Space Charge Limits and Single Turn Extraction in the 590 MeV Ring Cyclotron

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Introduction

- Concept of Ring Cyclotron with separate sectors by **Hans Willax** in 1962

- Early recognition of potential for „high currents" due to

  high voltage cavities, large radius and **strong vertical focusing**

$\Rightarrow$ goal of 100 µA for **meson factory**, considered "very ambitious" by experts

(at that time the record from synchro-cyclotrons was 0.1 µA !)

- critical point was **Injector**

  => Concept for Injector II (replacing Injector I) before ring cyclotron was commissioned !

- Today the record intensity is 2.4 mA, a **factor of 24 over its design goal !**

what are the key points for this success and

where is the limit to even higher currents?

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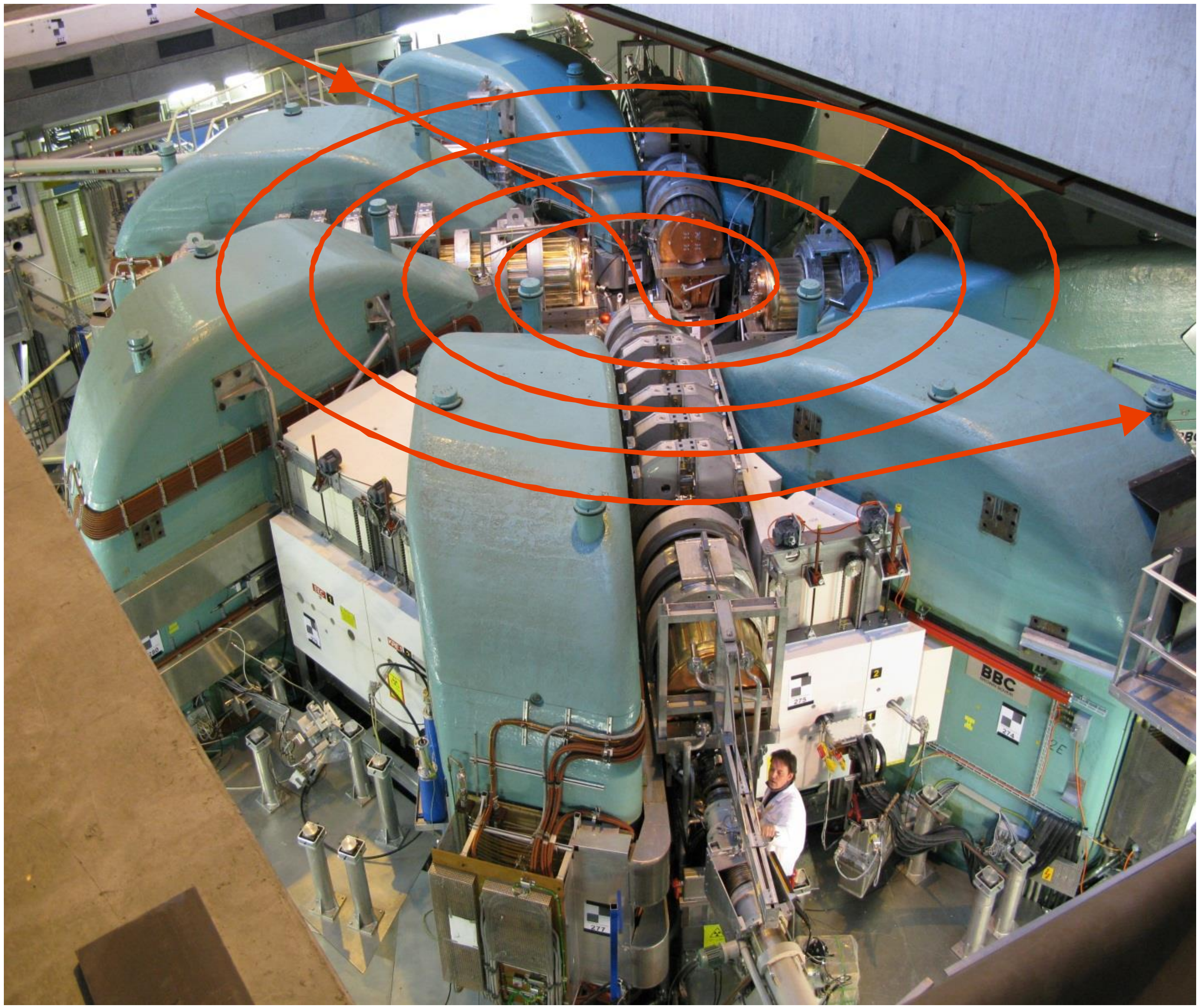
Ringcyclotron
4 new cavities 2008



590 MeV Protons

1.4 MW average Beam Power

(World Record!)

8 Magnets à 250 t

4 Cavities à 900 kV

Extraction 99.99%

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Key points for PSI Ring Cyclotron

- 4 **high voltage** cavities (initially 500 kV, now up to 900 kV) give high current limit
- 650 kW of RF power can be coupled into each cavity (350 kW goes to the beam)
- **Q_r at extraction** (drops from 1.75 to 1.5) is ideal to increase turn separation at extraction by a factor of ≈ 3 (from 6 to 17mm), using **eccentric injection**
- installation of a **flattop cavity** (at a later stage!) gives monoenergetic beam

=> single turn extraction with very low losses

- **„Panofsky" quadrupole** at extraction avoids excessive radial beam blow up from fringe field in last sector magnet
- drift space between magnets allows easy construction of **straight septa**
- **low power consumption** of magnets due to narrow magnet gap

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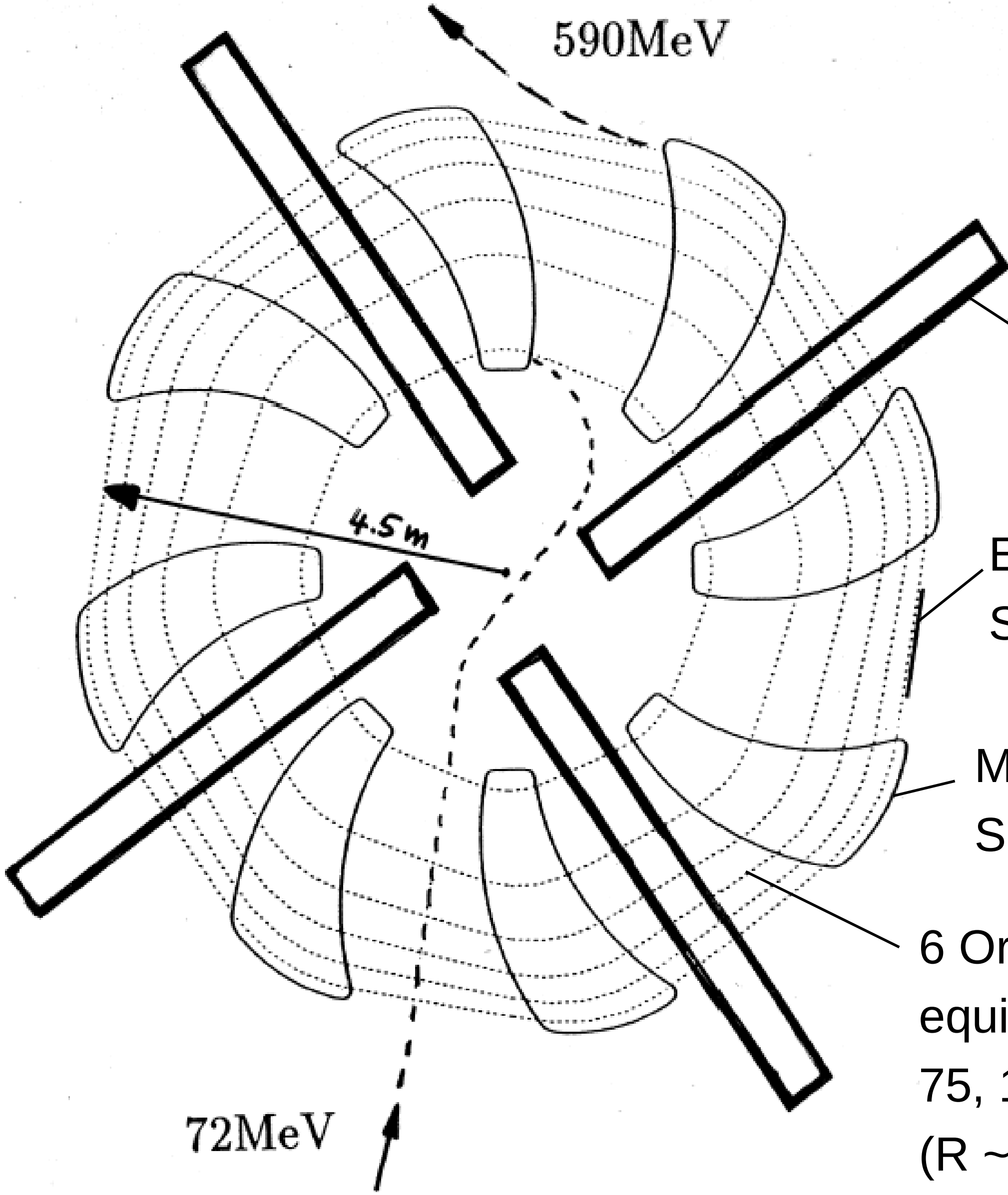
Orbits in Ring Cyclotron



RF Cavity

Extraction Septum

Magnet Sector

6 Orbits plotted at equidistant energies:
75, 177, 279, 381, 483, 585 MeV
(R ~ β)

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Key points for PSI Injector Cyclotrons

