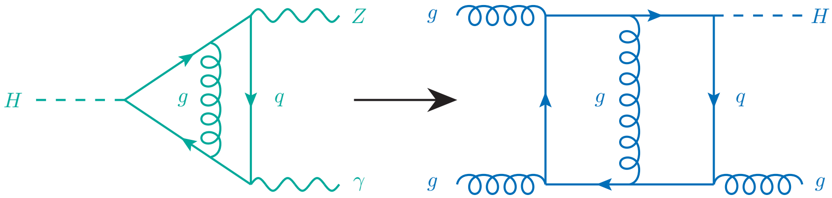
Two-loop QCD corrections to $H \to Z\gamma$
& steps towards $H + j$ with full m_t dependence

[arXiv: hep-ph/1505.00561]

Dominik Kara | August 27, 2015

PHD SEMINAR ZURICH - PAUL SCHERRER INSTITUT



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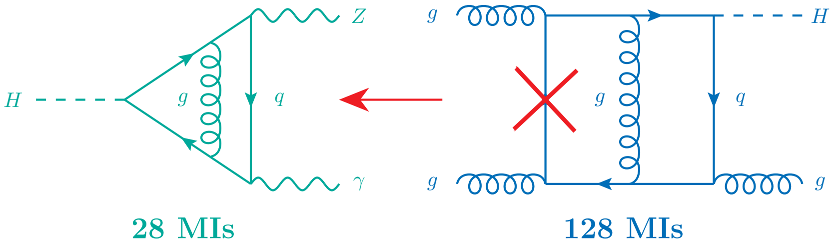

Motivation: $H + j$ production

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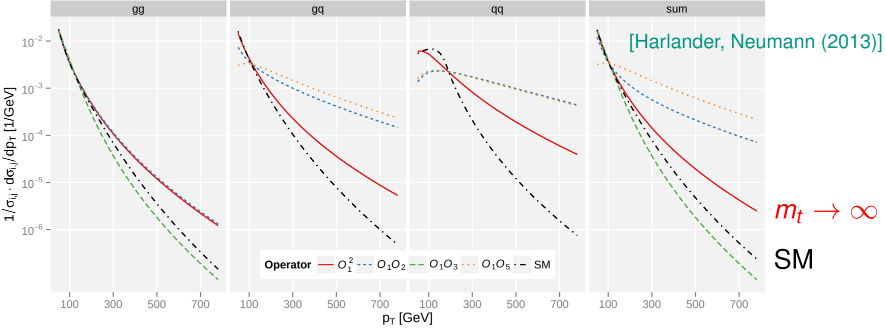
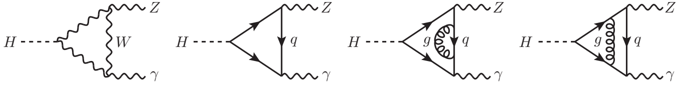


- NLO and NNLO result in QCD available in EFT with $m_t \to \infty$
- EFT is likely to break down at high p_T
- High-priority aim: NLO QCD corrections with full m_t dependence

$\Rightarrow$ **Two-loop integrals for $H \to Z\gamma$ pave the way**

Outline of the calculation

QGRAF ■ Generate Feynman diagrams for process $H(q) \to Z(p_1)\gamma(p_2)$



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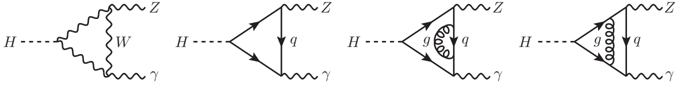

