q_T-subtraction for $t\bar{t}$ production at hadron colliders

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Outline

- Motivation
- q_T-resummation
- q_T-subtraction
- Results
- Summary

Top quark

Mass of the top quark obtained through combining the measurements at the Tevatron and LHC colliders is $m_t = 173.34 \pm 0.27\,(stat)\ \pm 0.71\,(syst)\,\mathrm{GeV}$ [ATLAS and CDF and CMS and D0 Collaborations (2014)].

- Strong coupling to the Higgs boson
- Crucial to the hierarchy problem
- Decays before hadronization, allowing for a better experimental measurements.

Top quark pair production

- The top quark pair production is the main source of the top quark events in the Standard Model (SM).
- Many New Physics models involve heavy top partners which then decay into a top quark pair.

The study of $t\bar{t}$ pair production at hadron colliders can

- shed light on the electroweak symmetry breaking mechanism.
- provide information on the backgrounds of many NP models.

QCD corrections

Theoretical efforts for obtaining precision predictions for $t\bar{t}$ production at hadron colliders started almost 3 decades ago

- NLO QCD corrections are calculated by [Nason, Dawson and Ellis (1988), Beenakker, Kuijf, van Neerven and Smith (1989), Beenakker, van Neerven, Meng, Schuler and Smith (1989)].
- Recently the calculation of the full NNLO QCD corrections was completed for the total cross section and for the $t\bar{t}$ asymmetry. [Barnreuther, Czakon, Mitov (2012), Czakon, Mitov (2012), Czakon, Mitov (2013), Czakon, Fiedler, Mitov (2013), Czakon, Fiedler, Mitov (2014)].
- Precision result for the invariant mass distribution is worked out in [Ahrens, Ferroglia, Neubert, Pecjak, Yang].
- Other computations of differential distributions are underway [Abelof, Gehrmann-De Ridder, Maierhofer (2014), Abelof, Gerhrmann-de Ridder (2014), Abelof, Gehrmann-De Ridder and Majer (2015)].

q_T distribution

- When $q_T^2 \sim M^2$, $\alpha_S(M^2)$ is small, and the standard fixed order expansion is theoretically justified.
- When $q_T^2 \ll M^2$ large logarithms of the form $\alpha_S^n \log(M^2/q_T^2)$ appear, due to soft and collinear gluon emissions. Effective expansion variable is the $\alpha_S^n \log(M^2/q_T^2)$, which can be ~ 1 even for small α_S. These large logarithms need to be resummed to all orders in α_S, in order to get reliable predictions over the whole range of the transverse momenta.

The resummation of large logs results in exponentiating these large logarithmic terms

$$\sigma^{(res)} \sim \sigma^{(0)} C(\alpha_S) exp \left\{ L g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \ldots \right\} .$$

hard-virtual LL NLL NNLL

Resummation for the $t\bar{t}$ production

- The first attempt to develop a q_T-resummation formalism at next-to-leading logarithmic (NLL) accuracy for $t\bar{t}$ production was done in [Berger, Meng (1994), Mrenna, Yuan (1997)]. However, they did not consider color mixing between singlet and oktet final states and missed the initial-final gluon exchange.
- Recently the resummation for the $t\bar{t}$ q_T spectrum, based on soft collinear effective theory (SCET), was performed at NNLL+NLO. [Zhu, Li, Li, Shao, Yang (2013)]. This work is limited to the study of the q_T cross section after integration over the azimuthal angles of the produced heavy quarks.
- Last year the q_T-resummation in QCD was performed at the fully-differential level with respect to the kinematics of the produced heavy quarks. [Catani, Grazzini, Torre (2014)].

