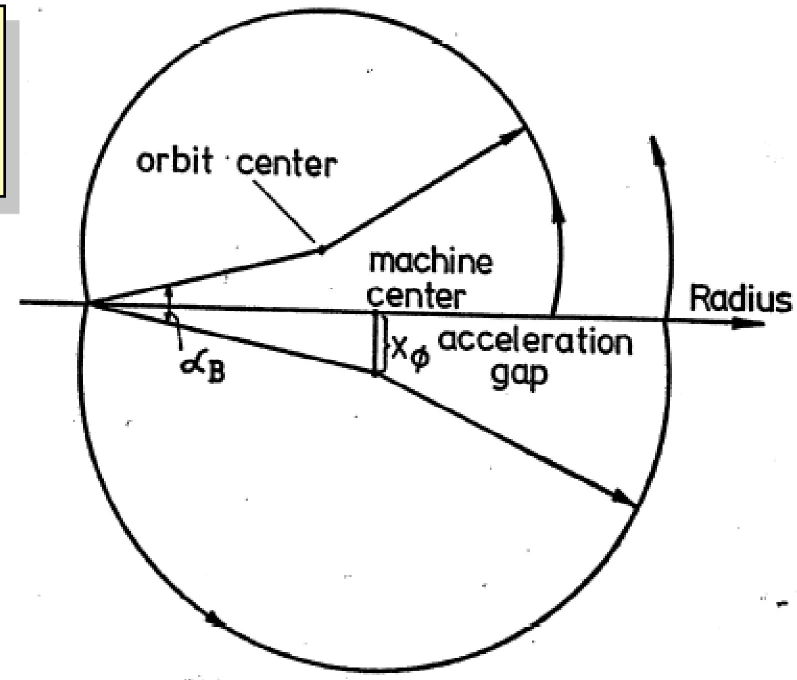
W.Joho, 9th Int. Cyclotron conference CAEN (1981)

maximum current in ring cyclotron



Effect of magnetic RF-Field on Phase of Cyclotron Beam





Electrical Field: $E(r,t) = E(r)\cos\Phi\,, \quad \Phi \equiv \omega_{rf}\, t$

Induction Law of Faraday: $\mathrm{rot}\,\vec{E} = -\dfrac{d\vec{B}}{dt}$

=> $B_z = -\dfrac{1}{\omega_{rf}}\dfrac{dE}{dr}\sin\Phi$

(magnetic field of a cavity produced by radial variation of E)

The magnetic field of the cavity gives the particle a radial kick.

=> change in path length

⇒ influence on phase!

⇒ phase compression or phase expansion

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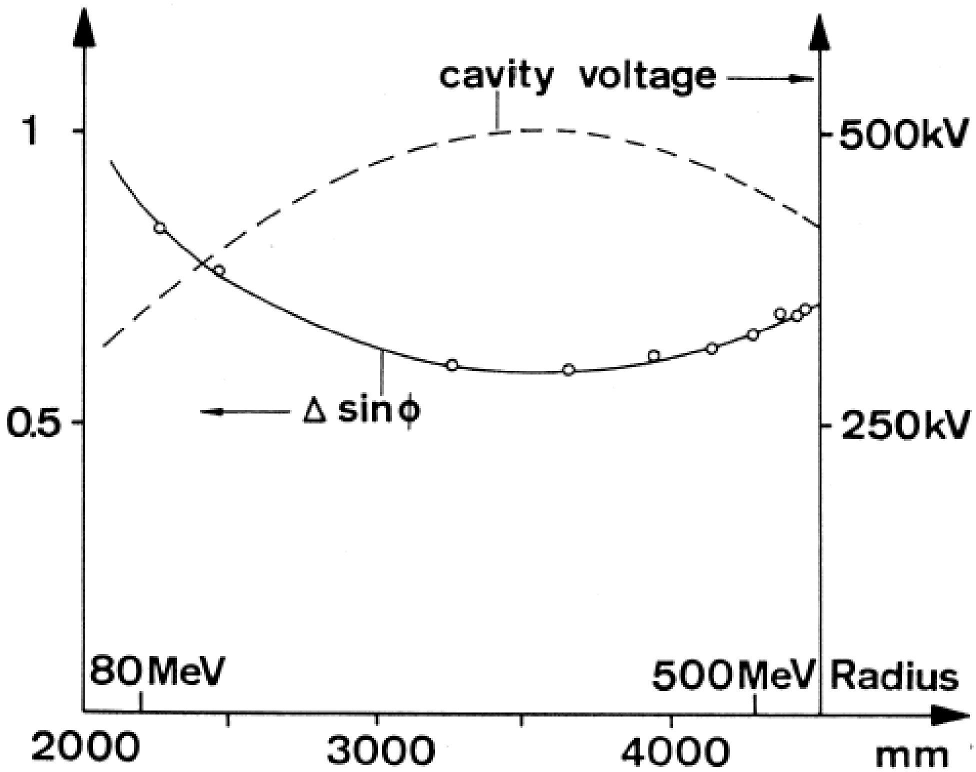
Phase Compression / Phase Expansion due to Variation in Cavity Voltage



The radial variation of the cavity voltage produces a phase dependent magnetic field. This effects the revolution time and thus the phase of a particle.

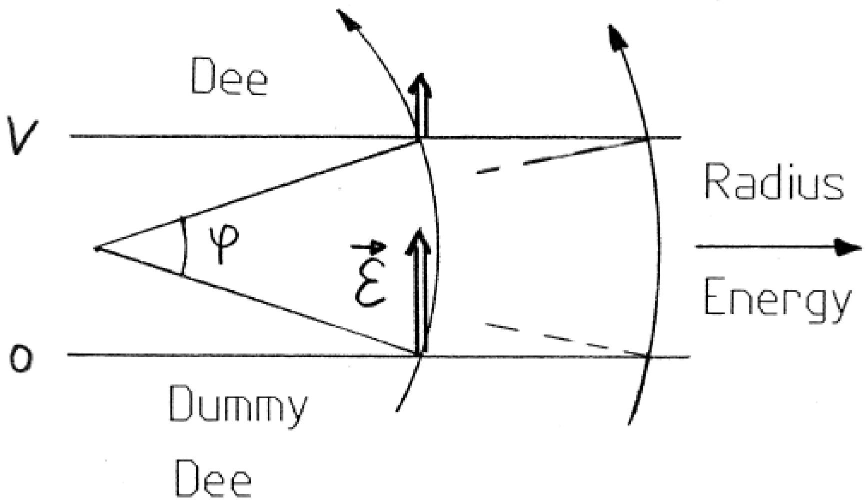
$$\boxed{E_G(R)\,\Delta\sin\Phi(R) = const.}$$