FORM
- Project relevant Feynman diagrams onto tensor structure

$$\mathcal{M} = \boxed{A}\, \epsilon_{1,\mu}(p_1, \lambda_1)\, \epsilon_{2,\nu}(p_2, \lambda_2)\, \frac{P^{\mu\nu}}{P^2}$$

with projector

$$P^{\mu\nu} = p_2^{\mu} p_1^{\nu} - (p_1 \cdot p_2) g^{\mu\nu}$$

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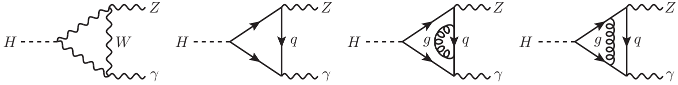

FORM
- Insert amplitude into decay width

$$\Gamma = \frac{G_F^2\,\alpha\, m_W^2}{4\, m_H^3\left(m_H^2 - m_Z^2\right)} \boxed{|A|^2}$$

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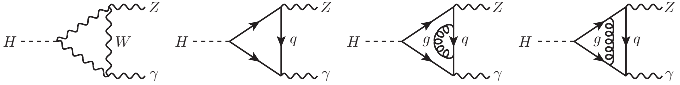

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REDUZE
- Reduce Feynman Integrals to set of MIs
  - Integration-by-parts identities (IBPs)
  - Laporta algorithm

$\Rightarrow$ **We are left with computation of Master Integrals (MIs)**

Parametrization

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Use Landau-type variables to absorb natural roots

$$m_H^2 = -m_q^2 \frac{(1-h)^2}{h}, \qquad m_Z^2 = -m_q^2 \frac{(1-z)^2}{z}$$

Differential equations

[Kotikov (1991)]
[Remiddi (1997)]
[Gehrmann, Remiddi (2000)]

1. Choose MI
2. Compute derivative of integrand with respect to internal mass and external invariants
3. Use IBPs to relate resulting integrals to original MI

Full system takes form of total differential

$$\mathrm{d}\vec{I}(h,z) = \sum_{k=1}^{N} R_k\left(\boxed{\epsilon}\right) \mathrm{d}\log(d_k)\, \vec{I}(h,z)$$

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Differential equations

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Canonical form

[Henn (2013)]

$$\mathrm{d}\vec{M}(h,z) = \boxed{\epsilon} \sum_{k=1}^{12} R_k \,\mathrm{d}\log(d_k)\, \vec{M}(h,z)$$

- ✓ reduced number of polynomials d_k
- ✓ can be integrated in terms of GHPLs:

$$G(w_1,\ldots,w_n;x) \equiv \int_0^x \mathrm{d}t \, \frac{1}{t-w_1}\, G(w_2,\ldots,w_n;t)$$

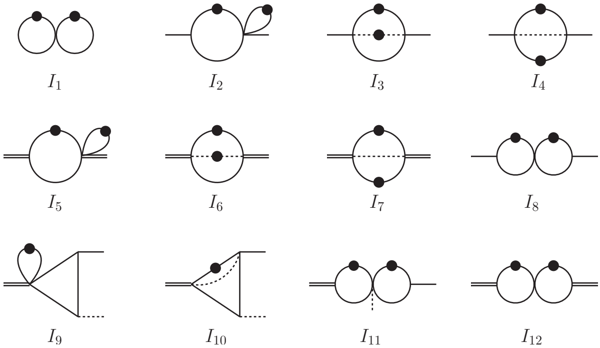
$$G\left(\vec{0}_n;x\right) \equiv \frac{\log^n x}{n!}$$

- ✓ leads to linear combinations of GHPLs of homogeneous weight

$\Rightarrow$ **Change basis from Laporta integrals $\vec{I}$ to canonical integrals $\vec{M}$**

Integral basis change

Follow algorithm from [Gehrmann, von Manteuffel, Tancredi, Weihs (2014)]