Philipps Injector

- **vertical collimators** in center of cyclotron give excellent beam quality

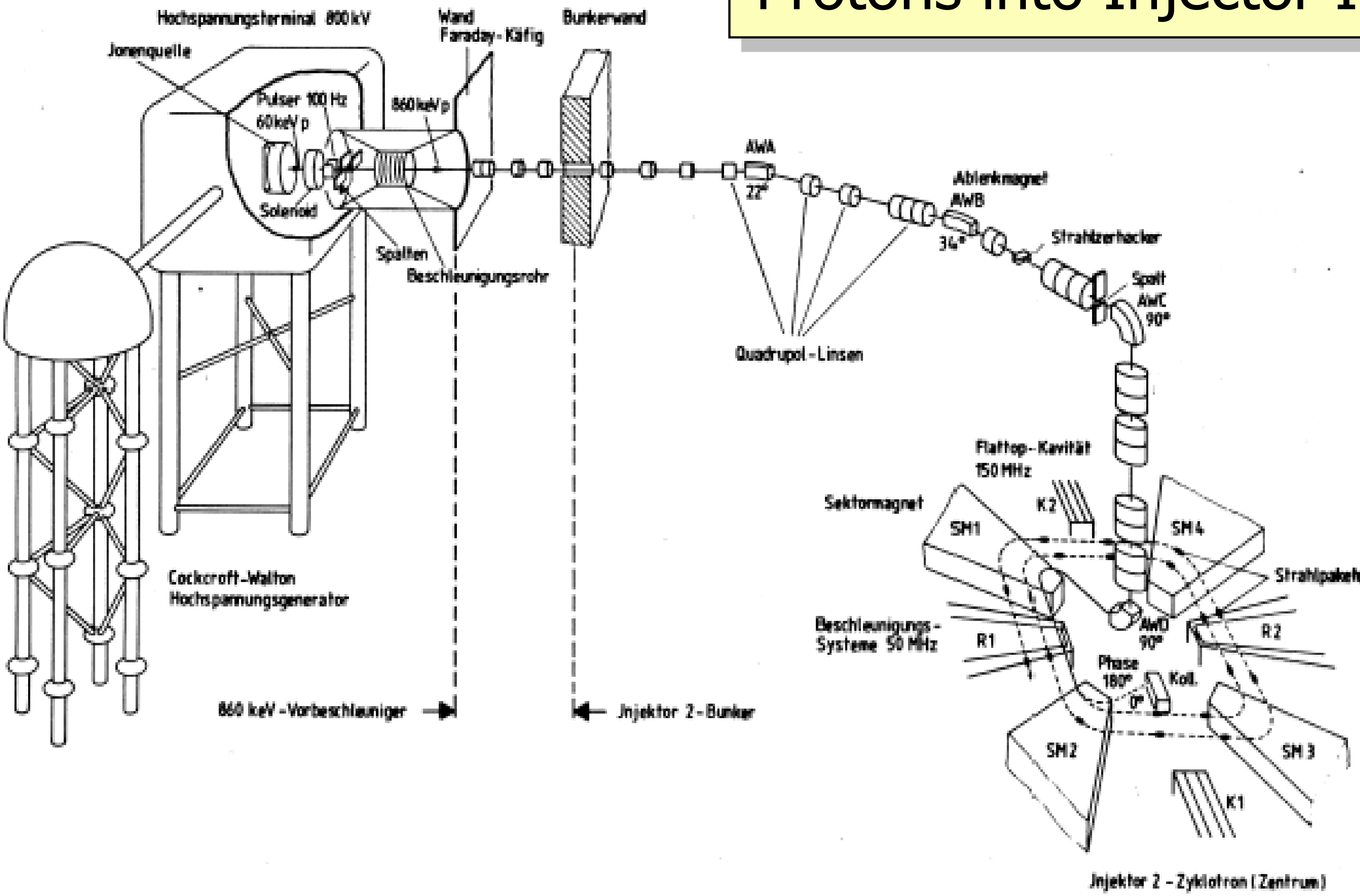
  => extraction efficiency jumped from 70% to 94% **(**100 **=> 200µA)**

Injector II

- space for new Injector II was **foreseen in initial layout**

  („provocation hole" in shielding wall !)
- large size of Injector II allows injection from **Cockkroft-Walton** at 870 keV
- large size gives **large turn separation** at extraction => very low losses
- **„spaghetti effect"** increases space charge limit by a factor of about 10 to ≈ **3 mA**

Injection of 870 keV Protons into Injector II



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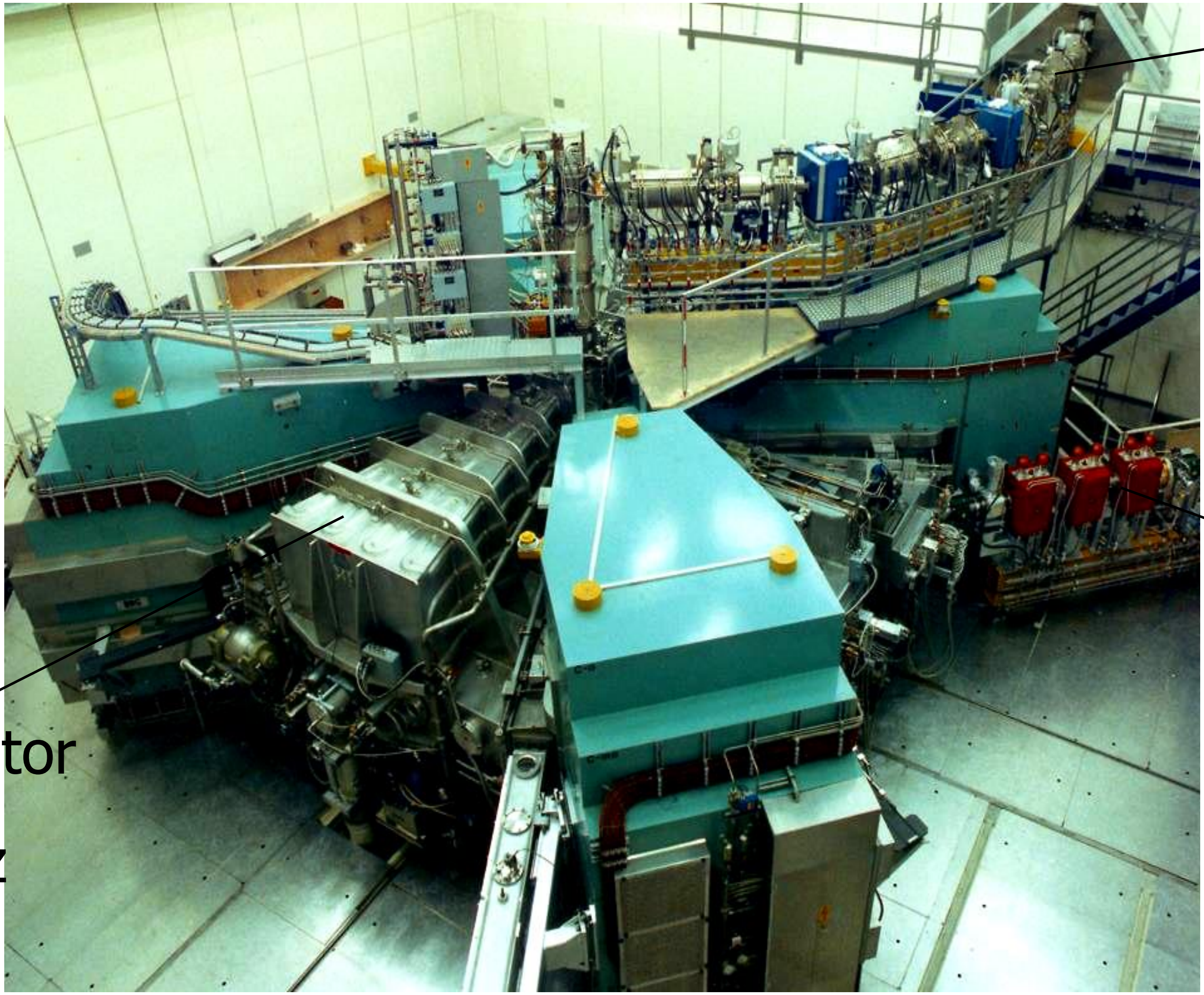
Injector II



Injection Line

870 keV

Resonator

50 MHz

Extraction Line

72 MeV Protons

(after 87 turns)

Current Limit

- **RF-System**: today 1.4 MW are delivered to the beam

  (100 kW are taken out again by the flattop system).

  => any further increase in current requires an upgrade of the RF system.

- **Activation of Cyclotron components**:

  Initially a loss of 5 µA at extraction was considered as acceptable !?

  Today the tolerance is about 0.5 µA in order to allow "hands-on" maintenance.

  => The losses at extraction determine the current limit

- **Transversal space charge forces**

  defocus the beam: The vertical tune is lowered and the beam size increases.

  For the Ring Cyclotron this effect becomes serious for currents above 10 mA

- **Longitudinal space charge forces**

  much more serious, because they increase the energy spread and thus

  the final beam size at extraction, increasing beam losses.

Model for Transversal Space Charge

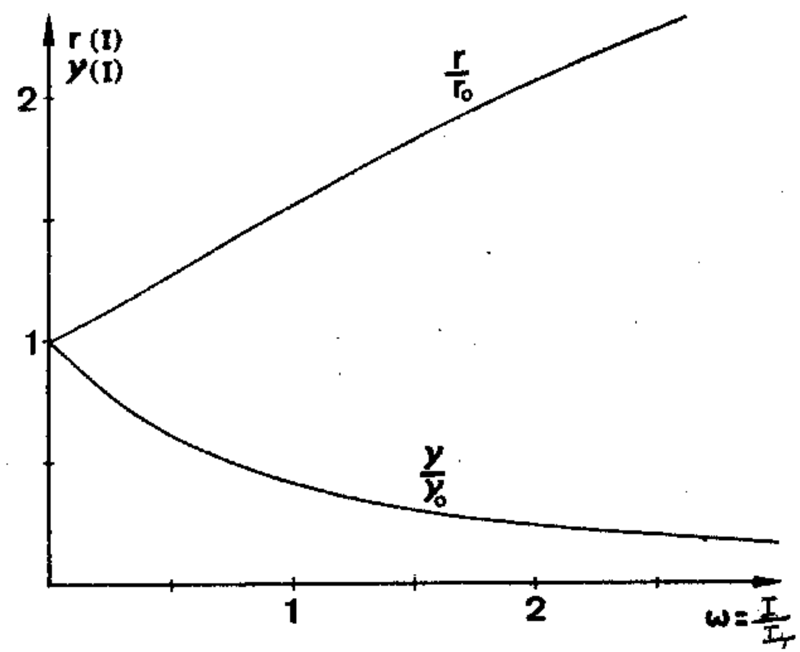
For a round beam the formula for the average tune $\nu(I)$ and beam radius r(I) is given by:

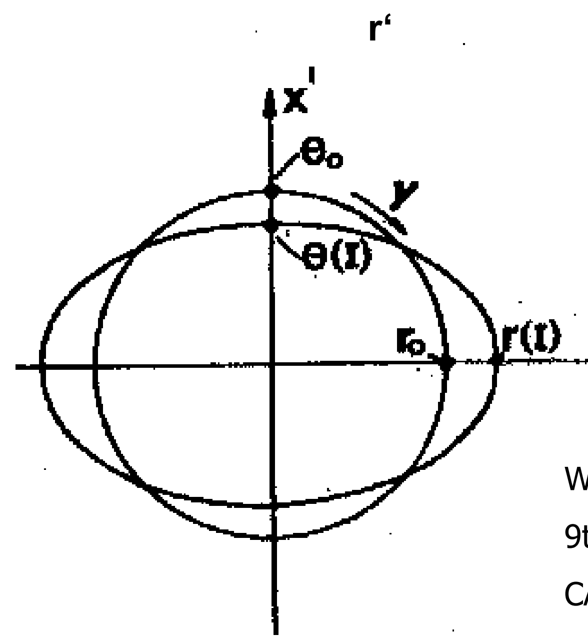
$$\frac{\nu(I)}{\nu_0} = \sqrt{1+\omega^2} - \omega , \qquad \left[\frac{r(I)}{r_0}\right]^2 = \sqrt{1+\omega^2} + \omega , \qquad \omega \equiv \frac{I}{I_T},$$

I is the average current,

for a parabolic longitudinal charge distribution with phase width $\Delta\Phi$:

$$I_T = \frac{2}{3}\frac{\Delta\Phi}{2\pi}\frac{\varepsilon\,\nu_0}{R}\,\tilde{p}^3\, I_0 , \qquad \varepsilon = \text{emittance}, \quad \tilde{p} \equiv \beta\gamma, \quad R = \text{orbit radius}, \quad I_0 = \frac{938\ \text{MV}}{30\ \Omega} = 31\ \text{MA}$$





W.Joho,
9th Int. Cyclotron conf.
CAEN (1981) p.337

Transversal Space Charge

At the injection energy of 72 MeV we have in the ring cyclotron:

$\tilde{p} = 0.4\,,\quad R = 2\,\mathrm{m},\quad \nu_0 \approx 1,\ \varepsilon \approx 2\,\mathrm{mm\,mrad},\ \Delta\Phi \approx 15^0,\ \Rightarrow I_T = 60\,\mathrm{mA}$

This gives for $I \approx 3\,\mathrm{mA}$: $\omega = 0.05\,,$

and thus only mild changes in r and ν:

$r(3\mathrm{mA}) \approx 1.025\, r(0)\,,\quad \nu(3\mathrm{mA}) \approx 0.95\,\nu(0)$

At an energy of 2 MeV we have in the injector II:

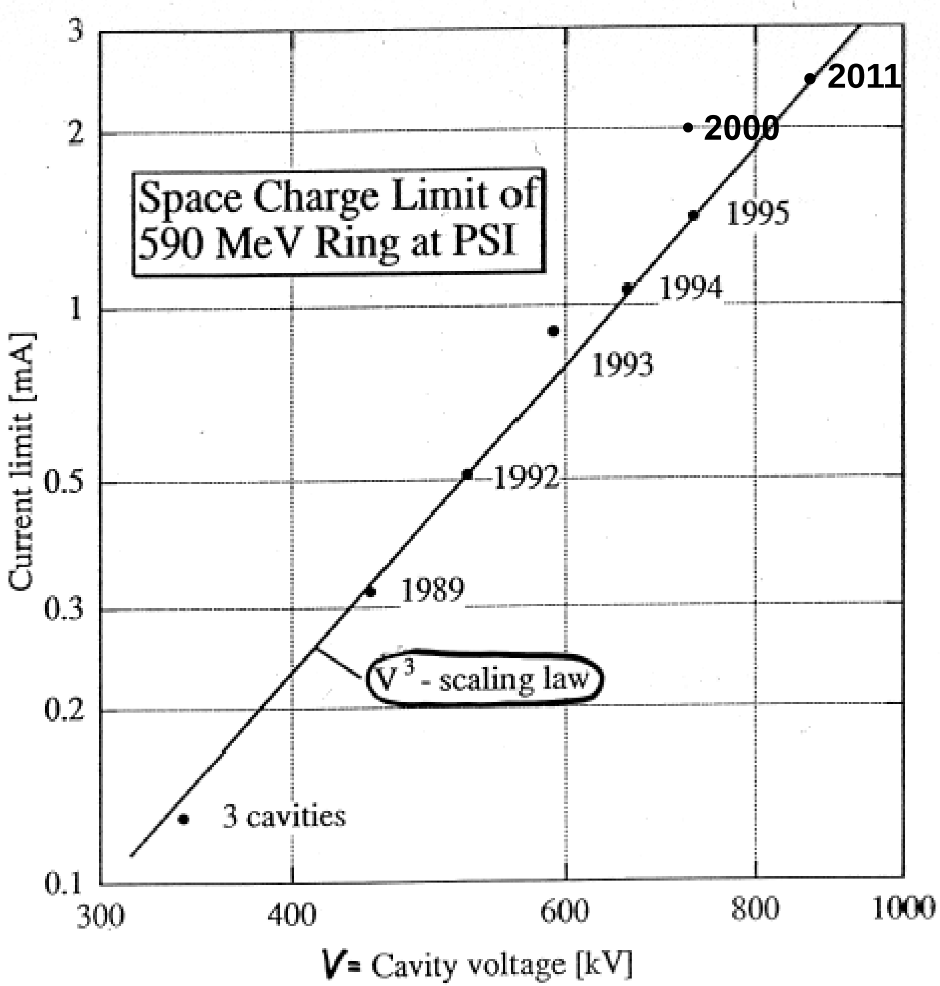
$\tilde{p} = 0.065,\quad R = 0.6\,\mathrm{m}\,,\quad \nu_0 \approx 1.3,\ \varepsilon \approx 11\,\mathrm{mm\,mrad}\,,\ \Delta\Phi \approx 9^0,\ \Rightarrow I_T = 3\,\mathrm{mA}$

This gives for $I \approx 3\,\mathrm{mA}$: $\omega = 1,$ and thus substantial changes in r and ν:

$r(3\mathrm{mA}) \approx 1.55\, r(0)\,,\quad \nu(3\mathrm{mA}) \approx 0.41\,\nu(0) \approx 0.5$

W.Joho, 9th Int. Cyclotron conference CAEN (1981) p.337

Longitudinal Space Charge



Longitudinal space charge forces increase the energy spread

=> higher extraction losses

=> limit on beam current

Remedy:

higher voltage V on the RF cavities

=> lower turn number **n** (V·n = const.)

current limit ~ V^3 ~ $1/n^3$!

There are 3 effects,
each giving a factor V(~1/n):

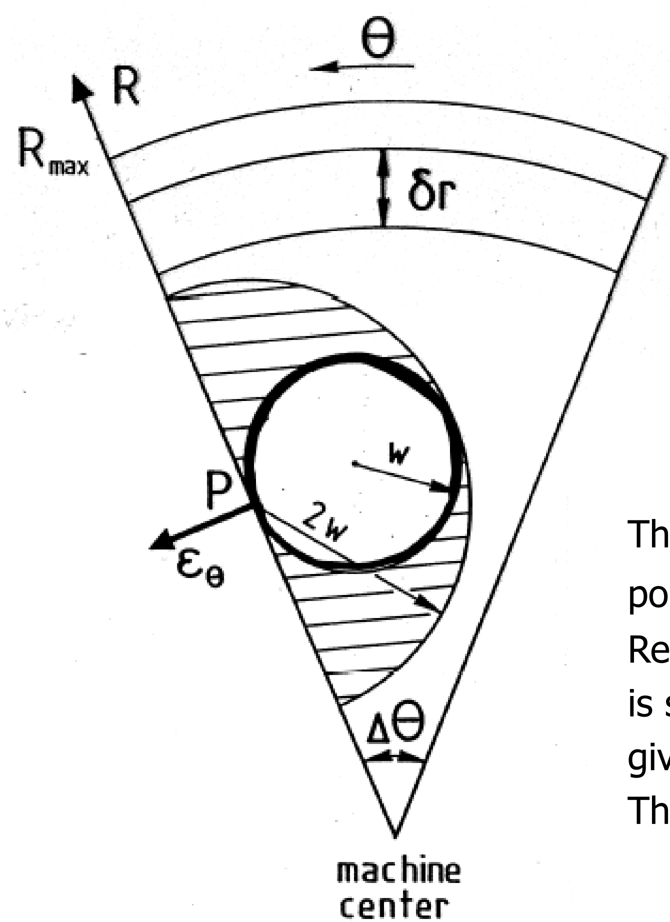
1) beam charge density ~ **n**

2) total path length in the cyclotron ~ **n**

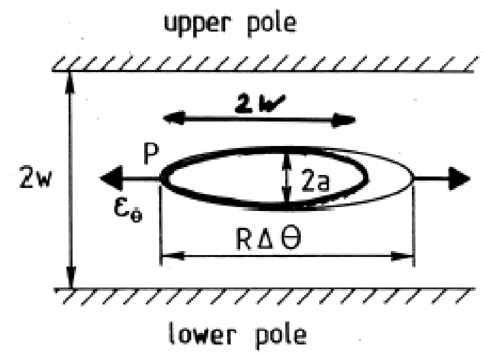
3) turn separation ~ **V**

W.Joho, 9th Int. Cyclotron conference CAEN (1981)

Space Charge Fields in Sector Model



top view



side view

circulating protons fill a cake-like piece with azimuthal extension Δθ. Neighbouring orbits are assumed to overlap radially.