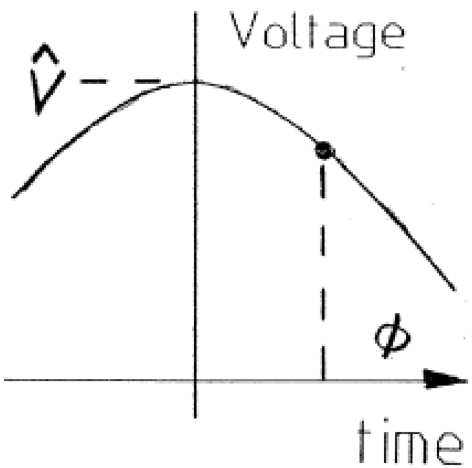
$$\mathrm{E_G} = \text{peak energy gain/turn}$$

$$\Phi = \text{phase of particle}$$

W.Joho, Particle Accelerators 1974, Vol.6, pp. 41-52

Bunching in the Center of a Cyclotron





Inside the dee gap the electric field is straight, whereas the orbit is curved.

This gives an inward radial kick at the entrance of the dee, and an outward one at the exit.
For a positive phase Φ the overall kick is inward
=> shorter path to next RF gap, => Φ decreases
=> **bunching**

$$\boxed{E_G(R)\sin\Phi(R) = const.}$$

$$\mathrm{E_G} = \text{peak energy gain/turn}$$

$$\mathrm{E_G} \propto \text{transit time factor } \mathrm{c}(\varphi)$$

$$\mathrm{c}(\varphi) = \frac{\sin(\mathrm{h}\varphi/2)}{(\mathrm{h}\varphi/2)}\ , (\mathrm{h} = \text{harmonic})$$

increases with radius => bunching

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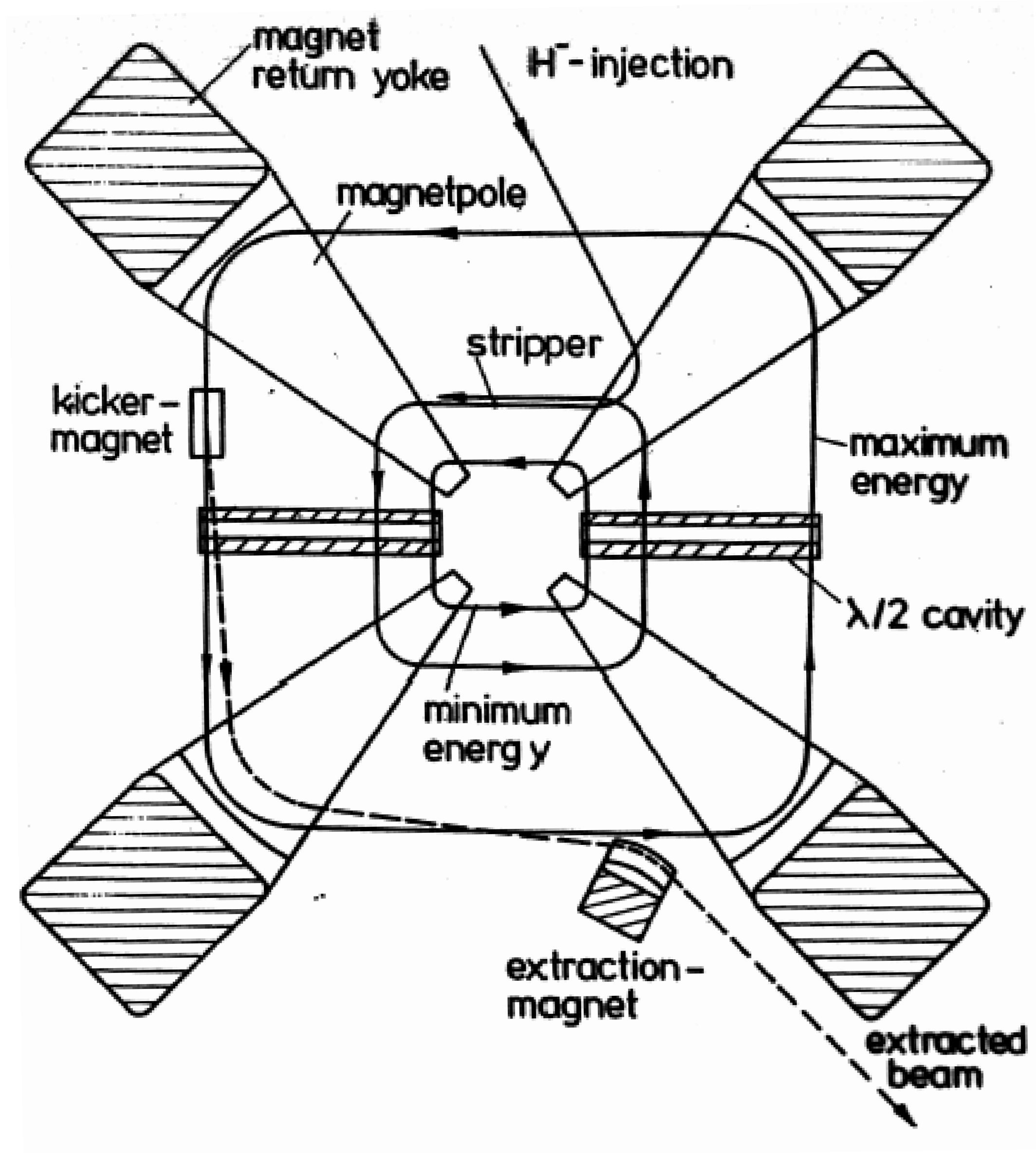
the Cyclotron as Storage Ring

