Differential diboson production in hadron collisions at NNLO: the real-virtual contribution.

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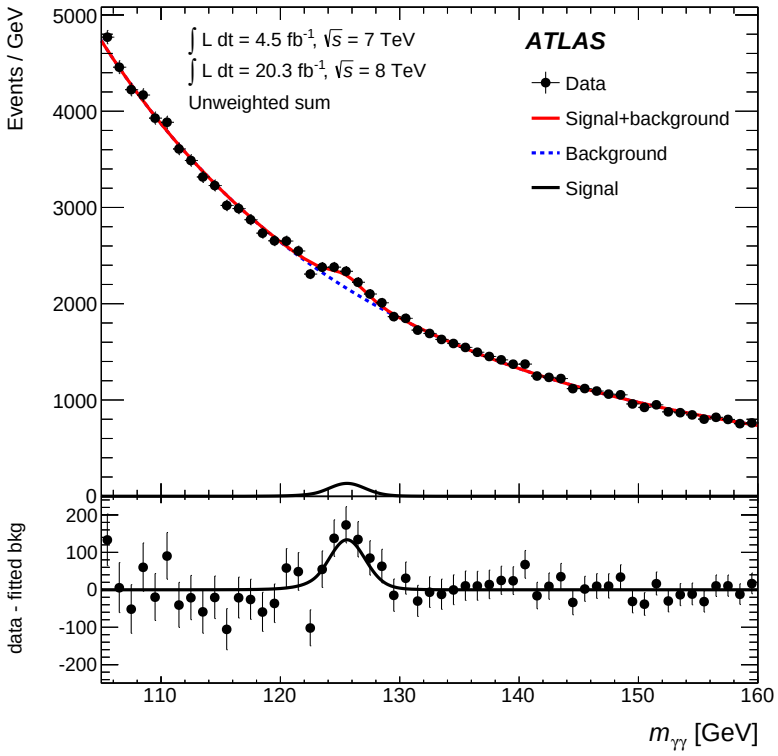
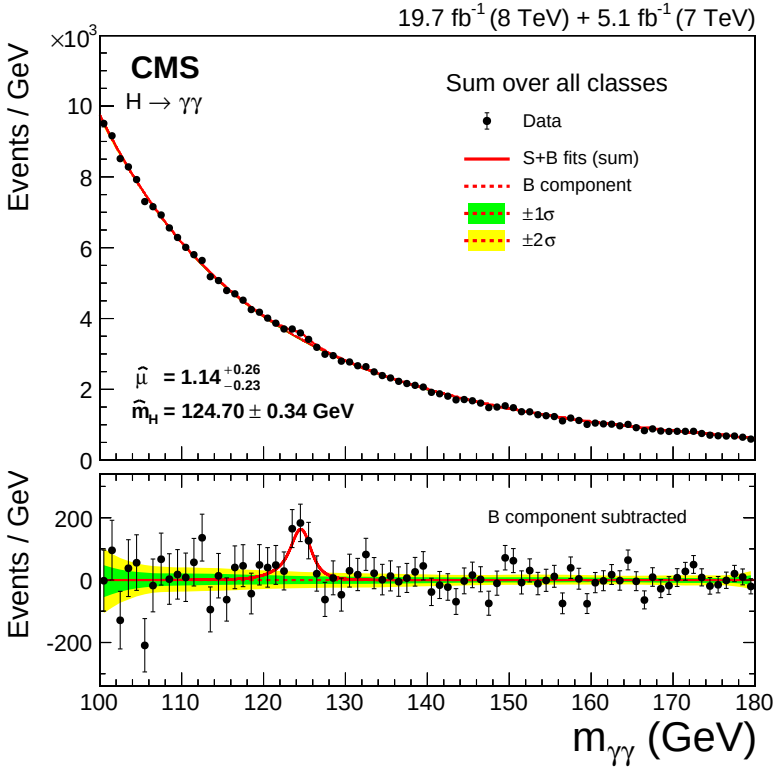
work in collaboration with C. Anastasiou and A. Lazopoulos

Why diboson production?





$$m_H \sim 125 \text{ GeV} \qquad \Rightarrow \qquad \begin{cases} H \to \gamma\gamma & \checkmark \\ H \to VV^* & \checkmark \end{cases}$$

QCD production of $\gamma\gamma$ and VV^*:

irreducible backgrounds for Higgs physics studies!

precision requires NNLO QCD, best if fully differential!

Why diboson production?

2013 Les Houches wish list

Process	known	desired	motivation
V	d\sigma(lept. V decay) @ NNLO QCD + EW	d\sigma(lept. V decay) @ NNNLO QCD + NLO EW MC@NNLO	precision EW, PDFs
V+j	d\sigma(lept. V decay) @ NLO QCD + EW	d\sigma(lept. V decay) @ NNLO QCD + NLO EW	Z+j for gluon PDF W+c for strange PDF
V+jj	d\sigma(lept. V decay) @ NLO QCD	d\sigma(lept. V decay) @ NNLO QCD + NLO EW	study of systematics of H+jj final state
VV'	d\sigma(V decays) @ NLO QCD d\sigma(stable V) @ NLO EW	d\sigma(V decays) @ NNLO QCD + NLO EW	bkg H → VV TGCs
gg → VV	d\sigma(V decays) @ LO	d\sigma(V decays) @ NLO QCD	bkg to H→VV
V\gamma	d\sigma(V decay) @ NLO QCD d\sigma(PA, V decay) @ NLO EW	d\sigma(V decay) @ NNLO QCD + NLO EW	TGCs
Vb\bar b	d\sigma(lept. V decay) @ NLO QCD massive b	d\sigma(lept. V decay) @ NNLO QCD massless b	bgk to VH(→bb)
VV'\gamma	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	QGCs
VV'V"	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	QGCs, EWSB
VV'+1jet	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	bkg to H, BSM searches
VV'+2j	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	QGCs, EWSB
\gamma\gamma	d\sigma @ NNLO QCD		bkg to H→ \gamma\gamma

2 mass scales are a technical obstacle yet to overcome ...

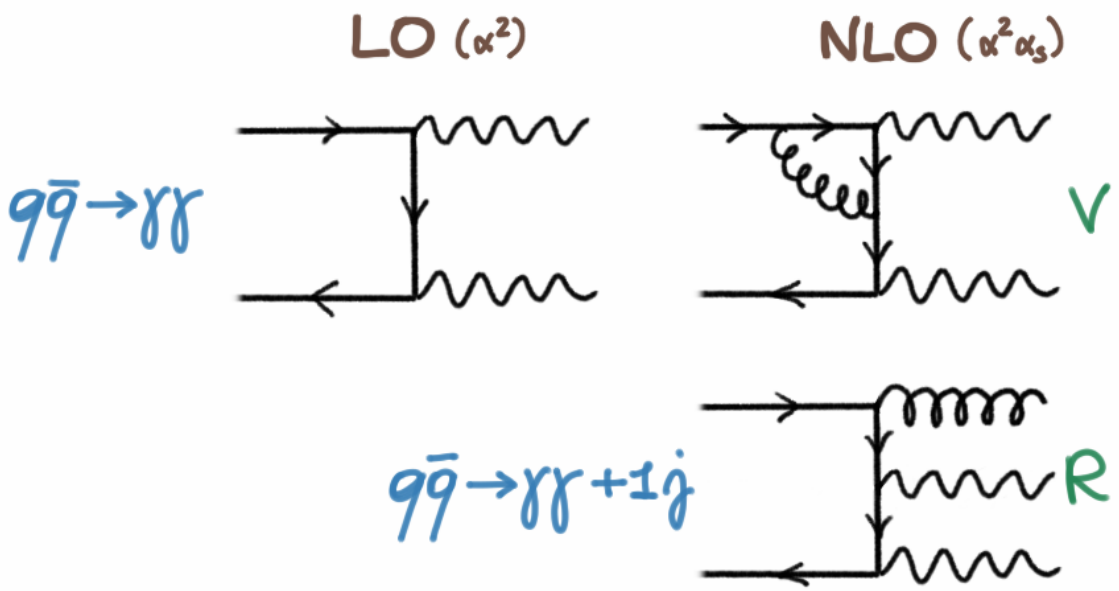
... we need a test process ...

... and it's right down here!

←

Real-virtual corrections to $q\bar{q} \to \gamma\gamma$

$pp \to \gamma\gamma$ to 2nd order in QCD: $q\bar{q}$ channel components



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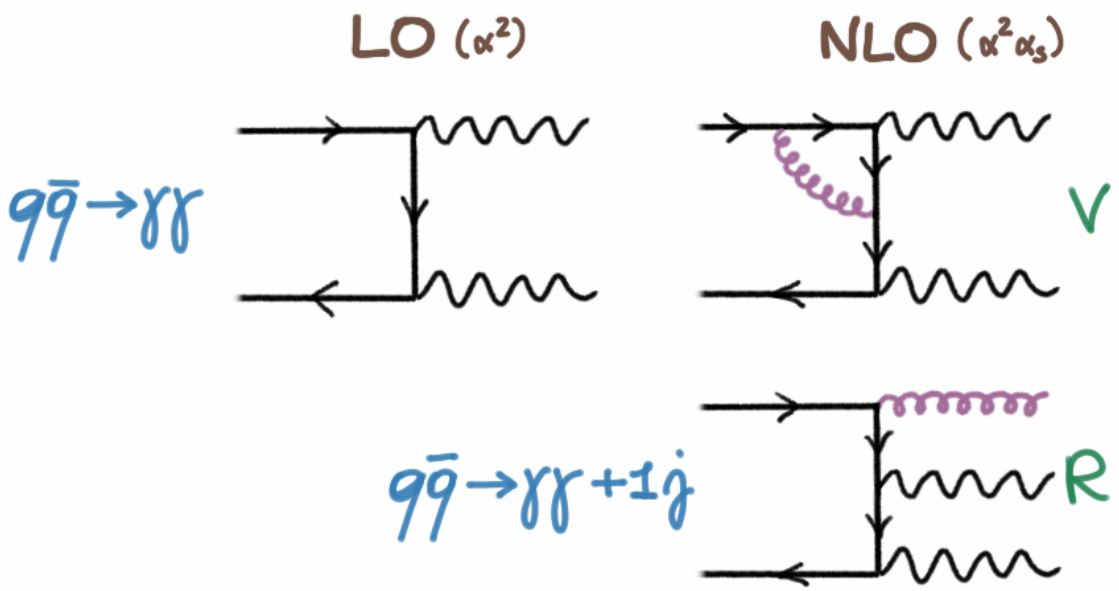