(W.Joho, 9th Int. Cyclotron Conf. Caen 1981, p.337)

The azimuthal electric field at the edge of the „piece of cake" at point P is approximated by the calculable field of a **Disc** with radius w.
Reasoning: the charge of the protons outside of the half circle around P is screened by the upper and lower poles and protons in the hashed area give only a small contribution to the azimuthal field ϵ_θ.
The proton at P gains through ϵ_θ an additional energy/turn:

$$dE/dn = 2\pi R \, \epsilon_\theta$$

This simple model predicts, that the intensity limit from longitudinal space charge forces increases with $\mathbf{V^3}$!!
(V=cavity voltage/turn)

Longitudinal Space Charge in Cyclotrons

Reference: W.Joho, 9th Int. Cyclotron Conference 1981, Caen, p.337

Basis: longitudinal space charge force in Sector Model , non relativistic approximation

starting point : formula (11) in reference paper :

$\Delta E_{sc}(\mathrm{lin}) = e\,\Delta U_{sc}(\mathrm{lin}) =$ induced energy spread from linear space charge forces

$$(11)\quad \Delta U_{sc}(\mathrm{lin}) = I_{peak}\, Z_I \frac{n^2}{\beta_f}$$

I_{peak} = peak current

$$\bar{I} = \frac{\Delta\Phi}{2\pi} I_{peak} = \text{average current}$$

$\Delta\Phi$ = average total phase width

$$Z_I = 2.8\,\mathrm{k\Omega} = g_{1c}\frac{64\pi}{3} Z_0\,, \qquad g_{1c} \approx 1.4 = \text{formfactor}, \quad Z_0 = \frac{1}{4\pi\varepsilon_0 c} = 30\,\Omega$$

n = number of turns

$E \equiv eU$ = kinetic energy, $\qquad \beta_f = v_f/c$ = final "velocity"

Current Limit in Cyclotrons

$\Delta E_{sc}(\text{lin.})$ can be compensated with a tilted flattop voltage;

there remains a nonlinear part $\Delta E_{sc}(\text{nonlin.})$:

$$f_n \equiv \frac{\Delta E_{sc}(\text{nonlin.})}{\Delta E_{sc}(\text{lin.})} = \text{ fraction which cannot be compensated}$$

separated turns at extraction requires :

$$\Delta E_{sc}(\text{nonlin.}) < \mu_n \Delta E_n \;, \qquad \Delta E_n = \text{energy gain per turn}$$

$$\mu_n \approx 1/3 \text{ for centered beam, } \quad \mu_n \approx 1 \text{ for an eccentric beam}$$

=> we obtain from this simple model the following current limit :

$$I_{max} = \frac{\mu_n}{f_n} \frac{(U_f - U_i)}{Z_I} \frac{\beta_f}{n^3} \frac{\Delta\Phi}{2\pi}$$

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Current Limit in 590 MeV Ring Cyclotron

$$I_{max} = \frac{\mu_n}{f_n} \frac{(U_f - U_i)}{Z_I} \frac{\beta_f}{n^3} \frac{\Delta\Phi}{2\pi} = \text{current limit from longitudinal space charge}$$

$U_i = 72\ \text{MV}, \quad U_f = 590\ \text{MV}, \quad \beta_f = 0.8$

$n = 188\ \text{turns} \quad (\Delta E_n \approx 3\ \text{MeV})\ , \ \Delta\Phi \approx 12^0$

$f_n \approx 1/4 \ \ \text{(rough estimate!)}$

$\mu_n \approx 1 \quad \text{for eccentric injection} \quad (\approx 1/3 \text{ for centered beam !?})$

$\Rightarrow \quad I_{max} \approx 3\ \text{mA}$

This estimate is remarkably close to the present limit of 2.4 mA (2011), given the crude assumptions in this sector model:

- non relativistic case (at high energies: the charge density $\rho \propto \gamma^3$, M.Gordon)
- no turn structure, no radial cut of charge sheet at injection and extraction
- uncertainties in the parameter f_n and μ_n

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Current Limit in 72 MeV Injector II

$$I_L = \frac{\mu_n}{f_n} \frac{(U_f - U_i)}{Z_I} \frac{\beta_f}{n^3} \frac{\Delta\Phi}{2\pi} = \text{current limit from long. space charge}$$

$U_f = 72\ \text{MV}$, $\beta_f = 0.37$

$n = 85 \quad (\Delta E_n \approx 0.75\ \text{MeV})$, $\Delta\Phi \approx 6^0$

$f_n \approx 1/4$ (rough estimate only !)

$\mu_n \approx 1/3$ for centered beam

$I_L \approx 0.3\ \text{mA}$ (present record = 2.7 mA !)

=> sector model fails due to phase mixing

(space charge forces produce spherical bunch, "spaghetti effect")

=> this lowers energy spread from longitudinal space charge

=> much higher current limit !

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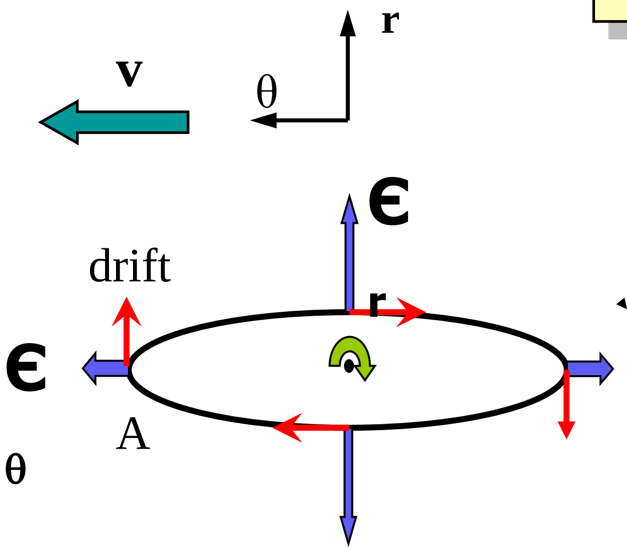
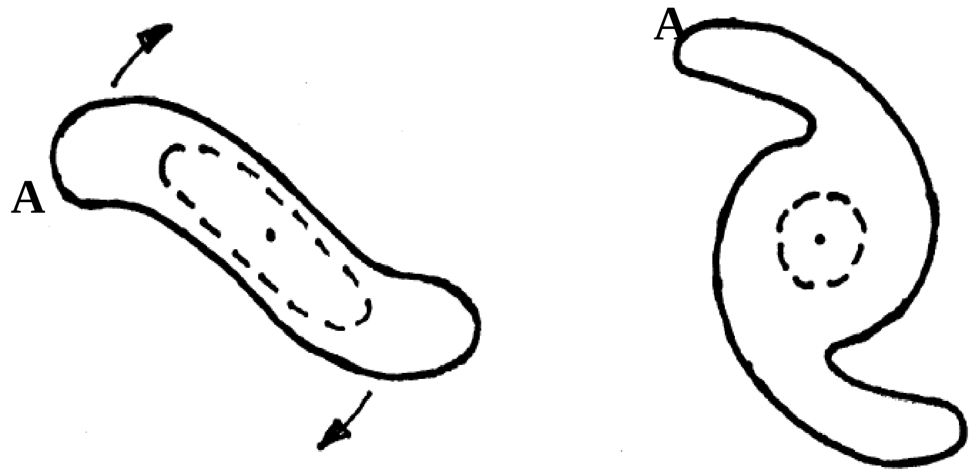
Longitudinal Space Charge in a Cyclotron Bunch





Particle at position A:

=> gains additional energy from space charge forces

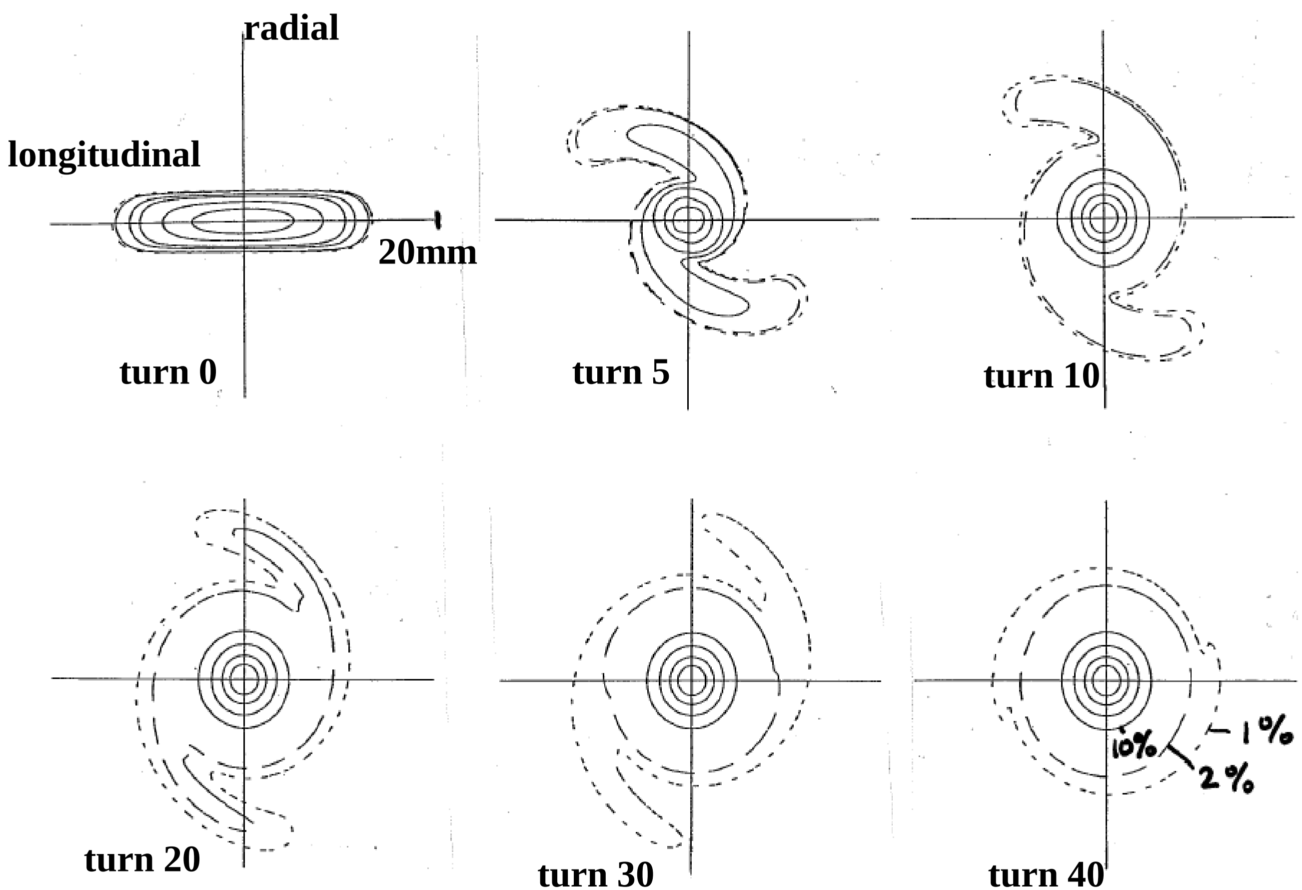
=> moves to higher radius due to isochronous condition

=> rotation of the bunch

=> nonlinearities produce spiral shaped halos

=> production of a **rotating sphere** (mixes phases)

Longitudinal Space Charge in Injector II Cyclotron



Simulation of a 1mA beam, circulating in Injector II at 3 MeV for 40 turns without acceleration.