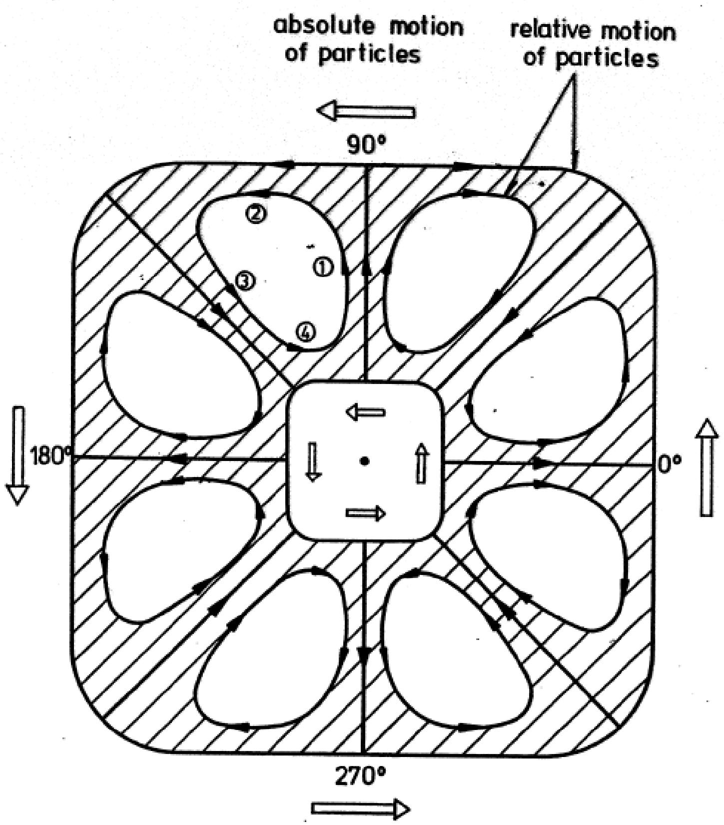


no electric field inside cavity at minimum and maximum radius

the magnetic rf_field detunes the phase and forces the particles to oscillate between the two extreme radii

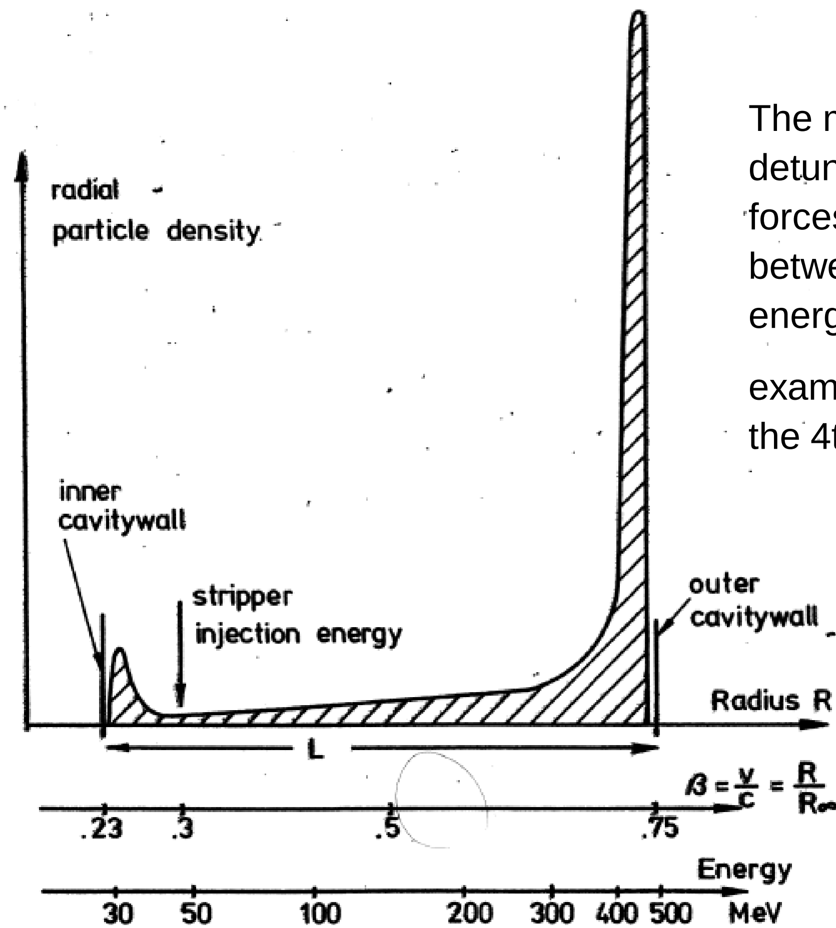
W.Joho Particle Accelerators 1974, Vol.6, p.41-52

Accumulation of Protons in a Cyclotron Storage Ring



Injection of H^- - Ions at 50 MeV

the majority of particles accumulate around the maximum radius at 450 MeV



The magnetic RF field detunes the particles and forces them to circulate between the 2 extreme energies.

example with operation on the 4th harmonic

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ASTOR =**A**cceleration and **STOR**age