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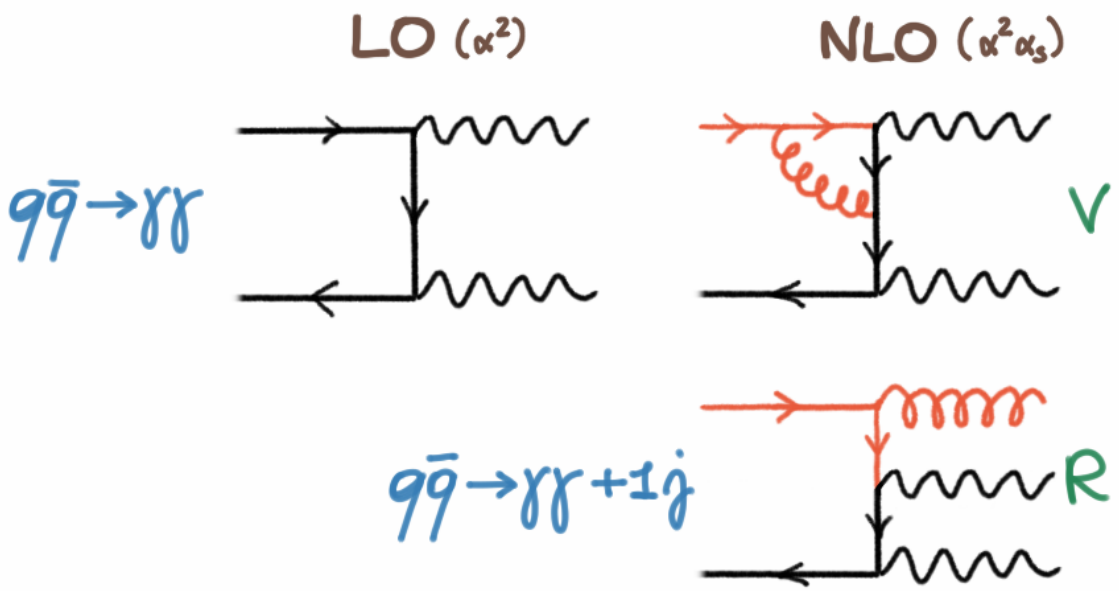

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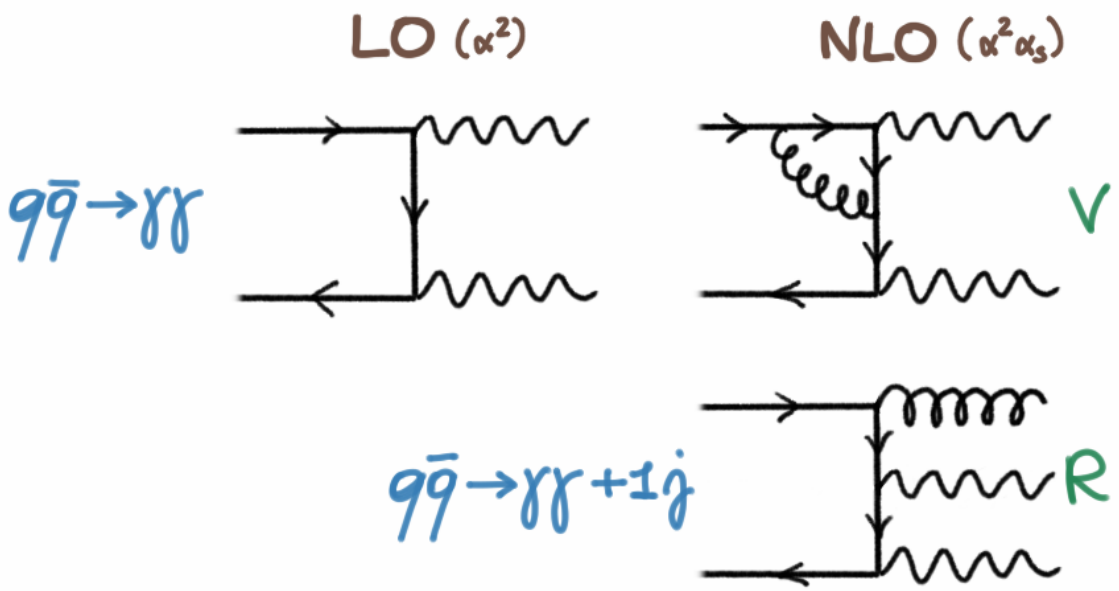

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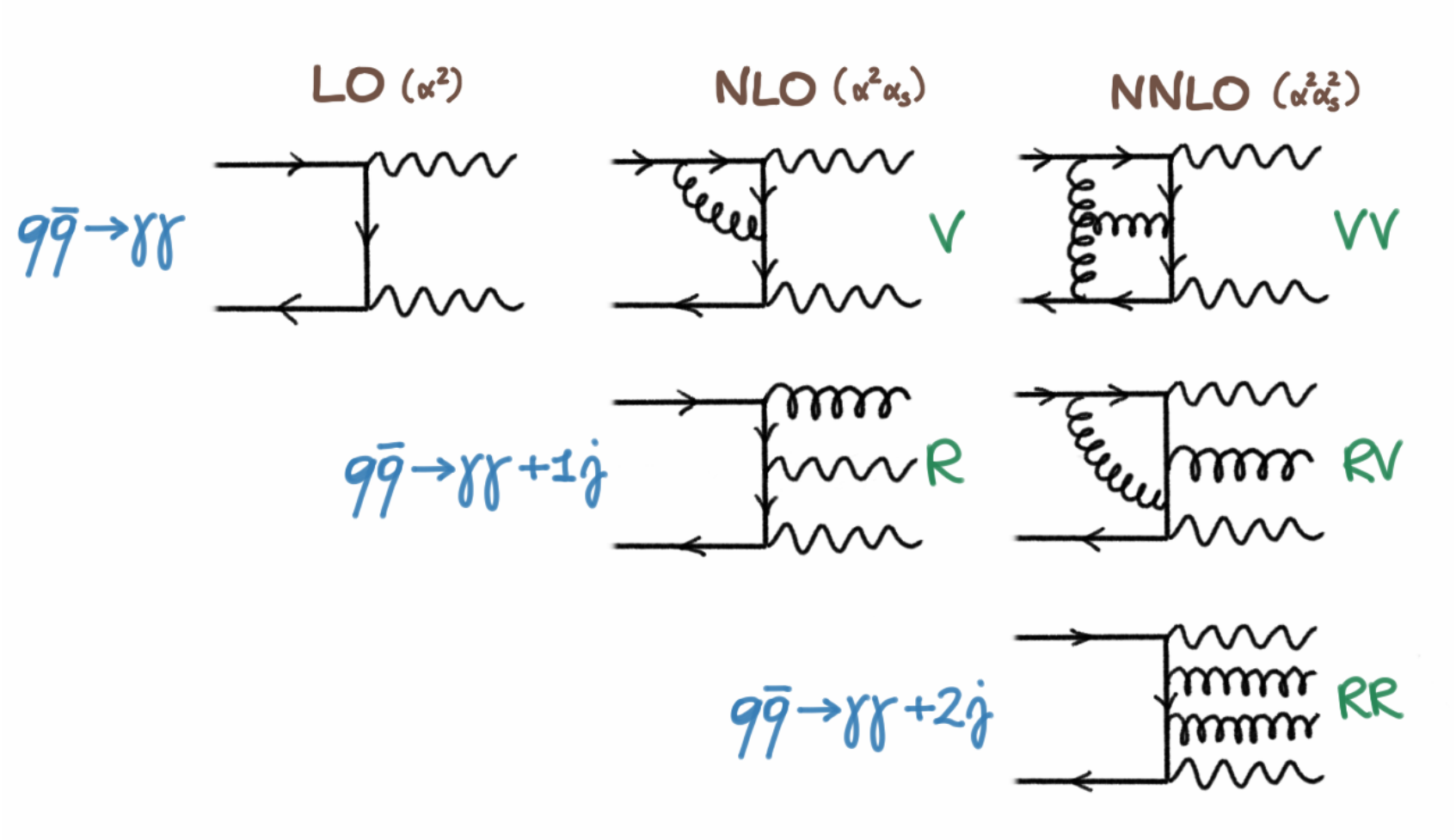