The all-order resummation formula

$$\frac{d\sigma^{(\mathrm{res})}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\Omega} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c}}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\mathbf{q_T}}\,S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\left[(\mathbf{H}\boldsymbol{\Delta})C_1\,C_2\right]_{c\bar{c};a_1a_2}\times$$

$$f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\,.$$

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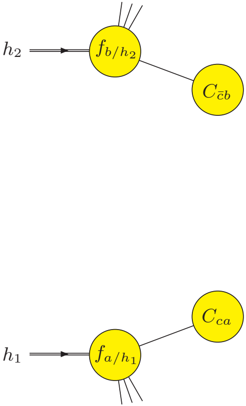

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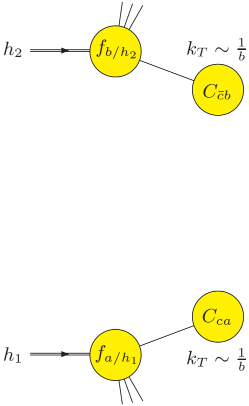

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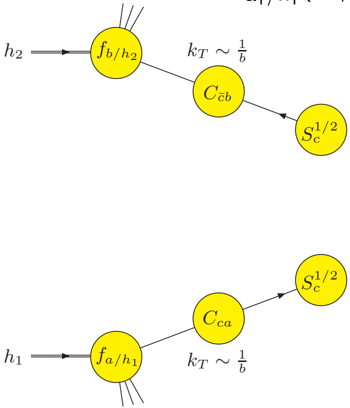

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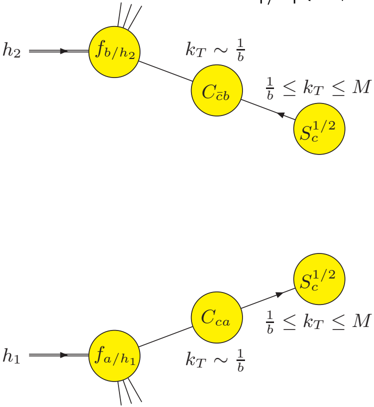

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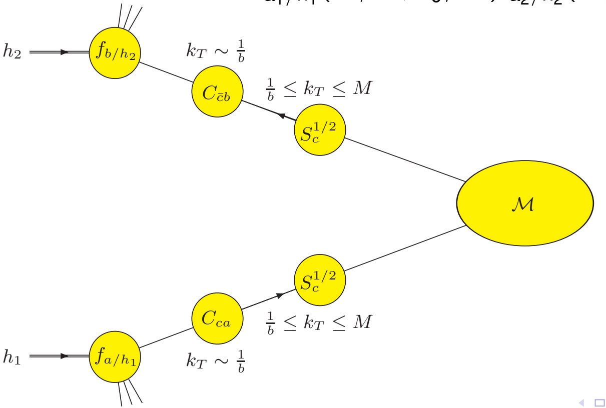

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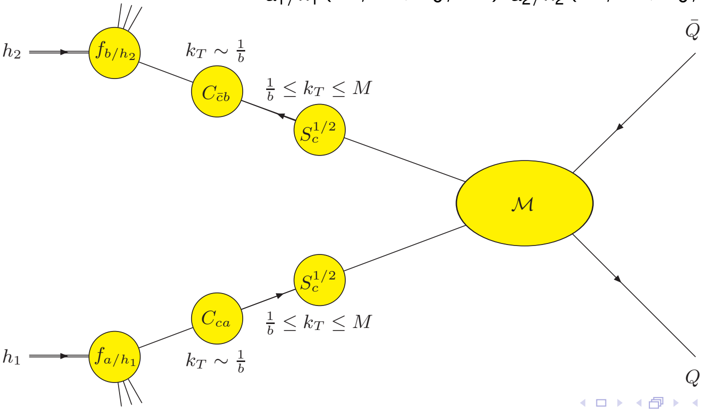