with 3 different cavities one can vary the

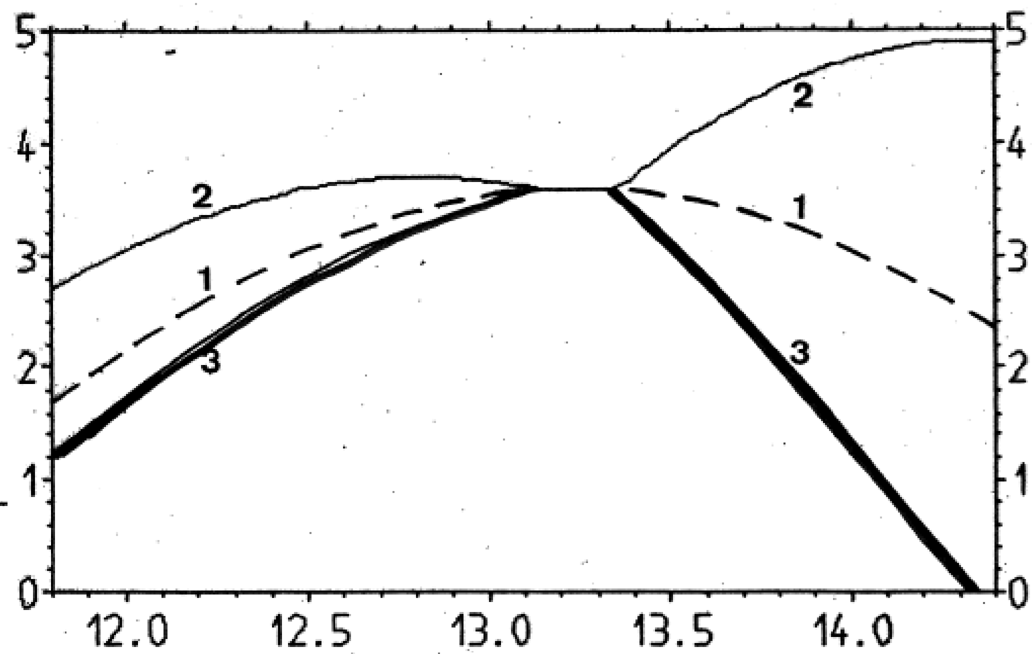
radial voltage profile in a cyclotron

1, 2 = cyclotron mode,

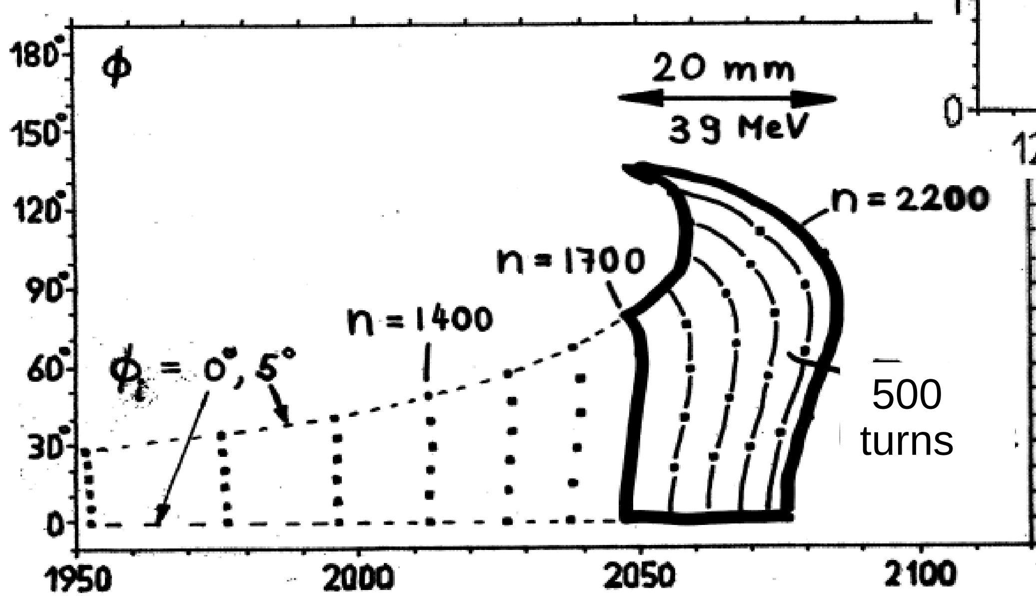
3 = storage mode ASTOR

W.Joho: US Particle Accel. Conference

Santa Fe, IEEE NS-30, 2083 (1983)



Radius [m]



E[MeV]

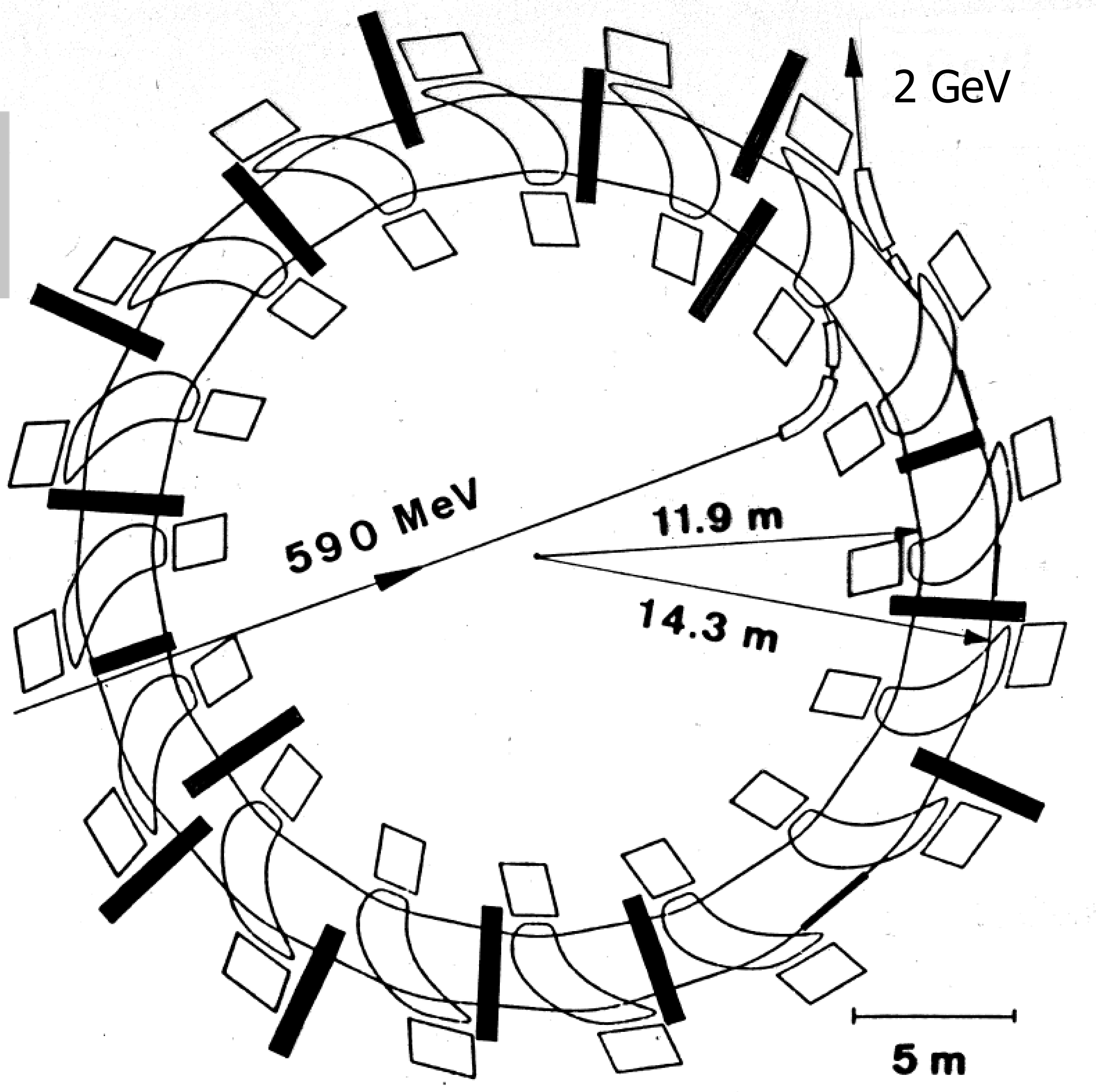
Astor mode:

Accumulation of 500 turns at
max. energy within 20mm
=> extraction with kicker

ASTOR Proposal for PSI
(W.Joho 1981)