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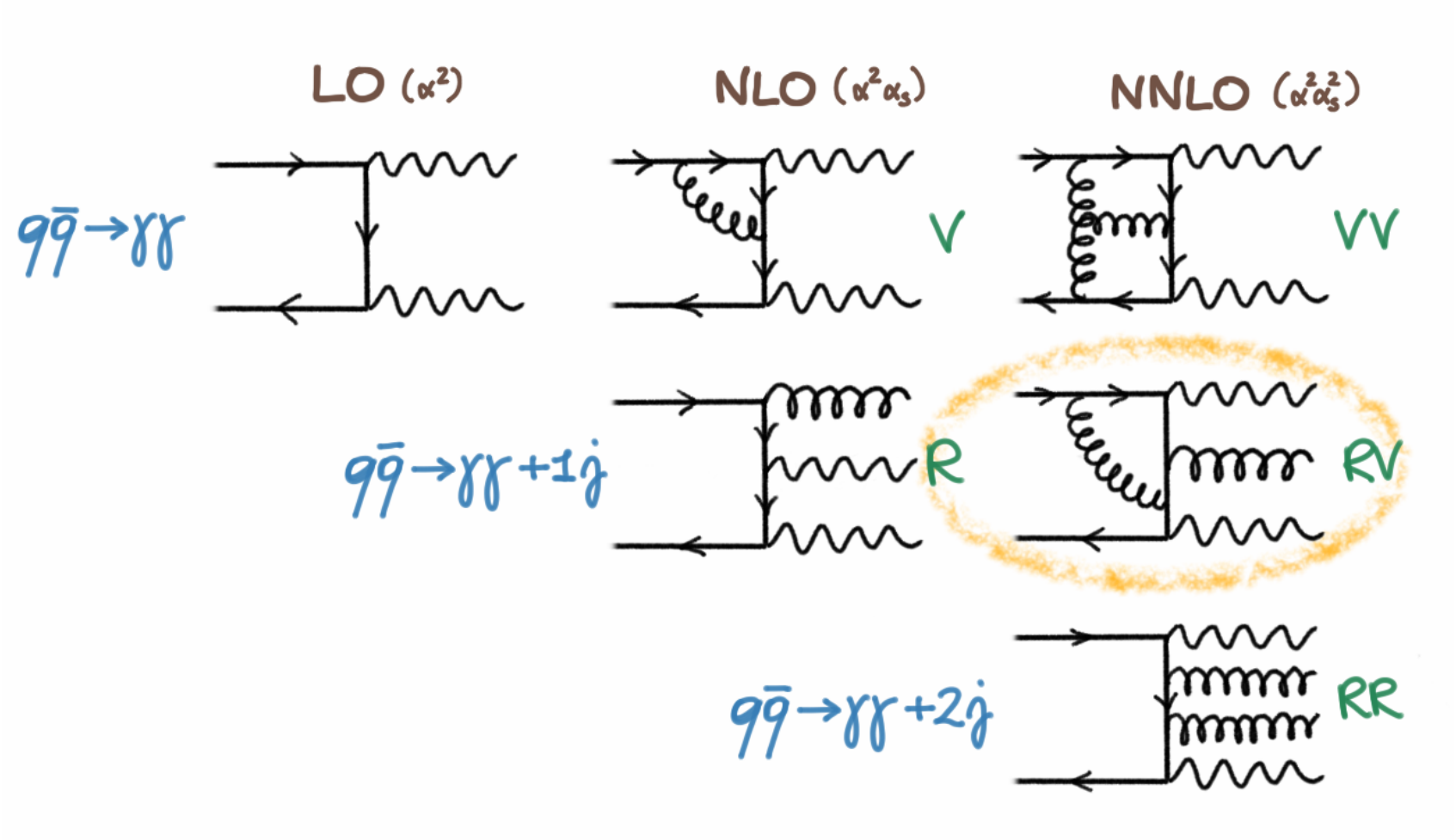
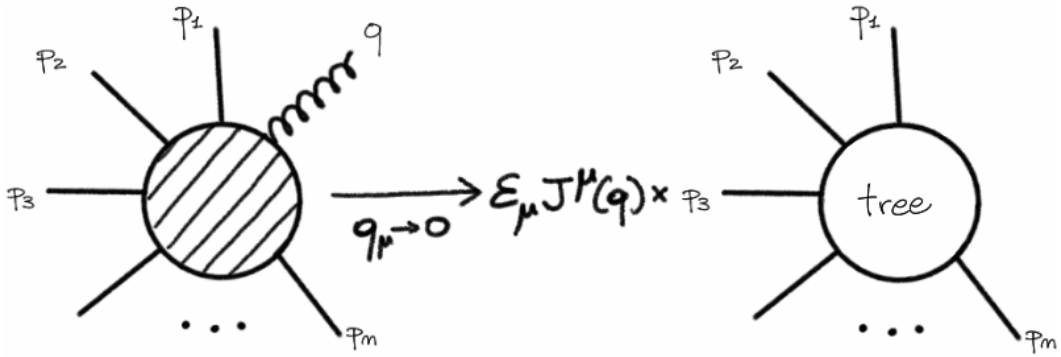
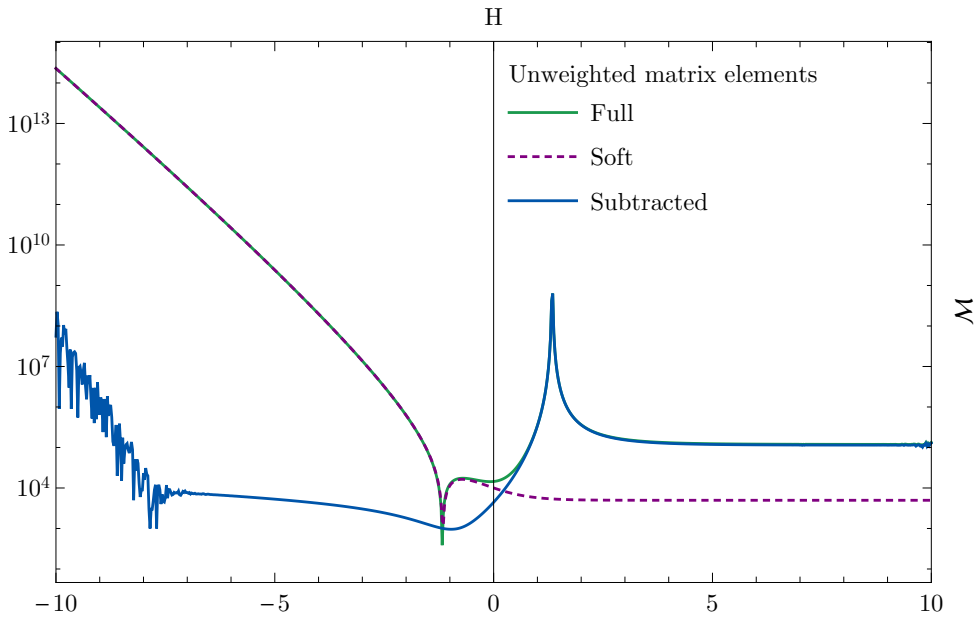


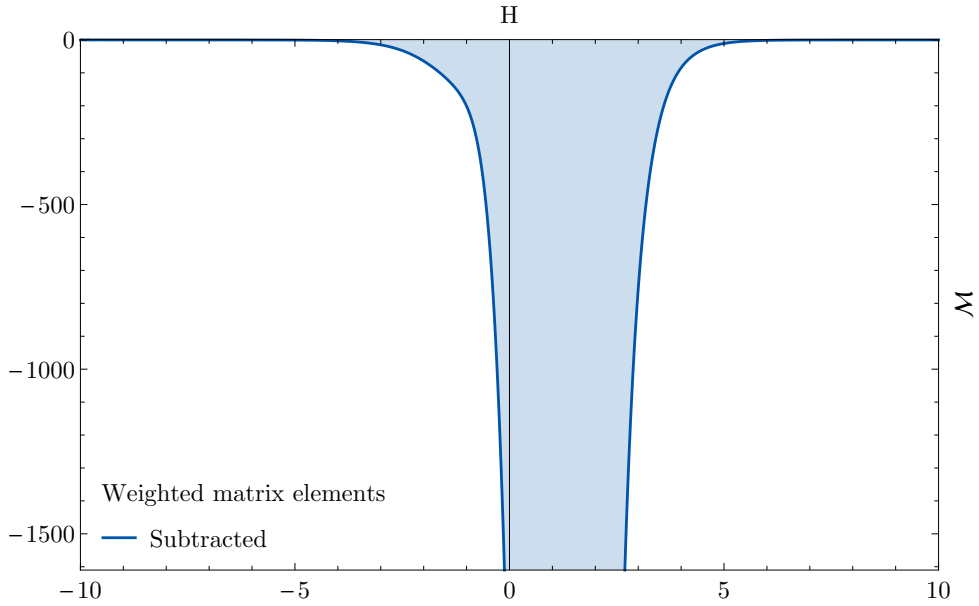
RV subtraction based on QCD factorization

Soft factorization



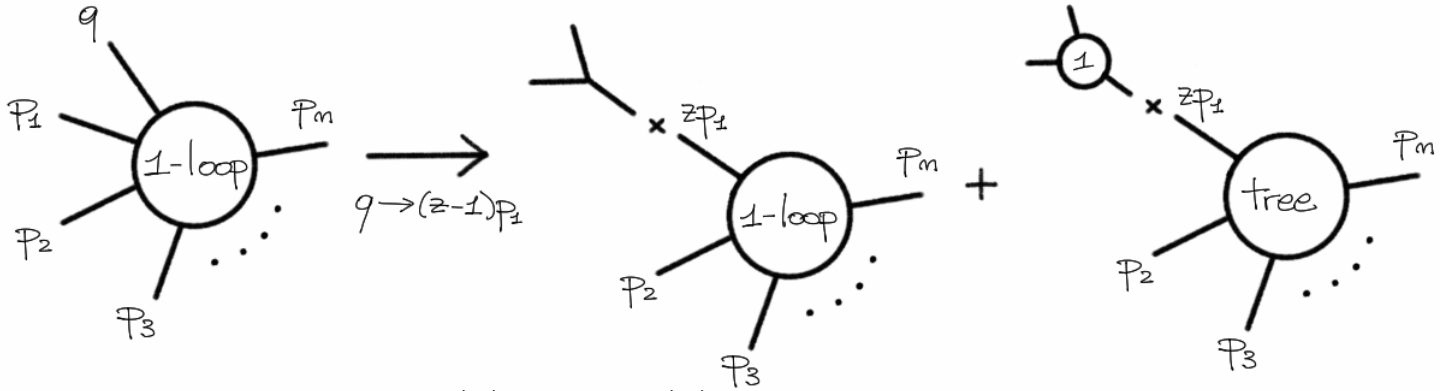
$$\mathcal{M}(p_1, \ldots, p_n; q) \sim \varepsilon^\rho(q) J^a_\rho(q) \mathcal{M}(p_1, \ldots, p_n) \quad \text{as} \quad q_\mu \to 0$$