The core stabilizes faster than the halos (calculations by Stefan Adam)

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Aristocracy ⇔ Democracy

Synchrotron

Linac

democratic:

a particle oscillates between head and tail

(phase focusing)

Cyclotron

aristocratic :

a particle „born ahead“ stays ahead !

(isochronism)

but **at high intensity**

a cyclotron becomes **democratic** !!

space charge mixes phases („spaghetti effect“)

=> higher current limit

Prediction of Current Limits in the PSI Cyclotrons

Predictions from 1978

(when I_{max} was 100 µA)

W.Joho, 8. Int. Cyclotron Conference

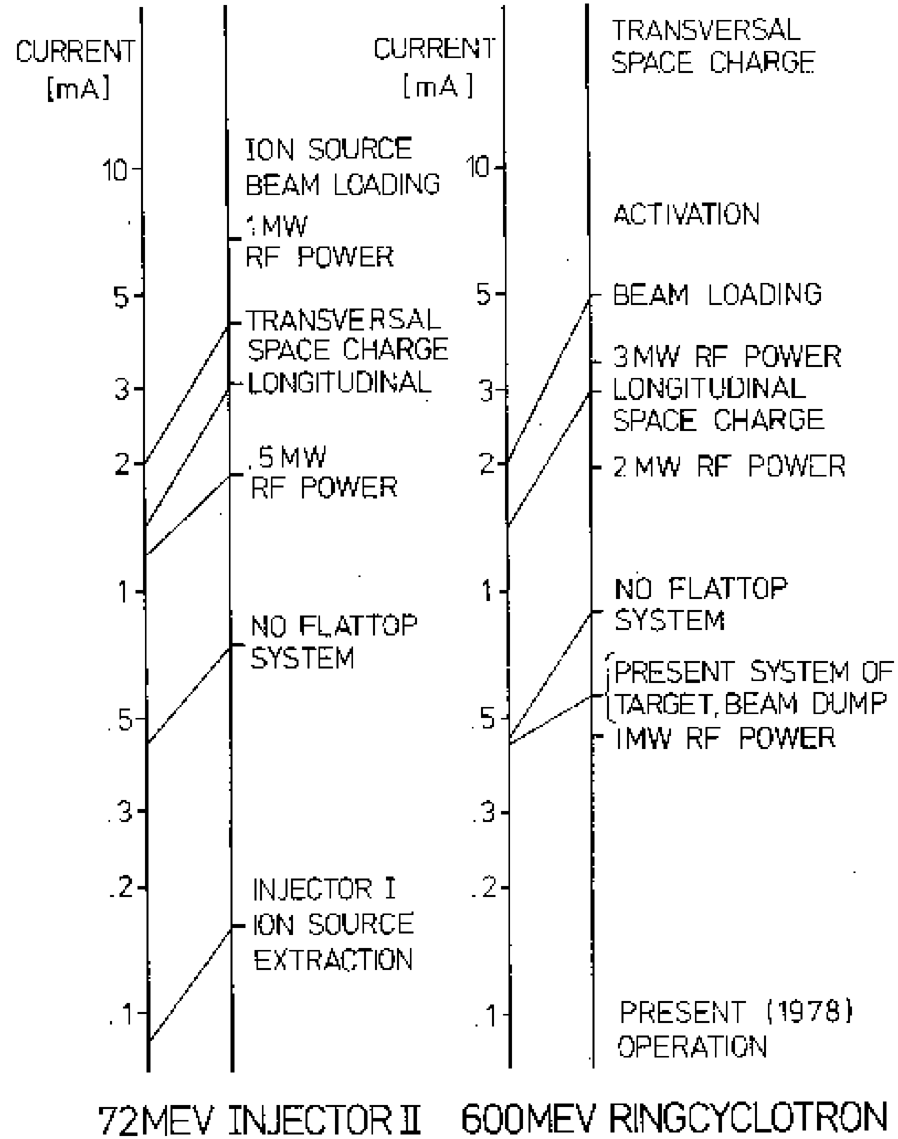
Indiana 1978 , p.1950

many limits are still valid.

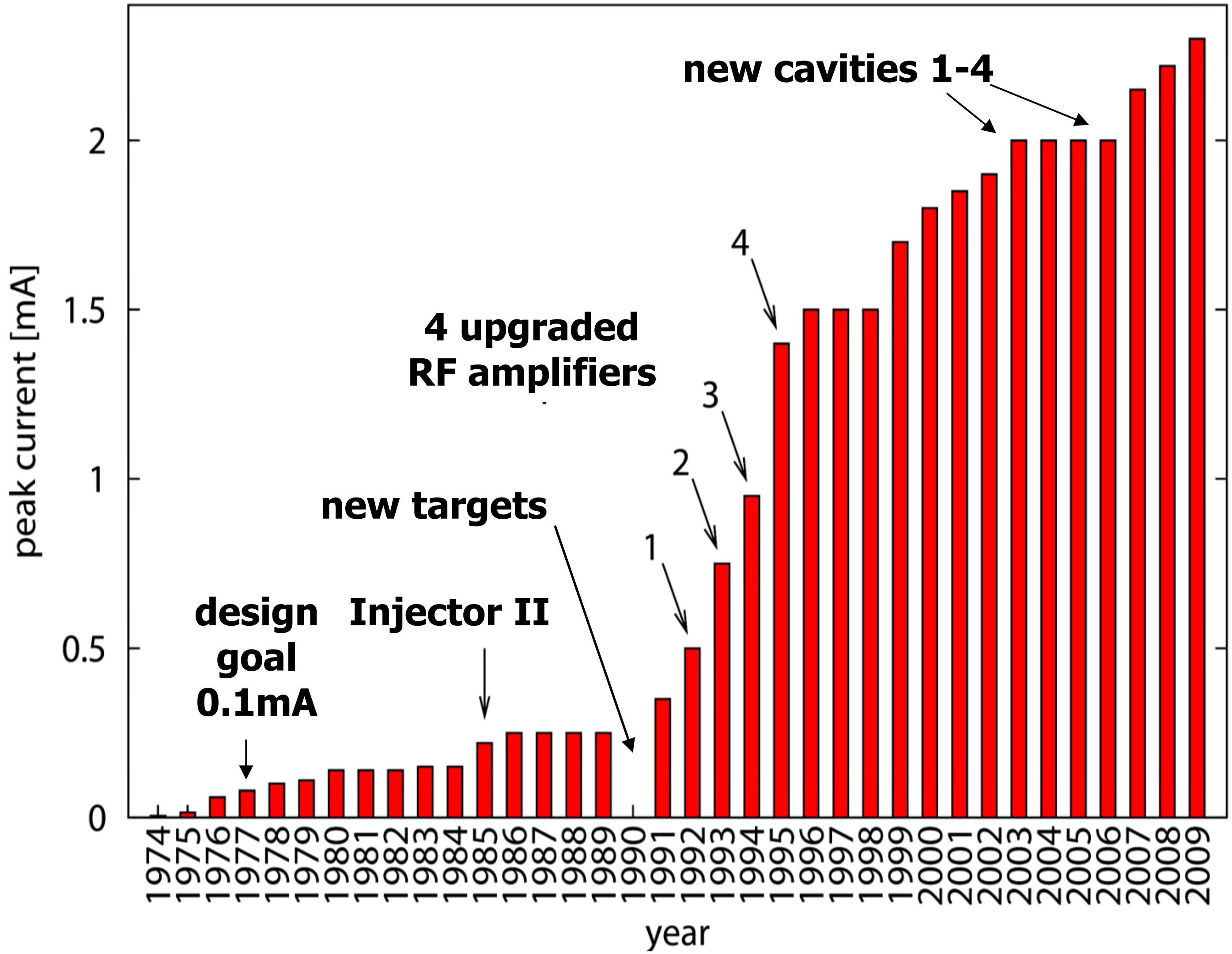
exception: even without flattop system

2.7 mA are possible in Injector II

due to „spaghetti effect" !