Injection of protons from 590 MeV Ring Cyclotron

Extraction at 2 GeV

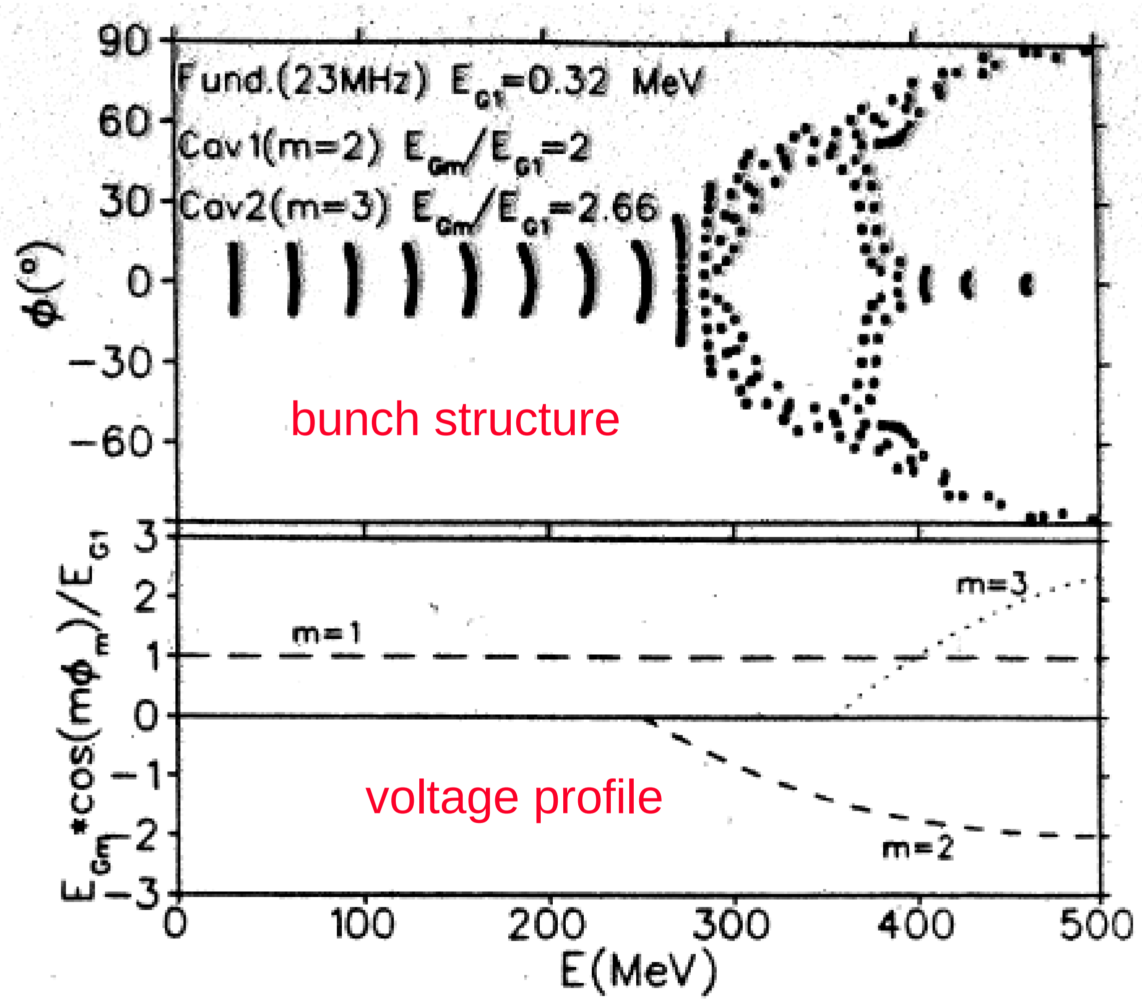


longitudinal beam splitting

Combination of cavities, operating at different frequencies

=> the magnetic RF fields can split up a bunch into several bunches, without beam losses !

Example for the TRIUMF Cyclotron:
A 23 MHz beam bunch (m=1) could be converted into 3 bunches of 69 MHz with 2 additional cavities operating at 46MHz (m=2) and 69 MHz (m=3)



R.E.Laxdal, W.Joho, Longitudinal Splitting of Bunches in a Cyclotron by Superposition of Different RF Harmonics EPAC92 Berlin, p.590

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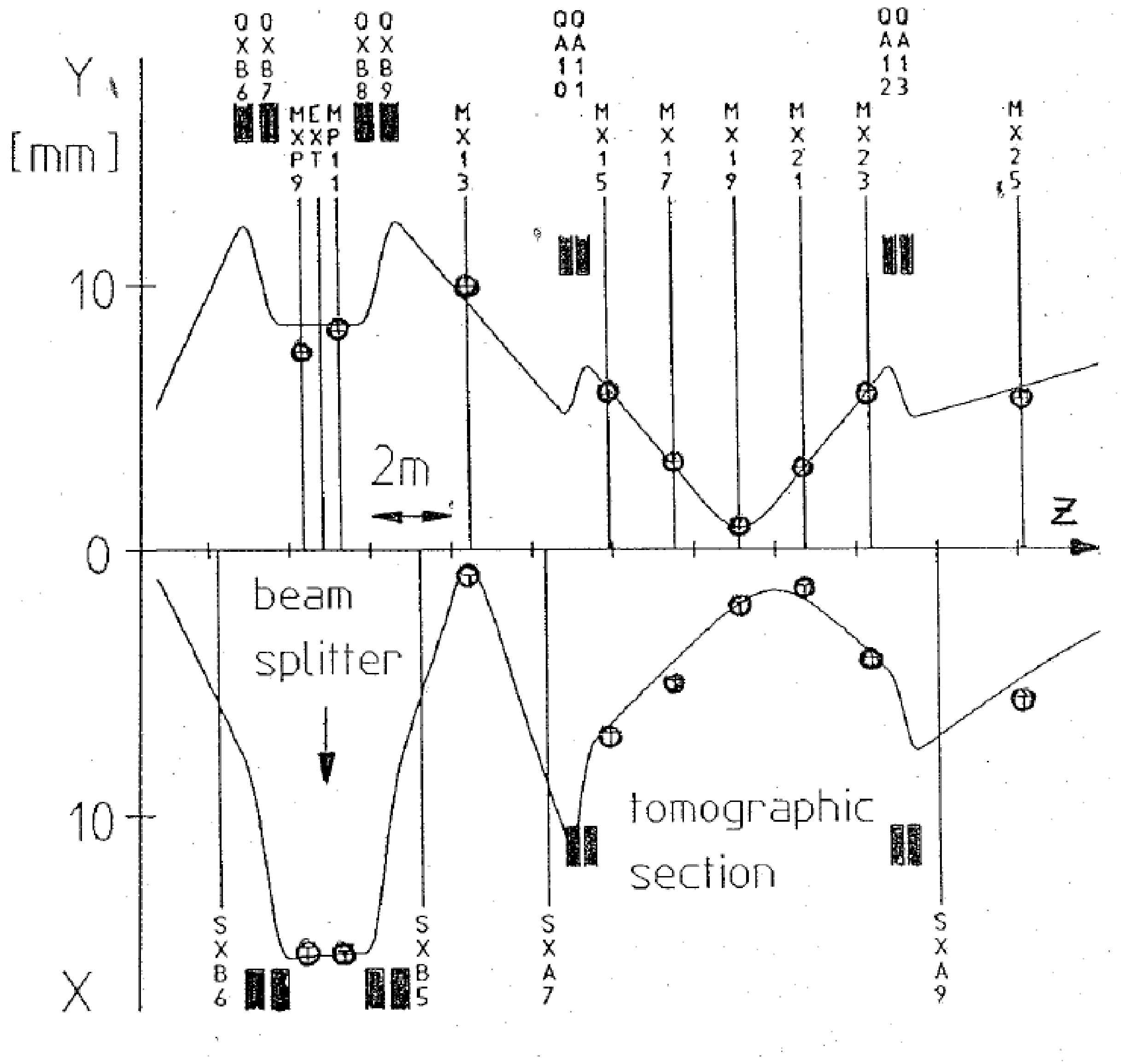
tomographic section in beamline