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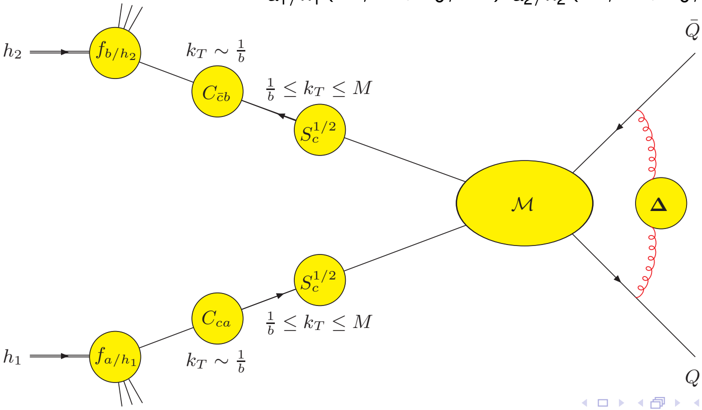

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$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\left[(\mathbf{H}{\color{red}\boldsymbol{\Delta}})C_1C_2\right]_{c\bar{c};a_1a_2}\times$$

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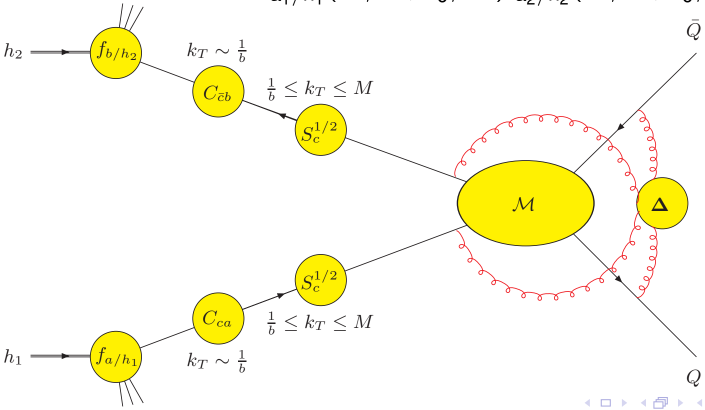

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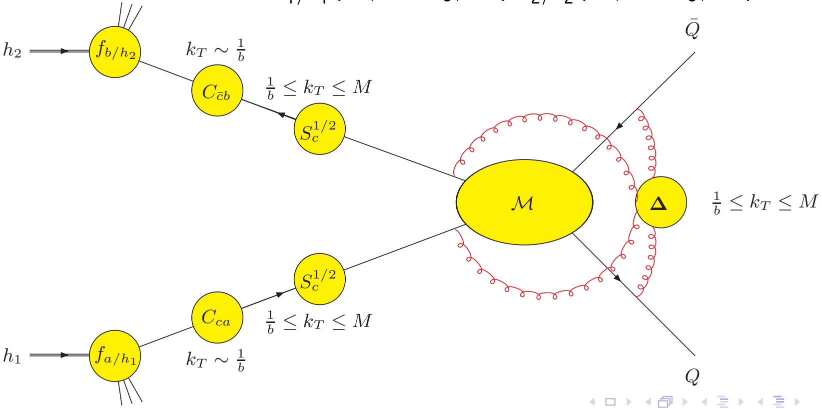


Azimuthal correlations

- Production of a colorless system
  - The gluonic collinear functions are the only source of azimuthal correlations

$$C_{ga}^{\mu\nu}(z;p_1,p_2,\mathbf{b}) = d^{\mu,\nu}(p_1,p_2)C_{ga}(z)+D^{\mu,\nu}(p_1,p_2;\mathbf{b})G_{ga}(z)$$

- Top-quark pair production

$$\boldsymbol{\Delta}(b,M;y_{34},\phi_3) = \mathbf{V}^{\dagger}(b,M;y_{34})\mathbf{D}\left(\alpha_S;\phi_{3b},y_{34}\right)\mathbf{V}(b,M;y_{34})\,.$$

  - Additional azimuthal correlations produced by the dynamics of soft-parton radiation, embodied in **D**.