History of the Peak current in the 590 MeV Ring Cyclotron



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Model of the last turns in the 590 MeV Ring Cyclotron

- the turn separation is proportional to the orbit radius **R**
  and the cavity voltage **V**

  => concept of a large ring cyclotron with many high voltage cavities

- Flattop cavity gives mono-energetic beam

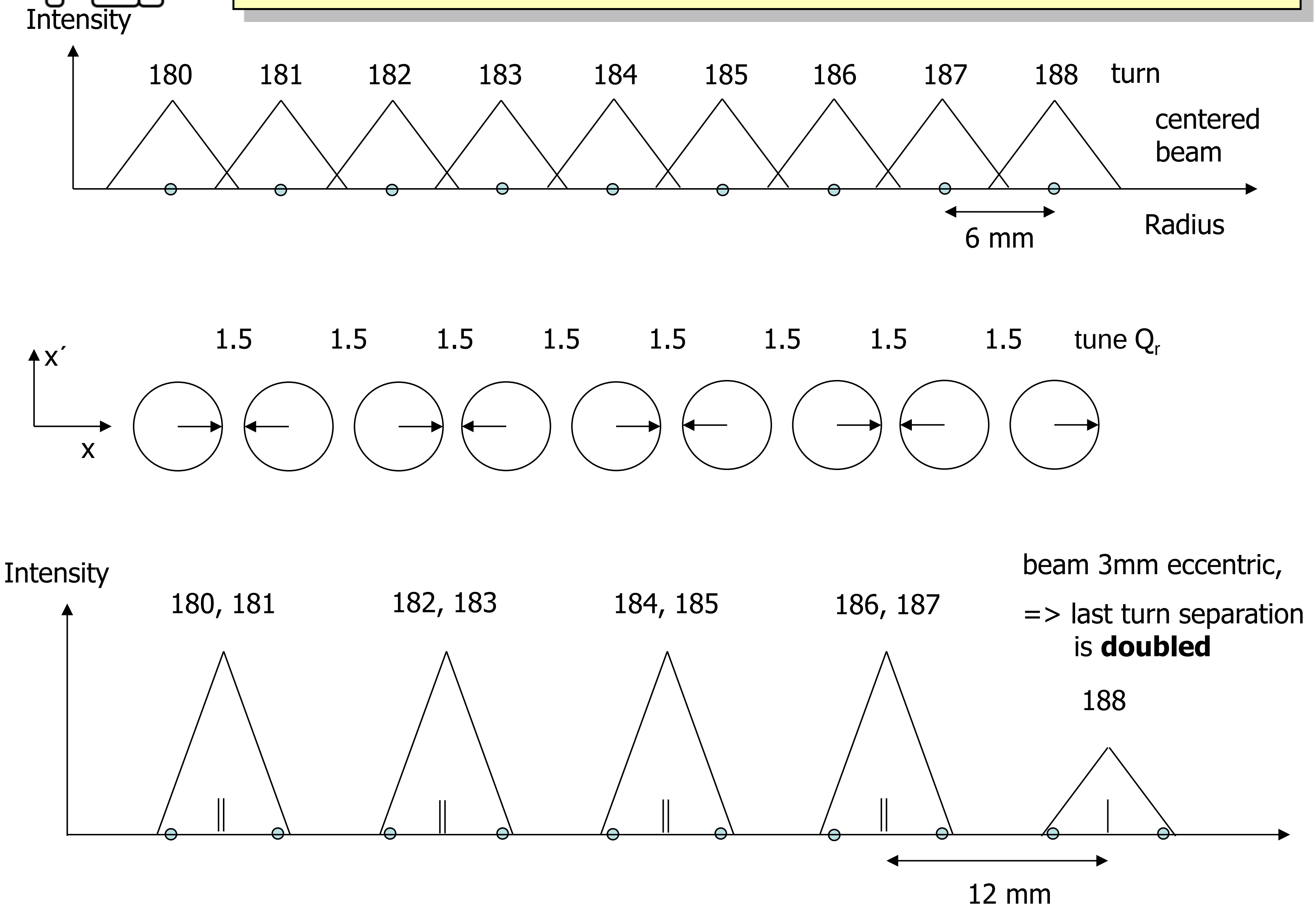
  => leads to single turn extraction

  => an eccentrically injected beam is still eccentric at extraction.

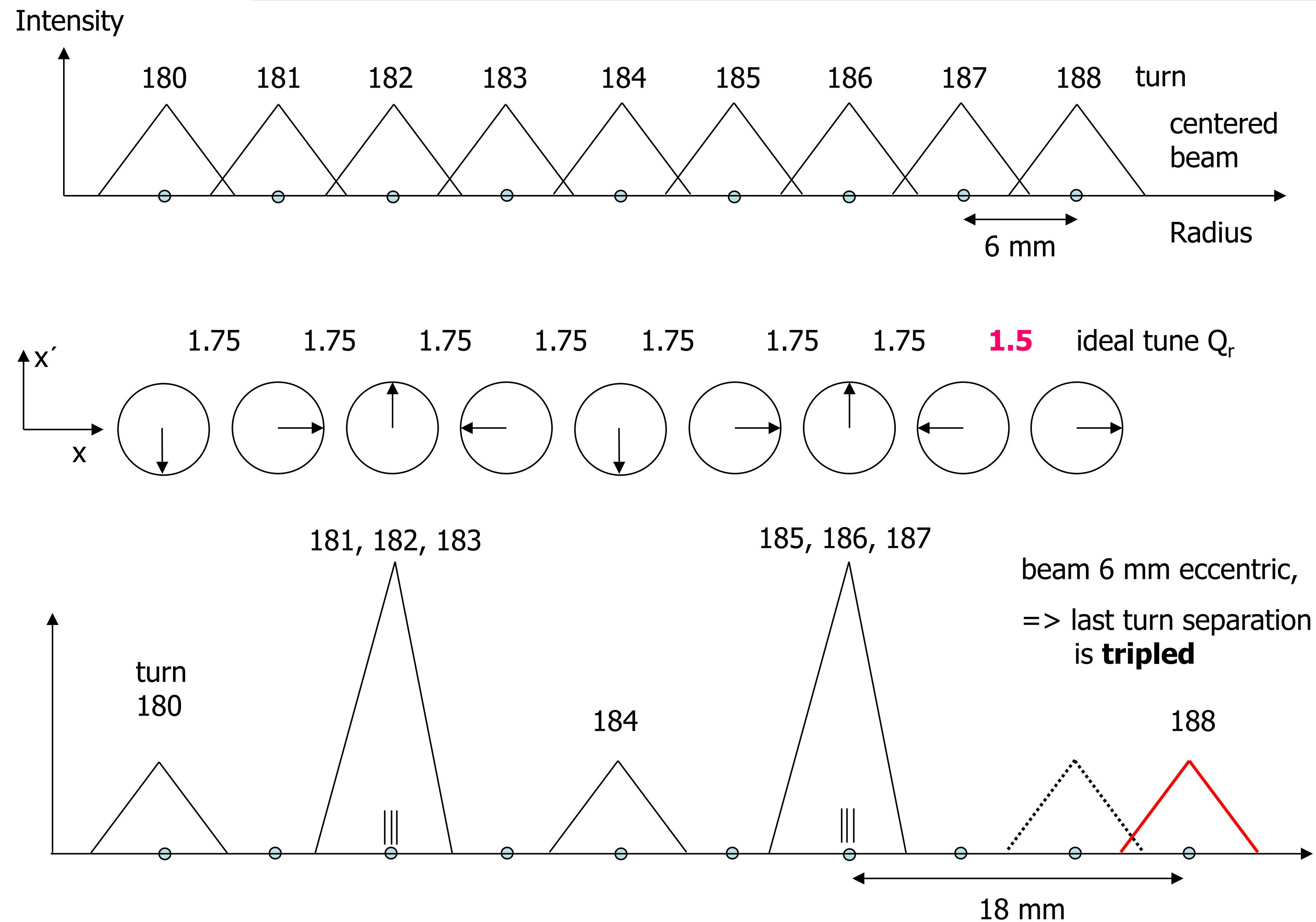
  => This can be used to increase the radial separation between the last two turns.

- In our simple model we assume an average turn separation of 6mm at extraction (energy gain 3 MeV/turn)

the last turns in the Ring Cyclotron,   model with half integer tune



the last turns in the Ring Cyclotron, model with ideal tune



beam 6 mm eccentric,

=> last turn separation is **tripled**

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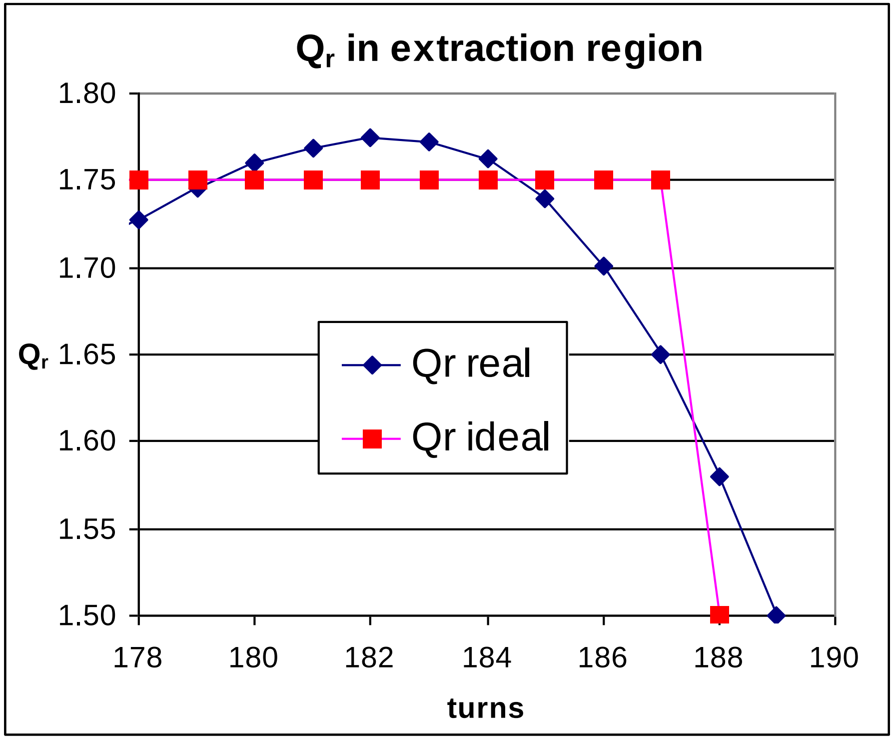
Tune Qr in Extraction Region