5 profile monitors in a drift
=>ideal to reconstruct phase space distribution of beam

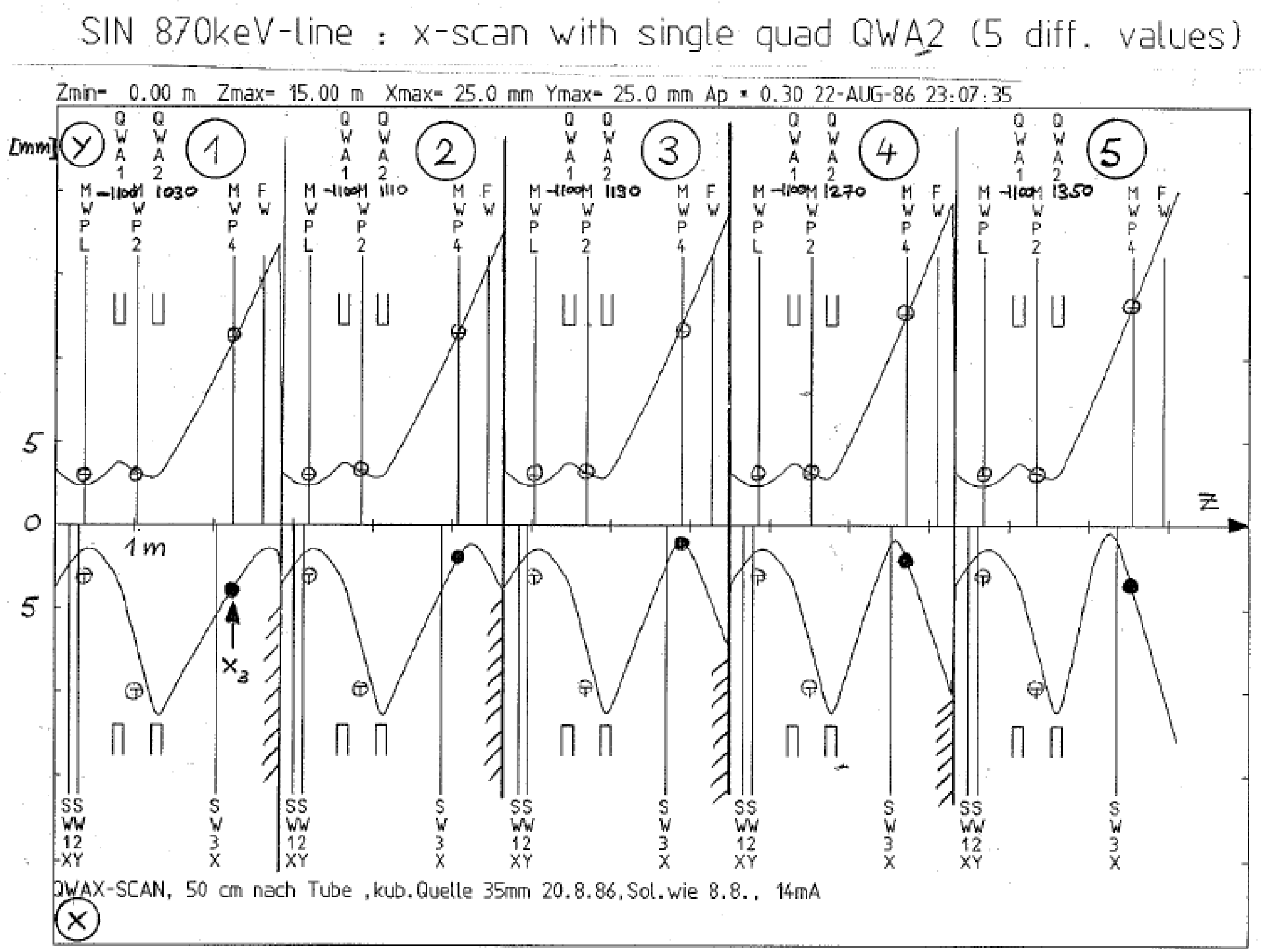


beam tomography

scanning the phase space with variation of a quadrupole (5 settings)

trick:

all 5 measurements are fitted **together** for the reconstruction of the initial beam ellipse



Some basics for cyclotrons

Force on a Particle

Newtons Law:

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}} \qquad \vec{p} = m\gamma\,\mathrm{v} \quad (= \text{momentum})$$

particle with charge q in electromagnetic field experiences force:

$$(1) \quad \overrightarrow{F_{el}} = q\,\vec{\mathcal{E}} \quad (\text{electrical field} \Rightarrow \underline{\text{acceleration}})$$

$$(2) \quad \overrightarrow{F_m} = q\,(\vec{v} \times \vec{B}) \quad (\text{Lorentz force} \Rightarrow \underline{\text{deflection}}\,,$$

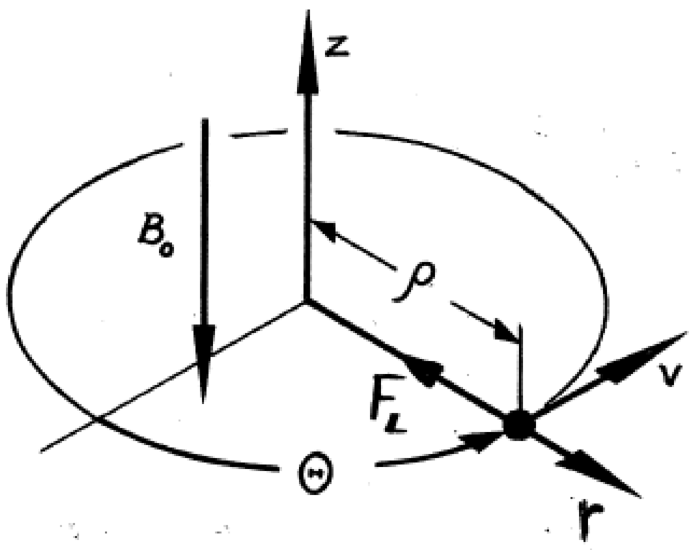
$$\text{energy stays const.})$$

comparison at v=c:

1 T ⇔ 300 kV/mm !!

circular orbit

In a homogeneous magnetic field B the particle has a circular orbit with radius ρ



Balance between Lorentz-force F_L and centrifugal force F_Z:

$$F_L = q\,v\,B\,, \qquad F_r = \frac{m\,v^2}{\rho} \qquad \text{(non relativ.)}$$

with $p = m\,v$:

$$\boxed{p = q\,B\rho}$$

valid relativistically!
(Bρ) = „magnetic rigidity"

Basis of all circular accelerators
(Cyclotron, Synchrotron, Storage Ring, Spectrometer etc.)

for electrons with E ≥ 10 MeV:

E[GeV] = pc= 0.3 Bρ [Tm]

homogeneous magnetic field

Particle with charge q and momentum p:

=> circular orbit with radius ρ in homogeneous magnetic field B

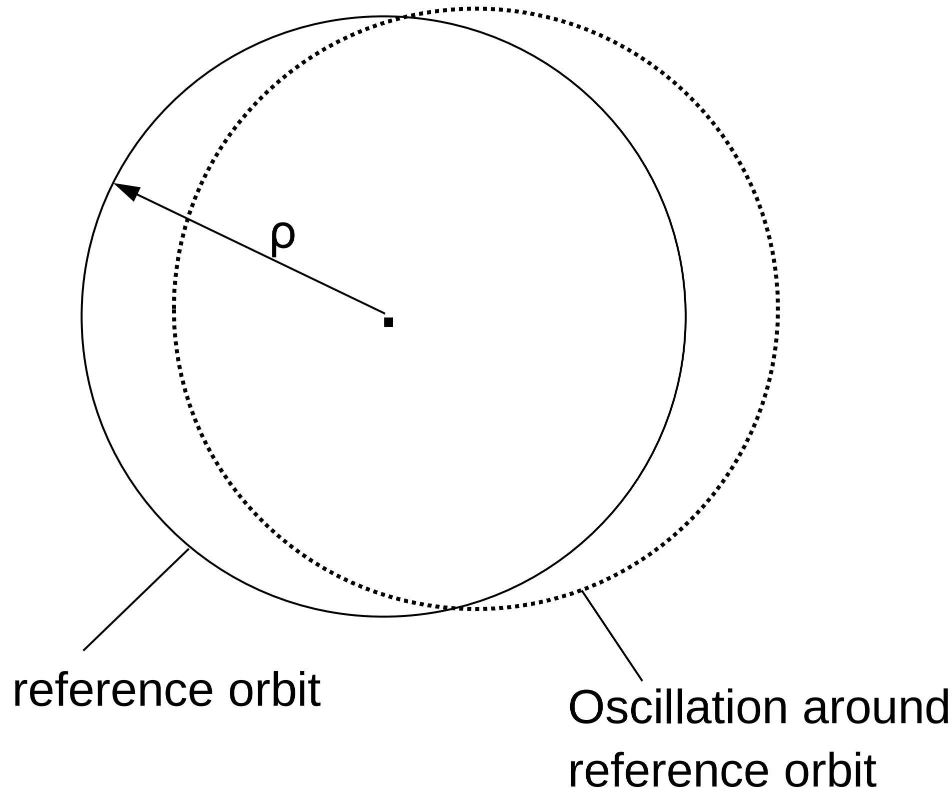
$$p = q\,B\,\rho$$

this circular orbit can be placed anywhere!

⇒ stable horizontal oscillation around reference orbit

with focusing frequency $Q_r = 1$

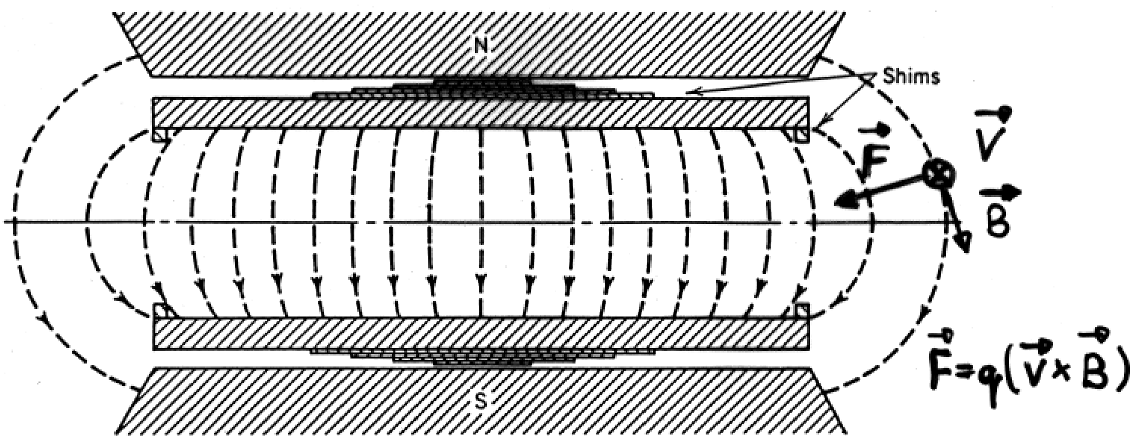
<u>no vertical stability</u> ($Q_y = 0$)



classical Cyclotron

In homogeneous magnetic field the circular orbits are vertically unstable

vertical stability with radially decreasing field B(r)



Definition of field index n with „logarithmic derivative“

$$\left(\frac{dB_0}{B_0}\right) \equiv -n \left(\frac{dr}{r}\right)$$

Focusing frequencies:

stable for $\boxed{0 < n < 1}$

$$Q_r = \sqrt{1-n}\,, \quad Q_y = \sqrt{n}\,,$$

$$Q_r^2 + Q_y^2 = 1$$

=> weak focusing, horizontally and vertically

W.Joho 2011

Scaling Laws in isochronous Cyclotrons

For each energy there exists a closed orbit with circumference

$L \equiv 2\pi R$ and constant revolution frequency ω_0

$$\omega_0 = \frac{q}{m} \; \frac{B_0(R)}{\gamma(R)} \; , \; with \;\; B_0(R) \equiv \frac{\int_0^L B_z(s) ds}{L}$$

since $\omega_0 = \dfrac{v}{R}$, $(v \equiv \beta \, c)$, we have the scaling laws