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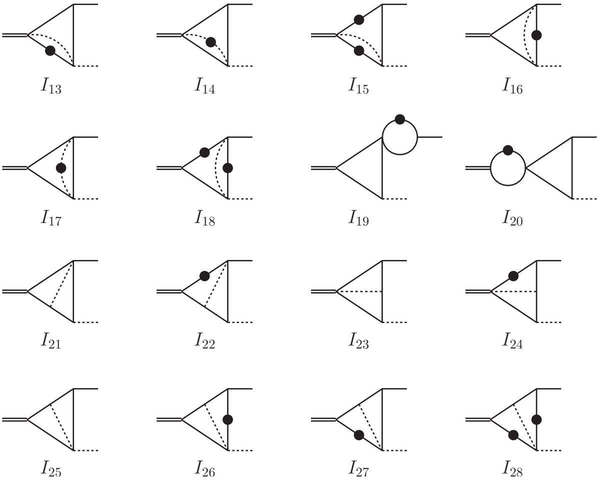

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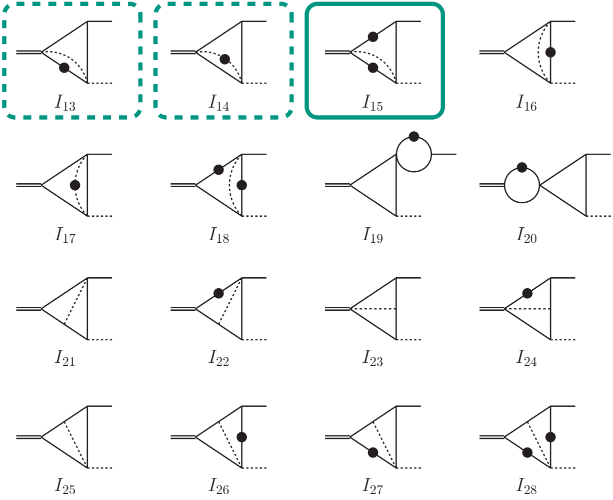

Integral basis

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$$
\begin{aligned}
\mathrm{d}\mathbf{M_{15}} = \epsilon \Bigg[& -\left(M_2 + \frac{3}{2} M_7 + 5\, M_{13} + M_{14} - 4\, \mathbf{M_{15}}\right) \mathrm{d}\log(d_1) \\
& + \left(\frac{3}{2} M_7 + 2\, M_{13} + M_{14} - 2\, \mathbf{M_{15}}\right) \mathrm{d}\log(d_2) \\
& - \left(M_2 + \frac{3}{2} M_7 + M_{13} - M_{14} - \mathbf{M_{15}}\right) \mathrm{d}\log(d_3) \\
& - \left(M_2 + 2\, M_6 + \frac{5}{2} M_7 + M_{13} - M_{14} - \mathbf{M_{15}}\right) \mathrm{d}\log(d_4) \\
& + 3\, M_7\, \mathrm{d}\log(d_6) \\
& + (M_2 + M_6 + 2\, M_7 + 3\, M_{13} - 2\, \mathbf{M_{15}})\, \mathrm{d}\log(d_7) \\
& + (M_2 - M_6 + M_7 + 3\, M_{13} - 2\, \mathbf{M_{15}})\, \mathrm{d}\log(d_8) \\
& - \left(\frac{3}{2} M_7 + 2\, M_{13} + M_{14} - \mathbf{M_{15}}\right) \mathrm{d}\log(d_9) \Bigg]
\end{aligned}
$$

Integration

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Results in terms of GHPLs up to weight four

$$G(a_1, \ldots, a_n; h) \quad \text{with} \quad a_i \in \left\{0, \pm 1, z, \frac{1}{z}, J_z, \frac{1}{J_z}, K_z^{\pm}, L_z^{\pm}\right\}$$

$$G(b_1, \ldots, b_n; z) \quad \text{with} \quad b_i \in \{0, \pm 1, c, \bar{c}\}$$

$$c = \frac{1}{2}\left(1 + i\sqrt{3}\right) \qquad K_z^{\pm} = \frac{1}{2}\left(1 + z \pm \sqrt{-3 + 2\,z + z^2}\right)$$

$$J_z = \frac{z}{1 - z + z^2} \qquad L_z^{\pm} = \frac{1}{2\,z}\left(1 + z \pm \sqrt{1 + 2\,z - 3\,z^2}\right)$$

