NNLO corrections to jet production in deep inelastic scattering

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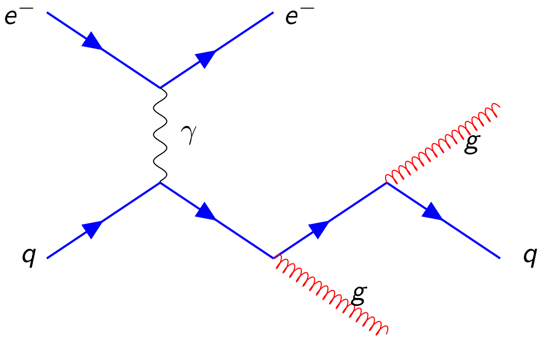
Overview

- Motivation
  - What is DIS?
  - Why QCD to higher orders?
- Theoretical Aspects
  - Infrared divergences
  - Antenna subtraction
- Outlook

Deep Inelastic Scattering



HERA

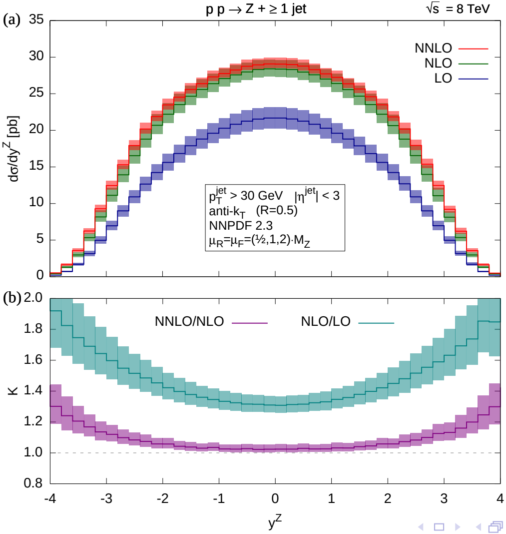
Why QCD to higher orders?

Last HERA analysis (2010):

$$\begin{aligned}\sigma^{dijet}_{meas}(ep \to 2\text{jets}) &= 73 \pm 2(\text{stat.}) \pm 7(\text{syst.})\text{pb},\\ \sigma^{dijet}_{theo}(ep \to 2\text{jets}) &= 77^{+25}_{-20}(\text{scale})^{+4}_{-14}(\text{PDF}) \pm 3(\text{had})\text{pb}.\end{aligned}$$

- Differential cross sections are important for determination of PDFs.
- Need to reduce renormalisation and factorisation uncertainties on theoretical predictions.

Visualisation



Jet Cross Sections

Cross sections at different orders:

$$\mathrm{d}\sigma_{\mathrm{LO}} = \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{B}}$$

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{NLO}}$$

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$$

IR divergences and KLN theorem

- IR divergences appear in dimensional regularisation as poles in $\frac{1}{\epsilon^n}$.
- They come from:
  - Virtual loops in Feynman diagrams,
  - Phase space integration: Regions where particle momenta are collinear or go to zero.
- Kinoshita-Lee-Nauenberg (KLN) theorem:
  - Sufficient inclusive quantities are IR finite, i.e. IR divergences cancel overall when all corrections are added togehter.

Problem: Direct numerical integration of phase space integrals is not possible due to divergences.

Jet Cross Sections

Cross sections at different orders:

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$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{NLO}}$$

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$$

The idea: Adding a clever zero

$$\overbrace{d\sigma_{\mathrm{NNLO}}}^{\text{IR finite}} = \int_{d\Phi_{m+2}} \overbrace{\left[d\sigma^{\mathrm{RR}}_{\mathrm{NNLO}} + d\sigma^{\mathrm{RR,S}}_{\mathrm{NNLO}} \right]}^{\text{IR finite}} + \int_{d\Phi_{m+1}} \underbrace{\left[d\sigma^{\mathrm{RV}}_{\mathrm{NNLO}} + d\sigma^{\mathrm{RV,T}}_{\mathrm{NNLO}} \right]}_{\text{IR finite}} + \int_{d\Phi_m} d\sigma^{\mathrm{VV}}_{\mathrm{NNLO}} + \int_{d\Phi_m} d\sigma^{\mathrm{VV,U}}_{\mathrm{NNLO}} + d\sigma_{MF}.$$

Subtraction terms have the same IR divergences as MEs
$\rightarrow$ integrand is IR finite
$\rightarrow$numerical integration is possible.

Antenna formalism

How does this work in detail?

Factorisation of IR divergences

- MEs in colour-order form follow universal IR divergence factorisation:

1. Soft radiation, $p_j \to 0$:
   $|\mathcal{M}^0_{m+1}(\cdots, p_i, p_j, p_k, \cdots)|^2 \to S_{ijk} |\mathcal{M}^0_m(\cdots, p_i, p_k, \cdots)|^2$
2. Collinear splitting $p_i || p_j$:
   $|\mathcal{M}^0_{m+1}(\cdots, p_i, p_j, p_k, \cdots)|^2 \to \frac{P_{ij\to I}}{s_{ij}} |\mathcal{M}^0_m(\cdots, p_{i+j}, p_k, \cdots)|^2$

Same idea at NNLO where up to two particles can be unresolved $\to$ more softkernels and collinear splitting functions, e.g. $P_{ijk\to K}$.