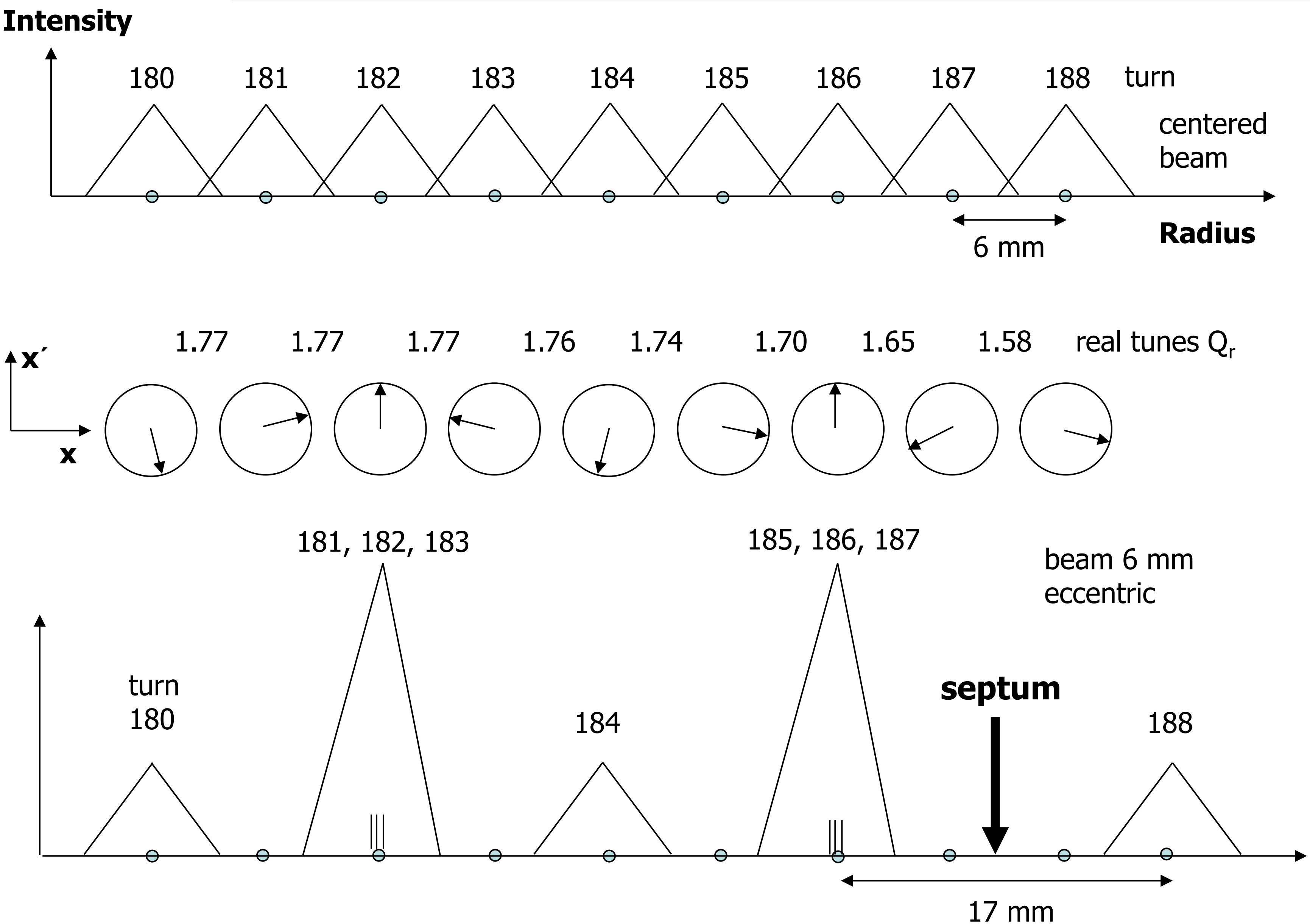


in the **ideal case** a tune of 1.75 puts 3 turns together, while with a fast drop to 1.5 the last turn is pushed away from the previous 3 turns.

In the fringe field region of the PSI ring cyclotron the **real tune** is close to the ideal one, giving an increase of the last turn separation from 6 to 17mm, using eccentric injection.

The drop of Qr in the fringe field helps to increase the turn separation even for a centered beam (dR ~ $1/(Qr^2)$)

the last turns in the Ring Cyclotron, model with real tune Q_r



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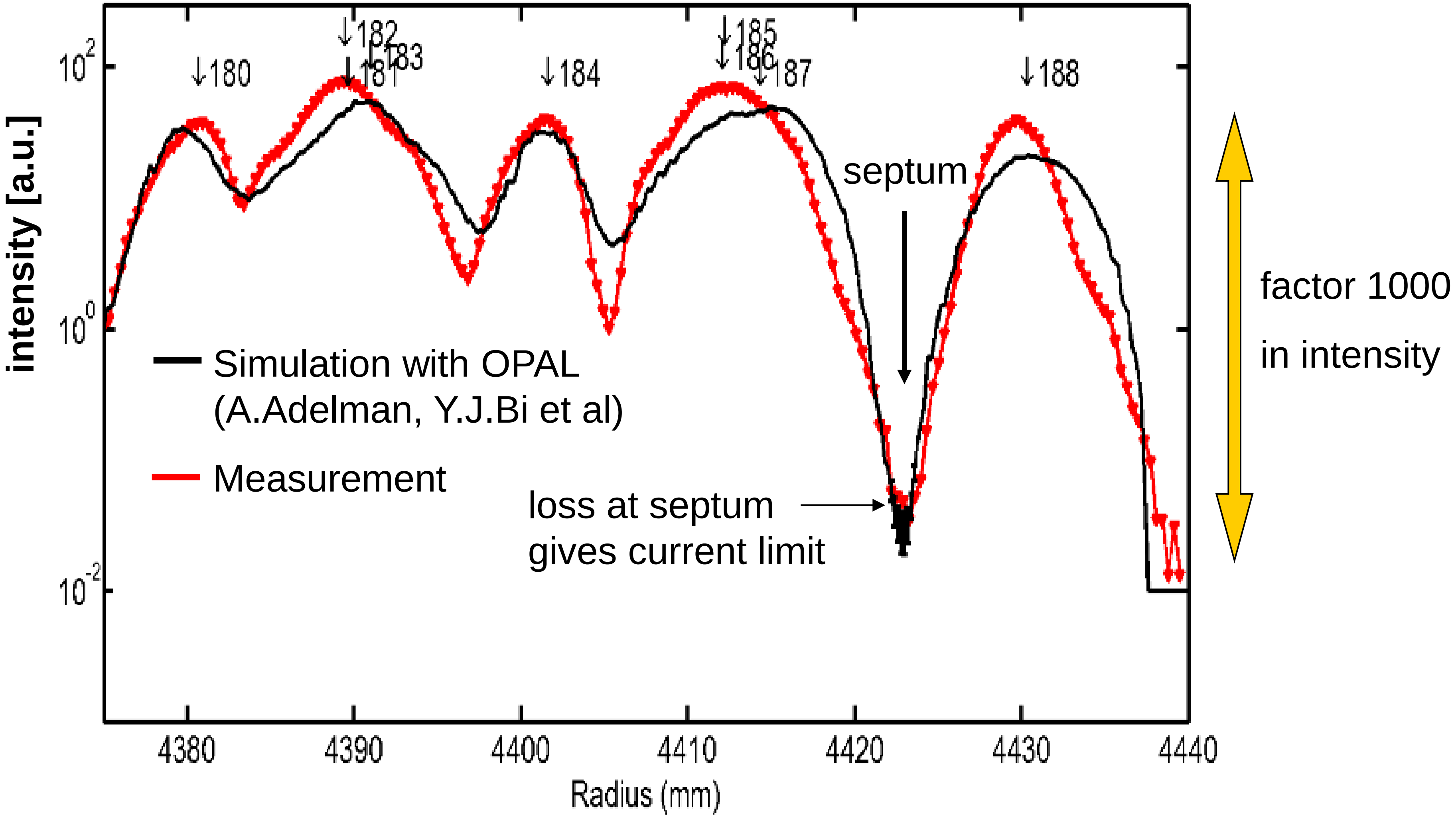
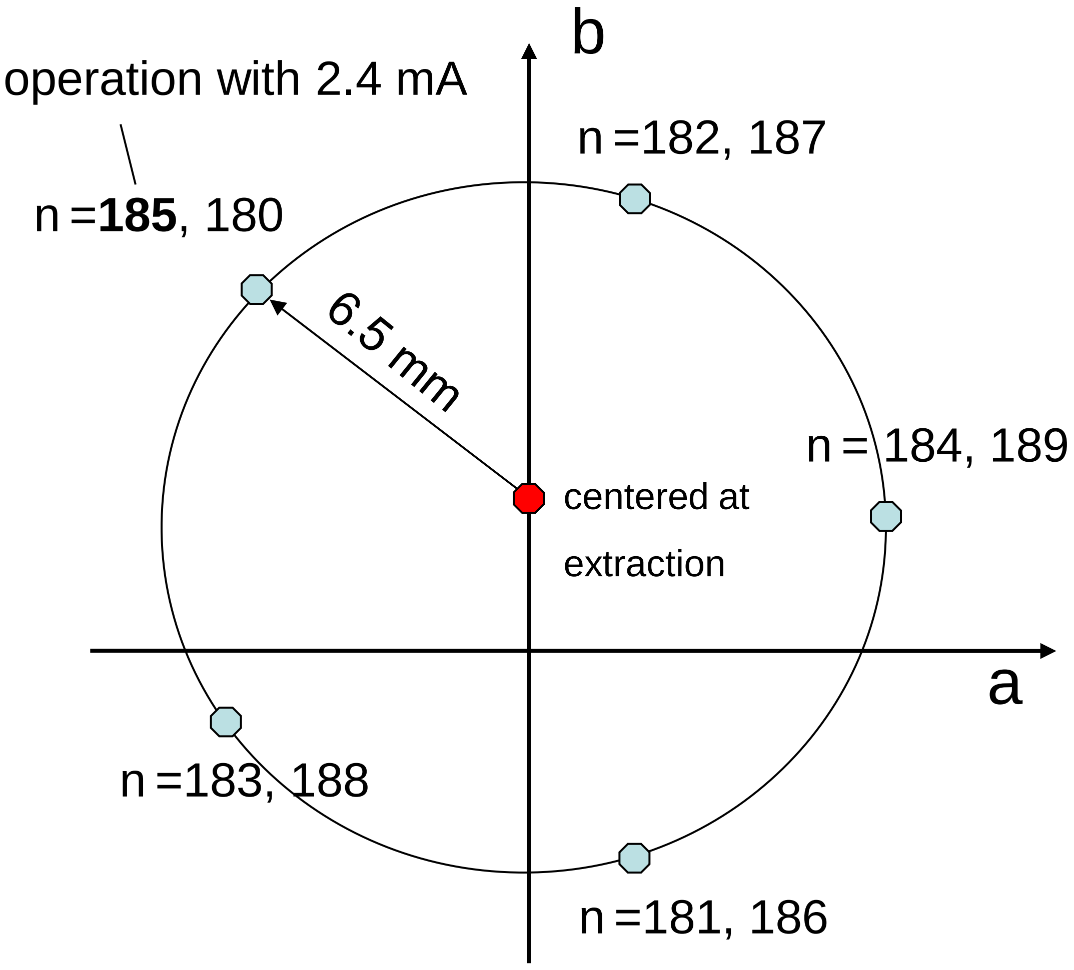
590 MeV Ring Cyclotron
last 9 of 188 turns for a 2 mA beam