✓ checked numerically against SecDec

[Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke (2015)]

Integration

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Example

$$
\begin{aligned}
M_{15} = \epsilon^2 \Big[& -\frac{\pi^2}{6} - G(\frac{1}{z}, 0; h) + G(z, 0; h) - G(0; z) G(\frac{1}{z}; h) - G(0; z) G(z; h) + G(0; z) G(0; h) \\
& -2G(0, 0; h) + G(1, 0; z) \Big] \\
+ \epsilon^3 \Big[& 5\zeta_3 + 2G(\frac{1}{z}, \frac{1}{z}, 0; h) - 2G(\frac{1}{z}, z, 0; h) + \frac{2\pi^2}{3} G(\frac{1}{z}; h) + 6G(\frac{1}{z}, -1, 0; h) \\
& - 3G(\frac{1}{z}, 0, 0; h) + 2G(\frac{1}{z}, 1, 0; h) + 2G(z, \frac{1}{z}, 0; h) - 2G(z, z, 0; h) + \frac{\pi^2}{3} G(z; h) \\
& - 6G(z, -1, 0; h) + 5G(z, 0, 0; h) - 2G(z, 1, 0; h) - G(K_z^-, \frac{1}{z}, 0; h) + G(K_z^-, z, 0; h) \\
& - \frac{\pi^2}{6} G(K_z^-; h) + 2G(K_z^-, 0, 0; h) - G(K_z^+, \frac{1}{z}, 0; h) + G(K_z^+, z, 0; h) - \frac{\pi^2}{6} G(K_z^+; h) \\
& + 2G(K_z^+, 0, 0; h) + \frac{\pi^2}{3} G(-1; z) + 2G(-1, 0; z) G(\frac{1}{z}; h) + 2G(-1, 0; z) G(z; h) \\
& - 2G(-1, 0; z) G(0; h) + G(-1, 0, 0; z) - 2G(-1, 1, 0; z) - G(0, \frac{1}{z}, 0; h) - \frac{\pi^2}{2} G(0; z)
\end{aligned}
$$

Integration

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Example

$$
\begin{aligned}
&+2G(0;z)G(\tfrac{1}{z},\tfrac{1}{z};h)+2G(0;z)G(\tfrac{1}{z},z;h)-2G(0;z)G(\tfrac{1}{z},0;h)+2G(0;z)G(z,\tfrac{1}{z};h)\\
&+2G(0;z)G(z,z;h)-2G(0;z)G(z,0;h)-G(0;z)G(K_z^-,\tfrac{1}{z};h)-G(0;z)G(K_z^-,z;h)\\
&+G(0;z)G(K_z^-,0;h)-G(0;z)G(K_z^+,\tfrac{1}{z};h)-G(0;z)G(K_z^+,z;h)+G(0;z)G(K_z^+,0;h)\\
&-G(0;z)G(0,\tfrac{1}{z};h)-G(0;z)G(0,z;h)+G(0,z,0;h)+G(0;h)G(1,0;z)+12G(0,-1,0;h)\\
&-G(0,0;z)G(\tfrac{1}{z};h)-G(0,0;z)G(z;h)-G(0,0;z)G(K_z^-;h)-G(0,0;z)G(K_z^+;h)\\
&+2G(0,0;z)G(0;h)+G(0,0;h)G(0;z)-G(0,0,0;z)-8G(0,0,0;h)+2G(0,1,0;z)\\
&+4G(0,1,0;h)-\frac{\pi^2}{3}G(1;z)-2G(1,-1,0;z)-2G(1,0;z)G(\tfrac{1}{z};h)-2G(1,0;z)G(z;h)\\
&+G(1,0;z)G(K_z^-;h)+G(1,0;z)G(K_z^+;h)+2G(1,0,0;z)-6G(1,0,0;h)+G(1,1,0;z)\Big]\\
&+\mathcal{O}\left(\epsilon^4\right)
\end{aligned}
$$