Factorisation of phase space

Phase space factorises for appropriate linear mappings $p_i, p_j, p_k \to p_I, p_K$:

$$\mathrm{d}\Phi_{m+1}(p_1, \cdots, p_{m+1}; q) = \underbrace{\mathrm{d}\Phi_m(p_1, \cdots, p_I, p_K, \cdots, p_{m+1}; q)}_{\text{reduced phase space}} \times \underbrace{\mathrm{d}\Phi_3(p_i, p_j, p_k; p_I + p_K)}_{\text{antenna phase space}}$$

- Mapping conserves 4 momentum.
- p_I and p_K "on mass-shell".
- Analogous factorisation of 4 particle phase space.

Subtraction method

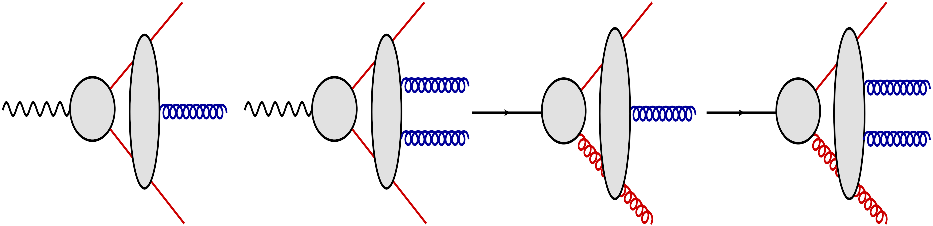
Using momentum map from $\{p_{m+1}\} \to \{\widetilde{p_m}\}$ with $\{p_X\} \subset \{p_{m+1}\}$, construct subtraction terms according to factorisation:

$$\mathrm{d}\sigma_{NNLO}^{RR,S} \approx \underbrace{X(\{p_X\})}_{\text{antenna}} \mathrm{d}\Phi_3(\{p_X\}) \times \overbrace{|\mathcal{M}(\{\widetilde{p_m}\})|^2}^{\text{reduced ME}} \mathrm{d}\Phi_m(\{\widetilde{p_m}\}) \times \underbrace{\mathcal{J}(\{\widetilde{p_m}\})}_{\text{jet function}}$$

Analytic Integration of antenna phase space
$\to$ IR divergences as explicit ϵ poles
$\to$ analytic cancellation with poles of virtual corrections.

Antenna functions

- Antennae have two hard radiators
- Antennae reproduce all IR divergences of QCD.
- Have all been analytically integrated over antenna phase space!
- Have some none QCD divergences.



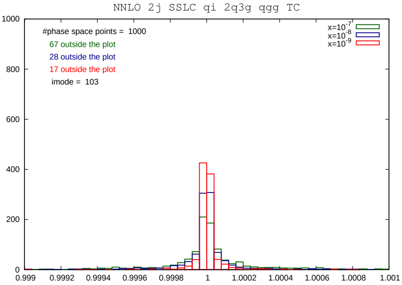
Recipe

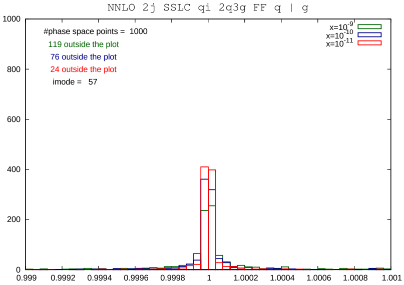
- construct RR subtraction terms. ✓
- construct RV subtraction terms. ✓
- construct VV subtraction terms. ✓
- do numerical integration; in progress.

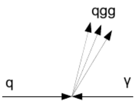
Correctness of RR,RV,VV are checked by pole cancellation and spike plots:

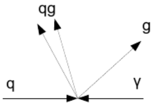
- spikeplots examine ratio of e.g. $\sigma_{\mathrm{NNLO}}^{\mathrm{RR,S}}/\sigma_{\mathrm{NNLO}}^{\mathrm{RR}}$.
- pole cancellation is analytic.

“Spike Plots”


NNLO 2j SSLC qi 2q3g qgg TC
#phase space points = 1000
67 outside the plot
28 outside the plot
17 outside the plot
imode = 103
x=10^-7
x=10^-8
x=10^-9
0
200
400
600
800
1000
0.999
0.9992
0.9994
0.9996
0.9998
1
1.0002
1.0004
1.0006
1.0008
1.001


NNLO 2j SSLC qi 2q3g FF q | g
#phase space points = 1000
119 outside the plot
76 outside the plot
24 outside the plot
imode = 57
x=10^-9
x=10^-10
x=10^-11
0
200
400
600
800
1000
0.999
0.9992
0.9994
0.9996
0.9998
1
1.0002
1.0004
1.0006
1.0008
1.001


qgg
q
γ


qg
g
q
γ

Outlook

- Results for NNLO DIS are expected to appear very soon!
- Other processes that are calculated using antennae are:
  - V+jet,
  - H+jet,
  - jet+jet,