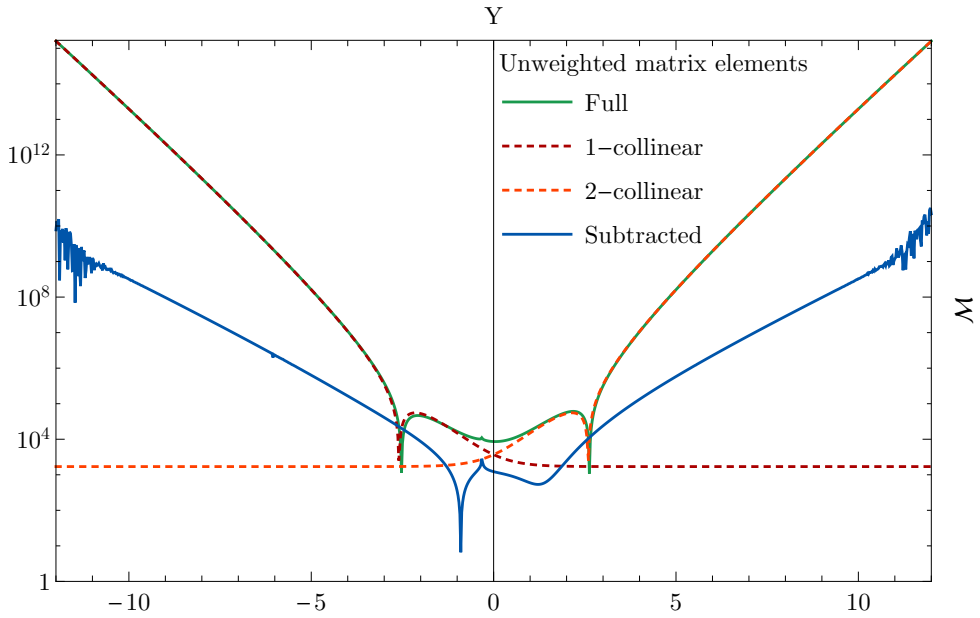


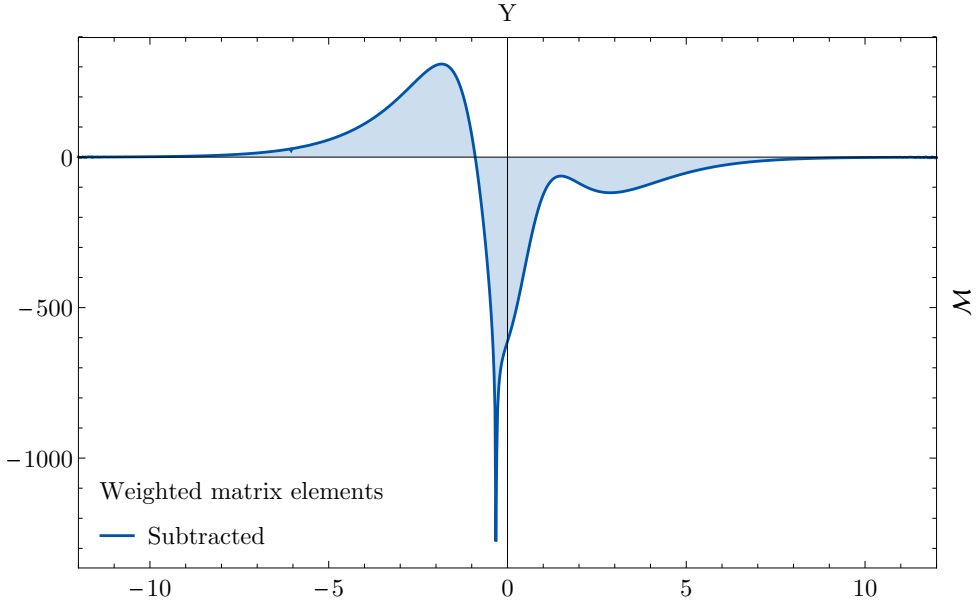
RV subtraction based on QCD factorization

Collinear factorization



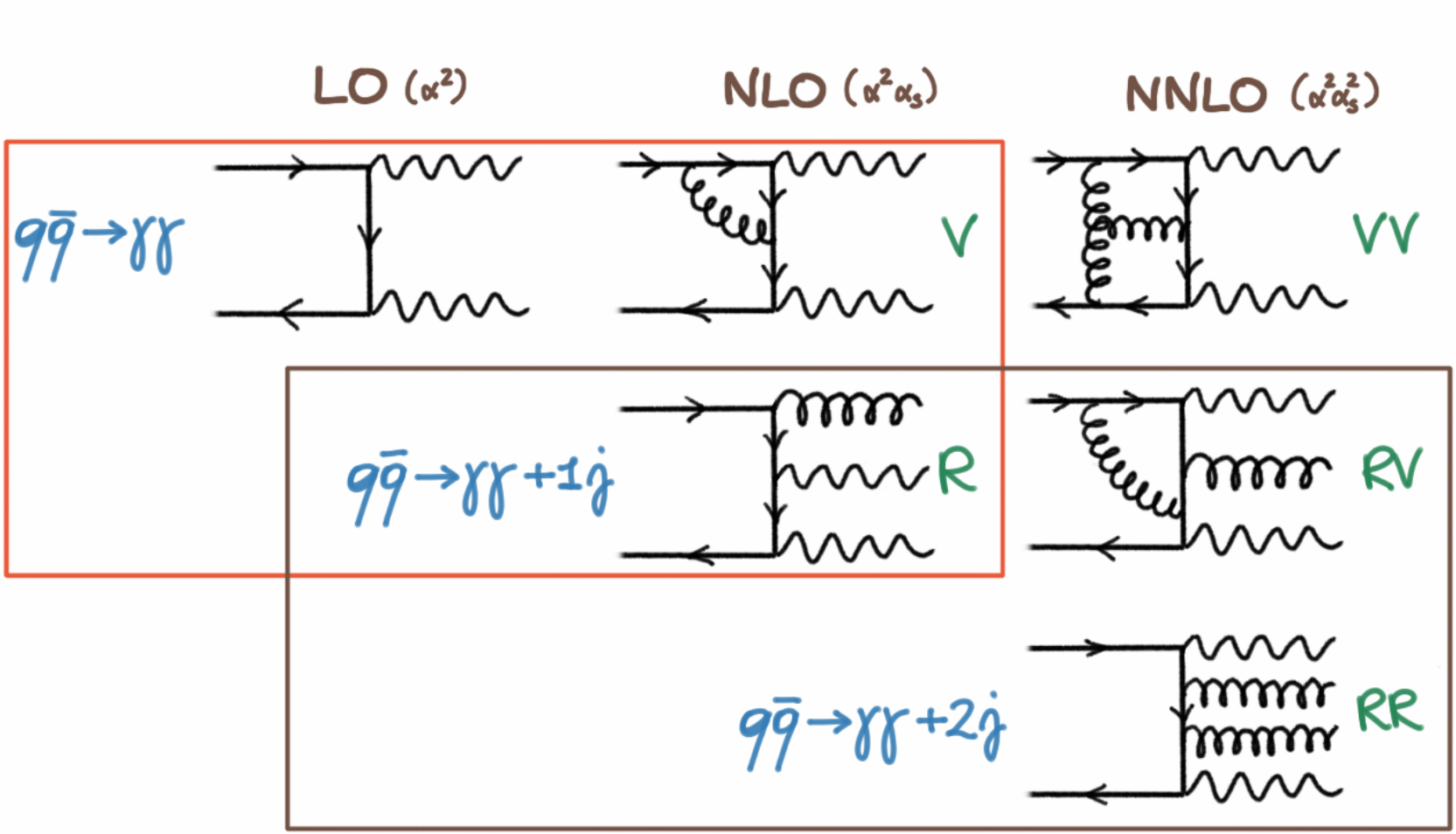
$$\mathcal{M}^{(1)}(p_1,\ldots,p_n;q) \sim \mathrm{Split}^{(0)} \times \mathcal{M}^{(1)}(zp_1,\ldots,p_n) + \mathrm{Split}^{(1)} \times \mathcal{M}^{(0)}(zp_1,\ldots,p_n) \quad \text{as} \quad q \to (z-1)p_1.$$





Is NNLO real-virtual HARDER than NLO virtual?