Eccentric Injection

for maximum turn separation at extraction septum



horizontal oscillation:

$$x = a \cos[(Q_r - 1)\, 2\pi\, n] + b \sin[(Q_r - 1)\, 2\pi\, n]$$

n= total turnnumber

Start at probe RRI2 ($\vartheta = 172^0$)

average tune from injection to extraction: Qr(av.) ≈ 1.4 => same eccentricity, modulo 5 turns (5*1.4 = 7 oscillations)

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maximum turn separation at Extraction Septum

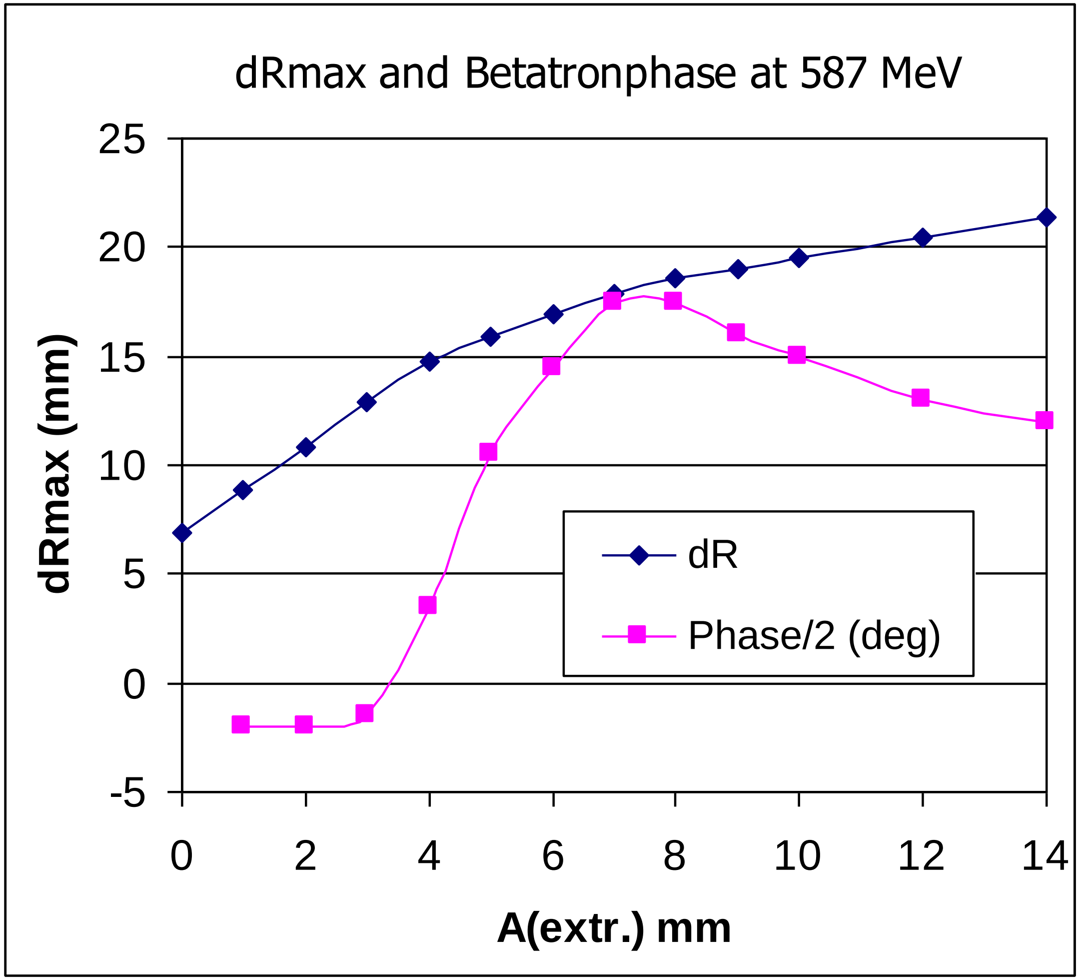
dRmax = max. separation between last turn and previous turns as a function of the eccentricity amplitude A at extraction. At injection this amplitude is about a factor 1.5 higher (effect of adiabatic damping).

dRmax starts to level off at an amplitude of 7mm, where the second last turn is now at a lower radius than the previous turns (turn crossing).

During 2011 with operation at 2.2 mA an amplitude of about 6.5 mm was used to obtain extraction losses of less than 0.01%.

Using much higher amplitudes leads to vertical losses due to the coupling resonance Qr = 2 Qz.

(This plot was obtained in collaboration with Herbert Müller).



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Reduction of Extraction losses

Cavity Voltage [kV]	Beam	Flattop Cavity	Losses [%]	Losses [µA]	Imax [µA]
500	Orig. Design	no	5	**5**	100
450	centered	no	1.2	0.5	40
450	eccentric	no	0.25	0.5	200
450	eccentric	yes	0.1	0.5	500
850	eccentric	no	0.06	0.5	800
850	eccentric	yes	0.02	0.5	2’400

Improvement by	Factor	
Beam Quality	~ 3	
eccentric Injection	~ 5	
Flattop Cavity	~ 3	
Cavity Voltage	~ 5	(≈ V^3 law)
total	**240**	

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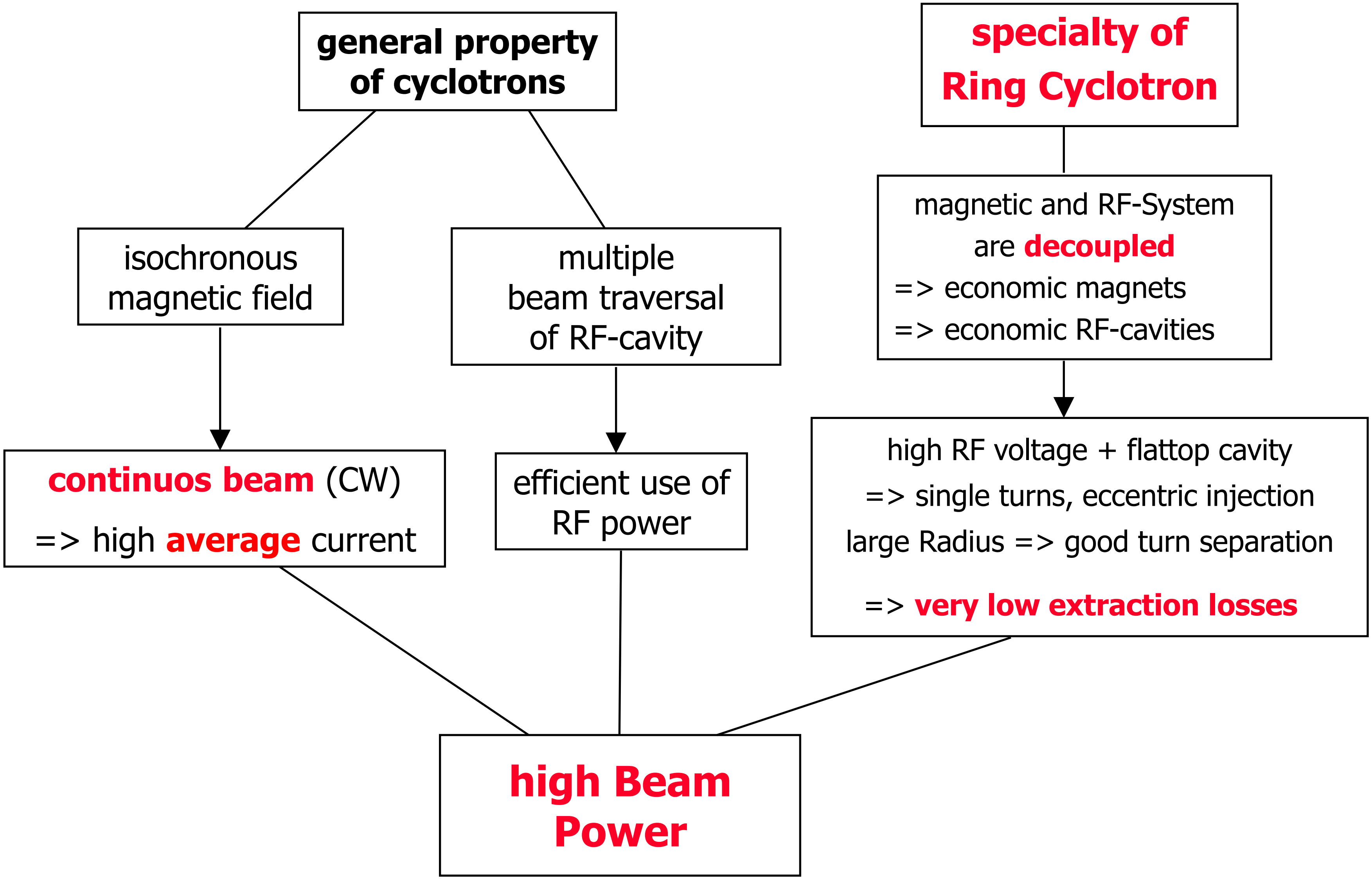
why is the PSI Ring Cyclotron such an efficient Accelerator ?



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Success Factors for PSI Ring Cyclotron

1. Magnets and RF-System are decoupled
2. 4 high voltage cavities → **I max ~ V³**
3. large Radius R → high turn separation dR/dn ~ R·V
   - fast acceleration into fringe field, where Qr drops: dR/dn ~ 1/Qr²
4. excellent beam from Injector → separated turns
5. Flattop Cavity → high phase acceptance ΔΦ
6. eccentric Injection → Qr at extraction 1.75 => 1.5
   - wins factor 3 in dR/dn: 6 => 17mm
7. straight electrostatic Septum with 0.1mm strips → Losses at extraction ≈ 10^{-4}
8. Continuous Beam (CW) → **1.4 MW Beam Power**

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