Numerical results

(a) Quark mass M_q and Yukawa coupling Y_q in OS scheme

(b) Quark mass M_q in OS, Yukawa coupling $\overline{y}_q$ in $\overline{\mathrm{MS}}$ scheme

(c) Quark mass $\overline{m}_q$ and Yukawa coupling $\overline{y}_q$ in $\overline{\mathrm{MS}}$ scheme

NLO decay width $\Gamma^{(2)}$ in renormalization schemes (a), (b) and (c)

$$\Gamma^{(2,a)} = \left[7.07533 + 0.42800\,\frac{\alpha_s(\mu)}{\pi}\right]\mathrm{keV} \overset{\mu=m_H}{=} 7.09072\,\mathrm{keV} \qquad \boxed{2‰}$$

$$\Gamma^{(2,b)} \overset{\mu=m_H}{=} \left[7.09409 + \frac{\alpha_s(m_H)}{\pi}\left(-0.53266 - 0.76661\,\log\frac{m_H^2}{\overline{m}_t^2(m_H)} + 0.01229\,\log\frac{m_H^2}{\overline{m}_b^2(m_H)}\right)\right]\mathrm{keV}$$

$$= 7.09403\,\mathrm{keV} \qquad \boxed{< 10^{-5}}$$

$$\Gamma^{(2,c)} \overset{\mu=m_H}{=} \left[7.05934 + \frac{\alpha_s(m_H)}{\pi}\left(0.64587 + 0.10597\,\log\frac{m_H^2}{\overline{m}_t^2(m_H)} + 0.01453\,\log\frac{m_H^2}{\overline{m}_b^2(m_H)}\right)\right]\mathrm{keV}$$

$$= 7.08438\,\mathrm{keV} \qquad \boxed{3‰}$$

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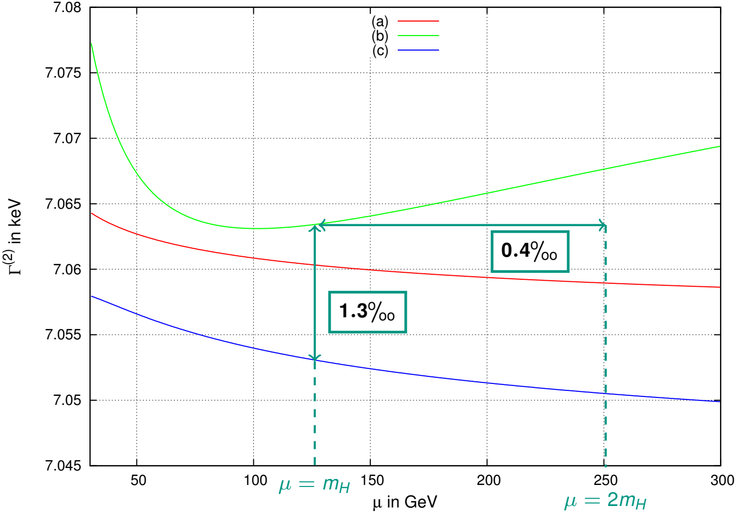

Conclusions

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Renormalization

- OS scheme
- $\overline{\mathrm{MS}}$ scheme
- hybrid scheme with OS mass and $\overline{\mathrm{MS}}$ Yukawa coupling

Numerical results

- Corrections in sub-per-cent range and consistent with each other
- Residual QCD uncertainty: 1.7‰

Checks

- Confirmation of previously available numerical OS result [Spira, Djouadi, Zerwas (1992)]
- Agreement with independent calculation [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov (2015)]

Master Integrals

- Two-loop three-point integrals with two different external legs and one internal mass derived analytically using differential equations
- Important ingredient to two-loop amplitudes of $H + j$ production

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