$pp \to \gamma\gamma$ to 2nd order in QCD: $q\bar{q}$ channel components



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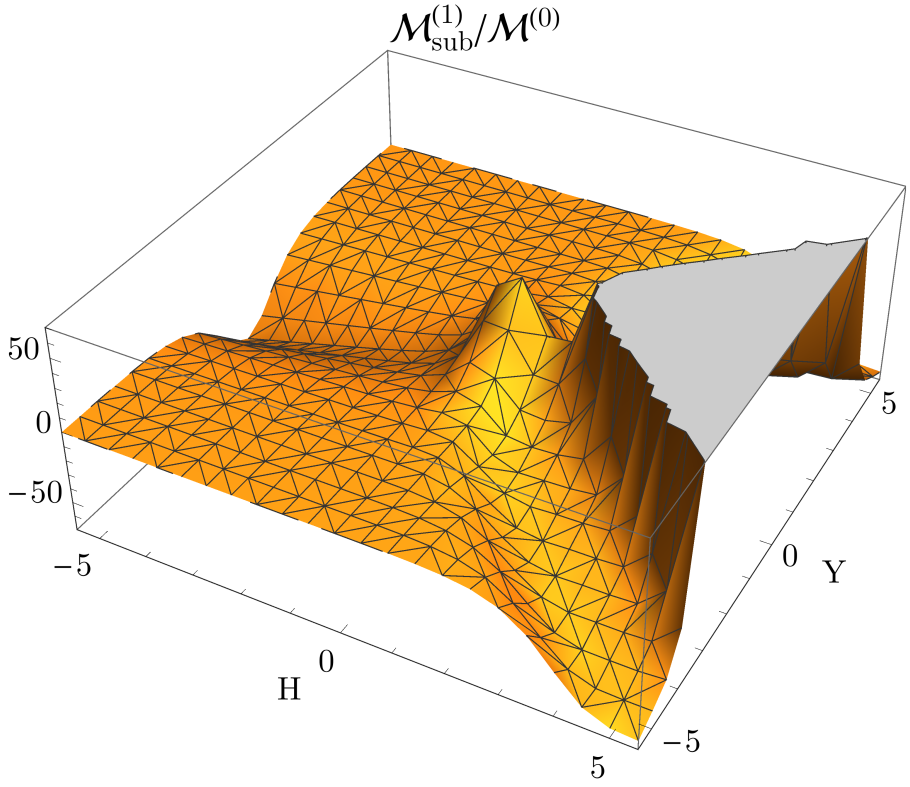
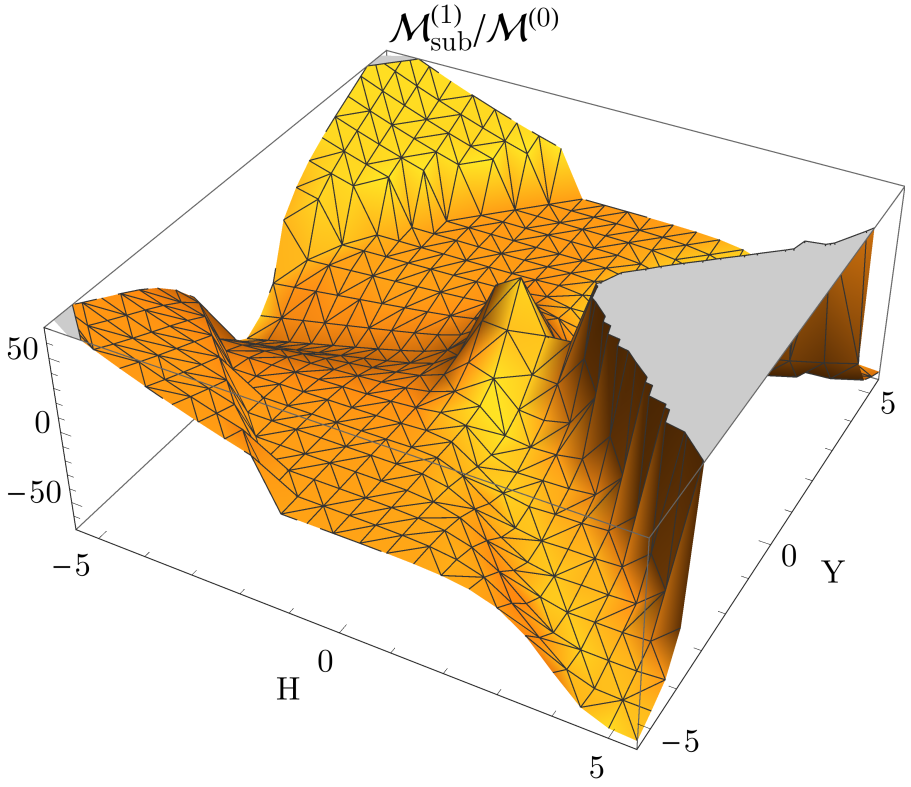


This work



MadGraph 5

Numerical integration

Ready to tackle cross-section integration in `ehixs`!

Need additional cuts for photons in the final state

(a) smooth cone isolation ($\delta_0 = \epsilon_\gamma = n = 1$);

(b) minimum $p_T^\gamma > 20$ GeV;

(c) minimum invariant mass for $\gamma\gamma$, $m_{\gamma\gamma} > 20$ GeV.

Hadronic σ requires 7-dim. integration $\Rightarrow$ adaptive Monte Carlo

```
[call_vegas] dimension of integration =  7
[vegas]   1     10000 pts : 2.90831e+05 +- 3.943e+04 (1.36e-01) chisq 0.0e+00 /   0
[vegas]   2     20500 pts : 2.75102e+05 +- 2.332e+04 (8.48e-02) chisq 2.4e-01 /   1
[vegas]   3     31500 pts : 2.70420e+05 +- 1.092e+04 (4.04e-02) chisq 3.0e-01 /   2
[vegas]   4     43000 pts : 2.69583e+05 +- 7.392e+03 (2.74e-02) chisq 3.1e-01 /   3
[vegas]   5     55000 pts : 2.74407e+05 +- 5.263e+03 (1.92e-02) chisq 1.2e+00 /   4
[vegas]   6     67500 pts : 2.75032e+05 +- 4.370e+03 (1.59e-02) chisq 1.2e+00 /   5
[vegas]   7     80500 pts : 2.76359e+05 +- 3.709e+03 (1.34e-02) chisq 1.5e+00 /   6
[vegas]   8     94000 pts : 2.77293e+05 +- 3.153e+03 (1.14e-02) chisq 1.8e+00 /   7
[vegas]   9    108000 pts : 2.79138e+05 +- 2.704e+03 (9.69e-03) chisq 3.1e+00 /   8
[.....]
[vegas]   1    100000 pts : 2.80845e+05 +- 2.060e+03 (7.34e-03) chisq 0.0e+00 /   0
[vegas]   2    200000 pts : 2.80793e+05 +- 1.396e+03 (4.97e-03) chisq 1.2e-03 /   1
[.....]
[ehixs] cross section
[ehixs] sigma = 280792.926486 +- 1395.97019995 prob = 0.027462252626 total # of points = 307989
```

Backup slides

Real-virtual corrections to $q\bar{q} \to \gamma\gamma$

Matrix element computation of $qq \to \gamma\gamma g$

Old-school Feynman diagrams

$+$ Integration By Parts reduction of loop integrals

Form of the result: $$\sum_{\text{masters } i} c_i(x_j) \times M_i(x_j)$$

- c_i are rational functions of kinematical variables x_j,
- M_i are Feynman integrals.

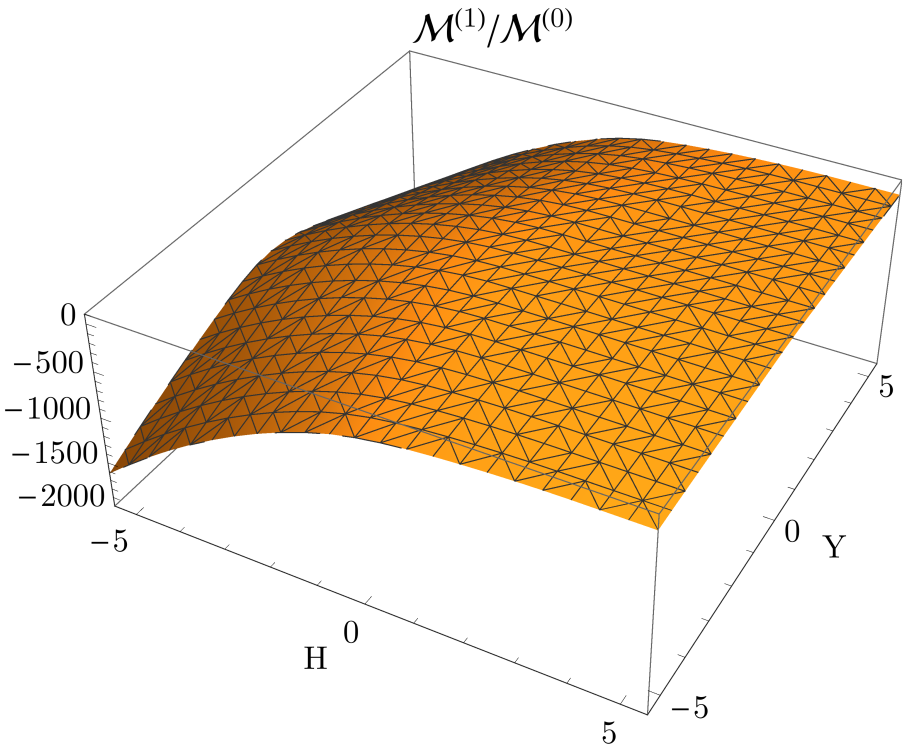
Partonic process has $2 \to 3$ kinematics:

for fixed $\hat{s}$ there are 5 (4) independent variables.

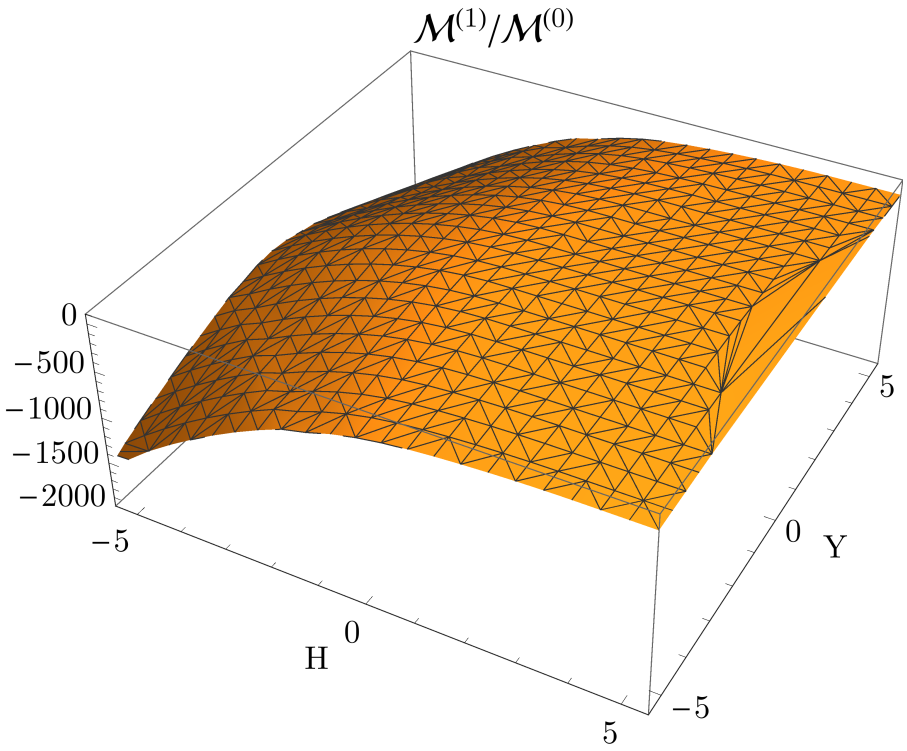
$$\lambda = \frac{p_1 \cdot q}{(p_1 + p_2) \cdot q}, \qquad \bar{z} = \frac{(p_1 + p_2) \cdot q}{p_1 \cdot p_2},$$

$$\mathrm{Y} = \operatorname{arctanh}(2\lambda - 1), \qquad \mathrm{H} = \operatorname{arctanh}(2\bar{z} - 1).$$

Real-virtual corrections to $q\bar{q} \to \gamma\gamma$



This work



MadGraph 5

[Warning: z axis includes an arbitrary scale factor. Only finite term $\mathcal{O}(\epsilon^0)$ plotted.]