$$\mathbf{D}\left(\alpha_S;\phi_{3b},y_{34}\right) = 1 + \frac{\alpha_S}{\pi}\mathbf{D}^{(1)}\left(\phi_{3b},y_{34}\right) + \mathcal{O}(\alpha_S^2)$$

$\langle\mathbf{D}\left(\alpha_S\left(b_0^2/b^2\right);\phi_{3b},y_{34}\right)\rangle_{\rm av.} = 1$ → vanishing contribution to $\langle\sigma\rangle_{\rm av.}$ at $\mathcal{O}(\alpha_S)$

But contributes at $\mathcal{O}(\alpha_S^2)$ due to the interference of the initial-final state azimuthal correlations

→ non-trivial integration over the azimuthal angle (computed analytically!)

q_T-subtraction

Knowledge of the low q_T limit is essential also for the fixed order calculation in the q_T-subtraction formalism.
q_T-subtraction formalism has been originally proposed for the production of colourless high-mass systems in hadron collisions. [Catani, Grazzini (2007)].
This subtraction formalism has been successfully applied to number of important processes of this class.

- $pp \to H$ [Catani, Grazzini (2007)].
- $pp \to V$. [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)].
- $pp \to \gamma\gamma$. [Catani, Cieri, Ferrera, de Florian, Grazzini (2011)].
- $pp \to WH$. [Ferrera, Grazzini, Tramontano (2011)].
- $pp \to Z\gamma$. [Grazzini, Kallweit, Rathlev, Torre (2013)].
- $pp \to ZZ$. [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)].
- $pp \to W^+W^-$. [Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi (2014)].
- $pp \to ZH$. [Ferrera, Grazzini, Tramontano (2014)].

q_T-subtraction for $t\bar{t}$

- The fully differential cross section at N(NLO):

$$d\sigma^{t\bar{t}}_{\mathrm{N(NLO)}} = \mathcal{H}^{t\bar{t}}_{\mathrm{N(NLO)}} \otimes d\sigma^{t\bar{t}}_{\mathrm{LO}} + \left[d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{N(LO)}} - d\sigma^{\mathrm{CT}}_{\mathrm{N(LO)}} \right] .$$

Regular as $q_T \to 0$

- $\mathcal{H}^{t\bar{t}}_{\mathrm{N(NLO)}}$ is the hard factor, which contains information on the virtual corrections to the LO process.
- $d\sigma^{t\bar{t}}_{\mathrm{LO}}$ is the Born cross section.
- $d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{N(LO)}}$ is the N(LO) cross section of $t\bar{t}$+jet(s) process.
- $d\sigma^{\mathrm{CT}}_{\mathrm{N(LO)}}$ is the counterterm, which can be derived by expanding the resummation formula.

Our implementation

Up to NLO our implementation is based on

- The scattering amplitudes and phase space generation of MCFM program.
- We use the corresponding routines of the Higgs boson production code HNNLO and the vector boson production code DYNNLO, suitably modified for $t\bar{t}$ production.

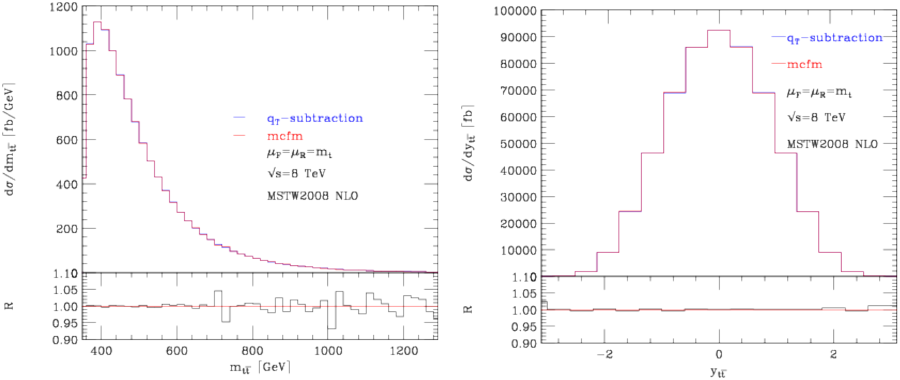
At NNLO accuracy the $t\bar{t}$ + jet cross section is evaluated by using the MUNICH code which provides:

- Fully automatic implementation of the NLO dipole subtraction formalism.
- Interface to the OPENLOOPS one-loop generator.