1) $R = \beta \, R_\infty \; , \quad R_\infty \equiv \dfrac{c}{\omega_0}$

2) $B_0(R) = B_{center} \;\; \gamma(R)$

the magnetic field index k in isochronous Cyclotrons

The local field index k is defined as :

$$\frac{dB_0}{B_0} \equiv k\,\frac{dR}{R}$$

k can be calculated from

$$p = q(B\rho) = q B_0(R) R$$

$$\frac{dp}{p} = \frac{dB_0}{B_0} + \frac{dR}{R} = (1+k)\frac{dR}{R}$$

on the other hand we have in general :

$$\frac{\mathrm{dp}}{\mathrm{p}} = \gamma^2 \frac{d\beta}{\beta}, \quad \text{which in a cyclotron gives} \quad \frac{\mathrm{dp}}{\mathrm{p}} = \gamma^2 \frac{dR}{R}$$

we have thus $\quad k(R) = \gamma^2(R) - 1$

the dispersion in isochronous Cyclotrons

From the field index k we get as an approximation for the horizontal focusing frequency Q_x :

$$Q_x(R) \approx \sqrt{1+k(R)} = \mathrm{f} \cdot \gamma(R) \quad (\mathrm{f} \approx 1.1\text{-}1.2 \text{ for Ring Cyclotrons})$$

The dispersion D is defined as

$$dR \equiv D\frac{dp}{p}, \qquad \text{since}$$

$$\frac{dR}{R} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}, \qquad \text{we get for the } \textbf{average} \text{ dispersion}$$

$$\mathrm{D} = \frac{R}{\gamma^2(R)}, \qquad \text{the momentum compaction factor is thus } \frac{1}{\gamma^2},$$

and due to isochronism the cyclotron is always on transition at all energies

The dispersion D varies thus strongly with energy

Larmor Frequency

Revolution frequency ω_0 in homogeneous magnetic field:

ω_0 = v/R ,   p = mv = q B R (non rel.) :

$$\omega_0 = \frac{q}{m} B \qquad (= \textit{Larmor frequency})$$

ω_0 is independent of radius R and energy E !

$\Rightarrow$ Basis for classical Cyclotron (non rel.)

relativistic formula for all energies, with $E_{tot} = \gamma\, mc^2$ and $\omega_0 \equiv 2\pi\nu_0$

$$\boxed{\nu_0 = \left(\frac{q}{2\pi m}\right)\frac{B}{\gamma}}$$

$$\frac{q}{2\pi m} = 15.25\ \mathrm{MHz/T} \quad \text{for protons}$$

$$\frac{q}{2\pi m} = 28\ \mathrm{GHz/T} \quad \text{for electrons}$$

Isochronism

Acceleration of a particle with RF frequency ν_{RF} on harmonic h:

$$\nu_{RF} = h\,\nu_0$$

If this RF frequency stays constant during acceleration, we talk about an **isochronous cyclotron**. The condition for this is an average field

which increases proportional to γ:

$$\Rightarrow B_0(R) \sim \gamma(R)$$

For an azimuthally symmetric field this leads to vertical instability. The way out is:

1) **magnetic sectors** give vertical focusing => $B(r,\vartheta)$, Thomas 1938

=> $B_0(R)$ = field averaged over the whole orbit

2) **synchro-cyclotron** with $\nu_{RF}(t)$ => pulsed beam, reduced intensity

Extraction from a Cyclotron

The intensity limit of a Cyclotron is given by the beam losses.

Important is the radial distance dR/dn between the last two turns before extraction

=> large turn separation with:

- high RF voltage (intensity limit ~ V^3 !!)
- large machine radius R !

=> compact cyclotrons (superconducting !)

have limited intensity

(1) $E = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 R^2 \sim R^2$

(non relativistic)

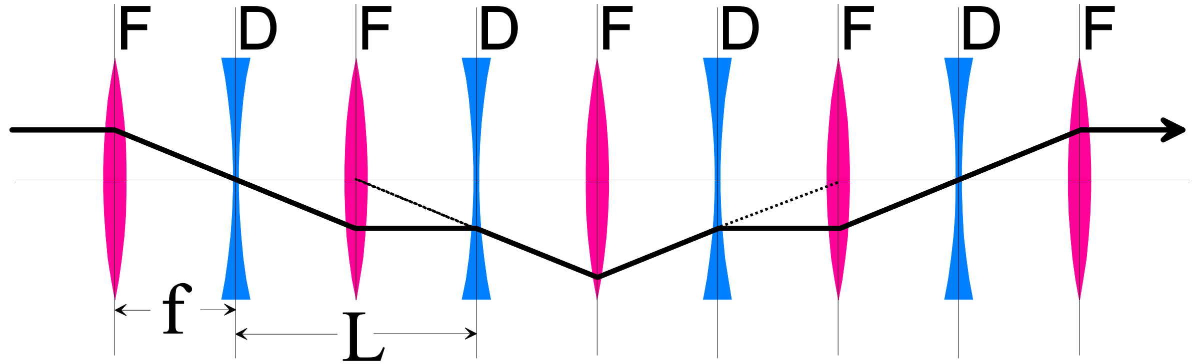
(2) $E \approx n q \bar{V} \quad \sim n$ (turn number),

$\bar{V}$ = average RF-voltage per turn

=> $R \sim \sqrt{n}, \qquad \frac{dR}{dn} = \frac{R}{2n}$

$$\frac{dR}{dn} = \frac{\gamma}{\gamma+1} R \frac{\bar{V}}{(E/e)} \frac{1}{Q_r^2} \quad \text{(exact)}$$

AG-Focusing



simple example of alternative gradient focusing:

$\Rightarrow$ FODO-lattice with thin lenses (focal length f)

if L = 2f => construction is possible by hand !

it takes 6 periods to get a 360^0-oscillation

i.e. the phase advance/period is $\psi = 60^0$

exact solution with transfer matrices gives

$$\sin\frac{\psi}{2} = \frac{L}{4f}$$

for L = 2f => $\psi = 60^0$ (graphic example)

for L = 4f => $\psi = 180^0$ (instability !)

References

More information on the PSI Accelerator Facilities can be found in: www.psi.ch

Some foils from talks by the author are found with google.ch : “WERNER JOHO PSI”

or in

http://indico.psi.ch/ Conferences Accelerator Talks by Dr. Werner Joho

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