NNLO differential calculations and subtraction

UV divergences of loop integrals are dealt with by renormalization, not a problem here...

IR divergences (using dimensional regularization in $d = 4 - 2\epsilon$)

- *explicit*: $1/\epsilon$ poles coming from loop integration;
- *implicit*: divergent matrix elements produce $1/\epsilon$ poles when they are integrated over phase space.

$$\hat{\sigma}^{\text{NNLO}} = \int_N d\hat{\sigma}^{VV} + \int_{N+1} d\hat{\sigma}^{RV} + \int_{N+2} d\hat{\sigma}^{RR}$$

NNLO differential calculations and subtraction

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$$\hat{\sigma}^{\text{NNLO}} = \int_N \left[d\hat{\sigma}^{VV} + \int_{+1} d\hat{\sigma}^{RV}_{sub} \right] + \int_{N+1} \left[d\hat{\sigma}^{RV} - d\hat{\sigma}^{RV}_{sub} \right] + \int_{N+2} d\hat{\sigma}^{RR}$$

NNLO differential calculations and subtraction

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IR divergences (using dimensional regularization in $d = 4 - 2\epsilon$)

- *explicit*: $1/\epsilon$ poles coming from loop integration;
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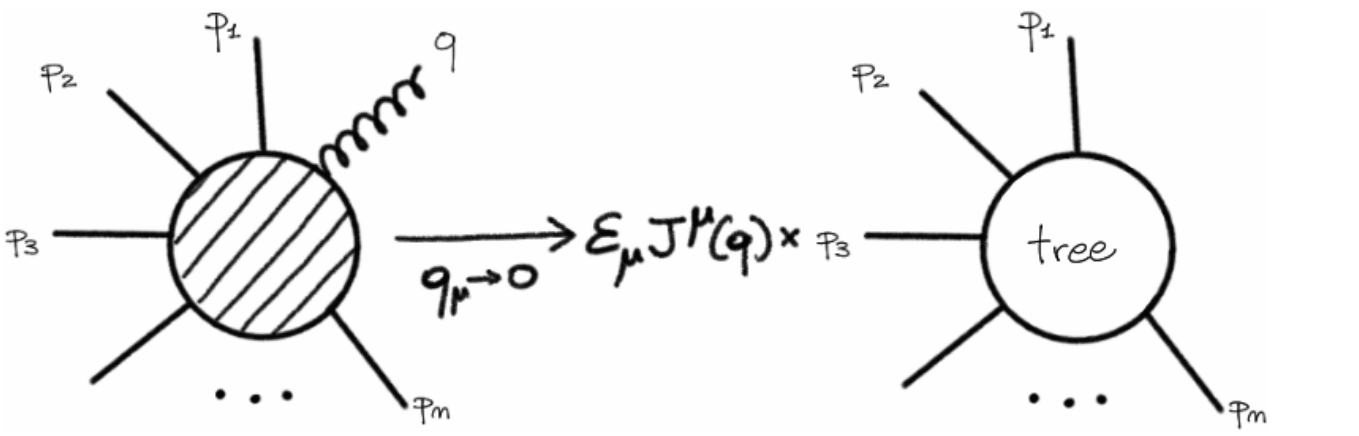
$$\hat{\sigma}^{\text{NNLO}} = \int_N \left[d\hat{\sigma}^{VV} + \int_{+1} d\hat{\sigma}^{RV}_{sub} + \int_{+2} d\hat{\sigma}^{RR}_{sub,2} \right]$$
$$+ \int_{N+1} \left[d\hat{\sigma}^{RV} - d\hat{\sigma}^{RV}_{sub} + \int_{+1} d\hat{\sigma}^{RR}_{sub,1} \right]$$
$$+ \int_{N+2} \left[d\hat{\sigma}^{RR} - d\hat{\sigma}^{RR}_{sub,1} - d\hat{\sigma}^{RR}_{sub,2} \right]$$

Soft current



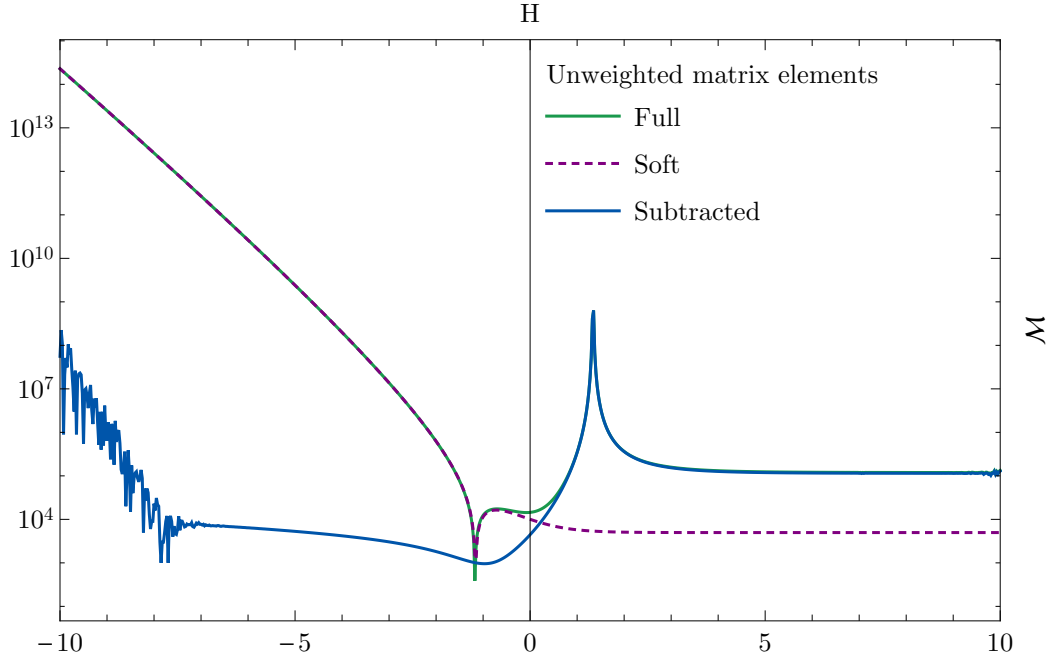
$$\mathcal{M}(p_1,\ldots,p_n;q) \sim \varepsilon^\rho(q) J^a_\rho(q) \mathcal{M}(p_1,\ldots,p_n) \quad \text{as} \quad q_\mu \to 0$$

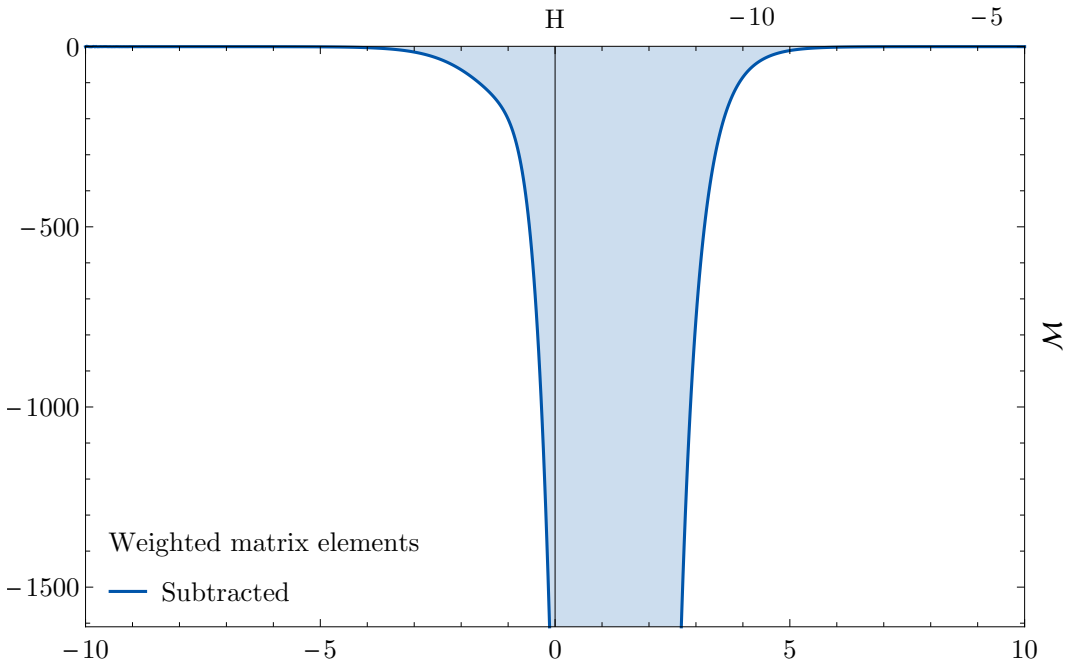
$$J^a_\rho(q) = g_s \mu^\epsilon \sum_{\ell=0}^{\infty} (g_s \mu^\epsilon)^{2\ell} J^{a(\ell)}_\rho(q),$$

$$J^{a(0)}_\rho(q) = \sum_i T^a_i \frac{p_{i\rho}}{p_i \cdot q},$$

$$J^{a(1)}_\rho(q) = -\frac{S_\epsilon}{16\pi^2} \frac{1}{\epsilon^2} \Gamma(1-\epsilon)\Gamma(1+\epsilon) i f^{abc} \sum_{i \neq j} T^b_i T^c_j \left(\frac{p_{i\rho}}{p_i \cdot q} - \frac{p_{j\rho}}{p_j \cdot q} \right) \left[\frac{(-s_{ij})}{s_{iq} s_{qj}} \right]^\epsilon .$$