We are grateful to D. Rathlev and S. Kallweit for their help with the MUNICH program.

Results at NLO

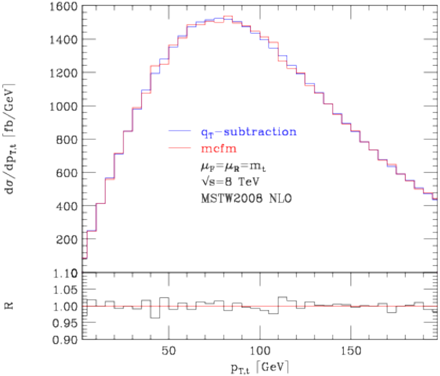
- Distributions for the $t\bar{t}$ system.

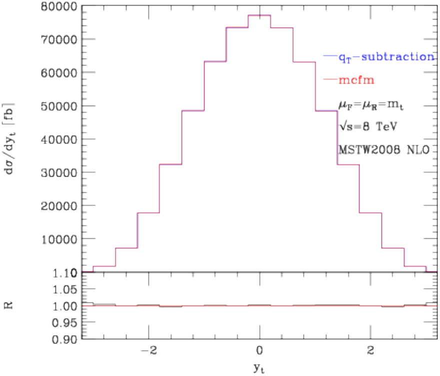


- Very good agreement!

Results at NLO

- Distributions for the top quark.





- Very good agreement!

Results at NNLO

Cross section [pb]	$\mathcal{O}(\alpha_S^4)_{qg}$	$\mathcal{O}(\alpha_S^4)_{q(\bar{q})q'}$
q_T subtraction	-2.25(5)	0.151(3)
Top++	-2.253	0.148

Table : $\mathcal{O}(\alpha_S^4)$ contribution to the total cross section for $t\bar{t}$ production at the LHC at $\sqrt{s} = 8\,\mathrm{TeV}$.

Cross section [pb]	$\mathcal{O}(\alpha_S^4)_{qg}$	$\mathcal{O}(\alpha_S^4)_{q(\bar{q})q'}$
q_T subtraction	-61.5(5)	1.33(1)
Top++	-61.53	1.33

Table : $\mathcal{O}(\alpha_S^4)$ contribution to the total cross section for $t\bar{t}$ production at the LHC at $\sqrt{s} = 2\,\mathrm{TeV}$.

$$qg = qg + \bar{q}g, \quad q(\bar{q})q' = qq + \bar{q}\bar{q} + qq' + \bar{q}\bar{q}' + q\bar{q}' + \bar{q}q'$$

Summary

- I have briefly discussed the all-order q_T-resummation for the heavy-quark production at hadron colliders, worked out in [Catani, Grazzini, Torre (2014)].
- We have used the knowledge of the low q_T behaviour of the amplitudes to extend the q_T subtraction method for $t\bar{t}$ production at hadron colliders.
- We have compared our results at NLO with MCFM program for various distributions. At NNLO we have compared our results for the total cross section in all the non-diagonal channels to TOP++ program. In both cases we have found a good agreement.
- The extension of our NNLO computation to include the missing $q\bar{q} \to t\bar{t} + X$ and $gg \to t\bar{t} + X$ channels requires the evaluation of the second-order hard-collinear functions $\mathcal{H}^{t\bar{t}}_{\mathrm{NNLO}}$, and an implementation of the two-loop virtual amplitudes, which, at present, are known only in numerical form.