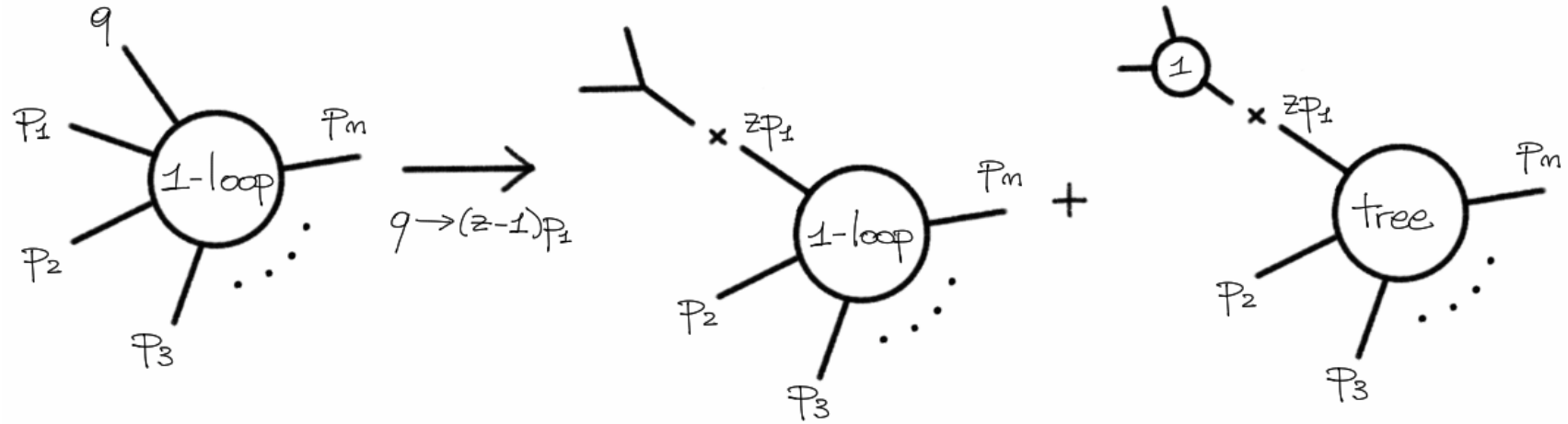
Soft subtraction





- the region $1.25 < \mathrm{H} < 1.40$ is cut by $p_T^\gamma > 20$ GeV;
- $H \to \infty$ always cut, but numerically large values can be reached.

Splitting amplitudes



$$\mathcal{M}^{(1)}(p_1,\ldots,p_n;q) \sim \mathrm{Split}^{(0)} \times \mathcal{M}^{(1)}(zp_1,\ldots,p_n) + \mathrm{Split}^{(1)} \times \mathcal{M}^{(0)}(zp_1,\ldots,p_n) \quad \text{as} \quad q \to (z-1)p_1.$$

$$\mathrm{Split}^{(\ell)}_{q\to qg} = \frac{1}{s_{1q}} \bar{u}(\vec{q}+\vec{p_1})\, \mathrm{Sp}^{\mu(\ell)}_{q\to qg}\, u(\vec{p_1}) \varepsilon^*_\mu(\vec{q})$$

$$\mathrm{Sp}^{\mu(0)}_{q\to qg} = \gamma^\mu,$$

$$\mathrm{Sp}^{\mu(1)}_{q\to qg} = r_3(z)\gamma^\mu + r_4 \frac{(p_1+q)^\mu \not{q}}{s_{1q}}.$$

Splitting amplitude details

The coefficients r_3 and r_4

$$r_4 = \left(N_c + \frac{1}{N_c}\right)\frac{1}{2}\frac{\epsilon^2}{(1-2\epsilon)(1-\delta\epsilon)}f_2;$$

$$r_3(z) = \frac{1}{2}\left[N_c(1-z)f_1(1-z) - \frac{1}{N_c}(zf_1(z) - 2f_2)\right] - r_4.$$

The special functions f_1 and f_2

$$\frac{1}{z}f_1\left(\frac{1}{z}\right) - 2f_2 = -c_\Gamma\frac{2}{\epsilon^2}{}_2F_1(1,-\epsilon;1-\epsilon;1-z);$$

$$\left(1-\frac{1}{z}\right)f_1\left(1-\frac{1}{z}\right) = c_\Gamma\frac{2}{\epsilon^2}\left[{}_2F_1(1,\epsilon;1+\epsilon;1-z) - \pi\epsilon\cot(\pi\epsilon)(1-z)^{-\epsilon}\right];$$

$$f_2 = -c_\Gamma\frac{1}{\epsilon^2}.$$

From splitting amplitudes to splitting functions

Collinear factorization for matrix elements squared

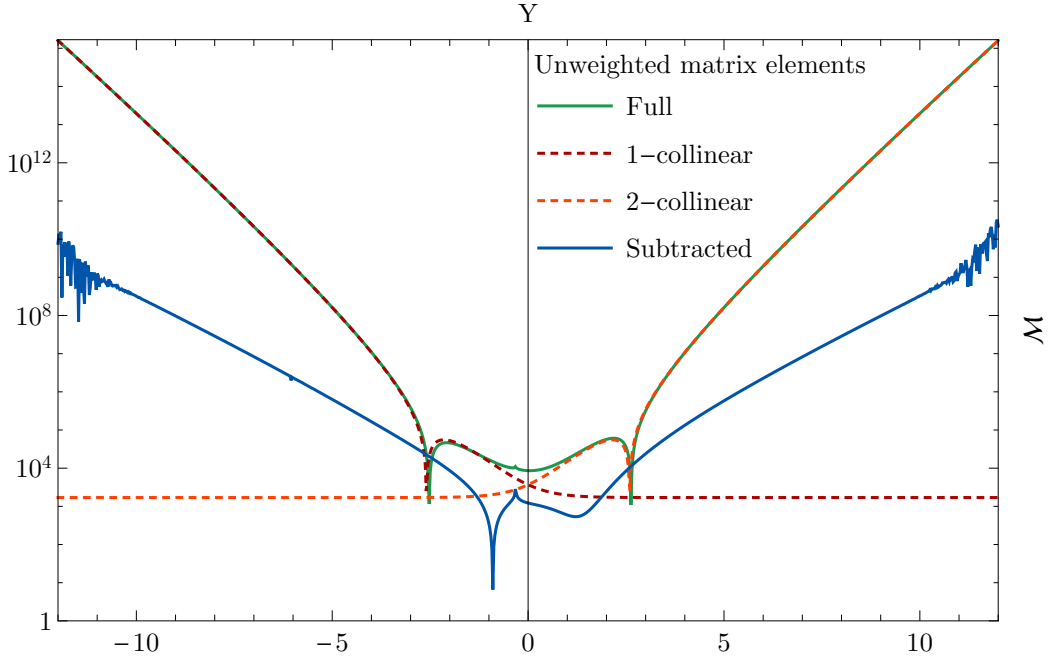
$$2\mathrm{Re}\left[\mathcal{M}^{(1)}_{q\bar{q}\to Fg}\mathcal{M}^{(0)*}_{q\bar{q}\to Fg}\right] \sim \frac{P^{(0)}_{qq}(z)}{z\, s_{qg}} 2\mathrm{Re}\left[\mathcal{M}^{(1)}_{q\bar{q}\to F}\mathcal{M}^{(0)*}_{q\bar{q}\to F}\right] + \frac{2\mathrm{Re}\left[P^{(1)}_{qq}(z)\right]}{z\, s_{qg}}\left(\frac{\mu^2}{-s_{qg}}\right)^{\epsilon}\left|\mathcal{M}^{(0)}_{q\bar{q}\to F}\right|^2.$$

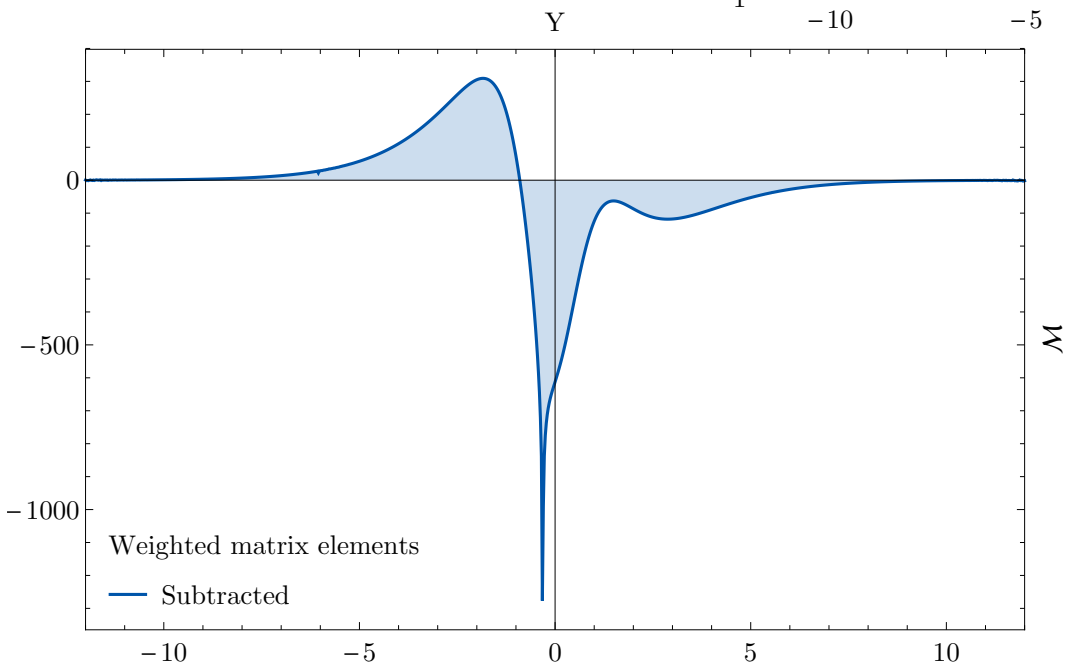
The tree-level and one-loop qq splitting functions

$$P^{(0)}_{qq}(z) = 2C_F g_s^2\left[\frac{1+z^2}{1-z} - \epsilon(1-z)\right],$$

$$P^{(1)}_{qq}(z) = r_3(z)P^{(0)}_{qq}(z) + r_4\left[2C_F\frac{1+z}{1-z}\right].$$

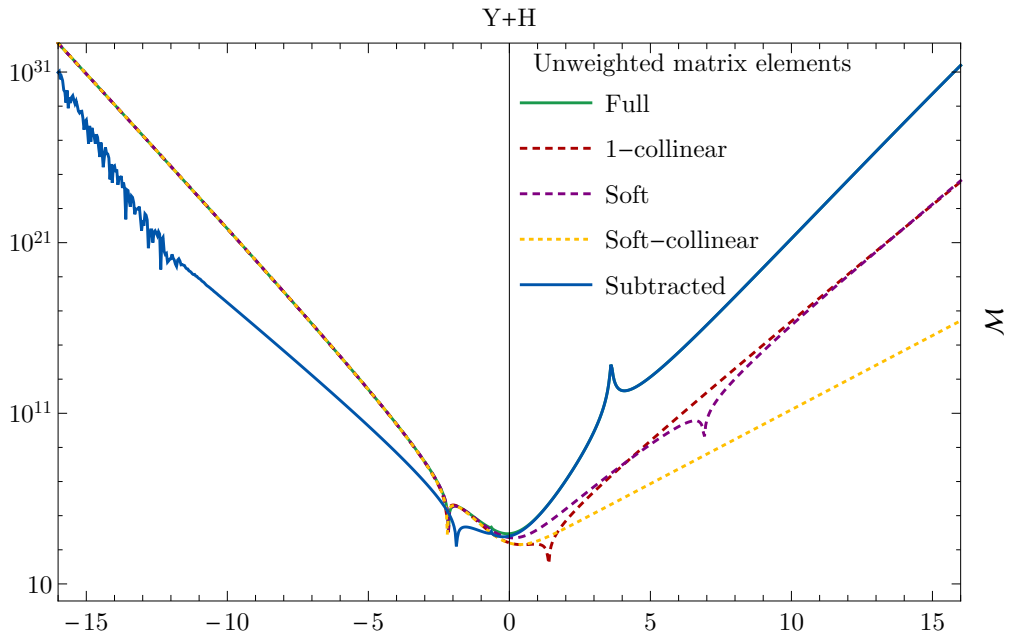
Collinear subtraction

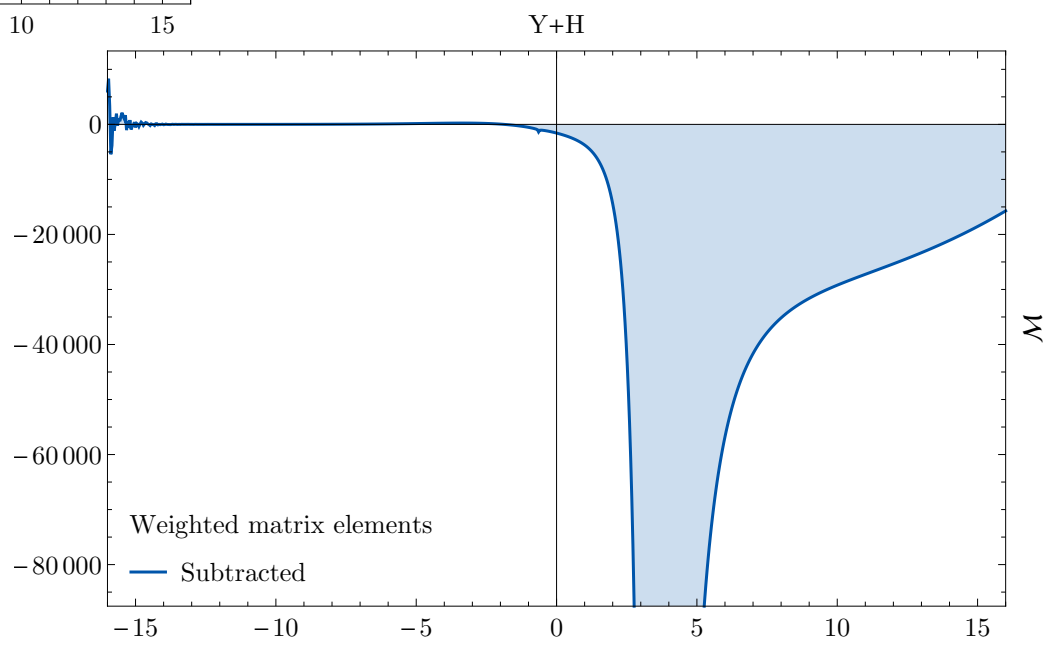




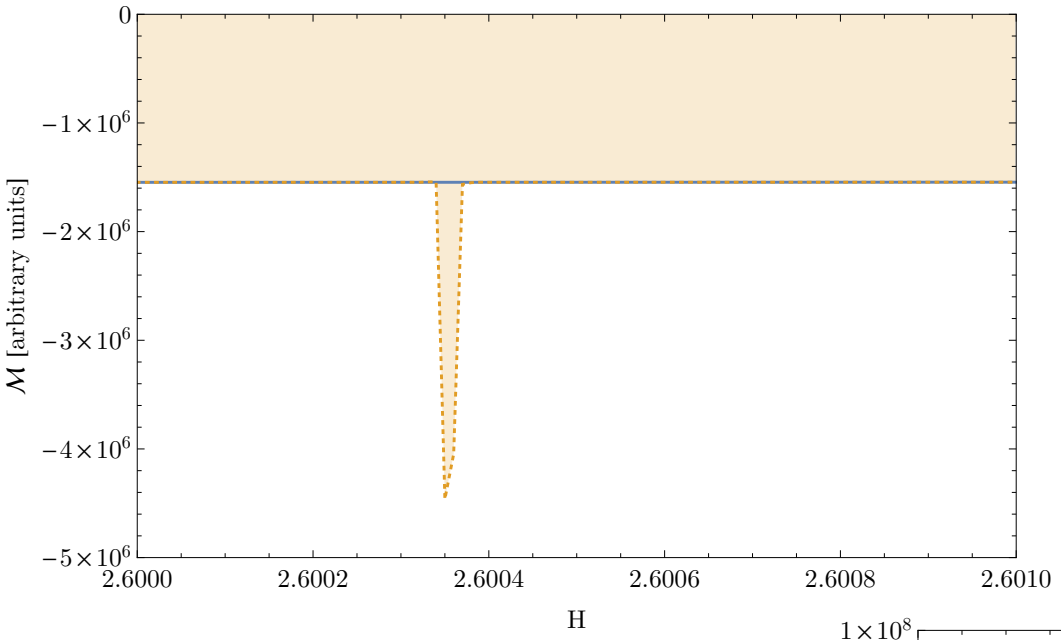
- the region $-1.15 < \mathrm{Y} < 0.45$ is cut by photon isolation;
- large cancellations require good control of singular regions.

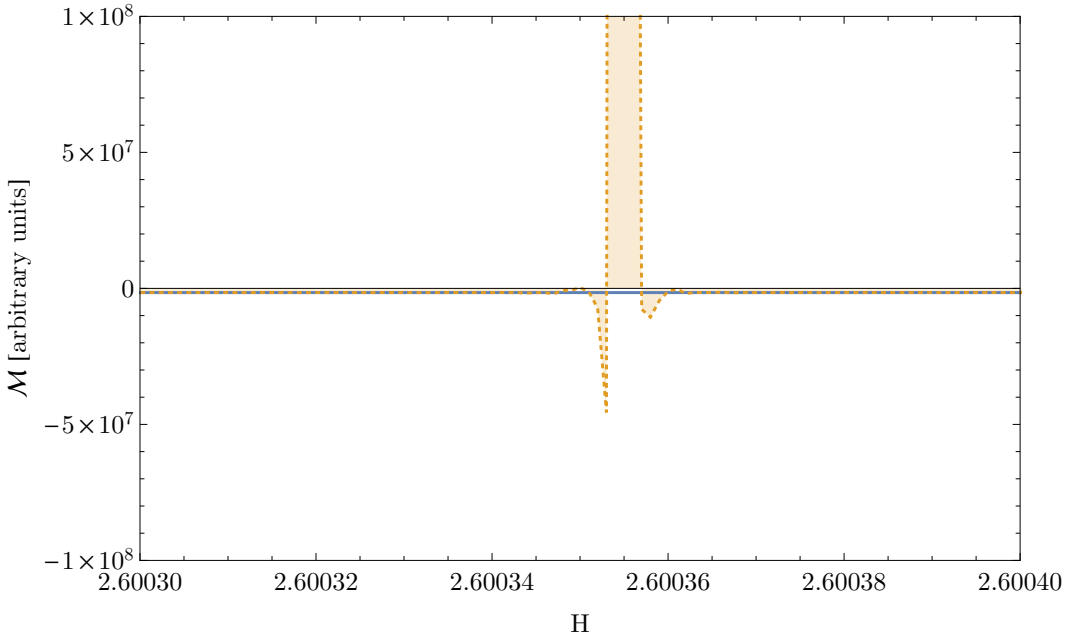
Soft-collinear subtraction term





Gram singularities inside phase space





Frixione isolation

Smooth isolation cones avoid the contributions coming from fragmentation functions of partons into photons.

The condition reads:

$$\sum_i E_i \theta(\delta - R_{i\gamma}) \leq \mathcal{X}(\delta) \qquad \text{for all } \delta < \delta_0,$$

where

$$R_{i\gamma} = \sqrt{(\eta_i - \eta_\gamma)^2 + (\phi_i - \phi_\gamma)^2},$$
$$\mathcal{X}(\delta) = E_\gamma \epsilon_\gamma \left(\frac{1 - \cos\delta}{1 - \cos\delta_0} \right)^n.$$

Can be reformulated as a measurement function factor

$$\mathcal{J} = \theta\left[\mathcal{X}\big(\min\{\delta_0, R_{i\gamma}\} \big) - \sum_{j=1}^{i} E_j \theta(\delta_0 - R_{j\gamma}) \right],$$

where partons are ordered in $R_{i\gamma}$.